Leverage vs. Feedback: Which Effect Drives the Oil Market?
Sofiane Aboura, Julien Chevallier

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Abstract

This article brings new insights on the role played by (implied) volatility on the WTI crude oil spot price. An increase in the volatility subsequent to an increase in the oil price (i.e. inverse leverage effect) remains the dominant effect as it might reflect the fear of oil consumers to face rising oil prices. However, this effect is amplified by an increase in the oil price subsequent to an increase in the volatility (i.e. inverse feedback effect) with a two-day delayed effect. This lead-lag relation between the oil price and its volatility is determinant for any type of trading strategy based on futures and options on the OVX implied volatility index, and thus is of interest to traders, risk- and fund-managers.

Keywords: WTI, Crude Oil Price, Implied Volatility, Leverage Effect, Feedback Effect.

JEL Codes: C4, G1, Q4.
However, contrary to equity markets, oil volatility seems to be positively correlated with past oil prices moves. While Geman and Shih (2009) have undoubtedly documented the existence of this inverse leverage effect in WTI prices, the authors mainly focus on the diffusion processes and neglect the identification of feedback effects. The same comment arises for the recent literature in energy economics (Agnolucci (2009), Larsson and Nossman (2011), Chang (2012)). In these contributions, the authors typically estimate various asymmetric GARCH models to take into account the inverse leverage effect in WTI prices, without a discussion on the feedback effects. Other authors have focused on the dynamic conditional correlations in WTI futures (see Lanza et al.(2006)).

The originality of this article stems from the fact that previous literature has neglected the investigation of leverage and feedback effects in WTI prices - the world’s most liquid commodity futures - while these effects have been investigated in-depth on equity markets. Hence, we aim at filling this gap. We formally test for the presence of feedback and leverage effects in the volatility of WTI prices by running OLS regressions in the spirit of Hibbert et al. (2008) and Fleming et al. (1995). To do so, this article uses an index of implied volatility applied to the oil market (equivalent to the VIX methodology) given that the asymmetry is stronger for implied volatility than for historical volatility (see e.g. Bollerslev and Zhu (2006)). Our study period goes from May 2007 to December 2011.

As a proxy for the Implied Volatility (IV) of the WTI price, we use the CBOE Crude Oil ETF Volatility Index (‘Oil VIX’, Ticker - OVX). The OVX measures the market’s expectation of 30-day volatility of crude oil prices by applying the VIX methodology to United States Oil Fund, LP (Ticker - USO) options spanning a wide range of strike prices. In previous studies (Agnolucci (2009), Larsson and Nossman (2011)), actual option prices were used to obtain the Black-Scholes implied volatility. To our best knowledge, this article is the first to make use of the CBOE OVX index. These CBOE indices based on the VIX methodology have been mainly used for equities (see Konstantinidi et al. (2008) for a recent contribution).

For the purpose of the econometric analysis, this article employs two additional volatility series. First, we extract the conditional volatility of the WTI spot price from an ARMA(1,1)-GARCH(1,1) model (following the examination of 25 competing GARCH models) as a proxy for the historical volatility. Second, we use intraday data on WTI futures prices contracts to obtain the realized volatility from the sum of intraday squared returns. The former time series is used to construct standardized returns, while the latter is used for robustness checks. Finally, we distinguish several sub-periods following the detection of structural breaks in the OVX index.

The article is organized as follows. Section 2 presents an analysis of the data. Section 3 opens with the methodology employed, while section 4 analyses the empirical results. Section 5 summarizes the main findings and concludes.

2 Data analysis

This section details (i) the data used, (ii) the filtering process to extract the historical GARCH volatility, (iii) the construction of the realized volatility, and (iv) structural break tests to detect instability in the Implied volatility series.

2.1 Data description

The database consists of 1,172 crude oil WTI Cushing daily spot closing prices and CBOE OVX Index spanning the period of time going from May 10, 2007 to December 30, 2011. The dataset
starts at the beginning of the availability of the OVX index, as provided by the CBOE. Hence, our
dataset corresponds to the most recent period of WTI price variation, characterized by the 2008 oil
price swing.

Insert Figure 1

Figure 1 plots the time series of raw WTI prices (solid black line), and the CBOE OVX Index
(dotted black line)\textsuperscript{1}. Various regimes are visible, where price and volatility seem to be either
positively or negatively correlated. During extreme periods of price and volatility, there seems
to appear a negative correlation since (i) when the volatility peaks at 100.42\% on December 11
2008, the corresponding WTI price per barrel is only 47.77\$, and (ii) when the WTI price peaks
at 145.31\$ on July 3 2008, the implied volatility declined to 44.39\%. This requires distinguishing
among volatility regimes (calm and turbulent). The gray rectangle represents the financial crisis
period as discussed in Section 2.4.

