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#### **Financial fragility in emerging market countries : firm balance sheets and the productive structure**

**Yannick Kalantzis**

**Codes JEL :E44, F32, F34, F43, O41**

**Mots clés :balance of payments crises, financial fragility,  
foreign currency debt, borrowing constraint, multiple  
equilibria.**

# Financial fragility in emerging market countries: firm balance sheets and the productive structure

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## Abstract

We build an overlapping generation model to study financial fragility in a two-sector small open economy. Firms are subject to a borrowing constraint and there is a currency mismatch in the balance sheets of the non-tradable sector. As a consequence, at a given point in time, multiple equilibria may arise, which makes self-fulfilling balance of payments crises possible. This state of financial fragility requires that firms producing non-tradable goods are sufficiently leveraged and that the relative size of the non-tradable sector is sufficiently large with regards to the tradable sector.

We study under what conditions the endogenous evolution of these two structural factors, firm balance sheets and the productive structure, along an equilibrium path, eventually leads to a financially fragile state.

*Keywords:* balance of payments crises, financial fragility, foreign currency debt, borrowing constraint, multiple equilibria.

*JEL Classification Numbers:* E44, F32, F34, F43, O41

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The opening of developing economies to international finance in the last three decades has led in a number of cases to balance of payments crises. The Southern Cone crises at the beginning of the eighties, the Mexican crisis of 1994, the Asian crises of 1997 and the Argentine crisis of 2001 all took place after the capital account had been liberalized. The suddenness of these crisis episodes, the fact that some of them were triggered by contagion phenomena and the absence of apparent changes in fundamentals before them have led many analysts to conclude that they were of a self-fulfilling nature.

Models with multiple equilibria seem to be the proper analytical tool to address the issue. In these models, the triggering of a crisis is an exogenous and arbitrary event, often represented by a change in the value of a *sunspot* variable. However, if the triggering of a crisis may be independent of fundamentals, the *possibility* of a crisis is not: fundamentals have not disappeared, they have just stepped backward from financial crises to financial fragility. Multiple equilibria models can thus be used to identify the structural characteristics that make crises possible.

A first hint was given by Krugman (1999): in a static model of a small open economy, he showed that self-fulfilling crises are possible when the foreign currency external debt is large enough relative to exports. But these possible structural characteristics are endogenous and evolve over time, especially after an economy has opened its capital account. Therefore, a dynamic framework is needed to give a full account of financial fragility in a developing economy opened to international finance.

Such a framework has first been developed by Schneider & Tornell (2004) in the case of boom-bust cycles. They model a dynamic economy with a finite number of periods where a credit boom is induced by an expected future increase in the demand for non-tradable goods. If the boom is large enough, it can make self-fulfilling crises possible. Their crisis mechanism relies on borrowing constraints and currency mismatches.

In this paper, we follow the same strategy and extend their framework in several directions:

- We build an overlapping generation model with an *infinite* number of periods and study financial fragility in steady state equilibrium paths. We are thus able to assess the long-run effect of capital account liberalization, independently of boom-bust cycles induced by transitory shocks.<sup>1</sup>
- We explicitly introduce two input-producing sectors: tradable and non-tradable, which we model in an entirely symmetric way. Any difference between the two sectors is an endogenous outcome of the model. The relative size of these two sectors plays a key role in our analysis.
- We use weaker technological assumptions: production functions are concave and the desired level of investment is a well-defined function. As a result, borrowing constraints need not bind in the equilibrium.

We show that a within-period sunspot equilibrium is possible when the debt repayments of firms producing non-tradable goods are high enough relative to their cash-flow and/or the sector producing non-tradable goods is large enough relative to the sector producing tradable goods. Financial fragility thus depends on *both financial and real* factors: the firm-level financial structure within the non-tradable sector and the cross-sectorial productive structure. Both of these characteristics evolve along equilibrium paths. The dynamic part of the model allows us to determine whether their evolution inevitably leads to a situation of financial fragility. We find that this is the case for sufficiently low levels of international interest rates, a sufficient degree of financial openness and sufficiently high growth rates.

The precise mechanism underlying the existence of multiple equilibria involves a self-reinforcing link between the real exchange rate and the level of investment expenditures. First, because of moral hazard and imperfect monitoring of loans, firms are subject to a borrowing constraint. The amount they

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<sup>1</sup>Based on empirical evidence, Kaminsky & Schmukler (2003) argue that the large amplitude of boom-bust cycles in the stock market following financial liberalization might be a transitory phenomenon and disappear in the long run.

are able to borrow is limited by their cash-flows, which yields a financial accelerator effect. Second, the economy is subject to *original sin* and firms cannot contract debt in domestic currency, which generates a currency mismatch in the balance sheets of firms producing non-tradable goods. Together, these two market imperfections create a balance sheet effect in the non-tradable sector, whereby movements in the real exchange rate affect firm's balance sheets, their capacity to raise external funds and their level of investment. Third, investment partly consists of expenditures in non-tradable goods so that an increase in investment provokes a real appreciation. Thus, a real appreciation increases the cash-flow of non-tradable firms and loosens their borrowing constraint so that they can invest more; the higher level of investment reinforces the real appreciation until the borrowing constraint does not bind any more. On the contrary, a real depreciation has a negative impact on their balance sheets, which limits the investment expenditures they can finance and further depreciates the real exchange rate until the non-tradable firms eventually default on their loans. To be possible, this reinforcing mechanism requires a sufficiently strong financial accelerator effect.

The stylized facts reproduced by the model are qualitatively consistent with the empirical evidence that has been documented in several papers. Our model predicts that an increase in the relative size of the non-tradable sector with regards to the tradable sector is one of the key factors inducing financial fragility. Tornell & Westermann (2002) indeed observe an asymmetrical evolution of the two sectors in an event study of twin crises: the relative size of the non-tradable sector increases before the crises and decreases after. According to our model, an increase in this relative size should go along with a real appreciation. A real appreciation is indeed observed in the phase of high growth preceding a crisis (Kaminsky & Reinhart 1999, Gourinchas, Valdés & Landerretche 2001, Tornell & Westermann 2002). The model also emphasizes the central role played by investment in the crisis mechanism. This is consistent with the fact that investment is the component of GDP displaying the largest variability in the crisis episodes, growing rapidly be-

fore and falling abruptly during the crisis (Gourinchas et al. 2001, Tornell & Westermann 2002). At last, the crises are often sudden, a fact accounted for by our multiple equilibria story.

There are other papers trying to model the instability of emerging market countries. Jeanne & Zettelmeyer (2002) propose a simple and unified framework that encompasses both Krugman's credit crunch model and other balance sheet approaches based on maturity mismatches and bank runs.

Rancière, Tornell & Westermann (2003) develop a growth model where self-fulfilling crises are possible in the long run. This work differs from ours in the modeling details. In their setting, the tradable sector has a standard production function with constant returns to scale and no borrowing constraint whereas the non-tradable sector has increasing returns and is subject to a borrowing constraint.<sup>2</sup> In our work, both sectors are treated symmetrically and any difference between them endogenously arise as a result of the model. Also, the crisis mechanism they use is different from ours.

Aghion, Bacchetta & Banerjee (2004*a*) construct a dynamic monetary model with multiple equilibria where a self-fulfilling crisis is possible in the first period if the subsequent productivity is sufficiently large. Therefore, their paper do not address the issue of long-run financial fragility. In another paper (2004*b*), the same authors develop a model of financial instability where endogenous cycles arise in the long run because of a balance sheet effect. They have the same perspective as the one we follow in this paper, in that they seek to determine the structural conditions of this cyclical instability but there are no crisis in their model and they do not discuss the effect of changes in the sectorial structure.

In previous works (Kalantzis 2004, 2005), we have constructed a dynamic two-sector growth model where self-fulfilling crises could occur in the long-

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<sup>2</sup>Contrary to the assumption made by these authors, it seems to us more natural, if one wants to treat the two sectors in an asymmetrical way, to assume increasing returns in the tradable sector alone. Increasing returns are indeed more likely to be observed in industrial sectors than in construction and services. See van Wijnbergen (1984) for an two-sector open-economy model with learning-by-doing in the tradable sector.

run. The dynamic part of that model relied on increasing returns and the crisis mechanism involved a myopic investment behavior of entrepreneurs whereas the present model has concave production functions and rational expectations.

Our work is also related to financial accelerator models in closed economies and the literature on borrowing constraints (Bernanke & Gertler 1989, Kiyotaki & Moore 1997, Holmstrom & Tirole 1996, Holmstrom & Tirole 1998), as well as to the literature on *original sin* (Eichengreen & Hausmann 1999, Eichengreen, Hausmann & Panizza 2003c).

The paper is organized as follows. We present the model in section 1. In section 2, we solve the within-period equilibrium and show that a sunspot equilibrium may exist. In section 3, we study the steady state of the dynamics and show to what condition it displays financial fragility. We calibrate the model in section 4 with data from Argentina in the nineties. In section 5, we examine the effects of different unexpected shocks and relate them with the condition of long-run financial fragility. Section 6 concludes.

## 1 The model

We consider a small open economy with overlapping generations. There are five kinds of agents: households, firms producing a final good for consumption, entrepreneurs producing tradable or non-tradable inputs and deep-pocket external investors. Time is discrete. Agents live two periods. There is one source of uncertainty: at each period, a sunspot variable  $S_t$  takes the value 1 with probability  $\omega$  and 0 with probability  $1 - \omega$  ( $\omega < 1$ ). When there are multiple equilibria,  $S_t = 0$  will correspond to crisis times.

### Financial openness

There is an international financial market with deep-pocket and risk-neutral external investors where one-period bonds are traded with an international

riskless interest rate  $R^*$  (in terms of tradable goods). Due to the small economy assumption,  $R^*$  is exogenous.

We assume that all agents in the economy have access to this market. However, there is an iceberg cost  $\tau \geq 0$  to international financial transactions. When an international investor lends  $1 + \tau$  units of tradable goods to a domestic agent, the domestic agent only gets 1 unit, and *vice versa*. As international investors are risk-neutral, the international riskless rate faced by a domestic agent *borrowing* abroad is  $R^D = (1 + \tau)R^*$ . Likewise, the international rate faced by a domestic agent *lending* abroad is  $R^*/(1 + \tau)$ .

$\tau$  is a measure of financial openness. The case  $\tau = 0$  corresponds to an economy entirely opened to international finance.

### The households

Households live two periods. They are endowed with one unit of labor in their first period of life and they consume final goods.  $L_t$  is the number of households born at time  $t$ . We assume a constant population  $L_t = L$ .

Denote  $c_t^y$  the consumption level of a young household born at time  $t$  and  $c_{t+1}^o$  the consumption level of the same household at time  $t + 1$ . Preferences are logarithmic:

$$U = \log(c_t^y) + \beta E_t[\log(c_{t+1}^o)] \quad (1)$$

where  $\beta \in ]0, 1[$  is a discount factor and  $E_t[\cdot]$  denotes the expected value at time  $t$ .

To transfer consumption from their first to their second period of life, households can buy three different kinds of assets: they can buy bonds on the international market or they can lend funds to the domestic entrepreneurs of either the tradable or the non-tradable sector. They choose the total amount of savings and the composition of their portfolio to maximize intertemporal utility subject to their budget constraint. We will solve this maximization program later, with the help of a simplifying assumption.

