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Tackling the Instability of Growth: A Kaleckian Model with Autonomous Demand Expenditures

Olivier ALLAIN

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TACKLING THE INSTABILITY OF GROWTH: A KALECKIAN MODEL WITH AUTONOMOUS DEMAND EXPENDITURES

Olivier Allain1

Abstract. This article presents a Kaleckian model enriched by introducing autonomous public expenditure which grows at an exogenous rate. It shows that the usual properties are not affected in the short run: growth is wage-led. But long run properties are strongly affected: public expenditure plays a role as an automatic stabilizer so that the accumulation rate converges on the growth rate of public expenditure. The effect of a change in income distribution on the growth rate is then only transient. However, the impacts on the level of variables (output, capital stock, labor, etc.) remain permanent. The research here also shows that this theoretical framework can provide a solution (depending on the parameters) to the 'second' Harrod knife-edge problem. In this case, Kaleckian outcomes are consistent with the convergence of the current utilization rate on the 'normal' rate, a result which has not been found in the existing literature.

RÉTABLIR LA STABILITÉ DE LA CROISSANCE : UN MODÈLE KALECKIEN AVEC UNE COMPOSANTE DE DEMANDE AUTONOME

Résumé. L’article présente un modèle kaleckien qui est enrichi par l’introduction d’une dépense publique autonome qui croît à un taux exogène. Il est montré que les propriétés des modèles kaleckiens ne sont pas modifiées à court terme : la croissance est tirée par les salaires. En revanche, les propriétés de long terme sont fortement affectées : la dépense publique joue un rôle de stabilisateur automatique tel que le taux d’accumulation converge vers le taux de croissance de cette dépense. L’effet d’un choc de répartition du revenu sur la croissance économique n’est donc que transitoire. En revanche, les effets sur les variables en niveau (production, stock de capital, emploi, etc.) restent permanents. L’analyse montra aussi que ce cadre théorique offre (en fonction de la valeur des paramètres) une solution au « second » problème de Harrod (fil du rasoir). Les résultats kaleckiens sont alors compatibles avec la convergence du taux d’utilisation des capacités vers le taux « normal », résultat qui n’a pas été mis en évidence dans la littérature existante.

Keywords: Kaleckian models, Utilization rate, Harrod instability, Income distribution, Automatic stabilizers.

Mots clés : Modèles kaleckiens, Taux d’utilisation, Instabilité harrodienne, Répartition du revenu, Stabilisateurs automatiques.

JEL codes: E12, E2, E25, E62

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1. Introduction

The aim of this paper is to contribute to the intense debate about the long run properties of income distribution and growth models. Models looking at this are based on three fundamental assumptions. Firstly, they are demand restricted, drawing on the Keynesian principle of effective demand. Secondly, national income distribution is assumed to affect economic activity, since the propensity to save out of wages is lower than the propensity to save out of profits. Thirdly, investment is assumed to be partly exogenous and partly endogenous, depending on capacity utilization as well as on profitability.

While there may be a consensus on the short run properties of these models (the paradox of thrift occurs and economic growth can be wage-led), contrasting positions can be identified in the long run between models which retain the short run (Kaleckian models) properties and those for which some short run properties are reversed mainly with growth becoming profit-led (Cambridgian and Marxian models). The key point behind this opposition is the pattern of the rate of capacity utilization. For Cambridgian or Marxian economists, this rate must converge in the long run on the normal rate. For the Kaleckian economists in contrast, the utilization rate remains endogenous: there is then a remaining gap between the current and normal utilization rates, except if the latter converges on the former.

In short, either long run growth is wage-led but the utilization rate cannot converge on the normal rate, or the utilization rate converges on the normal rate, but growth must be profit-led.

The debates have recently taken a new turn with the use of empirical arguments. Briefly, utilization rate data often show a greater stability than is expected using Kaleckian models. Of course, these empirical arguments are open to criticism. But they have already given rise to the question of whether it is possible to combine Kaleckian results formally in the long run with a rate of capacity utilization which converges on the normal rate. The present paper is a step in this direction.

This article presents a Kaleckian model which is just amended in order to introduce, near the autonomous component related to investment, another autonomous demand component characterized by its own growth rate. Such an amendment has not really been taken into account in the literature: for instance, government expenditure or public deficits are assumed

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3 The distinction between these currents of thought is beyond the scope of this article (see for instance Lavoie (2012). What they have in common (from the point of view of the present paper) is that the current utilization rate converges on the normal rate, in the long run. Some of these models are briefly presented further in the text.

to be proportional to capital stock and then to grow at the same rate.\textsuperscript{5} When exports are introduced, they are partly autonomous, but the results of the models do not fully take into account the consequences of this exogeneity.\textsuperscript{6}

Of course, the model’s conclusions may differ depending on the nature of the autonomous demand component that is taken into account and the source of its financing. This paper is based on autonomous public expenditure growing at an exogenous rate. Because of the complexity of the issue of debt interest, it is assumed for the sake of simplicity that government adjusts the tax rate endogenously, in order to preserve the budget balance.\textsuperscript{7} That is to say, emphasis is put on the impact of the autonomous demand component rather than on fiscal policy issues.

The model leads to two important and original outcomes. The first is that the rate of growth of public expenditure plays a stabilizing role on the rate of accumulation. The mechanism relates to the share of public expenditure in aggregate demand: a rise in the profit share results in a decrease of both the utilization and growth rates in the short run. But it also results in an increase in the share of public expenditure which exerts pressure both on the rate of utilization and on the propensity to save, and subsequently a turnaround in the utilization and growth rates. Eventually, it is the accumulation rate which adjusts to the rate of growth of public expenditure. However, although the impact on growth vanishes in the long run, its occurrence in the short run causes permanent effects on the level of variables (output, capital stock, labor, etc.). A rise in the profit share induces negative effects on economic activity, as in the canonical Kaleckian model, even if the long run rate of growth of capacity is not affected.

