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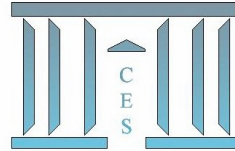
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## Wage bargaining with discount rates varying in time under exogenous strike decisions

Ahmet OZKARDAS, Agnieszka RUSINOWSKA

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# Wage bargaining with discount rates varying in time under exogenous strike decisions

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**Abstract.** In this paper, we present a non-cooperative wage bargaining model in which preferences of both parties, a union and a firm, are expressed by sequences of discount factors varying in time. We determine subgame perfect equilibria for three cases when the strike decision of the union is exogenous: the case when the union is supposed to go on strike in each period in which there is a disagreement, the case when the union is committed to go on strike only when its own offer is rejected, and the case when the union is supposed to go never on strike.

**JEL Classification:** J52, C78

**Keywords:** union - firm bargaining, strike, alternating offers, varying discount rates, subgame perfect equilibrium

## 1 Introduction

Collective wage bargaining between firms and unions (workers' representatives, workers' groups, etc.) is one of the most central issues in labor economics. Both cooperative and non-cooperative approaches to collective wage bargaining are applied in the literature; for surveys of bargaining models see e.g. Osborne and Rubinstein (1990); Muthoo (1999). By using a static (axiomatic) approach initiated by Nash (1950), some labor economists determine the levels of wage and employment between unions and firms; see e.g. McDonald and Solow (1981), Nickell and Andrews (1983). Following Binmore (1987); Binmore et al. (1986), also equilibria that are obtained can be used to measure the bargaining power of both parties (union and firm). Svejnar (1986) estimates bargaining powers derived from a Nash bargaining model as functions of exogenous variables. Doiron (1992) extends Svejnar's work and uses a generalized Nash bargaining (GNB) model to generate a wage and employment contract. The weights of the respective utilities in the GNB problem are the bargaining powers of the parties. The approach based on cooperative games is presented e.g. in Levy and Shapley (1997), where a wage negotiation is modeled as an oceanic game and the Shapley value (Shapley (1953)) is used for the solution concept. Some authors apply a dynamic (strategic) approach to wage bargaining and focus on the concept of *subgame perfect equilibrium* (that will be denoted here by *SPE*). Several

modified versions of Rubinstein's game (Rubinstein (1982); Fishburn and Rubinstein (1982)) to firm-union negotiations are proposed. Cripps (1997) who analyzes the model of investment, considers e.g. the alternating-offer bargaining game over binding long-term wage contracts and describes a stationary SPE of the game. Also Conlin and Furusawa (2000) consider a three-stage firm-union bargaining game and investigate SPE of the game.

Haller and Holden (1990) extend Rubinstein's model to incorporate the choice of calling a strike in union-firm negotiations. It is assumed that in each period until an agreement is reached, the union must decide whether or not it will strike in that period. It is shown that in this model there is no longer a unique SPE, and that strikes with a length in real time can occur in SPE. Both parties have the same discount factor  $\delta$ . Fernandez and Glazer (1991) consider essentially the same wage-contract sequential bargaining, but with the union and firm using different discount factors  $\delta_u, \delta_f$ . We will be referring to their model as the *F-G model*. The authors also show that there exist SPE in which the union engages in several periods of strikes prior to reaching a final agreement. Holden (1994) assumes a weaker type of commitment in the F-G model. He proves that if the union is committed to strike for two periods unless there is an agreement before that, then there is a unique SPE, although not always the same as the one of Rubinstein's model. In the F-G model, the union achieves the maximum-wage contract by threatening an alternating strike strategy (strike when the firm rejects an offer but continue working at the old contract wage when the firm makes an unacceptable offer). Bolt (1995) shows that this SPE only holds if  $\delta_u \leq \delta_f$ . For  $\delta_u > \delta_f$ , SPE is restored by modifying the alternating strike strategies. Houba and Wen (2008) apply the method of Shaked and Sutton (1984) to derive the exact bounds of equilibrium payoffs in the F-G model and characterize the equilibrium strategy profiles that support these extreme equilibrium payoffs for all discount factors. The authors emphasize that when applying the Shaked and Sutton's backward induction argument one must verify the presumption that continuation payoffs are bounded from above by the bargaining frontier.

There are other numerous works that study strikes in bargaining between unions and firms, both from a theoretical and an empirical point of view. In Hayes (1984) it is shown that although a strike seems to be a Pareto-inefficient outcome of bargaining, it can be the outcome of rational behavior of both agents. In a situation with asymmetric information, for instance, strikes can be used to gain more information. Sopher (1990) reports on an experiment on the frequency of disagreement (strikes) in a set of "shrinking pie" games in which parties bargain in consecutive periods over how to divide a quantity of money. Although bargaining theory predicts that no disagreement is involved in the outcome of a two-person pie-splitting game with complete information, in the experiment strikes occurred frequently in the games and they did not disappear over time. This can be supported by the joint-cost theory of strikes which attributes strikes to the costs of negotiation. Robinson (1999) uses the theory of repeated games to present a dynamic model of strikes as part of a constrained efficient enforcement mechanism of a labor contract. In particular, he shows that under imperfect observations strikes occur in equilibrium.

Although numerous versions of wage bargaining between unions and firms are presented in the literature, a common assumption is the stationarity of parties' preferences that are described by constant discount factors. In real bargaining, however, due to time preferences, discount factors of the parties may vary in time. Cramton and Tracy (1994)

emphasize that stationary bargaining models are very rare in real-life situations. They study wage bargaining with time-varying threats in which the union is uncertain about the firm's willingness to pay. Rusinowska (2000, 2001, 2002, 2004) generalizes the original model of Rubinstein to a bargaining model with non-stationary preferences.

The *aim of this paper* is to generalize the F-G model to the *wage union-firm bargaining* in which both parties have preferences expressed by *sequences of discount factors varying in time*. In the whole paper it is assumed that strategies of the parties are independent of the previous history of the game. We determine SPE for three cases when the strike decision of the union is *exogenous*: the case when the union is supposed to go on strike in each period in which there is a disagreement, the case when the union is committed to go on strike only when its own offer is rejected, and the case when the union is supposed to go never on strike. First, we show that in the exogenous 'always-strike' case, we cannot apply the generalization of Rubinstein's bargaining model investigated in Rusinowska (2000, 2001) to determine SPE. Next, we present the unique SPE for the considered case. We also determine the unique SPE for each of the two other cases. In order to find these SPE, we need to solve infinite series of certain linear equations, what we do with a help of infinite matrices theory.

The remainder of the paper is as follows. In Section 2 we briefly recall some basic information on infinite matrices that is used for proving our results. In Section 3 we present the generalized model and determine the conditions on the sequences of discount rates under which the generalization is well defined. Section 4 concerns the exogenous strike decision, when the union is supposed to go on strike in each period in which there is a disagreement. In Section 5 we analyze the exogenous strike decision, when the union goes on strike only after rejection of its own proposals. In Section 6 we determine SPE for the exogenous no-strike decision case when the union is supposed to go never on strike. We conclude in Section 7 with mentioning some possible applications of the model and our future research agenda.

