A world of interstices:
A fuzzy logic approach to the analysis of interpretative maps

C. Didelon[1], S. de Ruffray[2],
M. Boquet[3], N. Lambert[4]

Mots clefs: Carte mentale, Monde, Carte interprétative, Logique floue
Keywords: Mental map, World, interpretative map, fuzzy logic

Clarisse Didelon
Université du Havre, UFR LSH
25 rue Philippe Lebon
76 600 Le Havre
France

Sophie de Ruffray
UFR de lettres, département de géographie
Université de Rouen
I rue Thomas Becket
76 821 Mont-Saint-Aignan cedex
France

Mathias Boquet
UFR SHA
Université de Metz
Ile du Saulcy
57045 Metz cedex
France

Nicolas Lambert
UMS RIATE
Université Paris 7
UFR GHSS case 7001
75 205 Paris cedex 13
France

Abstract
This paper proposes a methodology based on fuzzy logic to analyze specific mental maps at world scale. Mental maps at all scales and especially at world scale raise specific issues related to the imprecision of the drawing and uncertainty linked to the object drawn. Here, the sample studied was asked to divide the world in regions. The interpretation dimension when building regions reinforce both the uncertainty and imprecision. The fuzzy logic is used here to focus on the world regions limits. It allows providing cartography of the vagueness of regions borders.

[1] UMR 6266 IDEES – CIRTAI – Université du Havre
[2] UMR 6266 IDEES – MTG – Université de Rouen
[3] CEGUM – Université de Metz
[4] UMS 2414 RIATE

Introduction

In order to understand how the world is perceived, spatial psychologists and geographers have traditionally sought to make surveys with mental maps at global scale. The objective was to capture a cognitive perception of space rather than produce an interpretation. Moving beyond this approach our aim is to analyse how places are clustered when interpretative maps are made. In the framework of a major research programme[5] we implemented a survey on the ways in which the world is perceived. While this approach is consistent with the heritage of mental map making it nevertheless raises a number of specific methodological problems which suggest resort to the relevant mathematical solutions proposed by fuzzy logic theory. The aim here is to identify and to characterise interstitial areas.

I. World scale "interpretative" mental maps.

In order to problematise the relation between territory, identity and space practice while moving beyond the more often studied intra-urban (Lynch, 1960, Cauvin, 1999) and regional (Gould & White, 1997, Frémont, 1976) scales, our approach is based on the hypothesis that the world level is a subjective space. This hypothesis may, however, initially seem paradoxical. Indeed one's knowledge of a space is related to the cognitive effort to make. The later is defined as more important when the "distance" increases (Moles & Rohmer, 1978). The world would be then the most difficult level to know. This assertion is however increasingly questionable as the world can be, at least theoreticaly, 'known' through the filters of education, television, the Internet and newspapers (Paulet, 2002). Indeed a general knowledge of the world is now as easy to gain as a general knowledge of one's own country or region. Therefore, if the knowledge of close at hand spaces relies mainly on experience, knowledge of far away spaces is made easier thanks to the availability of a multitude of new information sources, many of them numerical. Our approach is to be found at the crossroads of methods and concepts in relation to mental maps with the aim of capturing the various interpretations of the world that are revealed.

1.1. Mapping the subjective space : the polysemous concept of mental maps.

Psychologists were the first to focus on the notion of cognitive space forwarding the hypothesis that space is full of meanings and values. The later are specific by each individual, but with collective studies, it is possible to sum up more or less shared characteristics. The term "mental map" covers various notions taken from the field of "cognitive-behaviourism". This movement emerged in the USA in the 1970s (Gold, 1992), mixing the fields of perception and imagination, it sought to analyse the link between representation and action (Lynch, 1960, Gould & White, 1997). In the literature four rather different concepts and methods are generally included under the catch-all term "mental map".

The notion of "mental map" can refer to a **cognitive map**, i.e. an individual's un-mapped spatial knowledge, its subjective space (Kuipers, 1978). Such a map is created from a selection of elements in the space perception process, primarily physical factors that determine one's sensorial capabilities but also individual (gender, character) and sociocultural (scale of value) factors. It is then the result of the "reshaping" of space from its spatial perception and would be a kind of guideline to move and act through the space. This is why geographers are interested in "subjective space": the space is one factor in the shaping of this subjective space while similarly, the later is a factor in the shaping of

[5] The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement n°225260. (www.eurobroadmap.eu)

space. Doubts however remain over the potential for a real cartographical pattern to exist in the mind. Friedman & Brown (2000) stressed that the “mental map methaphor is a misleading analogy for the representation of geographical knowledge”. This then raises the problems of how to capture subjective space and how analyse it.

Mental map can also have the meaning of a “sketch map”. Such a map is realised in the framework of a survey where people are asked to draw a specific space on a blank page. The aim here is, primarily, to study people’s itineraries or how they position themselves in the space. Researchers often then try to identify and to explain the knowledge level or judgement on the space by highlighting the various perception filters in play. The analysis here can also include the construction of a statistical explanative model such as, for example, gravity models (Pinhiero, 1998). These models are based on the proxemics hypothesis according to which the closer one is from a space, the more information one obtains on it (Moles & Rohmer, 1978). They also introduce some information describing surveyed people. One of the major difficulties with this approach however concerns the actual interpretation of the maps (Cauvin, 1999). Indeed there is difference between the respondent’s level of knowledge on space and their ability to translate it in visual form. In addition, the implementation of such analysis is often complicated further because the landmarks and scales of the maps produced are rarely comparable. To produce a workable synthesis of invididual representations then is a difficult task.