The second result is that the model can provide a solution to the ‘second’ Harrod knife-edge problem. It is assumed that entrepreneurs react to the gap between the current and normal rates of capacity utilization by adjusting their rate of accumulation. As is well known, such behavior in a demand constraint model usually generates instability (the ‘second’ Harrod problem). In contrast, it can be shown in the present framework (and depending on the parameters) that this entrepreneurs’ behavior could be necessary in order for the current utilization rate to converge toward the normal rate. The intuition behind this is that instability is more than offset by the stability resulting from the first mechanism. As a consequence, Kaleckian outcomes can be consistent with the convergence of the current utilization rate on the ‘normal’ rate, a result which has not been found in the existing literature.

\textsuperscript{5} See Blecker (2002), Commendatore \textit{et al.} (2005), Sawyer (2012) or also You and Dutt (1996). In assuming exogenous public expenditure Lavoie (2000) is an exception. But he himself limits his analysis to the short run, whereas long run issues are examined here. See also Chatelain (2010).

\textsuperscript{6} See Blecker (1998, 2002, 2011). His 1998 model in particular is explicitly built on the contradictory assumptions that exports and national income growth rates may differ from each other, but that the share of exports in national income remains unchanged. That is the kind of contradiction tackled in this paper. However, open economies address specific issues which will be analyzed in further research.

\textsuperscript{7} You and Dutt (1996) take the debt dynamics into account but in a model where public expenditure is endogenous.
Section 2 is devoted to the canonical Kaleckian model with endogenous public expenditure and the assumption of a balanced public budget constraint. It includes a discussion about the long run equilibrium with a special focus on Harrod knife-edge instability problem. The model with autonomous public expenditure is presented in Section 3. A brief comparison between this model and Serrano’s Sraffian supermultiplier (Serrano, 1995A, 1995B) is proposed in Section 4.

2. **The canonical Kaleckian model with endogenous public expenditure and the assumption of a balanced public budget constraint**

2.1. **Model resolution and economic interpretation**

Let us assume a closed economy whose aggregate production function is given by:

\[ Y_t = qL_t = u_tK_t, \]  

where \( Y_t \), \( L_t \) and \( K_t \) correspond to production volume, labor input and capital stock; \( q \) is the fixed labor productivity coefficient, and \( u_t \) the current utilization rate of capacity. Aggregate demand \( (Y_t^d) \) is given by:

\[ Y_t^d = C_t + I_t + G_t, \]  

where \( C_t \), \( I_t \) and \( G_t \) represent consumption, investment and public expenditure. Public expenditure is assumed to be related to consumption rather than accumulation. It is given by:

\[ G_t = \beta Y_t, \]  

Tax revenue results from an income tax whose rate \( (\tau) \) is supposed to be the same for wages and profits:

\[ T_t = \tau Y_t. \]  

Of course, assuming of a balanced public budget constraint \( \beta = \tau. \)** In addition, workers are assumed to consume all their wages. Consumption and savings are then given by:

\[ C_t = (1 - \tau)(1 - s\pi)Y_t, \]  

\[ S_t = (1 - \tau)s\pi Y_t, \]  

where \( \pi \) is the profit share and \( s \) the propensity to save out of profits. Finally, if \( u_n \) is the ‘normal’ rate of capacity utilization from the entrepreneurs’ point of view, then the investment function can be written as:

\[ I_t = [\gamma + \gamma_u(u_t - u_n)]K_t, \]  

where \( \gamma_u \) corresponds to the sensitivity of the rate of capital accumulation \( (g_t = I_t/K_t) \) to the gap between the current and normal utilization rates. Of course, \( u_t = u_n \), results in \( g_t = \gamma \).

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8 See You and Dutt (1996) for a model which takes the public deficit and debt into account.
As a consequence, the $\gamma$ parameter can be understood as the average firms’ expectation of the secular rate of growth (subject to animal spirits).\(^9\) This function can be rewritten as:

$$I_t = \gamma_u Y_t + (\gamma - \gamma_u u_n) K_t.$$  \hspace{1cm} (8)

It clearly appears from this that all the aggregate demand components are endogenous, but the fraction of capital accumulation which relates on the existing capital stock, $(\gamma - \gamma_u u_n) K_t$.

Substituting $C_t$, $I_t$ and $G_t$ into the aggregate demand function and solving gives the goods market equilibrium utilization rate:\(^{10}\)

$$u_t^* = \frac{\gamma - \gamma_u u_n}{(1 - \tau)s\pi - \gamma_u}.$$ \hspace{1cm} (9)

The equilibrium rate of accumulation is then:

$$g_t^* = \gamma + \gamma_u (u_t^* - u_n),$$ \hspace{1cm} (10)

and the after-tax rate of profit:

$$r_t^* = (1 - \tau) \pi u_t^*.$$ \hspace{1cm} (11)

These are the main results of the canonical Kaleckian model. The main comparative static results are reported in Table 1. Every column gives the qualitative impact that a change in a given parameter (in columns) has on the short run equilibrium value of the endogenous variables $u_t^*$, $g_t^*$ and $r_t^*$ (in rows).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\gamma$</th>
<th>$s$</th>
<th>$\pi$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_t^*$</td>
<td>+</td>
<td>–</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>$g_t^*$</td>
<td>+</td>
<td>–</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>$r_t^*$</td>
<td>+</td>
<td>–</td>
<td>–</td>
<td>+</td>
</tr>
</tbody>
</table>

If entrepreneurs’ expectations (animal spirits) are more optimistic, then activity and growth both increase. A rise in the capitalists’ propensity to save pushes consumption down, and then results in a cut in the rate of utilization (paradox of thrift). For its part, a rise in the profit share causes a decline of activity and growth (stagnationist regime and wage-led economic growth) because it increases savings and reduces consumption. In addition, the Kaleckian model is such that a rise in $\pi$ induces a proportionally higher cut in the utilization rate and, consequently, a decrease in the after-tax rate of profit. As a result, the rise in $\pi$ is as detrimental to capitalists as it is to workers (paradox of costs). Eventually, in accordance with

\(^9\) For sake of simplicity and without loss of generality, profitability is not included in the investment function.

