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Competition in the quality of higher education: the impact of students' mobility*

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Abstract

This paper analyzes in a two-country model the impact of students' mobility on the country-specific level of higher educational quality. Individuals decide whether and where to study based on their individual ability and the implemented quality of education. We show that the mobility of students affects educational quality in countries and welfare in a very different way depending on the degree of return migration. With a low return probability, countries choose suboptimally differentiated levels of educational quality, or even no differentiation at all.

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1 Introduction

Worldwide we observe a substantial increase in students' mobility. More and more students decide to take up higher education in countries other than their home country. Since 1975 the number of students all over the world enrolled outside their country of citizenship increased from 0.8 million to 3 million in 2007. In OECD countries on average the percentage of foreign of all tertiary enrollment has risen from 4.9 to 9.6 since 2000. In the European Union the number of foreign students worldwide as percentage of all EU students went up from 5.3 in 2000 to 7.5 in 2006. Within the EU the share of all students studying in another member state changed from 2 percent in 1998 to 2.8 percent in 2007. Breaking down the figures to the country level, the number of foreign students in the EU12 countries is between 1.2 times and 2.6 times higher in 2006 than it was in 2000.[1]

The increasing mobility of students raises the question as to how it will affect the higher education programs of countries. The formation of human capital is an important factor of growth in developed economies. Since students are more mobile, countries may compete for students. Competition may take many forms and the incentives to compete for mobile students depend critically on various factors such as the degree of return migration after students have graduated, the financing system, the institutional constraints. In a tax-financed system, if foreign students return as educated workers to their home country, the incentives of a country to free-ride on the higher education system of other countries are considerable (see some references

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[1] These figures and other trends in students' mobility can be found in OECD (2009) and Eurostat (2009).

below). In this paper, we focus on a different dimension by examining competion in the quality levels of higher education. Quality is privately financed, which leaves aside the standard free riding problem. This can be understood as a focus on the quality of fee-financed education on top of a tax-financed basic higher education. Indeed, the introduction or extension of tuition fees in many countries provides an additional source of funding generated to improve educational quality.[2]

Higher student mobility extends the scope of countries to specialize in differing qualities of higher education. The rationale behind vertical differentiation is that individuals differ with respect to their innate ability to benefit from higher education. Thus the choice of an education quality level may be used as a way to attract or to repel individuals according to their ability. The incentives to do so depend on the government's welfare criterion.[3] To examine these incentives, we build up a simple game in which the decisions of both students and countries are derived through well-specified welfare criteria.

The analysis is conducted in a simple two-stage game with two countries At the first stage, each government chooses the education quality level. The level is restricted to be uniform and country-specific meaning that it applies to all students in the country. This restriction to a single educational level is a simplification which does not alter the main results, as discussed later on. At the second stage, individuals make their decisions about higher education and labor supply given the quality levels chosen by the governments. Specifically they decide whether and where to study on the basis of the expected lifetime income in each alternative. The quality level of education as well as the innate individual's ability generate the skill-units an individual is endowed with after having acquired education. Hence, the proposed education levels affect the individuals' choices and determine the structure of labor supply in the subsequent period through two channels: directly because it affects productivity and indirectly because it modifies the incentives to become skilled.

The welfare criterion of a country is given by aggregate wage income of skilled and unskilled workers net of educational cost. In general, the welfare function follows the residence principle by taking account of the residents in the country wherever they were born. However, the degree of return migration after students have graduated parameterizes the relative weight on natives and foreigners. The graduates return probability specifies the chances for a graduate student who has studied abroad to come back in his home country. At one extreme, with a return probability of unity the welfare function is equivalent to a welfare following the native principle where the government is concerned only with the natives.

We assume that the students' ability type are privately known and not observable by governments. With pure fee-financing, this assumption does not induce distortions in the individuals' choices because the private benefit and the social benefit of taking up a given educational level coincide. As a result, in a closed economy, a government that is concerned about its citizens' welfare chooses the same educational level with or without observing abilities and there is no welfare loss. Instead, in the case where economies are open and students freely choose where to study, the fact that ability is not observable plays a central role. Since governments have no full control on who studies and who works in their countries, the choice of the education level operates a selection of the students according to their ability. A country may prefer to provide a high quality of education in order to attract the best students, but at the cost of charging

[2]Kemnitz (2007) has shown that in a decentralized system of higher education allowing for fee-financing does not necessarily crowd out tax-financing and yields a higher quality of education.

