PARIS SCHOOL OF ECONOMICS
ECOLE D'ECONOMIE DE PARIS

WORKING PAPER N° 2012 – 39

Fair Accumulation under Risky Lifetime

Grégory Ponthière

JEL Codes : D63, E22, I12, J13

Keywords: Egalitarianism ; Differentiated Mortality ; Optimal Capital Accumulation ; Golden Rule ; Fertility

PARIS-JOURDAN SCIENCES ECONOMIQUES
48, BD JOURDAN – E.N.S. – 75014 PARIS
TÉL. : 33(0) 1 43 13 63 00 – FAX : 33 (0) 1 43 13 63 10
www.pse.ens.fr

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE – ECOLE DES HAUTES ETUDES EN SCIENCES SOCIALES
ÉCOLE DES PONTS PARISTECH – ECOLE NORMALE SUPÉRIEURE – INSTITUT NATIONAL DE LA RECHERCHE AGRONOMIQUE

Fair Accumulation under Risky Lifetime*

Gregory Ponthiere†

October 2, 2012

Abstract

Individuals save for their old days, but not all of them enjoy the old age. This paper characterizes the optimal capital accumulation in a two-period OLG model where lifetime is risky and varies across individuals. We compare two long-run social optima: (1) the average utilitarian optimum, where steady-state average welfare is maximized; (2) the egalitarian optimum, where the welfare of the worst-off at the steady-state is maximized. It is shown that, under plausible conditions, the egalitarian optimum involves a higher capital and a lower fertility than the utilitarian optimum. Those inequalities hold also in a second-best framework where survival conditions are exogenously linked to the capital level.

Keywords: egalitarianism, differentiated mortality, optimal capital accumulation, Golden Rule, fertility.

JEL classification: D63, E22, I12, J13.

*The author thanks Hippolyte d'Albis, David de la Croix, Marc Fleurbaey, Paola Gobbi, Paolo Piacquadio, Alice Schoonbroodt, Thomas Seegmuller, Bertrand Wigniolle, Stephane Zuber and one anonymous referee for their helpful comments on this paper.

†Ecole Normale Supérieure, Paris, and Paris School of Economics. Address: boulevard Jourdan, 48, building B, second floor, 75014 Paris, France. E-mail: gregory.ponthiere@ens.fr.

1 Introduction

Following early contributions by Phelps (1961, 1965) and Diamond (1965), the study of the optimal capital accumulation consists in deriving formal conditions under which the capital stock maximizes the consumption per head at the stationary equilibrium. In a two-period overlapping generations (OLG) model with no technological progress, the Golden Rule states that steady-state consumption per head is maximized when the marginal productivity of capital equals the sum of the cohort growth rate and the depreciation rate of capital.

The standard Golden Rule concerns the long-run dynamics of an economy. Given that the population is likely to vary strongly over long periods of time, it is natural to explore whether the Golden Rule remains valid in a more general framework with a varying demography. In a pioneer contribution, Samuelson (1975) showed that, when the social planner chooses the fertility rate in addition to capital, the Golden Rule still prevails, with the difference that the cohort growth rate in the Golden Rule formula takes now its optimal value. More recently, the Golden Rule was also shown to be robust to the inclusion of the survival rate in the social planning program (see de la Croix *et al.* 2012).

But the introduction of a variable population size raises serious difficulties regarding the selection of an adequate social objective. Those difficulties are general, and occur each time we face what Parfit (1984) called a "different number choice", including in the particular context of optimal capital accumulation.[1] For instance, the "optimal" fertility derived by Samuelson (1975) depends on the particular social objective - average utilitarianism - that he assumed. Other normative criteria could have been used instead, as there is no decisive argument supporting that particular social welfare criterion.[2]

The existence of risky and unequal lifetimes tends also to question the social objective to be pursued when considering optimal capital accumulation.[3] To see this, consider a 2-period OLG economy, where the survival from the first period (working period) to the second period (retirement) occurs with an exogenous probability π. Under standard assumptions on individual preferences, all young agents save, at the laissez-faire, some resources for their old days. The problem is that, at the next period, only a fraction π of savers will enjoy their savings, whereas a fraction $1 - \pi$ of the savers will not.[4] It follows from this that, at the laissez-faire, there exist substantial welfare inequalities across members of a given cohort, between agents who turn out to be long-lived and agents who turn out to be short-lived. The latter ones suffer not only from a shorter life, but, also, from the lost savings due to unanticipated death.

From a normative perspective, welfare losses due to longevity inequalities are problematic. Indeed, even if it is true that all agents did, at the young age, what they planned to do, and saved as much as they wanted, it remains nonetheless that those savings decisions were made *ex ante*, that is, before one knows one's own longevity, and, as such, had to rely on survival probabilities. Those probabilities are useful information when making decisions in front of risk, but the problem is that, at the individual level, those probabilities all

[1] See also Blackorby *et al.* (2005) and Arrhenius (2012) on population ethics.

[2] For instance, Renstrom and Spataro (2011) have recently studied the optimal capital and fertility under critical-level utilitarianism.

[3] See Broome (2004) on the selection of a social criterion under differentiated longevity.

[4] We assume here the Law of Large Numbers.

turn out to be false *ex post*. In the second period, each agent is either alive or dead, and if he had known, in advance, his actual longevity - rather than his expected longevity - he would probably have saved differently. One can thus, following Fleurbaey (2010), interpret situations of risk as situations of *incomplete information*: young agents who save resources for their old age do not know their actual length of life, and this lack of knowledge casts some doubt on the capacity of those agents to do what is the best for them.

Interpreting situations of risk as situations of incomplete information supports, as argued by Fleurbaey (2010), the replacement of (standard) *ex ante* social welfare criteria, which rely on individual expected outcomes, by *ex post* social welfare criteria, which are based on individual realized outcomes. Note that those two families of social criteria can, in some cases, lead to similar optimal allocations. For instance, as emphasized by Hammond (1981), the average utilitarian social objective can be interpreted either as the expected utility of a representative agent, or as the average realized welfare in the society. However, once there is some ethical concern for equality, the distinction between *ex ante* and *ex post* social welfare criteria becomes crucial.[5]

In the example of prematurely dead savers, where longevity inequalities are exogenous, there is a strong support for adopting an egalitarian social objective. Indeed, as argued by Fleurbaey (2008), welfare inequalities are acceptable only if these result from differences in responsibility characteristics, but not if these result from circumstance characteristics, for which individuals cannot be held responsible. Given that prematurely dead savers cannot be regarded as responsible for their realized longevity, there exists a strong ethical support for adopting *egalitarian* social objectives in that context. Therefore, if we interpret the situation of risky longevity as a situation of incomplete information, there is a strong support for adopting an *ex post egalitarian* social objective, which favors the reduction of inequalities in *ex post* lifetime welfare.