### The final good sector

At time  $t$ , the final good  $C_t$  is produced by a competitive sector using labor  $L_t = L$  and two types of intermediate goods: tradable inputs  $T_t^C$  and non-tradable inputs  $N_t^C$ . The production function is a Cobb-Douglas function with constant returns to scale:

$$C_t = [(N_t^C)^\mu (T_t^C)^{1-\mu}]^\alpha L^{1-\alpha} \quad (2)$$

with  $\alpha, \mu \in ]0, 1[$ . Profit maximization by firms give the usual first order conditions. Respectively denoting  $p_t$ ,  $p_t^C$  and  $w_t$  the price of the non-tradable input, the price of the consumption good and the wage, all of them in terms of tradable goods, one gets:

$$(1 - \alpha)p_t^C C_t = w_t L \quad (3)$$

$$\alpha \mu p_t^C C_t = p_t N_t^C \quad (4)$$

$$\alpha(1 - \mu)p_t^C C_t = T_t^C \quad (5)$$

Remark that  $p$  is a measure of the real exchange rate (a high value of  $p$  corresponds to an appreciated real exchange rate).

### The intermediate sectors

There are two kinds of intermediate goods: tradable and non-tradable. The tradable input is produced by the tradable sector (sector T) but can also be imported. Likewise, any excess production of this sector can be exported. We assume that the international demand for tradable inputs is infinitely elastic. The non-tradable input is exclusively produced by a domestic non-tradable sector (sector N) and the whole production has to be used domestically.

Each sector is composed by a continuum of firms of measure one. Each firm produces an intermediate good using the two kinds of inputs as capital. A firm of the sector N produces at time  $t + 1$  a quantity  $N_{t+1}$  of non-tradable intermediate goods using a capital of  $K_t^N$  tradable goods and  $J_t^N$

non-tradable goods.<sup>3</sup> The production function is a Cobb-Douglas function with diminishing returns:

$$N_{t+1} = A_{t+1}^N \left[ \left( \frac{K_t^N}{1-\eta} \right)^{1-\eta} \left( \frac{J_t^N}{\eta} \right)^\eta \right]^\delta \quad (6)$$

with  $\eta, \delta \in ]0, 1[$ . Both types of capital are fully depreciated from one period to the next.

Likewise, the production function for a firm of the sector T is:

$$T_{t+1} = A_{t+1}^T \left[ \left( \frac{K_t^T}{1-\eta} \right)^{1-\eta} \left( \frac{J_t^T}{\eta} \right)^\eta \right]^\delta \quad (7)$$

There is an exogenous and homogenous growth trend in the productivity of both sectors:

$$A_t^i = A_0^i [(1+g)^{1-\delta}]^t \quad i = N, T \quad (8)$$

In the following,  $g$  will be the steady-state growth rate of the economy.

Each firm in the intermediate sectors is run by successive generations of risk-neutral entrepreneurs. We assume that they do not consume during their first period of life and that they spend some fixed fraction  $\gamma \in ]0, 1[$  of their profits to consume final goods in their second period of life. Thus, an entrepreneur of the sector  $i$  born at time  $t$  invests her internal funds  $W_t^i$  in the first period of life, gets the return  $\Pi_{t+1}^i$  in the second period of life, keeps a fraction  $\gamma$  for her own consumption and gives the remaining proceeds  $W_{t+1}^i = (1-\gamma)\Pi_{t+1}^i$  to her successor. Moreover, at time  $t$ , a young entrepreneur can borrow some additional external funds to domestic households or international lenders and pay them back in the following period.

### The market for corporate debt

We now introduce two market imperfections. First, we assume that the economy is subject to the so-called *original sin*: there is no market for debt

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<sup>3</sup> $K_t^N$  mainly consists of machinery, transportation, ...  $J_t^N$  represents buildings but also all possible non-tradable goods and services necessary to the installation of tradable capital.

denominated in non-tradable goods.<sup>4</sup> Therefore, the entrepreneurs can only issue debt denominated in tradable goods. In particular, entrepreneurs producing non-tradable goods cannot insure against real exchange rate risk.<sup>5</sup> Second, because of moral hazard and imperfect monitoring, an entrepreneur might be subject to a borrowing constraint. We follow Schneider & Tornell (2004) and Aghion, Banerjee & Piketty (1999) for the detailed microfoundation of this borrowing constraint.

Let  $\frac{B_{t+1}^i}{R_t^i}$  denote the external funds lent to an entrepreneur of the sector  $i$  at time  $t$ . The entrepreneur has to repay at time  $t + 1$  an amount  $B_{t+1}^i$  in tradable goods.  $R_t^i$  may include a risk premium due to possible defaults. As in Schneider & Tornell (2004), the entrepreneur has the possibility, at time  $t + 1$ , if the firm is solvent (cf *infra*), to run away with the production without repaying its debt  $B_{t+1}^i$ . This, however, requires some special effort and costs her a disutility  $d \left[ W_t^i + \frac{B_{t+1}^i}{R_t^i} \right]$  proportional to the total funds of the firm. If she chooses to do this, the lender can try to find her and force her to repay her debt. He can choose the probability of success  $m$ , which can be thought of as the intensity of monitoring. But, as in Aghion et al. (1999), monitoring also requires some effort and cost him a disutility  $C(m) \frac{B_{t+1}^i}{R_t^i}$  proportional to the size of the loan,  $C(\cdot)$  being an increasing function. Therefore, if the entrepreneur has disappeared at the beginning of the period  $t + 1$ , the lender

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<sup>4</sup>See Eichengreen et al. (2003c) for an empirical investigation on the relevance of this concept. According to Eichengreen, Hausmann & Panizza (2003b), original sin might be the result of transaction costs in international finance which set a finite number of currencies in the world's portfolio: the cost to detain the marginal currency should compensate the benefit derived from risk diversification. As large countries offer more diversification than small ones, they argue that one should expect the currencies of large countries to be dominant in international portfolios.

<sup>5</sup>Several authors have proposed arguments to explain why domestic firms *choose* to take a risky position by issuing debt denominated in foreign currency: moral hazard induced by expected bail-outs (Schneider & Tornell 2004), borrowing constraints in the domestic financial system (Caballero & Krishnamurthy 2000), commitment problems (Jeanne 2000) or the lack of credibility of the domestic monetary policy (Jeanne 2003). In this paper, we consider that firms *could not* issue domestic currency debt, even if they wanted to. For a more detailed discussion on the difference between original sin and currency mismatches, refer to Eichengreen, Hausmann & Panizza (2003a).

will choose the intensity of monitoring  $m_{t+1}$  to maximize his expected utility  $m_{t+1}B_{t+1}^i - C(m_{t+1})\frac{B_{t+1}^i}{R_t^i}$ . The solution  $m_{t+1}^*$  of this maximization program is given by the first order condition:

$$C'(m_{t+1}^*) = R_t^i$$

This is anticipated by the entrepreneur who decides not to run away if the disutility from running away is higher than the debt repayment:

$$d\left(W_t^i + \frac{B_{t+1}^i}{R_t^i}\right) + m_{t+1}^*B_{t+1}^i \geq B_{t+1}^i$$

This is in turn anticipated by the lender at period  $t$ . As he prefers to be fully repaid, he lends the greatest possible amount satisfying the previous inequality. This sets an upper limit on borrowing, depending of the entrepreneur's internal funds:

$$B_{t+1}^i \leq \frac{d}{1 - m_{t+1}^* - \frac{d}{R_t^i}}W_t^i \quad (9)$$

Following Aghion et al. (1999), we now choose a particular functional form for the monitoring cost:  $C(m) = -c \log(1 - m)$ . We also make the following assumption to ensure the existence of a borrowing constraint:

**Assumption 1 (borrowing constraint).**

$$d < c$$

Then,  $m_{t+1}^* = 1 - \frac{c}{R_t^i}$  and equation (9) can be simplified to:<sup>6</sup>

$$\frac{B_{t+1}^i}{R_t^i} \leq \frac{W_t^i}{\frac{c}{d} - 1} \quad (10)$$

$\frac{d}{c}$  can be thought of as an indicator of the level of financial development of the economy. A high value of this ratio implies that it is difficult for the entrepreneurs to cheat and easy for the lenders to monitor their loans.

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<sup>6</sup>The functional form we have chosen for  $C(m)$  gives a credit multiplier independent of the interest rate. This is a special case whose only purpose is analytical tractability.

## Defaults

If the firm is not solvent, the entrepreneur cannot repay the debt in her second period of life: she defaults and does not get anything for her own consumption ( $\Pi_t^i = 0$ ). We assume that the entire production is then wasted as a bankruptcy cost, so that the lenders do not get anything either.<sup>7</sup> The next entrepreneur does not get anything from the incumbent one. We assume that she gets some exogenous endowment  $Z_t$  in tradable goods and has no access to the financial market. This conditional endowment grows at the rate  $g$ :  $Z_{t+1} = (1 + g)Z_t$ .

Entrepreneurs of the sector T know their future profits with certainty and invest at most up to the point where the marginal productivity of investment is equal to the interest rate. Therefore, they will always have positive profits and never default. On the contrary, firms of the sector N produce non-tradable goods and are indebted in tradable goods, so that they default when the real exchange rate is sufficiently depreciated. The proceeds from their sales at time  $t$  is strictly lower than their debt repayment when  $p_t < p_t^D$ , with:

$$p_t^D = \frac{B_t^N}{N_t} \quad (11)$$

We can now write the profits earned by entrepreneurs of both sectors

$$\Pi_t^T = T_t - B_t^T \quad (12)$$

$$\Pi_t^N = \begin{cases} p_t N_t - B_t^N & \text{if } p_t \geq p_t^D \\ 0 & \text{if } p_t < p_t^D \end{cases} \quad (13)$$

and the internal funds of entrepreneurs:

$$W_t^T = (1 - \gamma)\Pi_t^T \quad (14)$$

$$W_t^N = \begin{cases} (1 - \gamma)\Pi_t^N & \text{if } p_t \geq p_t^D \\ Z_t & \text{if } p_t < p_t^D \end{cases} \quad (15)$$

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<sup>7</sup>This assumption is not necessary to our results but yields a simple expression for the risk premium.

We will later construct a sunspot equilibrium where  $N$  firms default when the sunspot variable takes the value  $S_t = 0$  and make positive profits when  $S_t = 1$ . Denote  $F_t$  a dichotomic variable equal to 0 when there can never be any default, whatever the value taken by the sunspot, and to 1 when  $S_t = 0$  provokes defaults in the non-tradable sector.  $F_t$  is an indicator of financial fragility. The probability that an entrepreneur producing non-tradable inputs gets a positive return and repays its debt at time  $t$  is equal to:

$$\rho_t = 1 - (1 - \omega)F_t \quad (16)$$

The probability of default in the non-tradable sector is of course  $1 - \rho_t = (1 - \omega)F_t$ .