## 2 Preliminaries on regular triangular systems and infinite matrices

In this short section, we recall the basic definitions and facts on regular triangular systems and infinite matrices that will be used later on in the paper. For a detailed information on infinite systems and matrices, see, e.g., Combes (1957); Davis (1950); Cooke (1950).

Let us consider the following infinite system of equations:

$$\sum_{j=1}^{\infty} a_{ij}x_j = y_i \quad (i = 1, 2, \dots, n, \dots) \quad (1)$$

where  $a_{ij}, x_i, y_i \in \mathbb{R}$  and  $x_i$  are the unknown here. We can re-write this system in the matrix form

$$AX = Y \quad (2)$$

where  $A$  is the matrix of coefficients  $a_{ij}$ , and  $X, Y$  are the column vectors, i.e.,

$$A = [a_{ij}]_{i,j \in \mathbb{N}^+}, \quad X = [(x_i)_{i \in \mathbb{N}^+}]^T, \quad Y = [(y_i)_{i \in \mathbb{N}^+}]^T$$

with  $\mathbb{N}^+$  denoting positive integers and the upper index  $T$  denoting the transposition.

The *triangular system* of equations is the system (1) with  $a_{ij} = 0$  for all  $j < i$ . Additionally, if  $a_{ii} \neq 0$  then the system is called *regular triangular*. Without loss of generality we can assume that  $a_{ii} = 1$ . The corresponding matrices will be also called *triangular* or *regular triangular*.

One of the fundamental properties is the following:

*A regular triangular matrix  $A$  possesses a unique inverse matrix,*  
that is, a matrix  $B$  such that  $BA = I$ , where  $I$  is the infinite identity matrix  
(i.e.,  $I = [i_{jk}]_{j,k \in \mathbb{N}^+}$ ,  $i_{jj} = 1$  for each  $j \in \mathbb{N}^+$  and  $i_{jk} = 0$  for every  $j \neq k$ ).

In order to find the solution  $X$  of the system (2), we simply use  $X = BY$ . Let  $B = [b_{ij}]_{i,j \in \mathbb{N}^+}$ . If we suppose that  $a_{ii} = 1$  for each  $i \in \mathbb{N}^+$ , then the elements of the first row of  $B$  are determined by

$$b_{1,1} = 1, \quad b_{1,1}a_{1,2} + b_{1,2} = 0, \dots, b_{1,1}a_{1,j} + b_{1,2}a_{2,j} + \dots + b_{1,j} = 0, \dots \quad (3)$$

and the elements of the  $i$ th row (for  $i > 1$ ) are determined by

$$b_{i,1} = b_{i,2} = \dots = b_{i,i-1} = 0, \quad b_{i,i} = 1, \quad b_{i,i}a_{i,i+1} + b_{i,i+1} = 0, \dots \quad (4)$$

### 3 Wage bargaining with discount factors varying in time

The bargaining procedure between the union and the firm, as presented in Fernandez and Glazer (1991), and Haller and Holden (1990) is the following. There is an existing wage contract, that specifies the wage that a worker is entitled to per day of work, which has come up for renegotiation. Two parties (union and firm) bargain sequentially over discrete time and a potentially infinite horizon. They alternate in making offers of wage contracts that the other party is free to accept or reject. Upon either party's rejection of a proposed wage contract, the union must decide whether or not to strike in that period. The share of the union under the previous contract is  $w_0$ , where  $w_0 \in (0, 1]$ . By the new contract, the union and firm will divide the added value (normalized to 1) with new shares of the parties, where the union's share is  $W \in [0, 1]$  and the firm's share is  $1 - W$ . Figure 1 presents the first three periods of this wage bargaining.

ABOUT HERE FIGURE 1

The union proposes  $x_0$  (share for itself). If the firm accepts the new wage contract, the agreement is reached and the payoffs are  $(x_0, 1 - x_0)$ . If the firm rejects it, then the union can either go on strike, and then both parties get  $(0, 0)$  in the current period, or go on with the previous contract with payoffs  $(w_0, 1 - w_0)$ . If the union goes on strike, it is the firm's turn to make a new offer  $y_1$ , which assigns  $y_1$  to the union and  $(1 - y_1)$  to the firm. This procedure goes on until an agreement is reached, where  $x_{2t}$  denotes the offer of the union made in an even-numbered period  $2t$ , and  $y_{2t+1}$  denotes the offer of the firm made in an odd-numbered period  $(2t + 1)$ .

The key difference between the F-G model and our wage bargaining lies in preferences of the union and the firm and, as a consequence, in the payoff functions of both parties.

While Fernandez and Glazer (1991) assume stationary preferences described by constant discount rates  $\delta_u$  and  $\delta_f$ , we consider a wage bargaining in which preferences of the union and the firm are described by *sequences of discount factors varying in time*,  $(\delta_{u,t})_{t \in \mathbb{N}}$  and  $(\delta_{f,t})_{t \in \mathbb{N}}$ , respectively, where

$$\delta_{u,t} = \text{discount factor of the union in period } t \in \mathbb{N}, \quad \delta_{u,0} = 1, \quad 0 < \delta_{u,t} < 1 \text{ for } t \geq 1$$

$$\delta_{f,t} = \text{discount factor of the firm in period } t \in \mathbb{N}, \quad \delta_{f,0} = 1, \quad 0 < \delta_{f,t} < 1 \text{ for } t \geq 1$$

The *result* of the wage bargaining is either a pair  $(W, T)$ , where  $W$  is the wage contract agreed upon and  $T \in \mathbb{N}$  is the number of proposals rejected in the bargaining, or a *disagreement* denoted by  $(0, \infty)$  and meaning the situation in which the parties never reach an agreement. We introduce the following notation. Let for each  $t \in \mathbb{N}$

$$\delta_u(t) := \prod_{k=0}^t \delta_{u,k}, \quad \delta_f(t) := \prod_{k=0}^t \delta_{f,k} \quad (5)$$

and for  $0 < t' \leq t$

$$\delta_u(t', t) := \frac{\delta_u(t)}{\delta_u(t' - 1)} = \prod_{k=t'}^t \delta_{u,k}, \quad \delta_f(t', t) := \frac{\delta_f(t)}{\delta_f(t' - 1)} = \prod_{k=t'}^t \delta_{f,k} \quad (6)$$

The utility of the result  $(W, T)$  for the union is equal to

$$U(W, T) = \sum_{t=0}^{\infty} \delta_u(t) u_t \quad (7)$$

where  $u_t = W$  for each  $t \geq T$ , and if  $T > 0$  then for each  $0 \leq t < T$

$$\begin{aligned} u_t &= 0 && \text{if there is a strike in period } t \in \mathbb{N} \\ u_t &= w_0 && \text{if there is no strike in period } t. \end{aligned}$$

The utility of the result  $(W, T)$  for the firm is equal to

$$V(W, T) = \sum_{t=0}^{\infty} \delta_f(t) v_t \quad (8)$$

where  $v_t = 1 - W$  for each  $t \geq T$ , and if  $T > 0$  then for each  $0 \leq t < T$

$$\begin{aligned} v_t &= 0 && \text{if there is a strike in period } t \\ v_t &= 1 - w_0 && \text{if there is no strike in period } t. \end{aligned}$$

The utility of the disagreement is equal to

$$U(0, \infty) = V(0, \infty) = 0 \quad (9)$$

Since the utilities for both parties depend on the infinite series, the first question is under which sequences of discount rates these utilities are well defined, i.e., under which sequences the infinite series are convergent.