That rationale has led Fleurbaey *et al.* (2010) to study, in a two-period model with risky lifetime, the optimal allocation of resources, under a social objective focusing on the realized welfare of the individuals who turn out to be the worst-off *ex post*, that is, after individual longevities have been revealed.[6] In that paper, it is shown that the *ex post* egalitarian optimal allocation achieves a full compensation for premature death, that is, it makes, *ex post*, short-lived and long-lived agents equally well-off. That result is somewhat surprising: compensation for a premature death may seem *a priori* unfeasible, because short-lived individuals cannot be identified *ex ante*, and cannot be affected by any transfer *ex post*. However, it is possible to provide a full compensation to the short-lived by concentrating the consumption of resources early on the lifecycle.

Whereas that *ex post* egalitarian social objective may seem quite plausible in a static framework, it is not obvious to see what its consequences are for the optimal capital accumulation problem. How would a *fair* capital accumulation rule look like? To what extent would this differ from the standard Golden Rule? More precisely, does egalitarianism lead to a lower or, alternatively, to a higher optimal capital in comparison to standard average utilitarianism?

The goal of this paper is to reexamine the optimal capital accumulation

[5] On the distinction between equality of opportunity *ex ante* and equality of opportunity *ex post*, see Fleurbaey and Peragine (2012).

[6] Note that, in general, the worst-off individuals are those who turn out to have a *shorter* life than others. But, as we discuss below, this is not necessarily the case.

in an economy where individual lifetime is risky, and where individuals turn out to have unequal longevities. For that purpose, we consider a two-period OLG model where the probability of survival to the old age (second period) depends on the level of preventive health investment per worker made during the young age (first period). Within that framework, we characterize the optimal capital accumulation rule under two distinct long-run social optima: on the one hand, the average utilitarian criterion, which focuses on the steady-state average welfare; on the other hand, the *ex post* egalitarian criterion, which focuses on the welfare of the worst-off individuals born at the steady-state.

The reason why we introduce here an endogenous survival probability has to do with the recent emphasis on the determinants of mortality variations in the OLG literature.[7] Note, however, that the above discussion on the choice of an adequate social objective under unequal longevities is also relevant for that literature. For instance, models by Blackburn and Cipriani (2002), Chakraborty (2004) and Galor and Moav (2005) are frameworks where agents do not choose their life expectancy, which depends on the level of human capital (in Blackburn and Cipriani), on the level of physical capital (in Ckakraborty), or on the genetic type of the agent (in Galor and Moav). Other models, such as Bhattacharya and Qiao (2005) and Boucekkine and Laffargue (2010) include agents who can affect their life expectancy through some investment in their health, but still cannot choose a certain lifetime. In those models, agents with the *same* health investment can actually turn out to have *unequal* longevities.[8] Therefore, those economies, where agents can hardly be regarded as responsible for inequalities in realized longevity, invite an *ex post* egalitarian social objective. The present paper proposes to characterize optimal capital accumulation under such a social objective, and contrasts it with the one under standard average utilitarianism.

Anticipating on our results, we first show that, when the fertility rate is exogenously given, the egalitarian social optimum involves the same optimal capital as under average utilitarianism. However, once fertility is chosen by the social planner, the egalitarian optimum involves, under plausible conditions on production, a *higher* optimal capital and a *lower* optimal fertility than the average utilitarian optimum. Thus being egalitarian reinforces the necessity to accumulate capital. Finally, we show that our results remain globally valid in a second-best framework where longevity investments are not directly chosen by the social planner, but are exogenously related to the capital level.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 derives optimal capital and fertility under average utilitarianism. Section 4 characterizes the social optimum under an *ex post* egalitarian objective. Section 5 considers the second-best framework where health investments are exogenously linked to the capital level. Section 6 concludes.

[7] See the survey by Boucekkine *et al.* (2008).

[8] This is also the case in the real world, where longevity inequalities are largely due to factors on which individuals have no influence, such as genetic background (see Christensen *et al* 2006). Note also that, even though empirical studies, such as Kaplan *et al.* (1987), Contoyannis and Jones (2004) and Balia and Jones (2008), highlight the impact of some behaviors on the *average* (or *expected*) longevity, those studies identify only risk factors, but not factors affecting, for sure, the longevity *realized* at the individual level.

2 The model

We consider an OLG model with the same population structure and production technology as in the model developed by Chakraborty (2004). Time is discrete and goes from 0 to infinity. Longevity is risky: all agents live a first period (young age), but do not necessarily survive to the second period (old age).

Demography. The size of the cohort born at time t is denoted by L_t. Each young adult has n children ($n \geq 0$):

$$L_{t+1} = nL_t \tag{1}$$

All members of cohort t live the first period for sure, but only a proportion π_{t+1} of that cohort will survive to the second period. The proportion of survivors π_{t+1} depends on the amount of health expenditures per young adult h_t:

$$\pi_{t+1} = \pi(h_t) \tag{2}$$

We assume, as usual, that $0 < \pi(h_t) < 1$ for all $h_t \geq 0$, and that $\pi'(h_t) > 0$. It is also assumed that $\pi(0) = \tilde{\pi} > 0$ and that $lim_{h_t \to \infty} \pi(h_t) = \bar{\pi} < 1$.

Preferences. Agents have time-additive lifetime welfare, which depends on consumption per period. If the utility of being dead is normalized to zero, the lifetime welfare of the short-lived agents of cohort t, denoted by U_t^{sl}, is:

$$U_t^{sl} = u(c_t) \tag{3}$$

where we assume, as usual, that $u'(c_t) > 0$ and $u''(c_t) < 0$, that is, that temporal welfare is increasing and concave in consumption c_t.[9] For the long-lived members of cohort t, lifetime welfare, denoted by U_t^{ll}, is:[10]

$$U_t^{ll} = u(c_t) + u(d_{t+1}) \tag{4}$$

where d_{t+1} is the consumption at the old age.

When considering the optimal health investment, a central aspect of preferences concerns the intercept of the temporal utility function. Following the literature, we assume that there exists a consumption $\bar{c} \geq 0$ such that:[11]

$$u(\bar{c}) = 0 \tag{5}$$

Put it differently, there exists a welfare-neutral consumption level $\bar{c}$ that makes an individual indifferent between further life with that consumption and death.

Production. The production of an output Y_t involves labour L_t and capital K_t, according to the production function:

$$Y_t = F(K_t, L_t) \tag{6}$$

[9] We also assume $\lim_{c \to 0} u'(c) = +\infty$ (see de la Croix and Michel 2002 p. 5).
[10] For the simplicity of presentation, we abstract here from pure time preferences.
[11] See Becker *et al.* (2005).

where K_t denotes the total capital stock, while L_t is the labour force, which coincides here with the whole cohort at birth. The function $F(\cdot)$ is a positively-valued production function, increasing and strictly concave with respect to capital. Capital depreciates at a constant rate δ, with $0 < \delta \leq 1$.