Insert Table 1

Table 1 shows excess skewness and kurtosis with a strong rejection of the normality hypothesis
for all series. Non reported correlogram for the WTI residuals exhibits no more dependence since
the $Q$-statistics for the series are lower than the critical values. The Engle’s Lagrange multiplier
test statistic shows no more evidence of remaining ARCH effects.

2.2 Data filtering process

The study tests 25 specifications of ARMA($p,q$) + GARCH($p,q$) models with $p = 1, \ldots, 5$ and $q = 1, \ldots, 5$. The selection favors the most parsimonious model given four criteria used for comparison:
the log-likelihood value, the Akaike criterion, the autocorrelogram of residuals and squared residuals,
and the Engle-ARCH test. The ARMA(1,1) + GARCH(1,1) model produces the best fit. An
alternative model allowing for leverage effects was considered to evaluate the contribution of the
negative residuals in the ARCH effect. However, this ARMA(1,1) + TGARCH(1,1) model yields
a threshold parameter that is not statistically significant. These results are not reproduced in the
article to conserve space, but are available upon request.

Define the market log-returns as $\{R_t\}_{t=1, \ldots, T}$ with $T = 1,172$ daily observations. The ARMA(1,1)
+ GARCH (1,1) specification is then given as follows:

$$R_t = \mu + \phi_1 R_{t-1} + \theta_1 \epsilon_{t-1} + \epsilon_t$$

where the innovations $\epsilon_t$ being functions of $Z_t$ and $\sigma_t$:

$$\epsilon_t = Z_t \sigma_t$$

with the standardized log-returns $Z_t$ are independent and identically distributed, such as:

$$Z_t \sim F_Z(0, 1)$$

\textsuperscript{1}The graph of the historical GARCH and realized volatilities series is available upon request. July 2008-July 2009
corresponds to a period of high variability in both volatility series.
where $F_Z$ is an unknown distribution of $Z$. The purpose of the time-varying $\sigma_t$ is to capture as much as possible of the conditional variance in the residual $\epsilon_t$ in order to leave $Z_t$ approximately independently and identically distributed:

$$\sigma_t^2 = \omega + \alpha (Z_{t-1}\sigma_{t-1})^2 + \beta \sigma_{t-1}^2$$  \hfill (4)

The results for the maximum likelihood estimation of this model are displayed in the last column of Table 1. This model provides a very good fit according to the selected criteria, and all the parameters are highly statistically significant. We therefore extract the standardized returns $\{Z_t\}_{t=1,\ldots,T}$ using a time-varying volatility model. The discussion is hereafter restricted to the standardized returns maxima $+Z$ and minima $-Z$.

### 2.3 Realized volatility

In addition to daily data, intraday data was purchased from TickData for the WTI light sweet crude oil futures contract. As is usual, we consider the continuous series of the front month contract using a rollover procedure which selects the largest volume each day to jump from one contract to the next$^2$.

We choose to work with open-to-close returns because overnight returns have shown to follow a very different dynamics. In addition, including overnight returns may alter our analysis when standardizing returns as we work with volatility computed with intraday transaction data.

The total number of ticks for the continuous time series of the front month contract is equal to 16,941,854. The trading period for the WTI futures is from 9:00 AM to 2:30 PM, which should provide 60 intraday returns each day. The cleaning procedure involves removing days with less than 60 intraday returns, days with more than 15 zero-returns, and days with less than 700 registered ticks.

According to Andersen et al. (2003), the realized volatility (RV) is defined as the sum of returns at a frequency $1/\Delta$ (typically one-day horizon), or:

$$RV_{t+1}(\Delta) \equiv \sum_{j=1}^{1/\Delta} R_{t+j, \Delta}^2$$ \hfill (5)

Finally, note that the time series of realized volatility has been constructed with a sampling frequency of 5-minute returns. As WTI futures are highly liquid assets, this sampling interval corresponds to standard practice to minimize the impact of microstructure noise. This time series will be used in the robustness checks of our econometric analysis.

### 2.4 Breakpoints detection

The second stage of the analysis consists in detecting structural breaks. They occur in a time series when the structure of the data generating mechanism underlying a set of observations changes. The detection of structural breaks in the volatility series allows to identify sub-periods, which will be useful during the robustness checks of our results.