**Remark 1** *The necessary conditions for the convergence of the infinite series which define  $U(W, T)$  and  $V(W, T)$  in (7) and (8) are the following:*

$$\delta_u(t) \rightarrow_{t \rightarrow +\infty} 0 \quad \text{and} \quad \delta_f(t) \rightarrow_{t \rightarrow +\infty} 0 \quad (10)$$

*However, these are not sufficient conditions.*

**Proof:** The necessary conditions come immediately from the necessary condition of the convergence of the infinite series. In order to show that it is not a sufficient condition, let us consider  $\delta_{u,0} = 1$ ,  $\delta_{u,k} = \frac{k}{k+1}$  for each  $k \geq 1$ . Then

$$\delta_u(t) = \frac{1}{2} \cdot \frac{2}{3} \cdots \frac{t}{t+1} = \frac{1}{t+1} \rightarrow_{t \rightarrow +\infty} 0$$

If the agreement  $W$  is reached immediately, then

$$U(W, 0) = \sum_{t=0}^{\infty} \delta_u(t)W = W \sum_{t=0}^{\infty} \frac{1}{t+1} = W \left( 1 + \frac{1}{2} + \frac{1}{3} + \cdots \right)$$

which is divergent. Similarly, if  $W$  is reached in a certain period  $T > 0$ , then we have

$$U(W, T) = \sum_{t=0}^{T-1} \delta_u(t)u_t + \sum_{t=T}^{\infty} \delta_u(t)W = \sum_{t=0}^{T-1} \delta_u(t)u_t + W \sum_{t=T}^{\infty} \frac{1}{t+1}$$

■

**Remark 2** *If  $(\delta_{u,t})_{t \in \mathbb{N}}$  as well as  $(\delta_{f,t})_{t \in \mathbb{N}}$  are bounded by a certain number smaller than 1, i.e., if*

$$\text{there exist } \Delta_u < 1 \text{ and } \Delta_f < 1 \text{ such that } \delta_{u,t} \leq \Delta_u \text{ and } \delta_{f,t} \leq \Delta_f \text{ for each } t \in \mathbb{N} \quad (11)$$

*then the series which define  $U(W, T)$  and  $V(W, T)$  in (7) and (8) are convergent. The sufficient conditions given in (11) are not necessary conditions.*

**Proof:** Assume that there exist  $\Delta_u < 1$  and  $\Delta_f < 1$  such that  $\delta_{u,t} \leq \Delta_u$  and  $\delta_{f,t} \leq \Delta_f$  for each  $t \in \mathbb{N}$ . Let  $W$  be reached in period  $T$ . Suppose  $T = 0$ . Since  $\delta_{u,t} \leq \Delta_u$  for each  $t \in \mathbb{N}$ , we have

$$0 < \delta_u(t)W \leq (\Delta_u)^t W$$

Since  $\sum_{t=0}^{\infty} (\Delta_u)^t$  is the convergent geometric series,  $U(W, 0)$  is also convergent by virtue of the comparison test. Similarly for  $T > 0$ . The proof for the firm is analogous.

Consider the following sequence of discount rates:  $\delta_{u,0} = 1$ ,  $\delta_{u,k} = \frac{k}{k+2}$  for each  $k \geq 1$ . Obviously the sequence does not satisfy the condition (11). However, we have for  $t \geq 1$

$$\delta_u(t) = \frac{1}{3} \cdot \frac{2}{4} \cdot \frac{3}{5} \cdots \frac{t}{t+2} = \frac{2}{(t+1)(t+2)} \rightarrow_{t \rightarrow +\infty} 0$$



If the agreement  $W$  is reached immediately, then

$$U(W, 0) = \sum_{t=0}^{\infty} \delta_u(t)W = W + W \sum_{t=1}^{\infty} \frac{2}{(t+1)(t+2)}$$

which is convergent by virtue of the comparison test:  $\frac{1}{t^2} \geq \frac{1}{(t+1)(t+2)}$  and we know that  $\sum_{t=1}^{\infty} \frac{1}{t^2}$  is convergent. The proof is similar, if  $W$  is reached in a certain period  $T > 0$ . ■

**Remark 3** Every decreasing sequence  $(\delta_{u,t})_{t \in \mathbb{N}}$  ( $(\delta_{f,t})_{t \in \mathbb{N}}$ , respectively) gives the convergent series defined in (7) (defined in (8), respectively). Some increasing sequences lead to the convergent series as well.

**Proof:** It results immediately from the fact that every decreasing sequence satisfies (11). Take  $\delta_{u,0} = 1$ ,  $\delta_{u,k} = \frac{1}{2} - \frac{1}{2^{k+2}}$  for each  $k \geq 1$ . The sequence is increasing ( $\delta_{u,k+1} > \delta_{u,k}$  for each  $k \geq 1$ ), and  $\delta_{u,k} < \frac{1}{2}$ , so it satisfies the sufficient condition for convergence of the infinite series defined in (7). ■

**Remark 4** We restrict our analysis to the case in which the discount rates satisfy condition (11). Hence, in particular, for each  $t \in \mathbb{N}$ ,

$$\sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \leq \frac{\Delta_f}{1 - \Delta_f}, \quad \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) \leq \frac{\Delta_u}{1 - \Delta_u} \quad (12)$$

Moreover, we assume that the strategies do not depend on the former history of the game. In the whole paper we make the following assumption:

**Assumption 1** Let  $\overline{W}^{2t}$  denote an offer of the union in period  $2t$  ( $t \in \mathbb{N}$ ), and let  $\overline{Z}^{2t+1}$  denote an offer of the firm in period  $(2t+1)$ . We consider only the family of strategies  $(s_u, s_f)$ , where in each period  $(2t+1)$  the union accepts an offer  $y$  of the firm if and only if  $y \geq \overline{Z}^{2t+1}$ , and in each period  $2t$  the firm accepts an offer  $x$  of the union if and only if  $x \leq \overline{W}^{2t}$ . A strategy of the union specifies additionally its strike decision.

