The extreme downside risk of the S&P 500 stock index

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Abstract

Extreme value theory has been widely applied in insurance and finance to model rare events. Plenty of such events have occurred in financial markets during the last two decades, including stock market crashes, currency crises, or large bankruptcies. This article applies extreme value theory results to quantify the extreme downside risk of the S&P 500 stock index in light of the recent systemic banking crisis. The lower tail of the premier American stock index distribution reveals how deep the impact of the recent financial crisis is.

The financial markets throughout the world have been subject to financial disasters in the last twenty years due to the deregulation process that began in the early 1980’s. Recently, the U.S. mortgage credit market turmoil along with the bankruptcy of Lehman Brothers has brought the world’s major market economies to their knees. For this reason, the understanding of the expected frequency and magnitude of financial extremes constitutes the heart of modern risk management. This requires having precise statistical tools that do not generate downward biased measures of risk, which could be the case when extreme events are taken into account. Extreme value theory (EVT) provides the theoretical framework upon which financial theory can rely to model extreme events. It is applied in statistical analysis of genomic sequences, in quantile estimates of wind velocity, or in estimation of precipitation probabilities. The tail behavior of financial series has also been largely examined by many studies with various applications (Value-at-Risk, expected shortfall, upper limits in an open position on the foreign exchange market, etc.). A general discussion on the application of EVT to risk management is proposed by Embrechts et al. (1997), McNeil (1999), Coles (2001) and Beirlant et al. (2004).

Some of the main studies of univariate EVT in finance include the following: Longin (1996) finds that most of the extreme returns of shares listed on the NYSE obey a Fréchet distribution. He uses limits theorems for block maxima with raw data. McNeil (1997) fits the Generalized Pareto distribution to insurance losses that exceed high thresholds to show how this method is useful for estimating the tails of loss severity distributions. Danielsson and de Vries (1997) incorporate an estimate of the second order term of the tail expansion to improve the standard tail index and quantile estimator. They apply this methodology for modeling tails of very high frequency exchange rate time series and find that highest frequency observations are difficult to exploit for addressing economic issues. McNeil and Saladin (1998) fit the ‘peak over threshold’ model to insurance data to predict the magnitude of future large losses under various scenarios. Christoffersen et al. (1998) and Diebold et al. (1998) point out that sampling from the tail of the distribution should bring more accurate estimates with a quantile probability of 0.01% or lower. McNeil and Frey (1998) apply limit results for the excess distribution to study the residuals of a GARCH (1,1) model. Jondeau and Rockinger (2003) consider a large set of indices from mature and emerging countries including French CAC 40, and estimate the GPD over various returns samples. They find that the tail estimate for the S&P 500 predicts a possible drop of 9.79% instead of the 22.83% fall that was observed in October 1987 crash. Këllezi and Gilli (2000) model the tail of the oldest stock index in Switzerland from 1969 to 1989 and find that the loss experienced during the 1987 crash could be exceeded every 37 years using a peak over threshold model. Bali and Neftci (2002) specify the mean and the volatility parameters of the general Pareto distribution as a function of past information using U.S. Federal funds rates from 1954 to 2000. This distribution provides them with very good estimates of the VaR. LeBaron and Samanta (2004) address the differences between crashes and booms and between different markets. Danielsson and Morimoto (2000) recommend the use of EVT techniques on the Japanese Stock Exchange. Gençay et al. (2003) suggest the general Pareto distribution as a robust quantile forecasting tool for the Istanbul Stock Exchange. Mandira (2004) fits General Pareto distribution to the Indian Stock Index and discovers the presence of thickness in both tails and no evidence of asymmetric tails. Tolikas and Brown (2005) find that both the ‘general logistic’ and the ‘general extreme value’ distributions provide an adequate fit of the monthly returns’ minima for the Athens Stock Exchange. Gettingby et al. (2006) analyze the distribution of extreme returns in the U.K. stock market from 1975 to 2000 and find that the first and second best fitting distributions were respectively the generalized logistic distribution and the generalized extreme value distribution.

This article applies extreme value theory results to quantify the extreme downside risk of the S&P 500 stock index in light of the recent systemic banking crisis.