$^2$We do not build our continuous series using a fixed number of days prior to maturity, thus avoiding calendar effects.
Bai and Perron (1997, 1998, 2003) have developed very popular techniques to estimate and test linear models with multiple structural changes. By borrowing notations from Zeileis and Kleiber (2005), we briefly recall below the multiple linear regression model with $m$ breaks (or, equivalently, $m + 1$ regimes):

$$y_t = x_t^\top \delta_j + u_t, \quad t = T_{j-1} + 1, \ldots, T_j, \quad j = 1, \ldots, m + 1$$

with $T$ the sample size, $T_0 = 0$ and $T_{m+1} = T$. The goal of the analysis is to determine the number and location of the breakpoints $T_j$, $j = 1, \ldots, m$. A search over $m$ is conducted, for $m \leq m^*$, where $m^*$ is fixed by the researcher. The minimum number of observations per segment, $h$, can also be exogenously set by the user.

In our setting, we have the OVX at hand for the WTI (from the VIX methodology based on option prices). The procedure of breakpoints detection unfolds as follows. First, we specify a linear model in which the dependent variable (i.e. OVX) is modelled by just an intercept, and $h$ is set equal to 30. Second, the estimation output (not reproduced here because of its length) comprises a triangular matrix containing the residual sums of squares (RSS), the extraction of breakpoints, and the corresponding information criteria, coefficient estimates and confidence intervals as described by Bai and Perron. Third, we plot all models for a maximum of $m = 1, \ldots, 37$ breaks along with the corresponding values of the Bayesian Information Criterion (BIC) and the RSS.

Insert Figure 2

In Figure 2, we observe that the BIC selects two breaks $m = 2$ for OVX, and that the decrease of the RSS when passing from a two break to a three break model is significant. These results may be summarized as pointing to two competing models: $m = 2$ or $m = 3$. However, because information criteria are often downward-biased, Bai and Perron argue in favor of the presence of an additional break. That is why we proceed with the estimation of a three-break model for OVX.

The output (not shown here) returns coefficient estimates with standard errors utilizing a kernel heteroskedasticity and autocorrelation consistent (HAC) estimator. We are also able to compute the confidence intervals (at the default 95% level) for the three breakpoints.

The estimated breakpoints are for OVX:

1. March 6, 2008
2. October 2, 2008
3. April 15, 2009

---

3Note this may be considered as a bandwidth or trimming parameter.
4The same procedure of breakpoints detection has been applied to the historical volatility extracted from the GARCH model. The results are qualitatively similar, and are available upon request.
5With a quadratic spectral kernel, prewhitening using a VAR(1) model and an AR(1) approximation for the automatic bandwidth selection (see Zeileis and Kleiber (2005)).
6The confidence intervals are derived from the distribution of the argmax functional of a process composed of two independent Brownian motions with different linear drifts and scales (see Bai and Perron (1997) for further details on this nonstandard distribution). This cumulative distribution function depends on three parameters which are associated with ratios of quadratic forms in the magnitude of the shifts and weighting matrices defined as segment-wise covariance matrices (Zeileis and Kleiber (2005)).
There is little variation (7 to 30 days) in the confidence intervals\(^7\) corresponding to the break dates. Hence, we obtain considerable certainty as to the location of the three breaks for OVX.

Overall, the breakpoints identified in the OVX volatility series point towards the definition of four sub-periods:

1. **Sub-period #1** (S1 = 207 observations): [May 10, 2007 - March 6, 2008]
2. **Sub-period #2** (S2 = 148 observations): [March 7, 2008 - October 2, 2008]
3. **Sub-period #3** (S3 = 135 observations): [October 3, 2008 - April 15, 2009]
4. **Sub-period #4** (S4 = 681 observations): [April 16, 2009 - December 30, 2011]

The Sub-period #1 corresponds to the trend of rising WTI prices. The Sub-period #2 corresponds to the period of time during which WTI prices have been characterized by a strong surge followed by a dramatic correction. The Sub-periods #3 and #4 capture two different dynamics, respectively near the end of the WTI price correction and after April 15, 2009 when the effect of the WTI oil price swing seems to disappear. This information will be re-used in Section 4 for robustness checks. The main sub-period of interest is therefore S3 (which roughly goes from the Lehman-Brothers bankruptcy to the end of the recession according to the NBER Business Cycle Dating Committee). It has been represented in Figure 1 with a gray rectangle.

Next, we proceed with our econometric investigation of the leverage and feedback effects in these volatility series.

### 3 Methodology

Our econometric methodology is mainly based on two articles to test for the presence of leverage and feedback effects in the volatility of WTI spot prices. Concerning leverage effects, we follow the approach by Hibbert et al. (2008), who test various hypotheses of return-volatility relation with ordinary least squares (OLS). Contrary to them, this study does not make use of raw returns but standardized returns in order to neutralize the role of volatility in the return generating process.

