Diversity, Variability and Commonalities among Teaching Practices

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3. DIVERSITY, VARIABILITY AND COMMONALITIES AMONG TEACHING PRACTICES

INTRODUCTION

Researchers in mathematical didactics aim to understand and improve the teaching and learning of the discipline. However, the weak diffusion of research results into teaching practices prompts us to look closer at various teaching practices. Do institutional constraints and professional norms render these practices mostly homogenous? Do teachers have some amount of leeway, resulting in individual differences in styles? Are students' classroom activities completely determined by their teacher, or are teachers reciprocally affected by their students? And could this mean that students are themselves responsible for variation in their teachers' practices?

We will address these questions through the case of teaching decimal multiplication to French sixth graders (age 11), beginning with a study of the regularity and variability of mathematics teachers' practices. The "double approach" presented in Chapter 2 consists of understanding teachers' work as involving goals beyond student learning, taking into account their own professional objectives as well.

We will analyze the practices of four teachers who work under similar professional conditions. By examining commonalities in their practices, we will analyze the constraints under which these teachers work. This will allow us to both determine if all the originally anticipated scenarios are feasible, and to understand teachers' pre-class and in-class constraints. By examining the variability in individual practices, we intend to present coherences in teaching practices. It is the internal coherence in a teacher's practice that forbids the spontaneous adoption of another way of operating.

After specifying the research topic and the methodology adopted for the "double approach," we will present our results regarding the originally anticipated scenarios, the institutional constraints in place, and finally the scenarios deemed realizable under these constraints. We will then describe our observations of teachers in terms of the regularity, variability, and coherence of their teaching practices.

F. Vandebrouck (ed.), Mathematics Classrooms: Students' Activities and Teachers' Practices, 75–89.

A METHODOLOGY BASED ON THE DOUBLE APPROACH

We will specify the research topic and present the methodology used, developed under the framework of the "double approach."

A research topic aimed at interpreting the constant and variable aspects of practices, in terms of the constraints and flexibility afforded teachers

We have subdivided the overall investigation of regularity and variability in teaching practices into three subtopics. We will detail each topic and indicate briefly for each the approach used to resolve the issues involved.

The first subtopic concerns the various ways a class can be taught, in terms of the institutional and social components of teaching practices. This subtopic also includes the choices made by teachers from among these various possibilities. After evaluating the issues at play in teaching decimal multiplication, we will investigate the possible didactic transpositions in light of the numerous publications on this topic. We will then compare the observed practices to those that were originally anticipated.

The second subtopic, which includes personal, cognitive, and mediatory components, concerns the development of lessons, focusing on student work as a function of their teachers' activities. Our goal was to compare students' effective activities to the tasks as envisioned in the scenario, while also examining classroom interactions and assistance provided by teachers during the completion of the tasks.

Teacher constraints, the amount of freedom allowed within these constraints, and overall practice coherence constitute the third subtopic. Through a survey of official documents, we determine the constraints of the school system regarding the number of hours of class time as well as curriculum topics. Through interviews with teachers, we attempt to evaluate the weight of the constraints tied to the expectations of the school system, and of those tied to practicing the profession, in the classroom, with students. The variability of practices can be explained by the fact that teachers make different choices while operating within the leeway afforded them under these constraints. We attempt to define the limits of the leeway afforded teachers in order to specify the space of possible professional activities. Finally, between constraints and leeway lies the question of the coherence of teachers' choices. Even if, from the theoretical standpoint, this coherence of practice is a given, we are still interested in understanding how it is manifested. We look for indicators of this coherence by examining differences between the choices the teachers made during the preparation stage and the ones made during the actual classroom practice.

A corpus of published sources and classroom observations

The first subtopic involves determining the scenarios that are actually realizable in the classroom. This determination was conducted using studies on the mathematical topic and examining them in light of the institutional requirements and the constraints that stem from students' prior knowledge and their difficulties learning the topic. These studies rely on published sources such as curricula, manuals, evaluations of student competencies, publications intended for teachers, and research conducted in mathematical didactics.