Under constant returns to scale, the production function can be written as:

$$y_t = f(k_t) \tag{7}$$

where y_t is the output per worker, and k_t is the capital stock per worker, while $f(\cdot) = F(k_t, 1)$ denotes the production function in its intensive form. Under the above assumptions on $F(\cdot)$, we have, for $k_t > 0$, that $f(k_t) > 0$, $f'(k_t) > 0$ and $f''(k_t) < 0$.[12] The marginal productivity of labour is:

$$\omega(k_t) = f(k_t) - k_t f'(k_t) \tag{8}$$

From the concavity of $f(k_t)$, we have: $\omega(k_t) \geq 0$ and $\omega'(k_t) = -f''(k_t) > 0$.

Resource constraint The resource constraint of the economy, stating that what is produced is either consumed or invested, is:

$$F(K_t, L_t) = c_t L_t + h_t L_t + d_t \pi(h_{t-1}) L_{t-1} + K_{t+1} - (1-\delta) K_t \tag{9}$$

At the steady-state and in intensive terms, we have:

$$f(k) - k(n + \delta - 1) = c + h + \frac{\pi(h)}{n} d \tag{10}$$

Note the roles played by demographic variables in the resource constraint. The fertility rate n has two effects: on the one hand, n puts some pressure on the available resources (the LHS), by reducing, *ceteris paribus*, the sustainable capital available for each worker (i.e. the "Solow effect"); on the other hand, n also affects the spending part of the constraint (the RHS), by weakening the weight of the elderly's consumption (i.e. the "Samuelson effect"). The survival rate π only affects the economy's spending (RHS), by making the old's (aggregate) consumption larger *ceteris paribus*.

3 The utilitarian first-best problem

To characterize the optimal capital accumulation, we will focus here on a stationary equilibrium, where all per worker variables k_t, h_t and π_t are constant over time, whereas all aggregate variables, such as production Y_t, total capital stock K_t, and total consumption C_t grow at a rate n.[13]

We consider the problem of a utilitarian social planner, who selects capital, consumptions, health investment and fertility in such a way as to maximize the average lifetime welfare prevailing at the stationary equilibrium, subject to the resource constraint of the economy. That planning problem is:

$$\begin{aligned}
&\max_{c,d,k,h,n} u(c) + \pi(h) u(d) \\
&\text{s.t. } c + \frac{\pi(h)}{n} d = f(k) - k(n + \delta - 1) - h \\
&\text{s.t. } h \geq 0
\end{aligned}$$

[12] See de la Croix and Michel (2002) on this.

[13] Therefore we will, in the rest of this section, delete time indices.

The planner's problem can be rewritten by means of the following Lagrangian:

$$\max_{c,d,k,n,h} u(c) + \pi(h)u(d) + \lambda \left[f(k) - k(n + \delta - 1) - h - c - \frac{\pi(h)}{n} d \right] + \chi h$$

where λ is the Lagrange multiplier associated to the resource constraint, while χ is the Lagrange multiplier associated with the non-negativity constraint for h. The first-order conditions (FOCs) can be rewritten as:

$$\frac{u'(c)}{u'(d)} = n \tag{11}$$

$$f'(k) = \delta + n - 1 \tag{12}$$

$$k = \frac{\pi(h)}{(n)^2} d \tag{13}$$

$$u'(c) - \pi'(h)\left[u(d) - du'(d)\right] = \chi \tag{14}$$

as well as the conditions

$$\lambda \geq 0, f(k) - k(n + \delta - 1) - h \geq c + \frac{\pi(h)}{n} d \tag{15}$$

$$\chi \geq 0, h \geq 0 \tag{16}$$

with complementary slackness.

From condition (11), we see that the marginal rate of substitution between consumptions at the two ages of life should be equal to the optimal fertility rate n. Expression (12) is the standard Golden Rule for optimal capital accumulation, stating that the marginal productivity of capital should be equal to the sum of the depreciation rate of capital δ and the cohort growth rate $n - 1$.

Condition (13) states that the optimal fertility n should be such that the (negative) capital dilution effect (the LHS) equals the (positive) intergenerational effect (the RHS). Note that, on the basis of equation (13), nothing guarantees that the optimal fertility n is higher (or equal to) the replacement fertility level $n = 1$. Thus it is possible to have an optimal fertility n strictly below the replacement fertility rate ($n = 1$), with the corollary that the population becomes extinct asymptotically. That case is especially likely to prevail when the intergenerational redistribution effect is small in comparison with the capital dilution effect.

Condition (14) states that an interior optimal health investment h exists only if there exists some level of $h > 0$ such that the marginal welfare loss from foregone consumption (1st term of the LHS) equals the marginal welfare gain from health investment (2nd term of the LHS), yielding $\chi = 0$. That condition holds if and only if we have, under $h > 0$:

$$u'(c) = \pi'(h)\left[u(d) - du'(d)\right]$$

That equality requires that the second period is, at the optimum, worth being lived, that is, that $u(d) > 0$ or $d > \bar{c}$. This condition is satisfied either when individuals assign a high value to life (i.e. $\bar{c}$ is low), or when the economy is very productive, insuring that the optimal second-period consumption exceeds

$\bar{c}$.[14] That latter assumption is quite weak, and so we can expect, under general conditions, an interior optimal health investment.

Proposition 1 summarizes our results.

Proposition 1 *The average utilitarian first-best optimum* $\{c^*, d^*, k^*, n^*, h^*\}$ *is such that:*

$$\begin{aligned} \frac{u'(c^*)}{u'(d^*)} &= n^* \\ f'(k^*) &= \delta + n^* - 1 \\ k^* &= \frac{\pi(h^*)}{(n^*)^2} d^* \\ u'(c^*) &= \pi'(h^*) \left[u(d^*) - d^* u'(d^*) \right] \\ c^* + \frac{\pi(h^*)}{n^*} d^* &= f(k^*) - k^* (n^* + \delta - 1) - h^* \\ h^* &> 0 \end{aligned}$$

Proof. See the above FOCs. ■

The above conditions are necessary, but not sufficient for an *interior* optimum. As it is well-known, sufficiency requires to consider second-order conditions, which may not be satisfied under endogenous fertility and mortality. Given that the literature has already examined those second-order conditions, we will here assume that second-order conditions are satisfied, so that the above first-order conditions characterize the interior social optimum.[15]

Finally, it should be stressed here that the conditions mentioned in Proposition 1 may be satisfied by several distinct vectors $\{c^*, d^*, k^*, n^*, h^*\}$, so that those conditions do not guarantee the uniqueness of the social optimum. Actually, in the simple case where demography is fixed, i.e. $n = \bar{n}$ and $\pi = \bar{\pi}$, simple assumptions on the production function $f(k)$ [$f(k) > 0$, $f'(k) > 0$, $f''(k) < 0$ for $k > 0$] and on the utility function $u(c)$ [$u(c) > 0$, $u''(c) < 0$ for $c > 0$, as well as $\lim_{c \to 0} u'(c) = +\infty$], together with the assumption $f'(+\infty) < \delta + \bar{n} - 1 < f'(0)$, suffice to guarantee the uniqueness of the utilitarian social optimum (see de la Croix and Michel 2002 p. 78-79). However, once n and π are variables, the uniqueness issue becomes much trickier, since standard assumptions on production and utility functions do no longer guarantee the uniqueness of the social optimum. We do not explore the uniqueness issue further here, since our emphasis in this paper lies on the comparison of that utilitarian optimum with the egalitarian optimum, and, for that purpose, the uniqueness issue is not crucial. Actually, the comparison of the two optima, based on first-order conditions, keeps its entire significance even when the uniqueness of the social optima under comparison does not prevail, because we know that the optimum, whatever it is, must necessarily satisfy the associate set of first-order conditions.