Concerning feedback effects, we follow the approach initiated by Fleming et al. (1995). In what follows, these two methodologies are presented in details.

For the leverage effect, the analysis is based on two sets of models labelled \(M1\) and \(M2\):

- \(M1\) tests whether oil contemporaneous returns, oil lagged returns, and the absolute value of returns explain the current implied volatility log-changes.

\[
M1 : \Delta \log OVX_t = \alpha_0 + \alpha_1 Z_t + \alpha_2 Z_{t-1} + \alpha_3 \Delta \log OVX_{t-1} + \alpha_4 |Z_t| + \alpha_5 \epsilon_{t-1} + \epsilon_t \tag{7}
\]

with \(\Delta \log OVX_t\) the log-variation of the OVX index, \(\alpha_0\) the intercept, \(Z_t\) standardized returns on the WTI prices, \(Z_{t-1}\) standardized returns lagged one period, \(|Z_t|\) the absolute value of the standardized returns, \(\Delta \log OVX_{t-1}\) the AR(1) component, \(\epsilon_{t-1}\) the MA(1) component, and \(\epsilon_t\) the error term. The AR(1)-MA(1) process has been specified following the Box-Jenkins methodology.

\(^7\)Available upon request.
**M2** introduces two dummy variables to decompose the impact of lagged returns in positive \(Z_{t-1}^+\) and negative \(Z_{t-1}^-\) impacts.

\[
M2: \Delta \log OVX_t = \alpha_0 + \alpha_1 Z_t + \alpha_2 Z_{t-1}^+ + \alpha_3 Z_{t-1}^- + \alpha_4 \Delta \log OVX_{t-1} + \alpha_5 |Z_t| + \alpha_6 \epsilon_{t-1} + \epsilon_t \tag{8}
\]

Hence, \(Z_{t-1}^+\) and \(Z_{t-1}^-\) replace the influence of \(Z_{t-1}\) in the former model.

For the feedback effect, the analysis is based on two sets of models labelled **M3** and **M4**:

- **M3** tests whether contemporaneous implied volatility log-changes, lagged implied volatility log-changes, and the absolute value of implied volatility log-changes explain the current oil standardized returns.

\[
M3: Z_t = \alpha_0 + \alpha_1 \Delta \log OVX_t + \alpha_2 \Delta \log OVX_{t-1} + \alpha_3 \Delta \log OVX_{t-2} + \alpha_4 \Delta \log OVX_{t-3} + \alpha_5 |\Delta \log OVX_t| + \epsilon_t \tag{9}
\]

with \(\Delta \log OVX_{t-1}\) to \(\Delta \log OVX_{t-3}\) lagged values of the OVX index log-changes up to three days, and \(|\Delta \log OVX_t|\) the absolute value of contemporaneous implied volatility log-changes.

- **M4** introduces two dummy variables to decompose the impact of lagged implied volatility log-changes in positive \(\Delta \log OVX_{t-1}^+\) and negative \(\Delta \log OVX_{t-1}^-\) impacts.

\[
M4: Z_t = \alpha_0 + \alpha_1 \Delta \log OVX_t + \alpha_2 \Delta \log OVX_{t-1}^+ + \alpha_3 \Delta \log OVX_{t-1}^- + \alpha_4 |\Delta \log OVX_t| + \epsilon_t \tag{10}
\]

Taken together, models **M1** to **M4** allow to test explicitly for the presence of leverage and feedback effects in the volatility of WTI spot prices.

4 Results

In what follows, estimation results are presented and analyzed. Besides, we discuss some measures used as robustness checks.