We begin with an analysis of the mathematical concept at hand, decimal multiplication. The meaning of the multiplication must be understood in reference to the theory of the conceptual fields (Vergnaud, 1990). Studies of the mathematical issues involved in teaching decimal multiplication include the work of Brousseau (1987, 1998) and Douady & Perrin-Glorian (1986) on decimal numbers, and the work of Vergnaud (1979, 1981, 1983), Rogalski (1985) and Butlen (1985) on multiplication. Data extracted from these studies touch on multiplicative situations, properties of the multiplicative operation, calculation techniques, multiplicative written expressions and their potential transformations, and the connections between situations, properties, and their written forms. These data were then used to analyze the possible and observed teaching scenarios.

These possible scenarios were determined by assessing two constraints that have a strong influence on teachers' choices: The didactic transposition from the concept to the lesson, and students' difficulties in learning the subject. We first analyzed the diverse lesson plans proposed in didactic research, official curricula, and teachers' manuals. We also analyzed the results of various evaluations conducted by the Ministry of National Education and by the *Association des professeurs des mathématiques* (Mathematics Teachers Association) in order to better understand the difficulties on the part of students that teachers confront and that they therefore may keep in mind while planning their course.

The teachers whose lessons were observed were chosen according to precise criteria derived in accordance with the research topic. All variables concerning the lesson, except those tied to the teacher as an individual, were fixed. All the observed lessons involved experienced teachers using the same manual to teach the same topic to sixth grade classes who were at the same overall level, of similar size, and for similar lengths of time. In order to neutralize the time factor, each teacher was observed during all class periods dedicated to decimal multiplication. The term "sequence" refers to the set of these class periods.

The observable factors used for collecting data on teachers' lesson plans and class period activities are described in the two following sections. These factors were defined so as to be neither so fine that they hide commonalities, nor so broad that they mask differences.

The observable factors in scenario analysis

As indicated in Chapter 2, the planning of a lesson is called a “scenario,” both to acknowledge the fact that teachers picture themselves in class during lesson planning, and to differentiate the planned lesson from students’ actual activities. Three observable factors are used to analyze scenarios: the mathematical field, the teaching strategy, and the mathematical tasks assigned to students. The *mathematical field* describes the set of content introduced during the sequence: concepts, situations, symbolic representations and their transformations, properties, and theorems. The *teaching strategy* consists of the organization of a sequence’s mathematical content along a path chosen for mathematical or cognitive reasons. These reasons can vary with the teacher. Some teachers begin by providing the information to be learned before giving students mathematical problems to solve, while others choose the reverse strategy. We can also differentiate teachers by whether or not they institutionalize the mathematical knowledge that may or may not have been constructed by students through the in-class problems. Finally, the *mathematical tasks* are analyzed in reference to the criteria presented in Chapter 2.

The observable factors in lesson analysis

In order to analyze the events of a class period, three observable factors were defined: students’ effective activities, the assistance provided by teachers, and the order and organization of the lesson.

Recall that once a task has been assigned to the class, the *potential activity* is what the student ought to do to complete the task, the *real activity* is what the student does, and the *effective activity* is the reconstruction by the teacher of the probable real activity, as a function of the potential activity and of productions by the student (such as what the student says).

Below are three examples of tasks, together with the corresponding potential activities. All three can lead to the same effective activity: determining the product of two decimal numbers using a calculator.

- Task 1: Calculate 3.14×2.08. Potential activity: Apply the standard solving technique for calculating the product of two decimal numbers.
- Task 2: True or false? $3.14 \times 3 = 9.43$. Potential activity: Determine the last digit of the product of two decimal numbers.
- Task 3: Place the decimal point in the result of the equation $3.4 \times 2.5 = 85$. Potential activity: Determine the order of magnitude of the product of two decimal numbers.

The assistance provided to students by the observed teachers was primarily procedural, responding to what we call *didactic incidents*. As a result, this assistance was assimilated into incident management methods. The incidents considered here are not breaches of discipline, but actions that do not correspond to the possible correct responses. Four types of incidents were identified: Questions, errors, incomplete answers, and silences (when a student does not respond to a question asked by the teacher). Below are examples of the most common incidents.

