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Abstract

This paper provides a new estimation of an international poverty line based on a Bayesian approach. We found that the official poverty lines of the poorest countries are related to the countries' mean consumption level. This new philosophy is to be compared to the previous assumptions made by the World Bank in favour of an absolute poverty line. We propose a new international poverty line at \$1.48 per day (2005 PPP) based on a reference group consumption level. This figure is much higher than that proposed by the World Bank (\$1.25 in 2005 PPP), but still within a reasonable confidence interval. By this standard, there are more than 1.7 billion people living in poverty.

Keywords: Poverty line, Bayesian inference.

JEL codes: C11, C21, I32

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1 Introduction

In different countries and at different times, the definition of poverty changes according to people's living situations and to people's varying poverty perception. Even within a given society and at a given point of time, the critical level of income at which individuals are recognised as being poor is not perceived in the same ways by different income groups. The meaning of poverty differ between those groups as poverty can be, at least partly a social construction. This was illustrated for instance in van Praag (1971) as a preference drift.

Between different countries, the minimum basket of goods that ensures physical and even mental health is not the same, just because living standards are different and social characteristics and habits are different. The common view is that in the poorest countries, poverty is anchored to basic human needs, such as enough food, clean water, sanitation, clothing, shelter, health care and basic education. A poverty line in those countries is usually defined as an "absolute poverty line" that focus only on how much humans need for living, independently of the national income distribution. For richer countries, once the basic needs are satisfied, individuals tend to desire more expensive basket of goods, e.g. more varied diets, suitable cloth, comfortable shelter, better health and higher education, just to be like the others and take a decent part in social life. The definition of "poverty" in this case becomes more complex and is influenced largely by the perception of "economic inequality". An individual who considers himself as being poor may not face a problem of survival, but he's suffering from an envy comparison for what others have in his living society. The latter definition of poverty line is called "relative poverty line"

Where could we put the limit between these two definitions of a poverty line. What is the list of countries which are considered as being sufficiently rich in order to afford a relative poverty line and what is the list of the other countries? Ravallion et al. (1991) showed that official poverty lines varied little with mean consumption for poor countries while above a critical level of mean consumption, official poverty lines had a much stronger elasticity with respect to mean consumption. Based on that previous finding, Ravallion and Chen (2001) and Ravallion and Chen (2004) provided an international poverty line (a worldwide absolute poverty line) to be "$ 1 per day" ($1.08 at 1993 PPP). In a more recent paper, Ravallion et al. (2009) clearly identify two group of countries in a new data set covering 74 developing countries from 1988-2005. They estimate a non linear regression relating official poverty lines to mean consumption, allowing for a different coefficient between the two group of countries. From this model, the new international poverty line had risen to $1.25 per day at 2005 PPP. Greb et al. (2011) then re-revisited this study, using different econometric techniques and a different specification and found a higher international poverty line at \$ 1.45 per day. The difference between \$1.25 and \$1.45 does not seem important while it actually is. This adjustment means that 317.6 millions supplementary people would fall in poverty in 2005. This gives us enough reason to revisit the problem of setting an international poverty carefully.

We are going to use Ravallion et al. (2009)'s new data set as given at the end of their paper and adopt a Bayesian approach in order first to take fully into account the uncertainty of the estimated parameters and second to provide a posterior density for the obtained poverty line. We re-estimated the same empirical model used in Greb et al. (2011) and illustrate graphically where the difference lies between the two papers. We then show how to define a poverty line as a function of the mean level of consumption of a reference group of poor countries, the composition of that group being endogenously determined. We found in this latter analysis, that an international poverty line is eventually higher at \$1.48 per day. The posterior density of our international poverty line is much more concentrated than those obtained using any of the previous studies.

The paper is organised as follows. In section 2, we show how a subjective approach to poverty perception can be introduced, using macro cross country data. With section 3, we develop the econometric techniques involved by a Bayesian approach to our problem. With section 4, we show how a Bayesian approach can illustrate some misspecification problems and provide a rational route to derive the posterior density of a world poverty line. Section 5 concludes.

2 Poverty line and preference drift

A poverty line can be defined on a subjective basis. We refer to the minimum income question (MIQ) that can be found for instance in Kapteyn et al. (1988) and which be phrased as follows: *what is the minimum income that you would need in order to make the two ends meet?*. If z_i is the reported answer, y_i the actual income of the household and x_i a vector of characteristics of the household (such as its composition) then the following regression can be estimated

$$z_i = \alpha + \beta y_i + \gamma x_i + \epsilon_i$$

using individual data. An estimated subjective poverty line corresponds to a fixed point for every type of composition x and is given by:

$$z^* = \frac{\alpha + \gamma x}{1 - \beta}.$$

Individuals having an income below z^* would be classified as being poor. A poverty line at a country level cannot be determined independently of any subjective perception of poverty. A European comparison was made using this approach in for instance Van den Bosch et al. (1993). This approach can have however some bias just because it assumes that z^* is common to all individuals and does not depend on their income level. Using a fixed point for determining z^* eliminate any preference drift. So that in Van den Bosch et al. (1993) subjective poverty lines are much above official poverty lines. A preference drift, at least in developed countries is clearly identified, see for instance van Praag (1971) for Belgium. That means, that individual do not have the same perception of poverty. The perceived minimum necessary income rises with the level of own income.

In order to determine a revised world poverty line, Ravallion et al. (2008) start from a subjective definition of poverty, recognising that the perception of poverty can vary between countries because of different habits, different perceptions and social traditions. Absolute poverty lines are defined with respect to a consumption basket meant to provide the necessary calories to survive. But the composition of that basket is socially determined. Atkinson (1983) quotes the example of English workers who went on strike because tea was planned to be withdrawn from the official basket of goods for computing a poverty line in the nineteen century. Despite the fact that tea has no nutritional value, it had a social value. The preference drift transposed at the country level means that the minimum necessary income for an individual would depend on

$$z_{ij} = \alpha + \beta y_i + \gamma \bar{C}_j + \epsilon_{ij}$$

where $\bar{C}_j$ is the mean consumption level of country j. The same fixed point algorithm provides the level of the poverty line for country j:

