Compensating the dead? Yes we can!

Marc Fleurbaey, Marie-Louise Leroux, Grégory Ponthière

Compensating the dead?

Yes we can!

Marc Fleurbaey

Marie-Louise Leroux

Grégory Ponthière

Compensating The Dead?
Yes We Can!*

Marc Fleurbaey,† Marie-Louise Leroux‡and Gregory Ponthiere§

October 20, 2010

Abstract

An early death is, undoubtedly, a serious disadvantage. However, the compensation of short-lived individuals has remained so far largely unexplored, probably because it appears infeasible. Indeed, short-lived agents can hardly be identified *ex ante*, and cannot be compensated *ex post*. We argue that, despite the above difficulties, a compensation can be carried out by encouraging early consumption in the life cycle. In a model with heterogeneous preferences and longevities, we show how a specific social criterion can be derived from intuitive principles, and we study the corresponding optimal policy under various informational assumptions. We also study the robustness of our solution to alternative types of preferences and savings policies.

Keywords: compensation, longevity, mortality, fairness, redistribution.
JEL codes: D63, D71, I18, J18.

1

1 Introduction

It is undeniably true that an early death constitutes a serious loss, even when it is due to natural causes. Such a loss should, in a fair society, imply a compensation. However, the compensation of short-lived persons has remained so far largely unexplored in policy circles. The absence of debate on that issue is surprising, since longevity inequalities are widely documented. It is well-known that sizeable longevity differentials exist even within a given cohort, as shown by Figure 1.[1] Although all cohort members are, by definition, born in the same country and at the same epoch, there is a substantial dispersion of the age at death, some persons turning out to have longer lives than others.
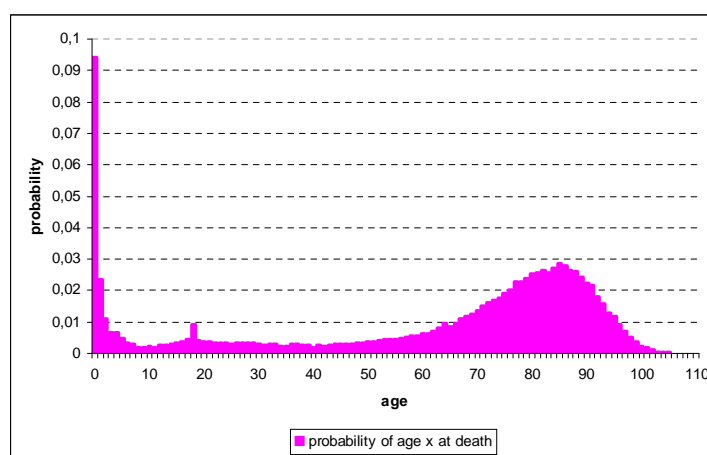
Figure 1: Distribution of the age at death: Swedish female (1900 cohort)

Given that longevity differentials are mainly explained by factors on which individuals have, on their own, little control, there exists a strong ethical intuition for compensating short-lived agents, who are, in some sense, victims of the arbitrariness of Nature.[2] Longevity inequalities due to differences in genetic backgrounds are the best illustration of this. According to Christensen *et al* (2006), about one quarter to one third of longevity inequalities within a cohort can be explained by differences in

---

[1]Sources: the Human Mortality Data Base (2009).

[2]Note that longevity is also influenced by individuals, for instance through their lifestyles (see Kaplan *et al* 1987), but those behavioural determinants of longevity (e.g. smoking, diet, physical activity, etc.) only explain one part of longevity differentials, the rest remaining out of individuals' control (e.g. genetic background, environmental determinants of longevity, etc.).

the genetic background. Hence there is a strong intuitive support for compensating the short-lived, who cannot be regarded as responsible for their genes.

But despite the sizeable — and largely arbitrary — longevity differentials, little attention has been paid to the compensation of short-lived agents. The reason behind that neglect lies in the apparent impossibility to compensate short-lived individuals. A first difficulty is that short-lived agents can hardly be identified *ex ante*. Life-tables statistics show the distribution of the age at death in a population or a subpopulation (e.g., by gender), but do not tell us what the longevity of each individual will be. Another difficulty is that, *ex post*, i.e., once a short-lived person is identified, its well-being can no longer be affected, so that little compensation can take place at that stage. Thus we face a non-trivial compensation problem: agents to be compensated cannot be identified *ex ante*, and cannot be compensated *ex post*. Such difficulties may explain why little attention has been paid to the compensation of an early death.

The goal of the present paper is to propose a way to overcome the above difficulties. For that purpose, the first part of this paper is devoted to the construction of a measure of social welfare. The social objective is derived from basic principles guaranteeing that compensating the agents who turn out to be short-lived would be desirable. Moreover, the approach, of the "egalitarian-equivalent" type, takes agents' preferences over longevity into account.[3] More precisely, the proposed social objective evaluates a particular social state by looking at the smallest consumption the individuals would accept in the replacement of their current situation, if they could benefit from some reference longevity level. In sum, it applies the maximin criterion to what we call the Constant Consumption Profile Equivalent on the Reference Lifetime (CCPERL). Hence we shall refer to the social objective we propose as the Maximin CCPERL.

Once the social objective is defined, it can be used to compute the optimal allocation of resources in various environments. In the second part of the paper, we compute the social optimum in a context in which the social planner knows each individual's preferences and life expectancy, as well as the statistical distribution of longevities in the population (but not individual longevities). We then also consider the more relevant second-best context, in which the planner knows the distribution of all variables (including longevity), but ignores each individual's preferences and life expectancy. It might seem that very little compensation for a short life can be made in this case, but the planner can nonetheless improve the lot of the short-lived agents by inducing everyone to save less than they spontaneously would. One of the key results of this paper is that it is even possible, in general, to eliminate welfare

---

[3]The egalitarian-equivalent approach to equity was first introduced by Pazner and Schmeidler (1978).

3

inequalities between short-lived and long-lived agents. Admittedly, the corresponding policy may look counterintuitive and is certainly not common. In the conclusion we discuss the prospects of application of this approach.

Throughout the second part of the paper, we also contrast our egalitarian-equivalent optimum with the standard utilitarian social objective. This allows us to show how the Maximin CCPERL avoids the counterintuitive redistributive implications of utilitarianism in the context of unequal longevities. As shown by Bommier *et al* (2009, 2010) and Leroux *et al* (2010), utilitarianism tends, under standard assumptions like time-additive lifetime welfare and expected utility hypothesis, to redistribute resources from short-lived agents towards long-lived agents, which is undesirable.[4] Our approach is, in this light, more intuitive and attractive than utilitarianism.

The rest of the paper is organized as follows. Section 2 presents the framework. Section 3 derives a social objective from ethical axioms. Section 4 compares the Maximin CCPERL solution with the utilitarian solution in the first best context. Then, Section 5 explores the second-best problem, in which the agents' characteristics are not known to the planner. Section 6 explores some generalizations and evaluates the robustness of the Maximin CCPERL solution to various assumptions, such as the reference longevity level and the savings policies. Section 7 concludes.

2 The framework

The model describes the situation of a given finite population of agents with ordinal preferences over lifetime consumption profiles. We consider a pure exchange economy, because the central tenets of the compensation problem can be captured in absence of production.

The set of natural integers (resp., real numbers) is denoted $\mathbb{N}$ (resp., $\mathbb{R}$). Let N be the set of individuals, with cardinality $|N|$. The maximum possible lifespan for any individual, i.e., the maximum number of periods that can ever be lived, is denoted by T, with $T \in \mathbb{N}$ and $T > 1$.

Each individual will have a particular *lifetime consumption profile*. Under the assumption of non-negative consumptions, a lifetime consumption profile for an individual $i \in N$ is a vector of dimension T or less, that is, it is an element x_i in the set $X = \bigcup_{\ell=1}^{T} \mathbb{R}_+^\ell$. The longevity of an individual i with consumption profile x_i is

---

[4]This anti-redistributive bias is due to Gossen's First Law, and is robust to various specifications of lifetime welfare. In particular, as shown by Leroux and Ponthiere (2010), representing lifetime welfare as a concave transform of the sum of temporal utilities only mitigates — but does not eradicate — the utilitarian tendency to redistribute resources to the long-lived.

defined by a function $\lambda : X \to \mathbb{N}$, such that $\lambda(x_i)$ is the dimension of the lifetime consumption profile, that is, the length of existence of individual i.

An *allocation* defines a consumption profile for each individual in the population N. Formally, an allocation for N is a vector $x_N := (x_i)_{i \in N} \in X^{|N|}$.

Each individual $i \in N$ has well-defined *preferences* over the set of lifetime consumption profiles X. His preferences are described by an ordering R_i (i.e. a reflexive, transitive and complete binary relation). For all $x_i \in X$, the indifference set at x_i for R_i is defined as $I(x_i, R_i) := \{y_i \in X \mid y_i I_i x_i\}$. For any lives x_i and y_i of equal length, preference orderings on x_i and y_i are assumed to be continuous, convex and weakly monotonic (i.e. $x_i \geq y_i$ implies $x_i R_i y_i$ and $x_i \gg y_i$ implies $x_i P_i y_i$).[5] Moreover, we assume that for all $x_i \in X$, there exists $(c, ..., c) \in \mathbb{R}_+^T$ such that $x_i I_i (c, ..., c)$, which means that no lifetime consumption profile is worse or better than all lifetime consumption profiles with full longevity. This excludes lexicographic preferences for which longevity is an absolute good or bad. Let $\Re$ be the set of all preference orderings on X satisfying these properties. A preference profile for N is a list of preference orderings of the members of N, denoted $R_N := (R_i)_{i \in N} \in \Re^{|N|}$.

Figure 2 shows an example of agents' preferences in a two-period setting, i.e. for $x_i \in \mathbb{R}_+ \cup \mathbb{R}_+^2$. An agent who lives the first period only remains on the horizontal axis (i.e. second period consumption is zero). The dashed line segments mean that the individual is indifferent between the two extreme points of the line segment. The upper end of the dashed segment gives the (constant) consumption that should be given to the agent in each period of a hypothetical two-period life to make him exactly as well-off as he is with a single period of life.

