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Integration, real exchange rate and growth *

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Abstract: *This paper deals with the relationship between real exchange rate and growth in the process of economic integration. Using a 2x2x2 model of overlapping generations, we show that growth depends on the real exchange rate (RER) through human capital accumulation. Integration leads to convergence in growth rates only in presence of cross-border externalities in human capital. Otherwise, divergence is likely to occur and integration may be good (bad) for growth if the integrated RER is higher (lower) than the autarky's RER. In reality, since capital mobility prevents the real exchange rate from adjusting, economic integration may lead to income divergence if countries are too different in terms of preference, altruism or productivity.*

Keywords: *Two-sector model, OLG model, real exchange rate, economic growth.*

Journal of Economic Literature Classification Numbers: E21, F15, F43, O41.

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1 Introduction

How does economic integration affect growth? This article addresses this question theoretically developing a 2-country 2-sector endogenous growth model with education. This framework allows to consider the relationship between real exchange rate (RER) and growth which is specially relevant in the process of european economic integration since RER still move even between eurozone members (Berka and Devereux (2010)).

In a recent work, Galor and Mountford (2008) highlight the influence of international trade on human capital. They show that trade exerts opposite effects in developed and developing countries. Education increases (decreases) in OECD (non-OECD) countries. Economic integration goes beyond trade integration allowing capital mobility between European countries. The model we develop highlights consequences of this capital mobility in a two-sector setting. We assume there are two sectors in the economy: a traded one and a nontraded one. The way capital mobility affect human capital accumulation depends on the traded Total Factor Productivity (TFP) gap between countries. This TFP gap reflects differences in relative prices of nontraded to traded goods which can be considered as an internal real exchange rate. We find that countries whose RER appreciates (depreciates) with integration accumulate more (less) human capital and experience higher (lower) economic growth than in autarky. In a general way, capital mobility prevents the RER from freely adjusting and may be growth damaging.

The literature on capital mobility, exchange rate and growth is not yet well documented. Aghion *et al.* (2009) show that exchange rate volatility matters for growth but especially for countries with relatively low levels of financial development. The literature on export led growth models often shows the benefits for growth in maintaining a low RER. First, Rodrik (2008) concludes that the depreciation is growth enhancing when there exist some economic distorsions in the traded sector. This explanation holds especially for Asian countries in which stages of growth accelerations have followed RER depreciation. Elsewhere, evidence is mixed and growth acceleration does not systematically follow episodes of depreciation. Eichengreen (2008) concludes that for developing countries - before the beginning of the growth process - depreciation is growth enhancing, driving resources out of the nontraded to the traded sector. The common starting point of these papers is that there exist some distorsions that prevent optimal resources allocation between the traded and nontraded sectors detrimental to the traded sector [Prasad *et al.* (2007)]. However, these models do insist on the fact that what is important for growth is to maintain the "correct" RER instead of targeting an artificially low level of the RER[1].

[1]Maintaining an artificially low level of the RER would generate costs in terms of inflation and reserve accumulation and maybe benefits in terms of temporary growth. Eichengreen (2008) states that the ratio between costs and benefits increases with the country's level of development.

Second, Harris (2001) shows empirically that the US Dollar appreciation against the Canadian Dollar in the 1990's is responsible for the technological spread between US and Canada. His argument is based on the fact that when RER appreciates, firms of the traded sector are forced to invest in R&D to maintain competitiveness despite the fact that their traded prices are higher. There were more incentives to invest for US firms than for Canadian ones. This may explain part of the technological difference. Finally, the relationship between relative prices and relative productivities is positive for developed countries [Canzoneri *et al.* (1999)]. This means that growth episodes in developed countries are frequently associated with real appreciations. This is not the case for developing countries [Ito *et al.* (1999)].

The model we develop in this paper is a two-sector generalization of the Michel and Vidal (1999) framework. In the Michel and Vidal one-sector model, determinants of growth were patience, altruism and the size of the external effect in human capital accumulation. Introducing a second sector of production, the RER matters for growth as well. The way the RER is determined depends on the assumption concerning capital mobility; so do growth determinants. In autarky, this RER clears the nontraded goods market and as a result depends on the propensity to consume services, the degree of altruism, the saving rate. In autarky, the more services the country consumes, the higher the relative price, the higher the return on education -if nontraded sector is labor intensive-, the higher the growth rate. Conversely, when capital is perfectly mobile between countries, RER no longer depends on demand and supply factors but only depends on TFP gap between traded and nontraded sectors. This derives from the fact that there are as many mobile factors as sectors in this economy. Then, the return on human capital -the wage- is no longer a function of global capital intensity. Instead it only depends on the RER and capital mobility implies that marginal products of capital are the same between countries. The higher traded productivity, the more appreciated the RER [Balassa-Samuelson effect].

Integration leads to convergence in growth rates only in presence of cross-border externalities.

Nevertheless empirical evidence does not suggest that integration in Europe has promoted convergence of growth rates. Indeed, the divergence of growth rates looks higher over the recent period (1991-2005) than it was before economic integration (1970-1990) [See Table 1].

When we reject the cross-border externality assumption, integration leads effectively to divergence: integration may be good (bad) for growth if the integrated RER is higher (lower) than the autarky's RER. The intuition of this result is that a higher RER stimulates human capital accumulation. Capital mobility prevents the RER from adjusting and may lead to an inapropriate resource allocation between the two sectors. In this case, it will not be growth enhancing. This result is in line with Eichengreen (2008) in the sense that the policy recommendation would be to allow the relative price to keep the autarky level which corresponds to the "correct"

Per Capita GDP growth rates	1970-1990	1991-2005
Austria	1.84	1.70
Belgium	1.70	1.57
Denmark	1.29	1.88
Finland	1.97	2.28
France	1.73	1.40
Germany	1.92	1.18
Greece	0.79	2.44
Ireland	2.43	5.61
Italy	2.19	1.02
Luxembourg	2.72	2.93
Netherland	1.31	1.62
Portugal	2.45	1.54
Spain	1.50	2.15
Sweden	1.20	1.93
United Kingdom	1.64	2.39
Average Deviation	0.41	0.69
Standard Deviation	0.52	1.09
Interquartile Range (25%-75%)	068	0.78

Table 1: European GDP growth rates. Source: Eurostat and Author's calculations.

level given the fundamentals (altruism, productivity, savings). In this model, capital mobility may lead to RER misalignment and growth reduction in the less altruistic country. Finally, this paper complements the existing literature. It points out that an additional cost of maintaining an RER too low - inflation and reserves accumulation are often blamed - decreases the incentive to accumulate human capital and is then bad for growth. Whereas the literature [Prasad et al. (2007) Rodrik (2008), Eichengreen (2008)] suggests that a significantly overvalued RER is growth reducing as well.

The rest of the paper is as follows. Section 2 presents the model. Section 3 deals with autarky whereas Section 4 deals with integration introducing capital mobility between the two countries. Section 5 concludes.

2 The model

We consider a two-country model that is an extension of Michel and Vidal (1999) in which we introduce two production sectors: a tradable sector and a non tradable sector. We normalize the traded good price to unity. In this setting, the relative price of the nontraded good, P_N, also denotes the domestic real exchange rate. We consider that the nontraded good is perishable and then is a pure consumption good.

The traded good is a mixed good which can either be consumed or invested. This two-sector production structure is a generalization of the standard two-sector setting in which one good is a pure consumption while the other is a pure investment good [Galor (1992), Venditti (2005)]. The world consists of two countries which accumulate human capital and experiment endogenous growth.

2.1 Production

The representative firm produces in two sectors: the tradable, and the non-tradable one. Production in the tradable (Y_T) and in the non-tradable (Y_N) sector resulting from two Cobb-Douglas production technologies, using two inputs, human capital H, and physical capital K. Let K_i and L_i, $i = T, N$, be respectively the quantities of capital and labor used by sector i, production is given by

$$Y_T = A_T K_T^{\alpha_T} H_T^{1-\alpha_T} \quad (1)$$

$$Y_N = A_N K_N^{\alpha_N} H_N^{1-\alpha_N} \quad (2)$$

with $\alpha_T, \alpha_N \in (0,1)$, $A_T > 0$ and $A_N > 0$.