\(^{10}\) The Keynesian stability condition is supposed to hold, that is $(1 - \tau)s\pi - \gamma_u > 0$. As a consequence, the $u_t^*$ numerator must also be positive $(\gamma - \gamma_u u_n > 0)$ for the rate of capacity utilization to be positive.
the Haavelmo theorem, the balanced growth of public expenses has a positive impact on economic activity and growth.

2.2. The rate of capacity utilization in the long-run

As the equilibrium utilization rate $u_t^*$ only depends on exogenous parameters, there is no guarantee about the equality $u_t^*$ and $u_n$. How could the current rate go back to its normal value? Intuitively, it can be expected that entrepreneurs adjust their expected secular rate of growth ($\gamma$ becomes $\gamma_t$). Starting from the accumulation function and assuming $u_t = u_n$, then:

$$g_{t|u_n} = \gamma_t,$$

so that:

$$g_t^* - g_{t|u_n} = \gamma_u(u_t^* - u_n).$$

The adjustment function then should be:

$$\dot{\gamma}_t = \psi \gamma_u(u_t^* - u_n),$$

with $\psi > 0$. But, as it is well known, such behavior worsens the situation because $\frac{\partial \dot{\gamma}_t}{\partial \gamma_t} > 0$: a fall in $u_t^*$ results in a decrease of $\gamma_t$ which induces another fall in $u_t^*$, etc. It is the Harrod knife-edge problem. The literature proposes other mechanisms that are here briefly surveyed, focusing on the long run responses to a rise in the profit share.\(^{11}\)

In the early Cambridge models (Robinson, 1956, 1962; Kaldor, 1955-56, 1957), the convergence between the current and normal rates of capacity utilization results from a price mechanism: entrepreneurs react to a fall in $u_t^*$ by decreasing goods prices via a cut in their profit margins. That allows an increase of aggregate demand and the restoration of $u_n$. In this framework, the utilization rate and profit share go hand-in-hand, as in a profit-led model, although income distribution is endogenous in the long run (while it is exogenous in the short run).

Another solution has been proposed by Duménil and Lévy (1999) who introduce the interest rate in the accumulation function and assume central banks have a goal in terms of economic activity: monetary authorities react to a fall in $u_t^*$ by cutting the interest rate because there is no risk of inflation. This policy boosts investment, there is a rise in aggregate demand and the utilization rate is brought back to its normal value. Eventually, the combination of a higher $\pi$, $u_t^* = u_n$, and a lower interest rate transforms the wage-led short run model into a profit-led long run model.

According to Skott (2010, 2012), the model may suffer from local Harrodian instability. However, the entrepreneurs’ behavior is finally supposed to restore stability: the fall in $u_t^*$ leads to an increase of unemployment. When unemployment is sufficiently high, entrepreneurs start accumulating capital again because they can easily hire workers to work on new equipment (formally, there is a rise in $\gamma$). In the opposite situation, an increase in $u_t^*$ makes it difficult for firms to hire workers, and this is followed by a decline in accumulation.

\(^{11}\) See Hein et al. (2011).
(a fall in $\gamma$). This mechanism gives rise to a limit cycle around a steady growth path which rests on growth in the supply of labor.

For his part, Shaikh (2007) makes a distinction between retained earnings and households’ savings. He also assumes that the retention ratio of firms (that is the share of retained profits) depends positively on the gap between the current and normal utilization rates. In this framework, a fall in $u_t^*$ leads to a decrease of the retention ratio, and then to a decline of the overall propensity to save. The rise of aggregate demand brings $u_t^*$ back to its normal value. Eventually, as for Duménil and Lévy, the wage-led short run model becomes a profit-led long run model.

In another article, Shaikh (2009) focuses on the accumulation function where the $\gamma$ parameter is replaced by the rate of growth of expected demand which is assumed to be perfectly foresight by entrepreneurs (that is with zero-mean errors). Perfect foresight combined with an accumulation process in order to return to the normal rate of capacity utilization provide stable adjustment around the Harrod-warranted path. As a result, paradoxes of thrift and costs occur in the short run, but they reverse in the long run.

At this stage, it is interesting to return to the Kaleckian authors to set out their position in detail, according to the long run rate of capacity utilization.\textsuperscript{12} Three main directions have been proposed. The first one is to question either the uniqueness of the normal utilization rate (some authors preferring the idea of a corridor of stability), or the pertinence of long run analysis (some authors preferring medium-run or provisional equilibria). A second approach consists in assuming firms have multiple targets whose realization may be mutually exclusive. As a result, entrepreneurs have to accept a lasting gap between $u_t^*$ and $u_n$ (Dallery and van Treeck, 2011). The third direction is to reverse the way to convergence: Lavoie (1996, p. 127) assumes for instance assumes entrepreneurs have adaptive behavior, setting the normal rate of capacity utilization according to past conventions and recent experiences.

In short, either long run growth is wage-led but the current utilization rate cannot converge on the normal rate, or the utilization rate converges on the normal rate but growth must be profit-led.\textsuperscript{13} In latter models, some short run outcomes (growth is wage-led) are reversed in the long run (growth becomes profit-led); and what appears to be a good policy (the expansion of wages) turns out to be bad.