Figure 2 illustrates that, to keep the same satisfaction level while raising the length of life, what is required may possibly be either a smaller or a larger consumption per period, depending on the consumption enjoyed while having a short life. For a short-lived individual whose consumption is high (i.e. at the right of the horizontal axis), the consumption that should be given to him in a two-period life to make him indifferent with its current state, which is given by the end of the dashed segment, would be much smaller than its current consumption. This reflects the attractiveness of a longer life for a person with a high current standard of living. On the contrary, for a short-lived agent whose consumption is low (i.e. at the left of the horizontal axis), the consumption that should be given to him in a two-period life to make him indifferent with its current state might be larger than his current consumption. His low current consumption puts him in such a misery that the lengthening of his life

---

[5]Note that we do not assume those properties for lives with *different* lengths. For instance, requiring that the three-periods life $(2, 2, 1)$ be necessarily better than the two-period life $(2, 2)$ would be too strong. One may prefer death to an additional period with consumption equal to 1.
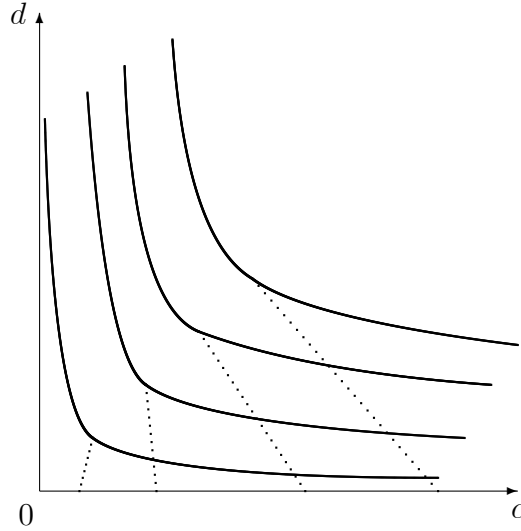
Figure 2: Indifference curves in (c, d) space

with the same consumption per period would make him worse off. Hence additional consumption per period is needed to compensate him for having a longer life.

Clearly, all allocations are not equivalent in terms of how short-lived agents are treated. Therefore, in order to provide theoretical foundations to the compensation of short-lived persons, it is necessary to define *social preferences* over allocations. Such social preferences will serve to compare allocations in terms of their goodness and fairness. Those social preferences will be formalized by a *social ordering function* $\succsim$ which associates every admissible preference profile R_N of the population with an ordering $\succsim_{R_N}$ defined on the set of all possible allocations for N, that is, an ordering defined on $X^{|N|}$. For all $x_N, y_N \in X^{|N|}$, $x_N \succsim_{R_N} y_N$ means that the allocation x_N is, under the preference profile R_N, at least as good as the allocation y_N. The symbols $\succ_{R_N}$ and $\sim_{R_N}$ will denote strict preference and indifference, respectively.

3   The social objective

This section aims at deriving a social objective that is adequate for the allocation of resources among agents having unequal longevities. As mentioned above, standard social objectives like utilitarianism do not do justice to basic intuitions supporting the compensation of the short-lived, so that we need to look for other objectives. Obviously, there exist many possible social preferences. The only way to select

6

reasonable social preferences is to impose some plausible ethical requirements that these should satisfy. Such ethical requirements will take here the form of four axioms.

The first axiom states that if all individuals prefer one allocation to another, then this should also be regarded as socially preferable to that alternative.

Axiom 1 (Weak Pareto) *For all preference profiles $R_N \in \Re^{|N|}$, all allocations $x_N, y_N \in X^{|N|}$, if $x_i P_i y_i$ for all $i \in N$, then $x_N \succ_{R_N} y_N$.*

That axiom can be justified on two grounds. First, it seems essential to respect individual preferences in order to address trade-offs between, for instance, consumption at different points in life. Second, the Pareto axiom is also a guarantee against the choice of inefficient allocations: it states that any unanimity in the population regarding the ranking of two allocations should be respected by social preferences.

The next axiom requires social preferences to use the relevant kind of information about individual preferences. More precisely, it states that, in order to compare allocations for a given individual, it is sufficient to look at the indifference sets of the individual at the consumption profiles under consideration.

Axiom 2 (Hansson Independence) *For all preferences profiles $R_N, R'_N \in \Re^{|N|}$ and for all allocations $x_N, y_N \in X^{|N|}$, if for all $i \in N$, $I(x_i, R_i) = I(x_i, R'_i)$ and $I(y_i, R_i) = I(y_i, R'_i)$, then $x_N \succsim_{R_N} y_N$ if and only if $x_N \succsim_{R'_N} y_N$.*

This condition, which was first introduced by Hansson (1973) and Pazner (1979), requires that social preferences over two allocations depend only on the individual indifference curves at these allocations. Note, however, that those indifference curves contain more information than individual pairwise preferences over these two allocations. This allows us to avoid Arrow's impossibility result.

The next two axioms are refinements of the Pigou-Dalton principle in the context of unequal longevities.

The Pigou-Dalton principle for Equal Preferences and Equal Lifetimes is an immediate translation of the Pigou-Dalton principle in the present context. It states that, if we take two allocations such that the consumption profiles are exactly the same under the two allocations for everyone *except* for two persons, then, if those two individuals have equal lifetimes and equal preferences, the allocation in which the two agents have, when alive, closer consumption profiles is more socially desirable than the one in which they have more unequal consumption profiles.

Axiom 3 (Pigou-Dalton for Equal Preferences and Equal Lifetimes) *For all $R_N \in \Re^{|N|}$, all $x_N, y_N \in X^{|N|}$, and all $i, j \in N$, if $R_i = R_j$ and if $\lambda(x_i) = \lambda(y_i) =$*

$\lambda\left(x_{j}\right)=\lambda\left(y_{j}\right)=\ell$, and if there exists $\delta \in \mathbb{R}_{++}^{\ell}$ such that

$$y_{i} \gg x_{i}=y_{i}-\delta \gg x_{j}=y_{j}+\delta \gg y_{j}$$

and $x_{k}=y_{k}$ for all $k \neq i, j$, then

$$x_{N} \succsim_{R_{N}} y_{N}.$$

That axiom is pretty intuitive: for agents who are identical in terms of everything (i.e. longevities, preferences) *except* their consumptions, a redistribution from the agent with the higher consumption to the agent with the lower consumption constitutes a social improvement.

While that refined version of the Pigou-Dalton principle is intuitive, it is nonetheless restricted to agents with equal preferences, which is a strong restriction. Actually, we would like also to be able to say whether a consumption transfer is a social improvement or not when agents have *different* preferences. Note, however, that making this kind of statement is not trivial, as it is not obvious to see in which case some consumption transfer from a rich to a poor could be regarded as a social improvement whatever individual preferences are.

In the following axiom, it is argued that, if the two agents in question have a longevity that is equal to a level of reference ℓ^{*}, then a transfer that lowers the constant consumption profile of the rich and raises the constant consumption profile of the poor constitutes a social improvement, whatever individual preferences are.

Axiom 4 (Pigou-Dalton for Constant Consumption and Reference Lifetime)
For all $R_{N} \in \Re^{|N|}$, all $x_{N}, y_{N} \in X^{|N|}$, and all $i, j \in N$, such that $\lambda\left(x_{i}\right)=\lambda\left(y_{i}\right)=$ $\lambda\left(x_{j}\right)=\lambda\left(y_{j}\right)=\ell^{*}$, and x_{i} and x_{j} are constant consumption profiles, if there exists $\delta \in \mathbb{R}_{++}^{\ell^{*}}$ such that
$$y_{i} \gg x_{i}=y_{i}-\delta \gg x_{j}=y_{j}+\delta \gg y_{j}$$
and $x_{k}=y_{k}$ for all $k \neq i, j$, then

$$x_{N} \succsim_{R_{N}} y_{N}.$$

The reference longevity level ℓ^{*} can be interpreted in the following way. An external observer could, when comparing the lives of two persons with the same length ℓ^{*}, say who is better off than the other by just looking at the constant consumption profiles of those agents, *without* knowing anything about their preferences. Thus, ℓ^{*} is the length of life such that if it is enjoyed by distinct persons, one can compare the well-being of those agents directly from their consumptions (provided they are

constant over time), without caring for their preferences. Note that this axiom is weak. It would be tempting to extend it to cases in which the longevity of the agents can take other values than a particular ℓ^* : isn't it intuitive that a greater constant consumption for any given longevity makes one better off? Unfortunately, such an extension would render the axiom incompatible with Weak Pareto.[6] This is why the axiom can be formulated for at most one reference level of longevity.

There is no need, at this stage, to assign a specific level to the reference longevity ℓ^*. Intuitively, it makes sense to set ℓ^* at the "normal" lifespan, that is, the lifespan that everyone — whatever one's life-plans are — would like to have, but it is not trivial to see which lifespan is the normal one. Note that the selection of ℓ^* may have important redistributive consequences, in combination with the Pareto axiom. Taking, for instance, a maximal lifespan of 120 years as the reference would imply giving priority to those who have a strong preference for longevity. This is because their situation is equivalent, according to their own preferences, to a situation in which they live for 120 years with a low consumption. Given that the "normal" lifespan may vary with the circumstances — in particular with the quality of life (health status) —, we will not fix it, and keep it as an ethical parameter.[7]

The four ethical principles that are presented above seem quite reasonable. We now have to investigate which kind of social preferences do satisfy these conditions. As we shall see, the answer to that question will be quite precise. But before providing that answer, let us first introduce what we shall call the Constant Consumption Profile Equivalent on the Reference Lifetime (CCPERL).

Definition 1 (Constant Consumption Profile Equivalent on the Reference Lifetime)
For any $i \in N$, any $R_i \in \Re$ and any $x_i \in X$, the Constant Consumption Profile Equivalent on the Reference Lifetime (CCPERL) of x_i is the constant consumption profile $\hat{x}_i$ such that $\lambda(\hat{x}_i) = \ell^*$ and

$$x_i I_i \hat{x}_i.$$

The CCPERL can be interpreted as a way to *homogenize* consumptions across individuals having different longevities, by converting consumptions under different longevities into some comparable consumptions. The intuition behind that homogenization exercise is the following. In the present context, where agents have unequal

---

[6]Such an incompatibility between the Pareto principle and the principle of transfer in the multidimensional context is well documented. See, e.g., Fleurbaey and Maniquet (2011). Intuitively, the problem stems from the fact that, at a low common level of longevity, making a *progressive* transfer from an individual who cares a lot about longevity to another who cares less about longevity may be Pareto equivalent to a *regressive* transfer at a greater level of longevity — their indifference curves crossing at an intermediate level of longevity.

[7]It is indeed likely that societies with a better health will consider that ℓ^* is larger.

longevities, looking at individual consumption profiles does not suffice to have a precise idea of individual well-being. However, the CCPERL does allow to have a more precise view, as it has, by construction, taken longevity differentials into account.

It is trivial to see that, if x_i is a constant consumption profile with $\lambda(x_i) = \ell = \ell^*$, then $\hat{x}_i = x_i$. However, if x_i is a constant consumption profile (with consumption level for each life-period equal to c_i) with $\ell < \ell^*$, then we have $\hat{x}_i \gtreqqless (c_i, ..., c_i)$, depending on whether c_i lies above or below the critical level making a longer life with that consumption worth being lived. The CCPERL of x_i always exists if $\ell^* = T$, by assumption made on $\Re$, but the existence of the CCPERL is not guaranteed if $\ell^* < T$. It may happen that x_i with high longevity is strictly preferred to all lifetime consumption profiles with lower longevity ℓ^*. When this happens, we adopt the convention that the CCPERL is infinite. This problem of non-existence is not very important as the social preferences highlighted here focus on the worst-off individuals.

