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A note on Leibniz's argument against infinite wholes

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1 Introduction

Leibniz had a well-known argument against the existence of infinite wholes that is based on the part-whole axiom: the whole is greater than the part.¹ The refutation of this argument by Russell and others is equally well known. In this note, I argue (against positions recently defended by Arthur, Breger and Brown) for the following three claims:

1. Leibniz himself had all the means to devise and accept this refutation.²

1. The part-whole axiom is also referred to as the 'Aristotelian principle' (e.g., Benci, Di Nasso, and Forti 2006) or 'Euclid's Axiom'. The former label is justified as it follows from what Aristotle says at *Metaphysics* 1021a4: 'That which exceeds, in relation to that which is exceeded, is "so much" plus something more' (Aristotle 1933, 263); the latter, to the extent that it figures as Common Notion 5 in Book I of Euclid's *Elements* from (at the latest) Proclus on (Euclid 1956, 232). See also Leibniz's 'Demonstratio Axiomatum Euclidis' (1679), Leibniz, 1923–, 6,4:167.

2. This contributes to showing that there is no intrinsic obstacle in Leibniz's philosophy to combining it with Cantorian set theory, as Kurt Gödel wished to do. For further details on this aspect of Gödel's thought, which provided the motivation for writing the present note, see van Atten 2009. Neither Gödel's published papers, nor, as far as I can tell from the currently existing partial transcriptions from Gabelsberger shorthand, his notebooks contain a direct comment on Leibniz's argument. However, in his paper on Russell from 1944, he wrote:

Nor is it self-contradictory that a proper part should be identical (not merely equal) to the whole, as is seen in the case of structures in the abstract sense. The structure of the series of integers, e.g., contains itself as a proper part. (Gödel 1944, 139)

Among other things, Gödel says here that it is consistent that an equality relation holds between a proper part and the whole. This entails a rejection of Leibniz's argument.

2. This refutation does not presuppose the consistency of Cantorian set theory.
3. This refutation does not cast doubt on the part-whole axiom.

A note on sources: although Leibniz's texts used below range from 1672 to after 1714, they all express the same view on the issue at hand. Of each the year will be given where possible. Emphasis in quotations from Leibniz is his. Translations without a reference are mine.

2 Leibniz's argument and its refutation

The following presentation of Leibniz's argument and its refutation is meant to establish claim (1) and to set the stage for the defence of claims (2) and (3).

Leibniz denied the existence of infinite wholes of any kind.³ For example, while he acknowledges that there are infinitely many numbers

For it cannot be denied that the natures of all possible numbers are really given, at least in God's understanding, and that as a consequence the multitude of numbers is infinite.⁴

he denies that this multitude forms a whole:

I concede [the existence of] an infinite multitude, but this multitude forms neither a number nor one whole. It only means that there are more terms than can be designated by a number; just as there is for instance a multitude or complex of all numbers; but this multitude

3. Friedman 1975, 338 suggests that even so, Leibniz might have been willing to accept the for him inconsistent concept of an infinite whole as a fiction that may prove useful in calculations, on a par with his acceptance of imaginary roots in algebra. To illustrate this point, Friedman refers to Leibniz (1705) 1882, 145. See also Leibniz's letter to Des Bosses of 1 September 1706: 'properly speaking, an infinity consisting of parts is neither one nor a whole, and can only be conceived of as a quantity by a mental fiction.' ('propre loquendo, infinitum ex partibus constans neque unum esse neque totum, nec nisi per fictionem mentis concipi ut quantitatem', Leibniz 1875, 2:314)

4. 'Neque enim negari potest, omnium numerorum possibilitatem naturas revera dari, saltem in divina mente, adeoque numerorum multitudinem esse infinitam.' (Leibniz 1875, Leibniz to Des Bosses, 11/17 March 1706; 2:305)

is neither a number nor one whole. (Leibniz 1849, Leibniz to Joh. Bernoulli, 21 February 1699; 3:575)^{5,6}

The point of departure for Leibniz's argument that an infinity cannot be a whole is the part-whole axiom, which he justifies by the following definitions and argument:⁷

If a part of one thing is equal to the whole of another, the former is called greater, the latter less. Hence the whole is greater than a part. For let the whole be A, the part B. Then A is greater than B, because a part of A (namely, B) is equal to the whole of B. This can be expressed in a syllogism whose major proposition is a definition, its minor an identity:

Whatever is equal to a part of A is less than A, by definition.
But B is equal to a part of A (namely, to B), by hypothesis.
Therefore B is less than A.

