The Enigma of the Inclined Plane from Hero to Galileo

Egidio Festa, Sophie Roux

The Enigma of the Inclined Plane from Heron to Galileo

Sophie Roux and Egidio Festa[1]

The law of the inclined plane states that the ratio between a weight and the force needed to balance this weight on a given inclined plane is equal to the ratio between the length and the height of this plane. With the peremptory tone for which he is known, Descartes affirmed that this law was known to "all those who write about mechanics".[2] Yet the problem of the inclined plane appears neither in Aristotle nor in Archimedes, and while writers such as Heron of Alexandria, Pappus of Alexandria, Leonardo da Vinci, Girolamo Cardano, and Colantonio Stigliola do indeed formulate it, they do not find the solution. At the end of the 16th century, the writers stating this law can be counted on the fingers of one hand: Jordanus de Nemore (and in his wake Niccolò Tartaglia), and later, within the space of some ten years, Michel Varron, Simon Stevin, and Galileo Galilei.

In the present article, painstaking work on the texts and an epistemological reflection will allow us to resituate Galileo's demonstration of the law of the inclined plane in a long history of mechanics. The brief version of Galileo's *Mecaniche* states the law of the inclined plane without giving the demonstration, writing instead: "it would be a bit more difficult study; let us thus set it aside here".[3] This is not a rhetorical clause employed by Galileo to get around providing a demonstration he was unable to supply: he had already provided the demonstration a few years earlier in the 23-chapter *De motu.*[4] If he omits the demonstration in the short version, it is likely

---

[1]Centre Alexandre Koyré (Egidio Festa) and "Philosophie, Langages et Cognition", Université Grenoble II (Sophie Roux). This article was written during the preparation of work on a critical edition, a translation and commentary on Galileo's *Mecaniche* to be published by Les Belles-Lettres. We have benefited from financial support from the program entitled "Formes d'articulation entre mathématiques et philosophie naturelle (XIV^{e}-XVII^{e} siècles)", within the CNRS Action Concertée Incitative "Histoire des Savoirs".

[2]Descartes to Mersenne, July 13, 1638, in Descartes*, Oeuvres*, vol. II, p. 232.

[3]Galilei*, Les mécaniques*, short version, p. 13, l. 5-10. The short version does not appear in Galilei, *Le opere*. Hereafter, we use the following abbreviations: "s.v." for "short version" and "l.v." for "long version".

[4]This is the name given to the manuscripts written in the hand of Galileo during his Pisan period, presented in Galilei, *Le opere*, vol. I, pp. 250-419. According to the indications given by Viviani, *Quinto libro degli elementi di Euclide,* pp. 104-105, these were several small notebooks (*quinternetti*) found together in a folder bearing the title *De motu antiquiora*, i.e., the older studies

because of the nature of this version: it was probably an oral lesson and it would have been normal, given the level of the students, to omit mathematical demonstrations. This was not the case in the long version, where after having exposed Pappus's error, Galileo gives a detailed demonstration of the law of the inclined plane.[5]

We shall begin by examining the texts of Heron and Pappus so as to understand the reasons for their failure to resolve the problem of the inclined plane. We shall then turn to the first successful demonstration of the law of the inclined plan, which provided by Jordanus de Nemore in his *De ratione ponderis*; unlike Pappus, Jordanus does not relate the force needed to move a body on an inclined plane to the force needed to move it on the horizontal: he compares it, so to speak, directly to the forces on two different inclined planes by reducing them to vertical movements. Thus are united the elements needed to understand the originality of the demonstration proposed by Galileo in *De motu*: like Jordanus, Galileo isolates the vertical movements; but he does this in a totally different fashion, by introducing a bent lever. We shall then show that the demonstration of the long version of the *Mecaniche* is geometrically similar to that of *De motu*, but that the introduction of the notion of "*momento*" shows an important conceptual and terminological reworking. Finally, we shall indicate the epistemological and historiographic conclusions that appear to arise from this study.

---

on movement; a first incomplete edition was presented under the title *Sermones de motu gravium* in the edition by Alberi, *Opere di Galileo Galilei*, vol. XI, pp. 83-125. Among the texts making up the writings of *De motu*, the chronological order of writing generally recognized today is as follows: an outline of projected work in Galilei, *Le opere*, vol. I, pp. 418-419, the *Dialogus*, *ibid.*, pp. 367-408; an *Essay* in 23 chapters, *ibid.*, pp. 251-340; a reworking of chapters 1 and 2 of this *Essay*, *ibid.*, pp. 341-343; an *Essay* in 10 chapters, *ibid.*, pp. 344-366; notes entitled "*Memoranda*", *ibid.*, pp. 409-419. On the vicissitudes of these manuscripts, see primarily Favaro, *Galileo Galilei e lo studio di Padova*, p. 3, Fredette, "Galileo's *De motu antiquiora*," pp. 321-350, Camerota, *Gli scritti* De motu antiquiora *di Galileo Galilei*, pp. 19-62, Giusti, *Elements for the relative Chronology of Galilei's* "De motu antiquiora", Fredette, "Galileo's *De motu antiquiora*: notes for a reappraisal", pp. 169-177.

[5] Galilei, *Les mécaniques*, l.v., pp. 59-63, l. 80-185. The long version is published in Galilei, *Le opere*, vol. II, pp. 149-190.

Ancient mechanics: the failure of the direct application of the model of the balance

How is it that a man who does not have the strength to raise a given weight can succeed in doing so when he pushes it up an inclined plane? This could have been the 36th or 37th question asked by Pseudo-Aristotle in his *Mechanica*: it would have been all the more interesting since the inclined plane cannot be so easily reduced to the balance as can other simple machines.[6] But there are only 35 mechanical questions, and the only authors of Antiquity who attempted to solve the enigma of the inclined plane were Heron and Pappus of Alexandria.[7] Their answers are interesting, despite their ultimate failure.

Hero of Alexandria: an intuition that falls short

One might be surprised that in an article devoted to the reading of Galilean text, we should analyze a text still unknown in the 15th century, Heron of Alexandria's *Mechanics.*[8] This is because Heron's attempt to find the law of the inclined plane is the first we know about, because we find it as such in Leonardo da Vinci and Stigliola, and because it employs an intrinsically interesting intuitive approach.

---

[6] In the following discussion, we take "model of the balance" to mean the idea that all mechanical systems (and in particular the inclined plane) can be understood from the starting point of weights balanced on a balance; we note that this idea does not prejudge the manner in which this balance is itself explained.

[7] Heron's list of simple machines, adopted by Pappus, does not include the inclined plane, undoubtedly precisely because it cannot be reduced to the balance (Heron, *Les mécaniques*, II 1, p. 115. Pappus, *Pappi Alexandrini collectionis quae supersunt*, VIII 31, vol. III p. 1115). The specificity of the inclined plane also appears in Pappus insofar as it constitutes, along with the Delian problem and a problem of how to construct a toothed wheel with a given number of teeth to fit another toothed wheel with a given number of teeth, as one of the three problems that had not been resolved by the Ancients that were later resolved (Pappus*, Pappi Alexandrini collectionis quae supersunt*, VIII, vol. III, p. 1028. These three problems are resolved in Heron, *Les mécaniques*, I 1, 11, and 23, resp. pp. 59-62, pp. 72-74, pp. 91-92).

[8] This work of Heron, written in the first century of the common era, is known to us by an Arabic translation by Qostâ ibn Lûkâ in the 9th century; in the 17th century, Golius brought back from the Levant a 16th-century copy of this translation; the first full translation and edition of this copy was that of the baron Carra de Vaux in 1893. For more on Heron, see more generally Schiefsky in this volume.

"Let us propose to draw upward a weight posed on an inclined plane", writes Heron.[9] One may suppose that the inclined plane is well polished and that the weight is a cylinder, represented in figure 1 by a right circular cross section.