[14] Note that assigning a high value to life coïncides with a low level of $\bar{c}$, and not with a high level of $\bar{c}$. The reason is that, if agents value strongly the mere fact of being alive, even extremely low consumption levels will make them regard life as better than death. Therefore the threshold $\bar{c}$ must be very low in that case.

[15] See Deardorff (1976) and Michel and Pestieau (1993) on the interiority of the optimal fertility rate. See de la Croix *et al* (2012) on the interiority of the optimum survival rate.

4 The egalitarian first-best problem

Let us now contrast the average utilitarian social optimum with the *ex post* egalitarian social optimum. Whereas average utilitarianism can be interpreted as the maximization of the expected welfare of a representative agent from an *ex ante* perspective (see Hammond 1981), the *ex post* egalitarian optimum considers, on the contrary, what maximizes the realized - rather than expected - welfare of the worst-off individuals at the stationary equilibrium.

The motivations for focusing on the worst-off individuals are the following. Clearly, the social objective behind the standard Golden Rule - i.e. the maximization of the steady-state average welfare - makes a lot of sense in the context where the Golden Rule was initially studied: a context where cohorts are all made of *identical* agents. However, once risky mortality is introduced, taking the maximization of average welfare seems less adequate from an ethical perspective. In our model, individuals are all identical *ex ante*, but some persons will, *ex post*, turn out to have a shorter life than others. Given that being long-lived or short-lived is here a matter of pure luck, it makes sense to take, as a social objective, not the maximization of average welfare, but, rather, the maximization of the welfare of the unlucky, worst-off individuals.[16]

Since the worst-off agents at the steady-state can be either the short-lived, who enjoy a lifetime welfare equal to $u(c)$, or the long-lived, who enjoy a lifetime welfare $u(c) + u(d)$, the social planner's problem is here:

$$\begin{aligned}
&\max_{c,d,k,h,n} \min \{u(c), u(c) + u(d)\} \\
&\text{s.t. } c + \frac{\pi(h)}{n} d = f(k) - k(n + \delta - 1) - h \\
&\text{s.t. } h \geq 0
\end{aligned}$$

That social planning problem is less straightforward than the standard utilitarian problem. The reason is that the objective function to be maximized is continuous but not differentiable. However, the identification of the worst-off agents in the population can help us to rewrite that planning problem in a more convenient manner.

Actually, whereas common sense identifies the worst-off individuals with the short-lived ones, this is not necessarily the case. Whether the short-lived or the long-lived is the worst-off depends on whether second-period consumption d exceeds the welfare neutral level $\bar{c}$ or not:

$$U^{sl} \lesseqgtr U^{ll} \iff u(d) \gtreqless 0 \iff d \gtreqless \bar{c}$$

In the light of that condition, we can rewrite the above social planner's problem under a much simpler form. Actually, the egalitarian planning problem can be rewritten as the maximization of the welfare of the short-lived individuals (i.e. $u(c)$) subject to the constraint that the lifetime welfare of the long-lived individuals is not inferior to the lifetime welfare of the short-lived ones (i.e. $u(c) + u(d) \geq u(c)$). Indeed, if we had $u(d) < 0$, long-lived agents would be worse off than short-lived agents, so that the objective of maximizing the welfare of the short-lived would not make sense. Hence the egalitarian social

[16] See Fleurbaey *et al* (2010) on the ethical foundations behind the maximization of the *ex post* welfare of the worst-off individuals under risky longevity.

planner's problem consists in maximizing the welfare of the short-lived subject to the constraint $u(d) \geq 0$, and can thus be rewritten as:

$$\max_{c,d,k,n,h} u(c) + \lambda \left[f(k) - k\,(n + \delta - 1) - h - c - \frac{\pi(h)}{n} d \right] + \mu\,[u(d)] + \chi h$$

where λ denotes the Lagrange multiplier associated to the resource constraint, μ is the Lagrange multiplier associated with the non-negativity constraint for welfare at the old age, whereas χ is the Lagrange multiplier associated with the non-negativity constraint for health investment. The first-order conditions are:

$$u'(c) = \lambda \tag{17}$$

$$\lambda \frac{\pi(h)}{n} = \mu u'(d) \tag{18}$$

$$f'(k) = n + \delta - 1 \tag{19}$$

$$k = \frac{\pi(h)}{(n)^2} d \tag{20}$$

$$\lambda \left(1 + \frac{\pi'(h)}{n} d \right) = \chi \tag{21}$$

as well as the conditions

$$\lambda \geq 0, f(k) - k\,(n + \delta - 1) - h \geq c + \frac{\pi(h)}{n} d \tag{22}$$

$$\mu \geq 0, u(d) \geq 0 \tag{23}$$

$$\chi \geq 0, h \geq 0 \tag{24}$$

with complementary slackness.

From FOC (19), it appears that the optimal capital accumulation remains determined by the standard Golden Rule. FOC (21) suggests that the optimal level of health investment h is a corner solution, and equal to 0.[17] That corner solution for health investment may seem surprising, but is actually quite intuitive: for the unlucky agents who do not reach the old age, preventive spending is a waste of resources, so that it is optimal to set those expenditures to zero. Regarding the last two FOCs, it is clear that it is optimal to exhaust the economy's resources, so that the budget constraint is, at the optimum, a strict equality. Moreover, regarding consumption at the old age, we must have, at the optimum, that second-period consumption d equals $\bar{c}$, in such a way as to obtain $u(d) = 0$.

Substituting for $h = 0$ and for $d = \bar{c}$ in (20), we obtain:

$$k = \frac{\tilde{\pi}}{(n)^2} \bar{c}$$

We see that, *ceteris paribus*, the capital dilution effect (i.e. the LHS) is the same as under average utilitarianism, but the intergenerational redistribution effect

[17] Indeed, an interior health investment would require the LHS of (21) to be equal to 0, that is:

$$u'(c) + \mu u'(d) d \frac{\pi'(h)}{\pi(h)} = 0$$

The first term of the LHS is necessarily positive, and the second term as well, so that this equality is always violated.