Investment instantaneously transforms a unit of tradable good into a unit of installed capital: $K_{t+1} = I_t$ and capital fully depreciates after one period. Both inputs are perfectly mobile between the two sectors provided that:

$$H_T + H_N \leq H, \qquad K_T + K_N \leq K \quad (3)$$

K being the total stock of physical capital and H the total amount of human capital. Let $k_i = Ki/Hi$ be the capital intensity of sector i, $h_i = H_i/H$ be the share of human capital allocated to sector $i, i = T, N$, and $k = K/H$ the physical to human capital ratio. Equations (2), (3) and (5) can be rewritten:

$$h_T + h_N \leq 1, \qquad k_T h_T + k_N h_N \leq k \quad (4)$$

$$y_T = A_T k_T^{\alpha_T} \quad (5)$$

$$y_N = A_N k_N^{\alpha_N} \quad (6)$$

where y_T and y_N are the production per unit of human capital in each sector.
Denoting w the wage rate, R the gross rental rate of capital and P_N the price of the non tradable good, profit maximization over the two sectors implies that production factors are paid their marginal product:

$$R_t = \alpha_T A_T {k_T}_t^{\alpha_T - 1} = P_{Nt} \alpha_N A_N {k_N}_t^{\alpha_N - 1} \quad (7)$$

$$w_t = (1-\alpha_T) A_T {k_T}_t^{\alpha_T} = P_{Nt} (1-\alpha_N) A_N {k_N}_t^{\alpha_N} \quad (8)$$

From which we derive the physical to human capital ratios as functions of the price of the non tradable good:

$$\begin{aligned} &k_{Tt} = B(P_{Nt})^{\frac{1}{\alpha_T-\alpha_N}} \\ &k_{Nt} = \tfrac{\alpha_N(1-\alpha_T)}{\alpha_T(1-\alpha_N)} B(P_{Nt})^{\frac{1}{\alpha_T-\alpha_N}} \\ &with\ B = \left(\tfrac{\alpha_N}{\alpha_T}\right)^{\frac{\alpha_N}{\alpha_T-\alpha_N}} \left(\tfrac{A_N}{A_T}\right)^{\frac{1}{\alpha_T-\alpha_N}} \left(\tfrac{1-\alpha_T}{1-\alpha_N}\right)^{\frac{\alpha_N-1}{\alpha_T-\alpha_N}} \end{aligned} \tag{9}$$

And thus the input prices are:

$$\begin{aligned} w_t &= (1-\alpha_T) A_T B^{\alpha_T} P_{Nt}^{\frac{\alpha_T}{\alpha_T-\alpha_N}} \equiv w(P_{Nt}) \\ R_t &= \alpha_T A_T B^{\alpha_T-1} P_{Nt}^{\frac{\alpha_T-1}{\alpha_T-\alpha_N}} \equiv R(P_{Nt}) \end{aligned} \tag{10}$$

2.2 Consumption, savings and children's education

The economy consists, in each country, of a sequence of three individual life periods. In the second period of his life, each individual gives birth to $1+n$ children so that population grows at rate n. We assume the population growth rate is the same in the two countries. Each generation born in period t consists of N_t identical individuals who make decisions concerning consumption, children's education, and savings. During childhood, individuals make no decision: their consumption is included in their parent's consumption. They are reared by their parents who decide on their level of educational attainment. When adult, they work and receive the market wage, consume, save, and rear their own children. When old they retire, and consume the proceeds of their savings.

Individuals care about their children's education. They exhibit a kind of paternalistic altruism whereby they value their child's human capital. Our modeling of intergenerational altruism follows Glomm and Ravikumar (1992) who assume that the parental bequest is the quality of education received by their children. The preferences of an individual belonging to generation t are represented by:

$$U(c_t, d_{t+1}, h_{t+1}) = (1-\beta) ln c_t + \beta ln d_{t+1} + \gamma ln h_{t+1} \tag{11}$$

where c_t, d_{t+1} and h_{t+1} are respectively consumption when adult, consumption when old, and the child's human capital; $\beta \in]0,1[$ denotes individuals' thrift and γ is the altruism factor. When adult, each agent born at t supplies inelastically h_{t+1} units of efficient labor. The level of human capital of each adult depends on his parent's decision on education during his childhood:

$$h_{t+1} = b_t e_t^a \tag{12}$$

where b_t is an externality, e_t the amount of resources a parent devotes to his child's education, and $a \in]0,1[$ the elasticity of the technology of human capital formation. Let $x = c, d$ denote individual consumption at each period of life, x_N and x_T be respectively the spending allocated to nontraded and traded goods. Instantaneous preferences over the two goods are defined according to:

$$x = x_T^{\mu} x_N^{1-\mu} \tag{13}$$

with $\mu \in (0,1)$. We denote π the consumer price index in terms of traded good. Adults distribute their earnings that consist of labor income, $w_t h_t$, among own consumption spending, investment in child's education, and savings, s_t,

$$w_t h_t = \pi_t c_t + e_t + s_t \tag{14}$$

When old, individuals retire and consume the proceeds of their savings:

$$R_{t+1} s_t = \pi_{t+1} d_{t+1} \tag{15}$$

An individual born in period $t-1$ is endowed with h_t units of human capital at the beginning of adulthood, and chooses e_t and s_t so as to maximize his life-cycle utility (11) under his budget constraints (12), (14) and (15). An individual's optimal choice is characterized by the first order conditions:

$$-\frac{1-\beta}{\pi_t c_t} + \frac{\beta R_{t+1}}{\pi_{t+1} d_{t+1}} = 0 \tag{16}$$

$$-\frac{1-\beta}{\pi_t c_t} + \frac{\gamma a}{e_t} = 0 \tag{17}$$

$$c_{Tt} = \mu \pi_t c_t$$

$$P_{Nt} c_{Nt} = (1-\mu) \pi_t c_t \tag{18}$$

$$\pi = \phi(\mu) \equiv \mu^{-\mu} (1-\mu)^{-(1-\mu)}$$

Equation (16) characterizes the optimal allocation of consumption for an individual over his lifetime. Equation (17) gives the optimal investment in the offspring's human capital. An adult reduces his consumption spending until his loss equates the increment in the utility he derives from his child's level of human capital out of altruism. Equations (18) give the static allocation of consumption spending between the two goods.

Plugging (14) and (15) into (16) and (17) yields:

$$s_t = \frac{\beta}{1+\gamma a} w_t h_t \tag{19}$$

$$e_t = \frac{\gamma a}{1+\gamma a} w_t h_t \tag{20}$$

As usual in overlapping generation models with paternalistic altruism, savings increase with individual's thrift and decrease with altruism. The more altruistic parents are, the more they invest in their offspring's education.

2.3 Cross-border external effects in human capital

Throughout the analysis, foreign variables are denoted by an asterisk. We assume cross-border externalities in human capital formation. An individual's investment in his child's human capital generates a positive externality for his country's fellows. We assume an externality of the form:

$$b_t = b(p\overline{e_t} + p^*\overline{e_t^*})^\lambda \overline{e_t}^{1-a-\lambda} \text{ and } b_t^* = b(p\overline{e_t} + p^*\overline{e_t^*})^\lambda \overline{e_t^*}^{1-a-\lambda} \tag{21}$$

where $b > 0$, $\lambda \in [0, 1-a]$, $p = N/(N+N^*)$ and $p^* = 1-p$. Since population grows at the same rate in the two countries, p and p^*, the shares of each country in the world population, are constant.
We denote respectively $\overline{e_t}$ and $\overline{e_t}^*$ the average levels of investment in children's human capital in the home and the foreign country. Since individuals are identical within each country, in equilibrium: $e_t = \overline{e_t}$ and $e_t^* = \overline{e_t}^*$. The magnitude of these cross-border external effects is given by λ. The term $(p\overline{e_t} + p^*\overline{e_t}^*)^\lambda$ is intended to capture the strength of international spillover of knowledge. The higher λ, the more the home country benefits from the foreign country's private expenditures in education.