Having defined the CCPERL, we can now present the following theorem, which characterizes the social preferences, or, more precisely, states that the Maximin on CCPERL is a necessary condition for social optimality.

Theorem 1 *Assume that the social ordering function $\succsim$ satisfies Axioms 1-2-3-4 on $\Re^{|N|}$. Then $\succsim$ is such that for all $R_N \in \Re^{|N|}$, all x_N, $y_N \in X^{|N|}$,*

$$\min_{i \in N}(\hat{x}_i) > \min_{i \in N}(\hat{y}_i) \implies x_N \succ_{R_N} y_N.$$

In other words, the social ordering satisfies the Maximin property on the Constant Consumption Profile Equivalent on the Reference Lifetime (CCPERL).

The proof is in the Appendix. It should be noted that this theorem does not give a full characterization of social preferences because it does not say how to compare allocations for which $\min(\hat{x}_i) = \min(\hat{y}_i)$.[8] All the theorem states is that if one allocation exhibits a higher minimum CCPERL than another, then it must also be socially more desirable. In other words, the theorem implies that maximizing $\min(\hat{x}_i)$ is a *necessary* operation, as the best social allocation is necessarily included in the set of allocations that maximize $\min(\hat{x}_i)$.

However, this result tells us a lot about social preferences. True, if the set of allocations that maximize $\min(\hat{x}_i)$ is not a singleton, looking at the minimum CCPERL only would not tell us which allocation is the most desirable. However, in more

---

[8]Clearly, given the postulated axioms, the equality of the $\min(\hat{x}_i)$ for two allocations does not necessarily imply social indifference between these allocations: an allocation could still be regarded as better than the other (on the grounds of other aspects of the distribution), and the theorem has nothing to say about that.

concrete problems, it is likely that the Maximin on CCPERL has, as a solution, a *unique* allocation, in which case that allocation must also be the most socially desirable allocation. When a unique solution is not obtained, it is natural to refine the Maximin into the Leximin, which extends the lexicographic priority of the worse-off to higher ranks in the distribution.

While the details of the proof are provided in the Appendix, its overall form can be briefly given here. The proof proceeds in two stages. In a first stage, it is shown that Weak Pareto, Hansson Independence and Pigou-Dalton for Equal Preferences and Equal Lifetimes imply Hammond Equity for Equal Preferences. That principle states that, if two persons i and j have the same preferences, but i lies on a higher indifference curve than j, pushing i on a lower indifference curve and j on a higher one is socially desirable. This embodies an absolute priority for the worst-off. In a second stage, Hammond Equity for Equal Preferences is then used to show that, if we add Pigou-Dalton for Constant Consumption and Reference Lifetime, we obtain Hammond Equity for Reference Lifetime. According to that principle, if two persons i and j, possibly with different preferences, have the same longevity equal to the reference ℓ^*, but i has a higher constant consumption profile than j, then lowering the constant consumption profile of i and raising the one of j is socially desirable.

Let us note that an alternative characterization can be made in a slightly different setting. Suppose for the rest of this section that longevity is a continuous variable, so that a lifetime consumption profile is now described as a function $x_i(t)$ defined over the interval $[0, T]$. We restrict attention to functions $x_i(t)$ which are strictly positive and continuous over an interval $[0, \lambda(x_i)]$ and null over the complement $(\lambda(x_i), T]$. The corresponding longevity is obviously $\lambda(x_i)$. Individual preferences over lifetime consumption profiles x_i can still be defined and assumed to be convex, continuous (with respect to the topology of pointwise convergence) and weakly monotonic. The axioms of Weak Pareto and Hansson Independence are immediately adapted to this setting. Let us now introduce a new axiom which states that, whatever the individual preferences, it is always socially desirable to reduce longevity inequalities among agents who enjoy the same consumption per life-period, when one agent lives longer than ℓ^* and the other has a shorter life. For this axiom not to be idle, it must be assumed that $0 < \ell^* < T$. A similar assumption was not needed with the axiom of Pigou-Dalton for Constant Consumption and Reference Lifetime.

Axiom 5 (Inequality Reduction around Reference Lifetime) *For all $R_N \in \Re^{|N|}$, all $x_N, y_N \in X^{|N|}$, and all $i, j \in N$, such that $\lambda(x_i) = \ell_i$, $\lambda(y_i) = \ell'_i$, $\lambda(x_j) = \ell_j$ and $\lambda(y_j) = \ell'_j$, and some $c \in \mathbb{R}_{++}$ is the same constant per-period level*

of consumption for x_i, y_i, x_j, y_j, if

$$\ell_j, \ell_j' \leq \ell^* \leq \ell_i, \ell_i' \ and \ \ell_j - \ell_j' = \ell_i' - \ell_i > 0$$

and $x_k = y_k$ for all $k \neq i, j$, then

$$x_N \succsim_{R_N} y_N.$$

That axiom is quite attractive: reducing longevity inequalities between long-lived and short-lived agents who enjoy equal consumptions per period can hardly be regarded as undesirable. Note, however, that the attractiveness of that axiom is not independent from the monotonicity of preferences in longevity. If consumption per period is so low that some agents may prefer having a short rather than a long life, reducing longevity inequalities by raising the longevity of the short-lived may be socially undesirable. Thus this axiom, unlike axioms 3 and 4, must be used in a subdomain of preferences satisfying a stronger monotonicity condition with respect to longevity.

Observe that by weak monotonicity, for every individual preference ordering R_i and every lifetime consumption profile x_i, there is a unique constant profile with same longevity such that every profile with greater consumption and same longevity is strictly preferred and every profile with lower consumption and same longevity is strictly worse. Therefore, by Weak Pareto one can restrict attention to constant lifetime consumption profiles and work with bundles having two dimensions, namely, per-period consumption and longevity. Formally, Inequality Reduction around Reference Lifetime is then similar to the Free Lunch Aversion Condition proposed by Maniquet and Sprumont (2004) in the context of public goods provision. It is then a simple adaptation of their analysis to show that the conclusion of Theorem 1 holds in this particular setting when it is required that the social ordering function must obey the axioms 1-2-5. The only minor difference is that longevity is here bounded between 0 and T, whereas the corresponding variable (contribution of private good to the production of public good) is unbounded in their model.[9]

---

[9]Although Inequality Reduction around Reference Lifetime is meaningful in the model of this paper introduced in Section 2, Th. 1 does not hold with axioms 1-2-5, even with stronger monotonicity assumptions about individual preferences. The reason is that if the worst-off gains very little, this may not be equivalent to gaining one period of longevity at any level of consumption. Axiom 5 is then powerless because it applies only when the worst-off in the "transfer" of longevity gains at least one period of additional longevity.

4 First-best optimum

The previous section showed that basic axioms on social preferences imply that the optimal allocation must maximize the minimum CCPERL in the population. What are the consequences of this result on the optimum allocation of resources? If, for instance, a social planner could have anticipated, in 1900, the distribution of longevities of Swedish women as shown on Figure 1, how should he have allocated a fixed amount of resources among the cohort members?

This section aims at characterizing the social optimum in a resource allocation problem when the axioms of Theorem 1 are satisfied. We will also contrast this social optimum with the utilitarian optimum, to see what kinds of compensations the Maximin on CCPERL implies in contrast with the utilitarian allocation.

For such purposes, let us consider a simple model where agents live either one or two periods.[10] The length of life of each agent is only known *ex post*. *Ex ante*, the social planner knows individual preferences and life expectancies, as well as the statistical distribution of longevity in the population, and looks for the optimum allocation of an endowment W of resources.

Individual lifetime welfare takes a standard time-additive form:

$$
\begin{aligned}
U_{ij}^1 &= u(c_{ij}), \\
U_{ij}^2 &= u(c_{ij}) + \beta_i u(d_{ij}), \\
EU_{ij} &= u(c_{ij}) + \pi_j \beta_i u(d_{ij}),
\end{aligned}
$$

where c_{ij} and d_{ij} denote first-period and second-period consumptions of an agent with a time preference factor β_i and a survival probability π_j, while U_{ij}^1 and U_{ij}^2 denote his actual lifetime utility if he lives respectively one or two periods, and EU_{ij} is the corresponding expected utility. Temporal utility $u(\cdot)$ takes the same form for everyone.[11]

Heterogeneity here takes the following form. *Ex ante*, agents differ in their attitude towards the future, i.e. in their time preferences, β_i, and in their survival probabilities, π_j, with two types for each parameter:

$$
\begin{aligned}
0 &< \beta_1 < \beta_2 < 1, \\
0 &< \pi_1 < \pi_2 < 1.
\end{aligned}
$$

---

[10]The assumption $T = 2$ is here made for analytical simplicity. We discuss below how robust our results are to assuming $T > 2$.

[11]As usual, we assume: $u'(c_{ij}) > 0$ and $u''(c_{ij}) < 0$.

Hence, there exist 4 types of agents *ex ante*, who are differentiated by their β_i and π_j. *Ex post*, there are 8 types of agents, as each *ex ante* type includes short-lived and long-lived agents.[12]

For the sake of presentation, we here adopt three assumptions which will be relaxed later on (see Section 6). First, we assume that $u(0) > 0$, so that it is always strictly better to be long-lived rather than short-lived. Note that this assumption is not the standard one (see Becker *et al.* 2005), but it greatly simplifies the computation of optimal allocations. Second, we assume that the social planner faces a unique intertemporal resource constraint, in the sense that he can allocate resources as first-period or second-period consumptions without any cost. Third, we assume that agents cannot transfer resources across periods, so that the bundles (received from the planner) have to be consumed in the same periods as they are received, without any possibility, at the individual level, to reallocate resources over time.

Within that framework, the problem of the social planner consists in offering four consumption bundles (c_{ij}, d_{ij}) to agents with time preference parameter β_i and survival probability π_j, for $i = 1, 2$ and $j = 1, 2$. Note that these bundles do not depend on whether agents live one or two periods, as the actual length of life is not known *ex ante* by the planner.

In the following, we first solve the problem faced by a utilitarian planner, and then, we contrast it with the Maximin on CCPERL, assuming that the planner can observe characteristics β_i and π_j. We relax this assumption in Section 5.