$$z_j^* = \frac{\alpha + \gamma \bar{C}_j}{1 - \beta}.$$

But this time there is a country specific effect. Ravallion et al. (2008) assumed that for very poor countries, the preference drift is zero so that $\gamma = 0$ for that group of countries. Working on a data set of 77 developing and developed countries, they identified a group of 15 very poor countries with an average level of private consumption per capita of less that \$60 a month. They estimate the following model

$$z_j = \mathbb{1}(C_j < \$60)\alpha_1 + \mathbb{1}(C_j > \$60)(\alpha_2 + \gamma C_j) + \epsilon_j \tag{1}$$

where z_j is an official poverty line in PPP dollars and C_j the average level of private consumption per capita in PPP dollars. The estimated poverty line is given by the regression coefficient α_1. This coefficient represents an estimate of the empirical mean of the z_j computed for the countries for which $C_j < \$60$. This group of countries is said to represent the reference group to compute the poverty line. Greb et al. (2011) estimate a slightly different model

$$z_j = \mathbb{1}(C_j < \theta)\alpha_1 + \mathbb{1}(C_j > \theta)(\alpha_2 + \gamma \log C_j) + \epsilon_j \tag{2}$$

where the reference group is endogenously determined by estimating θ.

2.1 Preference drifts among the poor

The main assumption made in Ravallion et al. (2009) and all the related works is that a poverty line has to be an absolute line for the poorest countries. When looking at the figures reported in the data base of Ravallion et al. (2009), there is a relation between z_j and C_j for the group of very poor countries, even if that relation is not of the same amplitude as for richer countries. For poorer countries (those with a mean C lower than \$60 a month), the poverty line represents on average 0.92 of the mean consumption while that factor drops down to 0.45 for the richer group of countries. This last figure is much more in accordance with the usual definition of a relative poverty line which corresponds usually to half of the mean income. The first figure of 0.92 can be completed by computing the average poverty line for the poorer group which is \$38 (\$12). So there is a large standard deviation which has to be explained. In this group the minimum and maximum poverty lines are \$19 and \$59. These figures confirm that the usual relative poverty line concept can be applied only to richer countries. However, there is a relation, even if it is of a different nature, between the average level of consumption and a reasonable poverty rate, which is not necessary the official one. To illustrate that point, we would like to report the controversy that took place recently in India around the decision of the Indian Government to reduce the level of the official poverty line, following the recommendation of the Tendulkar commission Tendulkar (2009). The official rate was reduced to Rs 28.65 per capita daily consumption in cities and Rs 22.42 in rural areas. The objective was to reduce the poverty rate which went down to 29.8% with these new figures (the world Bank estimates the rate of poverty in India to be 32.7% in 2010 with \$1.25 a day). The Indian press reported large protestation, which can be understood when we know that India is a very fast developing country. But certainly inequality is also very fast increasing. This is at odds with what was claimed by the commission who said that "*Fundamentally, the concept of poverty is associated with socially perceived deprivation with respect to basic human needs*". So there is a large gap between what society perceives and what the official agencies publish. This discrepancy between an official poverty line and what individuals perceive is not specific to developing countries. In Van den Bosch et al. (1993), we see that in countries of southern Europe, a subjective poverty line is also much higher than the official line. Our model should take that into account in order to define a world poverty line.

2.2 A new poverty line definition

The poverty line which is proposed both in Ravallion et al. (2009) and Greb et al. (2011) consists in computing the mean poverty line of a reference group when that reference group is given in Ravallion et al. (2009) or endogenously determined in Greb et al. (2011). The idea we would like to illustrate here is that a poverty line for the poorest countries has still to be determined as a function of the characteristics of a reference group, but that this poverty line should depend also on a reference income (or a reference consumption level) of the reference group. Consequently, the model that we shall estimate is

$$\begin{aligned} z_j &= s_j(\alpha_1 + \gamma_1 C_j) + (1 - s_j)(\alpha_2 + \gamma_2 C_j) + \epsilon_j \qquad (3) \\ s_j &= \begin{cases} 1 & \text{if } C_j < \theta \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

where θ is an unknown threshold. The new poverty line will be determined as a conditional expectation

$$\mathbf{E}(z_j|s_j = 1) = \alpha_1 + \gamma_1 \mathbf{E}(C_j|s_j = 1). \qquad (4)$$

In words, the poverty line we propose for developing countries is a function of a reference group consumption level which is taken to be equal to the mean consumption of that reference group. It is different from a usual relative poverty line in the sense that it depends not on the national mean consumption but on the mean consumption of a more general group, called the reference group. We call this new poverty line a subjective poverty line not because it depends on subjective data, but because it relates to a common group where countries are supposed to identify. They judge their poverty line by reference to that group. The notion of a reference group appeared in the happiness economic literature as a possible explanation to the Easterlin paradox. Individual satisfaction is a function not mainly of the level of income, but of the difference between their income and a reference income which is taken as the mean income of the reference group. See for instance Ferrer-i-Carbonell (2005) for an empirical investigation. We try here to translate that concept to countries and poverty lines.

The convenient way to both determine an estimate of the threshold parameter θ and to take into account the uncertainty in the determination of the reference group is to adopt a Bayesian approach, as we shall see in the next section.

3 Bayesian inference for regression models with a break

The generic model we want to estimate is a two regime model with a break determined if a variable is lower or greater than an unknown threshold:

$$\begin{aligned} \mathrm{E}(y_i|x_i) &= x_i'\beta_1 \quad \text{if } 1\!\!\mathrm{I}(w_i < \theta) && (5)\\ \mathrm{E}(y_i|x_i) &= x_i'\beta_2 \quad \text{if } 1\!\!\mathrm{I}(w_i > \theta). && (6)\end{aligned}$$

y_i is the dependent variable, x_i a set of exogenous variables and w_i is the regime shift variable which is supposed to be exogenous or predetermined. θ is a threshold parameter and $1\!\!\mathrm{I}(.)$ is the indicator function defined as

$$1\!\!\mathrm{I}(w_i < \theta) = \begin{cases} 1 & \text{if } w_i < \theta \\ 0 & \text{otherwise.}\end{cases}$$

The corresponding regression model is

$$y_i = 1\!\!\mathrm{I}(w_i < \theta)x_i'\beta_1 + (1 - 1\!\!\mathrm{I}(w_i < \theta))x_i'\beta_2) + \epsilon_i,$$

where the error term ϵ_i is supposed to be normal with zero mean and variance σ^2. For inference purposes, it is useful to define the following matrices

$$\begin{aligned} x_i'(\theta) &= [x_i' 1\!\!\mathrm{I}(w_i < \theta), x_{it}' 1\!\!\mathrm{I}(w_i > \theta)], && (7)\\ X(\theta) &= [x_i'(\theta)], && (8)\\ \beta' &= [\beta_1', \beta_2']. && (9)\end{aligned}$$

Thus we have the more compact form:

$$y = X(\theta)\beta + \epsilon.$$

where y is a vector containing the N observations of y_i.