In equilibrium, human capital depends both on domestic and foreign investment in education and on cross-border externality in human capital formation:

$$h_{t+1} = b_t {e_t}^a = b(pe_t + p^*e_t^*)^\lambda e_t^{1-\lambda} \tag{22}$$

Let $\rho_t = e_t^*/e_t$ be the ratio of foreign over home average investment in children's human capital and $g_t = e_t/e_{t-1} - 1$ the economy growth rate. Using equations (20), (22) and finally (10), we obtain:

$$1 + g_t = \frac{\gamma a b}{1+\gamma a}(1-\alpha_T) A_T {k_T}_t^{\alpha_T} (p + p^*\rho_{t-1})^\lambda \equiv G_t \tag{23}$$

2.4 The nontradable market clearing condition

Since there exists a nontraded good, we should consider a market clearing condition for that good:

$$P_{Nt} Y_t^N = N_t P_{Nt} c_{Nt} + N_{t-1} P_{Nt} d_{Nt} \tag{24}$$

This equation simply states that production equals total consumption in nontraded goods. We can rewrite this condition with only wage, interest factor and physical to human capital ratios:

Lemma 1 *The home country non tradable market clearing condition can be written*

$$\frac{1-\mu}{1+\gamma a}((1-\beta)w_t + \frac{\beta R_t}{1+n}\frac{1+\gamma a}{\gamma ab(p+p^*\rho^*_{t-1})^\lambda}) = P_{Nt}A_N D {k_T}^{\alpha_N - 1}(k_t - k_{Tt}) \quad (25)$$

With $D = \frac{(\alpha_N(1-\alpha_T))^{\alpha_N}(\alpha_T(1-\alpha_N))^{1-\alpha_N}}{\alpha_N - \alpha_T}$.

Proof: see Appendix 7.1.
It can be noted that expression D is the same for both countries as we assume home and foreign technologies have identical elasticities of substitution between production factors.

3 Autarky

As we first consider autarky, we rule out any interactions between countries. Investments in human capital in one country do not result in an external effect that enhances the formation of human capital in the other ($\lambda = 0$). The human capital externality depends only on the average level of education. From equation (21), we have with $\lambda = 0$: $b_t = b\overline{e_t}^{1-a}$. From equation (22), since individuals are identical, social returns on human capital investment are constant in equilibrium $h_{t+1} = be_t$. Young people's savings finance the following period's physical capital:

$$K_{t+1} = H_{t+1}k_{t+1} = N_t s_t \quad (26)$$

The labor market clears:

$$H_t = N_t h_t \quad (27)$$

Combining (19),(20), (22), (26) and (27), we obtain the constant equilibrium physical to human capital ratio:

$$k \equiv k^A = \frac{\beta}{\gamma ab(1+n)} \quad (28)$$

We then compute the equilibrium autarkic growth rate g^A.

Lemma 2 *The autarkic growth factor is:*

$$1 + g^A = \frac{\gamma ab}{1+\gamma a}(1-\alpha_T)A_T {k_T}^{\alpha_T} \quad (29)$$

with[2]

$$k_T = k^A \frac{\alpha_T}{1-\alpha_T}\frac{1-\mu\alpha_T-(1-\mu)\alpha_N}{\frac{1-\mu}{1+\gamma a}(1-\beta)(\alpha_N-\alpha_T)+\alpha_T} \quad (30)$$

[2]Notice that $k_T > 0$ if and only if $\alpha_N > \alpha_T$ or $\alpha_T > \alpha_N$ and $\mu < \frac{1-\alpha_N}{\alpha_T - \alpha_N}$. If technology is the same in both sectors $\alpha_T = \alpha_N$ and $k_T = k^A$.

Proof: see Appendix 7.2.
The following proposition contains some comparative statics results relating the long-run growth rate to preference parameters.

Proposition 1 *The more patient individuals are, the higher the growth rate. The growth rate is first increasing and then decreasing with γ reaching a maximum in $\bar{\gamma} = \frac{1-2\alpha_T+\sqrt{1+4((1-\mu)(1-\beta)(\alpha_N-\alpha_T))}}{2a\alpha_T}$. Moreover, if $\alpha_T > \alpha_N$ (respectively $\alpha_N > \alpha_T$), the growth rate is decreasing (respectively increasing) in μ.*

Proof: see Appendix 7.3.
The more altruistic individuals are, the higher their investment in children's education and the lower their consumption. We obtain, as in Michel and Vidal (1999), that excessive as well as weak altruism can lead to poor growth records.
When the tradable sector is capital intensive ($\alpha_T > \alpha_N$), the growth rate decreases with the preference for tradable goods (μ). This is a consequence of the Stolper-Samuelson Theorem. An increase in the propensity to consume the traded good (μ) leads to an RER depreciation. Since the nontraded good is labor intensive, the real depreciation entails a fall in the wage. Then, the return to human capital decreases and so does the growth rate. Hereafter, to fit empirical evidence [Ito *et al.* (1999)], we consider:

Assumption 1 $\alpha_N < \alpha_T$.

4 Economic integration and growth

We consider a two-country overlapping generations world in which countries differ in both levels of patience and altruism. We establish the growth implications of world economic integration.
In the integrated economy, as we assume no labor mobility between the two countries, the labor market clearing condition of the domestic country is given as in autarky by equation (27). Equation (25) gives the non tradable market clearing conditions for the home country. The foreign country equations are obtained if we denote by $*$ foreign variables.
In a two-country integrated world, there are capital flows between countries and the equality between domestic savings and domestic investment -equation (26)- no longer holds. The amount saved by adults in the home and the foreign country in period t is equal to the physical capital available as productive input in period $t+1$:

$$K_{t+1} + K^*_{t+1} = N_t s_t + N^*_t s^*_t \tag{31}$$

Dividing by the world population, the world capital market clearing condition is:

$$(1+n)\left(pk_{t+1}h_{t+1} + (1-p)k^*_{t+1}h^*_{t+1}\right) = ps_t + (1-p)s^*_t \tag{32}$$

4.1 Convergence of countries' growth rates

With perfect capital mobility, the interest rate is the same for both countries:

$$R_t = R_t^* \tag{33}$$

Using (10), we can determine the ratio between domestic and foreign relative prices:

$$\frac{P_{Nt}}{P_{N_t}^*} = \left[\frac{A_N^*}{A_N}\right]\left[\frac{A_T}{A_T^*}\right]^{\frac{1-\alpha_N}{1-\alpha_T}} \tag{34}$$

This ratio reflects the bilateral real exchange rate between these two countries. Using equation (20) we can compute the ratio of foreign over home average investment in children's human capital:

$$\rho_{t+1} = \frac{e_{t+1}^*}{e_{t+1}} = \frac{\gamma^*}{\gamma}\frac{1+\gamma a}{1+\gamma^* a}\frac{w_{t+1}^* h_{t+1}^*}{w_{t+1} h_{t+1}} \tag{35}$$

Integrating human capital accumulation from equation (22), we have:

$$\rho_{t+1} = \frac{\gamma^*}{\gamma}\frac{1+\gamma a}{1+\gamma^* a}\frac{w_{t+1}^*}{w_{t+1}}{\rho_t}^{1-\lambda} \tag{36}$$

Using expression of wages as functions of P_N and P_N^*, from equation (10), we obtain:

$$\rho_{t+1} = \frac{\gamma^*}{\gamma}\frac{1+\gamma a}{1+\gamma^* a}\left(\frac{P_{Nt+1}^*}{P_{Nt+1}}\right)^{\frac{\alpha_T}{\alpha_T-\alpha_N}}\left(\frac{A_N^*}{A_N}\right)^{\frac{\alpha_T}{\alpha_T-\alpha_N}}\left(\frac{A_T}{A_T^*}\right)^{\frac{\alpha_N}{\alpha_T-\alpha_N}}{\rho_t}^{1-\lambda} \tag{37}$$

Integrating equation (34), we can then determine the foreign relative to home growth on the steady path:

$$\rho_{t+1} = \frac{\gamma^*}{\gamma}\frac{1+\gamma a}{1+\gamma^* a}\left(\frac{A_T^*}{A_T}\right)^{\frac{1}{1-\alpha_T}}{\rho_t}^{1-\lambda} \tag{38}$$

To understand the importance of cross-border externalities, we consider first the case without cross-border external effects ($\lambda = 0$), which corresponds to the case where countries can exchange capital and goods but the education level in one country does not affect the human capital formation in the other country. Second, we focus on the case with cross-border external effects.