### The investment decision

An entrepreneur of the sector  $N$  who inherited internal funds  $W_t^N = (1 - \gamma)\Pi_t^N$  decides at time  $t$  how much to borrow and invest by maximizing the profits of the next period. As she will not get anything if the firm defaults at time  $t + 1$ , she solves the following maximization program in the case there is no default at time  $t + 1$ :

$$\begin{aligned} \max_{K_t^N, J_t^N, I_t^N, B_{t+1}^N} \quad & E_t[p_{t+1}|p_{t+1} \geq p_{t+1}^D] N_{t+1} - B_{t+1}^N \quad (17) \\ \text{s. t.} \quad & N_{t+1} = A_{t+1}^N \left[ \left( \frac{K_t^N}{1-\eta} \right)^{1-\eta} \left( \frac{J_t^N}{\eta} \right)^\eta \right]^\delta \\ & I_t^N = K_t^N + p_t J_t^N \\ & I_t^N = \frac{B_{t+1}^N}{R_t^N} + W_t^N \\ & \frac{B_{t+1}^N}{R_t^N} \leq \frac{W_t^i}{\frac{\varepsilon}{d}-1} \end{aligned}$$

$E_t[p_{t+1}|p_{t+1} \geq p_{t+1}^D]$  is the expectation at time  $t$  of  $p_{t+1}$  supposing that there is no default at time  $t + 1$ .  $I_t^N$  is the investment expenditure at time  $t$ . The four constraints are the production function, the definition of the investment expenditure, the budget constraint and the borrowing constraint. Of course,  $B_{t+1}^N$  could be negative, in which case  $R_t^N$  would be the rate of return on the

internal funds not invested in production, but we will only study situations where  $B_{t+1}^N \geq 0$  (see assumption 4 *infra*).

Two cases are possible. If the borrowing constraint does not bind, one has  $I_t^N = \bar{I}_t^N$ , with

$$\bar{I}_t^N = \left( \frac{\delta E_t [p_{t+1} | p_{t+1} \geq p_{t+1}^D] A_{t+1}^N}{p_t^{\eta\delta} R_t^N} \right)^{\frac{1}{1-\delta}} \quad (18)$$

If, on the contrary, the borrowing constraint does bind, the investment expenditure is limited by the internal funds:

$$I_t^N = \frac{1}{1 - \frac{d}{c}} W_t^N \quad (19)$$

This is a financial accelerator effect. Denote  $\lambda = \frac{1}{1 - \frac{d}{c}}$  the intensity of this financial accelerator. It increases with the level of financial development  $\frac{d}{c}$ . From assumption 1, it is strictly greater than 1. Summing up both cases, one ends up with:

$$I_t^N = \min(\bar{I}_t^N, \lambda W_t^N) \quad (20)$$

Denote  $p_t^B$  the value of the relative price  $p_t$  for which  $\bar{I}_t^N = \lambda W_t^N$ . The borrowing constraint binds for  $p_t < p_t^B$ , *i.e.* when the real exchange rate is sufficiently depreciated.

In both cases, one gets:

$$K_t^N = (1 - \eta) I_t^N \quad (21)$$

$$p_t J_t^N = \eta I_t^N \quad (22)$$

$$N_{t+1} = p_t^{-\eta\delta} A_{t+1}^N (I_t^N)^\delta \quad (23)$$

$$\frac{B_{t+1}^N}{R_t^N} = I_t^N - W_t^N \quad (24)$$

If the incumbent manager has defaulted, the young entrepreneurs start with the exogenous endowment  $Z_t$  and has no access to financial markets. In that case, equation (20) has to be replaced by:  $I_t^N = \min(\bar{I}_t^N, Z_t)$ . We suppose that  $Z_t$  is low enough so that, on the whole, the investment expenditure

in the sector N is given by:

$$I_t^N = \begin{cases} \bar{I}_t^N & \text{if } p_t \geq p_t^B \\ \lambda W_t^N & \text{if } p_t^D \leq p_t < p_t^B \\ Z_t & \text{if } p_t < p_t^D \end{cases} \quad (25)$$

The entrepreneur of the tradable sector faces a similar problem, except that the return on investment in tradable goods is certain, as both the debt repayment and the sales are tradable goods:

$$\bar{I}_t^T = \left( \frac{\delta A_{t+1}^T}{p_t^{\eta\delta} R_t^T} \right)^{\frac{1}{1-\delta}} \quad (26)$$

$$I_t^T = \min(\bar{I}_t^T, \lambda W_t^T) \quad (27)$$

$$K_t^T = (1 - \eta) I_t^T \quad (28)$$

$$p_t J_t^T = \eta I_t^T \quad (29)$$

$$T_{t+1} = p_t^{-\eta\delta} A_{t+1}^T (I_t^T)^\delta \quad (30)$$

$$\frac{B_{t+1}^T}{R_t^T} = I_t^T - W_t^T \quad (31)$$

### The “emerging market country” assumption

An emerging market country is a country where international capital is flowing into, after the capital account is liberalized. In technical terms, domestic savings are lower than domestic investment in an emerging country or, equivalently, the autarky interest rate (the rate of interest that would equalize supply and demand of loanable funds if the economy were closed to international finance) is higher than the international interest rate.

For analytical convenience, we make here a slightly more restrictive assumption: we assume that, at any period  $t$ , total savings from households  $\Sigma_t$  are lower than the demand for loanable funds from the tradable sector alone  $\frac{B_{t+1}^T}{R_t^T}$ .

This assumption allows us to determine the composition of the representative household’s portfolio and the equilibrium interest rates. First, even

if the household only invests in the tradable sector, the entrepreneurs from this sector will have to borrow some of their external funds abroad. Therefore, in any case, the interest rate  $R_t^T$  is set by the international risk-neutral investors:

$$R_t^T = (1 + \tau)R^* = R^D \quad (32)$$

As a domestic bond issued by an entrepreneur of the sector T gives a higher return to the household than a bond bought on the international market ( $(1 + \tau)R^* \geq R^*/(1 + \tau)$ ), the household portfolio will be exclusively composed of domestic assets.

Second, as the household is risk-averse, it will not hold a risky bond issued by the non-tradable sector unless there is a strictly positive risk premium over the interest rate charged by risk-neutral investors. Therefore, when the probability of a default in the next period is expected to be strictly positive, entrepreneurs from the non-tradable sector will borrow all their external funds abroad and the household's portfolio will only consist of bonds issued by the tradable sector. The interest rate charged to the non-tradable sector can then be deduced from the risk-neutrality of international investors:

$$R_t^N = (1 + \tau) \frac{R^*}{E_t[\rho_{t+1}]} = \frac{R^D}{E_t[\rho_{t+1}]} \quad (33)$$

We can now go back to the household's maximization program. A household born at time  $t$  saves by lending funds to the tradable sector (and to the non-tradable sector if  $E_t[\rho_{t+1}] = 1$ ) with a riskless rate of return  $R^D$  in terms of tradable goods. Note however that, because of the uncertainty on the future consumption price, the return in terms of consumption goods is also uncertain.

The Euler equation derived from the maximization program is:

$$\frac{1}{c_t^y} = \beta E_t \left[ \frac{R^D p_t^C}{c_{t+1}^o p_{t+1}^C} \right] \quad (34)$$

By using the budget constraint  $p_{t+1}^C c_{t+1}^o = R^D (w_t - p_t^C c_t^y)$ , it can be simplified to:

$$\frac{1}{c_t^y} = \frac{\beta p_t^C}{w_t - p_t^C c_t^y}$$

which yields the usual saving equation:

$$\frac{\Sigma_t}{L} = w_t - p_t^C c_t^y = \frac{\beta}{1 + \beta} w_t \quad (35)$$

$s = \frac{\beta}{1 + \beta}$  is the saving rate of young households.

We can now give a more precise formulation of our “emerging market country” assumption:

**Assumption 2 (emerging market country).** *The discount factor  $\beta$  is sufficiently small so that, at any time period  $t$ , one has:*

$$\Sigma_t \leq \frac{B_{t+1}^T}{R^D}$$

## 2 Within-period equilibrium

In this section, we take the past variable  $w_{t-1}$ , the predetermined variables  $N_t$ ,  $T_t$ ,  $B_t^N$ ,  $B_t^T$  and the expected variables  $E_t[p_{t+1}|p_{t+1} \geq p_{t+1}^D]$  and  $R_t^N = (1 + \tau) \frac{R^*}{E_t[\rho_{t+1}]}$  as given and determine the temporary equilibrium at time  $t$ .

The demand for non-tradable inputs stems from both the final sector and the investment expenditures from the intermediate sectors:  $N_t = N_t^C + J_t^N + J_t^T$ . From equations (4), (22) and (29), one gets:

$$p_t N_t = \alpha \mu p_t^C C_t + \eta (I_t^T + I_t^N) \quad (36)$$

The demand for final goods itself stems from young and old workers and old entrepreneurs of both sectors:

$$p_t^C C_t = (1 - s) w_t L + R^D s w_{t-1} L + \gamma (\Pi_t^T + \Pi_t^N)$$

Using equation (3), we can now write the reduced equation of the non-tradable market equilibrium:

$$p_t N_t = \frac{\mu}{1 + s \frac{1-\alpha}{\alpha}} [\gamma (\Pi_t^T + \Pi_t^N) + R^D \Sigma_{t-1}] + \eta (I_t^T + I_t^N) \quad (37)$$

where  $\Sigma_{t-1} = s w_{t-1} L$ .

Together with equations (12), (13) and (27), this equation yields a first relationship between the investment expenditure  $I_t^N$  and the real exchange rate  $p_t$ , represented by the NN schedule on figure 1. Denote  $I_t^N = f_{NN}(p_t)$  the equation of this first schedule. A second relationship between those two variables comes from the investment behavior of the N firms described by equation (25) and represented by the II schedule on figure 1. Denote  $I_t^N = f_{II}(p_t)$  the equation of this second schedule.

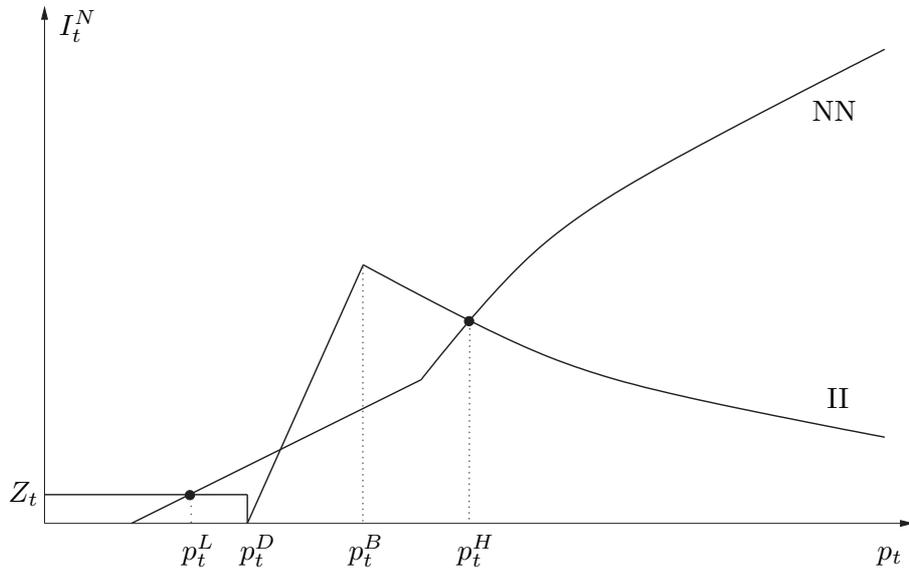


Figure 1: Within-period multiple equilibria

To compute the within-period equilibrium, we only have to determine the couple  $(p_t, I_t^N)$  as an intersection of the NN and II schedules. Once  $(p_t, I_t^N)$  is known, the within-period equilibrium values of all the other variables are easily determined :  $I_t^T$  is given by equation (27); equation (36) gives the total value  $p_t^C C_t$  of produced final goods, which, plugged into equation (3), yields the wage  $w_t$ . Equations (2), (3), (4) and (5) implicitly determine the price of the final good  $p_t^C$  as a function of  $p_t$  and  $w_t$ , from which the quantity  $C_t$  of final goods can be deduced.