(Leibniz 1969, 668 (after 1714))^{8,9}

5. 'Concedo multitudinem infinitam, sed haec multitudo non facit numerum seu unum totum; nec aliud significat, quam plures esse terminos, quam numero designari possint, prorsus quemqmodum datur multitudino seu complexus omnium numerorum; sed haec multitudo non est numerus, nec unum totum.'

6. Also: 'PH. ... Nothing is clearer than the absurdity of an actual idea of an infinite number. TH. I agree. But this is not because one couldn't have the idea of the infinite, but because an infinity cannot be a true whole' ('PH. ... rien n'est plus sensible que l'absurdité d'une idée actuelle d'un nombre infini. TH. Je suis du même avis. Mais ce n'est pas parcequ'on ne sauroit avoir l'idée de l'infini, mais parcequ'un infini ne sauroit estre un vrai tout', Leibniz (1705) 1882, 146)

7. Although an axiom is a proposition that is evident, Leibniz sees two uses for a demonstration, that is, a reduction to $A=A$: it contributes to the unification of the sciences and to the analysis of ideas. See Couturat 1901, 200ff.

8. 'Si pars unius sit aequalis alteri toti, illud vocatur Minus, hoc Majus. Itaque Totum est majus parte. Sit totum A, pars B, dico A esse majus quam B, quia pars ipsius A (nempe B) aequatur toti B. Res etiam Syllogismo exponi potest, cujus Major propositio est definitio, Minor propositio est identica:

Quicquid ipsius Q parti aequale est, id ipso A minus est, ex definitione, B est aequale parti ipsius A, nempe sibi, ex hypothesi, ergo B est minus ipso A.'

(Leibniz 1849, 7:20)

9. See also Leibniz 1903, 518/Leibniz 1969, 267 (around 1686); Leibniz, 1923–, 6/4:167 (1679); Leibniz 1875, 7:300; Leibniz 1849, 7:274 (1695, according to de Risi 2007, 82); Leib-

The major proposition shows that for Leibniz, ‘part’ means ‘proper part’, i.e., a part which is not equal to the whole. His argument from this axiom against infinite wholes runs as follows:¹⁰

There is no maximum in things, or what is the same thing, the infinite number of all unities is not one whole, but is comparable to nothing. For if the infinite number of all unities, or what is the same thing, the infinite number of all numbers, is a whole, it will follow that one of its parts is equal to it; which is absurd. I will show the force of this consequence as follows. The number of all square numbers is a part of the number of all numbers: but any number is the root of some square number, for if it is multiplied into itself, it makes a square number. But the same number cannot be the root of different squares, nor can the same square have different roots. Therefore there are as many numbers as there are square numbers, that is, the number of numbers is equal to the number of squares, the whole to the part, which is absurd. (Leibniz 2001, 13)¹¹

The *reductio* argument that Leibniz presents here can be reconstructed as follows:

1. The infinite multitude of the numbers forms a whole. (Assumption)
2. Every square is a number, but not vice versa. (Premise)
3. The multitude of the squares is equal to a part of the whole of the numbers. (1, 2)

niz 1849, 3:322 (1696).

10. Leibniz presented the same argument on various occasions: see Leibniz, 1923–, 2,1:226 and 228 (1672); Leibniz, 1923–, 4,3:403 (1675); Leibniz, 1923–, 6,3:463 (1675); Leibniz, 1923–, 6,3:168 (1676); Leibniz, 1923–, 6,3:550–3 (1676); his draft letter to Malebranche of 22 June 1679, Leibniz 1875, 1:338. See also the reference to it in a letter to Johann Bernoulli of 1698, Leibniz 1849, 3:535.