4.1 Utilitarian optimum

The problem of the utilitarian social planner amounts to selecting bundles (c_{ij}, d_{ij}) in such a way as to maximize social welfare *ex post*, subject to the resource constraint:[13]

$$
\max_{\substack{c_{11}, c_{12}, c_{21}, c_{22} \\ d_{11}, d_{12}, d_{21}, d_{22}}} \begin{aligned} & u(c_{11}) + \pi_1 \beta_1 u(d_{11}) + u(c_{21}) + \pi_1 \beta_2 u(d_{21}) \\ & + u(c_{12}) + \pi_2 \beta_1 u(d_{12}) + u(c_{22}) + \pi_2 \beta_2 u(d_{22}) \end{aligned}
$$
$$
\text{s.to } c_{11} + \pi_1 d_{11} + c_{21} + \pi_1 d_{21} + c_{12} + \pi_2 d_{12} + c_{22} + \pi_2 d_{22} \leq W
$$

From the first-order conditions, we obtain that the optimal allocation is such that

$$
c_{11}^* = c_{12}^* = c_{21}^* = c_{22}^* > d_{21}^* = d_{22}^* > d_{11}^* = d_{12}^*.
$$

For every agent, the first-period consumption exceeds the second-period consumption, since $\beta_i < 1$. Moreover, the first-period consumption is equalized across all

---

[12]For simplicity, we assume that there is a mass 1 of individuals in each of the *ex ante* groups.

[13]In the rest of this paper, we shall refer to classical utilitarianism as merely utilitarianism.

agents. On the contrary, the second-period consumption is differentiated according to their time preferences, but not according to their survival probabilities. Hence, the second-period consumption is higher for agents with a higher β_i, but is independent from π_j.[14]

Regarding the ranking of agents in terms of *ex post* lifetime welfare, it is clear that the worst-off agents are the short-lived, followed by β_1-type agents living two periods. The best-off agents are the β_2-type agents living two periods.[15] The utilitarian optimum thus tends to favour long-lived agents over short-lived agents, and patient agents (i.e β_2-type) over impatient agents (i.e. β_1-type).[16]

4.2 Maximin on CCPERL

Let us now contrast the utilitarian optimum with the Maximin on CCPERL. For that purpose, we will take the maximum length $\ell = 2$ as a reference level ℓ^*. By definition, the CCPERL for an agent of type (β_i, π_j) with an actual length of life $\ell = 1, 2$ is the constant consumption profile $\hat{x}_{ij\ell} = (\hat{c}_{ij\ell}, \hat{c}_{ij\ell})$ such that:

$$
\begin{aligned}
u\left(\hat{c}_{ij2}\right) + \beta_i u\left(\hat{c}_{ij2}\right) &= u\left(c_{ij}\right) + \beta_i u\left(d_{ij}\right) \\
u\left(\hat{c}_{ij1}\right) + \beta_i u\left(\hat{c}_{ij1}\right) &= u\left(c_{ij}\right)
\end{aligned}
$$

On the first line, $\hat{c}_{ij2}$ defines the consumption equivalent of an agent with time preference β_i and survival probability π_j who effectively lived two periods, while the second line defines a consumption equivalent $\hat{c}_{ij1}$ for a (β_i, π_j)-type agent living only one period. Note that since we take *ex-post* utilities on the right-hand side of these expressions, the CCPERL of an agent does not depend on his survival probability.[17]

Let us first define the consumption equivalent $\hat{x}_{ij\ell} = (\hat{c}_{ij\ell}, \hat{c}_{ij\ell})$ for each of the 8 groups of individuals that emerge *ex post*.

---

[14]This result is due to the fact that the survival probability π_j enters the social objective and the budget constraint in the same way.

[15]Note that since differences in survival do not affect the optimum, this ranking is similar to the one we would obtain if there was no difference in survival chances.

[16]This results follows from the additivity of (1) the utilitarian social welfare function and (2) individual lifetime welfare (see Bommier *et al.* 2009, 2010). Note, however, that relaxing (2) would not eradicate the utilitarian tendency to favour the long-lived (see Leroux and Ponthiere 2010).

[17]Thus we would obtain the same CCPERL if agents did not differ in survival chances.

β	π	ℓ	def. CCPERL $\hat{c}_{ij\ell}$
β_1	π_1	1	$u\left(\hat{c}_{111}\right)\left(1+\beta_1\right)=u\left(c_{11}\right)$
β_1	π_2	1	$u\left(\hat{c}_{121}\right)\left(1+\beta_1\right)=u\left(c_{12}\right)$
β_2	π_1	1	$u\left(\hat{c}_{211}\right)\left(1+\beta_2\right)=u\left(c_{21}\right)$
β_2	π_2	1	$u\left(\hat{c}_{221}\right)\left(1+\beta_2\right)=u\left(c_{22}\right)$
β_1	π_1	2	$u\left(\hat{c}_{112}\right)\left(1+\beta_1\right)=u\left(c_{11}\right)+\beta_1 u\left(d_{11}\right)$
β_1	π_2	2	$u\left(\hat{c}_{122}\right)\left(1+\beta_1\right)=u\left(c_{12}\right)+\beta_1 u\left(d_{12}\right)$
β_2	π_1	2	$u\left(\hat{c}_{212}\right)\left(1+\beta_2\right)=u\left(c_{21}\right)+\beta_2 u\left(d_{21}\right)$
β_2	π_2	2	$u\left(\hat{c}_{222}\right)\left(1+\beta_2\right)=u\left(c_{22}\right)+\beta_2 u\left(d_{22}\right)$

Table 1: Definition fo the consumption equivalents

To find the bundles that maximize the minimum CCPERL, we first need to identify the worst-off agents. We can first note that, as $u(0)>0$, short-lived agents are worse-off than long-lived agents of the same type, as death prevented them from enjoying the second period (which is positively valued). From this, it follows that the allocation that satisfies the Maximin on CCPERL is obtained by transferring second-period resources to the first period, i.e. by decreasing d_{ij} to 0:

$$d_{11}^* = d_{12}^* = d_{21}^* = d_{22}^* = 0.$$

Second, the CCPERL can be equalized among short-lived agents by increasing c_{2j} and decreasing c_{1j} until one reaches $\hat{c}_{111} = \hat{c}_{121} = \hat{c}_{211} = \hat{c}_{221}$. This is obtained by setting $c_{11}^* = c_{12}^* < c_{21}^* = c_{22}^*$. Hence, first-period consumption is larger for patient agents (i.e. with a high β_i) as they are more affected by a short life than impatient agents (i.e. with a low β_i). Thus, to compensate them, more consumption in the first period is needed. This justifies the differentiated treatment in terms of consumption between agents with different time preferences. We obtain the following ranking:

$$\hat{c}_{111} = \hat{c}_{121} = \hat{c}_{211} = \hat{c}_{221} < \hat{c}_{112} = \hat{c}_{122} < \hat{c}_{212} = \hat{c}_{222}$$

under our assumption $u\left(0\right)>0$. Thus, whereas the Maximin on CCPERL enables us to make some compensation of short-lived individuals, this does not, however, imply a *full* compensation, because the social planner cannot, *ex ante*, know the actual lengths of life. Moreover, as the above ranking shows, among the long-lived agents, there is also an inequality due to the larger benefit of living longer for patient agents.[18]

---

[18]Indeed one has, at the solution of the Maximin on CCPERL that,

$$u\left(\hat{c}_{112}\right)=\frac{u\left(c_{11}\right)+\beta_1 u\left(0\right)}{1+\beta_1}<u\left(\hat{c}_{212}\right)=\frac{u\left(c_{21}\right)+\beta_2 u\left(0\right)}{1+\beta_2}$$

16

Note that the social planner does not use the information on survival probabilities π_j to offer distinct consumption bundles to agents with different life expectancies. As mentioned earlier, this uniform treatment of agents with different π_j comes from the fact that survival probabilities do not influence the CCPERL, which is defined *ex post*, that is, once the risk of death has been resolved.[19] What we do here is to compensate short-lived agents, independently of how unlucky their death was.

All in all, this solution differs strongly from the utilitarian optimum, under which the optimal bundles included a positive consumption in the second period. The following proposition sums up the results of this section.

Proposition 1 *Assume that $u(0) > 0$, and that the social planner faces a unique intertemporal budget constraint. Under perfect information about ex-ante types (β_i, π_j):*

- *Utilitarianism equalizes first-period consumptions for all agents at a level that exceeds second-period consumptions. Agents with a low impatience benefit from a higher second-period consumption.*

- *Maximin CCPERL involves higher first-period consumptions for patient agents, and lower first-period consumptions for impatient agents. Second-period consumptions are all set to zero.*

- *Under both criteria, agents who differ in their survival probabilities but have the same preferences are treated identically. The introduction of heterogeneity in survival probabilities does not alter the optimal allocation.*

5 Second-best optimum

Up to now, we assumed that individual characteristics β_i and π_j were perfectly observable by the social planner. This section reexamines the utilitarian and egalitarian-equivalent solutions under asymmetric information, that is, when agents know their (β_i, π_j)-type, while the government only observes the distributions of types. The government can still propose different bundles to *ex ante* groups, but under the constraint of incentive compatibility.

---

with $c_{11} < c_{21}$ and $\beta_1 < \beta_2$. The same reasoning applies for $u(\hat{c}_{122}) < u(\hat{c}_{222})$.

[19]Note that this result is robust to the sign of $u(0)$, which does not need to be positive.

5.1 Utilitarian optimum

As we saw above, the utilitarian planner does not want, under perfect information, to give priority to agents on the basis of their survival probability: agents differing only in π_j were all treated equally in the first-best utilitarian optimum. However, under asymmetric information, one cannot exclude *a priori* a differentiation of bundles on the basis of π_j. Indeed, survival probabilities π_j now affect also the planner's problem by their presence in the incentive compatibility constraints.