3.1 Bayesian inference

Considering N observations, the likelihood function of the single variance model is:

$$L(\beta, \sigma^2, \theta; y) \propto \sigma^{-N} \exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}[y_i - X_i'(\theta)\beta]^2\right] \qquad (10)$$

Conditionally on θ, this is the likelihood function of a usual regression model, so that natural conjugate prior densities for β and σ^2 belong the normal inverted gamma2 family:

$$\begin{aligned}\pi(\beta|\sigma^2) &= f_N(\beta_0, \sigma^2 M_0^{-1}),\\ \pi(\sigma^2) &= f_{Ig}(\sigma^2|\nu_0, s_0) \qquad (11)\end{aligned}$$

The conditional posterior density of β and σ^2 are:

$$\pi(\beta|\theta, y) = f_t(\beta|\beta_*(\theta), s_*(\theta), M_*(\theta), \nu_*), \qquad (12)$$

$$\pi(\sigma^2|\theta, y) = f_{Ig}(\sigma^2|\nu_*, s_*(\theta)) \qquad (13)$$

where

$$\begin{aligned}M_*(\theta) &= M_0 + \sum_{i=1}^{N} X'(\theta)X(\theta),\\ \beta_*(\theta) &= M_*^{-1}(\theta)[X'(\theta)y + M_0\beta_0],\\ s_*(\theta) &= s_0 + \beta_o^{'} M_0\beta_0 + y'y - \beta_*'(\theta)M_*(\theta)\beta_*(\theta),\\ \nu_* &= \nu_0 + N\end{aligned}$$

The posterior density of θ is proportional to the inverse of the integration constant of the above Student density times the prior density of θ:

$$\pi(\theta|y) \propto |s_*(\theta)|^{-(N-k)/2}|M_*(\theta)|^{-1/2}\pi(\theta). \qquad (14)$$

As there is no conjugate prior for θ, we are free to use any form of parametric density. A convenient choice is to use a uniform prior between bounds or a non-informative prior. The marginal posterior densities of β and σ^2 have to be found using numerical integration as:

$$\pi(\beta|y) = \int f_t(\beta|\beta_*(\theta), s_*(\theta), M_*(\theta), \nu_*)\pi(\theta|y)d\theta,$$

and

$$\pi(\sigma^2|y) = \int f_{Ig}(\sigma^2|\nu_*, s_*(\theta))\pi(\theta|y)d\theta.$$

As the dimension of θ is one, we can use a traditional deterministic integration rule, like the Simpson rule in order to evaluate these densities. However, as we can also be interested in transformations of the parameters, a simulation method is better.[1] As (14) is a marginal density, we have simply to find a

[1] It is very easy to compute the posterior density of a transformation of a parameter when we have posterior draws from this parameter. We just have to take the transformation of each draw as draws from the posterior of the transformed parameter. This task would analytically very demanding.

feasible grid over which to evaluate it, compute numerically the cumulative and then use the inverse transformation method to draw a value for θ. The grid over which to evaluate (14) has to be chosen carefully. It should cover most of the probability, but it should also avoid identification problems as detailed in Bauwens et al. (1999, p. 235). To speak quickly, the grid should be chosen in such a way that there are enough observations per regime. The domain of definition of θ is given by $[\text{Min}(w_i), \text{Max}(w_i)]$. But its bounds cannot be reached, because otherwise, the model would not be identified. We draw a value of θ from $\pi(\theta|y_i)$. Using this draw, we draw a β from the conditional posterior $\pi(\beta|\theta, y)$ which is a Student density.

3.2 The two variance case

For modeling purposes, it will be useful to consider the possibility of having different variances in the two regimes. We keep the same dichotomous variable s_i as in the original model and assume this time that:

$$\text{Var}(\epsilon_i) = s_i\sigma_1^2 + (1 - s_i)\sigma_2^2 = \sigma_2^2(s_i\phi + 1 - s_i) = \sigma^2 h_i(\theta, \phi), \qquad (15)$$

as detailed in Bauwens et al. (1999, p. 236). Let us now divide the observations by $\sqrt{h_i(\theta, \phi)}$ in order to get a regression model with homoskedastic errors of variance σ^2:

$$\begin{aligned} y_i(\theta, \phi) &= y_i/\sqrt{h(\theta, \phi)}, & (16)\\ x_i'(\theta, \phi) &= [x_i'\mathbb{I}(w_i < \theta), x_{it}'\mathbb{I}(w_i > \theta)]/\sqrt{h(\theta, \phi)}, & (17)\\ X(\theta, \phi) &= [x_i'(\theta, \phi)], & (18) \end{aligned}$$

The final model is

$$y(\theta, \phi) = X(\theta, \phi)\beta + \epsilon,$$

its likelihood function being

$$\begin{aligned} L(\beta, \sigma^2, \theta, \phi; y) \propto\ & \sigma^{-N}\prod_{i=1}^{N} h_i(\theta, \phi)^{-1/2} & (19)\\ & \exp\left[-\frac{1}{2\sigma^2}(y(\theta, \phi) - X(\theta, \phi)\beta)'(y(\theta, \phi) - X(\theta, \phi)\beta)\right]. \end{aligned}$$

In this expression, $y(\theta, \phi)$ represents the N observations of $y_i(\theta, \phi)$. The conditional posterior densities of β and σ^2 are the same as before. We just have to replace y and $X(\theta)$ by $y(\theta, \phi)$ and $X(\theta, \phi)$ in the necessary expressions. The joint posterior density of θ and ϕ has the form

$$\pi(\theta, \phi|y) \propto \prod_{i=1}^{N} h_i(\theta, \phi)^{-1/2}|s_*(\theta, \phi)|^{-(N-k)/2}|M_*(\theta, \phi)|^{-1/2}\pi(\theta)\pi(\phi). \qquad (20)$$