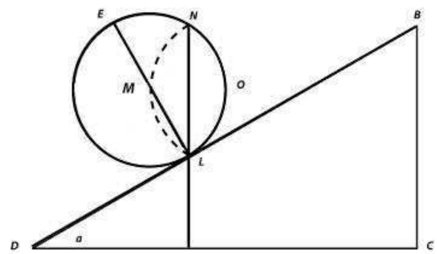

Fig. 1

As the circle touches the inclined plane (DB) at only one point (L), its natural tendency to roll downward will not be hindered. Let us consider the plane passing through L perpendicular to the inclined plane: it divides the cylinder into two equal parts. On the other hand, a second plane passing through L perpendicular to the horizon divides the cylinder into two unequal parts, the smaller, LON, above, and the larger, LEN, below; because the lower part is larger than the upper part, it draws the cylinder downward.[10] Here a common experience receives an intuitive explanation: it is harder to raise a weight on a steeper plane because the greater the inclination with respect to the horizontal the larger the lower part.

Heron goes a bit farther, but in addition to the fact that this extra step shows the limits of his physical intuition, it does not lead to a procedure that would allow him effectively to calculate the force necessary to raise a body on a given inclined plane. To the lune LON, he associates the lune LMN symmetric to it with respect to LN: according to him, they are in balance on the inclined plane. He consequently concludes that the cylinder will be in equilibrium on the inclined plane if

[9] Heron, *Les mécaniques*, I 23, p. 91. The context of this problem is interesting. I 20 establishes that, contrary to received opinion, the smallest force can move a body on a horizontal plane; I 21 analyzes the difference of behaviour between solids and liquids on an inclined plane; I 22 shows that a weight suspended from a pulley is moved by a power equal to it.

[10] Heron, *Les mécaniques*, I 23, pp. 91-92. The manuscript included no illustrations. Our figure is inspired by that found in Stigliola, *De gli elementi mechanici*, p. 41, and in Da Vinci, *Il codice atlantico*, f. 338r b.

the supporting force is equal to the weight of the cylinder less the weight of the part whose section corresponds to the two lunes. This force is not explicitly calculated.[11]

As we can immediately see in our figure 1, Heron thus gives an intuitive explanation for the reason a body goes down an inclined plane, as well as the reason why the steeper the plane, the more it goes down. The limits of this explanation appear when Heron affirms that the two lunes LON and LMN balance each other: without making it explicit, he reasons as if they were on the pans of a balance whose axis would be the perpendicular passing through the point of contact, whereas in fact they are on an inclined plane. It is undoubtedly the omnipresence of the model of the balance that explains that this same intuition, the same illustration and the same mistake are found in Leonardo da Vinci and Stigliola.[12]

Pappus of Alexandria: the failure of the direct application of the model of the balance

Contrary to Heron's *Mechanics*, book VIII of Pappus's *Mathematical Collection* was known at the end of the 16th century, in particular in Federico Commandino's Latin translation, finished before his death in 1575 and finally published by Guidobaldo del Monte in 1588.[13] The analysis of the inclined plane given in proposition 9 of book VIII was nonetheless known before 1588: in 1577, Guidobaldo del Monte indicated in his *Mechanicorum liber* that Pappus had reduced the inclined plane to the balance, without explaining how; in 1581, Filippo Pigafetta inserted into his Italian translation of the *Mechanicorum liber* a translation of this proposition, under the title "Problema di

[11] Cohen and Drabkin, *A Source Book in Greek science*, pp. 199-200, propose an illustration that allows a rapid calculation of F/P = (area of the circle – the two lunes)/area of the circle. By using the angle of inclination a of the plane BDC, we finally obtain $F/P = 2(a+\cos a \sin a)/\pi$. We can immediately see that Heron's result, false in general, is correct for $a = \pi/2$ (F = P) and for $a = 0$ (F = 0).

[12] Leonardo da Vinci noted that the farther the perpendicular to the horizon passing through the point of contact of the sphere and the inclined plane from the perpendicular to the horizon passing through the centre of the sphere, the heavier the body and the faster it descends (Da Vinci, *Il codice atlantico*, 338r b and 355r a, and Da Vinci, *Les manuscrits de Léonard de Vinci*, A 21v and A 52r). Stigliola, *De gli elementi mechanici*, "De rotte vettive", prop. 2, pp. 41-42

[13] On Commandino's translation and the vicissitudes of its publication, see Rose, *The Italian Renaissance of Mathematics*, pp. 209-212 and pp. 224-225. Passalacqua, "Le collezioni di Pappo".

Pappo Alessandrino nell'ottavo libro delle raccolte matematiche".[14] As we shall see, Galileo knew of Pappus's proposition when he wrote the long version of his *Mecaniche*.

Pappus poses the problem of the inclined plane in these terms:

> Given a weight drawn by a given force on a plane parallel to the horizon and given another inclined plane forming a given angle with the underlying plane, find the force by which the weight will be drawn on the inclined plane.[15]

Beginning with the idea that a given force is necessary to move the body on the horizontal plane, he proposes thus to evaluate as a function of the latter the force necessary to immobilize the body on the inclined plane. The instantiation of the problem is thus as follows (figure 2, based on Pigafetta 1581).

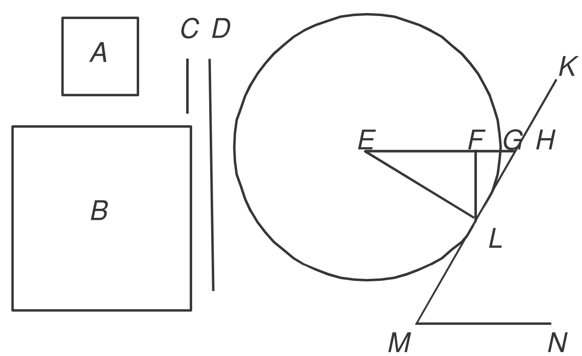

Fig. 2

The horizontal plane is MN, the inclined plane MK, the angle of inclination KMN. We are given the weight A and force C necessary to move the weight on the horizontal plane MN. We suppose that weight A is a homogeneous sphere concentrated in its centre E. We place the sphere on the inclined plane, with L being the contact point. The radius EL of the sphere is thus perpendicular to the inclined plane. We draw from E the line EH parallel to MN, we note H as the intersection of the inclined plane and EH, we call G the intersection of the sphere and EH, F the intersection of EH and

[14] Dal Monte, *Mechanicorum liber*, prop. 2, p. 124. Pigafetta 1581, p. 121.

[15] Pappus, *Pappi Alexandrini collectionis quae supersunt*, VIII. 9, vol. III, pp. 1055-1059.

the perpendicular passing through L. The question is to know what weight B will supply the necessary force D to maintain the sphere on the inclined plane MK.

Elementary geometric considerations show that we know the ratio FG/EF. The angle ELF is equal to the angle EHL (they are complementary to the same angle FLH) and the angle EHL is equal to the angle of inclination KMN (by construction). The angle ELF is thus equal to the angle KMN. Consequently the triangle LFE is given, thus the ratio EL/EF, which is equal to EG/EF. The final result is that we know the ratio FG/EF, which (by construction) is equal to (EG/EF) – 1.

We can thus arrange it so that weight B (and thus force D) is such that A is to B (and thus C to D) as FG is to EF. In these conditions, if we suspend weight A at E and weight B at G, A and B balance. GE thus acts like the beam of a balance fixed on the inclined plane KM by the fulcrum FL. But the weight of the sphere is A, situated at its centre E. Thus B is the weight which, placed at G, supplies the force D able to maintain the sphere on the inclined plane.

Now, to raise it, we must add C to D. In other terms:

$F_{total} = C+D = C+C/(EF/FG) = C(1+EF/FG)$.[16]

To conclude, Pappus does what he calls “the construction and the proof from an example”.[17] A body requiring a force C of 40 men to be moved on a horizontal plane could be raised, on an inclined plane of 60°, by a force seven and a half times as great, or by a force of 300 men.[18]

We know that Pappus knew of Heron, but we do not know if he knew of his analysis of the inclined plane. In any case, it is interesting to note the similarities, but also the differences, between

---

[16] If we call *a* the angle KMN equal to ELF, we have $D/C = EF/(EG-EF) = \sin a/(1-\sin a)$, from which $F_{total} = C+D = C/(1-\sin a)$. For a right angle, this gives $F_{total} = 1/0 = \infty$, or in other words, it requires an infinite force to raise any body vertically. For an angle of zero, this gives $F_{total} = C$, which fits the initial hypothesis.