3. **The introduction of autonomous public expenditure**

The introduction of autonomous public expenditure requires distinguishing between three equilibria which will be analyzed in three distinct sub-sections. In the short run, the share of public expenditure in aggregate demand (or in capital stock) is given and the rate of capacity utilization adjusts in order to balance the goods market (3.1).

\textsuperscript{12} See Hein et al. (2012).
\textsuperscript{13} The only exception is Skott (2010, 2012) whose model exhibits a cycle around a steady growth path.
But this public expenditure share has no reason to be stable from one period to the next, so every short run equilibrium is only temporary. It will be shown that the public expenditure share converges on a position of rest which is the medium run equilibrium set out here (3.2).

However the current and normal rates of capacity utilization may differ at this stage. The impact of entrepreneurs’ behavior is then analyzed by adjusting their accumulation rate in order to fill the gap between the two utilization rates. It was shown in the previous section that such behavior leads to instability (the Harrod knife-edge problem). Here it is shown that, depending on the parameters, this behavior can induce the convergence on the normal rate of capacity utilization; that is the long run equilibrium (3.3).

3.1. The short run temporary equilibrium

In the previous section, public expenditure was assumed to be completely endogenous, which is a strong assumption. Indeed, such expenditure may be expected to be partly autonomous. But the mix of the autonomous and endogenous components raises formal difficulties about deficit and debt issues. It is consequently assumed that public expenditure is completely autonomous, growing at an exogenous rate \( \alpha \), that is:

\[
G_t = G_0 e^{\alpha t}.
\] (15)

Tax revenue is specified as before, but now the tax rate must be endogenous in order to balance the public budget:

\[
\tau_t = \frac{\lambda_t}{u_t}
\] (16)

where \( \lambda_t = \frac{G_0 e^{\alpha t}}{K_t} \) represents the public expenditure’ share (relative to capital stock).

Taking these amendments into account, savings and accumulation respectively become:

\[
g^s = \frac{S_t}{K_t} = (u_t - \lambda_t)s\pi,
\] (17)

\[
g^i = \frac{I_t}{K_t} = \gamma + \gamma u(u_t - u_n),
\] (18)

where the after-tax rate of profit is now \( r_t = (u_t - \lambda G_t)\pi \). It should be emphasized that the average firms’ expected secular rate of growth \( \gamma \) may differ from the rate of growth of public expenditure \( \alpha \). The short run goods market equilibrium is then given by:

\[
u_t^* = \Phi (\gamma - \gamma_u u_n + s\pi \lambda_t),
\] (19)

where \( \Phi = (s\pi - \gamma_u)^{-1} \) is the Keynesian multiplier. Also:

\[
g^i_t = \gamma + \gamma_u (u_t^* - u_n).
\] (20)

The Keynesian stability condition is still supposed to hold, that is:

\[
s\pi - \gamma_u > 0.
\] (C1)

As a consequence, the term in brackets must also be positive for the rate of capacity utilization to be positive.
The comparative static results are reported in Table 2. Note that the sign of the derivatives with regard to \( s \) and \( \pi \) is that of \( \lambda_t - u_t^* \).\(^{14}\) For economic significance, \( \lambda_t \) must be lower than \( u_t^* \) (otherwise public expenditure would be greater than aggregate demand and the private demand, \( C_t + I_t \), would be negative), hence another restriction on parameters:

\[
\gamma - \gamma_u u_n > -\gamma_u \lambda_t. \tag{C2}
\]

Note that because of C1, C2 is more binding than the condition on a positive term in brackets in (19). However, C2 is less restrictive than in the canonical model because the expression \( \gamma - \gamma_u u_n \) can now be negative.

The comparative static results are summarized in Table 2. They are on the whole identical to those of the canonical Kaleckian model.\(^ {15} \)

### Table 2. Short run impact effects

<table>
<thead>
<tr>
<th></th>
<th>( \gamma )</th>
<th>( s )</th>
<th>( \pi )</th>
<th>( \lambda_t )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_t^* )</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>( g_t^* )</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>( \tau_t^* )</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>0</td>
</tr>
<tr>
<td>( r_t^* )</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>

3.2. **The medium run equilibrium**

The crucial point of the model is that the short run equilibrium (\( u_t^* \)) does not only depend on exogenous parameters. It now includes \( \lambda_t \) which varies with time as soon as the accumulation rate differs from the growth rate of public expenditure (\( \alpha \)), that is:

\[
\dot{\lambda}_t = \lambda_t (\alpha - g_t^*). \tag{21}
\]

As a consequence, the medium run equilibrium combines the goods market equilibrium with a position of rest (\( \dot{\lambda}_t = 0 \)). The stable medium run equilibrium is given by (see Appendix A for the proof):

\[
\begin{align*}
   u_t^{**} &= \frac{\alpha - \gamma}{\gamma_u} + u_n, \tag{22} \\
   \lambda_t^{**} &= u_t^{**} - \frac{\alpha}{\gamma u_t^{**}}, \tag{23} \\
   g_t^{**} &= \alpha, \tag{24} \\
   \tau_t^{**} &= 1 - \frac{\alpha}{\gamma u_t^{**}}, \tag{25} \\
   r_t^{**} &= \frac{\alpha}{s}. \tag{26}
\end{align*}
\]

Let us note that the necessary condition for \( u_t^{**} \) and \( \lambda_t^{**} \) to be positive is that:

\[14\text{ Actually, } d u_t^*/d s = \Phi(\lambda_t - u_t^*) \pi \text{ and } d u_t^*/d \pi = \Phi(\lambda_t - u_t^*) s.\]

\[15\text{ Note that the sign of } d \tau_t^*/d \lambda_t \text{ is given by } \gamma - \gamma_u u_n.\]
\[ \alpha > \Phi s\pi (y - y_u u_n). \]  

