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Insider Trading with Different Market Structures

Wassim Daher* Fida Karam† Leonard J. Mirman‡

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Abstract

We study an extension of Jain and Mirman (1999) with two insiders under three different market structures: (i) Cournot competition among the insiders, (ii) Stackelberg game between the insiders and (iii) Monopoly in the real market and Stackelberg in the financial market. We show how the equilibrium outcomes are affected by each of the market structure. Finally we perform a comparative statics analysis between the models. [1]

JEL classification: G14, D82

Keywords: Insider Trading, Cournot , Stackelberg, Correlated signals, Kyle model

1 Introduction

Insider trading is considered as one of the most notorious methods of stock fraud, in which an individual involved with a company whose stock is publicly traded, shares or sells information with regard to an event involving that company. Typically, this information is not only unavailable to the public, but is also expected to alter the behavior of the company's stock, resulting in profit opportunities of illegal nature. Insider trading has had some landmark incidents or scandals that helped define exactly what it is, starting with the Chase National Bank case in 1929. Newer scandals can

*Corresponding author, Department of Mathematics and Natural Sciences, Gulf University for Science and Technology. Email: daher.w@gust.edu.kw.

†Department of Economics and Finance, Gulf University for Science and Technology. Email: karam.f@gust.edu.kw.

‡Department of Economics, University of Virginia. Email: lm8h@virginia.edu.

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also be cited such as the Enron Corporation Case in 2004 and the New Castle Funds case in 2010. All these scandals reflect the different types of inside-relationship and outside-connection of corporations.

The recurrence of insider trading scandals has renewed the academic interest in this topic. Much of the theoretical research has centered on two issues: the public desire of implementing regulations and laws which prohibit insider trading (Dow and Rahi (2003), Leland (1992), Manove (1989)) and the dissemination of information that was captured by Kyle (1985) in his seminal work on insider trading.

The work of Kyle (1985) has constituted the cornerstone for subsequent works on the informational effect of insider trading. In the Kyle model, there is an insider who knows the value of the stock and a market maker who only knows the distribution of the values of the stock, gets information from the total noisy stock order flow, and sets the stock price in a way that his expected profits are zero . The main result is that the stock price reveals half of the inside information, regardless of the parameter values. When the marker maker is allowed to observe, in addition to the total order flow, another signal of the value of the asset, like in Jain and Mirman (1999), the stock price becomes more informative and the insider's profits decrease with respect to Kyle (1985).

Jain and Mirman (2000, 2002), Daher and Mirman (2006, 2007), Wang et al. (2009) and Wang and Wang (2010), then explored, the impact on information revelation, of various types of speculative markets in the spirit of Kyle (1985) and Jain and Mirman (1999), by modeling the financial and real sectors together. The idea behind the introduction of the real sector is that the insider's information is due to his participation in the real activities underlying the financial assets that are the object of insider trading. In Jain and Mirman (2000), the insider (or trader) is also the manager of the firm acts as a quantity-setting monopolist in the real sector. Cournot competition in the real sector is added to this model in Jain and Mirman (2002). Both papers show that the amount of information incorporated in the stock price, which is the same in both papers, increases with respect to Kyle (1985), but is the same as in Jain and Mirman (1999). In other words, the introduction of the real sector does not affect the amount of information revealed. However, the insider's profits are, in both papers, lower than Kyle (1985). The insider's profits and his compensation scheme are lower in Jain and Mirman (2002) with respect to Jain and Mirman (2000), due to Cournot competition in the real sector.

Daher and Mirman (2006, 2007), Wang et al (2009) and Wang and Wang (2010) extended Jain and Mirman (2000) and (2002) to include competition in the financial sector. Indeed, there are different types of insiders in the firm, some without any managerial responsibilities (the president and the members of the board of directors, for example), with the objective of maximizing their profits from trading the stock of the firm whose inside information they possess. Therefore, the competition among insiders is another form of competition that influence the amount of information disseminated in the stock price. Daher and Mirman (2006) and (2007) show that Cournot competition in the financial sector between the owner and the manager increases the amount of information incorporated in the stock price (which is the same between the two papers) with respect to Jain and Mirman (2000) and (2002). The profits of the manager sometimes increase and sometimes decrease, relatively to Jain and Mirman (2000, 2002), depending on the variances of the exogenous variables. However, the profits of the manager and the owner are less in the Cournot-real case (Daher and Mirman, 2006) than in the monopoly-real case (Daher and Mirman, 2007). Wang and Wang (2010) introduce Stackelberg competition in the real market to the model of Daher and Mirman (2007) and support the conclusion that competition in the real sector does not affect the degree of information revelation. In Wang et al (2009), the manager in the lower ladder of the organizational hierarchy takes the order from the owner and makes the decisions in the Cournot-real sector. He acts as a Stackelberg-follower in the financial sector to the owner who is high on the organizational hierarchy and who knows the manager's reaction function. The authors show that Stackelberg competition in the financial sector increases the amount of information revealed and the owner's profits, in comparison with Daher and Mirman (2006). The manager's profits may decrease or increase depending on the exogenous parameters of the model.