[3]Our model has some similarities with models of vertical product differentiation as developed by Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). However, the profit maximizing firms are replaced by countries whose objective is citizens welfare.

higher fees to finance better quality hence deterring some students, while the other country may decide to educate the less able students on a low quality level which can be provided at a low cost.To assess the relevance, efficiency, and stability of these strategies, we analyze the Nash equilibria of the game and their welfare properties as a function of the return probability of foreign students.

Optimality requires countries to differentiate their educational levels as this enables individuals to split according to their abilities. If all foreign students return to their home country the countries differentiate their educational levels optimally at the (unique) Nash equilibrium. The intuition is that governements do not compete effectively for students since all graduates return home. Hence, there is no distortion due to mobility and governments choose the efficient provision of education. If instead all foreign students stay in the country where they have been educated we show that at a symmetric equilibrium, the identical educational level is too high compared with the optimal one. Hence, if countries open up their borders students' mobility induces countries to over-provide higher education. The reason is that they compete for the highly able students. For intermediate values of the return rate, the equilibria result from the two forces at work: incentives for differentiation called by efficiency gains and incentives to compete for students. However, even if an asymmetric equilibrium with distinct quality levels exist, governments tend to be sub-optimally differentiated. This result can be traced back to external effects resulting from students' mobility.

Let us discuss now the constraint of a single educational level per country. This constraint cuts off the possibility for a government to achieve the first best policy which would be to provide a specific educational level for each ability type of students. Albeit strong, the restriction to a single educational level reflects the observed limited variability of the quality of educational programs within a country (at least relative to the variability across countries). Given the diversity of ability types in reality and positive costs for each differentiated educational level, governments can only provide a limited number of specific educational levels and cannot reach the first best educational levels by matching all ability types. The restriction to one educational level is thus only a simplification which does not alter the main insights of our results.

There is some literature about the effects of student's mobility on the provision of higher education. A branch focuses on tax-financed systems and the free-rider problem, which depends critically on the degree of graduates return migration. Del Rey (2001) assumes that all foreign students return home after accomplishing education and pay taxes only in their countries of origin. Mechtenberg and Strausz (2008) consider foreign students who acquire productive mulitcultural skills and stay in their host country with an exogenous probability. In both papers, governments underinvest in public education as long as some of their native students come back, due to the free-rider problem. Justman and Thisse (2000) analyze a model with endogenous labor mobility. Graduates take into account regional wage differentials among other factors when deciding in which country they prefer to work. Here the reason for underinvestment in education is that the emigration of some graduates in equilibrium generates positive external effects on the other region that are ignored by the local government. Lange (2008) extends the analysis by allowing some mobility for both skilled workers and students. Depending on the stay rate of graduates, over- or under-investment in education is possible. Our analysis sensibly differs since pure free-riding and spill-over effects are not present. Allowing for differentiated educational levels across countries, it is the competition for students that generates distortions and may result in inefficient equilibria if return probabilities of foreign students are sufficiently low.

There are two recent papers which are closest to our analysis. Kemnitz (2007) compares the impact of different funding reforms on teaching quality and welfare. He finds that uniform tuition fees and student grants fail to achieve the welfare optimum because the quality choices of universities are uniform and not differentiated according to abilities. In contrast, graduate taxes may be introduced in a way to implement the optimum. Haupt, Krieger and Lange (2010) analyze differentiated tax and fee-financed education policies of two countries which compete for students from the rest of the world. Countries set two policy instruments: educational qualities and tuition fees. They show that in equilibrium educational qualities are differentiated in order to relax tuition-fee competition. Furthermore, they find that the country with the higher educational level decreases this quality level with a lower return probability of foreign students. Our paper complements this analysis. We focus on the impact of students' mobility on the competition via educational qualities and assume private funding. In our model the decisions of students both whether and where to study are endogenous. Combined with return probabilities of foreign students this allows us to analyze equilibria with differentiated educational qualities as well as with symmetric ones. Furthermore, we find unambiguous welfare implications: In an asymmetric equilibrium the differentiation of educational levels is inefficiently low, in a symmetric equilibrium the educational level is too high.

The outline of the paper is as follows. Section 2 describes the model and analyzes a closed economy as a benchmark. Section 3 considers two open economies. First the optimal allocation is derived. Then we determine the Nash equilibria of the game in three cases: a) all foreign students return home, b) no foreign student returns home, c) some foreign students return home. We compare the Nash equilibria with the optimal allocation. Section 4 concludes.

2 Closed economy

2.1 The model

The analysis is conducted in a stationary overlapping generations model in which the population is constant. The economy is kept as simple as possible. There is a single consumption good that is produced by skilled and unskilled labor through a linear technology. The good cannot be stored and there is no capital. In each period higher education has to be financed via tuition fees by the students.[4] Since students do not have any income they have to borrow money in the first period in order to finance their higher education. Borrowing takes place between the individuals of one generation (not all decide to study) and possibly between generations. Credit markets are perfect and we are at the golden rule: the interest rate in the steady state without frictions is equal to the population growth rate, which is here equal to zero (see Gale, 1973).