(C3)

The comparative static results in Table 3 deserve attention because some of them seem to be at odds with Kaleckian results. Actually, they are not. It is argued here that they might represent a faithful extension of the short run Kaleckian model.

| Table 3. Medium run impact effects                                      |
|-------------------------------------------------------------|---|
| \( \gamma \)                  | \( s \) | \( \pi \) | \( \alpha \) |
| \( \lambda_t^* \)          | –     | +     | +     | +     |
| \( u_t^* \)                  | –     | 0     | 0     | +     |
| \( g_t^* \)                  | 0     | 0     | 0     | +     |
| \( \tau_t^* \)              | –     | +     | +     | –     |
| \( r_t^* \)                  | 0     | –     | 0     | +     |

The next figures help to explain the underlying mechanisms. The goods market equilibria correspond to the intersections between the two straight lines representing capital accumulation \( (g^i) \) and savings \( (g^s) \). The initial short run equilibrium \( (E_0) \) is assumed to be a position of rest \( (g_K = \alpha) \). Let us suppose an increase in the profit share \( (\pi) \). This shift will have distinct effects in short and medium run.

In the short run (Figure 1), the rise in \( \pi \) leads to a counter-clockwise rotation and a downward shift of the intercept of \( g^s \). The equilibrium moves to \( E_1 \). As in the canonical model, there is a decrease of \( u^* \) and \( g^* \).

![Figure 1. Short run impact of a rise in \( \pi \)](image)

But this solution is not stable because \( g_1^* < \alpha \). There is then a rise in the public expenditure share \( (\lambda) \) which supports aggregate demand and reduces savings. More precisely, a portion of
profits which was intended for capitalists’ saving is now redirected toward public expenditure via the increase of the endogenous tax rate: $g^s$ shifts downward on Figure 2, until the economy once again finds a position of rest in $E_0$.

\[ g_1^* \]

\[ g^s \]

\[ g^i \]

\[ u_1^* \]

\[ u^* \]

\[ u^{**} \]

\[ g \]

\[ \alpha \]

\[ -s \pi \lambda_2 \]

\[ E_1 \]

\[ E_0 \]

**Figure 2. Medium run impact of a rise in $\pi$**

Let us now suppose an increase in the entrepreneurs’ expected rate of growth ($\gamma$). It results in an upward shift of $g^i$ such that $u^*$ and $g^*$ both increase. But as $g^* > \alpha$, the public expenditure share decreases ($g^s$ shifts upward), relieving the pressure on capacity. Hence a decrease in both $u^*$ and $g^*$ which follows up until the economy finds a new position of rest with $g = \alpha$ but a lower rate of capacity utilization than in the initial situation. Eventually, the main result of the model is that the negative effects on capacity utilization and economic growth vanish in the medium run. Does this mean that the rise in the profit share has no impact in the medium run? Actually, the answer is no. Let us look at Figure 3 to see this. The normal straight line represents the temporal evolution of output assuming there is no shock to $\pi$. The bold straight line represents the evolution of capital stock assuming an increase in $\pi$ at time $t_0$. In the medium run, the economic growth rate is brought back to $\alpha$. But this rate is temporarily lower than $\alpha$. The output level thus remains permanently lower than it would have been with an unchanging profit share. In this sense, the medium run analysis does not contradict the short run one. The model is thus consistent with other Kaleckian models for which, in addition, the growth rate is durably affected (the dashed line). In contrast, the profit-led model (mixed line) leads to positive medium run effects on the output level, despite temporary short run negative effects.
Of course, the rise in the profit share causes an increase in the before-tax rate of profit ($\pi u_t^*$).
But it is entirely offset by the rise in the tax rate (resulting from the increase in $\lambda$), therefore
an after-tax rate of profit which is brought back to its initial value.

Finally, public expenditure plays a role as an automatic stabilizer of growth, although there is
no public deficit (whereas automatic stabilizers usually rest on public deficit variations
depending on tax income being more endogenous than expenditure). Stabilization here comes
from tax rate adjustments which makes it possible to transfer some income from capitalists’
saving to public expenditure, or conversely. Moreover, stabilization does not apply to the
production level which is permanently affected by exogenous shocks; stabilization only
applies to the rate of capital accumulation which is brought back to $\alpha$ in the long run. Note
that a higher $\alpha$ has a positive impact on every endogenous variable, even on the profit rate
thanks to public expenditure supporting activity.

### 3.3. **The long run equilibrium**

In accordance with equation (22), the medium run rate of capacity utilization $u_t^{**}$ may differ
from the normal rate $u_n$. The last step of the model consists in introducing an ‘Harrodian
mechanism’ assuming firms adjust their expected rate of growth (via $\gamma_t$) depending on the
gap between the current and normal utilization rates (see above):

$$\gamma_t = \psi y_a (u_t^* - u_n).$$

The *long run* equilibrium is given by (see the proof in Appendix B):

$$u_t^{***} = u_n,$$  \hspace{1cm} (27)

$$\lambda_t^{***} = u_n - \frac{\alpha}{sn},$$  \hspace{1cm} (28)

$$\gamma_t^{***} = \alpha,$$  \hspace{1cm} (29)

$$g_t^{***} = \alpha,  \hspace{1cm} (30)$$

![Figure 3. Short and medium run impacts of a rise in $\pi$](image)
\[ \tau_t^{**} = 1 - \frac{\alpha}{s\pi u_n}, \]

\[ r_t^{**} = \frac{\alpha}{s}. \]  

Note that \( \lambda_t^{**} \) is positive only if:

\[ s\pi u_n - \alpha > 0. \] \( \text{(C4)} \)

Moreover, the condition for the equilibrium to be locally stable is given by:

\[ \psi < s\pi u_n - \alpha \] \( \text{(C5)} \)

As a consequence, it is not possible to formulate a univocal conclusion. The best that can be said is that there is some room, depending on the parameters, for the system to converge on its long run equilibrium. For this to happen, \( \psi \) has to be small. That is an original outcome: while a positive \( \psi \) generates the Harrod knife-edge problem in the existing literature, it is here a necessary condition for the rate of capacity utilization to converge toward its normal level.