It is slightly more difficult to draw jointly θ and ϕ from this bivariate density than to draw θ from the univariate density (14). It is always possible in theory to decompose a bivariate density into

$$\pi(\theta, \phi|y) = \pi(\phi|\theta, y) \times \pi(\theta|y),$$

so that we first draw in the marginal density $\pi(\theta|y)$ and then in the conditional $\pi(\phi|\theta, y)$. To apply this method, we have first to determine a grid over θ and ϕ in order to fill up a matrix. From this matrix of points, we can determine numerically the marginal density $\pi(\theta|y)$. For a given draw of θ, we have to find the corresponding conditional $\pi(\phi|\theta, y)$. Of course, we will not have a draw of θ that corresponds exactly to a line of the initial matrix of points. So we shall have to proceed by linear interpolation between two lines as explained in the appendix.

4 Data and estimation

The data come from Ravallion et al. (2009) who have considered 74 developing countries. The data set includes national official poverty lines (PL) (or academic poverty lines in some cases) and Private Consumption Expenditures (PCE). These data report to different years from 1988 to 2005. They have been adjusted by the household consumption PPP's collected at the occasion of the international comparison program of 2005 (World bank, 2008). The PCE and PL are reported on a monthly basis. This data set is an improvement over the old data set used in Ravallion et al. (1991) which covered only 33 countries and had a weaker price adjustment.

4.1 Revisiting the initial model

Ravallion et al. (2009) estimate (1) while Greb et al. (2011), using the same data set, estimate (2). Both models include only a constant term in the first regime. They differ because Ravallion et al. (2009) adopt a formulation in levels while Greb et al. (2011) prefer to use a formulation in logs. Using a Bayesian approach provides us the adequate tools to discuss and compare those two alternative specifications. When we estimate both formulations with an unknown threshold θ, we observe that the model in levels provides a rather unprecise estimation for θ as we have $\mathbf{E}(\theta|y) = 98.77$ (44.36) while the model in logs provides a much higher value for θ as $\mathbf{E}(\theta|y) = 138.99$ (34.78), but also a more precise standard deviation (as given between brackets). As a consequence, the poverty line is better estimated with the model in logs. This is again well apparent if we examine the posterior density of θ in both models. From Figure 1, we see that in order to deliver a reasonable message

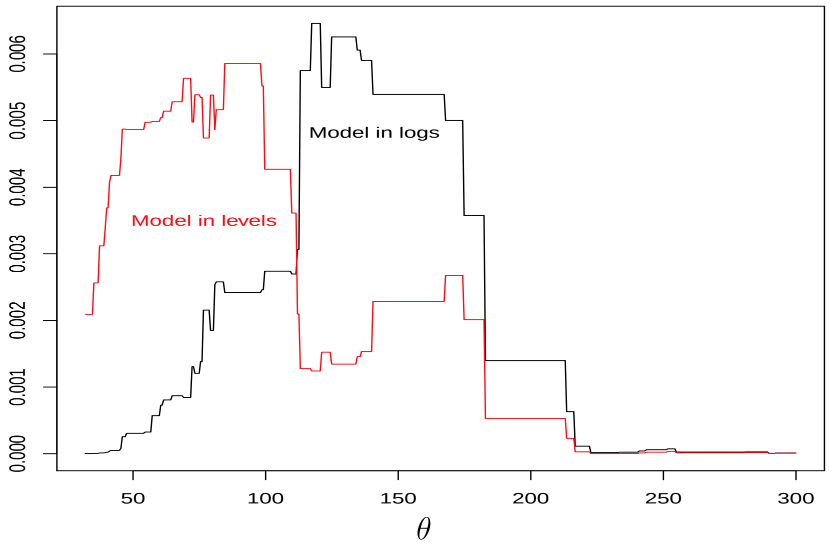

Figure 1: Why a model in logs is better

the model in levels has to be equipped with a strong prior on θ in order to limit its range to the first mode of the posterior density of θ, say [32,120]. The posterior corresponding to the model is logs is uni-modal, delivering thus a single message and does not need a correcting prior.

We report in Table 1 estimation results for the model using the log of consumption:

$$z_j = s_j\alpha_1 + (1 - s_j)(\alpha_2 + \gamma_2 \log(C_j)) + \epsilon_j, \tag{21}$$

using a non-informative prior and 5 000 draws. An estimate for a World Poverty Line is obtained by re-scaling the posterior density of α_1, considering $\alpha_1/365 * 12$. A graph is given in Figure 2. A 90% confidence interval is [1.05, 1.79]. This result is the similar to that reported in Greb et al. (2011) who found a 90% confidence interval of $[1.10, 1.72]$. Of course the Bayesian posterior interval is slightly larger, just because and as underlined in Hansen (2000), the distribution of the estimated threshold θ is not standard (see Figures 1 and 2) and thus using an asymptotic approximation as in Greb et al. (2011) is not the right method to report the empirical uncertainty. A Bayesian approach provides the small sample distribution of θ and thus allows to take into account uncertainty in the determination of an empirical

Table 1: Bayesian inference for initial model

	Estimate	std.error
α_1	43.44	7.02
α_2	-467.95	109.44
γ_2	104.45	10.56
θ	138.99	34.78
σ^2	1425.70	246.09
n	74	
Estimated IPL	1.43	0.23