The production sector The production sector in each country uses two kinds of input: labor supplied by individuals with and without higher education, L_s (skilled labor) and L_u (unskilled labor), respectively. Production takes place according to a linear technology where the wage rates of the skilled workers, w_s, and the unskilled workers, w_u, are assumed to be given and constant

$$F(L_u, L_s) = w_s L_s + w_u L_u. \tag{1}$$

Production is thus completely determined by the labor supply of skilled and unskilled workers which in turn is given by the individuals' decisions to acquire higher education.

[4]In a companion paper (Demange, Fenge and Uebelmesser, 2008), we have shown that pure fee-financing of higher education is optimal if credit markets are perfectly competitive. Since we model higher education as a private good without externalities this is a natural outcome.

The demand for higher education Individuals are distinguished by an ability parameter y which reflects individually different benefits from higher education. The distribution of abilities is identical in each country and assumed to be uniform in the range $[0, \overline{y}]$.

To be skilled, an individual must receive some education. Quality of education or the educational level, respectively, is denoted by e. The quantity of skilled labor provided by an educated worker with education level e depends on her ability y: it is given by ye. For simplicity, we assume that the amount of money spent for higher education per individual, given by $c(e)$, only depends on the educational level. Put differently, costs of education are proportional to the number of students for a given quality.[5] The cost function c is assumed to be increasing and strictly convex. Throughout the paper, to avoid corner solutions, we shall assume that marginal costs of education increase indefinitely with the level: $\lim_{e \to \infty} c'(e) = \infty$.

If an individual decides to study, she pays the educational costs as fees during the first period, $c(e)$, and earns no wage income. In the second period, the educated worker receives a gross wage rate w_s for each unit of effective labor supply so that the wage income depends on her ability y: $w_s ye$. Thus her lifetime income is

$$w_s ye - c(e) .$$

If the individual decides not to study she receives a wage income w_u in both periods. Hence, her lifetime income is

$$2w_u .$$

The individual compares both lifetime incomes and chooses the option which maximizes her income. The decision whether to study or not depends on the ability of the individual. The marginal ability type who is indifferent between both options is given by

$$y^u = \frac{2w_u + c(e)}{w_s e}. \tag{2}$$

Individuals with a lower ability, $y < y^u$, do not study and are employed as unskilled workers. Individuals with a higher ability, $y > y^u$, take up higher education in the first period and work as skilled workers in the second period.

Employment We describe here how the educational level e determines the supply of skilled and unskilled workers on the labor market.

As already mentioned, the population growth rate is assumed to be nil. In each period, employment consists of young and old unskilled workers and old skilled workers. Let an educational level e and a threshold ability level of skilled workers y^u be given. The number of unskilled workers per generation, denoted by N_u, is equal to y^u and the number of skilled workers, denoted by N_s, is equal to $\overline{y} - y^u$. The employment of unskilled labor is given by

$$L_u = 2 \int_0^{y^u} 1 \; dy = 2y^u = 2N_u \tag{3}$$

[5]Education is thus considered here to be a private good.

and the *effective* skilled labor by

$$\begin{aligned} L_s &= \int_{y^u}^{\overline{y}} yedy = e\left(\frac{\overline{y}^2-(y^u)^2}{2}\right) = (\overline{y}-y^u)\,e\left(\frac{\overline{y}+y^u}{2}\right) \\ &= N_s e\left(\frac{\overline{y}+y^u}{2}\right) \end{aligned} \tag{4}$$

which is equal to the number of skilled workers multiplied by their average ability and the educational level.

2.2 Optimal allocation

Under complete information on individuals' abilities, a social planner can determine the level of education and the ability of those who study. The welfare criterion is aggregate production net of education cost at a steady state, given by $F(L_s, L_u) - N_s c(e)$. This is the criterion that obtains in a fully fledged overlapping generations economy in which the planner treats all generations equally. In other words, we are at the golden rule with an implicit interest rate equal to the population growth rate, here zero (Gale, 1973).

The choice of the level of education and of the minimum ability of those who study, e and y respectively, fully determines skilled and unskilled labor from (3) and (4). Hence defining

$$W(y,e) = w_s L_s + w_u L_u - N_s c(e) \tag{5}$$

where from (3) and (4) L_u and L_s are functions of e and y and N_s is a function of y alone. The objective is to maximize $\underset{y,e}{Max}\ W(y,e)$.