[17] On the different functions of numerical examples and practical applications in Pappus, see Cuomo, *Pappus of Alexandria*, pp. 180-186, and for the particular case of the inclined plane, pp. 184-185.

[18] To calculate the ratio EF/FG, Pappus refers to the *Canon*, a manual table given in the first book of Ptolemy’s *Mathematics*. By using the value 0.866 for sin 60°, the formula established by Pappus allows him to conclude that the total force must be 7.46 times greater than C.

the two analyses. In both Pappus and Heron, only the components of the weights perpendicular to the horizontal plane are taken into account. Figure 2 allows us to understand that the reasoning of both Pappus and Heron are thus physically in error for a similar reason: just as Heron's lunes do not know that they are on an inclined plane, the balance GE being always parallel to the horizon, sphere E does not know that it is on an inclined plane. To express this more clearly in the case of Pappus, the calculation is formally correct, since the angle ELG is equal to the angle of inclination KLM, but it corresponds to nothing physically, because this same angle ELG does not intervene in the manner in which equilibrium is achieved. We can imagine that in both cases the error arose from the fact that the model of the balance was not adequately adapted to the particular problem of the inclined plane, but was so to speak directly applied to it.

The differences between the two approaches are no less significant: Pappus imagines himself to be both a better mathematician than Heron and closer to professionals.[19] On one hand, indeed, his proof is a canonical example of Euclidean reasoning, respecting what is announced in the introduction of book VIII of the *Mathematical Collection*: "I have found it appropriate to expound [these theorems] in a more concise and clearer manner and to establish [them] by a better reasoning than that of those who have previously written on this subject".[20] On the other hand, as the final numerical example indicates, he wants to satisfy an audience interested in the practical applications of the results obtained. It is undoubtedly this desire that leads him to take as his starting point the principle that a given force is needed to move a body on a horizontal plane, a principle that Heron had denounced as false.[21] This principle is testimony indeed to the concern for giving a method of calculation for those who use inclined planes: for craftsmen and engineers, pushing or pulling a material body on a horizontal plane, however smooth, requires that a certain effort be made; when they want to know the force needed to raise a weight on a given inclined plane, they

---

[19] On the representation of mathematics that Pappus could have had, see Cuomo, *Pappus of Alexandria*.

[20] Pappus, *Pappi Alexandrini collectionis quae supersunt*, VIII, vol. III, p. 1028.

[21] On the principle that the smallest force is sufficient to move a body on a horizontal plane, see Festa and Roux, "La moindre petite force".

can start from that which is needed to move it on a horizontal plane. Thus Pappus calculated the force required to move a weight on an inclined plane from the force needed to move it on a horizontal plane, whereas Heron did not include this force, since for him it was non-existent.

The problem of the inclined plane constitutes a particular challenge for the Archimedean as well as for the Aristotelian tradition: it cannot be immediately reduced to the balance, which are the fundamental model for these two traditions, whatever their other differences. This is why Heron and Pappus fail: they directly apply the model of the balance, the former doing so rather intuitively, the latter believing he can do so geometrically. As we will see, the response to this challenge did not come from a return to the direct experience of practitioners, but, in the case of Jordanus, from the exploitation of another mechanical system and, in the case of Galileo, from the adaptation of the model of the balance to the specificity of the problem considered.

Jordanus de Nemore: a new use for an old mechanical system

At the beginning of the last century, a certain number of historians held that Galileo's knowledge of the texts of Jordanus de Nemore inspired him in his demonstration of the law of the inclined plane.[22] We would certainly agree that in the second half of the 16th century the notion of positional gravity was known; it is found, for example, in the works of Cardano, Scaliger, Tartaglia, and Benedetti.[23] Nor can it be contested that the Jordanian demonstration of the law of the inclined plane was accessible at that time, in particular through Tartaglia's *Quesiti et inventioni diverse* (1546 for the first edition) and the edition he prepared of *De ratione ponderis* (published posthumously in 1565).[24] But no external clue allows us to affirm that Galileo had read this work by

---

[22] See, for example Duhem, *Les origines de la statique*, vol. I, pp. 251-252; and Caverni, *Storia del metodo sperimentale in Italia*, vol. IV, pp. 21-23, echoed by Vailati, "Il principio delle velocità virtuali", pp. 16-17, who affirmed furthermore that the texts of Jordanus served for the advanced study of mechanics.

[23] On the notion of *gravitas secundum situm,* see Moody and Clagett, *The Medieval Science of Weights*, pp. 94-95, pp. 123-124, pp. 150-151; and Galluzzi, *Momento*, pp. 70-73.

[24] The first text of the Jordanian tradition to be published was the *Liber de ponderibus* by Petrus Apianus in Nürnberg in 1533, but the analysis of the inclined plane can be found only in *De Ratione ponderis*, proposition 10. This last proposition was published the first time in 1546 by Tartaglia, *Quesiti et inventioni diverse*, book VIII, proposition 15, with an important addition (given

the time he wrote *De motu*, and more particularly his demonstration of the inclined plane; only an examination of the texts will allow us to determine if it is necessary, or even plausible, to suppose that Jordanus inspired Galileo on this point.

The demonstration of Jordanus de Nemore

The demonstration of the law of the inclined plane developed by Jordanus aimed to evaluate the notion of positional gravity (*gravitas secundum situm*) on an inclined plane. This notion results from an elementary physical observation: a body acts not only according to its weight, but also to the position at which it is placed. More precisely, the "suppositions" placed at the beginning of *De ratione ponderis* give two complementary determinations. First, the positional gravity of a body is physically characterized by the effects it produces: according to supposition 6, a body is lower in position than another when in going down it makes the other rise.[25] Second, positional gravity is evaluated geometrically: according to suppositions 4 and 5, the positional gravity of a body is inversely proportional to the obliquity of its descent, and this obliquity is measured by the ratio between the given segment of the descent and its vertical projection.[26] Bringing these two determinations together — which Jordanus does not do — we can say that the positional gravity of a body on an inclined plane is the part of its gravity that is effectively responsible for its descent along the plane and that must be counterbalanced so that the body remains immobile.

---

below in note 33 in the republication of the second edition [1554] of the *Quesiti*). In 1547-1548, Tartaglia was accused of plagiarism by Ludovico Ferrari, and perhaps for this reason prepared an edition of *De ratione ponderis*, which was published posthumously in 1565 by his Venitian publisher, Curzio Troiano, under the title *Jordani opusculum de ponderositate, Nicolai Tartaleae studio correctum*.

[25] Jordanus de Nemore, *Liber Jordani de ratione ponderis*, sup. 6, p. 174: "Minus grave aliud alio secundum situm, quod descensum alterius sequitur contrario motu".

[26] *Ibid.*, sup. 4: "Secundum situm gravius esse, cuius in eodem situ minus obliquus descensus ([A body] is heavier positionally when, in a given position, its descent is less oblique)". *Ibid.*, sup. 5: "Obliquiorem autem descensum, in eadem quantitatem minus capere de directo (A more oblique descent is that which, for a given distance, there is a smaller vertical component)". The expression "*minus capere de directo* (take less in the vertical)" simply means "descend less according to the vertical", or, as we translate it, "have a smaller vertical component"; by using the angle of inclination, l the part given for the descent and h its projection on the vertical, the obliquity is l/h = 1/sin *a*.

The problem of the inclined plane is explored in propositions 9 and 10. Proposition 9 states that for a given inclined plane, the weight is the same everywhere (figure 3).[27] This follows immediately from the fact that whatever the points D, E, chosen on the inclined plane, the right triangles DFK and EGM are equal, and thus so are their slopes.