(i.e. the RHS) is here weakened in comparison to what holds under utilitarianism. The reason is that, under an interior utilitarian optimum, we necessarily have $u(d) > 0$, implying $d > \bar{c}$.[18] As a consequence, we can expect, *ceteris paribus*, fewer children under egalitarianism than under average utilitarianism. Thus the possibility of an optimal fertility rate strictly smaller than the replacement rate $n = 1$ is, *ceteris paribus*, more plausible here than under the average utilitarian optimum. The possibility of an optimal fertility lower than the replacement rate is, *ceteris paribus*, larger when the minimum survival chance $\tilde{\pi}$ is low, and when the welfare neutral consumption level $\bar{c}$ is also low.[19]

Note, however, that this result is only true under a *given* k, so that further calculations will be necessary to have an unambiguous comparison of the two social optima. Prior to this, Proposition 2 summarizes our results.

Proposition 2 *The egalitarian first-best optimum* $\{c^*, d^*, k^*, n^*, h^*\}$ *is such that:*

$$\begin{aligned} c^* &= f(k^*) - k^* (n^* + \delta - 1) - \frac{\tilde{\pi}}{n^*}\bar{c} \\ d^* &= \bar{c} \\ f'(k^*) &= \delta + n^* - 1 \\ k^* &= \frac{\tilde{\pi}}{(n^*)^2}\bar{c} \\ h^* &= 0 \end{aligned}$$

Proof. See the above FOCs. ■

As under the utilitarian optimum, it may be the case that several vectors $\{c^*, d^*, k^*, n^*, h^*\}$ satisfy the above conditions. Hence the uniqueness of the egalitarian optimum is not guaranteed. However, in order to compare the utilitarian and the egalitarian optima, it is not necessary to be certain that each of those social optima is unique. Indeed, given that each social optimum necessarily satisfies a set of first-order conditions (mentioned in Proposition 1 for the utilitarian optimum, and in Proposition 2 for the egalitarian one), the comparison of the two optima based on the study of those first-order conditions leads to conclusions that hold whatever those optima are unique or not.

When comparing the egalitarian and utilitarian first-best optima, some obvious differences appear. First, the optimal level of prevention under the egalitarian optimum, which is equal to zero, is lower than under utilitarianism (when the utilitarian optimum is an interior solution). The reason is that, from the point of view of average welfare, it makes sense to spend some resources to raise the survival probability. On the contrary, from the point of view of a short-lived person, preventive health spending are just a waste of resources.

The comparison of optimal consumption levels is more complex, since it depends on the optimal level of capital and fertility under the two social objectives. Let us take, as a starting point, the case where fertility is not chosen by the social planner, but is equal to an exogenous level $n = \bar{n}$. In that case, the optimal capital satisfies, under utilitarianism and egalitarianism, the same

[18] See *supra*.

[19] Indeed, if $\bar{c}$ is very low, there is little welfare gain from fertility, and so the social optimum involves a fertility rate less than the replacement level.

condition, which is the standard Golden Rule:

$$f'(k^*) = \delta + \bar{n} - 1$$

Given that both δ and $\bar{n}$ are constant, the optimal capital is the same whatever the social objective is. That result is quite intuitive. Our egalitarian social objective cares about the worst-off individuals living at the steady-state, and, as such, focuses on *intragenerational* justice. On the contrary, the Golden Rule, which aims at maximizing consumption possibilities at the stationary equilibrium, concerns the *intergenerational* allocation of resources, and remains thus relevant under our (intragenerational) egalitarian objective. Indeed, the Golden Rule guarantees the largest aggregate consumption possibilities at the steady-state, and those consumption possibilities can be used differently depending on the adopted conception of intragenerational justice.

But that kind of intra- *versus* intergenerational separation result collapses once fertility is chosen by the social planner. Indeed, in that general case, the optimal capital and fertility levels satisfy respectively:[20]

$$\begin{aligned} f'(k^{*u}) &= \delta + n^{*u} - 1 \\ k^{*u} &= \frac{\pi(h^{*u})}{(n^{*u})^2} d^{*u} \end{aligned}$$

for the utilitarian optimum, and

$$\begin{aligned} f'(k^{*e}) &= \delta + n^{*e} - 1 \\ k^{*e} &= \frac{\tilde{\pi}}{(n^{*e})^2} \bar{c} \end{aligned}$$

for the egalitarian optimum. From those conditions, it can be shown that the two optimal capital and fertility levels differ under the two social objectives. While calculations are left to the Appendix, Proposition 3 summarizes our results.

Proposition 3 *Under the utilitarian and the egalitarian optima, we have:*

- *if* $f'(k) - \delta + 1 > 2k\omega'(k)$, *we have:* $k^{*u} > k^{*e}$ *and* $n^{*u} < n^{*e}$;
- *if* $f'(k) - \delta + 1 = 2k\omega'(k)$, *we have:* $k^{*u} = k^{*e}$ *and* $n^{*u} = n^{*e}$;
- *if* $f'(k) - \delta + 1 < 2k\omega'(k)$, *we have:* $k^{*u} < k^{*e}$ *and* $n^{*u} > n^{*e}$.

  Assuming a Cobb-Douglas technology $y_t = Ak_t^{\alpha}$, *we have, under a full depreciation of capital, that:*
- *if* $\alpha > 1/2$, *we have:* $k^{*u} > k^{*e}$ *and* $n^{*u} < n^{*e}$;
- *if* $\alpha = 1/2$, *we have:* $k^{*u} = k^{*e}$ *and* $n^{*u} = n^{*e}$;
- *if* $\alpha < 1/2$, *we have:* $k^{*u} < k^{*e}$ *and* $n^{*u} > n^{*e}$.

[20] Here we use the superscripts u and e to refer to, respectively, the utilitarian and egalitarian optima.

Proof. See the Appendix. ■

Hence, the egalitarian optimum may, in general, involve a lower or a higher capital than the utilitarian optimum once the fertility is also chosen by the social planner. The prevailing inequality depends on the level of the depreciation rate of capital δ, as well as on the properties of the production process.

As stated in the second part of Proposition 3, we can see that, under a full depreciation of capital $\delta = 1$ and a Cobb-Douglas production function with $\alpha < 1/2$ (as usually calibrated), the optimal capital under egalitarianism is superior to its level under average utilitarianism. Inversely, the optimal fertility is larger under utilitarianism. Note that this comparison does not tell us whether a particular social optimum involves a fertility rate strictly lower than the replacement fertility $n = 1$. However, we know for sure that, in the case of $\alpha < 1/2$, if the utilitarian optimum involves a fertility rate lower than the replacement level, this will be also the case for the egalitarian optimum, since this optimum involves a lower fertility than the utilitarian one.

Hence, even if the optimal capital remains, under egalitarianism, still determined by an accumulation rule that looks pretty much like the standard Golden Rule, i.e. $f'(k) = \delta + n - 1$, the mere fact that egalitarianism involves a lower optimal fertility implies also a higher optimal capital than under average utilitarianism. The reason is the following. Egalitarianism, by focusing on the worst-off individuals in the cohort, recommends a welfare-neutral consumption at the old age, which restricts the intergenerational gains from fertility. The "Samuelson effect" being reduced, the optimal fertility under egalitarianism is lower, implying a higher optimal capital. Thus, contrary to what one may expect, egalitarianism *reinforces* the necessity to accumulate capital.