## 4 Going always on strike under a disagreement

We analyze the case when the strike decision of the union is exogenous, and the union is supposed to go on strike in each period in which there is a disagreement. Fernandez and Glazer (1991) show that in such a case, if preferences are defined by constant discount factors, then there is a unique SPE of the wage bargaining game. It coincides with the SPE in Rubinstein's model and leads to an agreement  $\overline{W} = \frac{1-\delta_f}{1-\delta_u\delta_f}$  reached in period 0. Obviously, this equilibrium result does not hold if the parties' preferences are expressed by discount factors varying in time. We determine SPE in the model with the exogenous strike decision and discount factors varying in time, i.e., we generalize the equilibrium result obtained in Fernandez and Glazer (1991). Since for determining a SPE we must consider any possible subgame of the game, and the utilities of the parties are given by the infinite series and take into account any period till infinity, we have the following:

**Fact 1** Consider the generalized F-G model with preferences of the union and the firm described by the sequences of discount factors  $(\delta_{i,t})_{t \in \mathbb{N}}$ , where  $\delta_{i,0} = 1$ ,  $0 < \delta_{i,t} < 1$  for  $t \geq 1$ ,  $i = u, f$ . Assume that the strike decision is given exogenously and the union is supposed to go on strike in every period in which there is a disagreement. Then  $(s_u, s_f)$  in Assumption 1 is a SPE of this game if and only if the offers satisfy the following infinite system of equations: for each  $t \in \mathbb{N}$

$$\left(1 - \overline{W}^{2t}\right) + \left(1 - \overline{W}^{2t}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) = \left(1 - \overline{Z}^{2t+1}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \quad (13)$$

and

$$\overline{Z}^{2t+1} + \overline{Z}^{2t+1} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) = \overline{W}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) \quad (14)$$

In Rusinowska (2000, 2001) it is shown that in the generalized Rubinstein model with preferences of the players described by the sequences of discount factors  $(\delta_{i,t})_{t \in \mathbb{N}}$ , where  $\delta_{i,0} = 1$ ,  $0 < \delta_{i,t} < 1$  for  $t \geq 1$ ,  $i = u, f$ , such that  $\prod_{j=1}^{t+1} \delta_{u,2j} \delta_{f,2j-1} \rightarrow_{t \rightarrow +\infty} 0$ , there is only one SPE, where the offers of the players are as follows<sup>1</sup>:

$$\overline{W}^0 = 1 - \delta_{f,1} + \sum_{n=1}^{+\infty} \left( \prod_{k=1}^n \delta_{u,2k} \delta_{f,2k-1} \right) (1 - \delta_{f,2n+1}) \quad (15)$$

$$\overline{W}^{2t+2} = \frac{\overline{W}^{2t} + \delta_{f,2t+1} - 1}{\delta_{u,2t+2} \delta_{f,2t+1}} \quad \text{and} \quad \overline{Z}^{2t+1} = \overline{W}^{2t+2} \delta_{u,2t+2} \quad \text{for each } t \in \mathbb{N} \quad (16)$$

Unfortunately, we cannot apply this result to the generalized F-G model with the exogenous strike decision.

**Fact 2** The generalized F-G model in which the strike decision is given exogenously and the union is supposed to go on strike in every period in which there is a disagreement, does not coincide with the generalized Rubinstein model, and in general the SPE of the two models are different.

**Proof:** In order to find the SPE offers in the generalized Rubinstein model, we need to solve the following infinite system of equations: for each  $t \in \mathbb{N}$

$$\left(1 - \overline{W}^{2t}\right) = \left(1 - \overline{Z}^{2t+1}\right) \delta_{f,2t+1} \quad \text{and} \quad \overline{Z}^{2t+1} = \overline{W}^{2t+2} \delta_{u,2t+2} \quad (17)$$

which leads to the solution given in (15) and (16). In order to find the SPE offers in the generalized F-G model with the exogenous “going always to strike” decision, we need to solve the infinite system of equations given by (13) and (14), for each  $t \in \mathbb{N}$ .

For the model with constant discount rates  $\delta_u$  and  $\delta_f$ , these two infinite systems of equations are equivalent. For each  $t \in \mathbb{N}$

$$\sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) = \frac{\delta_f}{1 - \delta_f} \quad \text{and} \quad \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) = \frac{\delta_u}{1 - \delta_u}$$

<sup>1</sup> We keep here the same notation  $u$  and  $f$  for the players, although Rusinowska (2000, 2001) did not consider union-firm bargaining, but only a generalization of the original Rubinstein model in which preferences of players 1 and 2 were described by discount rates varying in time.

so inserting these sums into the system of equations (13) and (14) gives us equivalently the system of equations (17). However, these two infinite systems of equations are NOT equivalent if we consider the generalized F-G model, since for any  $t \neq t'$ , usually

$$\sum_{k=t}^{\infty} \delta_f(t, k) \neq \sum_{k=t'}^{\infty} \delta_f(t', k)$$

$$\sum_{k=t}^{\infty} \delta_u(t, k) \neq \sum_{k=t'}^{\infty} \delta_u(t', k)$$

As an illustrative example, consider a very simple model with the following discount rates:  $\delta_{f,1} = \delta_{u,1} = \frac{1}{2}$ , and  $\delta_{f,t} = \delta_{u,t} = \frac{1}{3}$  for each  $t \geq 2$ . Then

$$\sum_{k=1}^{\infty} \delta_f(1, k) = \frac{3}{4}, \quad \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) = \frac{1}{2} \text{ for each } t \geq 1$$

In other words, solving the system (17) gives  $\bar{W}^0 = \frac{5}{8}$ ,  $\bar{W}^{2t} = \frac{3}{4}$  for each  $t \geq 1$ ,  $\bar{Z}^{2t+1} = \frac{1}{4}$  for each  $t \in \mathbb{N}$ , but this solution does not satisfy the first equation of the system (13), i.e.,

$$\left(1 - \bar{W}^0\right) + \left(1 - \bar{W}^0\right) \sum_{k=1}^{\infty} \delta_f(1, k) \neq \left(1 - \bar{Z}^1\right) \sum_{k=1}^{\infty} \delta_f(1, k)$$

■

By solving the infinite system (13) and (14), we get the following:

**Theorem 1** *Consider the generalized F-G model with preferences of the union and the firm described by the sequences of discount factors  $(\delta_{i,t})_{t \in \mathbb{N}}$ , where  $\delta_{i,0} = 1$ ,  $0 < \delta_{i,t} < 1$  for  $t \geq 1$ ,  $i = u, f$ . Assume that the strike decision is given exogenously and the union is supposed to go on strike in every period in which there is a disagreement. Then there is the unique SPE of the form  $(s_u, s_f)$  in Assumption 1, in which the offers of the parties are given by*

$$\bar{W}^0 = \frac{1}{1 + \sum_{k=1}^{\infty} \delta_f(1, k)} +$$

$$\sum_{m=0}^{\infty} \frac{1}{1 + \sum_{k=2m+3}^{\infty} \delta_f(2m+3, k)} \prod_{j=0}^m \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{\left(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k)\right) \left(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)\right)}$$
(18)

and for each  $t \in \mathbb{N}$

$$\bar{W}^{2t+2} = \frac{\left[\bar{W}^{2t} \left(1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)\right) - 1\right] \left(1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)\right)}{\sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}$$
(19)

$$\bar{Z}^{2t+1} = \frac{\bar{W}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}{1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}$$
(20)