11. ‘Nullum datur Maximum in rebus, vel quod idem est Numerus infinitus omnium unitatum non est unum totum, sed nihilo aequiparatur. Nam si numerus infinitus omnium unitatum, seu quod idem est, Numerus infinitus omnium numerorum, sequetur aliquam eius partem esse ipsi aequalem. Quod est absurdum. Consequentiae vim ita ostendo. Numerus omnium Numerorum Quadratorum est pars Numeri omnium Numerorum: at quilibet Numerus est radix alicuius Numeri quadrati, nam si in se ducatur, fiet aliquis numerus quadratus; nec idem numerus potest esse radix diversorum quadratorum, nec idem quadratus diversarum radicum, tot ergo sunt Numeri, quot Numeri quadrati, seu Numerus Quadratorum aequalis est Numeri Numerorum, totum parti, quod est absurdum.’ (Leibniz, 1923–, 6,3:98 (1672–3))

4. There exists a bijection between the multitude of the numbers and the multitude of the squares. (Premise)
5. The multitude of the squares is equal to the whole of the numbers. (1, 4)
6. A part of the whole of the numbers is equal to the whole of the numbers. (3, 5)
7. The whole is greater than its parts. (Premise)
8. Contradiction. (6, 7)
9. Therefore, the infinite number of all numbers do not form a whole. (1, 8)

Leibniz holds the propositions in lines 2, 4 and 7 to be true; the source of the contradiction that arises in line 8 for him is the assumption made in line 1.

Russell and others have observed that Leibniz's argument is not correct because it rests on an equivocation on the concept of equality.¹² Clearly, in line 3 'is equal to' means 'is identical to', while in line 5 it means 'can be put in a bijection with'. While in their application to finite multitudes the concepts of equality given through these senses are equivalent, this is not so in the infinite case, for if we substitute the sense of 'is equal to' in line 3 for that of 'is equal to' in line 5, we obtain the falsehood that the multitude of the squares is identical to that of the numbers. Leibniz (who, at times, used the word 'term' instead of 'concept'¹³) defined

Same or coincident terms are those which can be substituted for each other anywhere without affecting truth ... Diverse terms are those which are not the same or in which substitution sometimes does not work. (Leibniz 1969, 371 (early 1690s))¹⁴

12. Russell 1919, 80–81. See also Benardete 1964, 47–48, and Levey 1998, 61–62.

13. 'By a term I do not understand the word but the concept or that which the word signifies, you could also say the notion or the idea.' ('Per Terminum non intellego nomen sed conceptum seu id quod nomine significatur, possis et dicere notionem, ideam', Leibniz 1903, 243 (c. 1680))

14. 'Eadem seu coincidentia sunt quorum alterutrum ubilibet potest substitui alteri salva veritate ... Diversa sunt quae non sunt eadem, seu in quibus substitutio aliquando non procedit.' (Leibniz 1875, 7:236)

Therefore, by Leibniz's own criterion, the concepts of equality in lines 3 and 5 are diverse or different, as substitution of the one for the other does not preserve truth here; but then, from lines 3 and 5, one cannot infer to line 6, for that inference presupposes that the two concepts are the same. Hence, Leibniz was in a position to see that the inference was incorrect.

Earlier in the text in which he states the proof of the part-whole axiom, Leibniz defines 'Equals are things having the same quantity' (Leibniz 1969, 667 (after 1714))^{15,16} About the concept of quantity, Leibniz there only remarks that it is essentially comparative: 'Quantity or magnitude is that in things which can be known only through their simultaneous compresence – or by their simultaneous perception' (Leibniz 1969, 667).^{17,18} However, that leaves unaddressed the fact that there are essentially different ways of comparing, resulting in correspondingly different concepts of equality.¹⁹

Breger has recently attempted to meet this objection to Leibniz's argument by suggesting that Leibniz was working strictly within a theory of finite multitudes: 'The fact that one finds objects outside the theory examined here for which both notions [of equality] are not equivalent is of no importance within the theory' (Breger 2008, 314). However, if that was what Leibniz was doing, then he could not have devised his argument against infinite wholes in the first place, for he then would have had no theory to apply

15. 'Aequalia sunt ejusdem quantitatis.' (Leibniz 1849, 7:19)

16. See also Leibniz, 1923–, 6,4:165 (1679) and Leibniz, 1923–, 6,4:406 (1680(?)).

17. 'Quantitas seu Magnitudo est, quod in rebus sola compraesentia (seu perceptione simultanea) cognosci potest.' (Leibniz 1849, 7:18 (after 1714))