In sum, the egalitarian social objective recommends, in comparison to average utilitarianism, a lower old-age consumption, and a lower preventive health spending. Those differences leave the optimal capital equal to its utilitarian optimal level when the fertility rate is exogenously given. However, once fertility is also a part of the social planner's optimization problem, the egalitarian optimum involves, in general, a higher capital and a lower fertility than under utilitarianism. Thus, it is through the optimal fertility decision that the shift to egalitarianism affects the optimal capital accumulation. Without that channel, egalitarianism would imply the same capital level as under utilitarianism.

5 The egalitarian second-best problem

The egalitarian first-best solution may face strong resistance among policy circles. Imposing a zero investment in health in the name of the - possibly few - unlucky individuals who turn out to be short-lived is questionable. This is the reason why we now turn to a second-best framework, where health expenditures take an interior value. In this section, we will consider the social planner's problem in a second-best context, where health spending are not directly chosen by the social planner. For that purpose, we consider three distinct cases:[21]

A1 Health spending per worker are constant: $h_t = \bar{h} > 0$.

A2 Health spending per worker are a constant fraction τ of labour productivity: $h_t = \tau\omega(k_t)$.

[21] Those cases are also studied in de la Croix and Ponthiere (2010), who focus on the utilitarian optimum.

A3 Health spending per worker are a constant fraction θ of the output per worker: $h_t = \theta f(k_t)$.

The case **A1** coincides with standard OLG models with fixed life expectancy $1 + \pi(\bar{h})$.[22] The case **A2** includes the economy considered by Chakraborty (2004). The case **A3** makes health spending depend on the whole social income.

Under case **A1**, the egalitarian social planner's problem can be rewritten as:

$$\max_{c,d,k,n} u(c) + \lambda \left[f(k) - k(n+\delta-1) - \bar{h} - c - \frac{\pi(\bar{h})}{n} d \right] + \mu [u(d)]$$

where λ is the Lagrange multiplier associated to the resource constraint, whereas μ is the Lagrange multiplier associated with the non-negativity constraint for old-age welfare $u(d)$. The first-order conditions are:

$$u'(c) = \lambda \tag{25}$$

$$\lambda \frac{\pi(\bar{h})}{n} = \mu u'(d) \tag{26}$$

$$f'(k) = n + \delta - 1 \tag{27}$$

$$k = \frac{\pi(\bar{h})}{n^2} d \tag{28}$$

as well as the conditions

$$\lambda \geq 0, f(k) - k(n+\delta-1) - \bar{h} \geq c + \frac{\pi(\bar{h})}{n} d \tag{29}$$

$$\mu \geq 0, u(d) \geq 0 \tag{30}$$

with complementary slackness.

From (27), it appears that, as in the first-best problem, the optimal capital accumulation remains determined by the standard Golden Rule. Old-age consumption remains equal to $\bar{c}$. Note also that, on the basis of (28), we obtain that, because of the higher preventive spending, there is now a higher intergenerational redistribution effect, which leads to a higher fertility *ceteris paribus*.

Turning now to cases **A2** and **A3**, the social planner's problem becomes:

$$\max_{c,d,k,n} u(c) + \lambda \left[f(k) - k(n+\delta-1) - h(k) - c - \frac{\pi(h(k))}{n} d \right] + \mu [u(d)]$$

where λ denotes the Lagrange multiplier associated to the resource constraint, whereas μ is the Lagrange multiplier associated with the non-negativity constraint $u(d) \geq 0$. The first-order conditions are:

$$u'(c) = \lambda \tag{31}$$

$$\lambda \frac{\pi(h(k))}{n} = \mu u'(d) \tag{32}$$

$$f'(k) - (n+\delta-1) - h'(k) - \frac{\pi'(h(k))}{n} h'(k) d = 0 \tag{33}$$

$$k = \frac{\pi(h)}{n^2} d \tag{34}$$

[22] The standard Diamond model relies on $\pi = 1$.

as well as the conditions

$$\begin{aligned} \lambda &\geq 0, f(k) - k(n + \delta - 1) - h(k) \geq c + \frac{\pi(h(k))}{n} d & (35) \\ \mu &\geq 0, u(d) \geq 0 & (36) \end{aligned}$$

with complementary slackness.

As above, second-period consumption equals the welfare-neutral level $\bar{c}$. Substituting for $d = \bar{c}$, one can rewrite (33) as:

$$f'(k) = n + \delta - 1 + h'(k) \left[1 + \frac{\pi'(h(k))}{n} \bar{c} \right]$$

That equality characterizes the optimal second-best capital level. Note that, under $h'(k) = 0$, we are back to the standard Golden Rule. However, once capital accumulation raises health investment, the optimal capital becomes, *ceteris paribus*, lower than under the standard Golden Rule. That comparison holds only *ceteris paribus*, that is, for an equal fertility level n. One can, nonetheless, expect the optimal fertility to differ in the first-best and in the second-best, simply on the grounds that a higher health investment h raises the intergenerational redistribution effect, inviting a higher optimal fertility for an equal level of capital k. The following proposition summarizes our results.

Proposition 4 *The egalitarian second-best optimum* $\{c^*, d^*, k^*, n^*\}$ *is such that:*

$$\begin{aligned} c^* &= f(k^*) - k^*(n^* + \delta - 1) - h(k^*) - \frac{\pi(h(k^*))}{n^*} \bar{c} \\ d^* &= \bar{c} \\ k^* &= \frac{\pi(h(k^*))}{(n^*)^2} \bar{c} \end{aligned}$$

Regarding optimal capital, we have:

$$\begin{aligned} \textit{under } \boldsymbol{A1} &: f'(k^*) = n^* + \delta - 1 \\ \textit{under } \boldsymbol{A2} &: f'(k^*) = n^* + \delta - 1 - \tau k^* f''(k^*) \left[1 + \frac{\pi'(h(k^*))}{n^*} \bar{c} \right] \\ \textit{under } \boldsymbol{A3} &: f'(k^*) = n^* + \delta - 1 + \theta f'(k^*) \left[1 + \frac{\pi'(h(k^*))}{n^*} \bar{c} \right] \end{aligned}$$

Proof. See the above FOCs. ■

If fertility was fixed to some exogenous level $n = \bar{n}$, the optimal capital at the second-best would be, under case **A1**, the same as it is at the first-best. Moreover, under cases **A2** and **A3**, the optimal second-best capital would be strictly inferior to its first-best level. The reason is that a higher capital tends here to foster health investment, and, hence, raises the inactive to active population ratio $\frac{\pi}{n}$, which reduces consumption possibilities *ceteris paribus*.[23] Hence, it follows that the second-best egalitarian optimal capital is, under case **A1**, equal to the one under utilitarianism, and, under cases **A2** and **A3**, it is lower than the utilitarian optimal capital.

[23] That negative effect of capital accumulation on consumption possibilities depends on the shape of the survival function, through the derivative $\pi'(h(k))$.