In this paper, we offer an in-depth study to the effect of the financial market structure on the revelation of information. We start by a natural extension of Jain and Mirman (1999) to include Cournot duopoly in the financial market (Model I). We find that each insider looses the market power and partially controls the stock price. Hence, the stock price reveals more information with respect to Jain and Mirman (1999). The unconditional profits of each insider also decreases. Those results are similar to Daher and Mirman (2007), when compared to Jain and Mirman (2000). We then try to add a little more reality to Model I, and assume that one of the insiders, the owner, is high in the organizational hierarchy and chooses the second insider, the manager, to serve his purpose. In other words, we model Stackelberg competition in the financial market, where the owner is the leader and knows the reaction function of the manager. This type of financial information asymmetry between the owner and the manager is widely observed in real life. We show that with Stackelberg competition in the financial market (Model II), the manager trades less and hence earns less than in the Cournot case. However, the owner, due to her role as leader, trades more than in the Cournot and earns more profits. We also notice that the price reveals more information in the Stackelberg than in Cournot structure. We finally take into consideration the real activities of the firm whose stock is traded and assume that the owner, who acts as a leader in the financial model, does not have any managerial responsibilities. Instead, the manager who acts as a follower in the financial market is a quantity-setting monopolist in the real market (Model III). We show that the introduction of the real market does not affect the amount of information revealed with respect to Model II. The same result holds in Jain and Mirman (2000), when compared to Jain and Mirman (1999).

The paper is structured as follows: in Sections 2, 3 and 4, we present respectively Model I, II and III with their comparative statics. In Section 5, we conclude.

2 Model I : The Cournot Case

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, all the random variables are defined with respect to this probability space. In this model we introduce Cournot competition between two insiders into the model of Jain and Mirman (1999).[2] Consider an economy with one financial asset, the stock of the firm. There are three types of agents trading in the financial market. First, there are two risk-neutral rational traders who know the realization z of $\tilde{z}$, the value of the stock. Second, there are (non-rational) noise traders, representing small investors with no information on z. The aggregate noise trade is assumed to be a random variable $\tilde{u}$, which is normally distributed with mean zero and variance σ_u^2. Finally, there are $K(K \geq 2)$ risk-neutral market makers who act like Bertrand competitors.

We assume, as in Jain and Mirman (1999), that the market makers observe two signals, a noisy signal about the value of the firm, denoted by $\tilde{q} = \tilde{z} + \tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is normally distributed with mean zero and variance σ_ε^2, and the total order flow signal. We assume that $\tilde{z}, \tilde{u}$ and $\tilde{\varepsilon}$ are pairwise independent.

Following Kyle (1985), the trading mechanism is organized in two steps. In step one, a linear pricing rule and optimal order rule are determined by the market makers and the insiders, respectively, as a Bayesian Nash equilibrium. The market makers determine a (linear) pricing rule p, based on their a priori beliefs, where p is a measurable function $p : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$. Each insider $i(i = 1, 2)$ chooses a stock trade function $\tilde{x}_i = x_i(\tilde{z})$, where

[2] It should be pointed out that the results can be automatically generalized to n insiders, but for the sake of comparison we restrict our attention to the two-insiders case.

$x_i : \mathbb{R} \longrightarrow \mathbb{R}$ is a measurable function. In the second step,[3] the insiders observe the realization z of $\tilde{z}$, and submit their stock order to the market makers based on the equilibrium stock trade functions. The market makers also receive orders from the noise traders, all these orders arrive as a total order flow signal $\tilde{r} = \sum_i x_i(\tilde{z}) + \tilde{u}$. The two signals are used by the market makers to set the price $\tilde{p} = p(\tilde{q}, \tilde{r})$, based on the equilibrium price function, to clear the market. The insiders know only the realization z of $\tilde{z}$ and does not know the values of $\tilde{u}, \tilde{\varepsilon}, \tilde{r}, \tilde{z} + \tilde{\varepsilon}$ before the order flow decisions is made. Moreover, each market maker does not know the realization z of $\tilde{z}$ but only knows its distribution. Finally, the market makers cannot observe either x_i, u or ε.

This is a game of incomplete information because the market makers, unlike the insider, do not know the realization of $\tilde{z}$. Hence, we seek a Bayesian-Nash equilibrium. A Bayesian-Nash equilibrium is a vector of three functions $[x_1(.), x_2(.), p(.,.)]$ such that:

(a) Profit maximization of insider i,

$$E(\tilde{z} - p(\tilde{z} + \tilde{\varepsilon}, \tilde{x}_i + \tilde{x}_{-i} + \tilde{u}))\tilde{x}_i \geq E(\tilde{z} - p(\tilde{z} + \tilde{\varepsilon}, \tilde{x}'_i + \tilde{x}_{-i} + \tilde{u}))\tilde{x}'_i \quad (1)$$

for any level of trading order $\tilde{x}'_i$ decided by the insider i and $\tilde{x}_{-i}$ is the trading order of the other insider ;

(b) Semi-Strong Market Efficiency: The pricing rule $p(.,.)$ satisfies,

$$p(\tilde{q}, \tilde{r}) = E[\tilde{z}|\tilde{q}, \tilde{r}]. \quad (2)$$

An equilibrium is linear if there exists constants μ_0, μ_1, μ_2 such that,

$$\forall q, r, \quad p(q, r) = \mu_0 + \mu_1 q + \mu_2 r. \quad (3)$$

Note that Condition (1) defines the optimal strategies of the two insiders while Condition (2) guarantees the zero expected profits for the market makers. The stock price, set by the market makers, is equal to the conditional expectation of the asset given the available information. We restrict our study to linear equilibrium. The normal distributions of the exogenous random variables, together with the particular expression of the demand, enable us to derive and to prove the existence of a unique linear equilibrium. Next in Proposition 1, we characterize the unique linear equilibrium.

[3] Random variables are denoted with a tilde. Realized values lack the tilde. The mean of the random variable is denoted with bar.