To study the utilitarian problem under asymmetric information, we shall, for simplicity, focus here on the case where the difference in patience looms larger than the difference in survival probabilities:

$$\pi_1\beta_1 < \pi_2\beta_1 < \pi_1\beta_2 < \pi_2\beta_2$$

The agents' preferences satisfy the single-crossing property in the corresponding order. If the social planner proposed the first-best allocation, β_1-type agents would like to mimic β_2-type agents, independently of their survival chances. Hence, using also the above inequalities, the relevant incentive compatibility conditions are:[20]

$$
\begin{aligned}
u\left(c_{11}\right) + \pi_1\beta_1 u\left(d_{11}\right) &\geq u\left(c_{12}\right) + \pi_1\beta_1 u\left(d_{12}\right), \\
u\left(c_{12}\right) + \pi_2\beta_1 u\left(d_{12}\right) &\geq u\left(c_{21}\right) + \pi_2\beta_1 u\left(d_{21}\right), \\
u\left(c_{21}\right) + \pi_1\beta_2 u\left(d_{21}\right) &\geq u\left(c_{22}\right) + \pi_1\beta_2 u\left(d_{22}\right).
\end{aligned}
$$

Under the single-crossing property, these three incentive compatibility constraints suffice to avoid *any* mimicking behaviour. Intuitively, it must be the case that the optimal second-best allocation is such that agents with type (β_1, π_1) receive the highest first-period consumption and the lowest second-period consumption, followed by the other types as follows:

$$
\begin{aligned}
c_{11} &\geq c_{12} \geq c_{21} \geq c_{22}, \\
d_{11} &\leq d_{12} \leq d_{21} \leq d_{22}.
\end{aligned}
$$

---

[20]If agents had the same survival chances, there would be only one incentive constraint,

$$u\left(c_1\right) + \pi\beta_1 u\left(d_1\right) \geq u\left(c_2\right) + \pi\beta_1 u\left(d_2\right),$$

This ensures that impatient agents would not mimic patient ones under asymmetric information. As usual in this type of problem, the allocation of the mimicker (with time preference β_1) would not be distorted. But the second-period consumption of β_2-type agents would increase and their first-period consumption would decrease with respect to the first-best, in such a way as to relax an (otherwise binding) self-selection constraint. Hence, we would have: $d_1^* < c_1^*$, $c_2^* < c_1^*$ and $c_2^* \lessgtr d_2^*$.

Let us now check whether the planner, in contrast with the first-best optimum, wants to differentiate between agents on the basis of their survival probabilities. The problem of the planner is equivalent to the first-best problem, to which we add the above incentive constraints. Rearranging the FOCs of the (β_1, π_1)-type agents, we obtain the usual result of no distortion at the top for the extreme mimicker:

$$u'(c_{11}) = \beta_1 u'(d_{11}).$$

This trade-off, which is similar to the one we had in the first-best, yields $c_{11} > d_{11}$. On the contrary, for other agents (and eliminating from these equations the Lagrange multiplier associated with the resource constraint), we now have:

$$\frac{u'(c_{12})}{\beta_1 u'(d_{12})} = \frac{1 - \mu_1 \frac{\pi_1}{\pi_2} + \mu_2}{1 - \mu_1 + \mu_2}$$

$$\frac{u'(c_{21})}{\beta_2 u'(d_{21})} = \frac{1 - \mu_2 \frac{\pi_2 \beta_1}{\pi_1 \beta_2} + \mu_3}{1 - \mu_2 + \mu_3}$$

$$\frac{u'(c_{22})}{\beta_2 u'(d_{22})} = \frac{1 - \mu_3 \frac{\pi_1}{\pi_2}}{1 - \mu_3}.$$

where μ_k, $k = 1, 2, 3$ are the Lagrange multipliers associated with the incentive compatibility constraints. The possibility of mimicking behaviors then leads the planner to give distinct bundles to agents who have not only different time preferences but also different survival chances. For instance, bundles (c_{11}, d_{11}) and (c_{12}, d_{12}) are different whenever the (corresponding) first incentive constraint is binding (i.e. $\mu_1 > 0$). However, from these equations it is impossible to exclude the possibility of bunching among the other agents.

Let us now see how these incentive constraints modify the optimal allocation of each type. As the right-hand sides of these expressions are all greater than one, incentive constraints push toward more consumption in the second period for all types, in such a way as to discourage these agents from pretending to be "apparently patient" (apparent patience may be due to a high β_i or a high π_j). To see this, let us study the first equation. The trade-off between (c_{12}, d_{12}) is distorted upward so as to avoid mimicking from (β_1, π_1)-type agents. Indeed, it is optimal to encourage second-period consumption for (β_1, π_2)-type agents as compared to the first-best trade-off, in order to discourage (β_1, π_1)-type agents from pretending to be of that type. By doing so, this bundle becomes less interesting to (β_1, π_1)-type agents, as they would obtain too much consumption in the second period and not enough in the first one, given that they face a lower survival chance $\pi_1 < \pi_2$. Hence, if $\pi_1 = \pi_2$, (β_1, π_1)-

19

type agents and (β_1, π_2)-type agents would be identical, and we would obtain the first-best trade-off.[21]

In comparison with the first-best, the relative difference between first-period and second-period consumptions is lower, as the introduction of incentive constraints pushes towards more consumption in the second period. Yet, very little can be said on welfare inequalities between agents with different types. As compared to the first-best, short-lived agents are no longer treated equally, as first-period consumptions may now be different for individuals with different (β_i, π_j).

Moreover, under asymmetric information, short-lived agents are still worse-off than the long-lived, and this inequality may even be increased by the introduction of incentive constraints, as it encourages consumption in the second period. Also, among long-lived agents, it is not sure who would end up with the highest welfare.

5.2   Maximin on CCPERL

As in the first-best, we take the maximum length $\ell = 2$ as the reference level ℓ^* for defining the CCPERL. Let us first recall that in the first-best, first-period consumptions are distributed among agents only according to their time preferences, and second-period consumptions are set to zero. Hence, under asymmetric information, independently of their survival chance π_j, only type-β_1 agents would like to mimic type-β_2 agents, so that the second-best allocation now has to also satisfy the following incentive compatibility constraint,

$$u\left(c_{1j}\right) + \pi_j \beta_1 u\left(d_{1j}\right) \geq u\left(c_{2j}\right) + \pi_j \beta_1 u\left(d_{2j}\right), \ \forall j$$

The first-best egalitarian-equivalent optimum, with $c_{11} = c_{12} < c_{21} = c_{22}$ and $d_{ij} = 0$ for all i, j, is not incentive-compatible because it violates the above condition. As the Maximin on CCPERL focuses on short-lived agents, it still makes sense to keep $d_{ij}^* = 0$, but we now need, in order to satisfy the incentive constraint, to equalize first-period consumptions. In sum, the second-best Maximin CCPERL solution is:

$$c_{11}^* = c_{12}^* = c_{21}^* = c_{22}^* > d_{11}^* = d_{12}^* = d_{21}^* = d_{22}^* = 0.$$

As a consequence, we have

$$\begin{aligned} \hat{c}_{211} &= \hat{c}_{221} < \hat{c}_{111} = \hat{c}_{121} < \hat{c}_{112} = \hat{c}_{122}, \\ \hat{c}_{211} &= \hat{c}_{221} < \hat{c}_{212} = \hat{c}_{222}. \end{aligned}$$

---

[21] The same reasoning applies for the other trade-offs.

20

In comparison with the first-best, it is no longer possible to equalize the consumption-equivalent of short-lived agents, as this would require consumption inequalities that violate the incentive-compatibility constraint. Moreover, it is no longer always true that $\hat{c}_{212}$ and $\hat{c}_{222}$ are the greatest, because now $c_{ij}^* = \bar{c}$ for all i, j, and $\beta_1 < \beta_2$.

Let us briefly sum up the results of this section.

Proposition 2 *Assume that $u(0) > 0$, and that the social planner faces a unique intertemporal budget constraint. Under asymmetric information about ex ante types (β_i, π_j):*

- *Utilitarianism gives a first-period consumption greater than the second period consumption to agents with a low patience or low survival probability. For the other agents, the introduction of incentive constraints pushes towards more consumption in the second period. Individual bundles should be differentiated according to their time preference but also according to their survival chance. We may have pooling for some types.*

- *Maximin CCPERL involves a perfect equalization of first-period consumptions for all agents. Second-period consumptions are all equal to zero.*

A common feature of the utilitarian and egalitarian-equivalent solutions is that consumptions are, under each social objective, not smoothed across time. Nevertheless, second-period consumptions are zero in the egalitarian-equivalent solution, whereas they are strictly positive under utilitarianism, so that the departure from a smoothed consumption is much larger under the egalitarian-equivalent solution. Hence welfare inequalities between short-lived and long-lived agents are larger under utilitarianism than under the egalitarian-equivalent solution. Moreover, under utilitarianism, first-period consumptions are not equalized across agents, whereas under the egalitarian-equivalent solution, first-period consumptions are equalized. This difference comes from the fact that, under utilitarianism, the distortion induced by the incentive constraints acts on both first-period and second-period consumptions. On the contrary, under the egalitarian-equivalent solution, the distortion must be only on the first-period consumption, as changing second-period consumptions would raise inequalities between short-lived and long-lived agents.

6   Extensions and generalisations

As shown in the previous sections, the Maximin on CCPERL yields rather extreme solutions. The optimal allocation is a corner solution, as it involves zero second-

period consumption. In this section, we propose to check whether the specific assumptions we made in Sections 4 and 5 are responsible for this result. For this purpose, in this section we will relax different assumptions successively, and discuss the robustness of the solution to those changes.

Firstly, we will consider more general preferences, by relaxing the assumption of a positive utility of survival (i.e. $u(0) > 0$). Secondly, we discuss the possibility of adopting a reference lifetime ℓ^* lower than maximum longevity (i.e. 2 periods). Finally, while we assumed so far that the social planner controlled the allocation of resources over time, we will consider the case where agents can freely transfer resources across periods.

6.1   The utility of survival

Let us first assume that the utility of zero consumption is zero: $u(0) = 0$. In this case, the individual does not gain any utility from his mere survival. Obviously, Table 1 is independent of the assumption on $u(0)$, so that our previous reasoning still holds. Thus, in the first-best egalitarian-equivalent optimum, we obtain that

$$
\begin{aligned}
c_{11}^* &= c_{12}^* < c_{21}^* = c_{22}^* \\
d_{11}^* &= d_{12}^* = d_{21}^* = d_{22}^* = 0
\end{aligned}
$$

The only difference with respect to the case where $u(0) > 0$ comes from the *ranking* of agents in terms of CCPERL. We now have that all CCPERL are equalized across agents with different lengths of life, different survival probabilities and different time preferences, unlike the case with $u(0) > 0$ (for which long-lived agents had higher consumption-equivalents). The assumption $u(0) = 0$ allows for a *complete* compensation of short-lived agents.

Let us now turn to the second-best (asymmetric information). As the first-best allocation under $u(0) = 0$ is identical to the one under $u(0) > 0$, incentive constraints are also identical, so that it still optimal to provide $c_{ij}^* = c^*$ and $d_{ij}^* = 0$. Again, only the ranking in terms of CCPERL changes:

$$
\hat{c}_{111} = \hat{c}_{121} = \hat{c}_{112} = \hat{c}_{122} > \hat{c}_{211} = \hat{c}_{221} = \hat{c}_{212} = \hat{c}_{222}
$$

which, looking at Table 1, is a direct consequence of the differences in time preferences. Our results are summarized in the following proposition.

Proposition 3 *Assume that $u(0) = 0$. In the first-best, the CCPERL is equalized across all agents by fixing second-period consumption to zero, and by giving a higher*

first-period consumption to patient agents. In the second-best, the optimal allocation consists in giving the same first-period consumption to all agents and zero second-period consumption.