The impact of a marginal increase in e keeping the set of students fixed is given by

$$\begin{aligned} \frac{\partial W}{\partial e} &= w_s\frac{\partial L_s}{\partial e} + w_u\frac{\partial L_u}{\partial e} - N_s c'(e) \\ &= (\overline{y}-y)\left[w_s\frac{\overline{y}+y}{2} - c'(e)\right]. \end{aligned} \tag{6}$$

It is equal to the effect of the quality level on the production of the skilled workers minus the increase in costs.

The impact of a marginal increase in the minimum ability level y keeping the education level fixed is given by

$$\begin{aligned} \frac{\partial W}{\partial y} &= w_s\frac{\partial L_s}{\partial y} + w_u\frac{\partial L_u}{\partial y} - c(e)\frac{\partial N_s}{\partial y} \\ &= -w_s e y + 2w_u + c(e). \end{aligned} \tag{7}$$

It is equal to the net impact on the productivity of a student of ability just equal to y from becoming skilled compared to remaining unskilled where the impact is measured at the steady state situation.

The objective function is concave in e and in y. At the optimum, assumed to be interior, the level of education and the threshold ability level are characterized by the following first-order conditions

$$w_s\frac{\overline{y}+y}{2} = c'(e) \tag{8}$$

$$w_s e y - 2w_u - c(e) = 0 \tag{9}$$

that is, the marginal gain from a change in educational quality on the average student, $w_s \frac{\overline{y}+y}{2}$, is equal to the marginal costs, and the net gain of education for the marginal student is null.

In the sequel, we put a superscript * to indicate the values at the optimum solution for the educational levels and the threshold ability. In the following, individuals' abilities are assumed to be unobservable (or not contractible) by governments.

2.3 Government's decision on the educational level e

Due to informational asymmetries, the set of students cannot be chosen in the same way as an omniscient social planner does. The government chooses the level of education taking account of the individual decisions which are determined by the threshold level of ability. The welfare criterion of the government is still the aggregate production net of education cost at a steady state.

Given an educational level, the ability threshold which determines who decides to study is denoted by $y^u(e)$ (see equation (2)). Thus, the government's objective is

$$\underset{e}{Max}\, W(y^u(e), e) = w_s L_s + w_u L_u - N_s c(e) \tag{10}$$

in which skilled and unskilled labor levels are those determined by the threshold ability level

$$N_s = \overline{y} - y^u(e), L_u = 2y^u(e), L_s = N_s e \left(\frac{\overline{y} + y^u(e)}{2} \right) \tag{11}$$

The impact on welfare due to a marginal change of education is composed of two terms: an indirect one through the selection of abilities and a direct one. Formally, the marginal change in welfare that results from an increase in the educational level chosen by the government is given by

$$\frac{dW}{de} = \frac{\partial W}{\partial y} \frac{dy^u}{de} + \frac{\partial W}{\partial e} \tag{12}$$

where $\frac{dy^u}{de}$ denotes the change in the threshold ability level and thus in the selection of abilities.

The key point is that individuals' choices are not distorted in our model under full fee financing. In other words, the optimal ability associated with a given educational level coincides with that chosen by individuals. Specifically $\frac{\partial W}{\partial y}(y^u(e), e)$ is identically null as can be seen from (2) and (7). An immediate consequence is that the optimal allocation and the maximal value for welfare can be reached even without observing abilities. By choosing the optimal level e^*, the associated optimal set of students is selected, those with ability larger than $y^* = y^u(e^*)$, and surely the government cannot do better.

3 Two open economies with mobile students

We study the same model as before - now, however, with two economies where students are mobile. We assume that unskilled workers are immobile whereas students and skilled workers are mobile across countries.[6] In particular, individuals who decide to study have no migration costs and choose the country where they attain higher education. Graduates may stay in the country where they have completed higher education or come back to their home country with

[6]This corresponds to empirical evidence according to which mobility increases with education. See, e.g., Ehrenberg and Smith (1993), Mauro and Spilimbergo (1999), Coniglio and Prota (2003) and Hunt (2006).

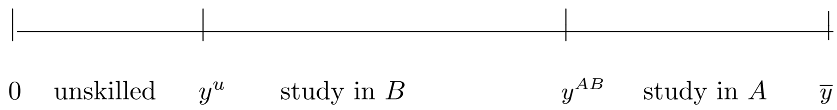

Figure 1: Threshold levels y^u and y^{AB} for $e^A > e^B$

some return probability as described below. As a benchmark, we start by analyzing the choice made by an omniscient social planner who can decide on the level of education in each country and on the ability of those who study and at which level. An alternative interpretation is that the two countries cooperate in their choice of the levels of education and have complete information on abilities. We then study a non cooperative game played by the two countries under various scenarii on the return rate of the graduates who have studied abroad.