**Proof:** By virtue of Fact 1, we need to solve the infinite system of equations (13) and (14), which can be equivalently written, for each  $t \in \mathbb{N}$ , as

$$\overline{W}^{2t} - \overline{Z}^{2t+1} \frac{\sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)}{1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)} = \frac{1}{1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)} \quad (21)$$

and

$$\overline{Z}^{2t+1} - \overline{W}^{2t+2} \frac{\sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}{1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)} = 0 \quad (22)$$

From (22) we get immediately (20), and inserting  $\overline{Z}^{2t+1}$  into (21) gives (19). In order to find  $\overline{W}^0$  we can use one of the following two methods:

#### Method 1

Note that the infinite system of (21) and (22) is a regular triangular system  $AX = Y$ , with  $A = [a_{ij}]_{i,j \in \mathbb{N}^+}$ ,  $X = [(x_i)_{i \in \mathbb{N}^+}]^T$ ,  $Y = [(y_i)_{i \in \mathbb{N}^+}]^T$ , where for each  $t, j \geq 1$

$$a_{t,t} = 1, \quad a_{t,j} = 0 \text{ for } j < t \text{ or } j > t + 1 \quad (23)$$

and for each  $t \in \mathbb{N}$

$$a_{2t+1,2t+2} = -\frac{\sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)}{1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)}, \quad a_{2t+2,2t+3} = -\frac{\sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}{1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)} \quad (24)$$

$$x_{2t+1} = \overline{W}^{2t}, \quad x_{2t+2} = \overline{Z}^{2t+1}, \quad y_{2t+1} = \frac{1}{1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)}, \quad y_{2t+2} = 0 \quad (25)$$

We know that any regular triangular matrix  $A$  possesses the (unique) inverse matrix  $B$ , i.e., there exists  $B$  such that  $BA = I$ , where  $I$  is the infinite identity matrix. The matrix  $B = [b_{ij}]_{i,j \in \mathbb{N}^+}$  is also regular triangular, and its elements are the following:

$$b_{t,t} = 1, \quad b_{t,j} = 0 \text{ for each } t, j \geq 1 \text{ such that } j < t \quad (26)$$

for each  $t \in \mathbb{N}$

$$b_{2t+1,2t+2} = \frac{\sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)}{1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)}, \quad b_{2t+2,2t+3} = \frac{\sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}{1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)} \quad (27)$$

and for each  $t, m \in \mathbb{N}$  and  $m > t$

$$b_{2t+2,2m+2} = \prod_{j=t}^{m-1} \frac{\sum_{k=2j+2}^{\infty} \delta_u(2j+2, k) \sum_{k=2j+3}^{\infty} \delta_f(2j+3, k)}{(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))(1 + \sum_{k=2j+3}^{\infty} \delta_f(2j+3, k))} \quad (28)$$

$$b_{2t+2,2m+3} = \prod_{j=t}^{m-1} \frac{\sum_{k=2j+2}^{\infty} \delta_u(2j+2, k) \sum_{k=2j+3}^{\infty} \delta_f(2j+3, k) \sum_{k=2m+2}^{\infty} \delta_u(2m+2, k)}{(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))(1 + \sum_{k=2j+3}^{\infty} \delta_f(2j+3, k))(1 + \sum_{k=2m+2}^{\infty} \delta_u(2m+2, k))} \quad (29)$$

$$b_{2t+1,2m+1} = \prod_{j=t}^{m-1} \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))} \quad (30)$$

$$b_{2t+1,2m+2} = \prod_{j=t}^{m-1} \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k) \sum_{k=2m+1}^{\infty} \delta_f(2m+1, k)}{(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))(1 + \sum_{k=2m+1}^{\infty} \delta_f(2m+1, k))} \quad (31)$$

Next, by applying  $X = BY$  we get  $\bar{W}^0$  as given by (18).

### Method 2

By virtue of (21) and (22), we have for each  $t \in \mathbb{N}$

$$\bar{W}^{2t} = \frac{1}{1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)} + \frac{\bar{W}^{2t+2} \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}{(1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k))(1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k))}$$

and hence, for each  $t \geq 1$

$$\begin{aligned} \bar{W}^0 &= \frac{1}{1 + \sum_{k=1}^{\infty} \delta_f(1, k)} + \\ &\sum_{m=0}^{t-1} \frac{1}{1 + \sum_{k=2m+3}^{\infty} \delta_f(2m+3, k)} \prod_{j=0}^m \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))} \\ &+ \bar{W}^{2t+2} \prod_{j=0}^t \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))} \end{aligned} \quad (32)$$

As it will be shown below, we have  $0 \leq \bar{W}^{2t+2} \leq 1$ , and by virtue of (12),

$$\begin{aligned} &\prod_{j=0}^t \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))} = \\ &\prod_{j=0}^t \left( 1 - \frac{1}{1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k)} \right) \left( 1 - \frac{1}{1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)} \right) \leq \\ &\leq (\Delta_f \Delta_u)^{t+1} \rightarrow_{t \rightarrow +\infty} 0 \end{aligned}$$

Hence, by virtue of the three sequences theorem, we get  $\bar{W}^0$  as given by (18).

Proving that  $\bar{W}^0, \bar{Z}^{2t+1}, \bar{W}^{2t+2} \in [0, 1]$  for each  $t \in \mathbb{N}$

Obviously  $\bar{W}^0 \geq 0$ . Let us consider the sequence of partial sums

$$S_t = \frac{1}{1 + \sum_{k=1}^{\infty} \delta_f(1, k)} +$$

$$\sum_{m=0}^t \frac{1}{1 + \sum_{k=2m+3}^{\infty} \delta_f(2m+3, k)} \prod_{j=0}^m \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))}$$

The sequence is obviously increasing, and also  $S_t \leq 1$  for each  $t \in \mathbb{N}$ , because

$$S_t = \frac{1}{1 + \sum_{k=1}^{\infty} \delta_f(1, k)} + \frac{\sum_{k=1}^{\infty} \delta_f(1, k) \sum_{k=2}^{\infty} \delta_u(2, k)}{(1 + \sum_{k=1}^{\infty} \delta_f(1, k))(1 + \sum_{k=2}^{\infty} \delta_u(2, k))(1 + \sum_{k=3}^{\infty} \delta_f(3, k))} + \dots +$$

$$+ \frac{\sum_{k=1}^{\infty} \delta_f(1, k) \sum_{k=2}^{\infty} \delta_u(2, k) \cdots \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}{(1 + \sum_{k=1}^{\infty} \delta_f(1, k))(1 + \sum_{k=2}^{\infty} \delta_u(2, k)) \cdots (1 + \sum_{k=2t+3}^{\infty} \delta_f(2t+3, k))}$$

and when putting all elements of the sum on the same denominator

$$\prod_{j=0}^t (1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))(1 + \sum_{k=2t+3}^{\infty} \delta_f(2t+3, k))$$

this denominator is greater than the corresponding nominator. Hence,  $S_t \leq 1$  for each  $t \in \mathbb{N}$ , and therefore  $\bar{W}^0 = \lim_{t \rightarrow +\infty} S_t \leq 1$ .