The third type of mental map, is characterised by the instructions provided during a survey on a particular space: one has to provide its appreciation on a space or to delimit a phenomenon. We suggest naming such maps, “interpretative maps”. The reasercher does not try to know how one draws a space nor to determine a respondent’s knowledge level but does desire to understand how the respondent defines a particular phenomenon in a space on the base map provided. The analysis then is easier to undertake than in the previous case because the researcher can quickly and easily compare the “real” and the “interpreted” space. This kind of approach is often used when we want analyse risks representations (Bonnet, 2004, Matei, Ball-Rokeach & Qiu, 2001). The level of knowledge and its effects certainly have an influence on the realisation of the maps, but the drawing of a space in map form allows us to substantially eliminate them and to employ a deeper level of analysis.

By extention then the cartographic synthesis of individual results obtained from a survey on the space (Paulet, 2002) is often also termed ‘a mental map’. This synthesis or reconstruction can be made from a classical survey questionnaire where people are asked about their knowledge or judgement on space. The mental map is then the synthesis of all answers and is a cartographic representation of the indicators of knowledge and judgement.

1.2. Mental maps at world scale.

Research initiated by T.F. Saarinen on world-scale sketch maps was often undertaken with a view to demonstrating the variety of points of view on the world or to test the geographical knowledge level on some sample of the world’s population (Saarinen, 1998, Saarinen & MacCabe, 1995). The material gathered during these surveys is analysed in two main approaches. The first focuses on the ‘centering’ of students according to their place of residence (Saarinen, 1987). The second, and more often utilised approach analyses the quality of mental maps i.e. the number or countries mapped, their relative proportions, their location etc. As such, in the literature we can source many authors and numerous places or surveys following this approach; see, for example, Saarinen & MacCabe in Finland (1989), in Germany (1990), in the USA (1995), Pinheiro in Brazil (1998), Boyowa in Africa (2003) etc. Beyond the descriptive approach some authors seek to explain the frequency of place representations via statistical approaches based on the gravity model. Even if they have hypothesis on the factors that could explain the differences in perception and knowledge, the link with the scholar or media knowledge produced (on the world) is rarely deepened and explanatory factors are often simply considered individually without linkage. Moreover, while such research may have been very fruitful from a theoretical perspective it is often disappointing from a methodological perspective : the rich vein of material accumulated is actually often underused. Beyond the setting up of statistical models, authors have never produced a world map synthesing their results allowing them to provide a global picture of the world. A further field of analysis emerged at the beginning of this century with researchers focusing on distortion in cognitive maps while exploring the explanatory factors for this (Friedman & Montello, 2006, Battersby, 2009, Battersby & Montello, 2009, Friedman, 2009).

1.3. Challenges raised by an “interpretative” survey at the global scale.

Our survey can be included in the interpretative mental maps category. The question (to divide the world into different regions and give each a name) led respondents to think about the world in order to construct their regions. As stated by Montello (2003) the creation of geographical regions is a necessary approach to effectively organising knowledge of the world. Indeed, when grouping places that they found similar and when separating others they found different respondent give their interpretation of the world and allow us to understand how they organise their knowledge of the world. The maps produced in this manner however raise specific challenges both from a theoretical and a methodological point of view.

Unlike the production of conventional mental maps, the starting point here is not the individual knowledge or experience but the space that has to be concieved, globaly, when doing the exercice. In addition, whatever the scale chosen, the combination of interpretation and incomplete knowledge of the world generally leads to the creation of imprecise and uncertain mental maps. Moreover, as the global scale is likely to be the that which is furthest from individual experience, a survey at this scale inevitably raises the level of level in the mapping of regions.

A second challenge here concerns the notion of regionalisation. Respondents are unlikely to have a precise idea of the limits between regions: they are unknown or fuzzy (Paulet, 2002). Regions are merely defined in terms of their visual structure, of specific activity, of ethnic composition and of other characteristics that are not strictly related to spatial cognition (Kuipers, 1978) and are built by using and mixing various sources of information (Friedman, 2009, Battersby & Montello, 2009) They are not necessarily precisely localisable particularly where multiple criteria are used. The regionalisation of the world then induces imprecision and uncertainty.