4.1 Empirical results

*Insert Table 2*

*Insert Table 3*

Tables 2 and 3 display the results respectively from models \(\{M1, M2\}\) and \(\{M3, M4\}\). By looking at the overall levels of the adjusted R-squared, the sets of regressions in the models \(\{M1, M2\}\) appear more satisfactory than for the models \(\{M3, M4\}\). Both tables show that the contemporaneous returns and volatility are negatively correlated, as judged by the \(\alpha_1\) parameter for \(Z_t\). In addition, the size effect parameter (i.e. absolute values) remains statistically significant, showing that the dependent variable is sensitive to the magnitude of shocks on the crude oil market.
Table 2 reveals for model M1 that the $\alpha_2$ coefficient for one-day lagged standardized returns $Z_{t-1}$ are positive and statistically significant at the 1% level during the full period, which characterizes the so-called ‘inverse leverage effect’ (i.e. that volatility is increasing following increasing returns). This effect was documented on the crude oil market by Geman and Shih (2009), among others, and we are able to confirm its existence with the OVX index. This stylized fact is significant in only the first sub-period, because of the turbulent nature of the overall period with various regime changes. Model M2 decomposes the one-day lagged standardized returns into positive and negative dummy variables to capture a possible sign effect. The inverse leverage effect parameter $\alpha_2$ for $Z_{t-1}^+$ becomes statistically significant in the first and last sub-period (S1 and S4), but not in the turbulent second and third sub-periods (S2 and S3). These latter results imply that the inverse leverage effect is not significant during periods of economic turmoil. In addition, the positive dummy variable remains significant contrary to the negative dummy variable, which means that only a higher oil price induces a higher implied volatility. To what extent can we explain this singular feature? A possible explanation is that if oil prices are driven by the supply-side of the market, the risk is expressed by a decrease of the price since this would affect negatively the revenues of oil-exporting countries. In that case, we expect the volatility to increase after a price decrease. As a consequence, this would manifest itself as a leverage effect much like on equity markets. On the opposite side, if oil prices are driven by the demand-side of the market, the risk increases when the price increases since this would affect negatively the expenses of oil-importing countries. Therefore, the volatility would increase consequently to a price increase. As a consequence, this would manifest itself as an inverse leverage effect (for a more detailed analysis of the supply and demand fundamentals on the crude oil market, see Chevillon and Riffart (2009)).

Table 3 displays the results for any type of feedback effects with models M3 and M4. Previous literature did not document this effect because it does not appear most of the time. For instance, in the model M3, the parameter $\alpha_2$ for $\Delta \log \text{OVX}_{t-1}$ is significant at the 10% level during the full period. However, precaution requires looking at the sub-periods and more particularly at the crisis period (S3). Indeed, this sub-period is represented in Figure 1 by a gray rectangle. It can be seen that it corresponds roughly to the financial crisis and the subsequent economic recession. More precisely, S3 (October 3, 2008 - April 15, 2009) begins two weeks after the Lehman Brothers bankruptcy and terminates six weeks before the end of the recession as defined by the NBER business cycle dating committee. During this crisis regime only, an inverse feedback effect (i.e. increasing returns following increasing volatility) is visible as the $\alpha_3$ coefficient for $\Delta \log \text{OVX}_{t-2}$ is significant at the 5% level and positive. Hence, model M3 reveals a delayed effect since an inverse feedback effect appears with a two-day lag. This delayed effect might be explained by the fact that volatility should be persistent enough to positively affect oil prices with little delay. In other words, the inverse leverage effect, which occurs with a one-day lag, should be significant enough to induce a regular feedback effect on the very same day, followed by an inverse feedback effect on the day after given the mean-reverting property of implied volatility. Finally, the model M4 displays the decomposition of the one-day lag volatility changes into positive and negative dummy variables. The results show that the $\alpha_3$ coefficient for $\Delta \log \text{OVX}_{t-1}$ is significant at the 5% level and negative during S3. This phenomenon is consistent with the regular (not inverse) feedback effect, since a significant decline in volatility induces a subsequent increase in the oil price. Hence, we have uncovered both regular and inverse feedback effects in the WTI crude oil spot price based on the OVX index of implied volatility. It is particularly interesting to notice that this result applies especially during the crisis period spanning October 2008-April 2009.

8See more information at http://www.nber.org/cycles/recessions.html
The understanding of the lead-lag relation between the crude oil price and the implied volatility index is of importance to modify the risk exposure and consequently the pay-off of any portfolio containing stocks of oil companies. While previous studies have described the inverse leverage effect as a stylized fact on the oil market (using realized or implied volatility), little attention has been paid to the existence of feedback effects during the recent crisis period and their trading implications. For that reason, our study contributes to the extant literature on the subject by using the OVX index of implied volatility. In line with our research, a trading strategy might hedge any long position on such a portfolio with OVX index futures contract. The futures contract value directly depends on the OVX value, since it is linear with the oil spot price. Therefore, it will reflect both the inverse leverage effect and the inverse feedback effect particularly during crisis periods. These results can therefore be of interest to traders, fund- and risk-managers alike.

4.2 Robustness checks

The sub-periods decomposition can be seen as a first round of robustness checks for the results obtained during the full-period. In addition, and similarly to Hibbert et al. (2008), we have introduced $RV_t$ in models $M1$ to $M4$ as an additional explanatory variable. These results, not reported here to conserve space but available upon request, did not change qualitatively the conclusions on the leverage and feedback effects reported in Tables 2 and 3.

