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STRUCTURE AND DYNAMICS OF TRANSPORTATION NETWORKS: MODELS, METHODS, AND APPLICATIONS*

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1 Introduction

The study of transport networks has long been at a crossroads between various scientific disciplines. Traditionally, transport networks are studied from a graph theory perspective, which is a branch of mathematics proposing concepts and measures about the topology of networks considered as sets of nodes (vertices) connected by links (edges). Limited computational power and the scarcity of relevant datasets limited the analysis to relatively small networks, most of which are planar networks (i.e. with no crossings between edges) and accessibility problems based on pure topology. Decreasing interest for structural approaches in social sciences partly caused the stagnation of transport network analysis since the 1970s, notwithstanding progress brought in by Geographical Information Systems (GIS) and and spatial economics from the 1980s onwards (Waters, 2006). Technology and information improvements as well as the wide popularization of the "network" concept contributed to the emergence of new analysis

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methods in the late 1990s. The research field of complex networks, mostly led by physicists eager to test their models and measures on real-world cases, provided many analyses of the structure and dynamics of large-scale networks of all kinds. Among them, transport networks have received much attention, thereby providing new ways of understanding their internal efficiencies and vulnerabilities.

However, most social scientists interested in transport networks, such as geographers, economists, and regional scientists, have long relied on classic graph theory and qualitative case studies. The integration of complex networks methods by transport specialists is very recent (Kuby et al., 2005) and comes as a complement to other approaches such as circulation routing and flow optimization which strongly focus on transport costs. So far, there remains little overlap and interaction between the different approaches. This state of affairs as well as the fast evolution of the field motivate this chapter, which proposes a review of existing studies of transport networks from a methodological point of view.

We propose distinguishing amongst the structure and the dynamics of transportation networks. The first section discusses the static dimension (structure) and reviews how transportation networks have been defined and analyzed with regard to their topology, geometry, morphology, and spatial structure. It presents a critical overview of main global (network level) and local (node level) measures and examines their usefulness for understanding transportation networks. The second section explores the dynamics of transportation networks, their evolution, and the properties underlying such evolutions. Each section provides a brief background of the relevant literature, concrete applications, and policy implications in various transport modes and industries, with an interdisciplinary focus. A discussion is provided evaluating the legacy of reviewed works and potential for further developments in transport studies in general.

2 Structure of Transportation Networks

Transport networks are mostly studied in a static fashion. Research in this field is very voluminous and diverse, but one central goal is to highlight the overall characteristics of the networks structure based on its topology, geometry, morphology, and traffic flows. We divide the literature review between the global level, which is interested in describing the entire network, and the sub-levels, which look either at groups or individual nodes within the network.

2.1 Global Level

Transport networks belong to a wider category of spatial networks, because their design and evolution are physically constrained (e.g. Euclidian distance) as opposed to non-spatial networks such as the Internet, scientific collaborations, multinational firms, social networks, and biological systems such as neuronal networks (Boccaletti et al., 2006; Gastner and Newman, 2004; Blumenfeld-Lieberthal, 2009). The physical grounding varies in relevance depending on the transport mode considered. Urban streets, roads, and railways are composed of track infrastructure, while maritime and air transports remain vaguely defined due to their higher spatial flexibility except for the location of terminals themselves (Rodrigue et al., 2009). Maritime networks remain more constrained than airline networks due to coastlines (Xu et al., 2007). River networks typically form basins and can be classified as trees or dendrograms (Banavar et al., 1999).

Most research on transport networks has focused on planar networks (e.g. roads, railways) due to easier access to infrastructure data allowing topological measurements and because the planning and operation of such networks concern everyday mobility at the urban (e.g. street, bus, subway) or regional level (e.g. highway, interstate lanes). Other transport networks without track infrastructure (e.g. air, sea) necessitate the use of traffic data to build a graph of ephemeral links from either vehicle movements or scheduled service. Partly due to the absence of comparable data for urban studies, airline networks have become central in the analysis of systems of cities on various levels. Maritime transport has recently received more interest due to the use of newly available data on carrier services and vessel movements (Ducruet et al., 2010).

In their review of the measures synthesizing the structure of communication networks, Béguin and Thomas (1997) introduce four different schools: graph theory, matrices of the shortest paths, geomorphometry (quantitative land surface analysis), and eigenvalues. Matrices of the shortest paths refer to the search for specific configurations in the network, such as the minimum cost tree, which is the part of the network in which all nodes remain connected by the lowest cost path. This approach can be at the global level to extract the optimal route from the network or at the local level to zoom to specific nodes using algorithms such as Kruskal, Floyd, or Dijkstra, among others (Ducruet, 2011). Geomorphometry mainly examines river networks based on several indicators such as the number of bifurcations and the length of the river, its number of junctions. There are two types of eigenvalues approaches: the application of multivariate statistical methods to adjacency matrices and the use of blockmodeling and structural equivalence to determine groups of nodes sharing similar connectivity patterns in the network (Beauguitte, 2011). Based on the simplification of real-world places and pathways into graphs, i.e. groups of nodes (vertices) connected by some links (edges), a wide set of concepts and measures have been proposed by graph theory since the seminal Seven Bridges of Koenigsberg problem proposed by Leonhard Euler in 1735. The main questions addressed are whether different transport networks share comparable properties and according to which criteria.