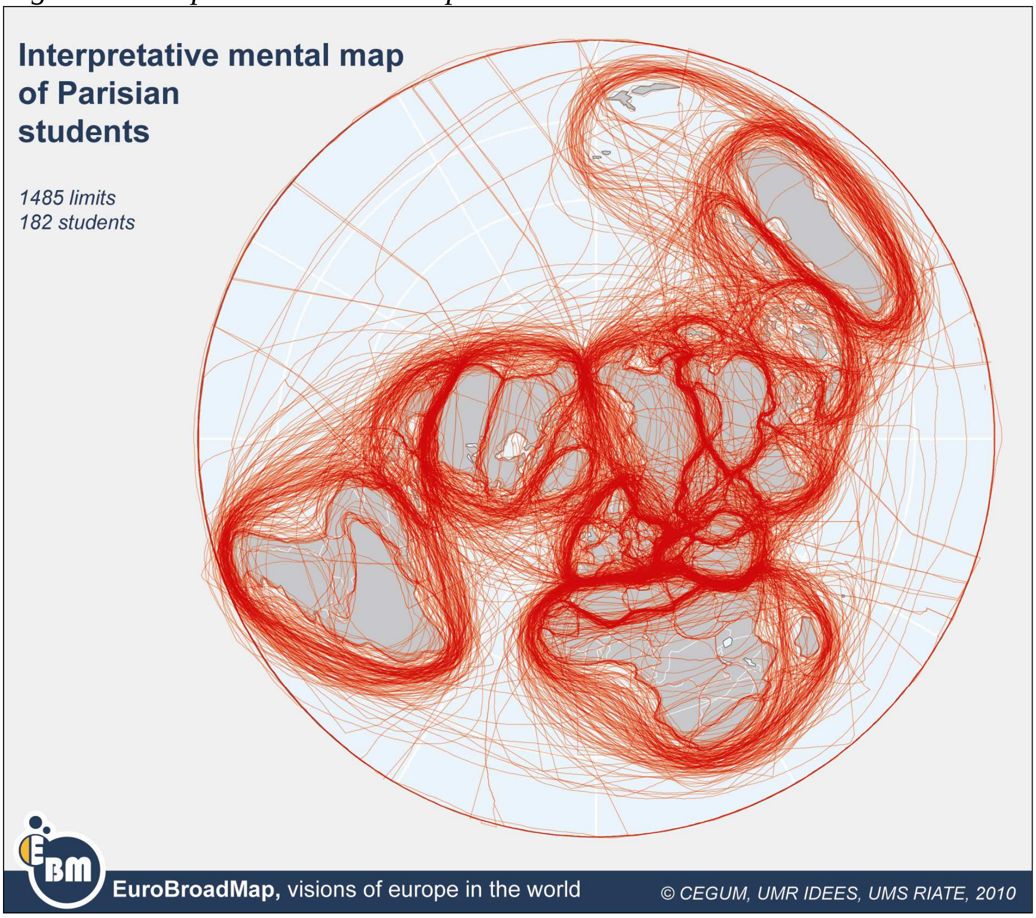
Last but not least, the transition from individual maps to a collective synthetic map is based on the assumption that, beyond the multiplicity of personal representations, common mechanisms are expressed in terms of knowledge and perception. Indeed, while individual respondents are seen as unique individuals they are nevertheless also socialised. This implies the existence of a relative level of consistency in terms of the mental pattern of the sample. The transition from individual to collective level is then made easier by the choice to use interpretative maps: clear instructions and a basemap were provided. If, however, individual representations are precise (limits are clearly draw) the transition to the collective level will inevitably introduce a level of imprecision.

II. Fuzzy logic: a methodological answer to the problem of analysing interpretative mental maps

2.1. Imprecision and uncertainty in interpretative mental mapping

The main feature of 'interpretative' mental maps is that the spatial objects respondents are asked to draw are deliberately undefined. Uncertainty in respect of the delimitation of regions here is a factor of the differing capability levels of the observers relating, primarily, to their education, character, culture, history etc. Moreover, respondents can have hesitations or doubts when faced with the difficult exercice of mental mapping. Imprecision emerges in respect of how world regions are composed when the interpretative mental maps are drawn. It is difficult, perhaps even impossible, to define a world region as the sum of spatial elements belonging to it without any ambiguity. In addition, the localisation of world region does not conform to an objective appreciation of geographical distance but rather to proximity and neighbourhood. Thus, the synthetic interpretative mental map of a group of people (figure 1) can be characterised as a fuzzy geographical space (Rolland-May, 2001, 2003).