When fertility is a part of the planner's choice, it is more complicated to compare the first-best and second-best egalitarian optima. To see this, let us consider the case **A1**. Comparing the condition for first-best (FB) fertility and second-best (SB) fertility, we have:

$$k^{*eFB} = \frac{\tilde{\pi}}{\left(n^{*eFB}\right)^2}\bar{c} \quad \text{and} \quad k^{*eSB} = \frac{\pi(\bar{h})}{\left(n^{*eSB}\right)^2}\bar{c}$$

it appears that, for a given k, the larger survival probability at the second-best raises the Samuelson effect (RHS), and, as such, the optimal fertility is, at the second-best, larger than at the first-best for a given k. That result is important, but, here again, the optimal second-best capital is a function of the optimal fertility, so that a *ceteris paribus* rationale is not satisfactory. Proposition 5 summarizes our results.[24]

Proposition 5 *Consider case **A1**. We have:*

$$k^{*eSB} \gtreqless k^{*eFB} \iff f'(k) - \delta + 1 \gtreqless 2k\omega'(k)$$

Assuming Cobb-Douglas production $y_t = Ak_t^{\alpha}$ and a full depreciation of capital, we obtain:

- *if $\alpha > 1/2$, we have: $k^{*eFB} < \left\{ \begin{smallmatrix} k^{*eSB} \\ k^{*u} \end{smallmatrix} \right.$ and $n^{*eFB} > \left\{ \begin{smallmatrix} n^{*eSB} \\ n^{*u} \end{smallmatrix} \right.$;*
- *if $\alpha = 1/2$, we have: $k^{*eFB} = k^{*u} = k^{*eSB}$ and $n^{*u} = n^{*eFB} = n^{*eSB}$;*
- *if $\alpha < 1/2$, we have: $k^{*eFB} > \left\{ \begin{smallmatrix} k^{*eSB} \\ k^{*u} \end{smallmatrix} \right.$ and $n^{*eFB} < \left\{ \begin{smallmatrix} n^{*eSB} \\ n^{*u} \end{smallmatrix} \right.$.*

Proof. See the Appendix. ■

Hence, if we take the realistic case where $\alpha < 1/2$, the optimal second-best capital under egalitarianism is lower than at the first-best, and the optimal second-best fertility is larger than at the first-best. The intuition is that the Samuelson effect is reinforced by the larger proportion of survivors associated with $\bar{h} > 0$, which pushes, at the end of the day, towards a higher optimal fertility and a lower optimal capital than in the first-best. Hence, the extent to which egalitarianism leads to a higher optimal capital than utilitarianism is lower at the second-best than at the first-best. Note, however, that, if $\alpha < 1/2$ and $\bar{h} < h^{*u}$, it is still the case, even at the second-best, that egalitarianism recommends a lower fertility and a higher capital than utilitarianism. The addition of the constraint $\bar{h} > 0$ reduces the gap, but it still exists.

In sum, the way in which the introduction of exogenous health investment affects the optimal accumulation depends on whether fertility is part of the planner's choice or not. This dependency on fertility is well illustrated in the case **A1** (fixed health investment). When fertility is exogenous, the second-best capital is equal to its first-best level. Once fertility is a choice variable, the second-best capital is generally inferior to its first-best level, because of a larger Samuelson effect, which pushes towards a larger fertility and a lower capital.

[24] For simplicity of presentation, we focus here on case **A1**, but similar conditions can be derived for cases **A2** and **A3**.

6 Concluding remarks

The goal of the present paper was to revisit the optimal capital accumulation in the context where lifetime is risky and where individuals turn out to have unequal longevities. For that purpose, we compared, in a two-period OLG model, two long-run social optima. On the one hand, the average utilitarian criterion, which consists of maximizing the average welfare at the stationary equilibrium; on the other hand, the *ex post* egalitarian criterion, which consists of maximizing the welfare of the worst-off born at the steady-state.

It has been shown that whether egalitarianism leads to a higher optimal capital or not depends on whether fertility is chosen by the social planner or not. When fertility is not chosen, the optimal capital is, under egalitarianism, exactly the same as under utilitarianism. Indeed, in that case, egalitarian concerns, which remain intragenerational, are not relevant for discussing the optimal capital accumulation, which is an intergenerational issue. However, that intragenerational *versus* intergenerational divide vanishes once fertility is chosen by the social planner. Indeed, in that case, the egalitarian optimum involves, under general conditions, a higher optimal capital than under utilitarianism, and, also, a lower optimal fertility. The reason is that egalitarianism, by implying lower old-age consumption, weakens intergenerational gains from fertility, i.e. the Samuelson effect, and, hence, the optimal fertility.

Thus egalitarian concerns do not reduce, but do actually *raise* optimal capital accumulation. That result is somewhat surprising, since one would expect egalitarian concerns to lower the optimal capital. Nonetheless, the present study shows that, contrary to what one may expect, capital accumulation becomes even more necessary when one is only concerned with the worst-offs. That result was also shown to be globally robust to the shift to a second-best set up where health investment is exogenously related to the capital level, even if the introduction of some constrained level of health investment reduces, in general, the optimal capital in comparison to the first-best.

Finally, let us notice some limitations of the present study. A first issue that remains to be explored consists of the *implementability* of the egalitarian optimum characterized here. That question can be formulated as follows: under which fiscal instruments could a government decentralize the egalitarian social optimum, and induce the fair capital accumulation? That question is not a trivial one, especially when fertility is also endogenous, since, in that case, the study of the decentralization is likely to be sensitive to the structure of parental preferences in terms of fertility. As such, the study of decentralization would require another paper on its own. Besides the implementation issue, one should also remind that the capital accumulation rule studied here is "fair" only in a particular sense. That capital accumulation rule, based on the maximization of the worst-off's welfare, is concerned with correcting for the arbitrariness of the time of *death*. However, there exists, from the point of view of history, another major source of welfare inequalities, which is also arbitrary: the time of *birth*. Given that the present study focused on the stationary equilibrium only, this could not deal with that issue, which would require to explore the Generalized Golden Rule and its egalitarian variants in the context of an endogenous demography. This task is also left on our research agenda.

7 References

Arrhenius, G. (2012): Population Ethics, Oxford University Press, forthcoming.

Balia, S. & Jones, A. (2008): "Mortality, lifestyles and socio-economic status", Journal of Health Economics, 27 (1), pp. 1-26.

Becker, G.S., Philipson, T. & Soares, R. (2005): "The quantity and the quality of life and the evolution of world inequality", American Economic Review, 95 (1), pp. 277-291.

Bhattacharya, J. & Qiao, X. (2005): "Public and private expenditures on health in a growth model", Iowa State University, Working Paper, Series 12378.

Blackburn, K. & Cipriani, G.P. (2002): "A model of longevity, fertility and growth", Journal of Economic Dynamics and Control, 26, pp. 187-204.

Blackorby, C., Bossert, W. & Donaldson, D. (2005): Population Issues in Social Choice Theory, Welfare Economics and Ethics. Econometric Society Monographs, Cambridge University Press.

