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Résumé

Cette note caractérise l’assiette optimale d’un impôt indirect lorsque la taxation d’une catégorie de biens implique un coût fixe d’administration. Nous montrons que, lorsque les taux d’imposition sont petits, l’assiette optimale comprend les biens dont l’indice de découragement est supérieur au rapport entre le coût d’administration impliqué par leur intégration dans l’assiette et l’impôt qu’ils permettent de collecter.

abstract

This note characterizes the optimal base for commodity taxation in the presence of administrative fixed costs varying across goods. For low tax rates, the optimal base comprises all commodities whose discouragement index is greater than the ratio of their administrative costs to the tax they yield.

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Mots-clés : Taxation indirecte, TVA, assiette fiscale, coûts de gestion et d’administration.

Keywords: indirect taxation, VAT, tax base, administrative costs.
1 Introduction

The theory of optimal taxation focuses on the excess burden of taxation as the main source of the social loss caused by taxation. The empirical evidence however suggests that the administrative costs required to collect taxes may be substantial (Slemrod, 1990). Most measures of such costs are based on staff salary or equipment costs, but little is known about their true underlying determinants and precise shape (Shaw, Slemrod and Whiting, 2010). Polinsky and Shavell (1982), Kaplow (1990) and Mayshar (1991) derived optimal tax rules when the administrative cost function displays usual continuity and convexity properties with respect to the level of tax rates. Still, as argued by Slemrod and Yitzhaki (1995) and Alm (1996), it is likely that this function exhibits significant discontinuities and/or nonconvexities.

Yitzhaki (1979) and Wilson (1989) have given early insights into the optimal indirect tax structure when these discontinuities are due to idiosyncratic fixed costs. Such a formulation obtains when a minimum number of employees is required for performing the administration of taxed goods. Otherwise, no tax is recovered because of, e.g., tax evasion and black market operations. In this circumstance the characterization of the tax base involves a discrete choice between taxation and exemption. As first shown by Yitzhaki (1979) in a simple Cobb-Douglas partial equilibrium economy with one representative consumer and uniform commodity taxation, this choice not only relies on the sensitivity of demand to price but also on the level of demand. The tax base actually comprises all the goods with a low enough ratio of administrative cost to the amount of tax they yield. Hereafter this ratio will be referred to as the ‘Yitzhaki ratio.’

This note provides a generalization of Yitzhaki (1979) and Wilson (1989) by allowing for nonuniform optimal tax rates, and heterogeneous households with equity considerations. It gives a general rule for including a good in the tax base. In the case of low tax rates, a good is included in the tax base only if its associated Mirrlees’ (1976) discouragement index is higher than its Yitzhaki ratio. There is therefore no specific effect of the equity concerns on the decision to tax. The influence of equity only transits through the optimal tax rates: a good whose demand should not be strongly discouraged, possibly because it is consumed by agents whose social value is high, is less likely to be taxed.
2 The setup

There is a continuum of categories of commodities $i \in C$ and a continuum of households $h \in H$. The preferences of household $h$ are represented by a utility function which is separable across consumption goods and labor,

$$\int_C u_i^h(x_i^h) d\mu_i - \ell^h.$$

The cdf $\mu$ captures the relative importance of the different categories of goods. The problem of household $h$ consists in choosing a bundle $(x_i^h)$ and a labor supply $\ell^h$ which maximize her utility subject to the budget constraint

$$\int_C (1 + t_i)x_i^h d\mu_i \leq \ell^h.$$

The demand function solution to this problem is $x_i^h = \xi_i^h(t_i)$ for every $i \in C$. Indirect utility is

$$\int_C \left( u_i^h(\xi_i^h(t_i)) - (1 + t_i)\xi_i^h(t_i) \right) d\mu_i \equiv \int_C v_i^h(t_i) d\mu_i.$$

The contribution of category $i$ goods to the welfare of household $h$ can therefore be measured by $v_i^h(t_i)$.