From (32) we have for each  $t \in \mathbb{N}$

$$\bar{W}^{2t+2} = \frac{1}{1 + \sum_{k=2t+3}^{\infty} \delta_f(2t+3, k)} +$$

$$\sum_{m=t+1}^{\infty} \frac{1}{1 + \sum_{k=2m+3}^{\infty} \delta_f(2m+3, k)} \prod_{j=t+1}^m \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))}$$

Obviously  $\bar{W}^{2t+2} \geq 0$ , and we get also  $\bar{W}^{2t+2} \leq 1$  by using the same method as the one for showing that  $\bar{W}^0 \leq 1$ .

Since  $0 \leq \bar{W}^{2t+2} \leq 1$ , from (20) we have  $0 \leq \bar{Z}^{2t+1} < 1$ . ■

**Example 1** When we apply our result to the wage bargaining studied by Fernandez and Glazer (1991), we get obviously their result. Let us calculate the share  $\bar{W}^0$  that the union proposes for itself at the beginning of the game. We have  $\delta_{f,t} = \delta_f$  and  $\delta_{u,t} = \delta_u$  for each  $t \in \mathbb{N}$ . Hence, for each  $j \in \mathbb{N}$

$$1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) = 1 + \delta_f + (\delta_f)^2 + \dots = \frac{1}{1 - \delta_f}$$

$$\frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k)}{1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k)} = \delta_f, \quad \frac{\sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)} = \delta_u$$

and therefore, by inserting this into (18), we get

$$\bar{W}^0 = (1 - \delta_f) + (1 - \delta_f) [\delta_f \delta_u + (\delta_f \delta_u)^2 + \dots] = \frac{1 - \delta_f}{1 - \delta_f \delta_u}$$

and  $\bar{W}^{2t+2} = \bar{W}^0$  for each  $t \in \mathbb{N}$ .

**Example 2** Let us analyze a model in which the union and the firm have the following sequences of discount factors varying in time: for each  $t \in \mathbb{N}$

$$\delta_{f,2t+1} = \delta_{u,2t+1} = \frac{1}{2}, \quad \delta_{f,2t+2} = \delta_{u,2t+2} = \frac{1}{3}$$

For each  $j \in \mathbb{N}$ ,  $\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) < +\infty$  and  $\sum_{k=2j+2}^{\infty} \delta_u(2j+2, k) < +\infty$ , and

$$\begin{aligned} \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) &= \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} + \dots = \\ &= \frac{1}{2} \left( 1 + \frac{1}{6} + \frac{1}{6^2} + \dots \right) + \frac{1}{6} \left( 1 + \frac{1}{6} + \frac{1}{6^2} + \dots \right) = \frac{4}{5} \\ \frac{1}{1 + \sum_{k=1}^{\infty} \delta_f(1, k)} &= \frac{1}{\sum_{k=2j+3}^{\infty} \delta_f(2j+3, k)} = \frac{5}{9}, \quad \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k)}{1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k)} = \frac{4}{9} \\ \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k) &= \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} + \dots = \\ &= \frac{1}{3} \left( 1 + \frac{1}{6} + \frac{1}{6^2} + \dots \right) + \frac{1}{6} \left( 1 + \frac{1}{6} + \frac{1}{6^2} + \dots \right) = \frac{3}{5} \\ \frac{\sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)} &= \frac{3}{8} \end{aligned}$$

Hence, by virtue of (18) the offer of the union in period 0 in the SPE is equal to

$$\bar{W}^0 = \frac{5}{9} + \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{5}{9} + \left( \frac{4}{9} \cdot \frac{3}{8} \right)^2 \cdot \frac{5}{9} + \dots = \frac{5}{9} \left( 1 + \frac{1}{6} + \frac{1}{6^2} + \dots \right) = \frac{2}{3}$$

Note again that if we would apply the generalization of the original Rubinstein model to this example, i.e., the formula given by (15), then we would get  $\bar{W}^0 = \frac{3}{5}$ .

## 5 Going on strike only after rejection of own proposals

Haller and Holden (1990) consider also another game with the strike decision taken exogenously, in which the union goes on strike only after its own proposal is rejected, and it holds out if a proposal of the firm is rejected. They analyze the model in which the union and the firm have the same discount factor  $\delta$  and show that in such a game there is the unique SPE with the union's offer equal to  $\bar{W} = \frac{1+\delta w_0}{1+\delta}$ . We generalize this game to discount rates varying in time.

**Fact 3** Consider the generalized F-G model with preferences of the union and the firm described by the sequences of discount factors  $(\delta_{i,t})_{t \in \mathbb{N}}$ , where  $\delta_{i,0} = 1$ ,  $0 < \delta_{i,t} < 1$  for  $t \geq 1$ ,  $i = u, f$ . Assume that the strike decision is given exogenously and the union is supposed to go on strike only after rejection of its own proposals. Then  $(s_u, s_f)$  in

Assumption 1 is a SPE of this game if and only if the offers satisfy the following infinite system of equations: for each  $t \in \mathbb{N}$

$$\left(1 - \overline{W}^{2t}\right) + \left(1 - \overline{W}^{2t}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) = \left(1 - \overline{Z}^{2t+1}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \quad (33)$$

and

$$\overline{Z}^{2t+1} + \overline{Z}^{2t+1} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) = w_0 + \overline{W}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) \quad (34)$$

By solving the infinite system (33) and (34), we get the following result:

**Theorem 2** Consider the generalized F-G model with preferences of the union and the firm described by the sequences of discount factors  $(\delta_{i,t})_{t \in \mathbb{N}}$ , where  $\delta_{i,0} = 1$ ,  $0 < \delta_{i,t} < 1$  for  $t \geq 1$ ,  $i = u, f$ . Assume that the strike decision is given exogenously and the union is supposed to go on strike only after rejection of its own proposals. Then there is the unique SPE of the form  $(s_u, s_f)$  defined in Assumption 1, in which the offers of the parties are given by

$$\begin{aligned} \overline{W}^0 &= \frac{1 + w_0 \sum_{k=1}^{\infty} \delta_f(1, k) + \sum_{k=2}^{\infty} \delta_u(2, k)}{(1 + \sum_{k=1}^{\infty} \delta_f(1, k))(1 + \sum_{k=2}^{\infty} \delta_u(2, k))} + \\ &\sum_{m=0}^{\infty} \frac{1 + w_0 \sum_{k=2m+3}^{\infty} \delta_f(2m+3, k) + \sum_{k=2m+4}^{\infty} \delta_u(2m+4, k)}{(1 + \sum_{k=2m+3}^{\infty} \delta_f(2m+3, k))(1 + \sum_{k=2m+4}^{\infty} \delta_u(2m+4, k))} \cdot \\ &\prod_{j=0}^m \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))} \end{aligned} \quad (35)$$

and for each  $t \in \mathbb{N}$

$$\begin{aligned} \overline{W}^{2t+2} &= \frac{\left[ \overline{W}^{2t} \left(1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)\right) - 1 \right] \left(1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)\right)}{\sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)} + \\ &\frac{w_0}{\sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)} \end{aligned} \quad (36)$$

$$\overline{Z}^{2t+1} = \frac{w_0 + \overline{W}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}{1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)} \quad (37)$$