We now focus on the determination of  $p_t$  and  $I_t^N$ . The II schedule is composed of three distinctive parts (see figure 1). For  $p_t < p_t^D$ , the N firms default. The new cohort of managers starts with the exogenous endowment

$Z_t$  and has no access to the financial market: therefore,  $I_t^N = Z_t$  on that interval. For  $p_t^D \leq p_t < p_t^B$ , the N firms have insufficient internal funds and face a borrowing constraint.  $I_t^N$  is linearly increasing with  $p_t$  on that interval. For  $p_t \geq p_t^B$ , the internal funds of N firms are sufficiently high so that the borrowing constraint does not bind. They borrow less than the maximum amount possible and invest the optimal quantity  $\bar{I}_t^N$ .  $I_t^N$  is decreasing with  $p_t$  on that interval.

As it can be seen from figure 1, it is possible that the II and the NN schedules intersect three times, with one intersection on each of these three intervals, thus yielding multiple equilibria. The equilibrium located in the interval  $[p_t^D, p_t^B]$  is then unstable (in the sense of implicit virtual out-of-equilibrium dynamics corresponding to the walrasian auctioneer's *tatonnement*) and we are left with two stable equilibria:

- a “good” equilibrium corresponding to tranquil times with an appreciated real exchange rate  $p_t^H$ , where the N firms have high internal funds and are not constrained,
- a “bad” equilibrium corresponding to crisis times with a depreciated real exchange rate  $p_t^L$ , where the N firms default on their loans.

This framework allows us to construct a crisis event as a transition from the good equilibrium  $p_t^H$  to the bad equilibrium  $p_{t+1}^L$ . The crisis manifests itself by a real depreciation, widespread defaults on external debt in the non-tradable sector and losses from external lenders. In this sense, it is a balance of payments crisis.

For this to happen, the slope of the II schedule has to be steeper than the slope of the NN schedule at their point of intersection on the interval  $[p_t^D, p_t^B]$ . A necessary condition for this is that:

$$\frac{\mu\gamma}{1 + s\frac{1-\alpha}{\alpha}} + \eta(1 - \gamma)\lambda > 1 \quad (38)$$

Therefore, the intensity of the financial accelerator effect  $\lambda$  has to be large enough for crises to be possible. As this coefficient increases with the level

of financial development, this means that the domestic financial system has to be sufficiently developed. The kind of crisis we are describing would not happen in an economy subject to financial repression.<sup>8</sup>

When there are multiple equilibria, we can construct a sunspot equilibrium where the agents use the sunspot variable  $S_t$  as a coordination device to choose between the “good” and the “bad” equilibrium. We say the economy is financially fragile when such a sunspot equilibrium exists, *i.e.* when both a good equilibrium with unbinding borrowing constraints and a bad equilibrium with defaults in the sector N exist at the same time. We suppose that this sunspot equilibrium is actually implemented whenever it is possible. The variable  $F_t$  defined above is equal to 1 when the economy is financially fragile and to 0 when it is not. The international lenders take into account the possibility of crisis by anticipating one period in advance the value of the variable  $F_{t+1}$  and incorporating it in the interest rate  $R_t^N$  (see equation (33)).

This short discussion is formalized by the following proposition:

**Proposition 1.** *When  $\frac{\mu\gamma}{1+s\frac{1-\alpha}{\alpha}} + \eta(1-\gamma)\lambda \leq 1$ , there cannot be at the same time an equilibrium characterized by  $p_t = p_t^L < p_t^D$  and an equilibrium with  $p_t \geq p_t^D$ .*

*When  $\frac{\mu}{1+s\frac{1-\alpha}{\alpha}}\gamma + \eta(1-\gamma)\lambda > 1$ , if  $f_{NN}(p_t^D) > Z_t$  and  $(f_{NN} - f_{II})(p_t^B) < 0$ , there is a sunspot equilibrium where  $p_t = p_t^H > p_t^B$  with probability  $\omega$  (when  $S_t = 1$ ) and  $p_t = p_t^L < p_t^D$  with probability  $1 - \omega$  (when  $S_t = 0$ ).*

*Proof.* See Appendix A.1. □

When there is no financial fragility ( $F_t = 0$ ), there is no uncertainty in the sector N so that profits have to be positive in that sector. Therefore, the equilibrium real exchange rate has to satisfy  $p_t \geq p_t^D$ . We also denote  $p_t^H$  this equilibrium with no default.<sup>9</sup>

The crucial condition for a crisis to be possible is of course  $f_{NN}(p_t^D) > Z_t$ , which states the existence of the bad equilibrium. Using the fact that  $I_t^T \leq$

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<sup>8</sup>This is a usual result of the literature on balance sheets and financial crises. See for example Aghion, Bacchetta & Banerjee (2004b) and Schneider & Tornell (2004).

<sup>9</sup>Remark that if the inequality (38) is not satisfied, the borrowing constraint may bind for the sector N in this equilibrium.

$\lambda(1 - \gamma)\Pi_t^T$ , we derive a sufficient condition for this existence:

$$\frac{B_t^N}{\Pi_t^T} > \frac{\mu}{1 + s\frac{1-\alpha}{\alpha}} \left[ \gamma + \frac{R^D \Sigma_{t-1}}{\Pi_t^T} \right] + \eta(1 - \gamma)\lambda + \frac{\eta Z_t}{\Pi_t^T} \quad (39)$$

This condition states that the bad equilibrium exists whenever the debt repayment of the sector N is large enough compared to the profits of the sector T. The corresponding threshold is increasing with the intensity of the financial accelerator effect  $\lambda$  and the amount of past savings from households. In tranquil times, the ratio  $\frac{B_t^N}{\Pi_t^T}$  can be decomposed in the following way:

$$\frac{B_t^N}{\Pi_t^T} = \frac{B_t^N}{\Pi_t^N} \frac{W_t^N}{W_t^T}$$

The first ratio relates debt service to cash-flow and captures the financial structure of N firms' balance sheets. The second one describes the relative size of both sectors and is an indicator of the productive structure of the whole economy. Thus, highly leveraged N firms and/or a productive structure largely oriented toward the production of non tradable goods are conditions that favor the possibility of crises. Those two quantities, which are predetermined in the within-period equilibrium, endogenously evolve with the model dynamics.

### 3 Financial fragility in the long run

We now study the dynamics of the model. We define an equilibrium path as a succession of within-period equilibria with rational expectations:

$$E_t[p_{t+1} | p_{t+1} \geq p_{t+1}^D] = p_{t+1}^H \quad (40)$$

$$E_t[\rho_{t+1}] = \rho_{t+1} = 1 - (1 - \omega)F_{t+1} \quad (41)$$

Remark that the good equilibrium has to exist at every period along an equilibrium path.<sup>10</sup>

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<sup>10</sup>Suppose there is only a bad equilibrium at time  $t$  so that all N firms default. On an equilibrium path, entrepreneurs with rational expectations should have anticipated it at time  $t - 1$  and decided to put their internal funds in bonds issued by the sector T or in the

Our aim is to investigate whether such an equilibrium path may run through a crisis, *i.e.* whether there is a period  $t$  such that  $F_t = 1$  and  $S_t = 0$ . Our strategy is to study the steady state of a particular equilibrium path where  $S_t$  happened to be always equal to 1 and therefore  $p_t$  always equal to  $p_t^H$ . If this steady state is financially fragile, then every possible equilibrium path becomes financially fragile for a large enough  $t$  and the economy eventually runs through a crisis with probability 1. In doing this, we assess the possibility of crises *in the long run*, independently of any shock that might trigger transitory dynamics.

From now on, we suppose that the inequality (38) is satisfied so that our economy may be subject to financial fragility.

**Assumption 3 (necessary condition for financial fragility).**

$$\frac{\mu\gamma}{1 + s\frac{1-\alpha}{\alpha}} + \eta(1 - \gamma)\lambda > 1$$

Let us consider the steady state of a particular equilibrium path where  $S_t$  has not taken the value 0 yet, so that the economy has always been in the good equilibrium where the borrowing constraint does not bind for the sector N. In this steady state, all quantity variables grow at the rate  $g$ . Ratios of quantities are therefore constant, and so are prices and interest rates. We drop the time subscript of these constant steady-state variables.

### The non-tradable sector

Equation (18) can be rewritten:  $R_t^N \bar{I}_t^N = \delta p_{t+1}^H N_{t+1}$ . This, together with equations (14), (13) and (24), allows us to write:

$$\begin{aligned} \frac{I^N}{W^N} &= \frac{(1+g)I_t^N}{(1-\gamma)[p_{t+1}^H N_{t+1} - B_{t+1}^N]} = \frac{(1+g)I_t^N}{(1-\gamma)\left[\frac{R^N \bar{I}_t^N}{\delta} - R^N(\bar{I}_t^N - W_t^N)\right]} \\ &= \frac{1+g}{(1-\gamma)\left[R^N(1/\delta - 1) + R^N \frac{W^N}{I^N}\right]} \end{aligned}$$

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international financial market where they get the riskless rate  $R^*/(1+\tau)$ . Had they done so, there would not be any N good produced at time  $t$  and the price  $p_t$  would be infinite, which is not consistent with a default.

which can be reduced to

$$\frac{I^N}{W^N} = \frac{\delta}{1 - \delta}(\rho\phi - 1) \quad (42)$$

where  $\phi$  is an exogenous reduced variable defined by

$$\phi = \frac{1 + g}{R^*(1 + \tau)(1 - \gamma)} \quad (43)$$

$\phi$  increases with financial openness (*i.e.* decreases with  $\tau$ ), technological progress ( $g$ ), the amount of international liquidity (*i.e.* decreases with  $R^*$ ) and captures part of the generational structure (the parameter  $\gamma$ ).

Likewise, we can compute the steady-state debt-to-internal funds ratio:

$$\frac{B^N}{W^N} = \frac{1}{1 - \gamma} \frac{\delta}{1 - \delta} \left( 1 - \frac{1}{\delta\rho\phi} \right) \quad (44)$$

We make the following assumption to ensure that the debt of the sector N is strictly positive in the steady state:<sup>11</sup>

**Assumption 4 (positive steady-state debt).**

$$\phi > \frac{1}{\delta}$$

This assumption also ensures that the steady-state value of  $\frac{I^N}{W^N}$  is positive.