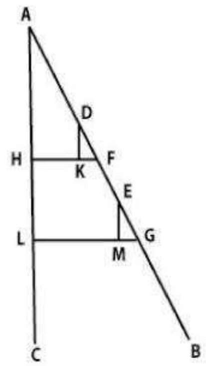

Fig. 3

In proposition 10, Jordanus wishes to demonstrate that if the weights are placed on planes whose slopes have the same ratios as the weights, they descend with the same force,[28] or in other words, that they have the same positional gravity.[29] Placing himself in the particular case of planes of different slope but with the same height, he calls the "proportion of declinations" the ratio between the lengths of these planes.[30]

---

[27] *Ibid.*, p. 188: "Equalitas declinationis identitatem conservat ponderis. (The equality of the declination conserves the identity of the weight)". The meaning of "declination" is given in prop. 10; see below, note 30.

[28] *Ibid.*, p. 190: "Si per diversarum obliquitatum vias duo pondera descendant, fueritque declinationum et ponderum una proportio eodem ordine sumpta, una erit utriusque virtus in descendendo (If two weights descend by paths with different declinations and if the declinations and the weights are directly proportional between them, the weights will have the same force when they descend)".

[29] Two bodies that have the same gravity descend with the same force: according to supposition 1, the force (*virtus*) of a body is its power to tend towards the bottom (*potentia ad inferiori tendendi*) and to resist contrary motion (*et motui contrario resistendi*); according to supposition 2, a heavier body descends faster (*quod gravius est, velocius descendere*).

[30] *Ibid.*, p. 190: "Proportionem igitur declinationum dico non angulorum, sed linearum usque ad æquidistantem resectionem in qua æqualiter sumunt de directo (By 'proportion of the declinations', I do not mean that of the angles, but that of the lines taken to their intersection with the horizontal when they have the same vertical component)".

Let ABC be a horizontal line, BD a vertical line and two inclined planes DC and DA of the same height but of different slopes (fig. 4). We place *e* and *h* two weights, whose ratio is the same as that of DC to DA – in other terms, the ratio of the slopes. We must show that *e* and *h* will have the same force in descending.

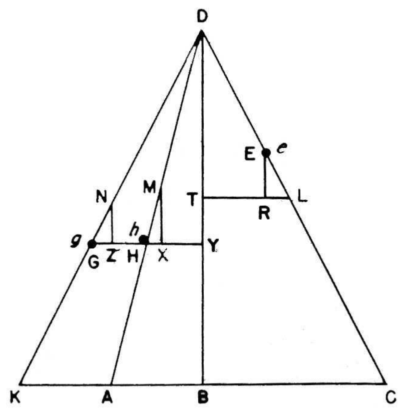

Fig. 4

As in the case of the demonstration of the principle of the lever, we suppose that there is no balance: "If that is possible, then *e* descends to L and thus pulls *h* to M".[31]

To simplify the geometric part of the demonstration, we use the vertical DB as a line of symmetry. We thus draw a line DK, symmetric to DC with respect to the vertical, and we then place on this line DK a weight *g* equal to weight *e*. We set a segment NG equal to EL and thus to MH; then let NZ, MX, ER, be the vertical projections of NG, MH, EL.

Next comes the geometric demonstration.

Given the similarity of the triangles NGZ, DYG, DBK, we have NZ/NG = DY/DG = DB/DK.

Likewise we have MX/MH = DB/DA.

We conclude by an elementary manipulation of the ratios MX/NZ = MH/NG x DK/DA.
But by construction, MH = NG; whence finally MX/NZ = DK/DA = *g*/*h*.[32]

---

[31] *Ibid.*, p. 190: "Si igitur possibile est, descendat e in L, et trahat h in M".

The equality of the ratios MX/NZ and DK/DA shows that we can replace movements on inclined planes by vertical displacements. Jordanus goes no further. But we can spontaneously give a physical interpretation of the inverse proportionality between weights *g* and *h*, and the vertical displacements NZ and MX: "that which suffices to raise *h* along XM also suffices to raise *g* along ZN" or, as Tartaglia writes, "the force or the power of *h* on the plane DA is equal to the force or the power of *g* on the plane DK".[33] This interpretation is not explicit in Jordanus, but his reasoning as follows seems to require it. In fact, he continues, since *e* cannot raise *g* (indeed, by construction, planes DC and DK have the same slope and weights *g* and *e* are equal to each other), *e* cannot raise *h* either. Thus *e* and *h* will remain in equilibrium.[34]

An ancient mechanical system and a new model

After having set certain physical definitions, Jordanus presents a "purely formal" mechanical proof; in one decisive point, it even avoids any involvement in a physical interpretation. It is nonetheless incomprehensible if we do not identify the underlying mechanical system: if *e* and *h* were not tied by a line passing over D (or better yet, through a pulley attached at D), there would be no sense in supposing that *e*, when it descends to L, would draw *h* to M. We do not know if certain manuscripts

---

[32] *Ibid.*, p. 190: "Quia igitur proportio NZ ad NG sicut DY ad DG, et ideo sicut DB ad DK, et quia similiter MX ad MH sicut DB ad DA, erit MX ad NZ sicut DK ad DA, et hoc est sicut g ad h".

[33] Tartaglia, *Quesiti et inventioni diverse*, fol. 97v: "E pero [*h* e *g*] si vengono ad egualiar in virtu, over potentia, E per tanto quella virtu, over potentia, che sara atta à far ascendere luno de detti dui corpi, cioe à tirarlo in suso, quella medesima sara atta, over sofficiente à fare ascendere anchora l'altro, adunque sel corpo *e* (per laversario) è atto, E sofficiente à far ascendere il corpo *h* per fin in M, el medesimo corpo *e* saria adunque sofficiente à far ascendere anchora il corpo *g* a lui equale, E inequale declinatione, la qual cosa è impossibile per la precedente propositione (Thus, if [*h* and *g*] come to be equal in force, or in power, for in as much as the force, or the power, which will be apt or sufficient to make the other rise, thus if the body *e* (according to the opponent) is apt and sufficient to make the body *h* rise to *m*, this same body *e* would be by this fact sufficient to make the body *g* also rise to equal to itself, and equal in inclination, which, according to preceding proposition, is impossible". Duhem fills in this gap by applying here an "implicit postulate" by which that which can raise a weight P by a height *h* can also raise *n*P by a height *h/n* (Duhem, *Les origines de la statique*, vol. I, p. 142, p. 147). This postulate is also made explicit in prop. 6 of *De ratione ponderis*, in *ibid.*, p. 182.

[34] *Ibid.*, p. 190: "Sed quia *e* non sufficit attollere *g* in *N*, nec sufficiet attollere *h* in *M*. Sic ergo manebunt".

of Jordanus de Nemore contained illustrations representing such an arrangement; in his notebooks, Leonardo da Vinci did associate analyses of the inclined plane clearly inspired by the Jordanian tradition with drawings of this system.[35] We will note in passing that the presence of this set up in a given technological system does not provide a sufficient condition for its use in the science of machines: the "Jordanus set up", if we may so name it, was usual in Antiquity, whether to raise loads or to prevent them from descending too quickly; yet neither Pappus nor Heron exploited it to analyze the inclined plane.[36] If Jordanus succeeded in resolving the problem of the inclined plane, it is first of all because he was able to see in an ancient scheme a new model.

We must also underline the fertility of the equation MX/NZ = g/h. In this equation, the segments MX and NZ, which express the variations of level, are the vertical projections of the oblique segments NG and MH respectively, equal to each other. As in the case of the balance, here too the heavier weight corresponds to the smallest vertical displacement. As has often been stressed, what characterizes Jordanus's demonstrations regarding the balance and the inclined plane is precisely the implementation of a method – that of vertical displacements – that will later lead to what has been called the principle of virtual work.[37] Jordanus's equation, which states explicitly the equilibrium condition as a function of vertical displacements, also contains implicitly the law of the inclined plane, as it would be established by Galileo. Let us write this equation in terms of positional gravity, that is to say in the form g x NZ = h x MX; since we have NG = MH, we can just as easily write g x NZ/NG = h x MX/MH. The positional gravity is thus correctly expressed as a function of the height/length ratio of the plane in question.