Furthermore, the other results of the previous section are preserved, especially the permanent cut in both capital stock and output due to a rise in \( s \) (see Appendix B for details).

Eventually, assuming stable long run equilibrium, the comparative static results are presented in Table 4.

<table>
<thead>
<tr>
<th>Table 4. Long run impact effects</th>
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<tbody>
<tr>
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<tr>
<td>( \gamma_t^{**} )</td>
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<tr>
<td>( \lambda_t^{**} )</td>
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<td>( u_t^{**} )</td>
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<td>( g_t^{**} )</td>
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<td>( r_t^{**} )</td>
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Accordingly, a change in saving or investment behaviors has no effect on the long run utilization and accumulation rates. As for the medium run, an increase in \( \pi \) (or in \( s \)) leads to a rise in \( \lambda \): more saving in the economy means a higher share of public expenditure. In addition, a rise in the rate of growth of public expenditure still induces greater accumulation and profit rates. Interestingly, because of its multiplier effect on consumption and investment, the higher \( \alpha \) reduces the share of public expenditure in aggregate demand.

4. **A comparison with Serrano’s Sraffian supermultiplier**

It is worth noting that the present model is close to Serrano’s Sraffian supermultiplier (Serrano, 1995A, 1995B). This supermultiplier also rests on the introduction of a non-capacity generating autonomous demand component in a Keynesian framework. In Serrano’s model, this component is not public expenditure but the lump of capitalists’ consumption and
the non-capacity generating part of investment. Its two main features are similar to those of the above long run equilibrium: i) the actual rate of capacity utilization is equal to the normal (or planned) one \( u_n \), and ii) the rate of growth of capacity and output is equal to that of the autonomous component \( \alpha \). As a result, a decrease of the marginal propensity to save (which can result from a cut in the profit share) “will have a positive long-run level (on capacity output), but will have no effect on the sustainable secular rate of growth of capacity” (Serrano, 1995B).

A core point of Serrano is his enlightened analysis of the properties of the saving function:

\[
g^s = (u_t - \lambda_t) s \pi,
\]

(33)

where the marginal propensity to save \( s \pi \) is given exogenously. According to Serrano, if the autonomous demand component is omitted \( \lambda_t = 0 \), any change in the rate of accumulation implies a change in the utilization rate. On the other hand, with the autonomous demand component, the adjustment could take place through \( \lambda_t \) rather than \( u_t \). It is then formally possible to combine an adjustment in saving with a rate of capacity utilization which remains at its normal level. But of course, what is possible is not necessarily what happens.

Actually, in order for the utilization rate to remain at its normal level, Serrano has to assume that “firms as a whole correctly foresee the evolution of effective demand” (Serrano, 1995B), that is to say that the expected secular rate of growth \( \gamma \) must be (on the whole and on average) equal to the rate of growth of the autonomous component \( \alpha \). In contrast, “if expectations do happen to have a systematic bias in any direction then the actual path of the economy in the long run will move systematically away from the path formed by the corresponding sequence of long-period positions, causing the average actual degree of utilization to deviate persistently from the planned degree” (Serrano, 1995A, p. 87).

Hence the question: which solution prevails? According to Serrano, it is the first one. But the author does not propose any formal demonstration. He only refers to “the stylized fact that (…) there seems to be, on average, a remarkable balance between the long-run trends of productive capacity and aggregate demand” (Serrano, 1995A, p. 68). In other words, what should have been a result is actually an ad hoc assumption: Serrano assumes the long run equality between the current and normal rates of capacity utilization. This leads him to give priority to the conclusion that “firms as a whole correctly foresee the evolution of effective demand”.

Furthermore, let us note that only a part of Serrano’s assertions are right. Indeed, the medium run equilibrium analyzed above in which:

\[
u_t^{**} = \frac{\alpha - \gamma}{\gamma_u} + u_n,
\]

(34)

16 In Serrano’s model, the autonomous component is wholly financed out of capital which is well-founded when the autonomous component relates to capitalists expenses. But according Serrano (1995A, f.n.1), government expenditures or total exports could as well be taken into account. It seems however that such types of developments necessitate some amendments in the model since government expenditures as well as exports are partly financed out of wages.
and:

\[ g_t^* = \alpha, \]  

(35)

confirms that systematic bias in firms’ expectations \((\gamma \neq \alpha)\) prevents the actual utilization rate from converging on the normal rate. But these bias do not prevent the rate of accumulation from converging on \(\alpha\).

Eventually and more fundamentally, Serrano’s supermultiplier model does not pay enough attention to the model dynamics. Hence the intensive debate between Sraffian economists.

Focusing on dynamics makes it possible to agree with Trezzini (1998, p. 66) when he concludes – in opposition to Serrano – that “the determining role played by aggregate demand in the accumulation process will generally manifest itself in the variability of the average utilization of productive capacity and is therefore, even in the long run, incompatible with the assumption of normal utilization”. That is the outcome of the short and medium run equilibria of the present paper.

However Serrano’s outcomes are consistent with those of our long run equilibrium, but Serrano must adopt the restrictive hypothesis that “firms as a whole correctly foresee the evolution of effective demand” (Serrano, 1995B). Our hypothesis is less restrictive since the expected secular rate of growth \((\gamma)\) can differ from the rate of growth of the autonomous component \((\alpha)\). The model dynamics shows how the divergence between the actual and normal rates of capacity utilization lead entrepreneurs to correct their expectations, and especially how this correction can restore the long run equilibrium whereas it is always connected with Harrodian instability.