Proposition 1 *In the insider trading Cournot model with two signals, a linear equilibrium exists and is unique. It is characterized by*

$$\tilde{x}_1 = \tilde{x}_2 = \frac{(1-\mu_1)(\tilde{z}-\bar{z})}{3\mu_2} \tag{4}$$

$$\mu_0 = (1-\mu_1)\bar{z}, \quad \mu_1 = \frac{\sigma_z^2}{\sigma_z^2 + 3\sigma_\varepsilon^2}, \quad \mu_2 = \frac{\sqrt{2}\sigma_\varepsilon^2\sigma_z}{\sigma_u(\sigma_z^2 + 3\sigma_\varepsilon^2)} \tag{5}$$

$$Var(\tilde{z}|\tilde{q},\tilde{r}) = \frac{\sigma_\varepsilon^2}{\sigma_z^2 + 3\sigma_\varepsilon^2}\sigma_z^2 \tag{6}$$

and

$$\pi_i(\tilde{z}) = \frac{\sigma_u\sigma_\varepsilon^2(\tilde{z}-\bar{z})^2}{\sqrt{2}\sigma_z(\sigma_z^2 + 3\sigma_\varepsilon^2)} \qquad i = 1, 2 \tag{7}$$

Proof: See Appendix A

Discussion of the equilibrium In this model we extend the Jain and Mirman (1999) model with one insider (monopoly) and two signals to the case of two insiders (Cournot) and two signals. The strategic competition between the two insiders alters the equilibrium outcomes. Indeed, Lemma 1 whose proof is omitted, shows that on the one hand, Cournot competition in the financial market decreases the trades of each of the insiders as well as their conditional profits. On the other hand, Cournot competition increases the total order flow and therefore increases the amount of information revealed by the stock price. In the process, the price function coefficients are also affected by Cournot competition. Formally,

Lemma 1 *The effects of Cournot competition on the equilibrium outcomes relative to the monopoly case are given by,*[4]

$$\mu_0 > \mu_0^{JM} \quad \mu_1 < \mu_1^{JM} \qquad \tilde{x}_i < \tilde{x}^{JM} \qquad \pi_i < \pi^{JM} \quad \forall\ i = 1, 2 \tag{8}$$

$$Var(\tilde{z}|\tilde{q},\tilde{r}) < Var(\tilde{z}|\tilde{q},\tilde{r})^{JM} \tag{9}$$

$$\mu_2 < \mu_2^{JM} \quad \text{if } \sigma_z^2 < \frac{3-2\sqrt{2}}{\sqrt{2}-1}\sigma_\varepsilon^2 \ \text{ and } \mu_2 > \mu_2^{JM} \text{ otherwise} \tag{10}$$

Before analyzing the results, let us highlight the relationship between the present model and the existing models. As noted, our model adds a second insider to Jain and Mirman (1999) and hence we study the effect of Cournot

[4] The superscript JM refers to Jain and Mirman (1999).

competition in the financial market. Daher and Mirman (2007) also studied Cournot competition in the financial market with respect to Jain and Mirman (2000), but both of these models incorporate decisions made in the real sector and therefor both models have two signals. Finally, Tighe (1989) and Holden and Subrahmanyam (1992) studied the Cournot extension of the Kyle model with only one signal the total order flow.

First note, adding another insider to Jain and Mirman (1999), decreases each the insider's trades and thus their conditional profits. But the total insider trade is increased as are the total profits. Moreover, this result also occurs in Daher and Mirman (2007) as compared to Jain and Mirman (2000), in both of these models, real decisions are endogenous. Moreover, Tighe (1989) and Holden and Subrahmanyam (1992) obtained the same result when they introduced Cournot competition between the insiders to the monopoly model of Kyle (1985) where the market maker observes only the total order flow signal.

Second, strategic competition among the insiders has a direct effect on information revelation. Indeed, with two insiders the stock price conveys more information than in the case of a single insider. Indeed, the expressions of the conditional variances measuring the amount of information revealed in the stock price in Jain and Mirman (1999) and in our model are:

$$Var(\tilde{z}|\tilde{q},\tilde{r})^{JM} = \frac{\sigma_\varepsilon^2}{\sigma_z^2 + 2\sigma_\varepsilon^2}\sigma_z^2 \quad \text{and} \quad Var(\tilde{z}|\tilde{q},\tilde{r}) = \frac{\sigma_\varepsilon^2}{\sigma_z^2 + 3\sigma_\varepsilon^2}\sigma_z^2$$

The key difference between these two conditional variances is the coefficient of σ_ε^2 in the denominator. The greater the value of this coefficient, the more the amount of information revealed in the stock price. The origin of this value is the aggregate orders of the insiders. The aggregate order in the Cournot case is greater than in Jain and Mirman (1999). [5] It should be pointed out that with additional real market structure together with Cournot competition among the insiders, Daher and Mirman (2007) showed that the stock price reveals more information than in the monopoly case studied in Jain and Mirman (2000), also with additional real market structure. Moreover, with only one signal (the total order flow) Tighe (1989) and Holden-Subrahmanyam (1992) found that under Cournot competition among the insiders, the stock price reveals more information than in the monopoly case studied by Kyle (1985).

Third, Cournot competition in the financial market alters the price function coefficients. In the presence of Cournot competition among the insiders and when the market maker observes zero value for both signals, he still receives

[5] $\tilde{x} = \tilde{x}_1 + \tilde{x}_2 = \frac{\sqrt{2}\sigma_u}{\sigma_z}(\tilde{z} - \bar{z})$ $\tilde{x}^{JM} = \frac{\sigma_u}{\sigma_z}(\tilde{z} - \bar{z})$

more information than in the monopoly case ($\mu_0 > \mu_0^{JM}$). On the other hand, the response μ_1 of the market maker to the noisy signal is lower in this model than in Jain and Mirman (1999). In order to understand this result recall the expression of μ_1 in Jain and Mirman (1999) and in our model, i.e.

$$\mu_1^{JM} = \frac{\sigma_z^2}{\sigma_z^2 + 2\sigma_\varepsilon^2} \quad \text{and} \quad \mu_1 = \frac{\sigma_z^2}{\sigma_z^2 + 3\sigma_\varepsilon^2}$$

which depends on the value of the firm, the noisy signal and the total order flow (through the coefficient of σ_ε^2). Hence, as the number of insiders increases, the total order flow increases. Thus the coefficient of the noise in the denominator increases which lowers the value of μ_1. This result reflects the fact that with two insiders there is more information in the order flow signal which gets more weights, and thus makes the value of μ_1 lower than in Jain and Mirman (1999).