**Proof:** The proof is very similar to the proof of Theorem 1. By virtue of Fact 3, we need to solve the infinite system of equations (33) and (34), which can be equivalently written, for each  $t \in \mathbb{N}$ , as

$$\overline{W}^{2t} - \overline{Z}^{2t+1} \frac{\sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)}{1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)} = \frac{1}{1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)} \quad (38)$$

and

$$\overline{Z}^{2t+1} - \overline{W}^{2t+2} \frac{\sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}{1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)} = \frac{w_0}{1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)} \quad (39)$$

From (39) we get immediately (37), and inserting  $\overline{Z}^{2t+1}$  into (38) gives (36). In order to find  $\overline{W}^0$  we can use again one of the following two methods:



### Method 1

The infinite system of (38) and (39) is also a regular triangular system  $AX = Y$  with  $A = [a_{ij}]_{i,j \in \mathbb{N}^+}$ ,  $X = [(x_i)_{i \in \mathbb{N}^+}]^T$ ,  $Y = [(y_i)_{i \in \mathbb{N}^+}]^T$ , where the matrix  $A$  is the same as in the case of Theorem 1, and is described by (23) for each  $t, j \geq 1$  and by (24) for each  $t \in \mathbb{N}$ . Moreover, we have

$$x_{2t+1} = \overline{W}^{2t}, \quad x_{2t+2} = \overline{Z}^{2t+1}$$

$$y_{2t+1} = \frac{1}{1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)}, \quad y_{2t+2} = \frac{w_0}{1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}$$

Hence, the only difference between the present case and the always-strike decision case lies in  $y_{2t+2}$ , but it obviously changes the solution  $X$ . Since we have the same regular triangular matrix  $A$ , its (unique) inverse matrix  $B$ , i.e.,  $B$  such that  $BA = I$ , is the same. Hence, we have  $B = [b_{ij}]_{i,j \in \mathbb{N}^+}$  given by the formula from (26) till (31). By applying  $X = BY$  we get  $\overline{W}^0$  as given by (35).

### Method 2

By virtue of (38) and (39), we have for each  $t \in \mathbb{N}$

$$\overline{W}^{2t} = \frac{1 + w_0 \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}{(1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k))(1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k))} +$$

$$+ \frac{\overline{W}^{2t+2} \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)}{(1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k))(1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k))}$$

and hence, for each  $t \geq 1$

$$\overline{W}^0 = \frac{1 + w_0 \sum_{k=1}^{\infty} \delta_f(1, k) + \sum_{k=2}^{\infty} \delta_u(2, k)}{(1 + \sum_{k=1}^{\infty} \delta_f(1, k))(1 + \sum_{k=2}^{\infty} \delta_u(2, k))} +$$

$$\sum_{m=0}^{t-1} \frac{1 + w_0 \sum_{k=2m+3}^{\infty} \delta_f(2m+3, k) + \sum_{k=2m+4}^{\infty} \delta_u(2m+4, k)}{(1 + \sum_{k=2m+3}^{\infty} \delta_f(2m+3, k))(1 + \sum_{k=2m+4}^{\infty} \delta_u(2m+4, k))}.$$

$$\prod_{j=0}^m \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))} +$$

$$+ \overline{W}^{2t+2} \prod_{j=0}^t \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))}$$

Similarly as in the proof of Theorem 1, since  $0 \leq \overline{W}^{2t+2} \leq 1$  and

$$\prod_{j=0}^t \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{(1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k))(1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k))} \xrightarrow{t \rightarrow +\infty} 0$$

by virtue of the three sequences theorem, we get  $\overline{W}^0$  as given by (35).

Proving that  $\overline{W}^0, \overline{Z}^{2t+1}, \overline{W}^{2t+2} \in [0, 1]$  for each  $t \in \mathbb{N}$

Since  $0 \leq w_0 \leq 1$ , from (37) it is obvious that  $0 \leq \overline{Z}_{2t+1} \leq 1$ . Moreover,  $\overline{W}^0 \geq 0$ . The proof that  $\overline{W}^0 \leq 1$  goes analogously as the proof of Theorem 1 by considering the sequence of partial sums  $S_t$  and showing that  $S_t \leq 1$  for each  $t \in \mathbb{N}$ . Similarly one can show that  $0 \leq \overline{W}^{2t+2} \leq 1$ . ■

**Example 3** Let us apply this result to the wage bargaining studied by Fernandez and Glazer (1991), i.e., we have  $\delta_{f,t} = \delta_f$  and  $\delta_{u,t} = \delta_u$  for each  $t \in \mathbb{N}$ . Hence, for each  $j \in \mathbb{N}$

$$\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) = \frac{\delta_f}{1-\delta_f}, \quad \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k) = \frac{\delta_u}{1-\delta_u}$$

$$\frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k)}{1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k)} = \delta_f, \quad \frac{\sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)} = \delta_u$$

and therefore, by inserting this into (35), we get

$$\begin{aligned} \overline{W}^0 &= (1-\delta_f)(1-\delta_u) \left( 1 + \frac{w_0\delta_f}{1-\delta_f} + \frac{\delta_u}{1-\delta_u} \right) [1 + \delta_f\delta_u + (\delta_f\delta_u)^2 + \dots] = \\ &= \frac{1-\delta_f + w_0\delta_f(1-\delta_u)}{1-\delta_f\delta_u} = w_0 + \frac{(1-\delta_f)(1-w_0)}{1-\delta_f\delta_u} \end{aligned}$$

and  $\overline{W}^{2t+2} = \overline{W}^0$  for each  $t \in \mathbb{N}$ .

If additionally we assume that  $\delta_f = \delta_u = \delta$ , then  $\overline{W}^0 = \frac{1+\delta w_0}{1+\delta}$ , which coincides with the result by Haller and Holden (1990).

**Example 4** We analyze a model presented in Example 2 in which the union and the firm have the following sequences of discount factors varying in time: for each  $t \in \mathbb{N}$

$$\delta_{f,2t+1} = \delta_{u,2t+1} = \frac{1}{2}, \quad \delta_{f,2t+2} = \delta_{u,2t+2} = \frac{1}{3}$$

Then we have

$$\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k) = \frac{4}{5}, \quad \frac{\sum_{k=2j+1}^{\infty} \delta_f(2j+1, k)}{1 + \sum_{k=2j+1}^{\infty} \delta_f(2j+1, k)} = \frac{4}{9}$$

$$\sum_{k=2j+2}^{\infty} \delta_u(2j+2, k) = \frac{3}{5}, \quad \frac{\sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)}{1 + \sum_{k=2j+2}^{\infty} \delta_u(2j+2, k)} = \frac{3}{8}$$

By virtue of (35) the offer of the union in period 0 in the SPE is equal to

$$\overline{W}^0 = \left( 1 + \frac{4}{5}w_0 + \frac{3}{5} \right) \cdot \frac{5}{9} \cdot \frac{5}{8} \cdot \left[ 1 + \frac{4}{9} \cdot \frac{3}{8} + \left( \frac{4}{9} \cdot \frac{3}{8} \right)^2 + \dots \right] = \frac{2+w_0}{3}$$

## 6 Going never on strike

In case of the exogenous never-strike decision of the union, the unique SPE leads to the minimum wage contract  $w_0$ . We have the following:

**Fact 4** Consider the generalized F-G model with preferences of the union and the firm described by the sequences of discount factors  $(\delta_{i,t})_{t \in \mathbb{N}}$ , where  $\delta_{i,0} = 1$ ,  $0 < \delta_{i,t} < 1$  for  $t \geq 1$ ,  $i = u, f$ . Assume that the no-strike decision is given exogenously and the union never goes on strike. Then there is the unique SPE of the form defined in Assumption 1, where  $\bar{W}^{2t} = \bar{Z}^{2t+1} = w_0$  for each  $t \in \mathbb{N}$ .