3 The decision to tax

Following Yitzhaki (1979), the tax authority is assumed to pay a fixed cost $c_i$ when it decides to tax (or subsidize) commodity $i$. Otherwise, this commodity remains tax-free, possibly in the informal sector. Whether a commodity should be taxed or exempted is a discrete decision to which the standard Lagrangian method does not directly apply. It can nevertheless be treated as a continuous decision by proceeding as if it were possible to tax a part $\pi_i$ of commodity $i$ and to exempt the remaining part of this commodity. The economically relevant solutions are such that $\pi_i$ is either 0 or 1.

With this new set of variables, the problem of the tax authority is to select $t_i$ and $\pi_i$ for every $i \in C$. Let the distribution of households be represented by
the cdf \( \nu \), and let \( \gamma^h \) stand for the social valuation of the welfare of household \( h \). At the optimum, the profile \( ((t_i), (\pi_i)) \) maximizes

\[
\int_{\mathcal{H}} \gamma^h \left( \int_{\mathcal{C}} \pi_i v^h_i(t_i) \, d\mu_i + \int_{\mathcal{C}} (1 - \pi_i) v^h_i(0) \, d\mu_i \right) \, d\nu^h
\]

subject to the budget constraint

\[
\int_{\mathcal{C}} \pi_i \left( t_i \int_{\mathcal{H}} \xi^h_i(t_i) \, d\nu^h - c_i \right) \, d\mu_i \geq R \quad (\lambda)
\]

and for every \( i \in \mathcal{C} \),

\[
\pi_i \geq 0, \quad (\rho_i) \\
\pi_i \leq 1. \quad (\sigma_i)
\]

The variables in brackets are the associated Lagrange multipliers associated to goods of category \( i \). The Lagrangian approach can now be used to give a necessary condition for commodity \( i \) to be taxed. The first-order condition for \( \pi_i \) to be a maximum is

\[
\int_{\mathcal{H}} \left( \gamma^h (v^h_i(t_i) - v^h_i(0)) + (t_i \xi^h_i(t_i) - c_i) + \rho_i - \sigma_i \right) \, d\mu_i \, d\nu^h = 0. \quad (3)
\]

In addition the Kuhn and Tucker exclusion conditions must be satisfied,

\[
\rho_i \geq 0, \quad \rho_i \pi_i = 0, \quad (4) \\
\sigma_i \geq 0, \quad \sigma_i (1 - \pi_i) = 0. \quad (5)
\]

Let

\[
\mathcal{L}_{i}(t_i, \lambda) \equiv \int_{\mathcal{H}} \gamma^h v^h_i(t_i) \, d\nu^h + \lambda \int_{\mathcal{H}} \xi^h_i(t_i) \, d\nu^h
\]

stand for the contribution of commodity \( i \) to social welfare (net of its associated administrative costs). The following result directly follows from the first-order condition (3).
Proposition 1. Assume that commodity $i$ is taxed (or subsidized) at rate $t_i$ when it belongs to the tax base. It should be included in the tax base only if $\mathcal{L}_i(t_i, \lambda) - \mathcal{L}_i(0, \lambda) > \lambda c_i$, or, equivalently,

$$
\int_{\mathcal{H}} \beta^h \left( v^h_i(t_i) - v^h_i(0) \right) d\nu^h + t_i \int_{\mathcal{H}} \xi^h_i(t_i) d\nu^h > c_i, \quad (6)
$$

where $\beta^h \equiv \gamma^h / \lambda$ is the marginal social valuation of the income of household $h$.