So far, we have computed these ratios by assuming that the good equilibrium exists in the steady-state. From proposition 1, however, this requires that  $(f_{NN} - f_{II})(p_t^B) < 0$ . This condition is satisfied in the steady state when the ratio  $\frac{I^N}{W^N}$  given by equation (42) is strictly lower than  $\lambda$ , so that the borrowing constraint does not bind for N firms in the computed equilibrium. Therefore, we have to assume that:

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<sup>11</sup> From equation (44), the debt of the sector N is positive when  $\phi \geq \frac{1}{\delta\rho}$ . Suppose  $\frac{1}{\delta} \leq \phi \leq \frac{1}{\delta\rho}$ . It is not possible to have  $\rho = \omega < 1$  in that case. Indeed, it would require on the one hand that  $F = 1$  and on the other hand imply a strictly negative steady state debt for the sector N, which would not be consistent with financial fragility and  $F = 1$ . Therefore,  $\frac{1}{\delta} \leq \phi \leq \frac{1}{\delta\rho} \Rightarrow \rho = 1$  and the condition for a strictly positive debt can be reduced to the one stated in assumption 4.

**Assumption 5 (existence of a good equilibrium in the steady state).**

$$\phi < \frac{1}{\rho} \frac{\delta + \lambda(1 - \delta)}{\delta}$$

As we shall see, the economy is financially fragile for a large enough  $\phi$ , so that  $\rho$  is a function of  $\phi$ . Therefore, this assumption implicitly restricts the possible values taken by  $\phi$ . Remark that assumptions 4 and 5 are consistent.

### The tradable sector

Let us now turn to the sector T. If the borrowing constraint does not bind for the sector T, the same calculations yield the following formulae:

$$\frac{I^T}{W^T} = \frac{\delta}{1 - \delta} (\phi - 1) \quad (45)$$

$$\frac{B^T}{W^T} = \frac{1}{1 - \gamma} \frac{\delta}{1 - \delta} \left(1 - \frac{1}{\delta\phi}\right) \quad (46)$$

From assumption 4, both quantities are strictly positive.

If, on the contrary, the borrowing constraint binds for the sector T, one has of course:

$$\frac{I^T}{W^T} = \lambda \quad (47)$$

$$\frac{B^T}{W^T} = \frac{1}{1 - \gamma} \frac{1}{\phi} (\lambda - 1) \quad (48)$$

By comparing equations (45) and (47), we can see that the borrowing constraint binds for T firms if  $\frac{\delta + \lambda(1 - \delta)}{\delta} < \phi$  ( $\leq \frac{1}{\rho} \frac{\delta + \lambda(1 - \delta)}{\delta}$ ). This is only possible when  $\rho < 1$ , *i.e.* when  $F = 1$  and there is financial fragility.

### The productive structure

By using equations (3), (36) and (37), we can reformulate the non-tradable market equilibrium in the steady state:

$$p_t N_t = \frac{\mu\gamma}{1 + s \frac{1-\alpha}{\alpha} \left[1 - \frac{1}{\phi(1-\gamma)}\right]} \frac{W_t^T + W_t^N}{1 - \gamma} + \eta (I_t^T + I_t^N)$$

and compute the steady-state sectorial ratio of internal funds:

$$\frac{W^N}{W^T} = \frac{\frac{\mu\gamma}{1 + s\frac{1-\alpha}{\alpha}\left[1 - \frac{1}{\phi(1-\gamma)}\right]} + \eta(1-\gamma)\frac{I^T}{W^T}}{\frac{1}{\delta\rho\phi}\frac{I^N}{W^N} - \frac{\mu\gamma}{1 + s\frac{1-\alpha}{\alpha}\left[1 - \frac{1}{\phi(1-\gamma)}\right]} - \eta(1-\gamma)\frac{I^N}{W^N}} \quad (49)$$

As we noticed earlier, this ratio characterizes the productive structure of the economy. Other interesting ratios can be easily derived from that one. For example:

$$\frac{I^N}{I^T} = \frac{I^N/W^N}{I^T/W^T} \frac{W^N}{W^T} \quad (50)$$

$$\frac{N}{T} = \frac{A^N}{A^T} \left(\frac{I^N}{I^T}\right)^\delta \quad (51)$$

$$\frac{\Sigma}{W^T} = \frac{s\frac{1-\alpha}{\alpha}}{1 + s\frac{1-\alpha}{\alpha}\left[1 - \frac{1}{\phi(1-\gamma)}\right]} \frac{\gamma}{1-\gamma} \left(1 + \frac{W^N}{W^T}\right) \quad (52)$$

When the borrowing constraint is not binding for the sector T, the steady-state value of the real exchange rate  $p^H$  is given by equations (18) and (26):

$$p^H = \frac{1}{\rho} \frac{A^T}{A^N} \left(\frac{I^N}{I^T}\right)^{1-\delta} \quad (53)$$

When it is binding,  $p^H$  can be deduced from equation (18) alone.

Remark that the steady state only exists if the ratio  $\frac{W^N}{W^T}$  is positive. This is the case whenever its denominator is positive. Denote  $Q_\rho(\phi)$  the denominator:

$$Q_\rho(\phi) = \frac{\rho\phi - 1}{\rho\phi(1-\delta)} - \frac{\mu\gamma}{1 + s\frac{1-\alpha}{\alpha}\left[1 - \frac{1}{\phi(1-\gamma)}\right]} - \frac{\eta(1-\gamma)\delta}{1-\delta}(\rho\phi - 1)$$

$Q_\rho(\phi)$  has either 1 or 3 zeros. As  $Q_\rho$  goes to  $-\infty$  when  $\phi \xrightarrow{\geq} 0$  and to  $+\infty$  when  $\phi \xrightarrow{\leq} \frac{1}{(1-\gamma)\left[1 + \frac{\alpha}{s(1-\alpha)}\right]}$ , it has at least one zero in the interval  $]0, \frac{1}{(1-\gamma)\left[1 + \frac{\alpha}{s(1-\alpha)}\right]}[$  [ and at most two zeros in the interval  $]\frac{1}{(1-\gamma)\left[1 + \frac{\alpha}{s(1-\alpha)}\right]}, +\infty[$ . Besides,  $Q_\rho(\phi)$  is continuous on that interval and goes to  $-\infty$  when  $\phi \xrightarrow{\geq} \frac{1}{(1-\gamma)\left[1 + \frac{\alpha}{s(1-\alpha)}\right]}$  and  $\phi \rightarrow +\infty$ . Let us make the following assumption:

**Assumption 6.**

$$\frac{s(1-\alpha)}{\alpha} \left[ \frac{\delta}{1-\gamma} - 1 \right] < \frac{1-\mu\gamma-\eta(1-\gamma)}{1-\eta(1-\gamma)}$$

This assumption is always satisfied when  $\delta < 1 - \gamma$ . When not, it sets an upper limit on the saving rate  $s = \frac{\beta}{1+\beta}$ .

Under this assumption,  $\frac{1}{\delta}$  is greater than  $\frac{1}{(1-\gamma)[1+\frac{\alpha}{s(1-\alpha)}]}$  and  $Q_\rho(\frac{1}{\delta\rho}) > 0$ . Therefore, there is a unique  $\tilde{\phi}(\rho)$  greater than  $\frac{1}{\delta\rho}$  such that  $Q_\rho(\tilde{\phi}) = 0$ . The denominator is positive on the interval  $[\frac{1}{\delta\rho}, \tilde{\phi}(\rho)[$ . As  $\frac{1}{\delta} \leq \phi \leq \frac{1}{\delta\rho} \Rightarrow \rho = 1$ ,<sup>12</sup> the denominator is positive on the whole interval  $[\frac{1}{\delta}, \tilde{\phi}(\rho)[$ .

**Assumption 7 (existence of the steady-state).**

$$\phi < \tilde{\phi}(\rho)$$

**Financial fragility**

From assumption 5, the existence of a good equilibrium is already ensured. Therefore, our steady state is financially fragile if and only if the bad equilibrium exists, *i.e.* if and only if  $f_{\text{NN}}(p_t^D) > Z_t$ . In the steady state,

$$f_{\text{NN}}(p_t^D) = \frac{1}{\eta} \left[ B_t^N - \frac{\mu\gamma \Pi_t^T}{1 + s\frac{1-\alpha}{\alpha} \left[ 1 - \frac{1}{\phi(1-\gamma)} \right]} - \eta I_t^T(p_t^D) \right]$$

Using the fact that  $I_t^T(p_t^D) \leq \lambda(1-\gamma)\Pi_t^T$ , we can write a *sufficient condition* of financial fragility in the steady state:

$$\frac{B^N}{\Pi^T} > \frac{\mu\gamma}{1 + s\frac{1-\alpha}{\alpha} \left[ 1 - \frac{1}{\phi(1-\gamma)} \right]} + \eta(1-\gamma)\lambda + \frac{\eta Z}{\Pi^T} \quad (54)$$

This condition is the long-run equivalent of the inequality (39). As we noticed in section 2:

$$\frac{B^N}{\Pi^T} = (1-\gamma) \frac{B^N}{W^N} \frac{W^N}{W^T}$$

Thus, the economy displays financial fragility in the long run when firms in the non-tradable sectors are sufficiently leveraged or when the productive structure is sufficiently oriented toward the non-tradable sector.

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<sup>12</sup>See footnote 11.

The following proposition shows that it is the case in the limit of a small enough conditional endowment  $Z/\Pi^T$  and for some values of  $\phi$ .

**Proposition 2.** *If  $\frac{Z}{\Pi^T}$  is small enough,  $\exists \phi^* \in ]\frac{1}{\delta}, \min(\tilde{\phi}(\omega), \frac{1}{\omega} \frac{\delta + \lambda(1-\delta)}{\delta})$  [ such that the inequality (54) holds for all  $\phi \in ]\phi^*, \min(\tilde{\phi}(\omega), \frac{1}{\omega} \frac{\delta + \lambda(1-\delta)}{\delta})$  [.*

*Furthermore, if  $\frac{Z}{\Pi^T}$  is small enough,  $\exists \omega^* < 1$ , such that for all  $\omega \in ]\omega^*, 1]$ ,  $\exists \phi^* \in ]\frac{1}{\delta}, \min(\tilde{\phi}(\omega), \frac{\delta + \lambda(1-\delta)}{\delta})$  [ such that the inequality (54) holds for all  $\phi \in ]\phi^*, \min(\tilde{\phi}(\omega), \frac{\delta + \lambda(1-\delta)}{\delta})$  [.*

*Proof.* See Appendix A.2. □

This proposition establishes that the steady state can always be financially fragile provided that the conditional endowment is small enough and  $\phi$  is large enough (though always within the limits set by assumptions 5 and 7). Thus, for these values of  $\phi$ , a balance of payments crisis will occur at some point in the future with probability 1. Looking back at the definition of  $\phi$  (equation (43)), we can see that the economy is likely to be subject to financial fragility along an equilibrium path when:

1. it is very opened to international capital flows (the iceberg cost to international transactions  $\tau$  is small),
2. there is a large amount of international liquidity (the international interest rate  $R^*$  is low),
3. there is a high growth (coming from large productivity gains).<sup>13</sup>

In general, it is possible that  $\phi^* \geq \frac{\delta + \lambda(1-\delta)}{\delta}$ . When this is the case, the firms of the tradable sector are credit-constrained in any financially fragile steady state. The second part of the proposition, however, shows that if the probability  $\omega$  is large enough, *i.e.* if the probability of default is sufficiently small, the threshold value  $\phi^*$  can be strictly lower than  $\frac{\delta + \lambda(1-\delta)}{\delta}$ , so that a steady state where T firms do not face a binding borrowing constraint can

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<sup>13</sup>This confirms the result by Rancière et al. (2003) that there might be a trade-off between high growth and recurrent financial crises, although in our framework the growth rate of productivity  $g$  is not the result of a choice.

also be financially fragile. Note that if  $\phi^* \geq \frac{\delta + \lambda(1-\delta)}{\delta}$ , there might not be an equilibrium path for all  $\phi^* \in [\frac{\delta + \lambda(1-\delta)}{\delta}, \phi^*]$ . Indeed, assumption 5 is not satisfied on this interval when  $F = 0$ .<sup>14</sup> This problem disappears when  $\omega$  is close enough to 1.