4.1.1 Divergence without cross-border external effects

Economic integration without cross-border external effects corresponds to the case where $\lambda = 0$ and thus $b_t = be_t^{1-a}$ as in autarky. Equation (38) becomes:

$$\rho_t = \frac{1+g_t^*}{1+g_t}\rho_{t-1} \tag{39}$$

With

$$\frac{1+g_t^*}{1+g_t} = \frac{\gamma^*}{\gamma}\frac{1+\gamma a}{1+\gamma^* a}\left[\frac{A_T^*}{A_T}\right]^{\frac{1}{1-\alpha_T}} \tag{40}$$

Convergence of growth rates means that $(1+g_t^*)/(1+g_t) = 1$. If the altruism factor is the same in both countries $\gamma = \gamma^*$, convergence is obtained if and only if $A_T^* = A_T$. Thus if countries differ with respect to their total factor productivity (TFP), convergence is no longer possible.
If altruism factors differ, convergence means:

$$\frac{\gamma^*}{\gamma}\frac{1+\gamma a}{1+\gamma^* a}\left[\frac{A_T^*}{A_T}\right]^{\frac{1}{1-\alpha_T}} = 1 \tag{41}$$

We have thus proved a first result concerning convergence of growth rates :

Proposition 2 *Without cross-border external effects, for identical altruism factors ($\gamma = \gamma^*$), if there exists a gap between traded TFP $A_T^* \neq A_T$, perfect capital mobility does not result in convergence in growth rates. Convergence in growth rates may occur for very special altruism factors only, namely*[3] $\gamma = \Gamma(\gamma^*)$ *with* $\Gamma(\gamma^*) = \left[\left(\frac{1}{\gamma^*}+a\right)\left(\frac{A_T}{A_T^*}\right)^{\frac{1}{1-\alpha_T}} - a\right]^{-1}$.

We may notice that if $(1+g_t^*)/(1+g_t) < 1$, then $\lim_{t\to\infty}\rho_t = 0$ meaning that the foreign education level e^* becomes in the long run very small compared to the home education level e. Symmetrically, if $(1+g_t^*)/(1+g_t) > 1$, then $\lim_{t\to\infty}\rho_t = +\infty$, meaning that e^* becomes much higher than e.
In the literature, the one-sector model concludes that integration systematically promotes convergence of growth rates when both countries have the same degree of altruism. This is no longer the case in a two-sector model. Introducing a 2-sector setting, we show here that what matters for economic growth is not only the altruism factors but also the traded productivity spread between the home and the foreign country which determines the RER through equation (34). The higher productivity in the traded sector, the higher the growth rate. This is an illustration of the Stolper-Samuelson theorem when the traded sector is capital intensive. Indeed, the RER affects wages and then education and growth. A higher RER increases return on human capital and stimulates education. The proposition states that if both countries have the same traded total factor productivity (TFP), the more altruistic the country, the higher the growth rate in an integrated world. Assume that there is a technological gap between countries and let the foreign country be the technologically "advanced" country. The foreign country will have the highest growth rate except if the domestic country has a sufficient degree of altruism ($\gamma > \Gamma(\gamma^*)$). Altruism can

[3] if $A_T \geq A_T^*$ or $\gamma^* < \left(a\left(\left(\frac{A_T^*}{A_T}\right)^{\frac{1}{1-\alpha_T}} - 1\right)\right)^{-1}$ then γ is positive.

be interpreted in terms of education spending. An "advanced" country continues to have the highest growth except if the other country invests enough in education to catch up on this technological gap.

4.1.2 Convergence with cross-border external effects

Introducing cross-border external effects, each country can benefit from a higher level of education in the other country. Using equation (38) with $0 < \lambda < 1 - a$, there exist two steady state values for ρ. The trivial long run value $\rho = 0$, and a positive one $\bar{\rho}$:

$$\bar{\rho} = \left(\frac{\gamma^*}{\gamma} \frac{1 + \gamma a}{1 + \gamma^* a} \left[\frac{A_T^*}{A_T} \right]^{\frac{1}{1-\alpha_T}} \right)^{\frac{1}{\lambda}} \tag{42}$$

As $\lambda \in (0, 1)$, the sequence of $\{\rho_t\}$ tends towards $\bar{\rho}$ and growth rates converge. The result is true for any value of altruism degrees, and especially with same altruism factors ($\gamma = \gamma^*$). We have thus proved:

Proposition 3 *With cross-border external effects, perfect capital mobility results in convergence in growth rates for any possible altruism factors.*

Thus the existence of cross-country external effects in human capital guarantees convergence of growth rates. Nevertheless, empirical evidence suggests that such external effects are not observed. As a result, cross border external effects are generally absent in models where labor is not mobile between countries. In line with this literature, we focus on the realistic case without cross-border effects.

4.2 Growth rates and integration without cross-border effects

Without cross-border effect, i.e. when $\lambda = 0$, Proposition 2 states that convergence in growth rates never occurs except for a very special ratio of altruism degrees. As a result, if there is a spread between technologies, the divergence in growth rates may last forever.

We now consider the world allocation of savings in the case without cross-border externalities. Denoting $x \equiv \beta^* \gamma / \beta \gamma^*$ then, from equation (28), $k_A^* = x k_A$. In this two-country two-sector model, only one good can be traded. Nontraded good is exclusively produced domestically. In some extreme cases, when the propensity to consume and/or the preference for non traded good are high, the representative firm tends to produce mainly nontraded goods, the traded good being imported. As we consider a human capital intensive non-traded production, to guarantee physical capital accumulation, we have to assume:

Assumption 2 $\frac{-\zeta}{1-\zeta} < \frac{x\eta}{\eta^*} < \frac{1-\zeta}{\zeta}$.

We obtain the following lemma which provides a simple expression of the world capital accumulation equation, and expressions of the physical to human capital ratios:

Lemma 3 *In an integrated world, the international capital market clearing condition is:*

$$pk_{t+1} + (1-p)k^*_{t+1}\rho_t = pk_A + (1-p)k^*_A\rho_t \tag{43}$$

and, under assumption 2 the physical to human capital ratios are:

$$k_t = \eta P_{Nt}^{\frac{1}{\alpha_T - \alpha_N}} + \zeta k_A \tag{44}$$

$$k^*_t = \eta^* P_{Nt}^{\frac{1}{\alpha_T - \alpha_N}} + \zeta x k_A \tag{45}$$

The domestic price of the nontraded good is:

$$P_{Nt+1}^{\frac{1}{\alpha_T - \alpha_N}} = \frac{k_A\left(\rho_t(1-p)x + p\right)(1-\zeta)}{\rho_t(1-p)\eta^* + p\eta} \tag{46}$$

with $\eta > 0$, $\eta^* > 0$ and ζ of the sign of $\alpha_N - \alpha_T$.[4]

Proof: see Appendix 7.4.
From equation (46), the price of the nontraded good P_{Nt+1} depends on time only through ρ_t. But equations (39) and (40) show that education levels between countries diverge with ρ_t tends to zero or infinity. We focus on the case of divergence, $\rho_t \to 0$, where education in the domestic country becomes very large with respect to education in the foreign country. The domestic growth rate $1+g$ increases with k_T through equation (23). Since k_T is increasing with the global physical to human capital ratio k through the price of the nontraded good[5] P_N, then, $1+g$ is increasing with k. By symmetry, $1+g^*$ is increasing with k^*.
We finally state:

Proposition 4 *Under Assumption 1 and 2, the home country converges to its autarkic physical to human capital ratio, and growth rates diverge iff $\gamma > \Gamma(\gamma^*)$ (Condition 1). Perfect capital mobility then results in a decrease (increase) in the foreign country growth rate iff $\frac{x\eta}{\eta^*} < 1$ (>) (Condition 2).*

Proof: see Appendix 7.5.

Let us define the "leading" country as the country with the higher education spending and/or the higher traded productivity; this leading country is the home country if $\gamma > \Gamma(\gamma^*)$. The proposition states on one hand that the growth rate with

[4]The expressions of η, η^*, and ζ are given in Appendix 7.4.

[5]From equation (9), k_T is increasing with P_N iff $\alpha_T > \alpha_N$ and from equation (44), P_N is increasing with k iff $\alpha_T > \alpha_N$.

integration corresponds to the autarky growth rate of the "leading" country. On the other hand, integration may be growth reducing for the foreign country.

To sum up, with a sufficient degree of altruism (education spending), integration does not change the home country's growth rate whereas consequences of integration are mixed in the foreign country. If A_T^*/A_T and/or β^*/β are high (low) enough[6], then the foreign country's growth rate is higher (lower) with integration.