Finally, the coefficient of the total order flow signal μ_2 is affected by the relationship between the exogenous variances σ_z^2 and σ_ε^2. When the noise signal is too noisy, i.e. when σ_ε^2 is large relative to σ_z^2, the market maker can not extract information from the noise signal and thus the model resembles to the Kyle type models with one signal, in which the market depth measure (the coefficient of the total order flow signal) decreases as the number of insiders increases (see Tighe (1989)). However, when the noise signal transmits valuable information to the market maker, (when σ_ε^2 is not relatively large with respect to σ_z^2) together with the zero expected profit condition, the value of the total order flow coefficient increases with respect to the monopoly case.[6]

3 Model II : The Stackelberg Case

In this model we introduce Stackelberg competition among the two insiders in the financial market. Specifically, we assume that one of insiders, the owner, is high on the organizational hierarchy and acts as a Stackelberg leader in the financial market. The second insider, the manager, is in the lower ladder of the organizational hierarchy and acts as a Stackelberg-follower in the financial sector to the owner who knows his reaction function before making any decision. Proposition 2 characterizes the unique linear equilibrium. Formally,

Proposition 2 *In the Stackelberg financial model with two correlated signals, a linear equilibrium exists and is unique. It is characterized by*

[6]For more details, see Daher and Mirman (2006), the discussion after Lemma 3, p. 540-541.

$$\tilde{x}_1 = \frac{(1-\mu_1)(\tilde{z}-\bar{z})}{4\mu_2}, \qquad \tilde{x}_2 = \frac{(1-\mu_1)(\tilde{z}-\bar{z})}{2\mu_2} \tag{11}$$

$$\mu_0 = (1-\mu_1)\bar{z}, \quad \mu_1 = \frac{\sigma_z^2}{\sigma_z^2+4\sigma_\varepsilon^2}, \quad \mu_2 = \frac{\sqrt{3}\sigma_\varepsilon^2\sigma_z}{\sigma_u(\sigma_z^2+4\sigma_\varepsilon^2)} \tag{12}$$

$$Var(\tilde{z}|\tilde{q},\tilde{r}) = \frac{\sigma_\varepsilon^2}{\sigma_z^2+4\sigma_\varepsilon^2}\sigma_z^2 \tag{13}$$

$$\pi_1(\tilde{z}) = \frac{\sigma_u\sigma_\varepsilon^2(\tilde{z}-\bar{z})^2}{\sqrt{3}\sigma_z(\sigma_z^2+4\sigma_\varepsilon^2)}, \quad \pi_2(\tilde{z}) = \frac{2\sigma_u\sigma_\varepsilon^2(\tilde{z}-\bar{z})^2}{\sqrt{3}\sigma_z(\sigma_z^2+4\sigma_\varepsilon^2)} \tag{14}$$

Proof: See Appendix B

In this model, we study the effects of Stackelberg competition in the financial market on the equilibrium outcomes. Wang et al. (2009) were also interested by Stackelberg competition among the two insiders but with the presence of Cournot competition in the real market.

Extending Jain and Mirman (1999) (monopoly) to the Stackelberg case has two effects. On the one hand, it enables us to study the effect of Stackelberg competition between the insiders as compared to the Cournot case studied in the first model of this paper. On the other hand, it allows us to perform a comparative static analysis with the Kyle-type models when Stackelberg structure is adopted.

It is well known in industrial organization theory that in the presence of Stackelberg competition, the leader produces more than in Cournot and thus makes more profits. The follower produces a lower quantity than in Cournot and earns less.[7] Lemma 3 provides a complete comparative static analysis between the Cournot competition model (the model I presented in the previous section) and the present model. It shows that the leader (the owner) trades more and earns greater profits than in the Cournot setting and that the follower (the manager) trades less and makes lower profits. Moreover, Stackelberg competition increases the amount of information revealed in the stock price. Finally, the stock price coefficients are also affected by Stackelberg competition. Formally

Lemma 2 *The effects of Stackelberg competition on the equilibrium outcomes, compared to the Cournot-duopoly case are given by,* [8]

$$\tilde{x}_1 < \tilde{x}_1^{M1}, \quad \pi_1 < \pi_1^{M1}, \quad \tilde{x}_2 > \tilde{x}_2^{M1}, \quad \pi_2 > \pi_2^{M1} \tag{15}$$

[7] For more details, see Julien (2011).

[8] The superscript M1 refers to Model I.

$$\tilde{x}_1 + \tilde{x}_2 > \tilde{x}_1^{M1} + \tilde{x}_2^{M1}, \qquad \pi_1 + \pi_2 > \pi_1^{M1} + \pi_2^{M1} \tag{16}$$

$$\mu_0 > \mu_0^{M1}, \qquad \mu_1 < \mu_1^{M1}, \qquad Var(\tilde{z}|\tilde{q},\tilde{r}) < Var(\tilde{z}|\tilde{q},\tilde{r})^{M1} \tag{17}$$

$$\mu_2 < \mu_2^{M1} \quad \text{if } \sigma_z^2 < \frac{4\sqrt{2} - 3\sqrt{3}}{\sqrt{3} - \sqrt{2}} \, \sigma_\varepsilon^2 \text{ and } \mu_2 > \mu_2^{M1} \text{ otherwise} \tag{18}$$

First, Stackelberg competition between the two insiders reduces the manager's trades and thus conditional profits decrease with respect to the Cournot case. However, the owner, due to his role as a leader, trades more than in the Cournot case and earns more. Moreover, the total trade of the insiders as well as the total insiders' conditional profits are greater in Stackelberg than in Cournot. All these results are consistent with the industrial organization theory.