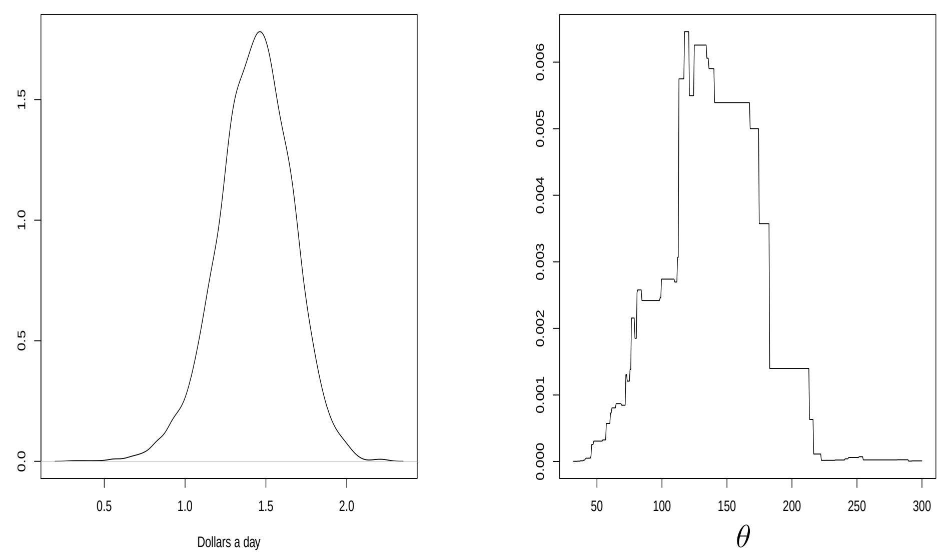

Figure 2: Rescaled posterior density of α_1 and posterior density of θ

poverty line in a rational way. Whatever the estimation method, (21) leads to the determination of a much larger reference group than that obtained in Ravallion et al. (2009). We have on average 38 countries when there was only 15 in Ravallion et al. (2009) where θ is fixed and equal to \$60. This leads to a slightly larger value for the poverty line.

A graph of the predictive density, as reported in Figure 3, suggests that the variance of the error term is not the same in the two regimes. A model with two variances is even more coherent with the theoretical model of Ravallion et al. (2009) as in the first regime, the poverty line is supposed to be constant and the level of consumption rather low. We thus consider the

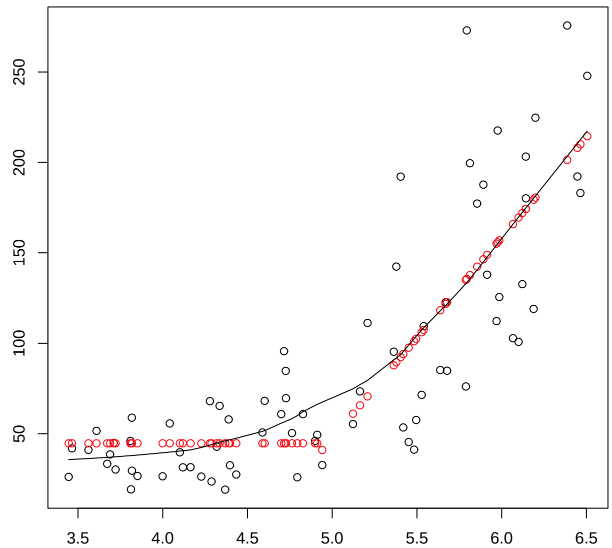

Figure 3: Posterior fit of the initial model in logs

alternative model:

$$z_j = s_j\alpha_1 + (1 - s_j)(\alpha_2 + \gamma_2 \log(C_j)) + s_j\epsilon_{1j} + (1 - s_j)\epsilon_{2j}. \qquad (22)$$

Table 2 validates the existence of two variances as the ratio between the variances of the two regimes is much lower than 1. α_1 is estimated in a much

Table 2: Bayesian inference the two variance case

	Estimate	std.error
α_1	43.16	3.93
α_2	-460.00	135.92
γ_2	103.16	9.99
θ	144.48	32.37
σ_1^2	312.06	100.4
σ_2^2	2518.2	647.4
ϕ	0.13	0.050
n	74	
Estimated IPL	1.42	0.13

more precise way with a standard deviation that goes down from 7.02 to 3.93, leading to a narrower 90% confidence interval of [1.20, 1.62] for the poverty line. However, the posterior density of θ becomes bimodal which leads us to look for a better model.

4.2 Preference drift

In the approach of both Ravallion et al. (2009) and Greb et al. (2011), the assumption is that for low income countries, the poverty line should have an absolute definition, which means that it is independent of income or consumption. When we look at Figure 3, we see that this assumption is not fully coherent with the data. In the first regime, the official poverty line seems to depend on the level of consumption, however with a much lower slope than in the second regime. We shall now estimate our preferred model (3) but including two variances so as to obtain:

$$z_j = s_j[\alpha_1 + \gamma_1 \log(C_j)] + (1 - s_j)\,[\alpha_2 + \gamma_2 \log(C_j)] + s_j\epsilon_{1j} + (1 - s_j)\epsilon_{2j}.$$

In Table 3, we have reported two versions of this model. Apparently we

Table 3: Model with preference drift and two variances

	Estimate	std.error	Estimate	std.error
α_1	-26.62	26.87	-	-
γ_1	16.92	2.40	10.63	0.67
α_2	-497.8	151.9	-496.2	142.9
γ_2	110.2	3.85	109.1	3.81
θ	172.0	19.29	167.2	18.24
σ_1^2	284.6	69.93	297.92	74.59
σ_2^2	2780.4	737.0	2668.0	692.8
ϕ	0.11	0.038	0.12	0.041

must have a parsimonious parametrisation in the first regime and suppress the constant term if we include the log consumption. So the final model is that corresponding to the second panel of this table. Compared to the initial two variance model (22), we have a slightly larger variance in the second regime, but the first regime which is of direct interest to us is by far more precisely determined. $\log(pce)$ does have an influence in determining the official national poverty line, but its impact is ten times lower than what it is in the second regime.

With equation (4), we have proposed a new type of absolute poverty line. It can be qualified as a subjective poverty line because it is computed as a function of the mean consumption of a group of reference, formed by