Proof. The first-order condition (3) can be rewritten as

$$
\mathcal{L}_i(t_i, \lambda) - \mathcal{L}_i(0, \lambda) - \lambda c_i + \rho_i - \sigma_i = 0.
$$

If $\mathcal{L}_i(t_i, \lambda) - \mathcal{L}_i(0, \lambda) - \lambda c_i > 0$, then $\rho_i \geq 0$ requires $\sigma_i > 0$, and (5) gives $\pi_i = 1$. If $\mathcal{L}_i(t_i, \lambda) - \mathcal{L}_i(0, \lambda) - \lambda c_i \leq 0$, then $\sigma_i \geq 0$ requires $\rho_i > 0$, and (4) shows that commodity $i$ must be exempted ($\pi_i = 0$). □

Proposition 1 characterizes the optimal tax base associated with arbitrary tax rates. The optimal tax rate $t^*_i$ on commodity $i$ maximizes $\mathcal{L}_i(t_i, \lambda)$, given the marginal social cost of public funds $\lambda$. It therefore satisfies the first-order condition

$$
-d^*_i \equiv \int_{\mathcal{H}} t^*_i \frac{\partial \xi^h_i(t^*_i)}{\partial t_i} d\nu^h = -((1 - \beta) - \beta \phi^*_i), \quad (7)
$$

with

$$
\beta \equiv \int_{\mathcal{H}} \beta^h d\nu^h, \quad \xi_i(t^*_i) \equiv \int_{\mathcal{H}} \xi^h_i(t^*_i) d\nu^h, \quad \phi^*_i \equiv \text{cov} \left( \frac{\beta^h}{\beta}, \frac{\xi^h_i(t^*_i)}{\xi_i(t^*_i)} \right).
$$

This is the familiar many-person Ramsey formula. At the optimum, the compensated demand for commodity $i$ must be reduced in proportion to the Mirrlees’ (1976) discouragement index, $d^*_i$.

In an optimal indirect tax structure, the tax base comprises goods satisfying (6), the tax rates ($t^*_i$) satisfy (7) and the marginal social cost of public funds $\lambda$ is determined by the budget constraint (2) of the tax authority.

In order to illustrate how the composition of the tax base given by inequality (6) and the Ramsey rule (7) interact, let us consider the empirically
plausible configuration of low rates of tax, i.e., \( t_i \) is close enough to 0 for all \( i \in C \). Then, appealing to Roy’s identity, a first-order Taylor expansion yields

\[
L_i(t_i, \lambda) \simeq L_i(0, \lambda) - t_i \left( \int_{\mathcal{H}} \gamma h \xi h_i(0) d\nu^h - \lambda \int_{\mathcal{H}} \xi h_i(0) d\nu^h \right).
\]

Thus, by Lemma 1, commodity \( i \) should be taxed (or subsidized) at some rate \( t_i \) only if

\[
-t_i \left( \int_{\mathcal{H}} \frac{\beta h \xi h_i(0)}{\xi_i(0)} d\nu^h - 1 \right) > \frac{c_i}{\xi_i(0)}.
\]

Note that

\[
\int_{\mathcal{H}} \frac{\beta h \xi h_i(0)}{\xi_i(0)} d\nu^h = \beta \left( 1 + \text{cov} \left( \frac{\beta h}{\beta}, \frac{\xi h_i(0)}{\xi_i(0)} \right) \right) \simeq \beta \left( 1 + \phi_i^* \right),
\]

where the last approximation is obtained by appealing to the assumption of a low tax rate. The rule (7) then yields the following result:

**Proposition 2.** Assume that commodity \( i \) is taxed at a low Ramsey tax rate \( t_i^* \) when it belongs to the tax base. It should be included in the tax base only if its associated discouragement index is greater than its Yitzhaki ratio, i.e.,

\[
|d_i^*| > \frac{c_i}{|t_i^*| \xi_i(0)}.
\]

Otherwise it should be exempted.

Proposition 2 gives a clear picture of the optimal tax base. Assume for instance that the Ramsey tax rate \( t_i^* \) is positive. Then, commodity \( i \) is more likely to be exempted whenever (1) it is costly to administrate \( (c_i \) is high), (2) it yields a low amount of taxes \( (t_i^* \xi_i(0) \) is low), and (3) its demand should not be strongly discouraged \( (d_i^* > 0 \) is low), i.e., the efficiency cost induced by taxation of commodity \( i \) is high \( (\beta \) is high) and this good is consumed by households with high social value \( (\phi_i^* \) is positive).
References