---

[35] This arrangement is, for example, represented when it is a matter of comparing weights on unequal slopes, in Da Vinci, *Il codice atlantico*, 354 vc et 375va, and in *Les manuscrits de Léonard de Vinci*, E 59r, E 75r, G 77r, G 79 r. The texts accompanying some of these representations attest to a knowledge of the Jordanian tradition; we can compare, for example, the statements of propositions 9 and 10 of the *De ratione ponderis* with Da Vinci, *Il codice atlantico*, 354 vc: "La equalita della diclinazione osserva la equalità de' pesi. (La proporzion de' pesi (po) posti in obliquiotà della medesima proporz). Se la proporzioni de' pesi e dell'obliqua, dove si posano, saranno equali, ma converse, (i pesi) essi pesi resteranno equali in gravità e in moto".

[36] See, for example, the drawing illustrating proposition 9 of book III of Heron's *Mechanics*, reproduced in Heron, *Les mécaniques*, p. 277.

[37] See first Vailati, "Il principio delle velocità virtuali", pp. 20-21.

However brilliant it is, Jordanus's solution remained isolated; in particular it is found neither in the *Liber de ponderibus*, nor in Blasius of Parma's treatises on statics. At the end of the 16th century, at least three solutions to the problem of the inclined plane emerged in less than ten years: those of Varron (published in 1584), of Stevin (published in 1586), and of Galileo (written between 1589 and 1592). To characterize the demonstrations of Varron and Stevin with respect to those of Jordanus, we could say that the former both have a common point with the latter: Varron considers vertical components, and Stevin uses the same mechanical system.

To evaluate the ratio of forces needed to raise a weight on different inclined planes, Varron proposes a figure in which the inclined planes have the same length and make up the radii of a circle; thus, he continues, the ratio sought is equal to the ratio between the tendencies to fall, evaluated by the vertical projection of the inclined planes (their lengths are equal, but their angles of inclination are different, as are the vertical projections). At the beginning of the treatise he set out the equality of between resistance to lifting and the tendency to fall.[38] The only problem is thus the evaluation of the tendencies to fall for the vertical projections. Varro immediately draws this from the initial definition of the line of inclination (*linea nutus*) as the straight line that goes from the place where the movement begins to the place to which it tends to move, that is to say, in the case of a body with weight, the vertical straight line.[39] Given the knowledge of the time, we can wonder if such an affirmation could have any demonstrative value: a body on an inclined plane cannot, in fact, move *vertically* downwards; and how can we know how far its movement, if it could take place, would take it?

Stevin uses the mechanical system we have seen used by Jordanus: he considers a triangle whose sides are inclined planes of the same height and whose hypotenuse is parallel to the horizon. The resemblance to Jordanus stops there, for we know that Stevin refused to explain equilibrium in terms of potential movement: if a necklace of identical and evenly-spaced beads surrounding our triangle were not in equilibrium, it would acquire a movement that "would have no end, which is

---

[38] See Varro, *De motu tractatus*, conclusions I-III, p. 15, in Camerota and Helbing, *All' alba della scienza galileana*, pp. 274-275, and their comments in *ibid.*, pp. 139-148.

[39] Def. IV, in *ibid.*, p. 250: "Linea autem recta quae est ab eo loco a quo motus fieri incipit ad illum ad quem tendit illius vis quae motus efficit, nutus dicetur".

absurd". For reasons of symmetry, he continues, we can eliminate the part of the necklace that is under the inclined plane; the remaining parts on the top and the slope are thus in equilibrium. Now the number of beads on a plane is proportional to its length, whence the conclusion that the "powers" of the beads on a plane are inversely proportional to the length of the planes.[40] Stevin's proof does not correspond to the canons of Euclidean geometry, but relies instead on a fruitful intuition, in which we can see a first step in the revelation of the parallelogram of forces.

Galileo (1): the model of the balance is adapted to the inclined plane in De motu

Galileo's *De motu* in 23 chapters is a criticism of the Aristotelian theses on movement developed from the notions of Archimedean hydrostatics and, of course, from the principle of the lever.[41] Galileo thus refers to the balance.[42] But as the title indicates, this is not a treatise on simple machines like the *Mecaniche*, but rather an essay on motion. In chapter 14, "in quo agitur de proportionibus motuum eiusdem mobilis super diversa plana inclinata (in which is examined the ratios between movements of the same moving object on different inclined planes)", it is not a matter of using the law of the inclined plane to explain the function of the screw as it will be in the *Mecaniche*, but rather to answer two questions concerning the movement of a body along an

---

[40]Stevin, *La statique*, I theor. 11, prop. 19, p. 448. The *De Beghinselen der Weeghconst* was published in 1586, the Latin translation by Willebrordus Snellius in 1605, the French translation by Albert Girard in 1634.

[41]Thus Archimedes is said to be more modern than Aristotle ("Archimedes Aristotele est multo recentior"), in Galilei, *Le opere*, vol. I, p. 303. On Galileo's training in Pisa, see Schmitt, "The Faculty of Arts at Pisa"; Helbing, "Mobilità della Terra"; and Camerota and Helbing, "Galileo and Pisan Aristotelianism". On the criticism of Aristotle and the relation between *De motu* and the teachings of the Collegio Romano, see Dollo, "Galileo e la fisica del Collegio Romano".

[42]See in particular chapter 6, "in quo explicatur convenientia quam naturalia mobilia cum libræ ponderibus habent" (*Le opere*, vol. I, p. 257). There is a "convenientia", that is, an analogy, between the natural movement of a body and the movement of a weight on a balance: in the first case, it is the excess gravity of weight with respect to the gravity of the counterweight that makes it move; in the second case, it is the excess of gravity of the body with respect to the gravity of its medium (see in particular chapters 7 and 8, in *ibid.*, pp. 260-273). This analogy is wrong: in balances with unequal arms, weight is not the only factor that must be taken into account. In fact, Galileo mixes here under the term "gravitas" two notions which we distinguish from each other: the specific weight of a body, which is involved in hydrostatics, and its weight, which is taken into account on a balance.

inclined plane. First, a qualitative question: why does a moving object descend very fast vertically and ever more slowly on an inclined line the smaller the angle of inclination? Second, a quantitative question regarding speed: how much faster is the moving object on the vertical than on the inclined path?[43] The resolution of these questions supposes a demonstration of the law of the inclined plane.

The first question

Galileo begins by recalling two of the physical hypotheses to which he subscribes at the time:

> i) The force with which a body falls is equal to the force required to raise it.[44]
>
> ii) The ratio of the forces required to raise a body vertically and on a given inclined plane is equal to the ratio between the respective gravities on these planes.[45]

If we know the gravity, we can answer the two initial questions. At the time of *De motu*, Galileo consistently assumes that the speeds follow the ratios of the gravities.[46]

---

[43] The term most frequently used in this chapter of *De motu* to designate the measurement of motion is not *velocitas*, but rather *celeritas*, which we translate here as "speed"; as we indicate below in § 3.2., if there is a problem, it concerns not the term used, but its meaning.

[44] Galilei, *Le opere*, vol. I, p. 297: "Prius hoc est considerandum, quod etiam supra animadvertimus: scilicet, quod manifestum est, grave deorsum ferri tanta vi, quanta esset necessaria ad illud sursum trahendum; hoc est, fertur deorsum tanta vi, quanta resistit ne ascendat". For previous occurrences of this hypothesis in *De motu*, see for example, Galilei, *Le opere*, vol. I, pp. 274-275. Regarding this idea, see Camerota and Helbing, *All' alba della scienza galileana*, pp. 139-148.

[45] *Ibid.*: "Sed tunc sciemus quanto minor vis requiratur ad sursum trahendum mobile per bd [inclined plane] quam per be [lesser inclined plane], quando cognoverimus quanto ejusdem mobilis erit gravitas in plano secundum lineam d, quam in plano secundum lineam be".

[46] See for example *ibid.*, pp. 295-296: "velocitates mobilium, in medio in quo moventur, gravitates; et proportiones consequenter velocitatum, gravitatum proportiones, sequ[u]ntur". Indeed, speed and motion cannot be distinguished from each other: gravity, the cause of motion, is also the cause of speed (*ibid.*, pp. 260-261).