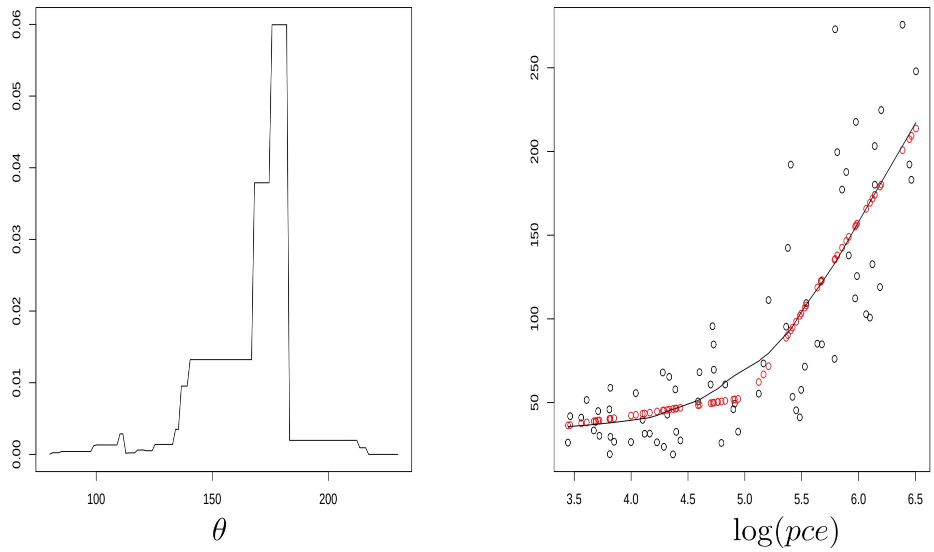

Figure 4: Posterior density of θ and posterior fit of the last model

the countries that look like the country under investigation. We have computed the posterior mean that a country belongs to this reference group. We have 26 countries for which this probability is equal to 1: Bangladesh, Benin, Burkina-Faso, Cambodia, Chad, Congo-Rep, Ethiopia, Gambia, Ghana, Guinea-Bissau, Malawi, Mali, Mongolia, Mozambique, Nepal, Niger, Nigeria, Rwanda, Senegal, Sierra-Leone, Tajikistan, Tanzania, Uganda, Vietnam, Yemen, Zambia. In this first group the maximum consumption is \$81 for Vietnam (to be compared to the \$60 of Ravallion et al., 2009). With a 95% bound, we get a group of 38 countries where those 12 countries are added: Cameroon, China, Cote d'Ivoire, Djibouti, India, Kenya, Kyrgyz, Lesotho, Mauritania, Moldova, Pakistan and the Philippines.

4.3 How to simulate the posterior density of IPL

We have to find a posterior density for IPL, based on the first regime characteristics. It is obtained as a transformation of the parameters. We define IPL as being

$$IPL = \gamma_1 \mathbf{E}(\log(C_j)|C_j < \theta),$$

in accordance with (4). In order to simulate IPL, we must have draws of γ_1 and θ. For each draw of θ, we determine the corresponding reference group and compute a value for the sample mean of $\log(C_j)$. The algorithm is as follows. We have kept draws from γ_1 and θ.

1. Start a loop in j
2. Given θ_j, determine a sample separation and n_j the sample size in the first regime
3. Compute $IPL_j = \gamma_j^1 \sum_{i=1}^n \log(C_i)\mathbb{I}(C_i < \theta_j)/n_j$
4. End loop

We get an IPL of 1.48 dollars a day with a standard deviation of 0.096. A 90% confidence interval is [1.32-1.64]. We have given in the previous subsection a list of 26 countries which had a probability equal to 1 to be included in the group of reference. We can compute the posterior density of the IPL when the reference group is limited to this group. There is no longer any uncertainty in the composition of this group. So the mean poverty line is of IPLs 1.39 dollars a day (0.086). We give in Figure 5 a graph of the posterior density of

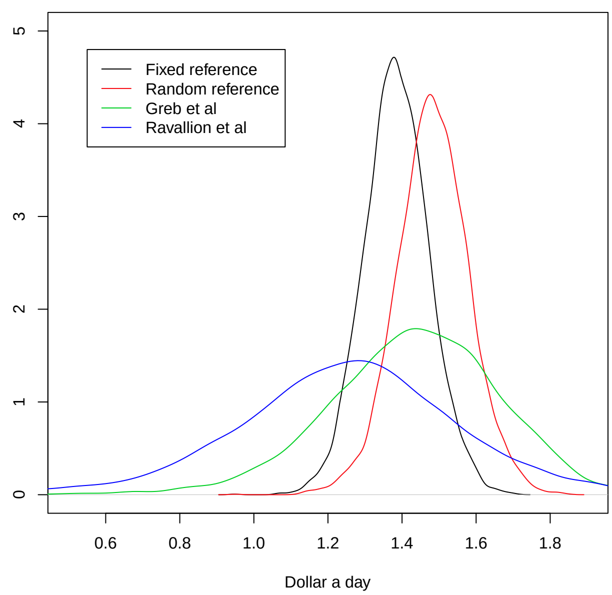

Figure 5: Posterior density of four poverty lines

these two possible poverty lines. We have also added the posterior density of the poverty line using the first model in logs as defined in Greb et al. (2011) as well as the poverty line corresponding to the approach of Ravallion et al.

(2009). For this last option, we had to specify a prior information θ which was compatible with range of θ representative of the approach of Ravallion et al. (2009). From Figure 1, we have used $\theta \in [32, 120]$, thus eliminating the secondary mode. We get a mean poverty line of \$1.26 (0.33), compared to the \$1.25 found in Ravallion et al. (2009).[2]

The four poverty lines presented in Figure 5 illustrate four different possible approaches.

1. The first poverty line of the plot corresponds to a fixed reference group. That group is used to compute a reference consumption level, common to that group of countries (however determined by the model). The common poverty line is defined as proportion of this reference consumption level. Uncertainty comes from that proportion γ_1. It corresponds to a sample -based prior for θ.

2. The second poverty line is slightly higher as it corresponds to a larger reference group, but it takes into account the whole uncertainty of the model. This is our proposed poverty line.

3. The last two poverty lines obey a different philosophy. They measure simply the mean of different national poverty lines, using a reference group which is determined by the model. In one case, we use a model formulated in logs, which was shown to correspond to a better specification. In the other case, we use a model specified in levels which corresponds to the initial model of Ravallion et al. (2009).

With a poverty line of \$1.25, following the data published in Chen and Ravallion (2008), we have 1.4 billion poor people in the developing World. With a poverty line of \$1.48 as we found, this figure goes up to more than 1.7 billion and the headcount index passes from 25.7 to 31.5.[3]

4.4 Gauging official national poverty lines

In Figure 6, we have listed a group of countries, ordered by increasing level of private consumption with the latter being represented by a continuous line. The top group of horizontal lines indicates a posterior lower probability bound for θ which determines the corresponding sample separation,

[2]If we had restricted the prior range of θ to [32-60] so as to follow more closely the options of that paper, we would have obtained a poverty line of \$1.23 with a larger standard deviation (0.44) and a reference group of 13 countries instead of 22.