3.1 Optimum

We consider the aggregate welfare over the two countries as the objective. The fact that there is a uniform educational level in each country is a constraint. This opens up the possibility of overall welfare gains if distinct educational levels are chosen in both countries and students are mobile. The omniscient social planner can choose two levels of education and the ability of those who study and at which level. Denote by e^A and e^B the educational levels (even though here an educational level is not necessarily attached to a specific country). With obvious notation overall welfare is

$$W = w_s L_s + w_u L_u - c(e^A) N_s^A - c(e^B) N_s^B \tag{13}$$

in which the number of students and skilled and unskilled labor are determined by the planner.

Arguing directly, it is optimal to split individuals according to their abilities. If $e^A \geq e^B$ for instance, let y^{AB} be the minimum ability of those who are assigned to a high educational level, and y^u the minimum ability of those who are allowed to study. Individuals with an ability between y^u and y^{AB} acquire the low level of education e^B and those with an ability between y^{AB} and $\overline{y}$ acquire the high level e^A, as depicted in Figure 1.

Hence the policy tools available to the planner are summarized by the levels of education levels e^A and e^B, and the thresholds, y^u and y^{AB}, which describe the abilities of those who acquire a given level. They fully determine the number of students in each program as well as skilled and unskilled labor:

$$N_s^A = 2\left(\overline{y} - y^{AB}\right), N_s^B = 2\left(y^{AB} - y^u\right) \tag{14}$$

$$L_s = \left(\overline{y} - y^{AB}\right)(\overline{y} + y^{AB})e^A + \left(y^{AB} - y^u\right)(y^{AB} + y^u)e^B,\ L_u = 4y^u. \tag{15}$$

Plugging these expressions as function of the policy tools in the welfare criterion W in equation (13) gives the objective to be maximized. The objective is concave, and with similar computations as in the case of a single quality level, the optimum is characterized by the following first-order conditions

$$w_s \frac{\overline{y} + y^{AB}}{2} - c'(e^A) = 0 \text{ and } w_s \frac{y^{AB} + y^u}{2} - c'(e^B) = 0 \tag{16}$$

$$-w_s e^A y^{AB} + c(e^A) = -w_s e^B y^{AB} + c(e^B) \text{ and } -w_s e^B y^u + c(e^B) = -2w_u \tag{17}$$

These conditions are easily interpreted. Conditions (16) say that the educational levels are optimal given the thresholds, that is given the set of students. The marginal gain from a change of the high educational level for the average student, $w_s \frac{\overline{y}+y^{AB}}{2}$, is equal to the marginal costs, $c'(e^A)$, and similarly for the lower level of education. Conditions (17) say that the ability of the students in each program is optimal given the education levels proposed. The net gain of top education relative to the lower level is null for the student with marginal ability y^{AB} and the net gain of the low level of education compared to remaining uneducated is null for the marginal student y^u.

Hence optimality calls for differentiation. We shall denote by $(\overline{e}^*, \underline{e}^*)$ the distinct optimal levels with $\overline{e}^* > \underline{e}^*$.

3.2 Game in educational levels

This section considers the situation with informational asymmetries and full mobility for students. As in a closed economy, a government chooses the level of education in its country taking account of the individual decisions. In an open economy, individuals face more choices and their decisions are affected by the education levels chosen by both countries. Mobility thus generates a game between the two countries. Before spelling out governments' criteria, we analyze individuals' decisions.

3.2.1 Individual choices

We consider the free choice of individuals when the two countries have chosen their education levels, e^A and e^B.

A young individual born in country I, $I = A, B$ now not only has to decide whether to study but also where to study. Since wages are constant, the lifetime income of a young who decides to study in I is $ye^I w_s - c\left(e^I\right)$. This implies that the maximum lifetime income of a y-young individual who decides to become skilled is

$$V_s(y) = \max[ye^A w_s - c\left(e^A\right), ye^B w_s - c\left(e^B\right)]$$

Similarly, the lifetime income of an unskilled worker is unchanged, given by $2w_u$ in both countries. The individual chooses to be skilled if $V_s(y) \geq 2w_u$.

In the symmetric case where educational levels are equal, $e^A = e^B = e$, individuals are indifferent between studying in either country. In that case we shall assume that they split equally (as occurs, for example, if they do not move at all).