Figure 1: interpretative mental map

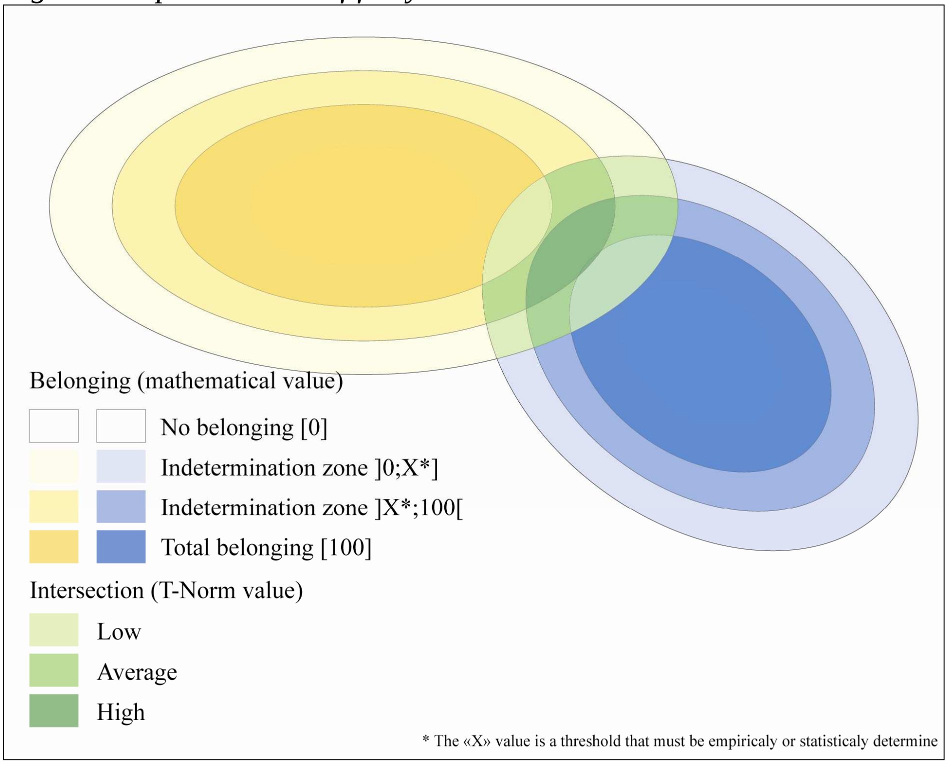


Given the objectives of the analysis, fuzzy sets and possibility theory (Zadeh, 1965) suits this survey. Indeed, it formalises the subjectivity, the uncertainty and the imprecision according to human perception, imagination and thinking from models and representations (Bouchon-Meunier, 1995, de Ruffray, 2007). Moreover, the fuzzy set concept has been introduced in order to deal with poorly defined limits, intermediate situations, the gradual transition from one property to another, approximate values and categories, thus avoiding the use of arbitrarily rigid boundaries.

2.2. The regionalisation of belonging: formalisation with fuzzy logic

Each region drawn by a group of people is a fuzzy geographical object that implies the definition of a membership function included in the [0,100] interval (Zadeh, 1965) for each spatial component of the geographical space, i.e. the elementary spatial unit that is here a grid cell (figure 2).

Figure 2: representation of fuzzy set.



- ⇨ When the membership function is "zero", the cell of the grid is not included in an interpretative region. This means it belongs to the non-perceived, or at least to areas not represented by respondents.
- ⇨ If the membership function is "100", the spatial unit is always included in an interpretative region of the world. However the characteristics of the region can vary.
- ⇨ In most cases the membership function of the spatial unit is included in the]0,100[interval. This reflects the gradation of the belonging of the cell to interpretative regions.

This approach allows us to take three important characteristics into account in the analysis: the differentiation of the region, the imprecision of limits, and the fact that regions overlap. The internal differentiation of geographical space is valuated thanks to the membership value of a world region. Indeed, a space is differentiated when the cells present different degrees of inclusion in a region, as in the "Europe" region (figure 4). The "core" is a homogeneous area of full membership. Differentiation of space introduces the idea of spatial gradient. The imprecision of its limits with its own environment is the second characteristic of the fuzzy geographical space. From a theoretical point of view, the limit of a fuzzy area is not defined. The lower value of the interval is the limit that can be discussed according to different thresholds that can be fixed both empirically and statistically. Thresholds are set up in order to differentiate the strongest and weakest level of membership. Mapping all the values in the]0,100[interval allows to visualise gradations and discontinuities as in the example presented below.

2.3. Focus on overlaps: fuzzy intersection operators

If non fuzzy spaces are disjointed, the principle of the excluded third is negated in a fuzzy space because a place can belong in different degrees to many geographical spaces. The space is therefore overlapped. In order to highlight the intersection between two regions fuzzy set theory provides a set of operators called *T*-norms that allow the mapping of the places that belong to two or more regions simultaneously. The *T*-norm functions are adapted operators to be used as intersection operators. Two main intersection operators will be used in this paper. First the Zadeh *T*-norm [$T = \min(x,y)$] which provides the minimum common membership value between two intersected regions and second, the probabilistic *T*-norm [$T = x*y$] which multiplies membership values. The highest values of the *T*-Norms are located where the memberships are most heavily shared and imprecision and uncertainty reaches the highest level (*figure 2*). This allows mapping of the interstice areas that characterise places of multi-membership. By definition, an area can belong to many fuzzy regions at the same time. The possibility of membership of two or more different spaces can be illustrated by reference, for example, to Turkey and Mexico which are often shared between two different world regions.