All are in reference to Task 4: Place the missing decimal point in the equation 1.35 × 42 = 5.67.

- Question. Raphael asks, "Can we say there is no missing decimal point?" Clearly, Raphael is counting digits after the decimal point. His question shows negative progress towards activity that would lead to the correct answer.
- Error. Maud says, "To place the decimal point, I added a zero. I wrote, '1.35 × 0.42 = 5.67'[1]." Maud's error is most likely a carry-over from decimal addition.
- Incomplete response. If Maud had only said, "To place the decimal point, I added a zero," her incomplete response would have been an incident. The class could then have wondered if Maud was thinking of .42, 4.02, 4.20, or 42.0, all of which could have corresponded to possible attempts to solve the problem.

An incident is managed through the subsequent intervention of the teacher. Methods of incident management that were observed during sequences led by the participating teachers were classified into two groups, depending on whether they tended to provoke students into re-tackling the task. We understand teachers' reception and management of incidents as factors that influence students' work, as well as, we hypothesize, student learning.

Class periods were divided into episodes characterized by the teacher's specific goals. This provided a chronology to the sequence. At a global level, this chronology allows us to analyze the organization of learning moments, as well as the dynamics between the class and the solving of problems. At a local level, this chronology feeds into the analysis of incident management, particularly regarding the influence of the passing of time on the interactions between students and teachers.

FROM POSSIBLE SCENARIOS TO REALIZABLE SCENARIOS

Using the previously referenced studies, we identified the possible ways to teach decimal multiplication. After evaluating the constraints, and examining teaching manuals, we determined the realizable scenarios.

A typology of possible scenarios

In the research literature, strategies for teaching decimal multiplication are differentiated by their representations of decimal numbers and by their global didactic choices. In terms of representations, decimal numbers can be considered as particular cases of rational numbers, or considered independently of fractions. This decision has consequences on the proposed tasks, particularly regarding rewriting and the available methods of justifying the solving technique. In terms of students' planned cognitive itinerary, three types of scenarios can be identified. In the first

[1] In France, we always write the ones digit, and thus write 0.42 and not .42.

type of scenario, the solving technique is first introduced by the teacher, and then applied by students to calculate products. These products may serve as answers to problems in which the multiplication is contextualized. In the second type of scenario, the teacher first presents an introductory problem. The solving technique is partially determined by students, and may be defined in terms of the example problem. The technique is then applied. In the third type of scenario, problems arising from multiplicative situations are given to students. The solving of these problems leads to the determination of the solving technique, which will be reinforced and reapplied to new problems.

All teaching manuals propose scenarios of the first two types, and consider decimals independently of fractions. The algebraic properties of the operation on which the solving technique relies always remain implicit. The study of multiplicative situations is largely neglected: The multiplication is always decontextualized, except when the problems involve price calculations. By contrast, literature aimed at teachers (generally written by teacher educators or researchers), as well as research in mathematical didactics, suggest only scenarios of the third type. Our analysis also shows that authors writing directly to teachers connect fractional and decimal representations, but do not always connect the meaning of the multiplication to the solving technique.

From possible scenarios to realizable scenarios: The effect of constraints

To develop a teaching scenario, teachers use published sources and their own mathematical knowledge. They also keep in mind certain constraints, with the most important being official requirements, students' current knowledge, and known difficulties in learning the specific topic.

In France, fractions are introduced in elementary school, but are not studied at much depth until later. Calculating with fractions is taught in secondary school. At the time this study was conducted, multiplication in elementary school was limited to multiplication of a decimal number by an integer. Multiplication of two decimal numbers was not taught until secondary school. The specific mathematical content to be taught was prescribed: Exploration of different methods of calculation (written, mental, reasoned, approximate, or with calculation tools) and a number of multiplicative situations. There was a strong time constraint. Considering the entire curriculum, we can estimate that overall 4 to 6 hours were spent on a sequence covering decimal multiplication (including solving problems arising from multiplicative situations).

The evaluations of student competency conducted at the end of elementary school or the beginning of sixth grade provide precise information on students' mastery of decimal numbers and solving techniques, but less information on their recognition of the multiplicative model within these problems.