Assume now that education levels are distinct. We take $e^A > e^B$. The return to education increases with ability. As a result, the individuals who choose to study in A and not in B are those with high enough ability and the individuals who stay unskilled are those with low enough ability. Specifically, let y^{AB} be the type of an individual who is indifferent between studying in A and B. It is the value defined by

$$y^{AB} e^B w_s - c\left(e^B\right) = y^{AB} e^A w_s - c\left(e^A\right). \tag{18}$$

or

$$y^{AB} = \frac{c\left(e^A\right) - c\left(e^B\right)}{w_s(e^A - e^B)}$$

Analogously, let y^u be the type of an individual who is indifferent between studying and not studying. It is defined by

$$y^u e^B w_s - c\left(e^B\right) = 2w_u. \tag{19}$$

Assuming $y^u < y^{AB} < \overline{y}$, young individuals partition themselves according to their abilities as depicted in Figure 1.

Comparing the conditions (18) and (19) with the optimality condition for the thresholds (17) given education levels (e^A, e^B), we see that there is no distortion. As in the case with a single level, the private benefit and the social benefit of taking up a given educational level or of remaining unskilled coincide. This yields the following Lemma:

Lemma 1 *The optimal partition of ability types of students between countries associated with given education levels coincides with that chosen by individuals.*

An immediate consequence is that the optimum can be obtained even without observing ability levels: If the optimal levels of education are implemented, it suffices to let individuals choose whether (and where) to study. These optimal levels can be thought as resulting from the decisions of a union of countries acting in a cooperative way. Thus any inefficiency that may result from a non-cooperative choice in educational levels is not due to individuals choices.

3.2.2 Return rate and Welfare criterion

The game between the governments is determined by the criterion on which governments base their choices. With migration, population is variable within a country, and a variety of welfare criteria may be considered (see e.g. the discussion in Blackorby, Bossert and Donaldson, 2006). We consider a residents' welfare criterion according to which the government is concerned with the residential workers (natives or foreigners). This criterion is affected by a 'graduate return probability'. As shown below, a native welfare criterion according to which the government only cares about its natives wherever they work coincides with the residential welfare criterion when all foreign graduates come back.

Skilled workers are indifferent between working in either country, so that any migration decision they may take is rational. Their behavior is described as follows. Students who have studied abroad come back to work in their home country with probability π, called the return rate. Those who study in their home country do not move afterwards and remain as skilled workers in their home country. The return rate π is independent of everything else, and in particular of the ability type. The return rate determines the skilled labor force in each country as follows.

To fix the idea, let us $e^A > e^B$. The top ability students from both countries study in country A. Their number is $2\left(\overline{y} - y^{AB}\right)$ as given by (14). Half of the students in A come from country B, and among those a proportion $\pi \in [0,1]$ comes back to the home country B as skilled workers. Hence only the fraction $1-\pi/2$ of skilled workers with education level e^A works in A and the remaining fraction, $\pi/2$, works in B. Similarly, the low ability students of both countries study in country B. Their total number is $2\left(y^{AB} - y^u\right)$, and among those a fraction $\pi/2$ will work in country A and the fraction $1-\pi/2$ in country B.

The residential welfare of a country is defined as the aggregate lifetime income of the residents. Since the cost of education is entirely borne by a student, the lifetime income of a skilled worker is defined as his wage income diminished by the cost of its education level. There are three types of workers: the skilled workers with education levels e^A or e^B and the unskilled workers. The lifetime income of all unskilled workers in a country is $2w_u y^u$. To simplify notation let Y^A denote the lifetime income of all skilled workers with education level e^A, and similarly for Y^B. We have

$$W^A(e^A, e^B) = (1 - \frac{\pi}{2})Y^A + \frac{\pi}{2}Y^B + 2w_u y^u, \qquad (20)$$

$$W^B(e^A, e^B) = \frac{\pi}{2}Y^A + \left(1 - \frac{\pi}{2}\right) Y^B + 2w_u y^u. \qquad (21)$$

The interpretation is simple : country A keeps $1 - \pi/2$ of the students who were educated in the country and $\pi/2$ of those who studied in B, and similarly for B.

It remains to spell out the values for Y^A and Y^B. Take $e^A > e^B$ (the case $e^A < e^B$ is symmetric). The average effective labour supply of a skilled worker with education level e^A is $\frac{\overline{y}+y^{AB}}{2}e^A$, which yields an average lifetime income equal to $w_s\frac{\overline{y}+y^{AB}}{2}e^A - c(e^A)$. Similarly the average effective labour supply of a skilled worker with education level e^B is $\frac{y^{AB}+y^u}{2}e^B$, which yields an average lifetime income equal to $w_s\frac{y^{AB}+y^u}{2}e^B - c(e^B)$. Weighting by the number of students in A, $2\left(\overline{y} - y^{AB}\right)$, or in B, $2\left(y^{AB} - y^u\right)$, we obtain for $e^A > e^B$

$$Y^A = 2\left(\overline{y} - y^{AB}\right)\left[w_s\frac{\overline{y} + y^{AB}}{2}e^A - c\left(e^A\right)\right] \qquad (22)$$

$$Y^B = 2\left(y^{AB} - y^u\right)\left[w_s\frac{y^{AB} + y^u}{2}e^B - c(e^B)\right]. \qquad (23)$$