## 4 The effect of capital account liberalization

To give an illustration of this result, we calibrate the model using data from Argentina in the nineties.<sup>15</sup>

The time period corresponds to the average duration of an investment plan. We set it to 10 years.

We set  $\alpha = 0.48$ ,  $\mu = 0.54$ ,  $\eta = 0.63$ ,  $\gamma = 0.3$ ,  $\delta = 0.8$  and  $\beta = 0.05$ <sup>16</sup>. With this set of parameters, assumption 6 is satisfied and assumption 3 is satisfied for  $\lambda > 1.92$ . We choose a conservative  $\lambda = 2$ . Finally, we set  $\omega = 0.99$ , so that financial fragility can occur for  $\phi < \frac{\delta + \lambda(1-\delta)}{\delta}$ .

Figure 2 displays  $\frac{B^N}{\Pi^T}$  as a function of  $\phi$  for this set of parameters. The

<sup>14</sup>We write “might not be” because the inequality (54) is only a sufficient condition of financial fragility and not a necessary one, so that the economy might be financially fragile for  $\phi \leq \phi^*$  after all.

<sup>15</sup>This country has implemented a reform package between 1989 and 1991, including the opening of the capital account. The economy has then experienced a decade of high growth (interrupted by the “Tequila” crisis of 1995) until the recession of 1999 that culminated in the collapse of 2001.

<sup>16</sup>The profit share  $\alpha$  is given by the 1993 National Accounts (Maia & Nicholson 2001).

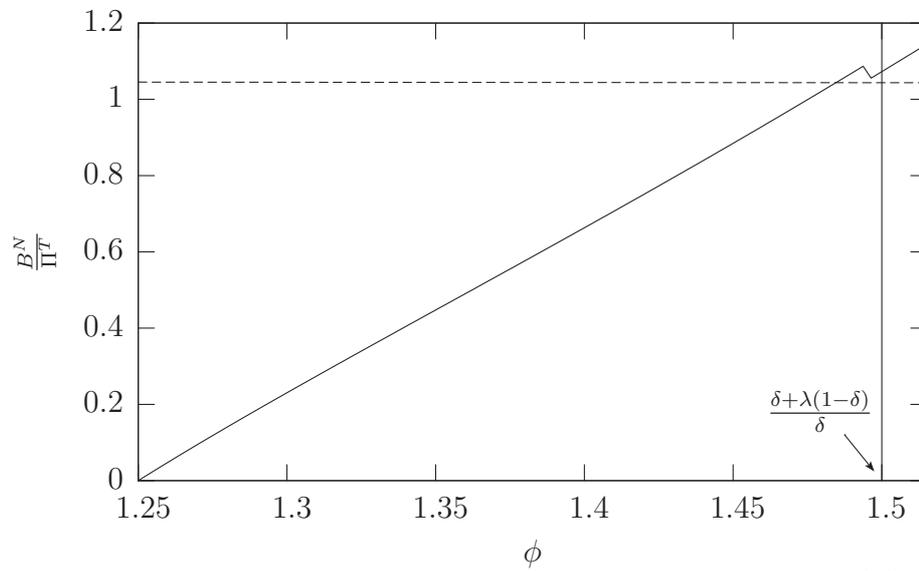
The share of non-tradables in consumption expenditures  $\mu$  is directly given by the composition of the Consumer Price Index in 1999.

The price of the composite used as capital in the intermediate sector is  $p_t^\eta$ . We proxy this price by the GDP price index and regress it (in logarithm) on price indices for goods, construction and services. We use the sum of the coefficients of construction and services as an estimate of  $\eta$ . The data come from the Ministerio de Economía (MECON) and the Instituto Nacional de Estadística y Censos (INDEC) and cover the period 1993-2003.

$\gamma = 0.3$  seems a reasonable value. For the sake of comparison, the average retention ratio for American firms is 60%, which yields  $\gamma = 0.4$  (Fazzari, Hubbard, Petersen, Blinder & Poterba 1988).

$\delta = 0.8$  corresponds to a share of pure profit equal to 20% in the intermediate sector. The same order in magnitude is used in another calibration exercise by Banerjee & Duflo (2004).

$\beta = 0.05$  yields a saving rate of households  $s = 4.8\%$  which accounts for the difference between aggregate savings (data from the Penn World Table (2002) for 1990-1996) and corporate savings (estimated for 1990-1996 by Bebczuk (2000)).



Note : This graph represents  $\frac{B^N}{\Pi^T}$  as a function of  $\phi$  on the interval  $]\frac{1}{\delta}, \frac{1}{\omega} \frac{\delta + \lambda(1 - \delta)}{\delta}[$  for  $\alpha = 0.48$ ,  $\mu = 0.54$ ,  $\eta = 0.63$ ,  $\gamma = 0.3$ ,  $\delta = 0.8$ ,  $\beta = 0.05$ ,  $\lambda = 2$ ,  $\omega = 0.99$  and  $\frac{Z}{\Pi^T} \rightarrow 0$ . The dotted line represents the threshold given by the inequality (54). The vertical line corresponds to  $\phi = \frac{\delta + \lambda(1 - \delta)}{\delta}$  and divides the plan in a zone where the sector T is not financially constrained (on the left) and a zone where it is (on the right).

Figure 2: Financial fragility in the long run

steady state is financially fragile when  $\frac{B^N}{\Pi^T}$  is greater than the threshold represented by the dotted line for  $\rho = \omega$ .<sup>17</sup> This happens for  $\phi$  greater than 1.494, which corresponds to an annual ratio  $(1 + g)/R^D$  equal to 1.0045. For an average annual growth rate of 4.79%,<sup>18</sup> the model predicts that the economy is financially fragile in the steady state when the annual riskless real interest rate is lower than 4.32%. This figure is low but not unusual for periods with large amounts of international liquidity and it seems plausible that Argentina was financially fragile in the nineties, indeed. The average real interest rate on external debt varied between 2.6 and 5.8% during the period 1990-1997<sup>19</sup> with an average of 4.32%.<sup>20</sup> Moreover, as this rate includes a risk premium, it is likely to overestimate the true value of the variable. In the following, we choose the slightly lower value of 4.30%.<sup>21</sup> Remark that this calibration exercise is meant for illustrative purpose only and should be taken with precautions: because of the large duration of an investment plan (10 years), the threshold of the financial fragility zone is very sensitive to the annual interest and growth rates.

The value of  $\frac{B^N}{\Pi^T}$  both reflects the steady state financial structure of the firms producing non-tradable goods ( $\frac{B^N}{\Pi^N}$ ) and the productive structure of the whole economy ( $\frac{W^N}{W^T}$ ). From equation (44), we know that the debt repayment to cash-flow ratio  $\frac{B^N}{\Pi^N}$  monotonically increases with  $\phi$ . This manifests the fact that the volume effect (firms contract more debt when the interest rate is low) overrides the price effect (for the same volume of debt, the debt repayment is lower when the interest rate is low). The way the productive structure

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<sup>17</sup>The discontinuity in the curve corresponds to the change in the value of the financial fragility indicator  $F$ .

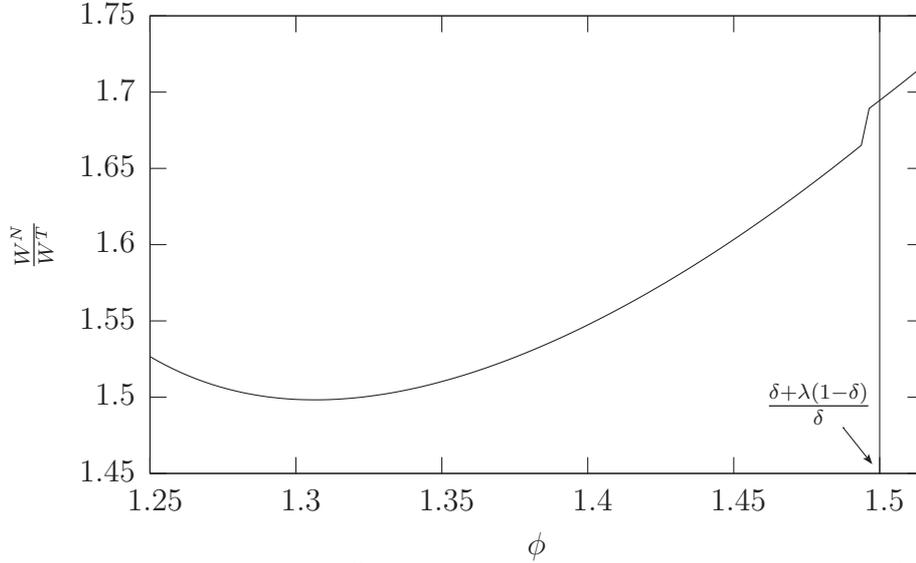
<sup>18</sup>This is the average GDP growth rate of Argentina over the period 1991-1998, obtained by regressing the logarithm of the GDP over a time trend (data from MECON).

<sup>19</sup>From 1998 on, real interest rates increase above 6%, partly as a result of the increase in the emerging market risk premium that follows the Russian crisis. Section 5 studies the effect of such an exogenous shock.

<sup>20</sup>The data on the nominal interest rate comes from the Institute of International Finance. We have deflated it by the US GDP price index given by the Bureau of Economic Analysis.

<sup>21</sup>We check that, for the corresponding value of  $\phi$ , assumption 2 is satisfied in the steady state:  $\frac{\Sigma}{W^T}$  is equal to 0.06 which is lower than  $\frac{B^{T+1}}{R^D W^T} = 0.98$ .

depends on  $\phi$  is however ambiguous. On figure 3, we plot the relative size of the two sectors  $\frac{W^N}{W^T}$  in the steady state as a function of  $\phi$ .



Note : This graph represents  $\frac{W^N}{W^T}$  as a function of  $\phi$  on the interval  $]\frac{1}{\delta}, \frac{1}{\omega} \frac{\delta + \lambda(1-\delta)}{\delta} [$  for  $\alpha = 0.48$ ,  $\mu = 0.54$ ,  $\eta = 0.63$ ,  $\gamma = 0.3$ ,  $\delta = 0.8$ ,  $\beta = 0.05$ ,  $\lambda = 2$ ,  $\omega = 0.99$  and  $\frac{Z}{\Pi^T} \rightarrow 0$ . The vertical line corresponds to  $\phi = \frac{\delta + \lambda(1-\delta)}{\delta}$  and divides the plan in a zone where the sector T is not financially constrained (on the left) and a zone where it is (on the right).