The integration of graph theoretical methods in geography and regional science with related applications to transport networks dates back to the works of Garrison (1960), Kansky (1963), Haggett and Chorley (1972), and Garrison and Marble (1974). The first set of studies was characterized by limited data, low computational power, and few modeling techniques (Xie and Levinson, 2007) but is still used to analyze the structure of transport networks. Most common measures include the number of nodes and links (i.e. network size), the length and diameter of the network, its connexity (continuity of the network in terms of the number of connected components), connectivity (more or less optimal distribution of links among the nodes), and nodality. Notably, the three main indices Alpha, Beta, and Gamma proposed by Kansky (1963) do not consider the real length, quality, and weight of the links;[1] networks of equal size may exhibit contrasted topological forms (Béguin and Thomas, 1997; Kurant and Thiran, 2006). However, they remain useful for describing the changing structure of one given network (Scott et al., 2005; Xie and Levinson, 2009). Using such indicators, Wang et al. (2009) demonstrate the phased development of the Chinese railway network from 1906 to 2000, highlighting its closeness with overall economic development and the formation of urban systems.

More recently, robust macroscopic measures have been proposed to refine those provided by graph theory (see the table in Appendix A: Complex Networks Measures). Barabási and Albert (1999) define scale-free networks using a high exponent (over 1 and/or between 2 and 3) of the power-law slope drawn based on plotting the frequency of nodes versus the degree centrality distribution (i.e. the number of edges connecting direct neighbors). Many nodes have a poor number of connections, while only a few nodes multiply them, thus making the network highly heterogeneous or disassortative (Figure 1). In comparison, small-world networks (Watts and Strogatz, 1998) are defined by a high average clustering coefficient and a short diameter, which illustrates a high density of links and the existence of many cliques based on the probability that two randomly chosen neighbors of a node are also direct neighbors of each other. The average shortest path length is a measure of network efficiency (Barabási and Albert, 2002). Considering actual account traffic, the rich-club index measures to what extent large degree nodes are strongly connected to each other. Scale-free and small-

[1] For a detailed explanation and illustration of graph theoretical measures and indices see: `http://people.hofstra.edu/geotrans/eng/ch1en/meth1en/ch1m3en.html`

world networks tend to concentrate at a few high-order connections, whereas road networks are characterized by a large diameter (i.e. maximum path length). Other indices have been proposed, such as the assortativity coefficient, which highlights to what extent nodes of comparable connectivity are connected to each other in the network. Such properties imply dynamics and evolutionary paths that are specific to transport networks.

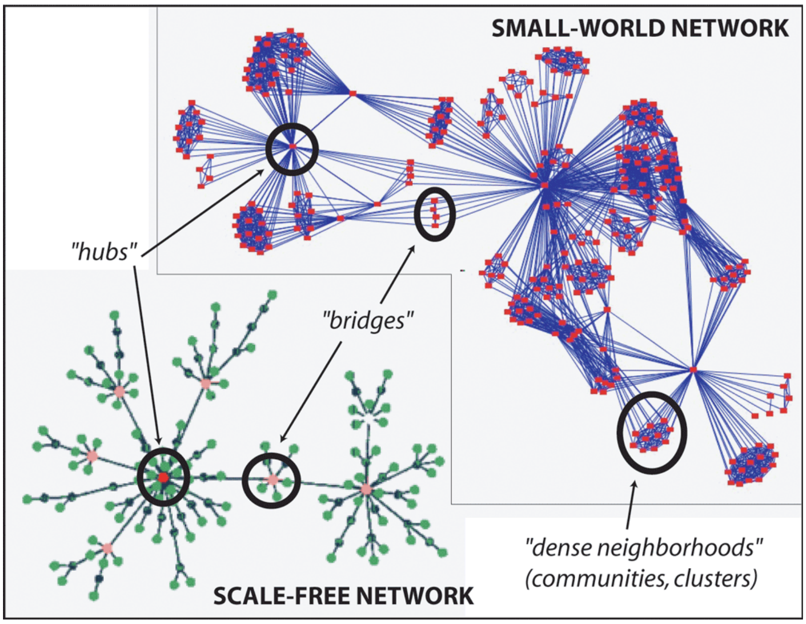


Figure 1: Two main network configurations

Several applications of complex networks have revealed interesting specificities and common grounds of different transport networks. Studies of the global maritime transport network (Hu and Zhu, 2009) and the global air transport network (Guimera et al., 2005) confirm their overall scale-free properties with regard to the existing literature on hub-and-spoke strategies of carriers (see (Fleming and Hayuth, 1994; O'Kelly, 1998)). While a relatively high vulnerability of the network lays upon few large nodes, the truncated distribution reveals the costs of creating new links at large nodes facing congestion problems, as seen in other case studies of air transport networks (Li and Cai, 2004; Guida and Maria, 2007), public transportation networks (Latora and Marchiori, 2002; Sienkiewicz and Holyst, 2005; Ferber et al., 2005; Xu et al., 2007) and railway networks (Sen et al., 2003).