18. See also Leibniz, 1923–, 6,4:168 (1679).

19. The example that Leibniz goes on to give in the same text, that of measuring either in inches or in feet, is not a case of two essentially different ways of comparing, because, as Leibniz himself points out (for a different purpose), they are interdefinable. Cantor remarks, after having shown that extending an infinite line (with one endpoint) by a finite line does not lead to a new line with more points, because there will be a 1–1 mapping:

Who here, and in the case of any actually infinite quantity, sees a violation of the principle of contradiction, is quite mistaken, by losing sight of the abstractive character of 'magnitude' and wrongly identifying it with the substantial entity of the quantity at hand.²⁰

Evidently, which abstractions are legitimate in defining a specific concept of magnitude depends on the context: there are perfectly good uses for the concept of magnitude that Cantor in this context rightly rejects. See also my concluding remark.

20. 'Wer hier wie überhaupt bei aktual-unendlichen Quantitäten einen Verstoß gegen das Widerspruchsprinzip findet, irrt durchaus, indem er den abstraktiven Charakter der "Größe" aus dem Auge verliert und sie fälschlich mit der substantiellen Entität des vorliegenden Quantum identifiziert.' (Cantor [1887] 1932, 393)

to the assumption by which he begins his argument. Leibniz is, on the contrary, working in a theory of parts, wholes and finite as well as infinite multitudes, and within that theory attempts to show that infinite multitudes cannot be wholes. Breger, moreover, holds that the refutation depends on a perspective that was developed only in the second half of the nineteenth century and that for Leibniz ‘it would have been absurd, absolutely unthinkable, to reject the equivalence [of the different notions of equality]’ (Breger 2008, 315). However, as the above reconstruction emphasises, the refutation uses no concept or technique that was not available to Leibniz.

3 The consistency of Cantorian set theory

Arthur, Breger and Brown hold that the refutation of Leibniz’s argument depends on whether Cantor’s theory of infinite sets is, in fact, consistent:

[The] argument (like those of Cantor, Russell, and Rescher before it) reduces to this: if with Cantor one assumes ... [the proposition C] that an infinite collection (such as the set of all numbers) is a whole or unity, then one can establish a consistent theory of infinite number; therefore Leibniz’s argument against it is unsound ... To say that Leibniz’s argument is unsound on the basis of the success of Cantor’s theory is to assume the truth [and hence the consistency] of C, and thus to beg the question (unless one has an independent argument for C, which Cantor does not) (Arthur 2001, 105).²¹

I suppose that one might argue that, for all we know and he knew, Leibniz’s argument against infinite number and wholes might be sound; for despite the fact that most mathematicians now seem to assume that Cantorian set theory is consistent in light of its long record of success and the absence of any proof of its inconsistency, it remains true that neither does there exist a general consistency proof for that theory. It remains at least possible, I suppose, that Leibniz’s argument against infinite number and wholes is sound, and that there really is

21. Leibniz (2001, 407n41) adds to this that ‘the paradoxes of the infinite still [beset] set theory’. But in the iterative concept of set, which has become the standard understanding of Cantorian set theory, no paradox has yet been found. See Gödel 1947, 518–519 and Wang 1974, 181–193.

some inconsistency lying dormant and undiscovered in the assumption that the part-whole axiom fails in the case of the infinite-and ultimately some inconsistency lying dormant and undiscovered in the Cantorian definitions of 'less than', 'greater than', and 'equal to' for infinite sets-so that infinite wholes are indeed impossible. (Brown 2005, 486)

It is striking that one already needs the existence (free of contradiction) of an infinite totality, which is precisely what is supposed to be proved or refuted, to show the non-equivalence of the two notions of 'having the same number' ... The two expressions are equivalent if and only if they are applied to finite multitudes. In other words: one can demonstrate the non-equivalence of the two notions if and only if one assumes the existence of infinite multitudes that as objects free of contradiction constitute a whole and are thus sets (as happens in the Zermelo-Fraenkel theory) or if one has already demonstrated this in some other way. As long as this has not happened, the objection that Leibniz is using two non-equivalent notions is false. (Breger 2008, 313–314)