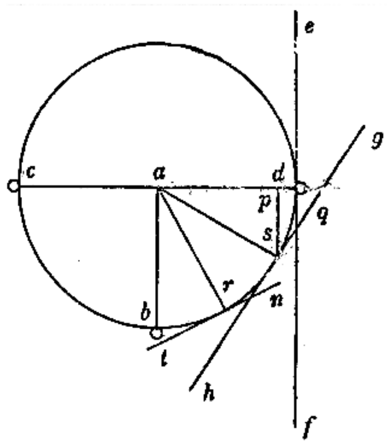

Fig. 5

Thus Galileo's purely geometric demonstration aims to determine the ratio between the gravities of a body on different inclined planes.[47]

To do so, the gravity of a body on an inclined plane is reduced to the gravity of a body suspended on a bent lever.[48] Galileo imagines (fig. 5) balance *cad* whose arms *ca* and *ad* are equal. We place at *c* and *d* two identical weights, then we hold *a* in place while turning *d* toward *b*: the suspended body is thus at one of the ends of a bent lever.[49] At each point of the quarter circle *db*, it is as if the moving object were at the tangent at this point, the descent following the gravity of the mobile object at this point. The successive positions of the bent lever thus correspond to a series of inclined planes.[50]

---

[47] *Ibid.*, p. 297: "Procedamus itaque ad inquisitionem talis gravitatis".

[48] Heron of Alexandria attributes the explanation of the bent lever to Archimedes (Heron, *Les mécaniques*, I 33, pp. 108-110); on it application by Jordanus de Nemore, see Moody and Clagett, *Liber Jordani de ratione ponderis*, pp. 136-137.

[49] Galilei, *Le opere*, vol. I, p. 297: "Et intelligatur libra cd, cuius centrum a, et in puncto c pondus æquale ponderi alii quod sit in puncto d. Si itaque intelligamus, lineam ad, manente puncto a, moveri versus b (…)".

[50] *Ibid.*, p. 297: ". . . in primo puncto d descensus mobilis erit veluti per lineam ef; quare per lineam ef descensus mobilis erit secundum gravitatem mobilis in puncto d. Rursus, quando mobile erit in puncto s, in primo puncto s suus descensus erit veluti per lineam gh; quare mobilis per lineam gh motus erit secundum gravitatem quam habet mobile in puncto s. Et rursus, quando mobile erit in puncto r, tunc illius descensus in primo puncto r erit veluti per lineam tn; quare mobile per lineam tn movebitur secundum gravitatem quam habet in puncto r".

If the gravity of the moving object at *r* is thus less than at *s*, and at *s* less than at *d*, then the speed of the object at *r* is less than at *s*, and at *s* less than at *d.* Now – and it is here that the law of the bent lever comes into play – the gravity at *r* is less than the gravity at *s*, and at *s* less than at *d*:

> The weight at point *d* counterbalances the weight at point *c*, since the distances *ca* and *ad* are equal; but the weight at point *s* does not counterbalance the weight at point *c*. Indeed, if at point *s* we draw the perpendicular line to *cd*, the weight at *s* is, with respect to the weight at *c*, as if it were suspended at *p*; now, at *p* the weight weighs less than the weight at *c*, since the distance *pa* is shorter than the distance *ac*.[51]

We thus find here the answer to the first question initially asked: if a moving object on an inclined plane descends with a smaller force when the angle of inclination is smaller, it is because the smaller the angle, the smaller its weight.[52]

The second question

The answer to the second question easily follows. First, the ratio between the speeds by *ef*, the tangent at *d*, and by *gh*, the tangent at *s*, is equal to the ratio between the gravities at *d* and at *s*, which is itself equal to the ratio between *da* and *pa*.[53] Second, the triangles *asp* and *sqp* being

---

[51] *Ibid.*, pp. 297-298: "Si itaque ostendamus, mobile in puncto s minus esse grave quam in puncto d, erit iam manifestum quod illius motus erit tardior per lineam gh quam per ef : quod si, rursus, ostendamus, in r mobile adhuc minus esse grave quam in puncto s, erit iam manifestum quod tardior erit motus per lineam nt, quam per gh. Atque iam manifestum est, mobile in puncto r minus gravare quam in puncto s; et in s, quam in d. Pondus enim in puncto d æqueponderat ponderi in puncto c, cum distantiæ ca, ad sint æquales: sed pondus in puncto s non æqueponderat ponderi c. Ducta enim linea ex puncto s perpendiculari super cd, pondus in s, respectu ponderis in c, est ac si penderet ex p; sed pondus in p minus gravat quam pondus in c, cum distantia pa sit minor distantia ac".

[52] *Ibid.*, p. 298: "Manifestum est igitur quod mobile maiori vi descendet per lineam ef quam per lineam gh, et per gh quam per nt".

[53] *Ibid.*: "Et quia tanto facilius descendit mobile per lineam ef quam per gh, quanto gravius est in puncto d quam in puncto s; est autem tanto gravius in puncto d quam in s, quanto longior est linea da quam linea ap; ergo mobile eo facilius descendet per lineam ef quam per gh, quo linea da longior est ipsa pa. Eandem ergo proportionem habebit celeritas in ef ad celeritatem in gh, quam linea da ad lineam pa".

similar and *as* being equal to *ad*, the ratio between *da* and *ap* is equal to the ratio between *qs* and *sp*, that is to say to the ratio between the oblique downward segment (or the length) and the vertical downward segment (or the height).[54] By calling l and h the length and the height of the inclined plane, V_l and V_h the speeds depending on the length and the height of the inclined plane, P_l and P_h the gravities depending on the length and height of the inclined plane, we have:

$$P_l/P_h = h/l \qquad (1)$$

$$\text{or } V_l/ V_h = h/l \qquad (2)$$

The analysis of the inclined plane thus first leads Galileo, in a text already obscured by the ambiguities of the term "*gravitas*", to construct a new understanding of this term: it is a propensity to descend, variable according to the inclination of the inclined plane, and consequently constant all along a given inclined plane.[55] By a physical hypothesis according to which speeds are like gravities, (2) then results from the extension of (1) to speeds. Obviously this extension is not legitimate if we understand "speed" in its classical sense: when a body is moving and accelerating as it descends an inclined plane, its speed is not constant. To save this extension, Pierre Souffrin supposes that the term "*celeritas*" refers here to what he calls a holistic measurement of speed. To say that the ratio of the speed along the plane to the speed along the vertical is equal to the ratio of their lengths would be equivalent to saying that the ratio between the spaces traversed in the same time along the plane and along the vertical is equal to the ratio between the height and the length of

[54] *Ibid.*: "Est autem sicut da ad ap ita qs ad sp, hoc est obliquus descensus ad rectum descensum: constat igitur tanto minori vi trahi sursum idem pondus per inclinatum ascensum quam per rectum, quanto rectus ascensus minor est obliquo; et, consequenter, tanto maiori vi descendere idem grave per rectum descensum quam per inclinatum, quanto maior est inclinatus descensus quam rectus".

[55] On the meaning of "gravitas" in *De motu*, see note 42 above. Galluzzi, *Momento*, p. 187, remarks that this understanding is hardly compatible with the hydrostatic model that permeates *De motu* and, pp. 195-196, formulates the hypothesis that Galileo became aware of the necessity of a lexical reorganization in these pages.

the plane, which is correct.[56] What remains, however, is that the notion of a holistic measurement of speed does not allow us to take into account the variations of *celeritas* on the inclined plane, and is thus unsatisfactory.[57]

We now have the elements needed to evaluate what Galileo may owe to his reading of his predecessors. According to his testimony, no philosopher had dealt with the question of the motion that he analyzes in chapter 14 of *De motu*; this declaration refers, to all appearances, to the question of the speeds of a mobile object on variously inclined planes, a question proper to the science of motion.[58] There is no obvious reason to doubt this testimony, for in general Galileo does not hesitate in his writings to cite the authors he opposes or those who inspire him; more importantly, the examination of the texts seems to confirm this. Galileo is in fact as far from the purely intuitive process of Heron, Leonardo and Stigliola as from the geometric analysis of Pappus.