However, when a real market is introduced to the model as in Wang et al. (2009), i.e. when real decisions are made endogenously, Stackelberg competition among the insiders does not always decrease the follower's (manager) profits when compared to Daher and Mirman (2006), where Cournot competition is introduced in both the real and financial markets. [9]

Second, Stackelberg competition among the insiders increases the amount of information revealed in the stock price compared to the Cournot case. To understand this result better, recall the expression of the total order flow signal,

$$\tilde{r} = \beta + \gamma\tilde{z} + \tilde{u}, \tag{19}$$

where the noise term $\tilde{u}$, in the total order flow signal is homoscedastic[10] (see equation 19). Hence, the greater the deterministic part γ of the total order flow signal, the more informative is the total order flow signal. Adding a second informed trader to Jain and Mirman (1999) (i.e, model I) or introducing Stackelberg competition between the two informed traders (model II), changes the values of the deterministic part of the total order flow signal. Since, the noise term ($\tilde{\varepsilon}$) in noise signal ($\tilde{q} = \tilde{z} + \tilde{\varepsilon}$) is heteroscedastic and multiplicative, any change in the financial market structure affects only the total order flow signal. Thus, such change alters the market makers' information about z only through the total order flow signal.

[9]See Wang el al (2009) page 129, Proposition 4.

[10]See Creane (1993).

Comparing the total order flow signal coefficients of model I and II, we find that γ in model II is greater than γ in model I.[11] Consequently, adding Stackelberg competition in the financial sector increases the level of information incorporated in the stock price set by the market makers with respect to the Cournot case.

Finally, note that both types of competitions (Cournot or Stackelberg) in the financial market alter the stock price coefficients. We have already shown that Stackelberg competition in the financial market leads to a wider dissemination of information through the stock price with respect to Cournot competition. This explains the lower value of the stock price noise signal μ_1 in Stackelberg than in Cournot. Moreover, due to the negative relationship between the noise signal coefficient μ_1 and the intercept coefficient μ_0 (which results from the zero expected profits of the market maker), we infer that the value of μ_0 in Stackelberg is greater than in Cournot.

Note also that the order flow signal coefficient μ_2 depends on the exogenous variances of the model. Lemma 3 shows that when the noise signal is relatively noisy, the total order flow coefficient of the stock price is greater in Stackelberg than in Cournot. This result is due to the fact that in the case of a noisy signal, the market maker only extracts information from the total order flow signal. To highlight the relationship between the value of μ_2, the exogenous variance, the market structure and the number of insiders, recall that the noise signal is heteroscedastic and multiplicative and given by

$$\tilde{q} = \tilde{z} + \tilde{\varepsilon}$$

It should be pointed out that this signal can not be manipulated by the insiders. In other words, competition (Cournot or Stackelberg) among the insiders in the financial market, does not alter the coefficient of the noise term (it is always equal to one). Thus, the market maker extracts the same information from the noise signal regardless the competition structure in the financial market. However, introducing competition in the financial market, alters the deterministic part γ, of the total order flow signal (see equation 19).

From equation 18, the variance of the stock value σ_z^2 and the variance of the noise signal σ_ε^2 are the determinants of the effect of the competition structure on the total order flow response μ_2. When σ_z^2 is relatively small with respect to σ_ε^2, the total order flow signal becomes more relevant to the market maker than the noisy signal. Thus, the value of μ_2 in the presence of Stackelberg competition is greater than the value of μ_2 in the presence

[11]Indeed, from Proposition 1, we infer that $\tilde{x}^{M1} = \tilde{x}_1^{M1} + \tilde{x}_2^{M1} = \frac{\sqrt{2}\sigma_u}{\sigma_z}(\tilde{z} - \bar{z})$ while Proposition 2 shows that $\tilde{x}^{M2} = \tilde{x}_1^{M2} + \tilde{x}_2^{M2} = \frac{\sqrt{3}\sigma_u}{\sigma_z}(\tilde{z} - \bar{z})$.

of Cournot. However, when both signals provide relevant information to the market maker, the zero profit condition makes the value of μ_2 in the Stackelberg case lower than the value of μ_2 in Cournot.

Now, we turn to the role of information sources in the Stackelberg case. In other words, we study the effects of information efficiency when the market maker observes two correlated signals instead of the total order flow signal alone. For this purpose, we present the benchmark model which is identical to our model except that the market maker only observes the total order flow signal. In other words, the benchmark model studies Stackelberg competition among the insiders in the Kyle-type models. Proposition 3 whose proof is omitted, characterizes the unique linear equilibrium of the benchmark model.