In what follows, we focus mainly on the S3 crisis period. First, the predictive power of $RV_t$ is confirmed for the models $M1$ and $M2$. Indeed, the coefficient estimates for $RV_t$ are negative and significant at the 10% level for model $M1$, and at the 5% level for model $M2$. This result is logical, since implied volatility remains a measure of uncertainty. Suppose that the OPEC cartel takes a decision about the daily production of oil barrels. The uncertainty associated with this decision should increase $\Delta \log OVX_t$, but not necessary $RV_t$. Once the decision is taken, the uncertainty disappears. If the decision is to increase production, the uncertainty should disappear, which might decrease $\Delta \log OVX_t$ while $RV_t$ should decline or remains stable. On the contrary, if the cartel decides to cut production, $\Delta \log OVX_t$ should decline but $RV_t$ might climb. Most of the time, implied volatility and realized volatility are negatively correlated.

5 Conclusion

The oil return-volatility relation is widely documented in the financial economics literature. However, few articles explore the economic meaning of the inverse leverage effect, which characterizes an increase in the volatility subsequent to an increase in the oil price. In addition, no previous study has documented the existence of any type of feedback effects (either regular or inverse) for crude oil prices.

This article establishes that the inverse leverage effect is the dominant effect driving the WTI crude oil spot price, with a data sample spanning May 2007-December 2011. This result might reflect the fear for the oil importing countries, mainly western and major emerging countries, to face the risk attached to climbing oil prices. To our best knowledge, this article uncovers for the first time both regular and inverse feedback effects in crude oil prices, which can be observed only during the October 2008-April 2009 crisis period. We argue that the existence of both types of feedback effects is consistent with the mean-reverting property of implied volatility, as the CBOE OVX index is used.
The main practical application of our work may be stated as follows. The lead-lag relation between the oil price and its volatility is determinant for any type of trading strategy involving oil company stock portfolios, where the risks and pay-offs can be modified by futures and options on the OVX implied volatility index.

References

Figure 1: Time series of WTI raw prices and OVX index

Figure 1 displays the raw prices of WTI, and the CBOE OVX Index for Implied Volatility (OVX) from May, 10 2007 to December, 30 2011.
Figure 2: Breakpoints detection: RSS and BIC criteria for models up to 37 breaks

Figure 2 displays the number of breaks according to Bai and Perron’s methodology based on the corresponding values of the BIC and the RSS. Results are shown for OVX.
Table 1: Descriptive statistics

Table 1 presents the descriptive statistics for the raw prices of WTI, the corresponding standardized daily log-returns ($Z_t$), the CBOE OVX Index for Implied Volatility ($\Delta \log OVX_t$), and the historical GARCH volatility ($\sigma^2_t$) from May 10, 2007 to December 30, 2011. The realized volatility is denoted RV, and goes from May 10, 2007 to January 15, 2010. The historical volatility model is an ARMA(1,1) + GARCH(1,1). The last column reports the parameter estimates of Eq.(1)-(4).

<table>
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<tr>
<th></th>
<th>$WTI_t$</th>
<th>$Z_t$</th>
<th>$\Delta \log OVX_t$</th>
<th>$\sigma^2_t$</th>
<th>$RV_t$</th>
<th>$\sigma^2_t$</th>
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<tr>
<td>Mean</td>
<td>83.3461</td>
<td>0.0004</td>
<td>42.2020</td>
<td>41.1640</td>
<td>0.0005</td>
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<td>Median</td>
<td>81.8700</td>
<td>0.0010</td>
<td>38.7000</td>
<td>34.37640</td>
<td>0.0003</td>
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<td>Max.</td>
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<td>0.1641</td>
<td>100.4200</td>
<td>120.8378</td>
<td>0.0040</td>
<td>$\theta_1$</td>
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<td>Min.</td>
<td>30.2800</td>
<td>-0.1282</td>
<td>24.6700</td>
<td>17.4440</td>
<td>0.0001</td>
<td>$\omega$</td>
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<td>Std. Dev.</td>
<td>21.0028</td>
<td>0.0282</td>
<td>14.6933</td>
<td>21.4723</td>
<td>0.0006</td>
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<td>0.0235</td>
<td>1.7123</td>
<td>1.8963</td>
<td>2.4333</td>
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<td>Kurt.</td>
<td>3.3692</td>
<td>7.4663</td>
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<td>5.7442</td>
<td>9.5824</td>
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<td>JB</td>
<td>14.58232</td>
<td>973.4039</td>
<td>918.8976</td>
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<td>1895.925</td>
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</tbody>
</table>

Note: Max. stands for Maximum, Min. for Minimum, Std. Dev. for Standard Deviation, Skew. for Skewness, Kurt. for Kurtosis, JB for the Jarque-Bera statistic, Prob(JB) for the p-value of the Jarque-Bera statistic, and N for the number of daily observations.
Table 2: Leverage effects: Estimation results