Let us now turn to the case in which $u(0) < 0$, and define d^* as the level of consumption such that $u(d^*) = 0$. The first-best optimum still equalizes CCPERL across all agents, but is now modified. Three cases should be distinguished, depending on how large the available resources W are.

If $W > d^* [4 + 2(\pi_1 + \pi_2)]$, it is optimal to fix second-period consumptions to d^*, and to give a higher first-period consumption to patient agents. That allocation equalizes CCPERL across agents with unequal longevities, because living a second period with d^* or not is a matter of indifference. Note that, while the positive second-period consumption induced by $u(0) < 0$ may seem to make the egalitarian-equivalent solution less "extreme" than before, it remains that this solution only assigns long-lived agents a consumption that makes their survival equivalent to death. Thus, even if the Maximin CCPERL seems less extreme than in the benchmark case, the underlying idea is the same: fully compensating short-lived agents implies making the survival of long-lived agents worthless. If, alternatively, $d^* 2(\pi_1 + \pi_2) < W < d^* [4 + 2(\pi_1 + \pi_2)]$, the Maximin CCPERL gives d^* as second-period consumption, and a first-period consumption lower than d^*, with a higher level for patient agents. Finally, if $W < d^* 2(\pi_1 + \pi_2))$, it is optimal to have a second-period consumption as close as possible to d^*, while fixing first-period consumption to 0.

Consider now the asymmetric information context, and focus on the case in which $W > d^* [4 + 2(\pi_1 + \pi_2)]$. The second-best optimum does not necessarily consist in giving d^* to all old agents and an identical bundle c^* to all young agents. Suppose, for instance that β_1 is close to zero. By offering a menu of two bundles, (c^*, d^*) and $(c^* + \varepsilon, 0)$, for a (not too) small ε, one may induce the impatient agents to choose the latter. This frees resources and makes it possible to achieve a larger c^* than if the same bundle (c^*, d^*) was offered to everyone. The worst-off agents are the patient agents who choose (c^*, d^*), whether they die young or survive, and it is then worth maximizing c^* in order to maximize the lowest CCPERL. Note that in this configuration compensation for a short life is over-achieved among impatient agents: those who die early are better off. Our results are summarized below.

Proposition 4 *Assume that $u(0) < 0$ and that $W > d^* [4 + 2(\pi_1 + \pi_2)]$. In the first-best, the CCPERL is equalized across all agents by fixing the second-period consumption to d^*, and by giving a higher first-period consumption to patient agents. In the second-best, the optimal allocation does not necessarily consist in giving the same consumption to all young agents and d^* to all old agents.*

23

Finally, it should be stressed that the intercept of the temporal utility function plays a more important role when we consider a general framework with $T > 2$ life-periods. Indeed, in that case, a strictly positive intercept $u(0) > 0$ would, under time-additive lifetime welfare, lead to large differentials between the welfares of short-lived and long-lived agents, since the utility from mere survival would accumulate over time for survivors. Hence, under $u(0) > 0$, the Maximin CCPERL would only provide *partial* compensation. However, if we make the more plausible assumption $u(0) \leq 0$, then the extension to $T > 2$ periods would not prevent a complete compensation to short-lived agents. In that case, the consumption at all ages beyond the first deaths would be equal to subsistence consumption d^*, yielding, by definition, no welfare gain from surviving one or many extra life-periods.

A graphical analysis is helpful. We shall focus here on the second-best optimum. Let us now assume a distribution of $\beta \in \left[\underline{\beta}, \overline{\beta}\right]$ and of $\pi \in \left[\underline{\pi}, \overline{\pi}\right]$.[22] Incentive constraints impose to give all agents (who cannot be distinguished *ex ante* by the planner) the same budget set, represented by the decreasing line in Figure 4 below. The optimistic $(\overline{\pi})$ patient $(\overline{\beta})$ agents choose the bundle (c, d), that is, from all chosen bundles, the farthest on the left, while the pessimistic impatient $(\underline{\beta}, \underline{\pi})$ agents choose the bundle that is the most on the right.

Let us first focus on the case in which $u(0) = 0$, which is illustrated in Figure 4. The CCPERL index, with $\ell^* = 2$, is computed as the solution to

$$\begin{aligned} u(\hat{c}) + \beta u(\hat{c}) &= u(c) + \beta u(d) &&\text{if the agent lives two periods,} \\ u(\hat{c}) + \beta u(\hat{c}) &= u(c) &&\text{if the agent lives one period.} \end{aligned}$$

We then compare the welfare of the agents who consume (c, d) and those who consume c and die. It is clear from above that, for given preferences, the latter are the worst-off since $u(d) > 0$ when $d > 0$. For the short-lived agents, the intersection between the indifference curve containing the point $(c, 0)$ and the 45° line gives the level of $\hat{c}$. Graphically, the worst-offs, *ex post*, are those who belong to the $(\overline{\beta}, \overline{\pi})$ group and die young, as they have the smallest $\hat{c}$ (i.e. the closest to the left). The arrow on the figure shows the CCPERL index for those individuals.[23]

Let us now consider the case in which $u(0) > 0$. Again, for given preferences, the worst-off agents are those who die young (because $u(d) > 0$). In order to visualize their situation, it is convenient to extend $u(\cdot)$ to negative values, and to find $d^* < 0$

---

[22] This does not change the analysis as long as we assume that there are agents with characteristics $(\overline{\beta}, \overline{\pi})$.

[23] In Figure 4, from point $(c, 0)$, we draw the indifference curves of short-lived individuals with same survival prospects but with different β, to make explicit that individuals with $\overline{\beta}$ are worse-off than if they had $\underline{\beta}$ (i.e., indifference curves are steeper in the latter case).
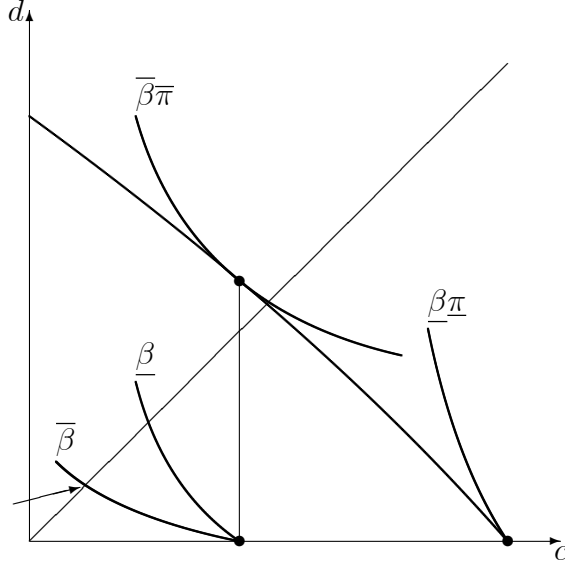
Figure 3: Illustration of the argument when $u(0) = 0$.

such that $u(d^*) = 0$. For short-lived agent one then computes the CCPERL by solving:

$$u(\hat{c}) + \beta u(\hat{c}) = u(c) = u(c) + \beta u(d^*).$$

Looking at Figure 5, this corresponds to finding the point (c, d^*) and drawing the indifference curve that goes through that point. The intersection with the 45° line gives the agent's CCPERL. Again, *ex post*, the worst-off are the $(\overline{\beta}, \overline{\pi})$ type who die young, as again their CCPERL are the closest to the left.

From this graphical analysis, it is easy to recover the analytical results of the previous section about the second-best policy. Moreover, one can also obtain a simple way to evaluate arbitrary policies, by observing that the CCPERL of the short-lived $(\overline{\beta}, \overline{\pi})$-type depends only on the first-period consumption of this type, which is the lowest by incentive compatibility.

Proposition 5 *If $u(0) \geq 0$, the comparison of two budget sets is made as follows: the better budget set is the one that induces the larger level of lowest first-period consumption.*
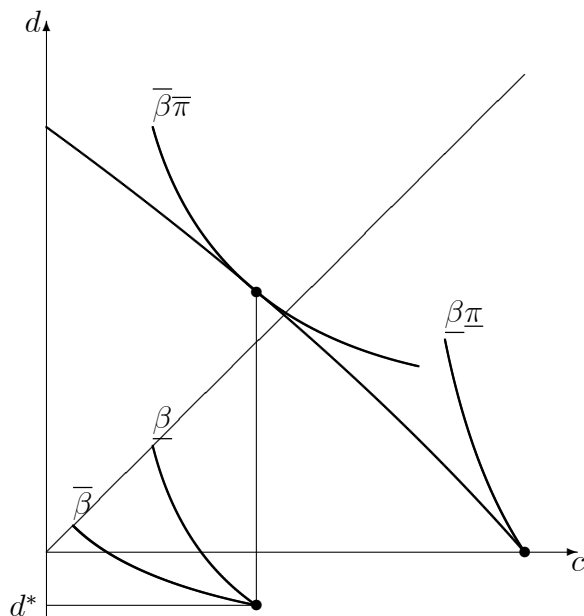
25

Figure 4: Illustration of the argument when $u(0) > 0$.

Let us finally illustrate the case in which $u(0) < 0$. We now define $d^* > 0$ such that $u(d^*) = 0$. We first consider the "normal" case in which all agents choose a point (c, d) such that $c \geq d^*$ and thus $u(c) > 0$. If $d > d^*$, then, for given preferences, the worst-offs are still those who die young, because $u(d) > 0$. Figure 6 illustrates that case. If, on the contrary, one had $d < d^*$, the worst-off agents would be the long-lived agents.

When some agents choose (c, d) such that $c < d^*$, the worst-off are not necessarily agents with characteristics $(\bar{\beta}, \bar{\pi})$, depending of the precise configuration, but certainly the worst-off agents are among those who choose in this way. Indeed, their CCPERL index is then less than d^* (whatever the reference longevity), whereas those who choose (c, d) such that $c \geq d^*$ have a CCPERL index at least as great as d^*.

6.2   Reference longevity

Let us now examine the sensitivity of the Maximin CCPERL solution to the longevity level chosen as a reference. As shown in Section 3, the CCPERL is constructed for a particular reference longevity level, at which comparisons in terms of dominance
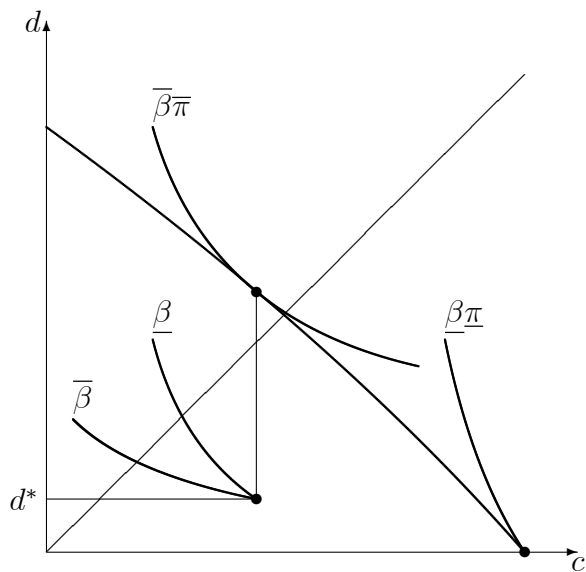
Figure 5: Illustration of the argument when $u(0) < 0$.

of consumption bundles can be made independently of the agents' preferences over longevity. Given that there are several candidates for the reference longevity level, it makes sense to study the robustness of our solution to this reference.