Decimal numbers remain, for some students, two integers of possibly different status separated by a decimal point. In French, to read the number 3.14 aloud, we do not say "three-point-one-four" but "three-point-fourteen," without reference to units and subunits[2]. The proportion of errors corresponding to the misconception that a decimal number is composed of two integers varies between 10% and 50%, depending on the problem. Problems involving multiplication of a decimal by a power of ten (10 and 0.1, 100 and 0.01, etc.) are solved correctly by 50% to 70% of students.

Integer multiplication problems are solved correctly by approximately three out of four students, depending on variables such as the presence of a zero in the multiplier, or the necessity of using a product from the multiplication table of two factors larger than five. This proportion remains approximately constant for the multiplication of a decimal by an integer. Exam questions given after the unit that involve multiplication of two decimals show certain difficulties in learning. The questions are solved correctly by only 35% to 55% of students. Twenty percent of the errors are in the placement of the decimal point.

We find few multiplicative situations on exams. Does this represent the actual intentions of the school system, or the assumptions of test creators as to teaching practices? In any case, the only situations covered on exams are size isomorphisms and finding the area of a rectangle. Otherwise, the results of the multiplication are largely unused. Situations involving a unit price and a quantity are recognized as multiplicative by 80% of students. Finding the area of a rectangle is a source of difficulties for more than half of students, who confuse the concepts of area and perimeter or their respective formulas.

Such results cannot help but affect teachers' choices. The task is considerable (effect students' acquisition of the concept of a decimal number, broaden the meaning of multiplication, and teach a solving technique which causes many students to stumble) and the teaching time is limited. It is therefore unlikely that a teacher will develop a scenario where multiplication is contextualized, where fractions and decimals are connected, and where students construct and justify their solving technique with reference to a multiplicative situation.

OVERALL SIMILARITIES IN SCENARIOS

The sequences of four teachers were compared, from the outline of their scenarios to classroom activities, in order to respond to the central research question concerning commonalities and variance among teaching practices. Results are presented in the two following paragraphs. The first discusses scenarios, and the second, activities in class.

The teachers are given names of mathematicians in order to distinguish them and to refer to them throughout the analyses. We call them Ms. Germain, Ms. Agnesi, Ms. Theano, and Mr. Bombelli. The reader should be aware that mathematics teachers at this level of schooling teach only this subject. They have studied mathematics for at least three years at the university level, and have received training analogous to that of future engineers or mathematics researchers.

[2] Translator's note: In France, the decimal separator is actually a comma. The number is written 3,14 and pronounced "three-comma-fourteen" (*trois-virgule-quatorze*).

Analysis of the mathematical field

Teachers' choices did not diverge widely. Their scenarios were all of the first or second type, as defined in the publications cited above, and decimal numbers were always treated independently of fractions.

The mathematical field is composed of the content studied: Calculation techniques, properties of the operation, symbolic representations of numbers, multiplicative situations, etc. Table 1 summarizes the comparative analysis of the mathematical fields.

Table 1. Mathematical fields of the observed sequences.

Mathematical content	Ms. Germain	Mr. Bombelli	Ms. Agnesi	Ms. Theano
Technique et properties				
Solving technique	♦	♦	♦	♦
Justification of the technique	♦	♦	♦	♦
Mental, reasoned, or approximate calculation	♦	♦	♦	♦
Multiplication by zero or one				
Algebraic properties of the operation	♦	♦	♦	
Effect of the multiplication on the order of magnitude	♦		♦	
Multiplication by a factor less than one	♦	♦	♦	♦
Representation of decimals				
Decimal notation	♦	♦	♦	♦
Fraction notation of decimals				
Representation using units of measure			♦	
Multiplicative situations				
Size isomorphisms	♦	♦	♦	♦
Product of lengths				
Operation on a length				
Composition of operators				

All teachers taught the solving technique, justified it, and presented alternate calculation methods to students, such as mental, reasoned, or approximate calculation. All teachers also treated the case of multiplication by a factor less than one. This case is crucial, as it challenges the idea that multiplication results in a larger number. This property, carried over from working with integers, is the source of numerous difficulties. The teachers were also unanimous in not discussing multiplication by zero or one. This unanimity disappeared, however, for the algebraic properties of multiplication and its effects on the order of magnitude. As for symbolic representations, all of the teachers covered the signification of decimal notation, but none made the link with fractional representations. Ms. Agnesi was the only one to propose a connection between decimal writing and changes in units of measurement. Teachers were completely unanimous in neglecting the study of multiplicative situations. The only problems in which decimal multiplication was contextualized were price problems within a numeric framework. No other situation was studied, and no other framework was called upon, even in the sixth grade classes. Teachers preferred to introduce these subjects later on in the school year, without specifically discussing multiplication.