This proposition means that integration may have dramatic consequences if the two countries are not similar enough in terms of productivity, altruism and saving rates. Indeed, it may lead to a growth reduction for the lowest traded TFP country - and/or the less altruistic country. This is a typically two-sector (2x2) setting result. In such a model, growth depends on the level of the RER. In autarky, this RER is directly linked to altruism, savings and nontraded consumption. Integration involves perfect capital mobility between countries. The 2x2 model becomes a 2x2x2 model. Capital mobility implies that relative prices no longer depend either on domestic savings, altruism or propensity to consume. Instead, relative prices only depend on relative traded and nontraded productivity through equation (34).
Let us consider that $\beta = \beta^*$ and $\gamma = \gamma^*$. Conditions (1) and (2) become $A_T^* < A_T$. In this case, Proposition 4 simply states that the growth rate with integration is the autarky growth rate of the home country which is the leading one in terms of productivity - since we have assumed identical altruism degrees- and that economic growth in the foreign country is worst in the case of integration.

In autarky, factor returns and growth rate are lower in the foreign country : $w^A/w^{*A} = R^A/R^{*A} = P_N^A/P_N^{*A} = \left(1+g^A\right)/\left(1+g^{*A}\right) = A_T/A_T^* > 1$. Capital mobility changes the way relative prices are determined: relative prices then result from equation (34). Relative prices depend only on relative traded TFP. Since $A_T > A_T^*$, we still have $P_N > P_N^*$ from equation (34) but the ratio between these two prices is higher[7]: $P_N/P_N^* = (A_T/A_T^*)^{(1-\alpha_N)/(1-\alpha_T)}$. This means that $P_N^* < P_N^{A*}$, $w^* < w^{A*}$ and the foreign country's growth rate is even lower in an integrated world.
The intuition is as follows. With capital mobility, the domestic and foreign returns on capital converge to a common world return on capital. This common world return is the one of the "leading" country - the Home country in our simple case. Then, integration consists of an increase in the foreign return on capital compared to the foreign autarky case. This increase in the foreign return comes from a decrease in the foreign relative price. With capital mobility, the foreign relative price is stuck at a too low level compared to what it would be in autarky. As a result, the foreign country's growth rate will be lower than in autarky for the foreign country ($g^* < g^{A*}$). This mechanism works when $\beta = \beta^*$ and $\gamma = \gamma^*$, as soon as there is a traded TFP spread between the two countries. We focused on the simplest case ($\beta = \beta^*, \gamma = \gamma^*$) but

[6]Notice that Condition 2 is equivalent to $\left(\frac{A_T^*}{A_T}\right)^{\frac{1}{1-\alpha_T}} < \frac{\beta\gamma^*(1+\gamma^* a)(\zeta(1-\alpha_T)(1-\beta)+(1+\gamma a))}{\beta^*\gamma(1+\gamma a)(\zeta(1-\alpha_T)(1-\beta^*)+(1+\gamma^* a))}$

[7]This ratio is higher when $\alpha_T > \alpha_N$.

conditions (1) and (2) allow all cases to be considered. The leading country can be the higher TFP country or the more altruistic country or the lower time-preference country, other things being equal.

5 Calibrations

Table 2 collects parameter values. Since we focus on the case without cross-border externality we set $a = 1$ and then $\lambda = 0$. Education technology b is assumed to be unity. As usual in two-sector models, we assume half of consumption is dedicated to traded goods: $\mu = 0.5$. In the model, $\beta/(1+\gamma a)$ denotes the saving rate whereas $\gamma a/(1+\gamma a)$ denotes education spending. Our calibration is consistent with figures collected in Table 3 where we assume each period of life lasts for 25 years. Using Beine et al. (2002), the rate of time preference is supposed to be around 3.7%, then we assume $\beta = 0.4$ and then $\gamma^* = 0.36$. In Figures 2, the discount rates β and β^* lie between 0.3 and 0.4 to consider differences in saving behavior between countries. Finally, population growth reflects French annual population growth: 0.4%. Notice that in Table 3, education spending represents the sum of public spending (4.64%) and private spending (0.67%).

α_T	α_N	R	μ	n	γ^*	β	β^*
0.65	0.2	1.64	0.5	0.105	0.36	0.3 or 0.4	0.3 or 0.4

Table 2: Parameter values

Education spending (% of GDP)	5.31%
Households saving rate (% of GDP)	5.8%

Table 3: Average on 27 European countries (2006). Source: Eurostat.

Assuming capital shares in production of traded and nontraded sectors respectively of 0.65 and 0.2, we obtain in autarky that production of traded goods corresponds to 40.5% of GDP. Factor allocation between the two sectors is such that $k_T = 1.58 k_N$ and the autarky interest rate is $R = 2.77$ which represents an annual real interest rate of 2.3%. These figures are consistent with empirical evidence since traded output represents 40% of total output [Mahbub Morshed and Turnovsky (2004)].

Figure 1 illustrates Proposition 2 which states that convergence in growth rates is obtained for very special rates of altruism only. The convergence line is the dotted line. If the two countries have the same traded TFP, identical rates of altruism guarantee convergence ($A_T/A_T^* = 1$, $\gamma = \gamma^* = 0.36$). If the domestic country has a higher traded TFP, then convergence occurs only for a domestic rate of altruism $\gamma = 0.305$

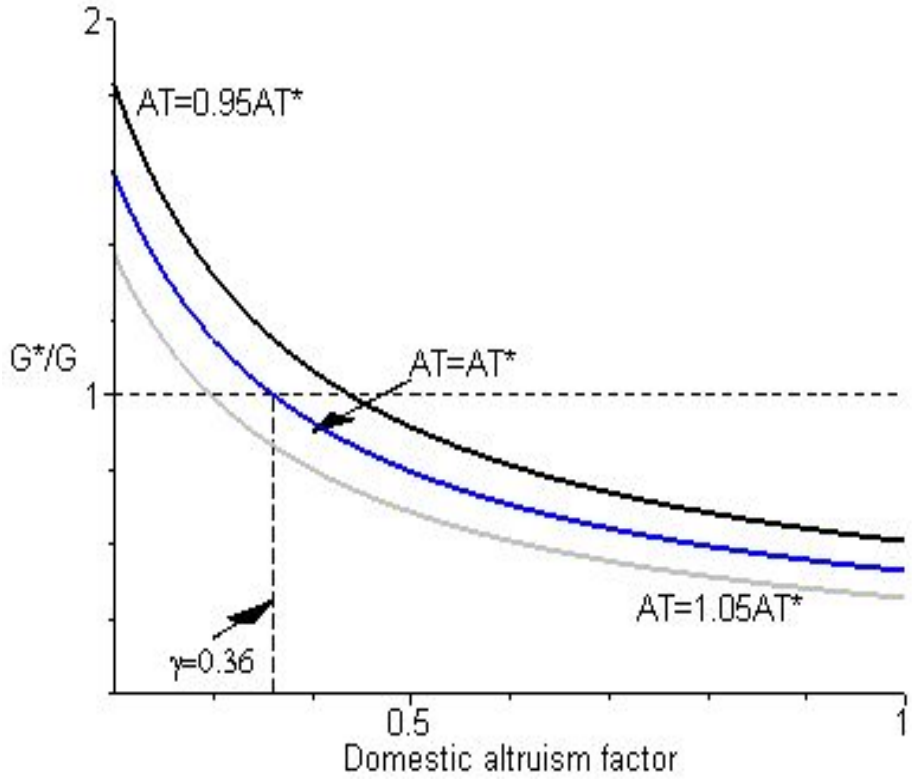


Figure 1: Domestic and foreign growth in an integrated world

which is lower than the foreign one ($A_T/A_T^* = 1.05$, $\gamma^* = 0.36$). The intuition is that in order to converge, the foreign country must offset the lag in technology by higher education spending. Similarly, if the domestic country has a lower traded TFP, then convergence occurs only for a domestic rate of altruism $\gamma = 0.429$ which is higher than the foreign one ($A_T/A_T^* = 0.95$, $\gamma^* = 0.36$).

Figures 2a, 2a', 2b and 2b' illustrate Proposition 4. If $\gamma > \Gamma(\gamma^*)$, then the domestic country converges to its autarky growth rate and growth rates diverge. Figures 2a and 2b plot the relative price - or more precisely the variable $Z = P_N^{1/(\alpha_T - \alpha_N)}$- as a function of the domestic rate of altruism γ. The dotted line represents the threshold $\Gamma(\gamma^*)$. Notice that on these figures, the domestic country is the leading country only when $\gamma > \Gamma(\gamma^*)$. Otherwise, the foreign country becomes the leading country.