Until now, we have assumed that the reference longevity was the maximum length of life, i.e. $\ell^* = 2$, and computed the CCPERL for all individuals on the basis of that reference longevity. Let us now assume, alternatively, that the reference longevity ℓ^* is the minimum longevity (i.e. 1 period). Under this assumption, Table 1 becomes:

β	π	ℓ	def. CCPERL $\hat{c}_{ij\ell}$
β_1	π_1	1	$u\left(\hat{c}_{111}\right) = u\left(c_{11}\right)$
β_1	π_2	1	$u\left(\hat{c}_{121}\right) = u\left(c_{12}\right)$
β_2	π_1	1	$u\left(\hat{c}_{211}\right) = u\left(c_{21}\right)$
β_2	π_2	1	$u\left(\hat{c}_{221}\right) = u\left(c_{22}\right)$
β_1	π_1	2	$u\left(\hat{c}_{112}\right) = u\left(c_{11}\right) + \beta_1 u\left(d_{11}\right)$
β_1	π_2	2	$u\left(\hat{c}_{122}\right) = u\left(c_{12}\right) + \beta_1 u\left(d_{12}\right)$
β_2	π_1	2	$u\left(\hat{c}_{212}\right) = u\left(c_{21}\right) + \beta_2 u\left(d_{21}\right)$
β_2	π_2	2	$u\left(\hat{c}_{222}\right) = u\left(c_{22}\right) + \beta_2 u\left(d_{22}\right)$

Table 2: CCPERL for reference longevity l*=1

For simplicity, we will assume here that $u\left(0\right) > 0$. In order to find the bundles maximizing the minimum CCPERL, we first need to identify the worst-off individuals. Here again it is clear that the long-lived individuals are better-off than the short-lived individuals. Therefore the optimal allocation must have

$$d_{11}^* = d_{12}^* = d_{21}^* = d_{22}^* = 0$$

It is also obvious that equalizing the CCPERL of the short-lived individuals is achieved by equalizing their consumption. One must therefore have

$$c_{11}^* = c_{12}^* = c_{21}^* = c_{22}^* = c^*$$

Note that this equalization of all first-period consumptions differs from what prevailed under $\ell^* = 2$, where patient agents received a higher first-period consumption than impatient agents. The reason is that, when the reference longevity is the minimum longevity (i.e. under $\ell^* = 1$), the CCPERL of the short-lived becomes independent from time preferences, contrary to what was the case when the reference longevity was assumed to be the maximum longevity. Actually, when the reference longevity is one period, all short-lived agents become equal, whether they are patient or not, and this explains why they all have the same compensation.

Note that our first-best optimum is also incentive compatible, and, therefore, optimal in the second-best context.

Proposition 6 *Assume that $u(0) > 0$, and that the social planner faces a unique intertemporal budget constraint. In the first-best, the Maximin CCPERL under $\ell^* = 1$ equalizes all first-period consumptions, and sets all second-period consumptions to zero. In the second-best, the Maximin CCPERL under $\ell^* = 1$ coincides with the first-best allocation and is exactly the same as under $\ell^* = 2$.*

In sum, this subsection reveals that the choice of a particular reference longevity level has some effects on the first-best egalitarian-equivalent solution, but is less crucial in the second-best context. All in all, one should not exaggerate the influence of the reference longevity on the Maximin CCPERL solution. Whatever ℓ^*, it keeps the property of decreasing optimal consumption profiles, in such a way as to compensate short-lived agents.[24]

6.3 Savings

Up to now, we have assumed that agents could not save resources from one period to the other, or, equivalently, that their savings were controlled by the social planner. In this section, we relax that assumption and assume that savings are completely free. For simplicity, we keep the assumption $u(0) > 0$.

Now the planner must, at the beginning of the first period, give an endowment to individuals, which they can freely allocate between their two periods of life. We denote by W_{ij} the amount of resources the social planner gives to individuals with time preference factor β_i and survival probability π_j.[25]

Hence, when the social planner provides W_{ij} to agents, they first decide how to allocate it between first-period and second-period consumptions, by solving the problem:

$$\max u\left(c_{ij}\right) + \beta_i \pi_j u\left(d_{ij}\right)$$
$$\text{s. to } c_{ij} + d_{ij} \leq W_{ij},$$

so that the indirect utility function of a (β_i, π_j)-type agent is

$$V_{ij}\left(W_{ij}\right) = u\left(c_{ij}\left(W_{ij}\right)\right) + \beta_i \pi_j u\left(d_{ij}\left(W_{ij}\right)\right),$$

where $c_{ij}\left(W_{ij}\right), d_{ij}\left(W_{ij}\right)$ are obtained from solving the agent's problem. It is clear from this maximization problem that, if $W_{ij} = \bar{W}$ for all i, j, we would have $d_{11} < d_{12}, d_{21} < d_{22}$, as impatient agents with a lower survival probability prefer to consume more in the beginning of their life. In this alternative setting, we redefine the consumption equivalent $\hat{c}_{ij\ell}$ for each of the 8 groups of agents:

---

[24]Similarly, one could explore variants to the reference to a constant consumption in the definition of CCPERL, without finding much change to the main policy conclusions.

[25]Note that, since these resources are allocated at the beginning of the first period, the social planner cannot distinguish between agents who live long and those who die in the end of the first period. This will has consequences on the optimal allocation, as we shall see.

β	π	ℓ	def. CCPERL $\hat{c}_{ij\ell}$
β_1	π_1	1	$u\left(\hat{c}_{111}\right)\left(1+\beta_1\right)=u\left(c_{11}\left(W_{11}\right)\right)$
β_1	π_2	1	$u\left(\hat{c}_{121}\right)\left(1+\beta_1\right)=u\left(c_{12}\left(W_{12}\right)\right)$
β_2	π_1	1	$u\left(\hat{c}_{211}\right)\left(1+\beta_2\right)=u\left(c_{21}\left(W_{21}\right)\right)$
β_2	π_2	1	$u\left(\hat{c}_{221}\right)\left(1+\beta_2\right)=u\left(c_{22}\left(W_{22}\right)\right)$
β_1	π_1	2	$u\left(\hat{c}_{112}\right)\left(1+\beta_1\right)=u\left(c_{11}\left(W_{11}\right)\right)+\beta_1 u\left(d_{11}\left(W_{11}\right)\right)$
β_1	π_2	2	$u\left(\hat{c}_{122}\right)\left(1+\beta_1\right)=u\left(c_{12}\left(W_{12}\right)\right)+\beta_1 u\left(d_{12}\left(W_{12}\right)\right)$
β_2	π_1	2	$u\left(\hat{c}_{212}\right)\left(1+\beta_2\right)=u\left(c_{21}\left(W_{21}\right)\right)+\beta_2 u\left(d_{21}\left(W_{21}\right)\right)$
β_2	π_2	2	$u\left(\hat{c}_{222}\right)\left(1+\beta_2\right)=u\left(c_{22}\left(W_{22}\right)\right)+\beta_2 u\left(d_{22}\left(W_{22}\right)\right)$

Table 3: Consumption equivalents when individuals can save.

Long-lived agents are better-off than short-lived ones given $u(0)>0$. The planner can equalize the CCPERL of the short-lived agents by distributing $W_{11}, W_{12}, W_{21}, W_{22}$ such that for some α, for all i,j,

$$\frac{u\left(c_{ij}\left(W_{ij}\right)\right)}{1+\beta_i}=\alpha.$$

As $c_{11}(W)>c_{12}(W)$ and $c_{21}(W)>c_{22}(W)$, this implies that

$$W_{11}<W_{12} \text{ and } W_{21}<W_{22}.$$

If $\beta_1\pi_2<\beta_2\pi_1$ (i.e. differences in patience are more important than differences in survival probabilities), one has $c_{12}(W)>c_{21}(W)$. In order to obtain

$$\frac{u\left(c_{12}\left(W_{12}\right)\right)}{u\left(c_{21}\left(W_{21}\right)\right)}=\frac{1+\beta_1}{1+\beta_2},$$

which implies $c_{12}\left(W_{12}\right)<c_{21}\left(W_{21}\right)$, one must have $W_{12}<W_{21}$. In sum, we have

$$W_{11}<W_{12}<W_{21}<W_{22}.$$

There are two main differences with respect to the standard case (Section 4.2). First, the planner now differentiates bundles also with respect to survival probabilities, contrary to the case where agents could not save. The planner now takes into account that, when agents can transfer resources to the second period, their actual consumption in the first period depends both on their time preferences and on their survival chances. Hence, agents who die early are more penalized by death when they had better survival prospects because they saved more. Second, inequalities in CCPERL between short-lived and long-lived agents are larger than in the standard

case. This is due to the fact that, since W_{ij} is given to agents *before* they know their length of life, agents always save some resources for the second period (thus $d_{ij} > 0$ for survivors). The compensation made by the planner is then limited by the possibility of individual savings. For the same reason, it is no longer possible to equalize the CCPERLs of long-lived agents with equal preferences.

Finally, we solve the problem under asymmetric information. If the planner were to propose the first-best bundles, individuals would always have interest in claiming to be a (β_2, π_2)-type agent. Hence, in order to solve the incentive problem, the optimum requires that the allocation is such that

$$W_{11}^* = W_{12}^* = W_{21}^* = W_{22}^*.$$

All possibilities of compensatory redistribution are gone in this context. This generates the following ranking among short-lived agents (assuming $\beta_1 \pi_2 < \beta_2 \pi_1$):

$$\hat{c}_{111} > \hat{c}_{121} > \hat{c}_{211} > \hat{c}_{221}.$$

Proposition 7 summarizes our results (compare with Proposition 2):

Proposition 7 *Assume that $u(0) > 0$, and that both the social planner and agents face a unique intertemporal budget constraint. In the first-best, the Maximin CCPERL differentiates individual endowments W_{ij} according to time preferences and survival probabilities. In the second-best, the Maximin CCPERL gives the same bundle to all.*

Assuming that agents can save at the same rate as the government nullifies the possibilities of compensation between long-lived and short-lived agents. This should be viewed as an extreme case, as the opposite extreme from the assumption that the agents cannot save at all. In the intermediate case in which the planner can tax savings and redistribute the proceeds, without being able (for technical or political reasons) to impose a prohibitive tax, the optimal policy under CCPERL would adopt the greatest admissible tax in order to maximize first-period consumptions.