Since most research focuses on one single network, there is a growing awareness that transport networks may be interdependent through issues of interconnectivity and the vulnerability of tightly coupled infrastructures regarding cascading failures, breakdowns, and attacks (Geenhuizen, 2000; Vespignani, 2010; Buldyrev et al., 2010). Notably, Zhang et al. (2005) argue that multi-layer infrastructure networks should be analyzed at various geographic levels with regard to the respective topology and function of individual networks and to potential inter-modal shifts and mutual influence. Examining the co-evolution of roads, canals, and ports during the English industrial revolution, Bogart (2009) reveals noticeable interdependencies among different nodes and networks over time based on spatial and functional proximity. This underlines that initial network developments are often done to support and then compete with an existing network by expanding geographically and topologically in ways unavailable to the prior network. Ducruet et al. (2011) also highlights the complementarity between air and maritime networks in the formation of a global urban hierarchy. This is also apparent in the work of Parshani et al. (2010) on the inter-similarity between coupled maritime and air transport networks, which shows that well-connected airports tend to couple with well-connected seaports in general, based on their respective geographic locations and on their topological attributes. Finally, Jin et al. (2010) propose an indicator of transport dominance applied simultaneously to freeway, railway and airline networks in China, stressing the importance of large urban concentrations and the role of distant cities serving as hubs for Western areas.

2.2 Local Level

Various local measures of networks have also been developed (Taaffe and Gauthier, 1973; Cliff et al., 1979; Dupuy and Stransky, 1996; West, 1996; Degenne and Forse, 1999). The goal is to compare the relative position of nodes in the network and to highlight groups of nodes in the network. While a full list of measures would run beyond the scope of this chapter, it is possible to organize them by those indicating the situation of the node in the entire network and those focusing on the relations of the node with its adjacent neighbors.

The most common global measures of a given node include:

- betweenness centrality: number of possible positions on shortest paths;
- eccentricity (or associated number, Koenig number): number of links needed to reach the most distant node in the graph;
- Shimbel index (or Shimbel distance, nodal accessibility, nodality): sum of the length of all shortest paths connecting all other nodes in the graph. The inverse measure is also called closeness centrality or distance centrality.

Other local measures are those looking at the neighborhood of a given node:

- degree (or degree centrality): number of adjacent neighbors. The weighted degree is the sum of weights on adjacent links;
- hub dependence: share of the strongest traffic link in total traffic, a measure of vulnerability;
- average nearest neighbors degree: indicates to what extent the node is surrounded by large or small nodes;
- clustering coefficient: proportion of observed closed triplets in the sum of all possible closed triplets, a measure of tightness and density.

Plotting together some of the aforementioned measures may highlight specific features of transportation networks. For instance, nodes with low degree centrality and high betweenness centrality reveal their role as bridges between different subgroups, i.e. as a strategic position or intermediacy. Some hubs may also act as a redistribution platform within their adjacent regions, thus multiplying their links in addition to their bridge role (e.g. feeder links in liner shipping, air or trucking). In maritime networks, Deng et al. (2009) underline the strong relation between actual throughput and degree centrality for container ports. The centrality of cities in air transport networks has received great attention in recent years. The relation between local measures and other local/regional indicators is also the focus of recent research on transportation networks. For instance, Wang et al. (2011) demonstrate the close relationship between the position of Chinese cities in airline networks (i.e. degree, closeness, and betweenness centralities) and their local socio-economic characteristics (i.e. total passenger traffic, urban population, and Gross Regional Product). Analyzing communication networks in the UK, Eagle et al. (2010) find strong interdependencies between the diversity of connections and the economic well-being of localities.

Local measures may or may not take into account the valuation of the edges concerned (i.e. weighted or non-weighted). This is of high relevance for transport studies, since transport networks are better understood by the usage level (e.g. number of passengers, tons, vehicles, capacity) than by their sole topology based on a binary state (i.e. presence or absence of links). The weighted degree centrality usually corresponds to the sum of edges traffic, sometimes called node strength. The hub dependence index, which corresponds to the percentage of the strongest traffic edge in the total traffic of each node, is a measure of vulnerability, notably highlighting the contrast for Shenzhen port between its traffic growth and its maintained dependence upon the Hong Kong hub (Ducruet et al., 2010). In their study of the worldwide air transport network, Barrat et al. (2004) demonstrated that the formation of cliques among major airports (also called rich-club phenomenon) is readable only using traffic weights. The inclusion of edge weight in the analysis can indeed reveal hidden patterns beyond the sole topology, such as the small-world structure of commuter flows in Italy (DeMontis et al., 2010) and the identification of different city types through the relative efficiency of their urban street patterns and length compared to optimal configurations of greedy triangulation and minimum spanning tree (Cardillo et al., 2006).