III. A world of fuzzy regionalisation

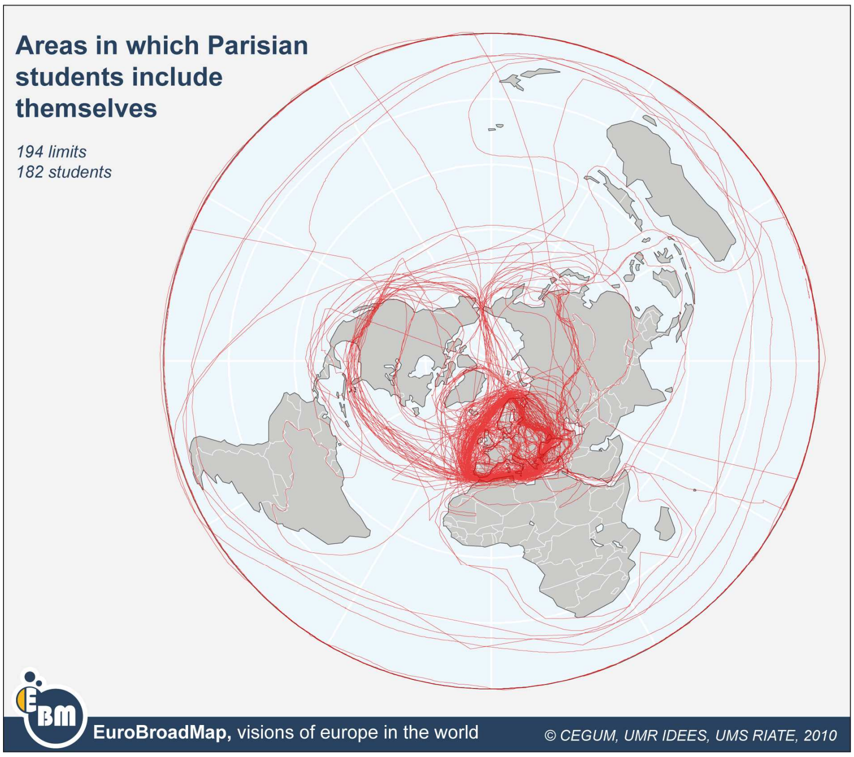
The maps presented in this section are built on a sample extracted from a worldwide survey made in 2009 on students in different fields (geography, medicine, engineering, arts, economics and political sciences) in 43 cities from 18 world countries[6]. In the survey students were asked to draw world regions (up to 15) and name them on a polar projection map. It is well known that every map projection induces many distortions in the earth representation and that they have an effect on “long term spatial memory and then on the cognitive map” (Battersby, 2009). We chose this projection because it was the most “unusual” for all students and because this projection does not lay stress on a particular area. This section shows the advantages of the fuzzy logic approach in the analysis of interpretative mental maps applied to the students of the first completed city of the survey, namely, Paris.

3.1. Topology and toponymy approaches

In choosing the regions we will analyse two main approaches are possible. The **topology approach** selects all the regions drawn around a specific place or a country without taking into account the various names given to the regions. The map obtained (figure 3), illustrates the cognitive indeterminacy related to the names used by respondents. The advantage of this approach is its focus on spatial concentration level of a region. In the example given, all the areas drawn around Paris have been represented. We can observe that the lines are concentrated around the European region but sometimes also include North America and Russia but rarely other spaces.

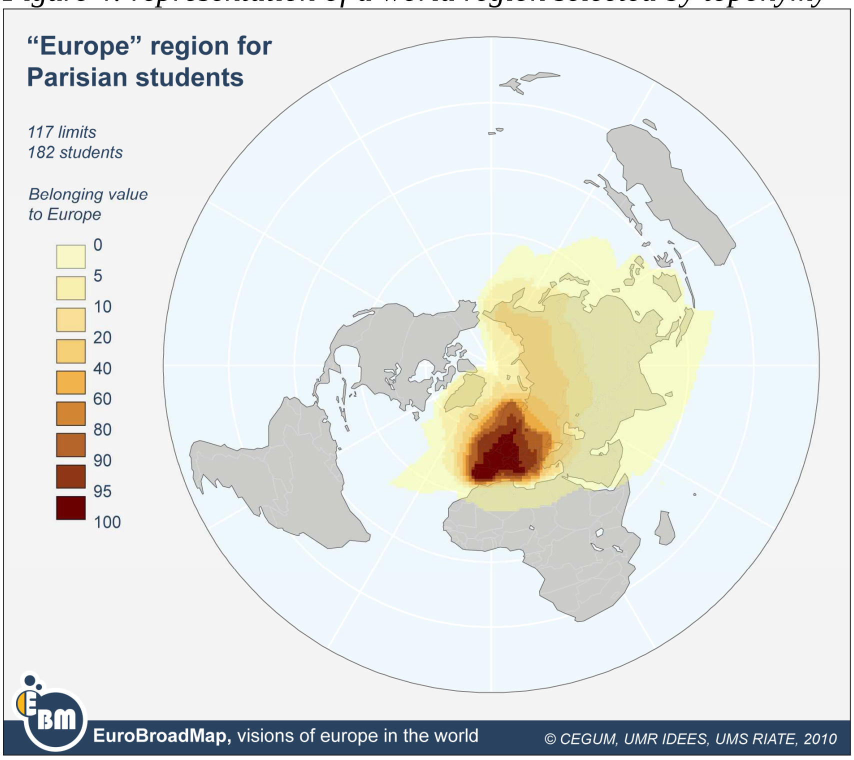
Figure 3: representation of a world region selected by topology