7   Concluding remarks

Can one compensate the dead? Such a compensation seems impossible: short-lived persons are hard to identify *ex ante*, and, once dead, it is too late. However, this study provides a positive answer: one solution is to allocate resources *ex ante* in such a way as to maximize the minimum Constant Consumption Profile Equivalent on the Reference Lifetime. The Maximin CCPERL solution involves, in general, declining

consumption profiles, and provides a full compensation to the short-lived when the utility from mere survival is non-positive.[26]

Is the Maximin CCPERL solution applicable, and by means of which instruments? Regarding pensions, our egalitarian-equivalent approach would recommend, under a Pay-As-You-Go pension system, to reduce contributions from the young, and to reduce pensions of the elderly to subsistence levels. There would also be little encouragement for savings, since savings always makes the short-lived worse off. Regarding the age of retirement, our approach would recommend a lifecycle that differs from the common one. Instead of leaving the enjoyment of retirement to ages not reached by the worst-offs, our approach would recommend an early "break" period which could be enjoyed by all. Recent protests against raising retirement age in France did prominently mention the unfair situation of categories of workers who die earlier than others.

Comparing the Maximin CCPERL solution with the actual consumption profiles, which exhibit, in general, an inverted-U shape (see Lee and Tuljapurkar, 1997), suffices to show how real economies depart from our egalitarian-equivalent approach to social justice. Clearly, the first, increasing part of the actual age-consumption profile is not hardly compatible with our results. While popular wisdom usually encourages youngsters to enjoy life when they can, it also carries an "ideal" of a long and constantly happy life. The ideas of discouraging savings and of favoring early gifts over bequests (that may come too late) are not common. Our analysis, however, reveals the hidden unfairness in the prevailing ideal of a long and happy life. While it is rational for every individual to pursue this ideal, letting all organize their life around it is the source of deep inequalities due to differential longevity, and ultimately runs against the *ex post*, true interests of the unlucky who die young. Policy-makers generally worry about myopic behaviors and seek ways to encourage or force people to save for their old age. Myopia, after all, might be the best ally of fairness for the short-lived.

Applying the Maximin CCPERL would thus lead to a radical change of perspective and a vast reorganization of the lifecycle. However, such changes may not be implementable in a political democracy, for several reasons. First, our egalitarian-equivalent approach would be beneficial, *ex post,* to the short-lived, but these are, by definition, unable to vote, *ex post*, in favor of this approach. In particular, young people may not adhere to the approach because death is a taboo. Even if death is the only certain event of a life, it remains a thing that people prefer to forget. Third, our societies are ageing societies, and thus older voters may prefer to improve their

---

[26]But welfare differentials between short-lived and long-lived remain otherwise, in particular under the possibility of individual savings.

lot, even if this reinforces arbitrary inequalities caused by Nature.

Two important issues have been left for future research. Some longevity inequalities result from individual behaviours, not simply from natural factors. One might think that, intuitively, risk-taking short-lived agents should be less compensated than other short-lived persons, as the former are *more responsible* for their short life than the latter. Incorporating this element of responsibility in the analysis of compensation for a short life requires a substantial extension of the setting and a reexamination of the social objective.

We have also ignored the possibility that the agents derive utility from leaving a bequest. If an agent who dies young leaves a greater bequest than an otherwise similar agent who lives longer, the presence of bequest utility achieves a partial compensation for dying young. If, on the contrary, living longer makes it possible to accumulate more and leave a greater legacy, then this reinforces the inequalities due to differential longevity. A serious study of this issue requires a richer model with production, and also a refinement of the Maximin CCPERL criterion.

8 References

Becker, G., Philipson, T., and R. Soares, 2005. 'The quantity and the quality of life and the evolution of world inequality', *American Economic Review*, 95(1), pp. 277-291.

Bommier, A., M-L. Leroux and J-M. Lozachmeur, 2009. 'On the public economics of annuities with differential mortality', CORE Discussion Paper 2009-30.

Bommier, A., M-L. Leroux and J-M. Lozachmeur, 2010. 'Differential mortality and social security', Canadian Journal of Economics, forthcoming.

Christensen, K., Johnson, T., and J. Vaupel, 2006. 'The quest for genetic determinants of human longevity: challenges and insights', Nature Reviews - Genetics, 7, pp. 436-448.

Fleurbaey, M. and Maniquet, F., 2011, A Theory of Fairness and Social Welfare, Cambridge: Cambridge University Press.

Hammond P.J. 1979, 'Equity in two-person situations', Econometrica, 47: 1127-1136.

Hansson, B., 1973. 'The independence condition in the theory of social choice', Theory and Decision, 4, pp. 25-49.

Human Mortality Data Base, 2010. Cohort Life Tables, Sweden, available online at http://www.mortality.org

Kaplan, G.A, T.E. Seeman, R.D. Cohen, L.P. Knudsen and J. Guralnik (1987). 'Mortality among the elderly in the Alameda county study: behavioral and demographic risk factors'. American Journal of Public Health, 77(3), pp. 307-312.

Lee, R. & Tuljapurkar, S., 1997. 'Economic consequences of aging for populations and individuals death and taxes: longer life, consumption and social security', Demography, 34 (1), pp. 67-81.

Leroux, M.L. and Ponthiere, G., 2010. 'Utilitarianism and unequal longevities: a remedy?', CORE Discussion Paper 2010-45.

Leroux, M.L., Pestieau, P. & Ponthiere, G., 2010. 'Optimal linear taxation under endogenous longevity', Journal of Population Economics, forthcoming.

Maniquet, F. and Sprumont, Y., 2004. 'Fair production and allocation of an excludable nonrival good', Econometrica, 72 (2), pp. 627-640.

Pazner, E., 1979. 'Equity, nonfeasible alternatives and social choices: a reconsideration of the concept of social welfare', in J.J. Laffont (ed.): Aggregation and Revelation of Preferences, North Holland, Amsterdam.

Pazner, E. & Schmeidler, D., 1978. 'Egalitarian equivalent allocations: a new concept of economic equity', Quarterly Journal of Economics, 92, pp. 671-687.

9   Appendix: Proof of Theorem 1

Lemma 1 *Assume that the social ordering function $\succsim$ satisfies Axioms 1-2-3 on $\Re^{|N|}$. Then $\succsim$ is such that for all $R_N \in \Re^{|N|}$, all x_N, $y_N \in X^{|N|}$, and all i, $j \in N$, if $R_i = R_j$ and*

$$y_i P_i x_i P_i x_j P_j y_j$$

and $x_k P_k y_k$ for all $k \neq i, j$, then

$$x_N \succsim_{R_N} y_N.$$

Proof. If $\lambda(x_i) = \lambda(y_i) = \lambda(x_j) = \lambda(y_j)$, the result follows from Fleurbaey and Maniquet (2011, Lemma A.1).

By assumption on $\Re$, there exist $\bar{x}_N, \bar{y}_N$ such that:

1) $\lambda(\bar{x}_i) = \lambda(\bar{y}_i) = \lambda(\bar{x}_j) = \lambda(\bar{y}_j) = T$;

2) $\bar{y}_i P_i' y_i P_i' x_i P_i' \bar{x}_i P_i' x_j P_j' \bar{x}_j P_j' \bar{y}_j P_j' y_j$;

3) $x_k P_k \bar{x}_k P_k \bar{y}_k P_k y_k$.

By Weak Pareto, $x_N \succ_{R_N} \bar{x}_N$ et $\bar{y}_N \succ_{R_N} y_N$. By the previous step, $\bar{x}_N \succsim_{R_N} \bar{y}_N$. By transitivity, $x_N \succsim_{R_N} y_N$. ∎

Lemma 2 *Assume that the social ordering function $\succsim$ satisfies Axioms 1-2-3-4 on $\Re^{|N|}$. Then $\succsim$ is such that for all $R_N \in \Re^{|N|}$, all x_N, $y_N \in X^{|N|}$, and all i, $j \in N$, if*

$$\hat{y}_i > \hat{x}_i > \hat{x}_j > \hat{y}_j$$

and $x_k P_k y_k$ for all $k \neq i, j$, then

$$x_N \succsim_{R_N} y_N.$$

Proof. The allocations constructed in this proof all involve a longevity equal to ℓ^* and a constant consumption for i, j (this will not be repeated below). Let R^* denote the Leontief preferences represented by $\min \{x_{ik} \mid x_{ik} > 0, \ 1 \le k \le T\}$.

Let z_N^1, z_N^2, z_N^3 be such that:

1) $z_i^1 > \hat{y}_i > z_i^2 > \hat{x}_i > z_i^3 > \hat{x}_j > z_j^3 > z_j^2 > z_j^1 > \hat{y}_j$;

2) $z_i^2 - z_i^3 = z_j^3 - z_j^2$;

3) $x_k P_k z_k^3 = z_k^2 P_k z_k^1 P_k y_k$ for all $k \ne i, j$.

Let R_N', R_N'' be such that:

1) $I(x_k, R_k') = I(x_k, R_k), \ I(y_k, R_k') = I(y_k, R_k)$ for $k = i, j$;

2) $I(x_k, R_k'') = I(x_k, R_k), \ I(z_k^1, R_k'') = I(z_k^1, R_k)$ for $k = i, j$;

3) $I(z_i^1, R_i') = I(z_i^1, R^*), \ I(z_i^2, R_i'') = I(z_i^2, R^*)$, and $R_i'' = R_j''$;

4) $R_k' = R_k'' = R_k$ for all $k \ne i, j$.

Suppose that, contrary to the desired result, one has $y_N \succ_{R_N} x_N$. By Axiom 2, $y_N \succ_{R_N'} x_N$. By Axiom 1, $z_N^1 \succ_{R_N'} y_N$ and by transitivity, $z_N^1 \succ_{R_N'} x_N$. By Axiom 2, $z_N^1 \succ_{R_N''} x_N$. By Lemma 1, $z_N^2 \succsim_{R_N''} z_N^1$ and by transitivity, $z_N^2 \succ_{R_N''} x_N$. By Axiom 4, $z_N^3 \succsim_{R_N''} z_N^2$ and by transitivity, $z_N^3 \succ_{R_N''} x_N$. The latter contradicts Axiom 1. Therefore $x_N \succsim_{R_N} y_N$. ∎

The rest of the proof of Th. 1 is a standard argument (see Hammond 1979 or Fleurbaey and Maniquet 2011).