The author to whom he is the closest is Jordanus de Nemore: like him, he attempts to evaluate the force of the body as it descends the inclined plane (a force called "*gravitas*" in *De motu* and "*virtus in descendendo*" in the *De ratione ponderis*), and there is a starting point that in each case allows a resolution of the problem. As we have seen, the equation of Jordanus contains the Galilean law of the inclined plane. Nonetheless, the geometric construction proposed by Galileo – the use of a bent lever with one fixed end and another whose successive positions follow a series of inclined planes – has nothing to do with that proposed by Jordanus. In this sense, although it is not materially impossible that Galileo had read the *De ratione ponderis* when he was in Pisa, the details of his demonstration prevent us from concluding, as did Caverni and Duhem, that he owes his demonstration of the law of the inclined plane to this reading.

---

[56] Among the many studies that Pierre Souffrin devoted to the holistic measurement of speed, the most explicit for our text is Souffrin, "Sur l'histoire du concept de vitesse". See also Souffrin, "Motion on inclined planes".

[57] On the theory of accidentally accelerated movement in *De motu*, see Galluzzi, *Momento*, pp. 182-187; Damerow, Freudenthal, McLaughlin and Renn, *Exploring the Limits*, pp. 138-144.

[58] Galilei, *Le opere*, vol. I, p. 296: "Quaestio, quam nunc explicaturi sumus, a philosophis nullis, quod sciam, pertractata est: attamen, cum de motu sit, necessario examinanda videtur illis, qui de motu non mancam tractationem tradere profitentur".

Galileo (2): The **Mecaniche** *introduces the notion of* **"momento"**

In the long version of the *Mecaniche*, Galileo begins by criticizing Pappus's demonstration; he did not do this in *De motu*, either because he did not know this demonstration or because he judged that it did not allow the resolution of the problem at hand, that of the speed of a body on the inclined plane. The criticism aimed at Pappus is that the principle he adopted as his starting point doomed him to failure. Since, according to Galileo, no force is needed to move a body on a horizontal plane, there is no sense in seeking to evaluate with respect to this force the force required to move a body on an inclined plane. What must be done instead is to suppose as given the force that moved the weight vertically and to evaluate with respect to *this* force that which will move the weight on the inclined plane.[59] Galileo thus merely makes explicit, in his criticism of Pappus, the procedure that he used in *De motu*; we shall show more generally that, if the demonstrations of *De motu* and the *Mecaniche* are identical from a geometric point of view, there is nevertheless a terminological and conceptual clarification in the *Mecaniche.*

Geometric identity…

Let us recall the process followed in the *Mecaniche*. We consider, as in *De motu*, a balance whose arms AB and BC are equal and from whose ends we suspend two equal weights: the system is in equilibrium on the vertical support BI (fig. 6).

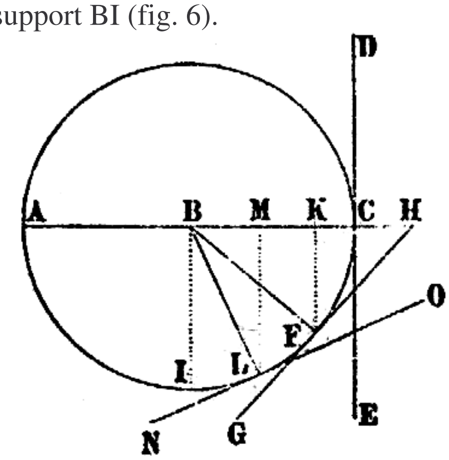

Fig. 6

If we hold AB fixed while BC can turn, we have a bent lever as in *De motu*. When BC turns, the equilibrium is broken, and according to the principle of the lever demonstrated at the end of chapter

[59] Galilei*, Les mécaniques*, l.v., p. 60, l. 5-17.

2 of the long version, the *momento* of the weight varies according to its distance from the point of support.[60] At F, for example, it is as if the weight were suspended from K, and its *momento* decreases by the ratio of BK to BC; at L, the *momento* decreases by the ration of BM to BC, etc.

Galileo makes explicit the reasons why we can go from a bent lever to an inclined plane, which he did not do in *De motu*:

> i) If the body that descends follows a single path, it does not matter whether it is suspended from a bent lever or supported by a circular inclined plane such as CFLI.[61]
>
> ii) When the moving body is at a given point of the circumference, it is as if it were at the tangent to this point; thus its inclination and hence its *momento* are the same on the inclined plane and on the tangent.[62]

We then return to the elementary geometric considerations of *De motu*. Given that the triangles BFK and KFH are similar and that BF = BC, we have BK/BC = KF/FH, from which we draw the ratio between the *momento* of descending the inclined plane and the *momento* of descending vertically: it is equal to the ratio between the height of the plane KF and its length FH.

What remains is merely to extend this result to the situation where a supporting force is required. How to do this is immediately apparent with the principle, already given in the study of the balance, by which there is no notable difference between the power of a weight to support another and its power to move it.[63] From this comes the final conclusion: the ratio of the force to

---

[60]*Ibid.*, pp. 35-38, l. 73-163. In the case of the bent lever, the distance is measured by the projection of the radius in the horizontal direction, here on the diameter AC this particular case is examined in l.v., p. 35, l. 59-72.

[61]*Ibid.*, p. 61, l. 131-140. In *De motu*, Galileo passes directly from the bent lever to the inclined plane, without any consideration at this point of anything related to motion. In this page of the *Mecaniche*, Galileo seeks to justify the assimilation of the circularly inclined plane by introducing notions supposing a movement (the body considered "moves" and follows a certain "path", l.v., pp. 61-62).

[62]*Ibid.*, pp. 61-62, l. 150-155.

[63]Referring to a balance on which two weights are in equilibrium, Galileo remarks that by adding the smallest of weights to one, the balance would move, which is obvious. What is much

support a weight is equal to the ratio of the height and length of the plane, which comes back to the relation noted as (1) above:

$$P_l/P_h = h/l$$

This result, as well as the construction and geometric structure of the demonstration, are thus identical to those of *De motu*.

… and conceptual differences

Three differences nonetheless separate the analysis of *De motu* and that of the *Mecaniche*:

1. *De motu* evaluated the speed of bodies descending an inclined plane; the *Mecaniche* refrain from any consideration of speed. This may be explained by the fact that the *Mecaniche* are not intended to deal with motion. It is undoubtedly also due to the fact that Galileo had no satisfactory result to propose. In *De motu*, the conclusion was that the speeds (or the spaces traversed in the same time, according to the interpretation of Souffrin) are as the gravities: this result does not take into account the trivial observation that a body accelerates when it descends an inclined plane. And if we admit that the speed does indeed increase, how can we obtain this result from a factor such as *momento*, which is constant on all points of a given inclined plane?[64] Perhaps Galileo had already understood at the time of the *Mecaniche* that the evaluation of the speed of a body descending vertically or along a plane constituted a problem that was important to solve.

2. *De motu* brought together under the name of gravity two distinct quantities, weight and the effectiveness of weight; the *Mecaniche* distinguishes them by calling "*gravità*" the weight, and "*momento*" the effectiveness of the weight, that is to say the force required to support it or move it. As we have seen, this *momento* is evaluated by the static moment of a bend lever, as a function of the weight and the distance from the centre; contrary to weight, it is thus variable according to the

---

less obvious is to oppose Guidobaldo dal Monte by affirming that one must make "no difference between the power that a weight has to support another and the power it has to move it" (*ibid.*, pp. 38-39; see also s.v. p. 6, l. 22-28).

[64] Galluzzi, *Momento*, p. 216.

inclination of the plane on which the body is placed. The term "*momento*" allows in this way a conceptual clarification.