However, it is Leibniz who makes the assumption at the beginning of his argument that an infinite whole exists. Of course, for him that is only an assumption towards a *reductio ad absurdum*, where the absurdity will arise when the part-whole axiom is brought in. The equivocation on the concept of equality occurs already before that stage of the argument is reached. When Leibniz makes this equivocation, the assumption has not yet been cancelled. Therefore, the need to make that assumption in order to be able to distinguish the two concepts of equality in play is no objection to the refutation of Leibniz's argument. Moreover, as the assumption is part of the argument itself, and not a presupposition of its refutation, the latter does not depend on whether Cantorian set theory in fact is consistent. (This also means that this refutation by itself does not show that there can be no correct arguments against infinite wholes; only that Leibniz's argument is not one.)

4 The part-whole axiom

It is often held that the distinction between different concepts of equality on which the refutation of Leibniz's argument depends also serves to show that the existence of infinite wholes and the part-whole axiom are incompatible.²² Here, too, means available to Leibniz can be indicated that enable one to see that, logically speaking, there is no such incompatibility. This turns on the fact that Leibniz defines the concept of proper part in terms of equality: a proper part is a part that is not equal to the whole. The idea now is that, as there are (extensionally) different concepts of equality, concepts whose definition involves the concept of equality may turn out to be equivocal, too. For example, a part of a whole may be proper with respect to one concept of equality and not with respect to another. Likewise, a part may be greater than another one with respect to one concept of equality and not with respect to another. If one accepts this view, then one can argue as follows.

Given that the part-whole axiom relates two concepts (proper part and being greater than) to each other that both involve the equivocal concept of equality, in any particular application of that axiom the same concept of equality should be used throughout. Now consider the axiom in its more explicit form: 'For all x and y , if x is a proper part of y , then y is greater than x '. (Leibniz recognised explication of this kind: 'A is B, that is the same as saying that if L is A, it follows that L also is B';²³ and 'The affirmative universal proposition Every b is c can be reduced to this hypothetical one: If a is b , then a is c '.²⁴) Instantiating the axiom in this form by taking for x the multitude of the squares and for y the whole of the numbers yields the simple conditional 'If the multitude of squares is a proper part of the whole of the numbers, then the whole of the numbers is greater than the multitude of the squares'. Any choice of concept of equality that, via the concept of 'proper part' it induces, makes the antecedent come out true, will, by the definition of 'greater than', make the consequent come out true as well; for example, the concept of equality defined in terms of elementhood. If, on the other hand, the chosen concept of equality makes the antecedent false, such as that defined in terms of a bijection, then it would be open to Leibniz to accept the conditional as vacuously true: the principle that

22. See, for example, the quotation from Brown 2005 in section 2, and Leibniz 2001, 406n41.

23. 'A est B, idem est ac dicere si L est A sequitur quod et L est B.' (Leibniz 1903, 260)

24. 'Propositio Universalis affirmativa Omne b est c reduci potest ad hanc hypotheticam Si a est b , a erit c .' (Leibniz, 1923–, 6,4:126 (1678/9(?)))

conditionals with false antecedents are true can be proved in (a rational reconstruction of) the logical calculus that Leibniz devised in 1690 (Leibniz 1903, 421–423).²⁵ In neither case does the conditional come out false. This shows that if one defines the concept of proper part in terms of equality, the part-whole axiom and the existence of infinite wholes are not logically incompatible.

5 Concluding remark

In Cantorian set theory, concepts of size are defined ones, and there is nothing against defining alternatives to Cantor’s own, or to working with several at the same time. In general, with different concepts of size will come different principles of arithmetic. (Indeed, Cantor himself has two different concepts of size, cardinality and ordinality, with different arithmetics.) For a mathematical exploration of Cantorian set theory equipped with a concept of size that respects the part-whole axiom non-vacuously for infinite sets, see Benci, Di Nasso, and Forti 2006, or, for an introduction with philosophical and historical background, Mancosu 2009.

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²⁵. ‘Fundamenta calculi logici’. With minor modifications, this system is equivalent to classical propositional logic. For an axiomatisation and a completeness proof, see Castañeda 1976.

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