Proposition 3 *In the Stackelberg model with one signal (the total order flow), a linear equilibrium exists and is unique. It is characterized as follows*

$$\tilde{x}_1 = \frac{(\tilde{z} - \bar{z})}{4\mu_1}, \qquad \tilde{x}_2 = \frac{(\tilde{z} - \bar{z})}{2\mu_1} \tag{20}$$

$$\mu_0 = \bar{z}, \quad \mu_1 = \frac{\sqrt{3}\sigma_z}{4\sigma_u}, \quad \tilde{p} = \mu_0 + \mu_1 \tilde{r} \tag{21}$$

$$Var(\tilde{z}|\tilde{r}) = \frac{1}{4}\sigma_z^2 \tag{22}$$

$$\pi_1(\tilde{z}) = \frac{\sigma_u(\tilde{z} - \bar{z})^2}{4\sqrt{3}\sigma_z}, \quad \pi_2(\tilde{z}) = \frac{\sigma_u(\tilde{z} - \bar{z})^2}{2\sqrt{3}\sigma_z} \tag{23}$$

A simple comparison between Propositions 2 and 3 shows that adding a signal that is correlated with the value of the asset and that can not be manipulated by the insiders, increases the amount of information revealed by the stock price. However, the additional signal does not affect the trading strategies of each insider. In other words, with one or two signals, each insider i trades the same quantity $\tilde{x}_i$ $(i = 1, 2)$. Moreover, adding a correlated signal to the informational structure always decreases the insiders' profits.

4 Model III : Monopoly in the real market

In this section, we study the effect of introducing decisions in the real sector on the results of model II, i.e., model III studies Stackelberg competition in the financial sector and monopoly in the real sector. It adds a real sector to model II and hence allows us to study the effect of the real market on equilibrium outcomes. Specifically, we consider an economy with one real good and one financial asset. The real good is produced by a monopolistic firm at no cost. The inverse demand function is linear and stochastic, i.e.,

$$q' = (a - by)\tilde{z} \quad a, b > 0 \tag{24}$$

where a and b are positive constants and $\tilde{z}$ is a random variable, normally distributed with mean $\bar{z}$ (assumed positive) and variance σ_z^2. The stock of the firm is publicly traded in a competitive financial market. The value of the stock is the net profits of the firm per share. We assume that the manager and the owner of the firm are the two insiders who trade in the stock market based on their inside information. Moreover the owner has no managerial responsibilities. As in the previous models, we suppose that there exist noise traders whose aggregate demand is denoted by the random variable $\tilde{u}$ with mean 0 and variance σ_u^2. We follow Jain and Mirman (2000) by allowing the market makers to observe two signals. The first, from the real market, is denoted by $\tilde{q} = (a - by)(\tilde{z} + \tilde{\varepsilon})$ where $\tilde{\varepsilon}$ is normally distributed with mean 0 and variance σ_ε^2. The second is the total order flow, i.e., $\tilde{r} = \tilde{x}_1 + \tilde{x}_2 + \tilde{u}$ where $\tilde{x}_1$ and $\tilde{x}_2$ are the manager's and the owner's trading orders, respectively.

Each insider knows only the realization z of $\tilde{z}$ and does not know the values of $\tilde{u}, \tilde{\varepsilon}, \tilde{r}, \tilde{z} + \tilde{\varepsilon}$ before his order flow decisions are made. Moreover, neither market maker knows the realization z of $\tilde{z}$ but both knows its distribution. Finally, the market makers cannot observe either x_i, u or ε.

The value per share of the firm is the net profit of the firm per share, i.e.,

$$v' = (a - by)y\tilde{z} \quad a, b > 0 \tag{25}$$

and the profit of the manager and the owner are respectively

$$\psi_1 = (v' - A - \tilde{p})\tilde{x_1} + A\tilde{x_1} \tag{26}$$

$$\psi_2 = (v' - A - \tilde{p})\tilde{x_2} \tag{27}$$

where A is the manager's compensation scheme.[12] We assume as in model II and Wang et al. (2009), that the owner moves first because he knows the manager's reaction function before making any decision. Proposition 4 characterizes the unique linear equilibrium, the amount of information revealed by the stock price and the conditional expected profits of the two insiders.

[12]Following Jain and Mirman (2000), the existence of such compensation scheme is to ensure the second order condition of the manager's maximization problem. For more details, see Jain and Mirman (2000) and the extended works thereafter.

Proposition 4 *In the insider trading Stackelberg model with a monopoly structure in the real market, a linear equilibrium exists and is unique. It is characterized by*

$$y = \frac{a + \mu_1 b}{2b} \qquad \tilde{x}_1 = \frac{(a - by)(y - \mu_1)\tilde{z}}{4\mu_2}, \qquad \tilde{x}_2 = \frac{(a - by)(y - \mu_1)(\tilde{z} - \bar{z})}{2\mu_2} \tag{28}$$

$$\mu_0 = 2A = \frac{(a - by)(y - \mu_1)\bar{z}}{2}, \qquad \mu_1 = \frac{a\sigma_z^2}{b(\sigma_z^2 + 8\sigma_\varepsilon^2)} = \frac{a}{b}k, \tag{29}$$

$$\mu_2 = \frac{a^2\sqrt{3}\sigma_\varepsilon\sqrt{(1-k)^3 k}}{4\sqrt{2}b\sigma_u}, \qquad k = \frac{\sigma_z^2}{\sigma_z^2 + 8\sigma_\varepsilon^2} \tag{30}$$

$$Var(\tilde{z}|\tilde{q}, \tilde{r}) = \frac{\sigma_\varepsilon^2}{\sigma_z^2 + 4\sigma_\varepsilon^2}\sigma_z^2 \tag{31}$$

$$\pi_1(\tilde{z}) = \frac{4a^2\sigma_u\sigma_\varepsilon^4 z^2}{b\sqrt{3}\sigma_z(\sigma_z^2 + 8\sigma_\varepsilon^2)}, \qquad \pi_2(\tilde{z}) = \frac{8a^2\sigma_u\sigma_\varepsilon^4(z - \bar{z})^2}{b\sqrt{3}\sigma_z(\sigma_z^2 + 8\sigma_\varepsilon^2)} \tag{32}$$

Proof: See Appendix C

The relationship between the two models, II and III, is similar to the relationship between Jain and Mirman (1999) and Jain and Mirman (2000). Indeed, Jain and Mirman (2000) added a real market to Jain and Mirman (1999) and investigated the effect of making decisions in the real market on the equilibrium outcomes. They found that several variables are affected by the real market structure. Specifically, in equilibrium, the order strategy of the owner and the amount of information revealed are both independent of the real market structure.