Analysis of teaching strategies

A certain pattern emerges in terms of teaching strategies, particularly regarding the construction of new knowledge: There was no non-didactic situation, no change of framework, and no tool/object dialectic. Thus, our assessment of the lack of didactic engineering in everyday teaching is confirmed. The teachers, like the authors of teaching manuals, have never imagined scenarios of the third type. We will see that one of the teachers designed a scenario of the first type, and the other three designed scenarios of the second type.

Despite these overall commonalities, we note different dynamics between the course and the exercises, exercises which are sometimes problems aiming at the introduction of new knowledge. For example, Ms. Germain introduced the topic by asking her students the question "How can we calculate the product of two decimals?" She let them produce rules that were effective for certain particular cases. At the end of the sequence, all of these rules allowed students to construct the usual technique. Mr. Bombelli, by contrast, began by presenting the solving technique, which he justified with the help of multiplicative operators. He then gave students exercises on which to apply the technique. Ms. Agnesi began with price problems in which the products of decimal factors could be calculated through conversions. These examples allowed students to infer the solving technique, and the rest of the sequence was dedicated to application problems and the systematic examination of multiplication properties. Ms. Theano introduced the calculation of the product of two decimals using orders of magnitude, allowing students to again infer the solving technique. Students could then check their conjectures with a calculator. Next were application problems and mental exercises that helped students begin to question the solving technique.

Overall, there was general homogeneity as to the content taught, and diversity as to the dynamics between constructing new knowledge and putting it into use for solving problems. What, then can we say about the mathematical tasks presented to students?

Analysis of mathematical tasks proposed to students

Among tasks proposed by teachers, we distinguish those that aim to introduce new knowledge, and those that lead to applications, to theoretical questioning, or to solving problems arising from mathematical situations. Table 2 summarizes our results.

Table 2. Tasks proposed to students as a function of the intended activity.

Mathematical tasks	**Ms. Germain**	**Mr. Bombelli**	**Ms. Agnesi**	**Ms. Theano**

Introduction of new knowledge				
Non-didactic situation				
Frameworks mobilized	Numeric	Numeric	Numeric	Numeric
Multiplication as a knowledge object	♦	♦	♦	♦
Multiplication as a tool				
Multiplicative situation			♦	
Potential activities				
Determination of a product	**75%**	**71%**	**50%**	**64%**
By written calculation or with tools	*17%*	*14%*	*17%*	*09%*
By mental, reasoned, or approximate calculation	*58%*	*57%*	*33%*	*55%*
Theoretical questions	**25%**	**29%**	**33%**	**18%**
Multiplicative situation	**00%**	**00%**	**17%**	**18%**

The analysis of proposed tasks confirms the uniformity in teachers' choices regarding the introduction of new knowledge: No non-didactic situations, no change of framework, and no tool/object dialectic. Only Ms. Agnesi proposed problems relying on a multiplicative situation. However, the solving technique was not constructed with reference to this situation.

We also noted a certain homogeneity concerning the exercises given to students. However, this result was not statistically significant due to the small population size. Many of these exercises (50% to 75%) led to a potential activity of calculating the product of two decimals, but applications of the solving technique (9% to 17%) were less frequent than mental, reasoned, or approximate calculations (33 to 58%). Other exercises led to theoretical questions (18% to 33%) or to solving problems arising from multiplicative situations (0% to 18%).