Boucekkine, R., Diene, B. & Azomahou, T. (2008): "Growth economics of epidemics: a review of the theory", Mathematical Population Studies, 15 (1), pp. 1-26.

Boucekkine, R. & Laffargue, J.P. (2010): "On the distributional consequences of epidemics", Journal of Economic Dynamics and Control, 34 (2), pp. 213-245.

Broome, J. (2004): Weighing Lives, Oxford University Press, New-York.

Chakraborty, S. (2004): “Endogenous lifetime and economic growth”, Journal of Economic Theory, 116, pp. 119-137.

Christensen, K., Johnson, T., & J. Vaupel, 2006. ‘The quest for genetic determinants of human longevity: challenges and insights’, Nature Reviews - Genetics, 7, pp. 436-448.

Contoyannis, P. & Jones, A. (2004): "Socio-economic status, health and lifestyle, Journal of Health Economics, 23, pp. 695-995.

Deardorff, A.V. (1976): "The optimum growth rate for population: comment", International Economic Review, 17 (2), pp. 510-515.

de la Croix, D. & Michel, P. (2002): A Theory of Economic Growth. Dynamics and Policy in Overlapping Generations. Cambridge University Press, Cambridge.

de la Croix, D., Pestieau, P. & Ponthiere, G. (2012): “How powerful is demography? The Serendipity Theorem revisited”, Journal of Population Economics, 25 (3), pp. 899-922.

de la Croix, D. & Ponthiere, G. (2010): “On the golden rule of capital accumulation under endogenous longevity”, Mathematical Social Sciences, 59, pp. 227-238.

Diamond, P. (1965): “National debt in a neoclassical growth model”, American Economic Review, 55 (5), pp. 1126-1150.

Fleurbaey, M. (2008): Fairness, Responsibility and Welfare, Oxford University Press, New-York.

Fleurbaey, M. (2010): “Assessing risky social situations”, Journal of Political Economy, 118 (4), pp. 649-680.

Fleurbaey, M., Leroux, M.L. & Ponthiere, G. (2010): "Compensating the dead? Yes we can", CORE Discussion Paper 2010-66.

Fleurbaey, M. & Peragine, V. (2012): "Ex ante versus ex post equality of opportunity", Economica, forthcoming.

Galor, O. & Moav, O. (2005): "Natural selection and the evolution of life expectancy", CEPR Minerva Center for Economic Growth Paper, 02-05.

Hammond, P.J. (1981): "Ex ante and ex post welfare optimality under uncertainty", Economica, 48, pp. 235-250.

Kaplan, G. A., Seeman, T. E., Cohen, R. D., Knudsen, L. P. & Guralnik, J. (1987): "Mortality among the elderly in the Alameda county study: behavioral and demographic risk factors, American Journal of Public Health, 77 (3), pp. 307-312.

Michel, P. & Pestieau, P. (1993): "Population growth and optimality: when does the serendipity hold", Journal of Population Economics, 6(4), pp. 353-362.

Parfit, D. (1984): Reasons and Persons, Oxford University Press.

Phelps, E. (1961): “The golden rule of accumulation: a fable for growthmen”, American Economic Review, 51 (4), pp. 638-643.

Phelps, E. (1965): “Second essay on the golden rule of accumulation”, American Economic Review, 55 (4), pp. 793-814.

Renström, T. & Spataro, L. (2011): "The optimum growth rate for population under critical-level utilitarianism", Journal of Population Economics, 24 (3), pp. 1181-1201.

Samuelson, P. (1975): "The optimum growth rate for population", International Economic Review, 16 (3), pp. 539-544.

8 Appendix

8.1 Proof of Proposition 3

Under utilitarianism, the optimal capital and fertility levels satisfy respectively:

$$\begin{aligned} f'(k^{*u}) &= \delta + n^{*u} - 1 \\ k^{*u} &= \frac{\pi(h^{*u})}{(n^{*u})^2} d^{*u} \end{aligned}$$

On the contrary, at the egalitarian optimum, these satisfy respectively:

$$\begin{aligned} f'(k^{*e}) &= \delta + n^{*e} - 1 \\ k^{*e} &= \frac{\tilde{\pi}}{(n^{*e})^2} \bar{c} \end{aligned}$$

Substituting for the Golden Rule in the optimal fertility condition, we have:

$$\begin{aligned} k^{*u} \left(f'(k^{*u}) - \delta + 1\right)^2 &= \pi(h^{*u}) d^{*u} \\ k^{*e} \left(f'(k^{*e}) - \delta + 1\right)^2 &= \tilde{\pi}\bar{c} \end{aligned}$$

Remind that $\pi(h^{*u}) > \tilde{\pi}$ and $d^{*u} > \bar{c}$, so that $\pi(h^{*u})d^{*u} > \tilde{\pi}\bar{c}$. Hence, if $\phi(k) \equiv k\left(f'(k) - \delta + 1\right)^2$ is increasing (resp. decreasing) in k, we have $k^{*u} > k^{*e}$ (resp. $k^{*u} < k^{*e}$). We have:

$$\phi'(k) = \left(f'(k) - \delta + 1\right)^2 + k2\left(f'(k) - \delta + 1\right) f''(k)$$

It is positive if and only if:

$$f'(k) - \delta + 1 > 2k\omega'(k)$$

Taking the example of the Cobb-Douglas production $y_t = Ak_t^{\alpha}$, we obtain:

$$\alpha A k^{\alpha - 1}(2\alpha - 1) > \delta - 1$$

Hence, under $\delta = 1$, $k^{*u} > k^{*e}$ holds if and only if $\alpha > 1/2$. The rest of Proposition 3 follows from similar rationales

8.2 Proof of Proposition 5

Under **A1**, the optimal capital under egalitarianism is still given by the standard Golden Rule, but the difference is that the optimal fertility rate n^* may differ significantly in the first-best and the second-best. To see this, note that, if one substitutes for the Golden Rule in the FOC for optimal fertility, one obtains:

$$k^{sb*}\left(f'(k^{sb*}) - \delta + 1\right)^2 = \bar{c}\pi(\bar{h})$$

Hence, if the LHS is increasing in k^{sb}, a higher RHS in the second-best in comparison to the first-best (i.e. $\bar{c}\pi(\bar{h}) > \bar{c}\tilde{\pi}$) must imply a higher optimal capital level. Therefore, if the condition:

$$f'(k) - \delta + 1 > 2k\omega'(k)$$

is true, then the second-best optimal capital is larger than the first-best optimal capital, and the second-best fertility is lower than the first-best fertility. Under a Cobb-Douglas technology and a full depreciation of capital, k^{*sb} exceeds k^{*fb} if and only if:

$$A\alpha k^{\alpha-1} > 2kA\alpha(1-\alpha)k^{\alpha-2} \iff \alpha > \frac{1}{2}$$

The rest of Proposition 5 follows from grouping that condition with the second part of Proposition 3.