We obtain similar results. First, the conditional variances in both models I and II are the same. In other words, introducing a real market in model II has no effect on information revelation. This is due to the non-randomness of the real output variable (y).

Second, simplifying the expression of the order strategy of the two owners in both models I and II (see equations 11 and 28), we obtain

$$\tilde{x}_2 = \frac{2\sigma_u(\tilde{z} - \bar{z})}{\sqrt{3}\sigma_z} \tag{33}$$

Hence, the owner's strategy is independent of the real market structure.

Note that, the introduction of the compensation scheme, decreases the manager's strategy compared with model II.[13] Moreover, it should de pointed out that going from one signal to two signal does not alter the insiders' strategies.

We analyze the effect of the real market structure on the equilibrium outcomes, i.e., we compare model III to Wang et al (2009) who modeled Stackelberg competition in the financial market with Cournot competition in the real market. Formally,

Lemma 3 *The differences between model III and Wang et al (2009) in terms of impact on the equilibrium outcomes are given by,* [14]

$$\mu_0 > \mu_0^{WWR}, \qquad \mu_1 > \mu_1^{WWR}, \qquad \mu_2 > \mu_2^{WWR}, \tag{34}$$

$$y_1 > y_1^{WWR}, \qquad y_1 < Y^{WWR}, \quad A > A^{WWR} \tag{35}$$

$$\tilde{x}_1 = \tilde{x}_1^{WWR}, \quad \tilde{x}_2 = \tilde{x}_2^{WWR}, \quad Var(\tilde{z}|\tilde{q},\tilde{r}) = Var(\tilde{z}|\tilde{q},\tilde{r})^{WWR} \tag{36}$$

$$\pi_1 > \pi_1^{WWR}, \qquad \pi_2 > \pi_2^{WWR} \tag{37}$$

Lemma 3 shows the effect of the real market structure on equilibrium outcomes in the presence of Stackelberg competition in the financial sector. The relationship between this model and Wang et al (2009) is similar to the relationship that exists between Daher and Mirman (2006) and (2007). Indeed, Daher and Mirman (2007) studied Cournot competition in the financial sector in the presence of a monopoly structure in the real sector while Daher and Mirman (2006) studied Cournot competition in both sides of the market, i.e. they added another competing firm in the real sector to Daher and Mirman (2007).

Cournot competition in the real sector generates no new information when compared to the monopoly case. This is due to the randomness of the real output variables in both the monopoly and duopoly case. Moreover competition in the real sector does not affect the trading strategy of the insiders. Jain and Mirman (2002) who studied Cournot competition in the real sector and monopoly in the financial market, obtained the same results compared

[13] In model II, the manager's strategy is an affine function while in our model it is a linear function passing through the origin.

[14] The superscript WWR refers to Wang et al (2009).

to Jain and Mirman(2000) who modeled a monopoly in both the real and financial market. Similarly, Daher and Mirman (2006) studied a Cournot competition in both the real and financial markets. They showed that the insiders' trades, as well as the information revealed are not affected by the real market structure compared to the monopoly case in the real market studied by Daher and Mirman (2007).

Second, Cournot competition in the real market decreases the profits of the firm with respect to the monopoly case. Such a decrease in profits must be incorporated into the stock price function of the market makers. This explains the lower values of the price coefficients in Wang el al (2009) compared to model III. Also, the compensation scheme is affected by the monopoly structure in the real sector. Since the value of the compensation scheme is half the value of μ_0, we deduce that the compensation scheme is greater in this model. In sum, competition in the real sector reduces the unconditional expected value of firm 1 and, thus, the coefficients of the stock price signals are less with the presence of Cournot competition in the real sector than in the monopoly case.

Finally, in the line with the industrial organization theory, we find that the amount of output produced in the monopoly case is less than in the Cournot case studied in Wang et al. (2009). We also find that the insiders' profits under Cournot duopoly are less than those under monopoly . It should be pointed out that these results also hold in Jain and Mirman (2002) which studied Cournot competition in the real sector and monopoly in the financial market when compared to Jain and Mirman(2000) which modeled monopoly in both the real and financial market. Similarly, Daher and Mirman (2006) when studying Cournot competition in both the real and financial markets, obtained the same results as compared to the monopoly case in the real market studied by Daher and Mirman (2007).

Appendix

In this appendix we recall the Theorem that we use to prove all the results in the three cases of the paper.

Theorem 1 *If the $p \times 1$ vector Y is normally distributed with mean U and covariance V and if the vector Y is partitioned into two subvectors such that* $Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$ *and if* $Y^* = \begin{pmatrix} Y_1 \\ Y_2^* \end{pmatrix}$

$$U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \text{ and } V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

are the corresponding partitions of Y^*,U and V, then the conditional distribution of the $m \times 1$ $(m < p)$ vector Y_1 given the vector $Y_2 = Y_2^*$ is the multivariate normal distribution with mean $U_1 + V_{12}V_{22}^{-1}(Y_2^*$ - $U_2)$ and covariance matrix $V_{11} - V_{12}V_{22}^{-1}V_{21}$.

Proof: See Graybill, Theorem 3.10 pp 63.