Table 2 presents the estimation results for the leverage effects. The dependent variable is the current implied volatility log-changes $\Delta \log OV_X_t$.

| Leverage | Period | Adj. $R^2$ (%) | Intercept | $Z_t$ | $Z_{t-1}$ | $|Z_t|$ | AR(1) | MA(1) | $Z_{t-1}^+$ | $Z_{t-1}^-$ |
|----------|--------|---------------|-----------|-------|-----------|--------|-------|-------|-----------|-----------|
| $M1$     | Full   | 21.4          | -0.0200*** | -0.0137*** | 0.0043*** | 0.0265*** | 0.5728*** | -0.7796*** |
|          |        |               | (0.0017)   | (0.0013) | (0.0013) | (0.0020) | (0.0645) | (0.0494) |
| S1       | 6.0    | -0.0106**     | -0.0048*   | 0.0098*** | 0.0141*** | -0.5075  | 0.4997  |        |
|          |        |               | (0.0049)   | (0.0029) | (0.0029) | (0.0049) | (1.2011) | (1.2095) |
| S2       | 25.1   | -0.0198***    | 0.0015     | 0.0008  | 0.0265*** | 0.6320*** | -0.9925*** |
|          |        |               | (0.0030)   | (0.0026) | (0.0026) | (0.0036) | (0.0662) | (0.0096) |
| S3       | 18.6   | -0.0202***    | -0.0150*** | 0.0022  | 0.0245*** | 0.1805   | -0.5864*** |
|          |        |               | (0.0072)   | (0.0055) | (0.0055) | (0.0086) | (0.1971) | (0.1604) |
| S4       | 33.4   | -0.0192***    | -0.0196*** | 0.0024  | 0.0250*** | -0.7712*** | 0.7280*** |
|          |        |               | (0.0025)   | (0.0017) | (0.0017) | (0.0025) | (0.0492) | (0.0590) |
| $M2$     | Full   | 21.5          | -0.0220*** | -0.0136*** | 0.0257*** | 0.5840*** | -0.7905*** | 0.0077*** | 0.0011 |
|          |        |               | (0.0021)   | (0.0013) | (0.0021) | (0.0624) | (0.0472) | (0.0025) | (0.0024) |
| S1       | 6.3    | -0.0155**     | -0.0047*   | 0.0145*** | -0.4176  | 0.4176   | 0.0150*** | 0.0032 |
|          |        |               | (0.0065)   | (0.0029) | (0.0049) | (1.4150) | (1.4175) | (0.0052) | (0.0062) |
| S2       | 24.6   | -0.0199***    | 0.0015     | 0.0263*** | 0.6318*** | -0.9924*** | 0.0010   | 0.0005 |
|          |        |               | (0.0034)   | (0.0026) | (0.0045) | (0.0666) | (0.0100) | (0.0052) | (0.0053) |
| S3       | 18.1   | -0.0224***    | -0.0151*** | 0.0229** | 0.1602   | -0.5704*** | 0.0067   | -0.0021 |
|          |        |               | (0.0086)   | (0.0055) | (0.0092) | (0.2000) | (0.1647) | (0.0108) | (0.0106) |
| S4       | 33.4   | -0.0214***    | -0.0197*** | 0.0248*** | -0.7728*** | 0.7287*** | 0.0058*  | -0.0004 |
|          |        |               | (0.0031)   | (0.0017) | (0.0025) | (0.0489) | (0.0587) | (0.0032) | (0.0029) |

Note: Adj. $R^2$ (%) stands for the Adjusted R-squared in percentage. Full stands for Full period. S1 to S4 stand for the sub-periods 1 to 4. ***, **, * stand for statistical significance at the 1%, 5% and 10% levels, respectively. Standard errors are reported in parentheses below the parameter estimates. In this Table, we present the results for two different models to test the leverage effect in the implied volatility index for WTI spot prices:

$$M1: \Delta \log OV_X_t = \alpha_0 + \alpha_1 Z_t + \alpha_2 Z_{t-1} + \alpha_3 \Delta \log OV_{X_{t-1}} + \alpha_4 |Z_t| + \alpha_5 \epsilon_{t-1} + \epsilon_t$$

$$M2: \Delta \log OV_X_t = \alpha_0 + \alpha_1 Z_t + \alpha_2 Z_{t-1}^+ + \alpha_3 Z_{t-1}^- + \alpha_4 \Delta \log OV_{X_{t-1}} + \alpha_5 |Z_t| + \alpha_6 \epsilon_{t-1} + \epsilon_t$$
Table 3: Feedback effects: Estimation results