[6] For more information : www.eurobroadmap.eu



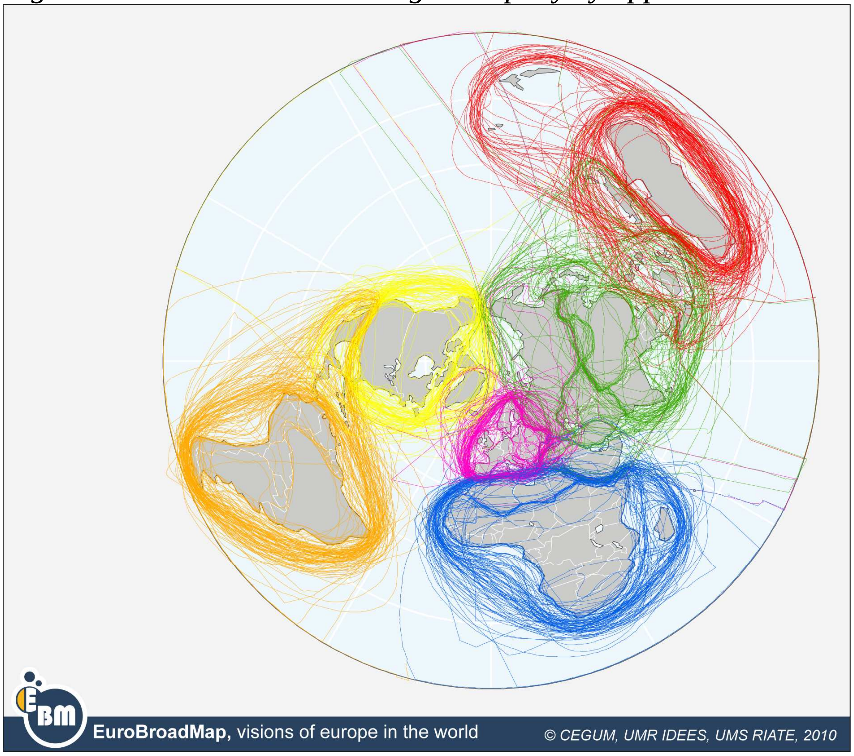
The **toponymy approach** selects the region according to the names they received. The heterogeneity character appears in mapped space limits. Figure 4 shows the fuzzy region called "Europe". The most interesting aspect of this approach is its ability to represent the limits of the region and to highlight the discontinuities. The core of the "Europe" region here shows an important discontinuity on its southern edge (the Straits of Gibraltar) while on the Eastern edge membership values are slowly decreasing.

Figure 4: representation of a world region selected by toponymy



In order to map a fuzzy world, we chose the toponymy approach. We analysed the frequency of the names given to the different regions produced retaining the six world regions with the highest frequency. It is striking that those frequently identified regions match the traditional continental vision of the world (Grataloup, 2010): Europe, Africa, North America, South America, Asia and Oceania are most often quoted (figure 5). Despite the toponymy selection, we observe that the limits used vary significantly and that overlaps are numerous. This approach will later allow us to compare the regions obtained with the relative size and position of those in a classical world regions map as suggested by Battersby & Montello (2009).

Figure 5: selected areas utilising the toponymy approach



3.2. A world of interstices

For each operator overlapping areas where the *T*-norm values were the highest are mapped in four maps. A global map showing all the values has also been produced. The Zadeh *T*-Norm (figures 6 & 7) offers a minimalist vision of the intersections highlighting the areas where the overlap is strongest. The highest level of overlap is localised to Mexico, which is often shared by the respondents between North and South America. A second are of interest in this context is the Mediterranean Sea area between Europe and Africa. These overlapping areas are not very large, but their cores are quite dense and discontinuities are magnified. This illustrates a clear belonging in the context of some spaces, mainly in the north. The United States never belongs to the southern part of America and Europe never belongs to Africa. In the south large area of gradation can however be observed. They show the hesitation on the membership of Mexico, Central America and the Caribbean to the North American region or of North Africa to the European region. The intersection between Europe and Asia is the largest. It covers all of Eurasia but its core, localised on Russia, is not very intense. The memberships between the two regions are less marked and the indeterminate area is thus larger. The global map (figure 7), illustrates that only three spaces in the world are not characterised by membership indeterminacy: North America, South America and Sub-Saharan Africa.

Figure 6: main overlapping areas with Zadeh T-Norm.