Figure 2a (2b) corresponds to the case where the domestic country has a lower traded (higher) TFP but a higher (lower) discount rate (β). The consequences of capital mobility on the relative price depend both on the technological gap between the two countries and on the rates of time preference. If the domestic country integrates with a high-TFP-low-saving foreign country and $\gamma < \Gamma(\gamma^*)$, its relative price depreciates (black line) compared to what it would be in autarky relative price (grey line). Conversely, if the domestic country integrates with a low-TFP-high-saving foreign country and $\gamma < \Gamma(\gamma^*)$, its relative price appreciates (black line) compared

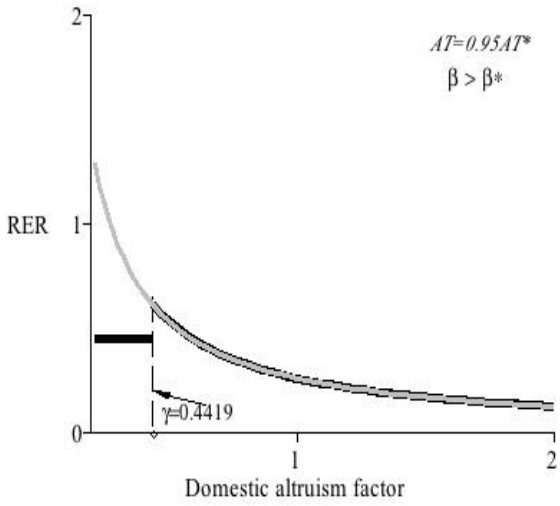


Figure 2a

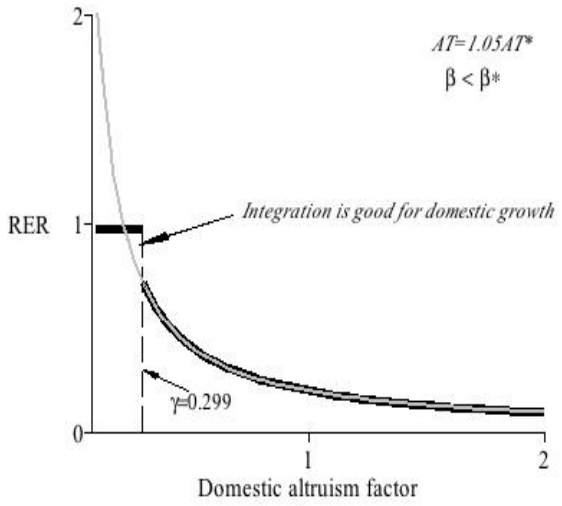


Figure 2b

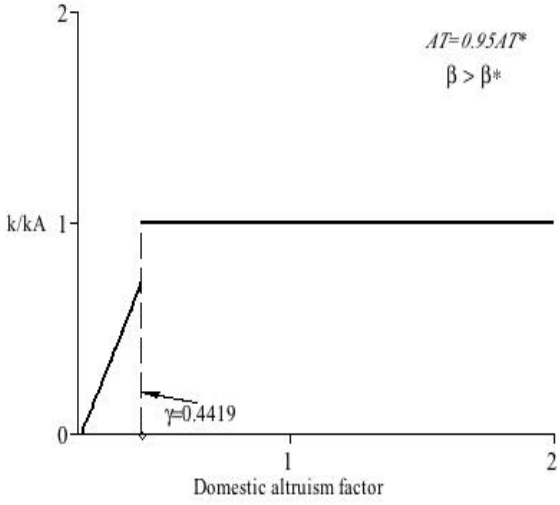


Figure 2a'

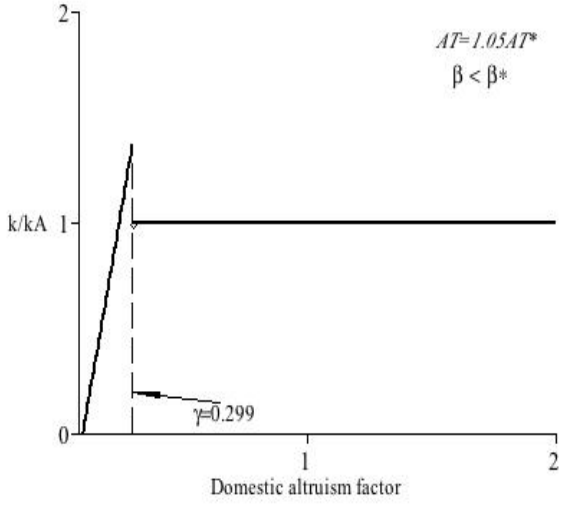


Figure 2b'

Figure 2: Benefit from integration, Real Exchange Rate and Domestic altruism

to what it would be in autarky (grey line). Otherwise $\gamma > \Gamma(\gamma^*)$ and the domestic country converges to its autarky equilibrium (black line) in both cases.

In this two-sector model, economic growth depends only on the RER. Figure 2a' (2b') depicts the ratio k/k_A[8] to compare the benefits of integration on growth when the domestic country integrates with a high-traded-TFP-low-saving country (low-traded-TFP-high-saving country). We focus on the case where $\gamma < \Gamma(\gamma^*)$. The advantage of integration depends on the gap between the grey and the black lines on Figures 2a and 2b. On Figure 2a (2b), integration leads to an RER depreciation

[8]Notice that from equations (23), (29) and (30), $G/G_a = (k/k_A)^{\alpha_T}$.

(appreciation) compared to the autarky relative price; thus, integration is growth damaging (enhancing): $G/G_a < 1$ ($G/G_a > 1$). In a standard one-sector model, capital mobility affects economic growth in each country because growth no longer depends on the domestic saving rate. In this two-sector model, economic growth also depends on the relative price, and then on the traded TFP gap between the two countries. When a low-TFP-high-saving domestic country integrates with a high-TFP-low-saving foreign country, capital mobility entails a depreciation, and growth rate with integration (G) is lower than the autarky growth rate (G_A). Capital mobility is then growth reducing when the rate of altruism is too low ($\gamma < \Gamma(\gamma^*)$) in a low-TFP-high saving country (see Figure 2a'). This means that a low-education-spending-low-traded-TFP-high-saving country has a higher growth rate in autarky.

This calibration exercise shows that capital mobility prevents the relative price from adjusting. Sometimes this real exchange rate sluggishness may be damaging for growth. The only case where growth may be better with capital mobility is in the case of a low-altruism-high-traded-TFP-low-saving domestic country as depicted by Figure 2b'.

6 Conclusion

This paper deals with capital mobility, RER and growth. Introducing a 2-sector structure in Michel and Vidal (1999) we show that opening borders to trade and factor mobility does lead to divergence between countries growth rates except if there exist cross-border externalities in human capital accumulation. In this 2x2x2 model, consequences of integration are mixed and highly dependent on both altruism and traded TFP gap between countries. Convergence is far from being the rule even with capital mobility. When a low-traded-TFP-low-altruism domestic country integrates with a high-TFP foreign country, capital mobility entails a depreciation and a growth deterioration in the domestic country (compared to autarky). This means that a low-education-spending-low-traded-TFP country has a higher growth rate in autarky. When a high-traded-TFP-high-altruism domestic country integrates with a low-TFP foreign country, capital mobility does not affect the domestic growth rate but improves the foreign one (compared to autarky).

This paper concludes that capital mobility may lead to real exchange rate misalignments which decrease growth in the less altruistic country with resulting income divergence. Education policy may be an appropriate instrument to make convergence more likely to occur. We leave this issue for future research.

7 Appendix

7.1 Proof of Lemma 1

Substituting equations (2) and (18) in equation (24), and dividing by N_t, we obtain:

$$(1-\mu)\pi_t(c_t + \frac{d_t}{1+n}) = P_{Nt}A_N {k_{Nt}}^{\alpha_N} h_t h_{Nt} \tag{47}$$

Integrating the budget constraints (14), (15) and the optimal level for s_t and e_t from equations (19) and (20) gives:

$$\frac{1-\mu}{1+\gamma a}((1-\beta)w_t h_t + \frac{\beta R_t w_{t-1} h_{t-1}}{1+n}) = P_{Nt}A_N {k_{Nt}}^{\alpha_N} h_t h_{Nt} \tag{48}$$

Moreover, from the optimal choice of investment in children's education(20) we know:

$$\frac{h_{t-1}}{h_t} = \frac{1+\gamma a}{\gamma a b w_{t-1}} \frac{e_{t-1}^{1-a}}{b_{t-1}}$$

And thus using equation (22) and dividing (48) by h_t we get:

$$\frac{1-\mu}{1+\gamma a}((1-\beta)w_t + \frac{\beta R_t}{1+n}\frac{1+\gamma a}{\gamma a b (p+p^*\rho_{t-1})^\lambda}) = P_{Nt}A_N k_{Nt}^{\alpha_N} h_{Nt} \tag{49}$$

As from equation (4), $h_N = \frac{k-k_T}{k_N-k_T}$, the non tradable market clearing condition is:

$$\frac{1-\mu}{1+\gamma a}((1-\beta)w_t + \frac{\beta R_t}{1+n}\frac{1+\gamma a}{\gamma a b (p+p^*\rho_{t-1})^\lambda)}) = P_{Nt}A_N {k_{Nt}}^{\alpha_N} \frac{k_t - k_{Tt}}{k_{Nt}-k_{Tt}} \tag{50}$$

From equations (9), we finally get the condition of the lemma.