Motivated by practical and policy implications, transport networks analysis has also benefited from further developments in accessibility studies. Various algorithms, such as the Koenig number and the Shimbel index, have been used to describe nodal accessibility in numerous empirical studies. The Shimbel index was used in several case studies to reveal nodal accessibility for European regions in the context of integration (Gutierrez and Urbano, 1996) and Belgian crossroads (Lannoy and Oudheusden, 1991); the impact of deregulation and hub formation on air accessibility in China (Choua, 1993; Shaw, 1993) and in Southeast Asia (Bowen, 2000, 2002); and the impact of railway development on urban accessibility in Japan (Murayama, 1994). Examining the Indianapolis city road network, (Gleyze, 2007) refines topological measures of centrality and eccentricity by distinguishing between network and spatial effects. Chapelon (2005) applies the widely used Floyd algorithm of the shortest path, taking into account topological features and circulation constraints such as national regulations (e.g. speed limits), highway tolls, the types and capacities of main roads, and the travel time including loading /unloading delays at multimodal junctions (e.g. ferry crossings). Based on such criteria, Chapelon (2005) calculated the topological accessibility of ports to population and regional wealth potentials within a 6, 36, and 72 hour drive (Figure 2). The results clearly the privileged situation of North European Range ports regarding hinterland coverage and connectivity, while the correlation between road accessibility level and port throughput volume remains only moderately significant.

The search for subgroups of nodes or clusters in a network is another central issue in network analysis with related applications on transport networks. Sociologists "were the first to formalize the idea of communities, to devise mathematical measures of the number and cohesion of communities, and to develop methods to identify the subgroups of individuals within the network" (Boccaletti et al., 2006). Such methods also allow for simplifying large datasets in order to make their structure more readable. In their general works on graph theory, Berge (1973) and Bollobas (1998) define subgraphs (or subgroups) as the union of a subset of nodes and the edges linking them. All methods seek

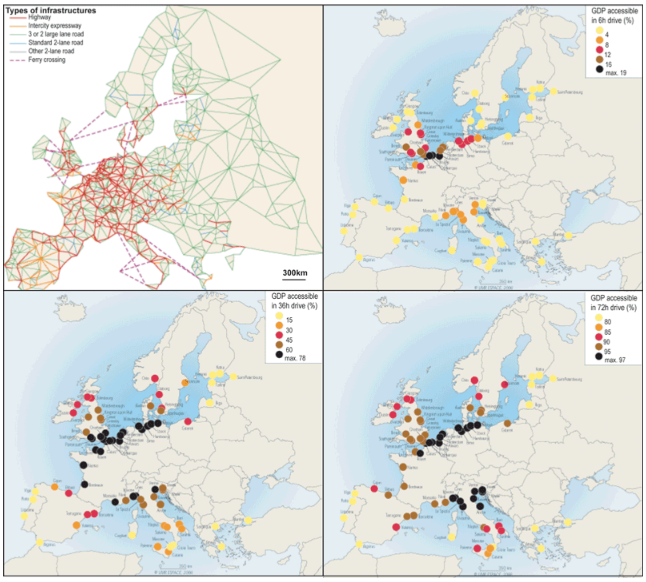


Figure 2: Example of accessibility measures. European ports in the road network (Source: adapted from Chapelon (2005)

to find relevant measures in order to bisect a given graph based on closeness within resulting groups (see (Wasserman and Faust, 1994; Moody and White, 2003)). Resulting subgroups exhibit various characteristics such as adhesion, with a centralization upon a leader node, cohesion, with more harmonized ties among nodes, cliquishness, exhaustiveness, reachability, and density (Gleyze, 2010). The identification of cut-sets allows the identification of the edges to be removed between subgroups, focusing on connections existence, quality and robustness within and between subgroups, with reference to the concept of clique. The notion of modularity refers to the optimization of intra-group cohesiveness (measured by distance, structural similarity or dissimilarity among nodes) with regard to inter-group linkages.

One of the first methods introduced for delimitating coherent sub-regions was the Nystuen-Dacey or dominant flow algorithm (Nystuen and Dacey, 1961), first applied to telecommunication flows among Oregon cities. Its numerous applications on weighted transport networks have revealed barrier effects in air and rail transport flows (Cattan, 1995), which cities dominate the hierarchy of air flows (Grubesic et al., 2008), and which ports act as pivots in maritime systems (Ducruet and Notteboom, 2011). Retaining only the maximum flow link between each node simplifies the networks architecture and highlights the existence of so-called nodal regions. Independent nodes are those in which dominant flow connects a smaller node in terms of individual traffic size; subordinate nodes always connect a larger sized node. Notably, Cattan (2004) depicts the lowering integration level of the worldwide airline system, observing its growing polarization by fewer large hubs and the disappearance of sub-regional systems formerly well connected internally. Figure 3 gives a more concrete illustration with the evolution of the nodal region of Busan, South Korea based on inter-port container traffic flows, showing how the hub strategy has resulted in an extension of its tributary area from mostly Japanese satellites to a number of Chinese secondary ports.