Overall, the lessons of the observed teachers were convergent in terms of content introduced and tasks prescribed, but were distinguished in part by the teaching strategies used. Following this assessment, we will attempt to determine if, despite the similarity of tasks, students' activities will differ, particularly regarding knowledge construction. The analysis of classroom activities will allow us to evaluate this prediction.

DIFFERENCES IN CLASSROOM ACTIVITIES

The study of classroom activities has two parts: Analyzing the effective activities of students and analyzing the assistance given by teachers.

Before analyzing the activities, we should note that observed sequences lasted between 2.5 and 5 hours, not including evaluation. The estimated timespans from lesson plans were thus respected. Presumably, no teacher spent longer on these multiplication lessons as a result of being included in this study.

A larger variety of effective activities than potential activities

The passage from potential to effective activities requires some methodological explanations. Once students have difficulties with a task, teachers can provide assistance that will guide students to different activities. For example, a teacher who has assigned Task 4, "Place the missing decimal point in the equation

1.35 × 42 = 5.67," may ask students to find the result of 1.35 × 42. This will provoke student activity, leading to a response of 56.70. The teacher can then prompt students to apply this result to the original task, which will this time lead to using a reasoned calculation to deduce that 1.35 × 4.2 = 5.67. But a teacher who asks students to think of orders of magnitude will provoke very different activities. This example demonstrates why the effective activities arising during the observed class periods were both more numerous than, and different than, the potential activities identified during task analysis. It is exactly the effect of the teacher on this transformation that we aim to evaluate and interpret. Our results are given in Table 3.

Table 3. Classification of potential and effective activities.

Potential and effective activities	Ms. Germain	Mr. Bombelli	Ms. Agnesi	Ms. Theano
Potential activities				
Written or tool-based calculation	17%	14%	17%	09%
Mental, reasoned, or approximate calculation	58%	57%	33%	55%
Theoretical questions	25%	29%	33%	18%
Problem (contextualized multiplication)	00%	00%	17%	18%
Effective activities				
Written or tool-based calculation	9%	62%	27%	40%
Mental, reasoned, or approximate calculation	58%	25%	50%	44%
Theoretical questions	33%	13%	16%	13%
Problem (contextualized multiplication)	00%	00%	07%	03%

The table reveals differences between the scenario and the in-class activities of each teacher, as well as between the in-class activities of the four teachers. These differences are confirmed through statistical analysis: A number of chi-squared tests of independence with a 1% threshold were performed on the raw data that produced the above table. These tests confirmed both a significant difference for each teacher between the potential and effective activities, as well as a teacher effect on the effective activities.

Examining the respective teaching strategies allows us to interpret these results. Ms. Germain's strategy was to let students develop rules for calculating products, with the intention that these rules would lead them to the solving technique. For her students, technical exercises were often enriched by complementary questions favoring reasoned strategies or student introspection. Mr. Bombelli's strategy, by contrast, was to present the solving technique and have the students apply it. This teacher reinforced written calculation activities over mental, reasoned, or approximate calculation activities, and over theoretical questions. Ms. Agnesi chose to introduce the solving technique through price problems, leading her students to reasoned calculation activities. Finally, Ms. Theano asked her students to place the decimal point by determining the order of magnitude of the product, and then to check this result with a calculator, leading to approximate and tool-based calculation activities.

To conclude our analysis, we note that students' effective activities show a wider variety of practices that the potential activities would have allowed us to predict. A teacher's in-class work therefore seems to determine students' activities. During the lesson, the teacher modifies the proposed tasks appropriately in accordance with a teaching strategy.

Didactic incidents and teacher assistance

To consider students' actions in class, and their management by teachers, we will present didactic incidents observed in class, and the assistance provided by teachers in response to these incidents.

The number of incidents per class hour varied as a function of the teacher. Overall, incidents were frequent. Mr. Bombelli, who had the fewest incidents, had an average of one incident every three minutes. Ms. Agnesi, who had the most, had double this incident rate. Incident classifications are given in Table 4. While the effect of the teacher on incident classifications was not significant, we observe four values that are noticeably different from the average values (highlighted in bold). These values will inform our interpretations of teaching practices.