In the following we analyze scenarii with differing return probability rates of students. For this, it is useful to note that *total* welfare is independent on that rate. Thus the return rate matters only because it determines the share of this total welfare assigned to a country. Specifically, let $TW(e^A, e^B)$ denote the total welfare associated to two education levels, e^A, e^B. We have

$$TW(e^A, e^B) = Y^A + Y^B + 4w_u y^u \qquad (24)$$

and the identity

$$TW(e^A, e^B) = W^A(e^A, e^B) + W^B(e^A, e^B). \qquad (25)$$

Since the return rate determines the share of the total welfare assigned to a country, it affects a country's incentive to choose an education level. In particular, the larger the return rate is, the less the skilled labor force in a country depends on the country's decision. At one extreme, where the return rate is null ($\pi = 0$), each country ends up with the skilled labor force that graduated in the country. This opens up the possibility of competition for students. At the other extreme case where the return rate is one ($\pi = 1$), each country ends up with the same labor force. In the intermediate case, with π between 0 and 1, it turns out that the welfare of each country is a combination of the welfare obtained in each extreme case. From (20) we can write

$$W^A(e^A, e^B) = (1 - \pi)[Y^A + 2w_u y^u] + \pi[\frac{1}{2}(Y^A + Y^B) + 2w_u y^u].$$

The first term in square brackets is the welfare of A for π equal to zero, and the second term the welfare of A for π equal to one. Thus W^A can be written as $(1-\pi)W^A_{|\pi=0} + \pi W^A_{|\pi=1}$ where $W^A_{|\pi=0}$ and $W^A_{|\pi=1}$ are the welfare levels in the two extreme cases $\pi = 0$ and $\pi = 1$ respectively. This is also true for B, so we obtain:

$$W^I(e^A, e^B) = (1 - \pi)W^I_{|\pi=0} + \pi W^I_{|\pi=1}(e^A, e^B) \;\; I = A, B. \qquad (26)$$

We will analyze first in detail the two extreme cases where all students who take up higher education in a foreign country either return after graduation to their home ($\pi = 1$) or stay in the foreign country ($\pi = 0$). We are especially interested in whether countries want to match their levels of education leading to a symmetric equilibrium or whether they aim at differentiating their levels, and what impact this has on total welfare. In that purpose, it is useful to determine students' behavior for almost identical educational levels in both countries, and to understand the benefits to differentiation.

Scope for differentiation Marginal changes in education levels in a neighborhood of a symmetric situation have a dramatic selection effect on the students. Starting from identical levels (e, e), if a country say A increases slightly its educational level above e, the set of students split into two parts with A attracting the top ability students, and B attracting the low ability ones. Thus, a massive reallocation of students takes place. To make this precise, consider the type of an individual who is indifferent between studying in A and B, y^{AB}, as e^A and e^B are very close. Making the education levels converge to e, the limits of y^{AB} from above or below e are given by:

$$\lim_{e^A \to e^+} \frac{c\left(e^A\right) - c\left(e\right)}{w_s(e^A - e)} = \lim_{e^B \to e^-} \frac{c\left(e\right) - c\left(e^B\right)}{w_s(e - e^B)} = \frac{c'(e)}{w_s}.$$

We denote this limit by $y^{lim}(e)$:

$$y^{lim}(e) = \frac{c'(e)}{w_s}. \tag{27}$$

Starting from identical levels, countries share equally the students, the individuals with ability above y^u. By marginal increasing its level e^A above e, country A attracts all top ability students, those with ability larger than $y^{lim}(e)$, and B attracts those with ability between y^u and $y^{lim}(e)$. If a country say B decreases its level below e, the analysis is similar but there is also a marginal change in the unskilled level y^u since it is determined by the smallest level e^B (as given by (19)).

Such an analysis is true provided e is not too extreme. Specifically the thresholds given by (18) and (19) are valid if $\overline{y} > y^{AB} > y^u$. These inequalities are satisfied for education levels close to e if

$$\overline{y} > \frac{c'(e)}{w_s} > \frac{2w_u + c(e)}{w_s e}. \tag{28}$$

When inequalities (28) hold we say that there is *scope for differentiation* at e. In that case it turns out that not only differentiation is possible but it is beneficial, as stated in the following lemma.