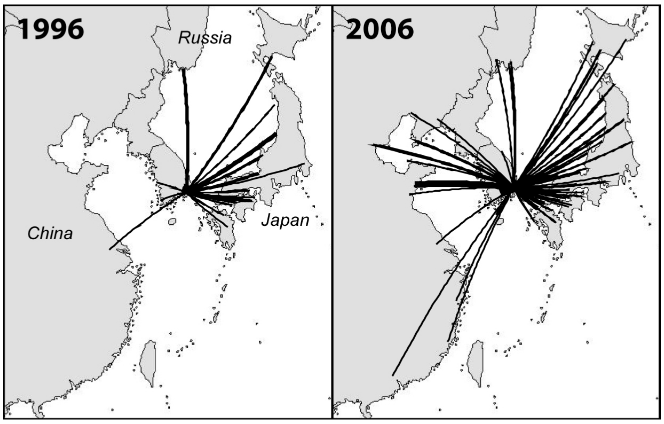


Figure 3: Example of a nodal region. The case of Busan, South Korea (Source: adapted from Ducruet et al. (2010).

Because the Nystuen-Dacey algorithm neglects a large number of secondary links, other methods of graph clustering (or partitioning) have been preferred in order to take into account the complexity of the network (Gleyze, 2010) and its multi-level organization. For instance, the application of strength clustering on commuter flows in France reveals the polycentrism, cohesion and fragmentation of urban areas (Tissandier, 2011). It has also been applied to air passenger flows among cities worldwide (Amiel et al., 2005), showing the importance of geographic proximity in the formation of clusters and the multi-level relations among them. For instance, Asian cities tend to form dense neighborhoods, while some global hubs do not belong to a specific cluster due to their intermediate role between regions. Other measures include the z-score (share of the cluster in a nodes connections) and the participation coefficient (connections to other clusters) applied to worldwide air traffic (Guimera et al., 2005). The availability of air traffic data on city pairs has helped foster a vast number of studies on the centrality of cities in such networks (see Shin and Timberlake (2000)). The bisecting K-means algorithm applied on all weighted maritime links among Atlantic ports verifies a strong interdependency within the Le Havre-Hamburg or North European range as well as strong ties among Iberian Peninsula ports and Brazilian ports. Other port groups are better defined by internal geographic proximity and peripherality from the whole system (Ducruet et al., 2010). Overall, the only problem related to clustering methods is that there is no guarantee to reach the best result as it depends on the edge weight selected (e.g. traffic), and there is no indication about the right number of iterations needed unless it verifies valid hypotheses about the number, quality, and size of expected groups.[2] Simpler methods have been proposed, such as deleting edges with high betweenness centrality (Newman and Girvan, 2004) or removing the highest or lowest degree centrality nodes (Zaidi, 2011). Other approaches include clustering junctions on road networks (Mackechnie and Mackaness, 1999; Gleyze, 2007); the use of structural equivalence, blockmodeling, and lambda-set methods to study the regionalization of the US Internet backbone infrastructure network (Gorman and Kulkarni, 2004); and measures of node vulnerability in the Swedish road network (Jenelius, 2009).

3 Dynamics in Transportation Networks

Understanding transport development requires analyzing complex and dynamic processes that produce temporal changes in the transport system. Two main issues are addressed in the literature: how the spatial structure of transportation networks evolves over time and the mechanisms modifying this structure. Several techniques have been developed to describe, compute, and simulate large-scale features of transport networks. Geographers describe topological transformations in the network based on intuitive mechanisms to replicate network geometries (Kansky, 1963; Taaffe et al., 1963; Garrison and Marble, 1962; Morrill, 1965); economists and urban planners compute and design statistical models for choosing the best design of the network from an explicit set of available variables and alternatives (Schweitzer et al., 1998; Leblanc, 1975; Yang and Bell, 1998; Gastner and Newman, 2004; Barthélemy and Flammini, 2006); and engineers and physicists simulate emergent attributes of networks applying the concept of self-organization and agent-based models (ABM) (Barabási, 2002; Newman, 2003; Lam and Pochy, 1993; Helbing et al., 1997; Yamins et al., 2003; Yerra and Levinson, 2005; Fricker et al., 2009; Levinson and Yerra, 2006; Xie and Levinson, 2009).[3]

In this section, we review briefly the application of Agent-Based Models (ABMs) to analyze the dynamics in transport networks. Although such an application is relatively new in the field of transportation networks, it provides

[2]See in-depth reviews of clustering methods by Fortunato (2010) and Schaeffer (2007).
[3]For a complete review of literature see Xie and Levinson (2009).

important advantages over analytical models. For example, ABMs describe individual interrelationships and form emergent properties in the system. Therefore, ABMs are a class of computational models for simulating systems of autonomous components, called agents, and the relationships between them and their environment. Based on a set of simple behavioral rules, agents interact and produce unexpected collective behavior (Newman, 2003). From the perspective of complex systems, the self-organization process shows how global patterns or structures in a system appear without the influence of a central authority or planning entity (Vicsek, 2000). Therefore, ABMs are a good option to study the dynamics in transportation networks, because they are evolutive models and allow for flexible application to diverse theoretical approaches.