In Mr. Bombelli's class, questions predominated, while in Ms. Agnesi's class incomplete answers were the most common. This difference provides evidence of a pedagogical divergence: While Ms. Agnesi values student participation, Mr. Bombelli's students are expected to answer completely and correctly. Thus, when Mr. Bombelli's students are unsure, they prefer to ask questions rather than answer incompletely. We note also numerous student answers to questions posed by Ms. Theano that indicated that the questions were completely out of reach for students. Ms. Theano focused predominantly on orders of magnitude, despite this concept posing numerous theoretical problems.

Table 4. Classification of incidents in the observed sequences.

Didactic incidents	**Total**	**Ms. Germain**	**Mr. Bombelli**	**Ms. Agnesi**	**Ms. Theano**
Error	25%	27%	28%	21%	26%
Question	18%	16%	**32%**	15%	20%
Incomplete response	38%	36%	**16%**	**49%**	36%
Silence	9%	12%	8%	6%	7%
Question out of reach	4%	1%	0%	4%	**11%**
Disagreement	6%	7%	16%	5%	0%

Examining the assistance provided by teachers in response to in-class incidents reveals both their practices and the effect on students' activities. Table 5 shows the classification of each teacher's incident management methods into those that provoke students to further activity and those that do not.

Table 5. Incident management by teachers.

Incident management	Ms. Germain	Mr. Bombelli	Ms. Agnesi	Ms. Theano
Provokes further student activity	72%	21%	42%	50%
Does not provoke further activity	28%	79%	58%	50%

Ms. Germain's incident management provokes further activity in students in more than 70% of cases. By contrast, Mr. Bombelli prefers, almost 80 times out of 100, to not pass the activity back to students but instead to complete the proposed task himself. The management methods of Ms. Agnesi and Ms. Theano fall between these two extremes. The substantial differences that appear between teachers in Table 5 are confirmed by statistical analysis: A chi-squared test of independence at a 1% threshold was conducted, which allowed us to conclude that teachers have a significant effect on incident management. Incident management methods therefore appear to be a personal aspect of teaching practices.

In conclusion, our analyses show that teaching scenarios are overall constrained, particularly by institutional factors, but that there remains a certain amount of leeway that teachers use as much for designing a cognitive path for students as for managing in-class interactions. Their choices conform to their conceptions of teaching and learning.

SOCIAL, PERSONAL, AND COHERENT PRACTICES

The ergonomic approach, by considering teachers' practices as simultaneously personal and as taking part in a professional arena, allows us to propose several hypotheses for interpreting the results discussed above, in terms of the overall commonalities of practices, as well as their local variations.

Between the scenario and the in-class activities: Results in the form of hypotheses

Whenever teachers make the same choices in their work, we must ask what professional necessities their choices are reflecting. Our analyses and interviews have prompted several hypotheses. As teachers act as if they were all respecting principles of professional necessity, we will describe these hypotheses in terms of principles.

The observed teachers respected the curriculum's content, as well as its rhythm. They thus responded to a "principle of conformity to official curricula," which assures them professional legitimacy in encounters with students and their parents, with colleagues who will teach the same students in the following year, and with inspectors who are charged with implementing instructions from the ministry.

Two other principles allow us to better understand commonalities between teachers in terms of the field of mathematical content to be taught. The "principle of pedagogical efficacy" reflects the fact that teachers do not introduce mathematical content with which students show difficulties unless it is indispensible to the sequence. We can see this principle at work in the omission of problems involving fractions and finding the area of a rectangle. In addition, the "principle of an enclosed mathematical field" leads teachers to avoid teaching content that is too directly tied to the omitted concepts. As a result, the mathematical objects that remain within the field of the sequence connect to each other, but do not depend (or depend only slightly) on non-integrated objects. These principles are surprising, as they apparently lead to excluding from instruction those topics that students find the most difficult! In fact, these two principles lead to a hierarchy of content, and to avoiding subjects that threaten to pose difficulties that the teacher cannot handle without deviating from the intended path and risking confusion that will not be beneficial to student learning. This guarantees a strong guideline that keeps teachers within what Rogalski (2003) calls "the envelope of acceptable trajectories."