The extreme value analysis

There are two main approaches to study the extreme behavior of a returns series. The first is the block maxima and the second is the peak over threshold. The distributional theories are equivalent. In this article, I discuss the extreme downside risk behavior of the S&P 500 stock index. The peak over threshold model is applied since we are interested explicitly in analyzing the left tail component of the distribution. To identify this tail region, let us define a threshold denoted by u that we estimate in the next section. Balkema and de Haan (1974) and Pickands (1975) show that when the threshold u is sufficiently high, the distribution function of the excess beyond this threshold can be approximated by the generalized Pareto distribution (GPD). This limit distribution has a general form given by:

$$G_{\xi,\mu,\sigma}(x) = \{1 - (1+\xi x/\sigma)^{-1/\xi} \text{ for } \xi\neq 0;\ 1 - \exp(-x/\sigma) \text{ for } \xi=0\}$$

where $\sigma \geq 0$ and where $x \geq 0$ when $\xi \geq 0$ and where $0 \leq x \leq -\sigma/\xi$ when $\xi<0$. σ is a scaling parameter and ξ is the tail index. This distribution encompasses other type of distributions. If $\xi>0$ ($\xi<0$) then it is a reformulated version of the ordinary Pareto distribution (Pareto type II distribution) and if $\xi=0$, it corresponds to the exponential distribution. It has been shown that the maximum likelihood estimates of the GP distribution parameters are consistent and asymptotically normal for large samples provided that the tail index value is above -0.5.

The analysis of the extreme

Data description

The sample size corresponds to 14,845 raw daily log-returns computed from the closing prices of the U.S. S&P 500 stock index. The time period covered is from January, 3rd 1950 to December 31st 2008. Let us denote X as the log-return series, +X as the return maxima and –X as the return minima. Our study focuses only on –X. **Table 1** summarizes the descriptive statistics of the data. It exhibits a return time series with excess skewness and kurtosis indicating a non-normal distribution.

The threshold selection for GPD estimation

We follow Beirlant et al. (2004), who propose a criterion for which the AMSE of the Hill estimator of the extreme value index is minimal for the optimal number of observations in the tail. Optimal detection allows for a tradeoff between finding a high threshold where the tail estimate has a high variance or a low threshold where the tail estimate has a reasonable variance. Optimal threshold is around 0.010 for –X. The critical threshold value computed from the optimal algorithm responds to the criteria of stability and sufficient exceedances with minimum variance. The selection of the critical threshold is required for the GP distribution estimation.

Table 2 displays the results for the GP distribution when considering the optimal threshold value. The maximum likelihood estimators of the GP distribution are the values of the two parameters (ξ, σ) that maximize the log-likelihood. The tail index parameter value of 0.2097 for –X is statistically significant. The magnitude along with the positive sign confirms the fat-tailness. This result is fully consistent with the quantile plot. The scale parameter of –X is statistically significant. Note that unreported results show a substantial increase in the weight of the tail index due to the recent systemic banking crisis. Indeed, the tail index estimate has a value of 0.1537 for the period ending in 2006, which is highly statistically significant, while it is equal to 0.2097 when including the 2007-2008 financial crises. This represents a substantial surge of +26.72%.

Extreme downside risk analysis

A simple examination of the data can revealed the stock returns extreme behavior. Out of the 100 strongest declines of the S&P 500 stock index log-returns, 28 occurred during 2008. Half of the 10 strongest declines also occurred in 2008. Therefore, a natural question to ask would be, has 2008 increased the downside risk of this major US stock index? In some extent, the intuitive answer is yes, because it contributes to almost 50% of the highest extreme events of the index.

The extreme behavior of the S&P 500 stock index negative log-returns (-X) can be observed within **Figure 1** where four plots are displayed. The probability plot, the quantile plot, the return level plot and density plot. They are useful for model presentation and validation. The three first plots are based on a comparison of theoretical GP distribution and empirical estimates of the distribution. The last plot compares the density function of the fitted model with a histogram of the threshold exceedances. However, this graph is relatively less informative than the others. Both probability and quantile plots have the same information but expressed on a different scale. Probability plot displays a linear fit of the empirical distribution against the GPD distribution. This encouraging graph is complemented by the quantile plot. The reported quantile plot exhibit a slight departure from the unit diagonal due to the 1987 Black Monday (–22.90%, 1987-10-19), which correspond to the highest negative return. But when fitted against an exponential distribution[1], a concave departure from a straight line is observed as a sign of heavy-tailed behaviour. This graphical analysis confirms the presence of heavy tails and means that the extreme distribution of –X belongs to the Fréchet domain of attraction. The return level plot consists of plotting the theoretical quantiles as a function of the return period with a logarithmic scale for the x-axis so that the effect of extrapolation is highlighted. For example, the 100-year return level is the level expected to be exceeded once every 100 years. Our results[2] indicate that the 22-year return period has an upper confidence interval corresponding to the second highest negative return (–9.47%, 2008-10-15).

Conclusion

Financial markets throughout the world have experienced a number of financial disasters during the last century. Predicting and hedging against such rare events seems difficult in practice. However, exploring the tail region of the extreme negative log-returns distribution reveals rich information on their individual extreme behavior. This study discusses the extreme downside risk of the premier American stock index. It uses 58 years of negative log-

[1] The Q-Q plot is available upon request.
[2] The results are available upon request.

returns of the Standard & Poor's 500 for extreme value analysis. This article quantifies the impact of the recent crisis by computing the tail index before and after the current banking crisis. The main conclusions of this study can be summarized as follows:
(1) the crisis increased the extreme nature of the Premier US index,
(2) the 1987 Black Monday remains the highest one-day crash in the history,
(3) the highest shock return of 2008 has can exceeded once every 22 years,
(4) half of the 10 strongest declines also occurred in 2008,
(5) the Generalized Pareto distribution offers a good description of the S&P log-returns.