For the purpose of analysis, we present two different methods to conduct an ABM application: generative and degenerative processes. Then, we point out some important factors that analyze the change and growth in the spatial configuration of the network. Finally, we suggest some significant features to be considered when designing ABMs for transport networks.

3.1 Generative vs Degenerative Methods

When the goal of the analysis is to design, create, and compare a simulation model to empirical evidence in transportation networks, there are two general procedures to analyze dynamics. The first is a generative method of network formation, which explains how nodes connect by edges over time and which situations are considered rest-points in this process. Starting from an unconnected graph, the process begins to connect each node based on different mechanisms related to edges, for example, the cost of building or maintaining the link. Another process to connect nodes is based on their hierarchical attributes. For example, a link is created between two nodes based on their degree of connectivity. This process is known as "preferential attachment" and explains the dynamics of networks that generate a power law degree distribution in a system (Newman, 2003). Applying such a process to transportation systems may reveal some emergent behaviors in planar and non-planar networks, such as hub-and-spoke configurations (e.g. airlines and shipping lines) and the limitations imposed by the spatial structure of transport networks (e.g. number of connections per node for road and railway paths). Therefore, preferential attachment not only explains the dynamics of a system based on nodes connectivity but also highlights the consideration of hierarchical attributes of nodes in order to explore the evolution of networks. The generative method has been used in various scientific fields, for example, in economics (Goyal and Joshi, 2002; Yamins et al., 2003; Kirman, 2011), geography (Jiang and Yao, 2010), and urban planning (Wilson and Nuzzolo, 2009).

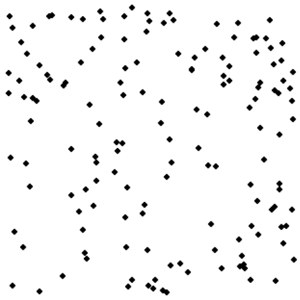
On the other hand, the degenerative method proposes starting the analysis from a complete network, where all nodes are connected to each other. All edges represent possible paths that can be used in the system, but only the more valuable ones reinforce its existence and modify its attributes. For example, some local roads can reinforce their importance based on the number of users and the cost of maintenance as well as the hierarchy of nodes to which they are connected. Therefore, edges that are less valuable decrease in importance and in their probability to be considered part of a representative path. Civil engineering is the most common field of application for such a method (see Yerra and Levinson (2005), Levinson and Yerra (2006), and Xie and Levinson (2009)).

Both methods have the same purpose: explaining an endogenous change and growth that produces a particular network structure. For example, Figure 4 shows a planar network road system consisting of 150 nodes that are uniformly and randomly distributed in an array of 400x400 cells, where the initial configuration of the system in the generative method is shown in 4a, and the degenerative method is displayed in 4c. Phase 4b presents one possible outcome in the final structure of the network; the process behind this structure is a simple distance mechanism that connects shorter edges or deletes large edges to nodes.

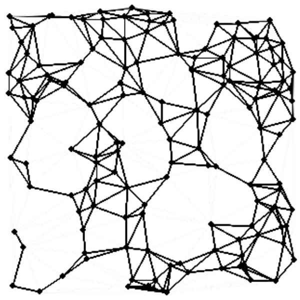
Even though both methods start from different initial conditions and apply inverse distance mechanisms, they converge to explain the final configuration of the network. In other words, there are different stories to tell about a process that creates particular network configurations. With this in mind, pre-knowledge of the network structure, such as global/local measures, and its contextualization, such as social and economic conditions, make it possible to define a mechanism that well explains the dynamics behind such a configuration. Then, the dynamics can be related to local mechanisms particularly market areas, traffic flows, and socio-cultural characteristics (Christaller, 1933). Therefore, the next step in the process to make a simulation model is selecting those factors that guide the explanation of the evolution in the transportation network and specify them as parameters in the model.

3.2 Mechanisms for Changing and Growing

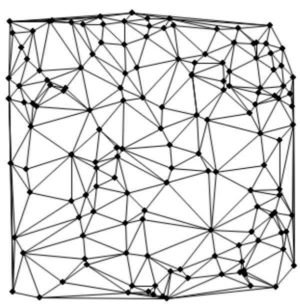
The evolution of the transportation network is closely related to the change and growth of central places. For example, Christaller (1933) explains the existence, characteristics, and evolution of such places. Transportation networks



(a) Generative method displays an unconnected graph



(b) A complex patter resulted by connecting or deleting links with small and large distance respectively



(c) Degenerative method shows a complete planar graph

Figure 4: Generative vs Degenerative Methods

are one of the principal factors to describe the number, size, and patter of spatial distribution of these places. The co-evolution process of central places and transport networks points out the strong connection between the dynamics of urban and transport systems, as seen in the works of Batty (2005, 2008) and Blumenfeld-Lieberthal and Portugali (2010).