□

7.2 Proof of Lemma 2

Considering autarky, and thus $\lambda = 0$ and $k_t = k_A$, the non tradable market clearing condition (25) is:

$$\frac{1-\mu}{1+\gamma a}((1-\beta)w_t + k_A R_t(1+\gamma a)) = P_{Nt}A_N D {k_T}^{\alpha_N - 1}(k_A - k_{Tt}) \tag{51}$$

Substituting P_{Nt} from equations (9):

$$\frac{1-\mu}{1+\gamma a}((1-\beta)w_t + k_A R_t(1+\gamma a)) = A_T \frac{\alpha_T(1-\alpha_T)}{\alpha_N - \alpha_T} {k_T}^{\alpha_T - 1}(k_A - k_{Tt}) \tag{52}$$

With factor prices from equations (7) and (8), we get:

$$\begin{aligned} \tfrac{1-\mu}{1+\gamma a}\left((1-\beta)(1-\alpha_T)A_T {k_T}^{\alpha_T} + k_A(1+\gamma a)\alpha_T A_T {k_T}^{\alpha_T - 1}\right) \\ = A_T \tfrac{\alpha_T(1-\alpha_T)}{\alpha_N - \alpha_T} {k_T}^{\alpha_T - 1}(k_A - k_{Tt}) \end{aligned} \tag{53}$$

Dividing by $k_T^{\alpha_T-1}$:

$$\frac{1-\mu}{1+\gamma a}\left((1-\beta)(1-\alpha_T)A_T k_T + k_A(1+\gamma a)\alpha_T A_T\right) = A_T\frac{\alpha_T(1-\alpha_T)}{\alpha_N-\alpha_T}\left(k_A - k_{Tt}\right) \tag{54}$$

From straightforward computations, we finally obtain equation (30). The last task is to compute the equilibrium growth rate g^A. Using equation (23), we readily obtain equation (29).

□

7.3 Proof of Proposition 1

As $k_T = \frac{\beta}{\gamma ab(1+n)}\frac{\alpha_T}{1-\alpha_T}\frac{(1-\mu\alpha_T-(1-\mu)\alpha_N)(1+\gamma a)}{(1-\mu)(1-\beta)(\alpha_N-\alpha_T)+\alpha_T(1+\gamma a)}$, we can define the growth factor as a function of β and γ:

$$1+g^A = ab(1-\alpha_T)A_T\frac{\gamma}{1+\gamma a}k_T^{\alpha_T} \equiv G^A(\beta,\gamma,\mu)$$

The logarithmic derivative of G^A with respect to β is:

$$\frac{\partial lnG^A(\beta,\gamma,\mu)}{\partial\beta} = \alpha_T\left(\frac{1}{\beta} + \frac{(1-\mu)(\alpha_N-\alpha_T)}{(1-\mu)(1-\beta)(\alpha_N-\alpha_T)+\alpha_T(1+\gamma a)}\right) \tag{55}$$

Which is positive if and only if

$$\frac{\alpha_N(1-\mu)+\alpha_T(\gamma a+\mu)}{(1-\mu)(1-\beta)(\alpha_N-\alpha_T)+\alpha_T(1+\gamma a)} \geq 0 \tag{56}$$

The numerator is always positive. The denominator is positive if and only if

$$\beta(1-\mu)(\alpha_N-\alpha_T) \leq \alpha_N(1-\mu)+\alpha_T(\gamma a+\mu) \tag{57}$$

which is true iff $\alpha_N \leq \alpha_T$ or $\alpha_N \geq \alpha_T$ and $\beta \leq \bar{\beta}$ with $\bar{\beta} = \frac{\alpha_N(1-\mu)+\alpha_T(\gamma a+\mu)}{(1-\mu)(\alpha_N-\alpha_T)}$. Since $\bar{\beta} > 1$, the growth factor is always increasing with β.
Concerning the variation of the growth rate with γ. The logarithmic derivative of G^A with respect to γ is:

$$\begin{aligned}\frac{\partial lnG^A(\beta,\gamma,\mu)}{\partial\gamma} &= \frac{1-\alpha_T}{\gamma} - \frac{a(1-\alpha_T)}{1+\gamma a} - \frac{a{\alpha_T}^2}{(1-\mu)(1-\beta)(\alpha_N-\alpha_T)+\alpha_T(1+\gamma a)} \\ &= \frac{(1-\alpha_T)((1-\mu)(1-\beta)(\alpha_N-\alpha_T)+\alpha_T(1+\gamma a))-a{\alpha_T}^2\gamma(1+\gamma a)}{\gamma(1+\gamma a)((1-\mu)(1-\beta)(\alpha_N-\alpha_T)+\alpha_T(1+\gamma a))}\end{aligned} \tag{58}$$

Which is zero for a unique positive value of γ, $\bar{\gamma} = \frac{1-2\alpha_T+\sqrt{1+4((1-\mu)(1-\beta)(\alpha_N-\alpha_T))}}{2a\alpha_T}$, positive for $0 \leq \gamma \leq \bar{\gamma}$ and negative for $\gamma > \bar{\gamma}$.
Concerning the variations of the growth rate with μ. As

$$\frac{\partial lnG^A(\beta,\gamma,\mu)}{\partial\mu} = \alpha_T(\alpha_N-\alpha_T)\left\{\frac{(1-\beta)(1-\alpha_T)+\alpha_T(1+\gamma a)}{((1-\mu)(1-\beta)(\alpha_N-\alpha_T)+\alpha_T(1+\gamma a))(1-\mu\alpha_T-(1-\mu)\alpha_N)}\right\} \tag{59}$$

The denominator is positive from the previous analyses and the positivity of k_T. Thus this derivative is of the sign of $\alpha_N - \alpha_T$.

□

7.4 Proof of Lemma 3

From equations (19) and (20), we obtain $s_t = \frac{\beta e_t}{\gamma a}$ and $s_t^* = \frac{\beta^* e_t^*}{\gamma^* a}$ and thus the world capital market clearing condition (32) becomes:

$$(1+n)\left(pk_{t+1}h_{t+1} + (1-p)k_{t+1}^* h_{t+1}^*\right) = p\frac{\beta e_t}{\gamma a} + (1-p)\frac{\beta^* e_t^*}{\gamma^* a} \tag{60}$$

Substituting the individual level of human capital for equation (22) :

$$(1+n)b\left(pk_{t+1}e_t + (1-p)k_{t+1}^* e_t^*\right) = p\frac{\beta e_t}{\gamma a} + (1-p)\frac{\beta^* e_t^*}{\gamma^* a} \tag{61}$$

dividing by e_t to write the equation as a function of $\rho_t = \frac{e_t^*}{e_t}$, we obtain:

$$(1+n)b(p+p^*\rho_t)\left(pk_{t+1} + (1-p)k_{t+1}^*\rho_t\right) = p\frac{\beta}{\gamma a} + (1-p)\frac{\beta^*}{\gamma^* a}\rho_t \tag{62}$$

From equation (28), we have $k^A = \frac{\beta}{(1+n)\gamma ab}$ and $k_A^* = \frac{\beta^*}{(1+n)\gamma^* ab}$, we finally get the expression of the lemma, giving implicitly ρ_t.