5. Conclusion

This article presents a Kaleckian model which includes autonomous public expenditure growing at an exogenous rate. The only restrictive assumption is that the tax rate adjusts for the public budget to remain balanced. The model confirms the well-known positive role of public expenditure on activity and growth.

Moreover, it has been shown that such public expenditure plays an automatic stabilizing role in economic growth (rather than in the level of activity). Changes in, say, capitalists’ propensity to save or in the profit share have a transient effect on growth: in the long run, the rate of capital accumulation is brought back to its initial value which is given by the exogenous rate of growth of public expenditure. The mechanism in question is based on the adjustment of the endogenous tax rate which results in an income transfer between capitalists and government: a portion of profits which was intended for capitalists’ saving is now redirected toward public expenditure via the increase of the endogenous tax rate. However, the changes in propensity to save or in income distribution have permanent effects on capital and output levels; and these effects are in the same direction as in the Kaleckian models.

---

Consequently, this automatic stabilizer mechanism has been combined here with entrepreneurs’ accumulation adjustment behavior, which is well known for being unstable. It has been shown that, depending on the parameters, the former mechanism can be strong enough in order to preserve the model’s stability and to provide a solution to the Harrod knife-edge problem. In this case, the current rate of capital utilization converges on its normal value. In other words, contrary to the existing literature, economic growth does not have to be profit-led for the current utilization rate to converge on the normal rate.

Of course, since it depends on the parameters, this solution to the Harrod knife-edge problem remains fragile. But it opens a door that has never been opened before.

In addition to the efforts to improve the robustness of the present conclusions, this model could be enriched in at least two ways. Firstly, its realism could be improved by relaxing the no public deficit and endogenous tax rate assumptions. But this task is not so easy because it is necessary to include the interest paid to capitalists, and then to take into account another dynamics, namely that of public debt.

Secondly, the model could be extended to other autonomous demand components, especially to exports. Blecker (1998, 2002, 2011) among others developed the Kaleckian model in an open economy framework. But his models cannot highlight the role of exports as an automatic stabilizer, because the share of exports in aggregate demand is assumed to be given exogenously. Some further explorations about the potential role of exports as a growth stabilizer should also refuel the debate between Kaleckian models and other post Keynesian models, such as the export-led cumulative causation or the balance-of-payment constraint models.18

Lastly, the above model does not take account of the labor market and unemployment. It thus provides no solution for the ‘first’ Harrod problem, except in arguing that government sets the rate of growth of public expenditure in accordance with demographic growth, in order to stabilize or eliminate unemployment. This question deserves more attention in further research.

6. Appendix

A. The medium run equilibrium

The dynamics of the public expenditure’ share is:

$$\dot{\lambda}_t = \lambda_t (\alpha - g_t^*)$$  \hspace{1cm} (21)

where $g_t^*$, depending on $u_t^*$, is given by (20). The medium run equilibrium is given by the system:

$$\begin{cases} 
\dot{\lambda}_t = 0 \\
\dot{u}_t^* = \Phi (Y - \gamma u_n + s\pi \lambda_t) 
\end{cases}$$  \hspace{1cm} (36)

18 See Cornwall (1977) or Setterfield and Cornwall (2002) for the former and Thirlwall (1979) for the latter. See also Blecker (2010) for a critical survey of the two approaches.
which has two solutions:

\[
\begin{align*}
\{u_t^- &= \Phi(y - \gamma_u u_n) \\
\lambda_t^- &= 0
\end{align*}
\] (37)

and:

\[
\begin{align*}
\{u_t^{**} &= \frac{\alpha - \gamma}{\gamma_u} + u_n \\
\lambda_t^{**} &= u_t^{**} - \frac{\alpha}{\pi}
\end{align*}
\] (38)

The former corresponds to the assumption that \( G_0 = 0 \) and is of little interest here (it refers back to the canonical Kaleckian model). For the latter to have positive values, the necessary condition is (restrictions are more binding on \( \lambda_t^{**} \) than on \( u_t^{**} \)):

\[
\alpha > \Phi s \pi (y - \gamma_u u_n). \tag{C3}
\]

Consequently C3 is fulfilled if \( y - \gamma_u u_n \) is weakly negative (such that C2 holds in every transitory period). Eventually, the condition \( \lambda_t^{**} < u_t^{**} \) holds whatever the value of the parameters.

The stability conditions depend on the first and second derivatives:

\[
\frac{d\dot{\lambda}_t}{d\lambda_t} = \alpha - \Phi s \pi (y - \gamma_u u_n) - 2\Phi y u_s \pi \lambda_t, \tag{39}
\]

\[
\frac{d^2\dot{\lambda}_t}{d\lambda_t^2} = -2\Phi y u_s \pi < 0. \tag{40}
\]

The second derivative being negative, the function \( \dot{\lambda}_t(\lambda_t) \) is an inverted u-shaped relationship with two roots, \( \lambda_t^- = 0 \) and \( \lambda_t^{**} \) (see Figure 4). The first derivative for \( \lambda_t = \lambda_t^- = 0 \) is:

\[
\left. \frac{d\dot{\lambda}_t}{d\lambda_t} \right|_{\lambda_t=0} = \alpha - \Phi s \pi (y - \gamma_u u_n), \tag{41}
\]

which is positive if C3 is fulfilled. Assuming C3, \( (u_t^-, \lambda_t^-) \) is unstable and the system converges toward its stable medium run equilibrium \( (u_t^{**}, \lambda_t^{**}) \).