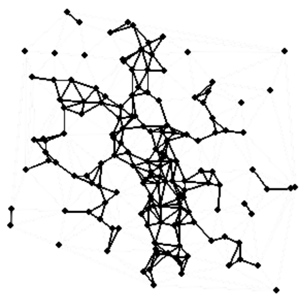
The market factor applied to transport networks explains the importance of the transportation cost between places in order to maximize the trade-off between demand and supply of goods and services. The creation and maintenance of transportation systems depend on the distance between central places; the higher the distance between places, the more expensive it is to sustain the transportation system. In other words, short links in transportation are preferable. Therefore, the market area affects the transportation cost and shapes some emergent patters in the structure of the network Fujita et al. (1999).

In addition to the market factor, traffic is another important element that affects the structure of transportation systems. This explains the importance of the flow of individuals, goods, and services between central places. Components that describe this factor include the total travel time and speed on links (Xie and Levinson, 2009). Large-scale properties of the network, such as the existence and hierarchy of links in the system, are analyzed by the traffic factor (de Dios and Willumsen, 2001; Levinson and Yerra, 2006).

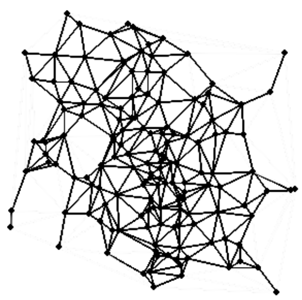
The socio-cultural factor is a hierarchical order between central places showing the level of embedded systems across a particular geographical area, for example, an increased number of activities coordinated inside a city makes it more important compared to other cities. Economic and demographic variables, such as the Gross Domestic Product (GDP) and the urban population, valuate and describe the rank of a place in a system. Consequently, transportation links receive the effects of this factor, because more activities need to expand and distribute their influence to other places (Krugman, 1996; Pumain and Tannier, 2005; Batty, 2005).

As a result, depending on the spatial structure of the network, dynamics can be studied through the above factors and translated into parameters. Continuing with our example of a random planar network, links are classified into different levels according to the transportation cost. Each link has a specific cost associated to the distance between a node pair; the closer the nodes are, the easier it is to build and maintain their connection, and the less likely longer pairs are to exist. Defining a parameter beta, which provides a threshold value for selecting shorter links (distance of one link divided by the larger link), and modifying its value provides the results shown in Figure 5.

In the figure, 5a shows a topological structure when beta is defined as 0.15, meaning that links of bigger size (i.e. more than 15%), are not used to connect



(a) Transportation network topology with high preference to have shorter links, in this case links less than 0.15.



(b) Transportation network topology when it is more desire to have larger links, in this case links bigger than 0.25.

Figure 5: Variation in the transportation cost parameter and the emergent transportation network

nodes or are deleted from the system. Conversely, 5b displays a dissimilar structure, because it includes larger links, where the threshold value is set at 25%. Hence, depending on the value of the parameter, different structures emerge in the system.

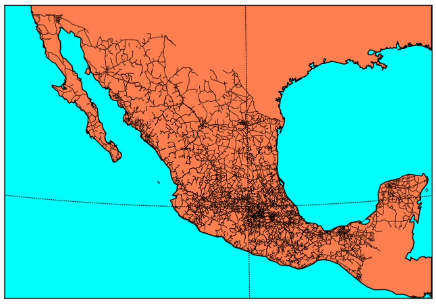
3.3 Applications

Almost any transportation network shows properties of complex systems, and its structures can be explained by the self-organization process. Hence, ABMs are a natural method to search explanations of the evolution of such systems, because they provide flexible and novel applications. For instance, Martens et al. (2010) simulate parking behaviors in a city, and Blumenfeld-Lieberthal and Portugali (2010) develop an urban simulation model to study the dynamics of a system of cities. After specifying the method and the relevant parameters of the analysis, the next step is to design and implement simulation experiments.

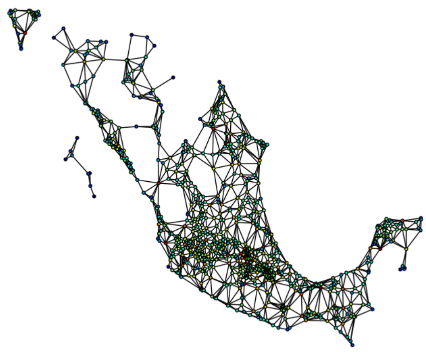
At a glance, ABMs are computational methods conforming to a collection of autonomous agents, each of which has specific attributes and behavioral rules; their interrelationships produce large-scale structures in the system. In order to apply ABMs to transportation networks, it is necessary to define the scale of the analysis and the type of agents in the system. The scale can be related to a region, a metropolitan area, or a city; the type of agents can be specified by nodes such as cities or terminals; and edges represent transportation links among them. The key point is to define an autonomous agent who has particular attributes and specific methods to interact with other agents and its surroundings. Attributes are intrinsic characteristics of the agent, such as its name, age, or location, and methods are mechanisms or behavioral rules that make possible the interaction with others and modify its attributes, for example, a method that changes the agent's location.