Figure 3: Productive structure in the long run

The resulting curve is U-shaped, decreasing for low enough value of  $\phi$  and increasing for higher  $\phi$ . To get an intuitive interpretation of this shape, it is useful to discuss the case of zero household savings ( $\beta = 0$ ). In that case,  $\phi = \frac{1}{\delta}$  corresponds to a closed economy where firms have zero debt and finance all their investment expenditures by using their internal funds. After a capital account liberalization, the interest rate decreases and  $\phi$  increases. When  $\phi < \frac{\mu\gamma + \eta(1-\gamma)}{\eta(1-\gamma)}$ ,  $\frac{W^N}{W^T}$  is lower in the steady state than in the closed economy. This roughly corresponds to the case of a negative financial transfer:<sup>22</sup> firms are repaying more to external lenders than what they receive

<sup>22</sup>Strictly speaking, the financial transfer from abroad is negative when  $\phi < \frac{1}{(1-\gamma)}$ , *i.e.* when  $1 + g < R^D$ . Since there are heterogenous agents in our model (the entrepreneurs who repay the debt and consume are not the same as the ones who contract new debt and invest), the limit  $\frac{\mu\gamma + \eta(1-\gamma)}{\eta(1-\gamma)}$  takes into account the difference between the shares of non-tradable in consumption and investment expenditures.

from them. This induces a shift of resources from the non-tradable sector to the tradable sector. On the contrary,  $\phi \geq \frac{\mu\gamma + \eta(1-\gamma)}{\eta(1-\gamma)}$  roughly corresponds to the case of a positive financial transfer. The higher resources made available to the country increase the level of consumption and investment expenditures and the demand for both tradable and non-tradable inputs. The adjustment comes from larger imports of tradable goods and a larger relative size of the non-tradable sector.<sup>23</sup> Therefore, if we think of emerging market economies as receiving a positive financial transfer from abroad, we should expect them to experience an increase in  $\frac{W^N}{W^T}$  as they open their capital account to international flows. It is not surprising, then, that an emerging market country should be prone to self-fulfilling balance of payments crises.

Let us come back to our calibration exercise. For an annual growth rate of 4.79% and an annual real interest rate of 4.30%, the model predicts that  $\frac{W^N}{W^T}$  is 11% higher than what it would be for  $\phi = \frac{1}{8}$ , which approximately corresponds to a closed economy. From equations (50), (51) and (53), this corresponds to a ratio  $\frac{p^H N}{T}$  higher by 9%. This is rather well supported by the empirical evidence. According to the available data,<sup>24</sup> the relative size of the non-tradable sector has increased by 8% between 1993 and 2000. Remark that according to equations (50) and (53), the steady state real exchange rate should also be higher than its “autarky” value, which is a well-established stylized fact.

The effect of that shift in the productive structure towards non-tradable goods is essential in explaining financial fragility. Would the ratio  $\frac{W^N}{W^T}$  remain equal to the value it has when  $\phi = \frac{1}{8}$  (the “autarky” case),  $\frac{B^N}{\Pi^T}$  would be equal to 0.96, which is lower than the threshold 1.04. Thus, financial factors alone are insufficient to account for financial fragility and the change in the sectorial allocation of resources is a major mechanism at work in financial crises.<sup>25</sup>

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<sup>23</sup>The mechanism described here is similar to the so-called “Dutch disease” phenomenon. See Kalantzis (2004, 2005) for a model specifically relying on this effect.

<sup>24</sup>We use the ratio of sectorial GDP at 1993 prices for  $\frac{p^H N}{T}$ . N is defined as services and construction. T is defined as manufacturing, agriculture and mining.

<sup>25</sup>This result is consistent with the empirical evidence presented by Tornell & Westermann (2002). They observe that the size of the non-tradable sector increases with regards

## 5 The effect of exogenous shocks

In section 3, we have shown that an equilibrium path eventually leads to a crisis under certain conditions on the exogenous parameters of the model. In particular, assumption 3 must hold, which means that the financial system has to be sufficiently developed or, in other words, the borrowing constraint has to be weak enough. Moreover, from proposition 2, the reduced variable  $\phi$  must be greater than some threshold. This implies, among other things, a sufficiently low level of the international interest rate.

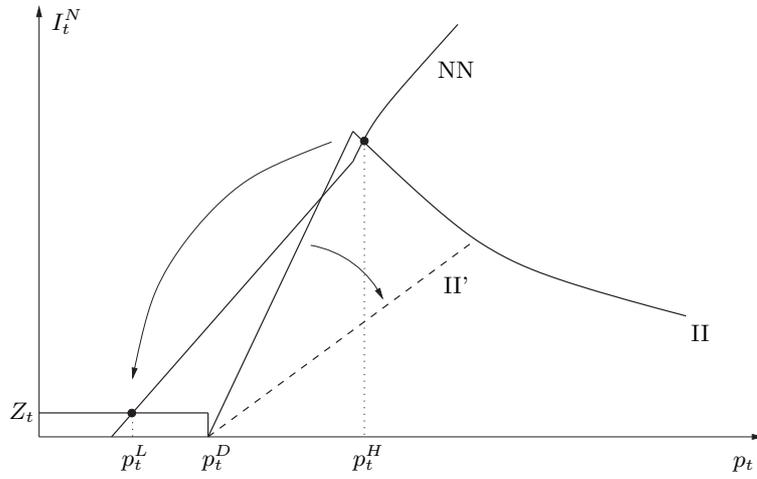
Those results were derived under the assumption that the parameters have a constant value over the long run, so that the productive structure of the economy ( $\frac{W^N}{WT}$ ) and the financial structure of the non-tradable sector ( $\frac{B^N}{\Pi^N}$ ) have time to change. But we have said nothing about the short-run effect of a change in these parameters, which is likely to be very different. The empirical literature on emerging market crises indeed suggests that crises are for example often associated with *increases* in the international interest rate and with sudden stops, *i.e.* sudden *increases* in the intensity of the borrowing constraint.

In this section, we briefly introduce different kinds of unexpected shocks in the framework we have developed so far. We examine their short-run effect, *i.e.* whether they can trigger a crisis, and the relationship between this effect and the conditions of financial fragility in the long run derived in section 3.

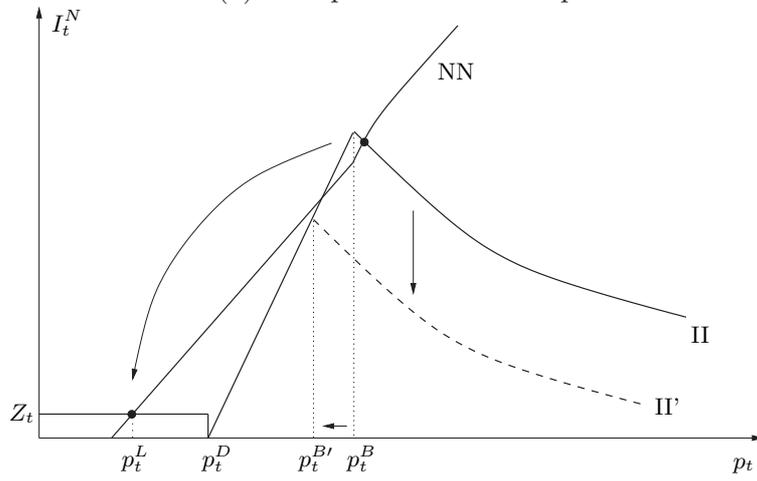
Suppose the economy is in the steady state studied in section 3 at time  $t-1$  and that  $S_t = 1$ , so that no crisis would occur at time  $t$  in the absence of shocks. At time  $t$ , an unexpected shock hits one of the model's parameters. To make things simple, we assume the shock is known after agents have formed their expectations of future prices, so that the expected relative price is still equal to the steady state good equilibrium price:  $E_t[p_{t+1} | p_{t+1} \geq p_{t+1}^D] = p^H$ , where  $p^H$  is given by formula (53).

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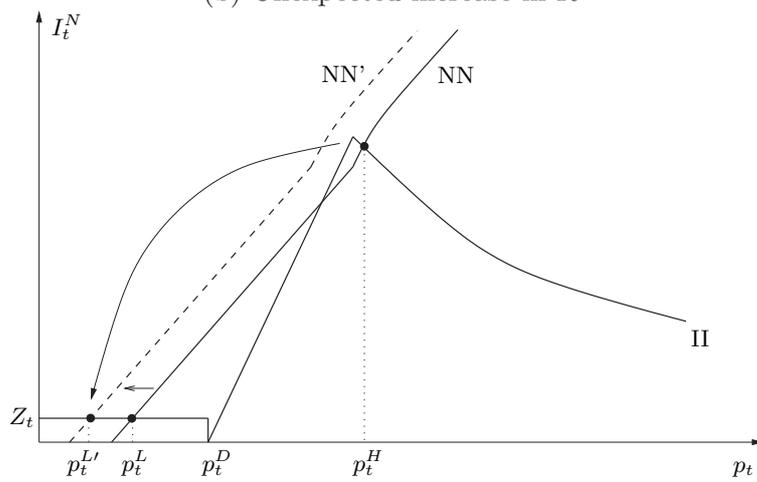
to the tradable sector in the few years preceding a twin crisis.



(a) Unexpected sudden stop



(b) Unexpected increase in  $R^*$



(c) Unexpected negative shock on terms of trade

Figure 4: Crises triggered by unexpected shocks

## Sudden stops

We can model sudden stops in a rather crude way by assuming that firms suddenly lose their access to finance due to an infinitely strong borrowing constraint and have to entirely rely on their internal funds. Therefore,  $\lambda$  unexpectedly takes the value 1. If the steady state was financially fragile ( $F_{t-1} = 1$ ), the II and NN schedules do not intersect any more on the right of  $p^D$  (see figure 4, panel (a)), the good equilibrium ceases to exist and the economy jumps to the bad equilibrium  $p^L$ . On the contrary, if  $F_{t-1} = 0$ , the bad equilibrium does not exist. Two cases are possible, then. If the NN schedule is far on the right so that it intersects the new II' schedule in its unconstrained part, the change in  $\lambda$  has no effect. If the NN schedule is not so far on the right so that it intersects the new II' schedule in its constrained part, the economy jumps to an equilibrium where the borrowing constraint binds for N firms. Because the inequality (38) is not satisfied anymore, this equilibrium is now stable. The change of equilibrium then provokes a decrease in  $I_t^N$  and a real depreciation but firms do not default on their loans: the sudden stop provokes a recession, but no crisis with widespread defaults and losses for external lenders.

Therefore, a sudden stop provokes a crisis if and only if the economy is financially fragile. The condition of long-run financial fragility derived in section 3 is also a condition such that sudden stops lead to balance of payments crises with defaults.

## Shocks on $R^*$

Consider now the case of a positive shock on  $R^*$ . This increases the opportunity cost of investment in the intermediate sectors and therefore decreases the unconstrained level of desired investment  $\bar{I}_t^N$  and  $\bar{I}_t^T$ : the right part of the II schedule moves down (and the unconstrained part of the NN schedule moves up) and the price  $p^B$  decreases to  $p_t^{B'}$  (figure 4, panel (b)). If  $p_t^{B'}$  is lower than the price corresponding to the unstable equilibrium, the good equilibrium no longer exists and the economy jumps to the bad equilibrium.

First, a necessary condition for this to happen is of course that the bad equilibrium exists, *i.e.* that the economy is financially fragile ( $F_{t-1} = 1$ ).