[3]In fact the figures given in Chen and Ravallion (2008) are for a range of poverty lines which are \$1.00, \$1.25,\$1.45, \$2.00 and \$2.50.

and consequently the reference group used for computing an international poverty line. The vertical lines indicate the national official poverty line of each country while the bottom group of horizontal lines correspond to a posterior confidence interval of our new international poverty line. We have skipped the upper part of the sample for clarity. From Figure 6, we see that

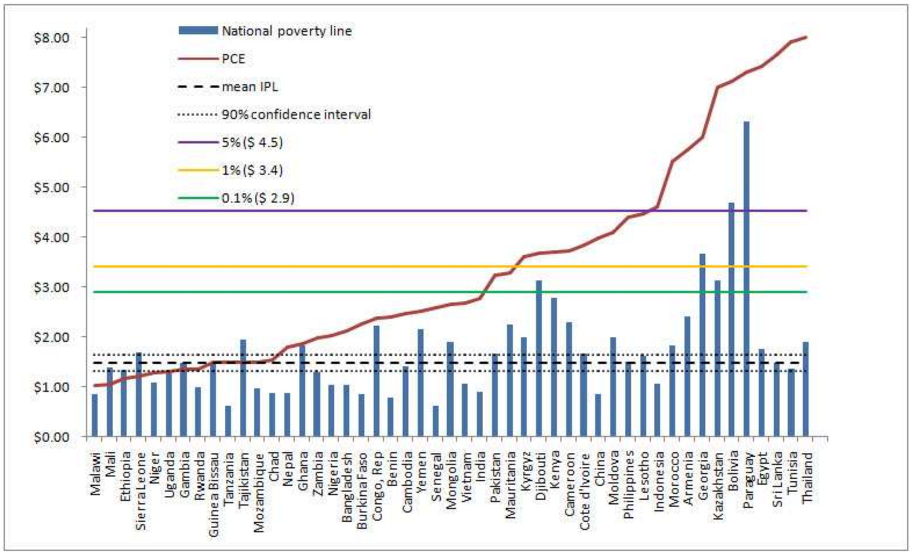

Figure 6: Official poverty lines

in the group of very poor countries, there are some countries which report an official poverty line which is both greater than our proposed poverty line and greater than their own PCE. It is difficult to find a rationale behind this, except suspecting the way these data are elaborated, in particular concerning the 2005 PPP.

There are emerging countries (members of the BRICS group), India and China which report an official poverty line which is well below either the new poverty line of Ravallion et al. (2009) and of course of our proposed poverty line. There is also one country, Indonesia which belongs to the edge of the reference group (countries with a probability between 0.90 and 0.95 of belong to that group) that is not so poor and which report a national poverty line lower than ours. Finally, there is Tunisia which is not in the reference group and which report a very low poverty line. We give the list in Table 4. It is not surprising to find India in this list, if we remember the reference we made to the Tendulkar (2009) report. And the official poverty of China is even lower than that of India, despite the fact that it has a higher PCE than India. Using the figures provided in Chen and Ravallion (2008), we can compute by difference the evolution of the number of poor for various levels of the

Table 4: Richer countries with a low OPL in 2005

country	Prob	OPL	C	Gini
China	0.9738	0.85	120.78	44.9
India	0.9992	0.90	84.24	36.8
Indonesia	0.9316	1.07	139.96	37.6
Tunisia	0.0000	1.35	240.63	39.8

poverty line in China. This will provide an explanation for the behaviour of those countries.

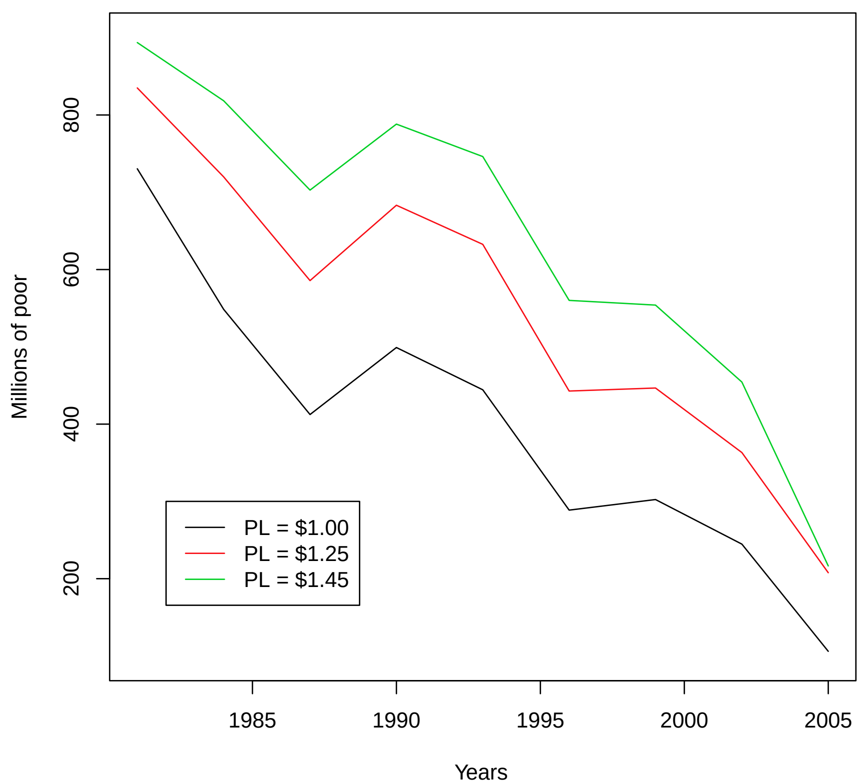

Figure 7: Number of poor in China

With Figure 7, we see all the interest there is for an official agency to monitor the official poverty line. Poverty in China had dropped a lot between 1981 and 2005, but this drop was not regular. Adopting a poverty line of $1.25 or $1.45 does make a difference. At the beginning of the period the decrease in the number of poor is much stronger with the \$1.00 poverty line than with the two other options. The increase in 1987 is steeper with the \$1.25 line. At the end of the period the number of poor is roughly the same with the \$1.25 and \$1.45 lines, but of course it remains much larger than with the \$1.00 line.