5. Appendix

Table 1: Descriptive Statistics

Table 1 presents the descriptive statistics of the S&P 500 stock index daily log-returns from January, 3rd 1950 to December, 31st 2008.

	X
Mean	0.000269
Median	0.000441
Maximum	0.109572
Minimum	-0.228997
Std. Dev.	0.009514
Skewness (Z-statistic, p-value)	-1.1628*** (-30.8516, 2.2e-16)
Kurtosis (Z-statistic, p-value)	35.1689*** (55.7282, 2.2e-16)
Jarque-Bera (p-value)	643252.3*** (0.0000)
Number	14845

*, ** and*** denotes parameter significantly different from one at the 90%, 95% and 99% confidence level.

Table 2: Parameters estimates for the GPD model

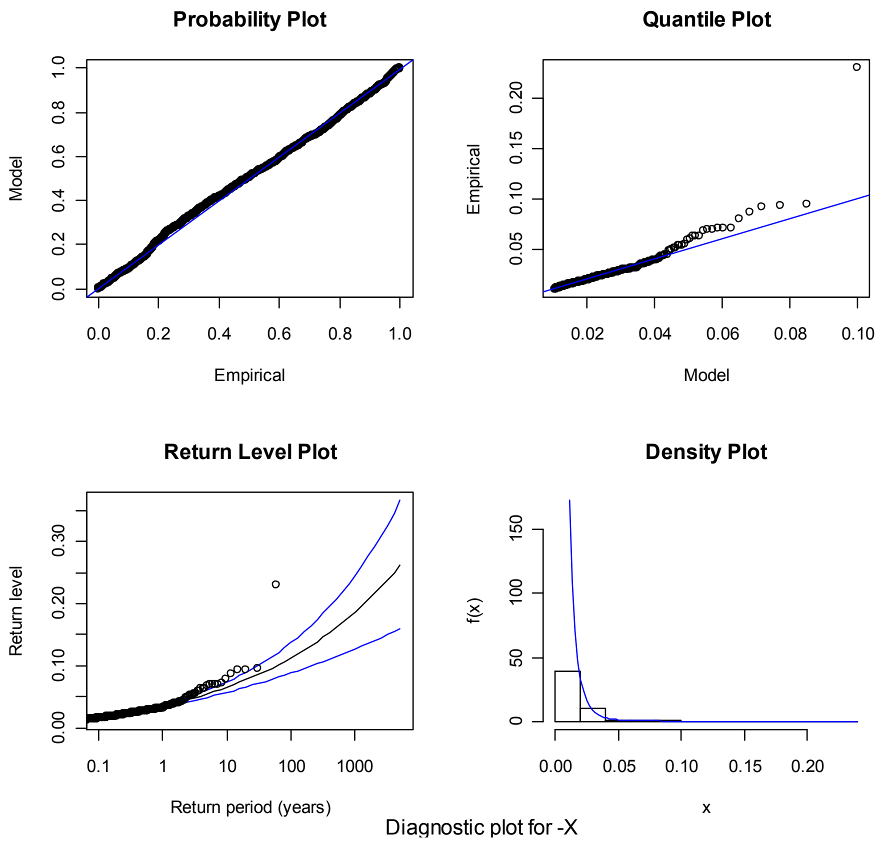
Table 2 gives parameter estimates of the General Pareto Distribution fitted to the lower tail (-X) of the S&P 500 stock index. The generalized Pareto distribution is fitted to excesses over the optimal threshold. The vector parameters are estimated by the maximum likelihood method. Number of exceedances corresponds to the number of observations in the tail. Percentile is the percentage of observations below the threshold. Neg. Lik is the negative logarithm of the likelihood evaluated at the maximum likelihood estimates.

	-X
ξ	0.20977***
(s.e)	(0.03039)
σ	0.00561***
(s.e)	(0.00021)
Threshold	0.01073561
Nb. Exceedances	1260
Percentile	0.91511
Neg. Lik.	-5004.479

*** denotes parameter significantly different from one at the 99% confidence level.

Figure 1: Diagnostic plots

Figure 1 displays the lower tail (-X) of the S&P 500 stock index returns. The Diagnostic plots compare the observed quantiles with the theoretical ones. From upper left to lower right corner: probability, quantile, return level and histogram with fitted GPD density. Quantile and return level plots are for the negative transformed minima.



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