Second, for a shock on  $R^*$  of a given size, the corresponding shift of the II schedule is sufficient to make the good equilibrium disappear if  $p^H$  is close enough to  $p^B$ , *i.e.* if the borrowing constraint is close enough to binding. This happens when  $\phi$  is close enough to its upper bound  $\frac{1}{\omega} \frac{\delta + \lambda(1-\delta)}{\delta}$ .

Therefore, the model predicts that a *positive* shock on  $R^*$  will be likely to lead to a crisis if the value of  $R^*$  prior to the shock was *low* enough, so that  $\phi$  was high enough.

### Terms of trade shocks

We may model a negative shock on terms of trade as a decrease in the productivity  $A_t^T$  of the sector T (with respect to an unchanged productivity of sector N). The effect of this shock is to decrease the production of tradable goods. Therefore,  $\Pi_t$  and  $W_t$  decrease so that the NN schedule moves to the left (figure 4, panel (c)). If the shock is large enough, the shift of the NN schedule makes the good equilibrium disappear and the economy jumps to the bad one, triggering a crisis.

As in the previous case, given the size of the shock, the good equilibrium disappears if  $p^H$  is close enough to  $p^B$ . If not, it leads to a real depreciation ( $p_t^H$  decreases) and to a higher investment in the non-tradable sector.

Therefore, here again, a negative shock on the terms of trade triggers a crisis if  $\phi$  is close enough to its upper bound  $\frac{1}{\omega} \frac{\delta + \lambda(1-\delta)}{\delta}$ .

To sum it all up, a high enough value of  $\phi$  does not only lead to financial fragility in the long run, as we proved in section 3, but also makes the economy sensitive to different kinds of shocks: adverse movements in the terms of trade, sudden increases in the international interest rates or sudden stops. All of these shocks trigger a crisis if the interest rate (before the shock) is low enough and if the rate of technical progress and the financial openness are high enough.

## 6 Conclusion

We have built an overlapping generation model of financial fragility in a small open economy. The production of the consumption good requires tradable and non-tradable intermediate goods produced by two different sectors. In both sectors, a continuum of firms led by entrepreneurs produces the intermediate good with concave production functions.

Because capital is fixed one period in advance in the intermediate sectors, production is predetermined in any given time period. Therefore, a decrease in investment expenditures which diminishes both the demand for tradable and non-tradable inputs has to be met by a decrease in imports (an increase in net exports) and a decrease in the relative price of the non-tradable good: a real depreciation. The real depreciation has a negative effect on the balance sheets of firms producing non-tradable goods and limits their investment capacity through a financial accelerator effect. This validates the initial decrease, up to the point where they cannot meet their debt repayments and have to default on their loans. Our model therefore allows for self-fulfilling balance of payments crises, where a sudden real depreciation goes along with a sharp drop in investment and widespread defaults in the non-tradable sector.

The balance sheet effect through which changes in the real exchange rate have an impact on investment expenditures comes from two market imperfections. First, firms cannot contract debt denominated in non-tradable goods so that there is a currency mismatch in the non-tradable sector. Second, because of moral hazard and the impossibility of lenders to perfectly monitor their loans, the debt contracts limit the amount borrowed by entrepreneurs to an amount depending on their internal funds.

However, such a self-fulfilling crisis is not always possible and requires certain conditions to be satisfied. First, the self-reinforcing mechanism requires the financial accelerator effect to be sufficiently strong. This would not happen in an economy subject to financial repression. Second, an equilibrium with defaults only exists when the balance sheets of firms producing non-

tradable inputs are sufficiently leveraged and/or if the productive structure of the whole economy is sufficiently oriented toward the production of non-tradable goods. Therefore, a self-fulfilling balance of payments crisis, though a contingent event in itself, depends on two structural factors: a firm-level financial factor and an economy-wide real factor. We have shown that the evolution of these two factors along any possible equilibrium path leads to financial fragility provided that the real riskless interest rate is low enough with regards to the growth rate of the economy, which happens when sufficiently large amounts of international liquidity are available and the economy is sufficiently opened to foreign capital flows. As an example, we have shown in a calibration exercise that this might have been the case of Argentina in the nineties.

Moreover, the same kind of conditions (low international interest rate, high growth rate, high financial openness) also ensure that adverse unexpected shocks trigger a crisis of the same kind as described above. This is the case of sudden stops, increases in the international interest rate and adverse movements in the terms of trade.

This paper has several policy implications for an emerging market country wishing to prevent balance of payments crises. First, it has to pay attention to mismatches in firm balance sheets, a lesson that is now widely agreed on.<sup>26</sup> In particular, if a reform of domestic financial liberalization is believed to lead to a more developed financial system, an appropriate regulation should be implemented along with this reform to diminish the financial accelerator effect. Second, it also has to pay a strong attention to changes in the productive structure. An increase in the relative size of the non-tradable sector with regards to the tradable sector might make the economy financially fragile and prone to crises. In times of high international liquidity, a tax on capital flows set at an adequate level can diminish such an increase while allowing the economy to reap parts of the benefits of foreign capital inflows.

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<sup>26</sup>See for example Allen, Rosenberg, Keller, Setser & Roubini (2002).

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## A Appendix

### A.1 Proof of proposition 1

An equilibrium real exchange rate is a zero of the function  $f_{\text{NN}} - f_{\text{II}}$ . Suppose the condition expressed in equation (38) is *not satisfied*. Then, when  $p_t \geq p_t^D$ , one has:

$$\begin{aligned} \frac{d}{dp_t}(f_{\text{NN}} - f_{\text{II}}) &= \frac{N_t}{\eta} \left[ 1 - \frac{\mu\gamma}{1 + s\frac{1-\alpha}{\alpha}} \right] - \frac{\partial I_t^T}{\partial p_t} - \frac{df_{\text{II}}}{dp_t} \\ &\geq \frac{N_t}{\eta} \left[ 1 - \frac{\mu\gamma}{1 + s\frac{1-\alpha}{\alpha}} - \eta(1 - \gamma)\lambda \right] \\ &\geq 0 \end{aligned}$$

$f_{\text{NN}} - f_{\text{II}}$  is a continuous and increasing function on the interval  $[p_t^D, +\infty[$ . Besides, this function goes to  $+\infty$  when  $p_t$  goes to  $+\infty$ . Therefore, it has a zero on this interval if and only if  $(f_{\text{NN}} - f_{\text{II}})(p_t^D) = f_{\text{NN}}(p_t^D) \leq 0$ . When this is the case,  $f_{\text{NN}}$ , which is increasing on  $[0, p_t^D[$ , is negative on this interval, so that  $f_{\text{NN}} - f_{\text{II}} = f_{\text{NN}} - Z_t < 0$  when  $p_t < p_t^D$ . There cannot be at the same time an equilibrium with  $p_t \geq p_t^D$  and another with  $p_t < p_t^D$ . This proves the first part of the proposition.

Suppose now that  $f_{\text{NN}}(p_t^D) > Z_t$  and  $(f_{\text{NN}} - f_{\text{II}})(p_t^B) < 0$ . As  $f_{\text{NN}}$  is strictly increasing on  $[0, p_t^D[$  and negative when  $p_t = 0$ , there is a unique

$p_t^L < p_t^D$  such that  $f_{\text{NN}}(p_t^L) = f_{\text{II}}(p_t^L) = Z_t$ . Similarly,  $f_{\text{NN}} - f_{\text{II}}$  is continuous and strictly increasing on  $[p_t^B, +\infty[$ , and goes to  $+\infty$  when  $p_t$  goes to  $+\infty$ . Therefore, there is a unique  $p_t^H > p_t^B$  such that  $(f_{\text{NN}} - f_{\text{II}})(p_t^H) = 0$ .

## A.2 Proof of proposition 2

Let  $\Psi_\rho(\phi) = \frac{B^N}{\Pi^T} - \frac{\mu\gamma}{1 + s^{\frac{1-\alpha}{\alpha}} \left[1 - \frac{1}{\phi(1-\gamma)}\right]} - \eta(1-\gamma)\lambda$ . The inequality (54) holds when  $\Psi_\rho(\phi) > \frac{\eta Z}{\Pi^T}$ . When this is the case for  $\rho = \omega$ , the economy is financially fragile so that  $F = 1$  and  $\rho = \omega$ .

$\Psi_\rho$  is a continuous function of  $\phi$  on the interval  $\left[\frac{1}{\delta\rho}, \min\left(\tilde{\phi}(\rho), \frac{1}{\rho} \frac{\delta+\lambda(1-\delta)}{\delta}\right)\right]$  defined by the assumptions 4, 5 and 7. Furthermore,  $\Psi_\rho\left(\frac{1}{\delta\rho}\right) < 0 < \frac{\eta Z}{\Pi^T}$ . Therefore, we only have to show that  $\Psi_\omega(\phi) > 0$  when  $\phi \rightarrow \min\left(\tilde{\phi}(\omega), \frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}\right)$  to prove the first part of the proposition.

Suppose  $\tilde{\phi}(\omega) \leq \frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}$ ; then  $\Psi_\omega(\phi) \xrightarrow[\phi \rightarrow \tilde{\phi}(\omega)]{} +\infty$ . If, on the contrary,  $\tilde{\phi}(\omega) > \frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}$ , let us show that  $\Psi_\omega\left(\frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}\right) > 0$ . To simplify notations, we define  $x = \frac{s^{(1-\alpha)}}{\alpha}$ ,  $a = \mu\gamma$ ,  $b = \eta(1-\gamma)$  and  $A(\phi) = \frac{a}{1+x \left[1 - \frac{1}{\phi(1-\gamma)}\right]}$ .

$$\begin{aligned} \Psi_\omega\left(\frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}\right) &= \frac{(\lambda-1) \left[ A\left(\frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}\right) + b\lambda \right]}{\frac{\lambda}{\delta} - \left[ \frac{\delta+\lambda(1-\delta)}{\delta} \right] \left[ A\left(\frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}\right) + b\lambda \right]} - \left[ A\left(\frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}\right) + b\lambda \right] \\ &= \frac{\delta+\lambda(1-\delta)}{\delta} \frac{\left[ A\left(\frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}\right) + b\lambda \right] \left[ A\left(\frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}\right) + b\lambda - 1 \right]}{\frac{\lambda}{\delta} - \left[ \frac{\delta+\lambda(1-\delta)}{\delta} \right] \left[ A\left(\frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}\right) + b\lambda \right]} \\ &> 0 \end{aligned}$$

because  $A\left(\frac{1}{\omega} \frac{\delta+\lambda(1-\delta)}{\delta}\right) + b\lambda - 1 > \frac{a}{1+x} + b\lambda - 1$  from assumption 6 and  $\frac{a}{1+x} + b\lambda - 1 > 0$  from assumption 3.

The second part of the proposition comes from the fact that

$$\lim_{\omega \rightarrow 1} \Psi_\omega\left(\frac{\delta + \lambda(1-\delta)}{\delta}\right) = \Psi_1\left(\frac{\delta + \lambda(1-\delta)}{\delta}\right) > 0$$

by continuity of the function  $\omega \mapsto \Psi_\omega\left(\frac{\delta+\lambda(1-\delta)}{\delta}\right)$ .