**Proof:** Suppose that the union never goes on strike. In order to find the SPE offers  $\bar{W}^{2t}$  and  $\bar{Z}^{2t+1}$ , we solve for each  $t \in \mathbb{N}$

$$\left(1 - \bar{W}^{2t}\right) + \left(1 - \bar{W}^{2t}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) = (1 - w_0) + \left(1 - \bar{Z}^{2t+1}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \quad (40)$$

and

$$\bar{Z}^{2t+1} + \bar{Z}^{2t+1} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) = w_0 + \bar{W}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) \quad (41)$$

Obviously,  $\bar{W}^{2t} = \bar{Z}^{2t+1} = w_0$  for each  $t \in \mathbb{N}$  is a solution of this system of equations, and we know from the infinite matrices theory that this system has the only one solution. ■

## 7 Concluding remarks

There are several issues in our agenda for future research on the generalized F-G model. As mentioned in the Introduction, Houba and Wen (2008) apply the method of Shaked and Sutton (1984) to derive the exact bounds of equilibrium payoffs in the original F-G model. We intend to apply their method to find the maximum wage in our generalized F-G model. Since we assume that the sequence of discount rates of a party can be arbitrary, with the only restriction that the infinite series that determines the utility of a result for the given party must be convergent, we will first describe the conditions in a general case for the supremum of the union's SPE payoffs in any even period and for the infimum of the firm's SPE payoffs in any odd period. Then, we will be solving the conditions for particular cases of the sequences of discount rates varying in time.

Several authors analyze the issues of bargaining power, both in the standard bargaining models and in the wage bargaining, where the parties have constant discount rates. Since discount rates are usually crucial in determining bargaining power of parties, it is of importance to study the bargaining power issues also in our framework, i.e., in the generalized models with preferences of the union and the firm described by discount rates varying in time.

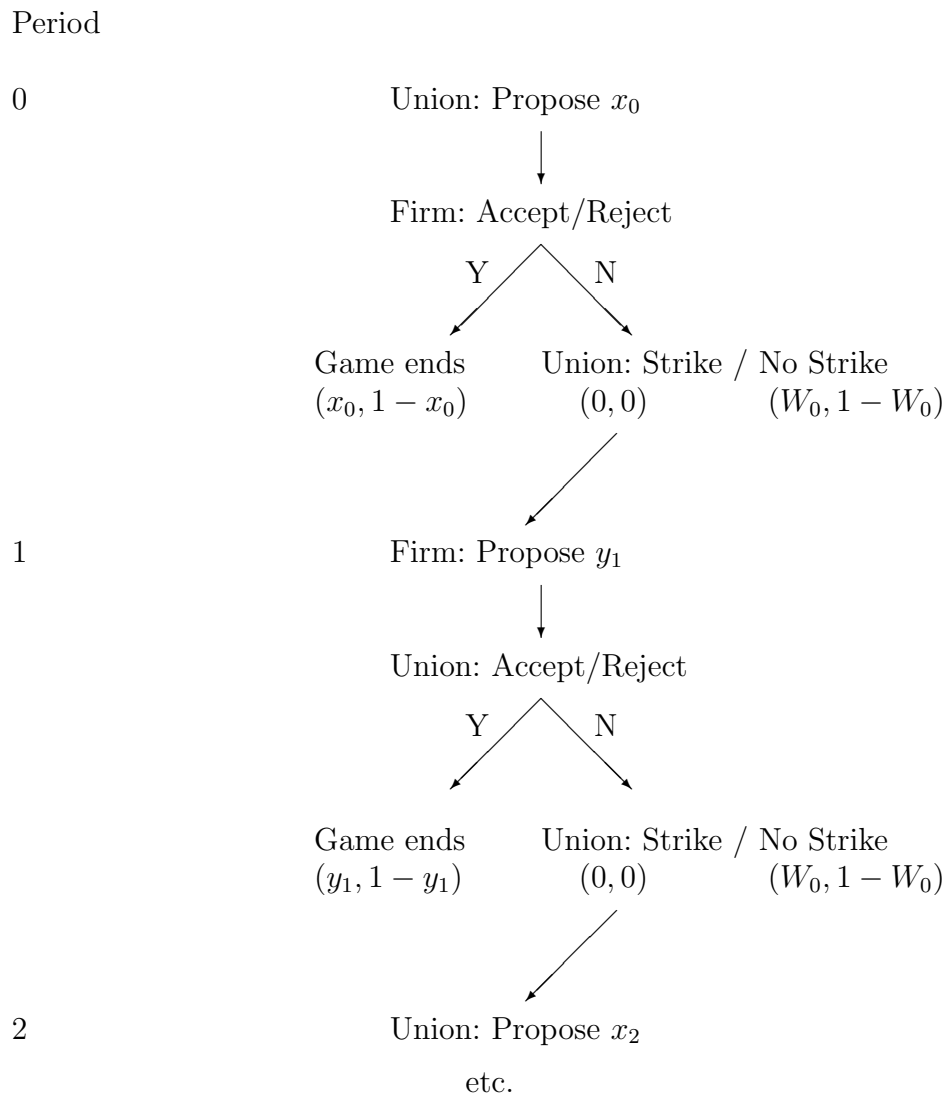
We would like also to provide a detailed analysis of some applications of the generalized F-G model to real-life situations. Bargaining with discount rates varying in time, and its generalized wage bargaining version in which utilities of bargainers are of the type (7) and

(8), can model bargaining faced in reality (much) better than the analogous bargaining with constant discount rates. Patience of parties may obviously be changing over time, due to many circumstances, e.g., economic, financial, political, social, environmental, health or climatic issues. Moreover, in many situations, the utility of an agreement is counted not only in one step (the given period when the agreement is achieved), but is the long-term utility. If we negotiate wage for workers or a price of a pharmaceutical product, the agreement is valid for a longer time. Even if the time of implementing the given agreement is finite, its expiration time might be not known, and therefore it is appropriate to define the utilities by the type (7) and (8). Consequently, since the discount rates are varying in time, it is the generalized F-G framework that is more suitable than the original model.

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**Figure 1:** Non-cooperative bargaining game between the union and the firm