Appendix A: proof of Proposition 1

First note, in the Cournot case the equilibrium is symmetric, i.e. $\tilde{x}_1 = \tilde{x}_2$. Taking the first and the second order conditions of the insider maximization problem (equation 1), we find

$$\tilde{x}_1 = \tilde{x}_2 = \frac{(1-\mu_1)(\tilde{z}-\mu_0)}{3\mu_2} \tag{38}$$

and $\mu_2 > 0$. Substituting the expression of $\mu_0 = \bar{z} - \mu_1\bar{q} - \mu_2\bar{r}$ into equation (38), we obtain the insiders strategies in (4). Now, applying Theorem 1 to the normal random vector $B = (\tilde{z}, \tilde{q}, \tilde{r})$ with $p = 3$ and $m = 1$. By identification, we have $Y_1 = \tilde{z}$ and $Y_2 = \begin{pmatrix} \tilde{q} \\ \tilde{r} \end{pmatrix}$. $U_1 = \bar{z}$, $U_2 = \begin{pmatrix} \bar{q} \\ \bar{r} \end{pmatrix}$ and

$$V = \begin{pmatrix} \sigma_z^2 & \sigma_{zq} & \sigma_{zr} \\ \sigma_{zq} & \sigma_q^2 & \sigma_{qr} \\ \sigma_{zr} & \sigma_{qr} & \sigma_r^2 \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

Where $V_{11} = \sigma_z^2, V_{12} = (\sigma_{zq}, \sigma_{zr})$, $V_{21} = \begin{pmatrix} \sigma_{zq} \\ \sigma_{zr} \end{pmatrix}$ and $V_{22} = \begin{pmatrix} \sigma_q^2 & \sigma_{qr} \\ \sigma_{qr} & \sigma_r^2 \end{pmatrix}$.

Note that

$$V_{22}^{-1} = \frac{1}{D}\begin{pmatrix} \sigma_r^2 & -\sigma_{qr} \\ -\sigma_{qr} & \sigma_q^2 \end{pmatrix},$$

where D is the determinant of V_{22}, that is $D = \sigma_q^2\sigma_r^2 - \sigma_{qr}^2$.

$$\mu_1 = \frac{\sigma_{zq}\sigma_r^2 - \sigma_{zr}\sigma_{qr}}{D} \tag{39}$$

$$\mu_2 = \frac{\sigma_{zr}\sigma_q^2 - \sigma_{vq}\sigma_{qr}}{D} \tag{40}$$

Substituting the corresponding values for the variances and covariances, we find

$$\mu_1 = \frac{\sigma_z^2\sigma_u^2}{D} \tag{41}$$

$$\mu_2 = \frac{2(1-\mu_1)\sigma_z^2\sigma_\varepsilon^2}{3\mu_2 D} \tag{42}$$

Combining equations (41) and (42) we obtain

$$3\mu_2^2 = \frac{2(1-\mu_1)\mu_1\sigma_\varepsilon^2}{\sigma_u^2} \tag{43}$$

Now computing for the expression of D, one can find that

$$3\mu_2^2 D = \frac{4(1-\mu_1)^2\sigma_z^2\sigma_\varepsilon^2}{3} + 3\mu_2^2\sigma_u^2(\sigma_z^2+\sigma_\varepsilon^2) \tag{44}$$

Arranging for (42) and (44), we obtain

$$3\mu_2^2 = \frac{2(1-\mu_1)(1+2\mu_1)\sigma_z^2\sigma_\varepsilon^2}{3\sigma_u^2(\sigma_z^2+\sigma_\varepsilon^2)} \tag{45}$$

Solving for (43) and (45) we obtain

$$\mu_1 = \frac{\sigma_z^2}{\sigma_z^2 + 3\sigma_\varepsilon^2} \tag{46}$$

Plugging (46) into (43) we obtain the expression of μ_2 in (5). Applying Theorem 1 to compute the conditional variance we find that

$$\begin{aligned} Var(\tilde{z}|\tilde{q},\tilde{r}) &= \sigma_z^2 - \mu_1\sigma_{zq} - \mu_2\sigma_{zr} \\ &= \frac{\sigma_\varepsilon^2}{\sigma_z^2 + 3\sigma_\varepsilon^2}\sigma_z^2 \end{aligned}$$

Finally, substituting the values of $\tilde{x}_i, \mu_1$ and μ_2, we find the expression of the insiders conditional profits in (7) □

Appendix B: proof of Proposition 2

In this appendix, the main change with respect to appendix A is the reaction curves derivation of each insider. The remainder is similar and thus omitted. Indeed, following Stackelberg structure, the manager moves second and hence his reaction curve is given by

$$\tilde{x}_1 = \frac{z - \mu_0 - \mu_1 z - \mu_2\tilde{x}_1}{2\mu_2} \tag{47}$$

Now plugging (47) into the profit function of the owner and taking the first order condition we find,

$$\tilde{x}_2 = \frac{z - \mu_0 - \mu_1 z}{2\mu_2} \tag{48}$$

which implies that

$$\tilde{x}_1 = \frac{z - \mu_0 - \mu_1 z}{4\mu_2} \tag{49}$$

□

Appendix C: proof of Proposition 4

First note that the value of the firm is now $\tilde{v}$ and thus adjusting Theorem 1 with this new information and using structures detailed in appendices A and B, we derive equations (28) and (29). Second, the conditional variance becomes

$$Var(\tilde{z}|\tilde{q},\tilde{r}) = \sigma_z^2 - \frac{\mu_1}{(a-by)y}\sigma_{zq} - \frac{\mu_2}{(a-by)y}\sigma_{zr}$$

and equation (31) follows. Finally, equation (32) follows automatically after submitting all the required expressions. □

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