Finally, the "Principle of the necessity of success by steps" explains how teachers segment their instruction in such a way that students regularly engage in the activity of applying what they have just learned. Without making use of any complete model of learning dynamics, teachers use isolated simple technical tasks to evaluate the impact of their instruction as they go along.

How coherent are teaching practices?

The assessment of commonalities and variance of teaching practices raises the question of the coherence of these practices for each teacher. Analyzing each sequence, and comparing the different results obtained, allows us to identify levels of coherence of practices. These levels may seem unjustified, as they are based on only four examples of practices. We mention these results due to their confirmation by other research presented in this volume.

We have repeatedly remarked on the contrast between Mr. Bombelli's sequence and that of Ms. Germain. The factor dividing these teachers seems to be tied to their conceptions of the classroom. For Mr. Bombelli, it is a place for demonstration and application of knowledge. For Ms. Germain, the classroom is a place for construction of knowledge by students. These conceptions give coherence to their respective practices. In a classroom that is conceived as a place of demonstration and application of knowledge, the demonstration of knowledge takes place very early. The effective activities are primarily applications. Incidents are mainly questions or errors, and their management rarely provokes further activity from students, as the teacher can, if necessary, show an example. In a classroom conceived as a place of knowledge construction, knowledge is established fairly late. Research activities predominate, and incident management provokes further activity by students.

Ms. Agnesi's practice does not fall under one of these extremes. This is a teacher who would like her students to express themselves easily. She tries to involve them as much as possible in the class, and encourages their activity. Her conceptions of teaching and learning lead her to expect her classroom to be above all a place of exchange between teacher and students. Her students respond to this expectation. The number of didactic incidents in her class is substantial, particularly for incomplete responses, for which the rate is markedly higher than those found in the other classes.

Before concluding, we should note that this study also shows the variability of each teacher's practice. Despite the constraints and the conceptions that organize their instruction, teachers are continuously adapting their actions in class. One result particularly concerns the effect of time pressure on the practices of certain teachers. The classroom conceptions of Ms. Germain and Ms. Agnesi, as a place of knowledge construction or of exchange, require giving plenty of time to students. However, to respect the rhythm imposed by the principle of conformity to official programs, once the first half of the sequence is over, teachers find themselves obligated to adopt a more closed style of student interaction.

CONCLUSION

This study of the teaching practices of mathematics teachers is a clinical study. The results refer only to the work of four teachers, which limits the applicability. Nevertheless, these results have not been invalidated by a large number of studies on teaching practices, several of which are presented in this volume.

The observed regularities show that the school system, in fixing the content to teach and the length of the lesson, constrains teaching practices from initial lesson preparation to the eventual in-class activities with students. Other research shows that it is often gaps in the curriculum that constrain teaching practices. The study presented in this volume by Julie Horoks gives one example, in the case of similar triangles. Our research on histograms (Roditi, 2009) provides another. In addition, the conditions of the profession lead teachers to share several general principles, and, consequently, to make overall analogous choices as to content and the organization chosen to transmit it. These invariants outline an envelope that contains the observed teaching practices, but that does not contain all the scenarios that would be *a priori* imaginable, if only criteria tied to student learning were taken into account. These results have implications for teacher training.

Nevertheless, practices are varied. Teachers use the leeway available to them beyond the constraints, and the range of observed differences includes the inferred activities of students as much as the assistance provided by teachers. The observed diversity can be explained by the personal component of practices, whose connection to conceptions of teaching and learning was shown above. The research presented in Nathalie Sayac's chapter shows the greater or lesser influence on practices of other personal characteristics of teachers, such as age, gender, and initial training. In addition, linguistic analyses of teachers' speech, such as the study presented in this volume by Monique Chappet-Pariès, Aline Robert and Janine Rogalski, show the specificity and global stability of a teacher's speech patterns. Hence, not everything is possible for a single teacher, and the numerous choices a teacher makes seem to center around a pre-determined logic, while constantly adapting during every instant of class.

Overall, this research has highlighted elements related to individuals that explain the diversity of teaching practices. It has also shown that teachers share certain elements, and that this commonality homogenizes their practices. These elements are undoubtedly tied to institutional constraints, but also, more largely, to their profession.

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