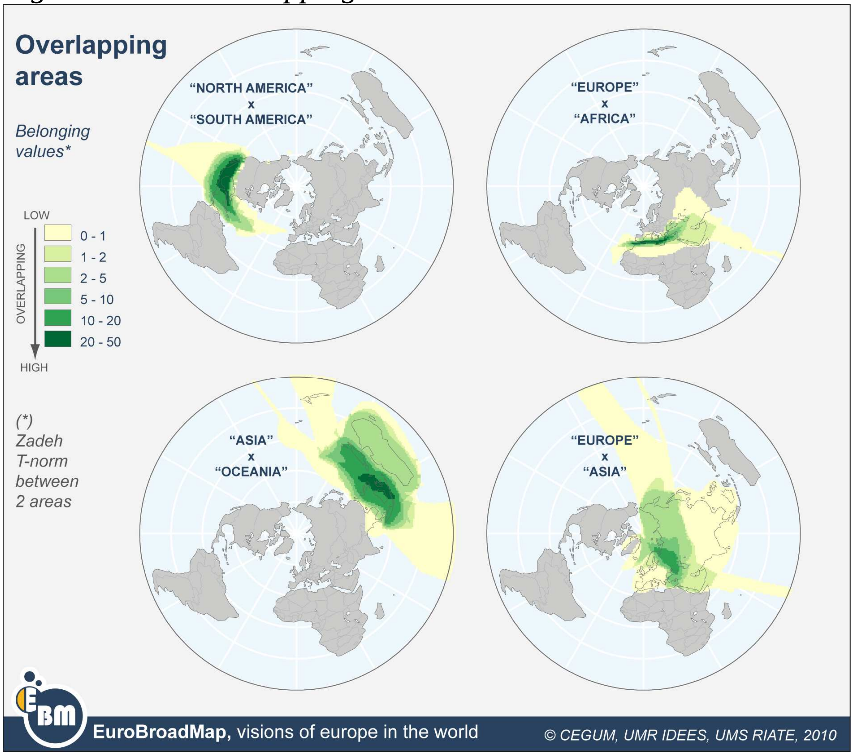
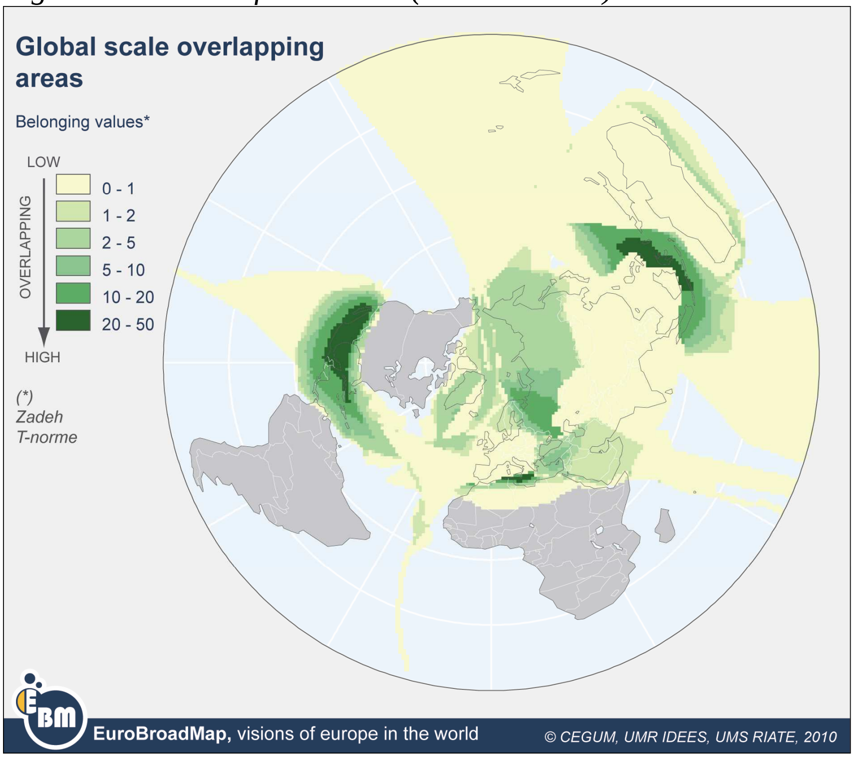


Figure 7: a world of interstices (Zadeh T-norm)



The probabilistic *T*-norm (figures 8 & 9) highlights the intersection where the membership values to two or more areas are close and high, and therefore areas of strong respondent hesitation. Indeed, if a spatial unit attains a membership value of "50" for Europe and "50" for Asia, the *T*-Norm value is 2500. If the membership value is "5" for Europe and "90" for Asia, the *T*-norm value will be only "450". With this *T*-norm, the maximal value can be observed on the Eastern Mediterranean Sea, more precisely near Crete, between Turkey and Greece. This place is the maximum overlapping area, highlighting the intersection between three membership regions inherited from the European classical world vision (Grataloup, 2010): Africa, Asia and Europe. As for the previous *T*-norm mapping, the strongest indeterminacy areas fit with strong development level discontinuities. Discontinuities are marked in the North while less precise in the South. This *T*-norm, for example, accentuates the indeterminacy between Australia and the surrounding spaces.