Applying such methods to the Mexican road system specifies two types of autonomous agents: cities as nodes and roads as edges. The former is defined as a central place with the attributes of location and population. The latter is defined as transportation links with the attributes of location and distance. The first method compares the distance between links and returns a group of the shortest ones; the second method compares the distance between links and the number of habitants between nodes in order to return short links to connect more populated cities. In Figure 6, 6a displays the road system of Mexico, 6b shows the simulation result based on short roads, and 6c exhibits a simulation output regard to short roads and large cities.

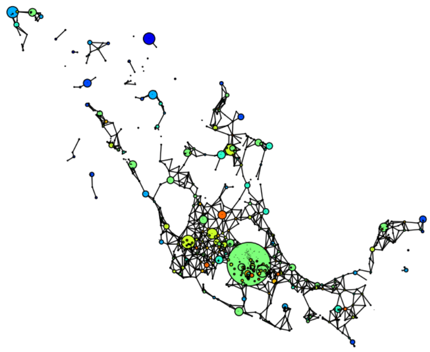
Although this example is very simple, note that the process behind emergent patterns is an auto-organization between agents without a central authority



(a) Road system of Mexico in 2005 (Source: Instituto Nacional de Estadística y Geografía (INEGI) (2010)



(b) Short roads based on a distance less than 10% compare to its maximum.



(c) Short roads based on a distance less than 6% and a number of habitants bigger than $10,000$.

Figure 6: Application of ABMs to road networks: the case of the road system at Mexico

or planner that can be applied to the analysis of the existence, geometry, or hierarchy in road networks.

In addition to the rigorous application of ABMs to transportation networks, the results must be communicated in a straightforward, structured, and scientific way. Simulation models are usually validated through comparison to empirical data. The simulation also needs to achieve other requirements, such as analyzing temporal changes in the topology of the network based on different network paths, measuring structural properties of the network, and making a sensitivity analysis where emergent patterns in the network are related to dissimilar initial conditions (Xie and Levinson, 2009). Furthermore, there are some interesting proposals to create a basic framework for communicating ABMs results, the most useful of which is the ODD protocol (Overview, Design Concepts, and Details). Based on the work of Grimm et al. (2006), this protocol is applied to study large-scale system properties that emerge from the adaptive behavior of agents.

4 Conclusion

Transportation networks are at the center of network analysis in natural and social sciences. Recent trends show more interdisciplinary approaches and fast developments of new analytical methods explored by physicists. While transport network analysis still has a lot to gain from such advances, it maintains its specificity by bridging abstract measures and theories with knowledge of the transport industry and broader socio-economic and spatial issues. On another hand, natural sciences give increasing importance to the spatial and social dimension of networks, thereby calling for further collaboration between natural and social sciences. One interesting research path lies in the field of intermodalism, which integrates local studies of planning and strategies with global-level approaches to multiple network topologies and interdependencies. Additionally, a closer look at meso-level measures would certainly foster understanding of how transportation networks are structured and how they evolve with regard to the combination of their local and global properties.

The study of dynamics in transport networks is improved through the application of ABMs. Although such models are relatively new in the field of transport networks, they help to understand endogenous mechanisms that produce unexpected collective behavior in the system. In order to apply ABMs to the analysis of network dynamics, it is important to define the method and mechanism of analysis based on theoretical and empirical evidence. In conclusion, applying complex systems to transport networks is preferred to complement theories and applications in different scientific fields.

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A Complex Networks Measures

Global Measure	Basic Definition	Formula
Hierarchy	Exponent of the slope for the power-law line drawn in a bi-log plot of node frequency over degree distribution	$y = \alpha x^h$
Transitivity	Ratio between the observed number of closed triplets and the maximum possible number of closed triplets in the graph	$C_i = \frac{\lambda_G(v)}{\tau_G(v)}$
Average shortest path length	Average number of stops between two nodes in the graph	$l_G = \frac{1}{n(n-1)\sum_{i,j} d(v_i,v_j)}$
Assortativity coefficient	Pearson correlation between the degree of nodes at both ends of each link (edge) in the network	$\tau = \frac{M^{-1}\sum_i j_i k_i [M^{-1}\sum_i \frac{1}{2}(j_i+k_i)]^2}{M^{-1}\sum_i \frac{1}{2}(j_i^2+k_i^2) - [M^{-1}\sum_i \frac{1}{2}(j_i+k_i)]^2}$
Local Measure		
Degree centrality	Number of adjacent nodes	$k_i = C_D(i) = \sum_j^N x_{ij}$
Eccentricity	Number of links needed to reach the most distant node in the graph	$e(x) = max_{y \in X} d(x,y)$
Shimbel index	Sum of the length of all shortest paths connecting all other nodes in the graph	$A_i = \sum_{j=1}^N d_{ij}$
Betweenness centrality	Number of times a node is crossed by shortest paths in the graph	$C_B(i) = \frac{g_{jk}(i)}{g_{jk}}$
Average nearest neighbors degree	Average degree of adjacent nodes	$k_{nn,i} = \frac{1}{k_i}\sum_j \alpha_{ij} k_j$

Common measures used by complex networks (compiled from various sources)