Denoting $x \equiv \frac{\beta^*\gamma}{\beta\gamma^*}$ then $k_A^* = xk^A$, and we can compute k_t, k_t^* as functions of P_{Nt} for which we also give an expression.
Using equations (9) and (10) in equation (25), we obtain the nontraded goods market clearing condition for the home country:

$$\begin{aligned}\tfrac{1-\mu}{1+\gamma a}\left[(1-\beta)\,w\left(P_{Nt}\right) + (1+\gamma a)k^A R\left(P_{Nt}\right)\right] \\ = P_{Nt}A_N D\left[k_T\left(P_{Nt}\right)\right]^{\alpha_N - 1}\left(k_t - k_T\left(P_{Nt}\right)\right)\end{aligned} \tag{63}$$

Meaning that

$$k_t = \frac{(1-\mu)\left(\frac{1-\beta}{1+\gamma a}w\left(P_{Nt}\right) + k^A R\left(P_{Nt}\right)\right)}{P_{Nt}A_N D\left[k_T\left(P_{Nt}\right)\right]^{\alpha_N - 1}} + k_T\left(P_{Nt}\right) \tag{64}$$

Substituting the expression of k_T from equation (9), the factor prices w_t and R_t from equations (10), and simplifying by $P_{Nt}^{\frac{\alpha_T - 1}{\alpha_T - \alpha_N}}$, we obtain:

$$k_t = (1-\mu)\frac{A_T}{A_N}\frac{B^{\alpha_T - \alpha_N}}{D}\left[\frac{(1-\beta)}{(1+\gamma a)}(1-\alpha_T)BP_{Nt}{}^{\frac{1}{\alpha_T - \alpha_N}} + k^A\alpha_T\right] + BP_{Nt}{}^{\frac{1}{\alpha_T - \alpha_N}} \tag{65}$$

and similarly for the foreign country:

$$k_t^* = (1-\mu)\frac{A_T^*}{A_N^*}\frac{B^{*\alpha_T - \alpha_N}}{D}\left[\frac{(1-\beta^*)}{(1+\gamma^* a)}(1-\alpha_T)B^* P_{Nt}^{*\,\frac{1}{\alpha_T - \alpha_N}} + k^{A^*}\alpha_T\right] + B^* P_{Nt}^{*\,\frac{1}{\alpha_T - \alpha_N}} \tag{66}$$

But as

$$\frac{A_T}{A_N}\frac{B^{\alpha_T - \alpha_N}}{D} = \frac{A_T^*}{A_N^*}\frac{B^{*\alpha_T - \alpha_N}}{D} = \frac{\alpha_N - \alpha_T}{\alpha_T(1-\alpha_T)}$$

and B can be rewritten $B = \Delta\left(\frac{A_N}{A_T}\right)^{\frac{1}{\alpha_T-\alpha_N}}$, $B^* = \Delta\left(\frac{A_N^*}{A_T^*}\right)^{\frac{1}{\alpha_T-\alpha_N}}$ where $\Delta \equiv \left(\frac{\alpha_N}{\alpha_T}\right)^{\frac{\alpha_N}{\alpha_T-\alpha_N}}\left(\frac{1-\alpha_T}{1-\alpha_N}\right)^{\frac{\alpha_N-1}{\alpha_T-\alpha_N}}$ is the same for both country.

Then we obtain the expressions for k_t and k_t^* given by equations (44) and (45), where:

$$\eta \equiv \Delta\left(\frac{A_N}{A_T}\right)^{\frac{1}{\alpha_T-\alpha_N}}\left[(1-\mu)\frac{\alpha_N-\alpha_T}{\alpha_T}\frac{1-\beta}{1+\gamma a}+1\right]$$

$$\eta^* \equiv \Delta\left(\frac{A_N}{A_T^*}\right)^{\frac{1}{\alpha_T-\alpha_N}}\left(\frac{A_T^*}{A_T}\right)^{\frac{1-\alpha_N}{(\alpha_T-\alpha_N)(1-\alpha_T)}}\left[(1-\mu)\frac{\alpha_N-\alpha_T}{\alpha_T}\frac{(1-\beta^*)}{(1+\gamma^* a)}+1\right]$$

$$\zeta \equiv (1-\mu)\frac{\alpha_N-\alpha_T}{1-\alpha_T}$$

Finally, substituting those expressions in the world market clearing condition (43), we obtain the expression of P_{Nt} given by equation (46).
It is clear that ζ is of the sign of $\alpha_N - \alpha_T$. Thus if $\alpha_N < \alpha_T$, we have $\zeta < 0$ and we need conditions to get $k > 0$ and $k^* > 0$.
If $E(\rho_t) \equiv \frac{\rho_t(1-p)x+p}{\rho_t(1-p)\eta^*+p\eta}$, then from equations (44), (45) and (46), and the monotonicity of $E(.)$:

$$k_t > 0 \Leftrightarrow E(\rho_t) > \frac{-\zeta}{\eta(1-\zeta)} \Leftrightarrow \begin{cases} E(0) > \frac{-\zeta}{\eta(1-\zeta)} & (a) \\ \lim_{\rho_t\to+\infty} E(\rho_t) > \frac{-\zeta}{\eta(1-\zeta)} & (b) \end{cases}$$

$$\text{and} \quad {k_t}^* > 0 \Leftrightarrow E(\rho_t) > \frac{-\zeta x}{\eta^*(1-\zeta)} \Leftrightarrow \begin{cases} E(0) > \frac{-\zeta x}{\eta^*(1-\zeta)} & (c) \\ \lim_{\rho_t\to+\infty} E(\rho_t) > \frac{-\zeta x}{\eta^*(1-\zeta)} & (d) \end{cases}$$

Conditions (a) and (d) always hold when ζ is negative, and under assumption 2, conditions (b) and (c) hold.

7.5 Proof of Proposition 4

If $\gamma > \Gamma(\gamma^*)$ then $\frac{1+g^*}{1+g} < 1$ and ρ_t converges to zero as t goes to infinity.
But as from equation (43):

$$\rho_t = \frac{p}{1-p}\frac{k_A - k_{t+1}}{k_A^* - k_{t+1}^*}$$

then the capital stock of the home country k_t converges to k^A. From equations (45) and (46) with $\rho = 0$, we have:

$$k_t^* = k_A^*\left[\zeta + (1-\zeta)\frac{\eta^*}{\eta x}\right] \tag{67}$$

In order to understand the effect of economic integration in the foreign country, it is important to compare the long term value of k^* to the autarkic value k_A^*.

As we have assumed that $\alpha_N < \alpha_T$, we know $1-\zeta > 0$, and Condition 2 follows from $k^* > k_A^*$ iff $\frac{\eta^*}{\eta x} > 1$. Since $\zeta(1-\alpha_T)\frac{1-\beta}{1+\gamma a} + \alpha_T > 0$ and $\zeta(1-\alpha_T)\frac{1-\beta^*}{1+\gamma^* a} + \alpha_T > 0$ from the proof of Proposition 1 and using the expressions of η, η^* and x, we can rewrite Condition 2 as :

$$\left(\frac{A_T^*}{A_T}\right)^{\frac{1}{1-\alpha_T}} > \frac{\beta^*\gamma(1+\gamma^* a)\left(\zeta(1-\alpha_T)(1-\beta)+\alpha_T(1+\gamma a)\right)}{\beta\gamma^*(1+\gamma a)\left(\zeta(1-\alpha_T)(1-\beta^*)+\alpha_T(1+\gamma^* a)\right)}$$

But as $1+g^* < 1+g$ means

$$\left(\frac{A_T^*}{A_T}\right)^{\frac{1}{1-\alpha_T}} < \frac{\gamma(1+\gamma^* a)}{\gamma^*(1+\gamma a)}$$

or equivalently:

$$\frac{x\eta}{\eta^*} > \frac{\beta^*}{\beta}\frac{\zeta(1-\alpha_T)(1-\beta)+\alpha_T(1+\gamma a)}{\zeta(1-\alpha_T)(1-\beta^*)+\alpha_T(1+\gamma^* a)}$$

This case is obtained under Assumption 2 when

$$Max\left(\frac{-\zeta}{1-\zeta}, \frac{\beta^*}{\beta}\frac{\zeta(1-\alpha_T)(1-\beta)+\alpha_T(1+\gamma a)}{\zeta(1-\alpha_T)(1-\beta^*)+\alpha_T(1+\gamma^* a)}\right) < \frac{x\eta}{\eta^*} < 1$$

with γ such that:

$$\frac{\beta^*}{\beta}\frac{\zeta(1-\alpha_T)(1-\beta)+\alpha_T(1+\gamma a)}{\zeta(1-\alpha_T)(1-\beta^*)+\alpha_T(1+\gamma^* a)} < 1$$

□

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