Figure 8: main overlapping areas with probabilistic T-norm

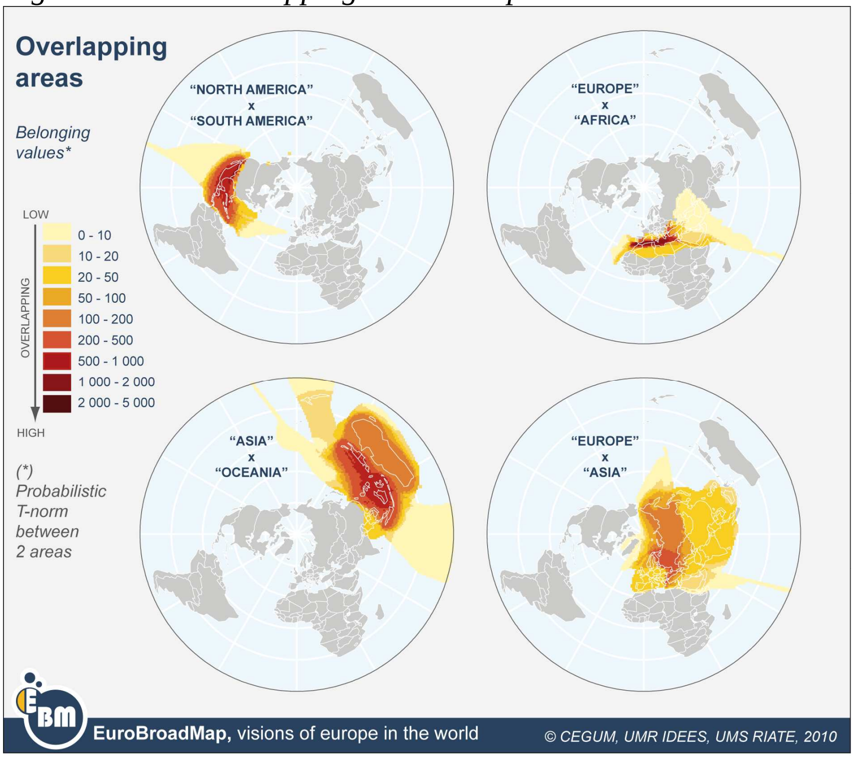
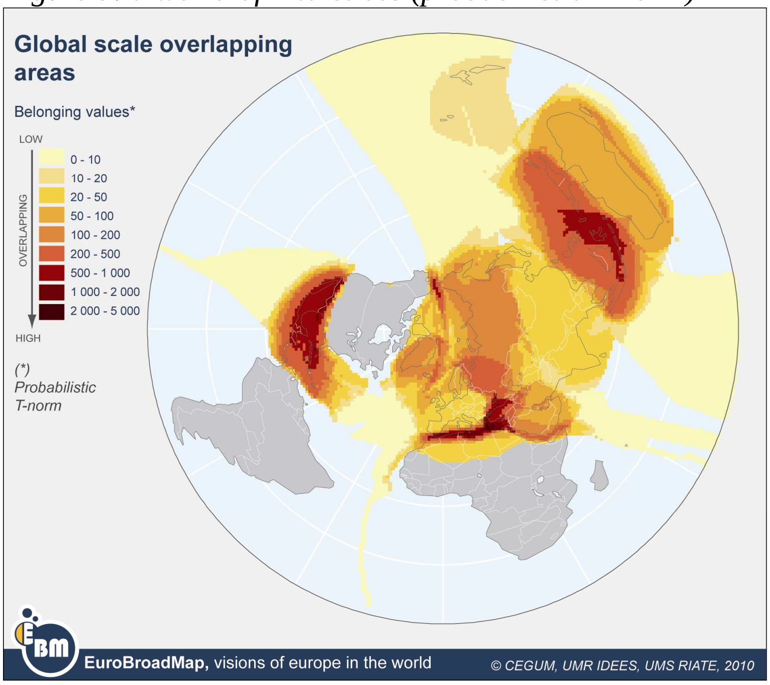


Figure 9: a world of interstices (probabilistic T-norm)



This approach first stresses the existence of three world regions of strong membership values, even if their borders are fuzzy: South and North America and Sub-Saharan Africa. All other places in the world are characterised by the possibility (at least) of double membership. The cartography of overlaps shows a smooth decrease in membership level (Mexico-USA border to the south) and strong discontinuities (Mexico-USA border to the north): for the respondents the membership uncertainty between two regions varies according to place, but also according to direction. The vagueness of such regional limits cannot therefore be described by means of a linear relationship. These results show how much the definition of world regions by the respondents is blurred, even when the same names are used to describe them. This confirms that various knowledge and information sources are mobilised by respondents to draw regions and to delineate the otherness level of two regions. The results of this cartographic experiment should nevertheless be explored further to provide an explanation of this 'dissymmetry of vagueness' on each part of the border. Moreover, this preliminary analysis must be now extended to other places in the world. The results produced here nevertheless represent an important step in the process of realising that the simplistic regionalisation of the world learned at school or proposed by the media is not "fixed" in the public mind as our respondents tended to mobilise various information forms in their choice determinations and thus, perhaps, that rather more fuzzy representations of the world region could be experimented with and proposed to the general public.

Conclusions & perspectives

From a theoretical point of view the survey proves that the world scale is actually perceived as a subjective space: students seem to have a general knowledge of the world when deciding on how to divide it into specific regions. The preliminary results obtained from the use of the fuzzy logic approach in relation to world interpretative mental maps thus appear to be quite promising. It allows the identification of "hard" and "soft" continents (Grataloup, 2010) where membership values to one single zone are high while also localising and characterising overlapping and indeterminate areas. This methodological approach is quite interesting because it allows us to build a world level map of regional representations that stresses that regional borders are more or less blurred in the representations. It may also prove fruitful to explore the reasoning behind the construction of world regions and to compare them with the world region patterns that already exist. Perhaps of particular interest in this respect would be a comparison of the results produced here with other rather sharper divisions such as that made famous by Huntington. As noted by Montello (2003) the setting of such regional borders is in itself an exercice that implies vagueness; the approach forwarded here however allows us to identify variations in the vagueness level in space and will be further enriched to help us explore the factors that can explain this vagueness distribution. This method allows quite interesting perspectives to be developed from both a methodological and a theoretical point of view. It is important to continue this work by analysing the large number of world interpretative maps already in existence.

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