**Figure 4.** Dynamics of \( \lambda_t \) (phase diagram)
B. The long run equilibrium

It is assumed that firms adjust their expected rate of growth depending on the gap between the current and normal utilization rates:

\[ \gamma_t = \psi \gamma_u (u_t^* - u_n). \]  

(14)

The long run equilibrium is the solution of the system:

\[
\begin{align*}
\dot{\gamma}_t &= 0 \\
\dot{\lambda}_t &= 0 \\
u_t^* &= \Phi(\gamma_t - \gamma_u u_n + s\pi \lambda_t)
\end{align*}
\]  

(42)

where \( \dot{\lambda}_t \) is given by (21). The unique solution of this system is:

\[
\begin{align*}
u_t^{***} &= u_n, \\
\lambda_t^{***} &= u_n - \frac{\alpha}{s\pi}, \\
\gamma_t^{***} &= \alpha.
\end{align*}
\]  

(27) (28) (29)

The condition for a positive \( \lambda_t^{***} \) is:

\[ s\pi u_n - \alpha > 0. \]  

(C4)

The local stability conditions depend on the dynamics of both \( \gamma_t \) and \( \lambda_t \). These conditions can be analyzed by means of the Jacobian matrix which (after linearization) is given by:

\[
J = \begin{pmatrix}
\frac{\partial \dot{\gamma}_t}{\partial \gamma_t} & \frac{\partial \dot{\gamma}_t}{\partial \lambda_t} \\
\frac{\partial \dot{\lambda}_t}{\partial \gamma_t} & \frac{\partial \dot{\lambda}_t}{\partial \lambda_t}
\end{pmatrix} = \begin{pmatrix}
\psi \Phi \gamma_u & \psi \Phi \gamma_u s\pi \\
-\Phi(s\pi u_n - \alpha) & -\gamma_u \Phi(s\pi u_n - \alpha)
\end{pmatrix}
\]

For the equilibrium to be stable, the matrix determinant must be positive whereas the trace must be negative. The determinant is:

\[ \text{Det}(J) = \psi \Phi \gamma_u (s\pi u_n - \alpha). \]

This leads to the result that \( \text{Det}(J) \) is positive, as soon as \( \lambda_t^{***} \) is positive (see C4 above). On the other hand, the trace is given by:

\[ \text{Tr}(J) = -\gamma_u \Phi(s\pi u_n - \alpha - \psi). \]

It can therefore be deduced that:

\[ \text{Tr}(J) < 0 \iff \psi < \psi^- \text{ with } \psi^- = s\pi u_n - \alpha \]  

(C5)

In summary, assuming C4 is fulfilled, the necessary condition for the system to converge on its long run solution is \( \psi < \psi^- \) (C5). The system’s trajectory depends on the discriminant of its eigenvalues, that is:

\[ \Delta = \text{Tr}(J)^2 - 4\text{Det}(J). \]

It can be shown that:

\[ \Delta = 0 \iff \psi^2 - 2(1 + \rho)\Omega \psi + \Omega^2 = 0 \]
where $\Omega = s_n \pi u_n - \alpha$ and $\rho = \frac{2}{\Phi \gamma u}$. The roots of this quadratic function rest on the value of $\psi$:

$$
\psi_1 = \Omega \left[(1 + \rho) - \sqrt{(2 + \rho)}\right], \\
\psi_2 = \Omega \left[(1 + \rho) + \sqrt{(2 + \rho)}\right].
$$

Given that the terms in brackets are respectively lower and higher than unity, then $\psi_1 < \psi < \psi_2$. Assuming $Det(J) > 0$, the results can be summarized as follows (see Figure 5):

a. $\psi < \psi_1 \Rightarrow \Delta > 0$ and $Tr(J) < 0$: $\lambda_t$ and $\gamma_t$ converge monotonically toward their long run equilibrium (stable node).

b. $\psi_1 < \psi < \psi \sim \Rightarrow Tr(J), \Delta < 0$: $\lambda_t$ and $\gamma_t$ converge via damped oscillations (stable focus).

c. $\psi = \psi \sim \Rightarrow Tr(J) = 0$ and $\Delta < 0$: oscillations are not damped (equilibrium is centre).

d. $\psi \sim < \psi < \psi_2 \Rightarrow Tr(J) > 0$ and $\Delta < 0$: the system diverges since $\lambda_t$ and $\gamma_t$ oscillations are unstable (unstable focus).

e. $\psi_2 < \psi \Rightarrow Tr(J), \Delta > 0$: $\lambda_t$ and $\gamma_t$ monotonically diverges (unstable node).

In Figure 5, $\dot{\gamma} = 0 \Leftrightarrow \gamma_t = s\pi(u_n - \lambda_t)$ (1), whereas $\dot{\lambda}_t = 0 \Leftrightarrow \gamma_t = \frac{\alpha}{\Phi\gamma\pi} + \gamma_u u_n - \gamma u \lambda_t$ (2).

It can be shown that the slope is higher for (1) than for (2), and conversely for the intercepts.

As for the medium run, a rise in $\pi$ leads to a permanent cut in both capital stock and output. Assuming configuration (a) holds, the rise in $\pi$ causes a decrease in both $u^*$ and $g^*$. As
entrepreneurs react in reducing $\gamma_t$. That induces another decrease in $g^*$, which slows down the equilibrium restoration (compared with the medium run dynamics) resulting from the increase in $\lambda_t$. The convergence being monotonically, capital stock and output are permanently lower than they should have been without the initial change in $\pi$. Now if the equilibrium is a center (configuration c), $g^{***}$ is also a center whose decreases are strictly offset by the increases and vice versa. Intuitively, both capital stock and output should oscillate around their initial path. By deduction, the intermediate configuration (b) should show the same permanent cuts in capital stock and output than in configuration (a): the decreases in $g^*$ being not totally offset by its increases, capital stock and output stabilize on a lower path than the initial one.

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20


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