5 Conclusion and comments

Defining an international poverty line is an important objective, because it leads to measure the number of poor countries and of poor people in the world. Knowing these numbers and the localisation of the poor, it is easier to devise an economic policy and to evaluate the results of these policies later on.

We have seen in this paper that it is not an easy task to devise a poverty level. The one dollar a day line had to be reformed and Ravallion et al. (2009) was a major attempt to do this. Their newly proposed poverty line is the lowest of the different poverty lines we have reviewed. But we have shown that a large uncertainty is attached to them. In fact, the posterior density of our poverty line covers all the other point alternatives. They all rely a different definition of a reference group.

The final poverty line we obtain (1.48 dollar a day) is larger than the 1.25 of Ravallion et al. (2009). But it is well in a reasonable confidence interval. Due to the way it is computed, it compels to the logic of a subjective poverty line.

The final point we would like to make concerns the Bayesian approach. We have used quite standard tools, even if they could seem complex for a reader not familiar with the field. With these tools we have visualised the origin of some questions concerning the model to be used and its specification. The posterior density of the break point θ was particularly useful in this respect. And finally, we could compare various assumptions concerning the determination of an international poverty line.

References

Atkinson, A. B. (1983). *The Economics of Inequality.* Clarendon Press, Oxford.

Bauwens, L., Lubrano, M., and Richard, J.-F. (1999). *Bayesian Inference in Dynamic Econometric Models.* Oxford University Press.

Chen, S. and Ravallion, M. (2008). The developing world is poorer than we thought, but no less successful in the fight against poverty. Policy Research Working Paper 4703, World Bank, Washington, DC.

Ferrer-i-Carbonell, A. (2005). Income and well-being: An empirical analysis of the comparison income effect. *Journal of Public Economics*, 89(5-6):997–1019.

Greb, F., Klasen, S., Pasaribu, S., and Wiesenfarth, M. (2011). Dollar a day re-revisited. Discussion Paper 91, Georg-August-Universitat, Gottingen.

Hansen, B. (2000). Sample splitting and threshold estimation. *Econometrica*, 68(3):575–603.

Kapteyn, A., Kooreman, P., and Willemse, R. (1988). Some methodological issues in the implementation of subjective poverty definitions. *Journal of Human Resources*, 23:222–242.

Ravallion, M. and Chen, S. (2001). How did the world's poorest fared in the 1990s. *Review of Income and Wealth*, 47:283–300.

Ravallion, M. and Chen, S. (2004). How did the world's poorest fared since the early 1980s. *World Bank Reserch Observer*, 19(2):141–170.

Ravallion, M., Chen, S., and Sangraula, P. (2008). Dollar a day revisited. Working Paper S4620, The World Bank Development Research Group Director's office.

Ravallion, M., Chen, S., and Sangraula, P. (2009). Dollar a day revisited. *The World Bank Economic Review*, 23(2):163–184.

Ravallion, M., Datt, G., and van de Walle, D. (1991). Quantifying absolute poverty in the developing world. *Review of Income and Wealth*, 33:667–702.

Tendulkar, S. D. (2009). Report of the expert group to review the methodology for estimation of poverty. Technical report, Government of India Planning Commission.

Van den Bosch, K., Callan, T., Estivill, J., Hausman, P., Jeandidier, B., Muffeis, R., and Yfantopoulos, J. (1993). A comparison of poverty in seven European countries and regions using subjective and relative measures. *Journal of Population Economics*, 6:235–259.

van Praag, B. (1971). The welfare function of income in Belgium: An empirical investigation. *European Economic Review*, 1:337–369.

World bank (2008). Global purchasing power parities and real and real expenditure 2005. Technical report, World Bank, Washington DC.

Appendix
Simulating from a bivariate density

Let us consider a bivariate posterior density:

$$\pi(\phi, \theta|y) = \pi(\phi|\theta, y) \times \pi(\theta|y)$$

We know the analytical form of the joint density $\pi(\phi, \theta|y)$, but neither its marginal $\pi(\theta|y)$ nor its conditional $\pi(\phi|\theta, y)$. We want to draw random numbers for the joint posterior density. To do so, we are first going to evaluate this bivariate density on a grid, filling a matrix F where the rows will corresponds to θ and the columns to ϕ. From this matrix of points, we can determine numerically the marginal density $\pi(\theta|y)$ by summing over the columns. Using this marginal density and using the inverse transformation method, we can draw a value for θ. For a given draw of θ, we have to find the corresponding conditional density $\pi(\phi|\theta, y)$ as a row of matrix F. Of course, the draw will not correspond exactly to one of the predetermined point of the grid in θ. So we shall have to proceed by linear interpolation between two lines.

1. Compte numerically the cumulative and then use the inverse transformation method to draw θ_j from $\pi(\theta|y)$
2. Find the two nearest points of θ_j on the grid of θ, denoted as θ_{j-} and θ_{j+}
3. Calculate the differences: $a = \theta_j - \theta_{j-}$, $b = \theta_{j+} - \theta_j$ and $c = |\theta_{j+} - \theta_{j-}|$
4. Obtain the conditional posterior densities $\pi(\phi|\theta_{j-}, y)$ and $\pi(\phi|\theta_{j+}, y)$ from the joint posterior matrix (20)
5. Compare each point of the two above conditional posterior densities in order to get $\pi(\phi|\theta^i, y)$ by line interpolation:

$$\sum_{k=1}^{k} \pi(\phi_k|\theta, y) = 1$$

$$\pi(\phi|\theta_j, y) = \begin{cases} \pi(\phi_k|\theta_{j-}, y) + a \times (\pi(\phi_k|\theta_{j+}, y) - \pi(\phi_k|\theta_{j+}, y))/c & \text{if } \pi(\phi_k|\theta_{j+}, y) \geq \pi(\phi_k|\theta_{j-}, y) \\ \pi(\phi_k|\theta_{j+}, y) + b \times (\pi(\phi_k|\theta_{j-}, y) - \pi(\phi_k|\theta_{j+}, y))/c & \text{otherwise} \end{cases}$$

knowing that ϕ_k is the kth point on the grid of ϕ

6. Compte numerically the cumulative and then use the inverse transformation method to draw ϕ_j from $\pi(\phi|\theta_j, y)$
7. Record the ith joint draw : θ_j and ϕ_j