Table 3 presents the estimation results for the feedback effects. The dependent variable is the standardized daily log-returns $Z_t$.

| Feedback | Period | Adj. $R^2$ (%) | Intercept | $\Delta \log OV_{X_t}$ | $\Delta \log OV_{X_{t-1}}$ | $\Delta \log OV_{X_{t-2}}$ | $\Delta \log OV_{X_{t-3}}$ | $|\Delta \log OV_{X_t}|$ | $\Delta \log OV_{X_{t-1}}^+$ | $\Delta \log OV_{X_{t-1}}^-$ |
|----------|--------|---------------|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| **M3**   | Full   | 7.8           | 0.0669*** | -5.2995*** | -0.9222*** | -3.7012*** | 0.1299*** | -1.0038*** |                  |                  |
|          |        |               | (0.0402)  | (1.0263)     | (0.5596)     | (0.4871)     | (0.4303)     | (1.0893)     |                  |                  |
|          | S1     | 2.3           | 0.2767*** | -1.3919     | -1.7013     | 0.6050       | -1.0048     | -3.2939     |                  |                  |
|          |        |               | (0.0942)  | (1.9502)     | (1.7341)     | (1.4559)     | (1.6339)     | (2.4678)     |                  |                  |
|          | S2     | 1.78          | 0.0272    | 0.2250      | 0.9178      | -2.7475     | -0.1173     | -0.4277     |                  |                  |
|          |        |               | (0.1167)  | (3.4588)     | (2.0503)     | (2.3849)     | (2.1018)     | (3.4876)     |                  |                  |
|          | S3     | 6.5           | -0.1203   | -3.7121**   | -0.8381     | 2.8191**    | 0.8970      | 0.1637      |                  |                  |
|          |        |               | (0.1180)  | (1.5009)     | (1.5015)     | (1.2250)     | (1.0456)     | (1.7441)     |                  |                  |
|          | S4     | 16.1          | 0.0436    | -7.4010**   | -1.0493     | -0.7309     | 0.1110      | -0.8221     |                  |                  |
|          |        |               | (0.0515)  | (1.6176)     | (0.6926)     | (0.5771)     | (0.4909)     | (1.5387)     |                  |                  |

| **M4**   | Full   | 7.8           | 0.0723*** | -5.3174*** | -0.9803     | -0.9809     | -0.6782     |                  |                  |
|          |        |               | (0.0420)  | (0.5763)     | (0.7739)     | (0.8717)     | (1.0194)     |                  |                  |
|          | S1     | 2.1           | 0.2496*** | -1.2649     | -3.4259     | -0.6921     | -2.9461     |                  |                  |
|          |        |               | (0.1178)  | (1.7443)     | (2.5774)     | (2.6373)     | (3.5263)     |                  |                  |
|          | S2     | 2.1           | 0.0415    | 1.5399      | -0.8636     | 1.5320      | 2.6992      |                  |                  |
|          |        |               | (0.1461)  | (2.4346)     | (3.2170)     | (3.9348)     | (4.2802)     |                  |                  |
|          | S3     | 6.3           | -0.2152*  | -3.6955*** | -0.6570     | 1.0351      | -4.7716**   |                  |                  |
|          |        |               | (0.1255)  | (1.3066)     | (1.7423)     | (2.1596)     | (2.1970)     |                  |                  |
|          | S4     | 16.1          | 0.0673    | -7.5475*** | -0.5823     | -1.7143*    | 0.1326      |                  |                  |
|          |        |               | (0.0505)  | (0.6953)     | (0.9208)     | (1.0349)     | (1.2160)     |                  |                  |

**Note:** Adj. $R^2$ (%) stands for the Adjusted R-squared in percentage. Full stands for Full period. S1 to S4 stand for the sub-periods 1 to 4. ***, **, * stand for statistical significance at the 1%, 5% and 10% levels, respectively. Standard errors are reported in parentheses below the parameter estimates. In this Table, we present the results for two different models to test the feedback effect in the standardized returns of WTI spot prices:

$M3: Z_t = \alpha_0 + \alpha_1 \Delta \log OV_{X_t} + \alpha_2 \Delta \log OV_{X_{t-1}} + \alpha_3 \Delta \log OV_{X_{t-2}} + \alpha_4 \Delta \log OV_{X_{t-3}} + \alpha_5 |\Delta \log OV_{X_t}| + \epsilon_t$

$M4: Z_t = \alpha_0 + \alpha_1 \Delta \log OV_{X_t} + \alpha_2 \Delta \log OV_{X_{t-1}}^+ + \alpha_3 \Delta \log OV_{X_{t-1}}^- + \alpha_4 |\Delta \log OV_{X_t}| + \epsilon_t$