3. Whereas, as we have just recalled, any reference to speed is absent from the *Mecaniche*, *momento* appears there as the "*momento di discendere" (momento* of descending) and as "*impeto* to go downward", that is to say, as the cause of downward movement, the motive force that draws a body toward the centre of the universe.[65] The importance of the notion of "*momento di discendere*" for the later developments in Galilean science has already been noted. As Paolo Galluzzi has shown, Galileo no longer considered speed as proportional to the weight of the body, but he has nonetheless not given up studying, from the basis of statics, the characteristics of the motion of a body on an inclined plane, and in particular its speed. A certain number of letters and fragments indicate in fact that, during his entire stay in Padua, Galileo attempted to found a science of accelerated motion "*senza trasgedire i termini meccanici*", and that he based this on the notion of *momento* of speed, constructed by analogy with the notion of *momento* of gravity, and not yet designating an instantaneous speed.[66] Thus, for Galluzzi, the notion of *momento* of descending is a milestone in the long series of metamorphoses of the notion of *momento*. Maurice Clavelin gives even more decisive conceptual importance to the notion of *momento* of descending. For him, this is the equivalent of dissociating the "gravific" function of gravity (accounting for weight as the pressure exerted by a body at rest) and the "motor" function of gravity (accounting for movement toward the centre). Once this dissociation is accomplished, it is impossible to suppose that gravity as weight can measure gravity as motive force; thus the latter would become a physical entity unto

---

[65]The use of the expression "*impeto* to go downwards" as a double for the term "*momento*" already appears in the definition of "*momento*", see Galilei, *Les mécaniques*, l.v., p. 34, l. 22-24. According to Galluzzi, *Momento*, p. 206, *impeto* represents so to speak the physical consequences and the concrete effects of variations of the effectiveness of weight abstractly recorded by *momento*. For other occurrences of "*impeto*", see Galilei, *Les mécaniques*, l.v. p. 40, p. 45, p. 59, p. 61, p. 62, p. 66.

[66]For an overview of this period, see Galluzzi, *Momento*, pp. 261-308, who comments on the letters to Guidobaldo dal Monte from 29 November 1602 and to Sarpi from 16 October 1604 in Galilei, *Le opere*, vol. X, pp. 97-100 and pp. 115-116, the fragments found in *ibid.*, vol. VIII, p. 378 ff. and p. 417 ff., and their repetition in the *Discorsi*, in *ibid.*, vol. VIII, p. 222 and p. 262.

itself requiring a specific analysis, which would be indispensable for arriving at the idea that there must be a law of falling bodies that applies to all bodies.[67]

To repeat the various stages of the evolution examined by Galluzzi and Clavelin would take us far beyond the purpose of the present article. To conclude our examination of the *Mecaniche*, let us merely stress that, in addition to a conceptual effort that this work shares with *De motu* and which allows for a physically effective mathematical treatment of the problem of inclined plane, the *Mecaniche* is a testament to the conceptual work that followed this mathematical treatment. It is not only a matter of reflecting on a proof and on a figure to draw out their physical significance, but also to invent a terminology that will allow the stabilization of new concepts.

Conclusion

The law of the inclined plane constitutes a significant advance of Galilean science. It will be used, for example, to demonstrate that the degrees of speed acquired on the planes, with different slopes but the same height, are equal when the mobile object arrives on the horizontal plane, and that their value depends only on the height of the plane.[68] The first edition of the *Discorsi* (1638) made of this statement a principle; an objection from Viviani led Galileo to give a demonstration using, among other elements, the law of the inclined plane.[69] He formulated it at this time in the following manner:

---

[67]Clavelin, *La philosophie naturelle de Galilée*, pp. 173-175, remarks in particular that the *gravitas secundum situm* of Jordanus does not lead to this dissociation, but merely indicates the ratio of the total weight to the reduced weight (*ibid.*, p. 174, n. 143). Considering the intellectual conditions that would allow the emergence of this new conception of gravity, Clavelin, "Le copernicianisme padouan de Galilée" also holds that the notion of *momento* of descending is the hint of a "silent Copernicanism" in Galileo's mechanics during the Paduan period; for reasons which we will not present here, we do not share this interpretation.

[68]Galilei, *Le opere*, vol. VIII, p. 218: "I gradi di velocità d'un mobile descendente con moto naturale dalla medesima sublimità per piani in qualsivoglia modo inclinati, all'arrivo all'orizzonte son sempre eguali, rimossi gli impedimenti". The statement of this principle can already be found in the *Dialogo*, in Galilei, *Le opere*, vol. VII, p. 47.

[69]Galileo to Benedetto Castelli, 3 December 1639, in Galilei, *Le opere*, vol. XVIII, p. 126. Viviani wrote down this demonstration for Galileo who was already blind, and had it inserted in the first edition of the *Opere* of Galileo (Bologna, Manolessi, 1656, pp. 132-134). On the role of this

. . . The moments (*i momenti)* or speeds of a same moving object vary with the different inclination of the planes . . . so that the *impeto*, the power (*il talento*), the energy (*l'energia*) or, as we wish to say, the moment (*il momento*), of descending, are decreased in the mobile object by the plane on which they are supported and they descend.[70]

Because Galileo would later be able to articulate this law to obtain certain results in the science of accelerated motion for which he is famed, it may appear natural for the historian to trace it back to its beginnings; but as we have just seen, this is a complex task. More precisely, in the light of the questions raised in this volume, the three following points can serve as a conclusion:

1. The geometric techniques are identical in all the demonstrations: it is a matter of comparing similar triangles and manipulating proportions. The advances do not come here so much from a greater or lesser mathematical sophistication as from a work of conceptualization, both before and after the geometric demonstration. Before the geometric demonstration, by the mental manipulations that allow the reduction of the inclined plane to mechanical systems already understood. After the geometric demonstration, in particular by work on the language, allowing the sedimentation or crystallization in a precise terminology of what had heretofore been mixed together, for example by the distinction between *gravità* and *momento* that characterizes the *Mecaniche*.

2. The problem of the inclined plane constitutes a challenge to the Archimedean tradition as well as to the Aristotelian tradition: it cannot be immediately reduced to the balance, the fundamental model of these traditions, whatever their other differences. As we have seen, the failures of Heron, Pappus, Leonardo da Vinci, or Stigliola come from a direct application of this model; the success of Jordanus, Stevin, or Galileo comes from the fact that they relied on another model or managed to adapt the model of the balance to the problem at hand, by means of the bent lever. In light of this inventiveness, the distinction often made since Duhem among different mechanical traditions is not pertinent: faced with a new problem, mechanical physicists make the

---

principle in Galileo's new science of motion, see Laird, "Renaissance Mechanics and the New Science of Motion".

[70] Galilei, *Le opere*, vol. VIII, p. 215.

most of the tools they have to hand. This conclusion could be verified historically in a certain number of 16th-century authors, who did not consider Pseudo-Aristotle, Archimedes, or Jordanus as fathers of opposing traditions, but as representatives of a common knowledge that was to be revived.

3. The few certainties that we have regarding the material paths followed by these texts are rare, and often negative: we know, for example, that the Pseudo-Aristotelian *Mechanica* was not directly known in the medieval Latin world, and that Heron's *Mechanics* was unknown in 16th-century Italy. In such conditions, how can we interpret the identity between two or more demonstrations? We will offer here two elementary rules. First, it is indispensable to establish that the two demonstrations are in fact identical: contrary to what has often been written, for example, the demonstration of the law of the inclined plane proposed by Jordanus is not identical to that of Galileo. Second, when this identity has been established, historians must distinguish as best they can (and for this, they surely need as much tact as patience) what cannot be explained without genuine knowledge of a text from what may be a mere coincidence. Let us suppose, for example, that Galileo (1) and Galileo (2) are two distinct individuals and it is possible, but not established, that the latter knew the works of the former: the identity of their geometric demonstrations of the law of the inclined plane would lead us to think that the latter had indeed consulted the works of the former. Conversely, the identity of the intuitive analyses of the inclined plane given by Heron, Leonardo da Vinci, and Stigliola is inconclusive, for there is a weak identity, sufficiently explained by the omnipresence of the model of the balance until the end of the 17th century.