Lemma 2 *Total welfare TW is continuous in educational levels. It is maximum at $(\overline{e}^*, \underline{e}^*)$. Furthermore, assume that there is scope for differentiation at e, i.e. that (28) holds. Then*

$$\lim_{e^A \to e^+} \frac{\partial TW}{\partial e^A}(e^A, e) = w_s \left(\overline{y} - \frac{c'(e)}{w_s}\right)^2, \lim_{e^B \to e^-} \frac{\partial TW}{\partial e^B}(e, e^B) = -w_s \left(\frac{c'(e)}{w_s} - y^u\right)^2 \tag{29}$$

where $y^u = \dfrac{2w_u + c(e)}{w_s e}$. As a result there are strong benefits to differentiation.

Proof. See the proof section. ■

The fact that the optimum can be obtained even without observing ability levels explains why the maximum of TW is obtained at $(\overline{e}^*, \underline{e}^*)$. We emphasize the continuity property of total welfare even at symmetric levels (e, e): even though the students reallocate between the two countries, each individual lifetime income is continuous in the education levels, hence the total welfare as well. Instead, when we consider countries' welfare the reallocation of students will generate discontinuities in their levels, and the jumps will be in the opposite direction (since the sum of the countries' welfare, TW, is continuous). According to (29), $\frac{\partial TW}{\partial e^A}(e^A, e)$ is positive for e^A sufficiently close to e but larger than e: increasing slightly the education level e^A above e a symmetric situation typically increases welfare. Similarly, since $\frac{\partial TW}{\partial e^B}(e, e^B)$ is negative for e^B sufficiently close to e but smaller than e, decreasing slightly the education level of B below e increases welfare (of course similar results obtain by exchanging the roles of A and B, i.e. if e^A is decreased or e^B is increased). Hence, there is a benefit to differentiation, whatever the direction, as long as differentiation is possible.

3.3 Graduates return to their home country ($\pi = 1$) or the native welfare criterion

When all students come back to work home, each country ends up with an identical number of skilled and unskilled labor. Countries' welfare are therefore identical, each one equal to half the total welfare TW:

$$W^A_{|\pi=1}\left(e^A, e^B\right) = W^B_{|\pi=1}\left(e^A, e^B\right) = \frac{1}{2} TW\left(e^A, e^B\right) \tag{30}$$

$$= \left(\overline{y} - y^{AB}\right)\left(w_s \frac{\overline{y} + y^{AB}}{2} e^A - c\left(e^A\right)\right) + \left(y^{AB} - y^u\right)\left(w_s \frac{y^{AB} + y^u}{2} e^B - c\left(e^B\right)\right) + 2 w_u y^u.$$

Such a criterion is also obtained under the 'native's principle' according to which a government is concerned with the well-being of the natives even those who have left the country, and does not care about the immigrants. Hence, the results obtained in this section apply if both governments act according to the native's principle.

How total welfare reacts to a change in an educational level gives insight on the incentive for a country to differentiate its education level when the graduate return probability is one (since a country's welfare is given by half the total welfare). Taking the point of view of country A when B's educational level is fixed at e, we have that $e^A \to TW(e^A, e)$ has a local minimum at $e^A = e$ with a kink. To see this, Lemma 2 gives, exchanging the role of A and B in the second inequality of (29):

$$\lim_{e^A \to > e} \frac{\partial TW}{\partial e^A}(e^A, e) = w_s \left(\overline{y} - \frac{c'(e)}{w_s}\right)^2, \quad \lim_{e^A \to < e} \frac{\partial TW}{\partial e^A}(e^A, e) = -w_s \left(\frac{c'(e)}{w_s} - y^u\right)^2 < 0.$$

Hence, a country benefits by decreasing its level below that of the other country and also by increasing it. There is a strong force towards differentiation, and a symmetric situation is surely not an equilibrium. Actually, countries choose the optimal differentiation levels at equilibrium, as stated in the following proposition.

Proposition 1 *If all graduates return to their home country (or if countries maximize the welfare of their natives), optimal differentiated educational levels,* $(\overline{e}^*, \underline{e}^*)$ *or* $(\underline{e}^*, \overline{e}^*)$*, form a Nash equilibrium. There is no other Nash equilibrium.*

Proof. The proof that optimal differentiation is a Nash equilibrium is straightforward. By recognizing that in each country welfare is just half of total welfare it is obvious that the incentives of both countries and the social planner are aligned. As a result, if one country chooses one of the optimal levels, say the largest one $\overline{e}^*$, the other country optimal choice is the lowest $\underline{e}^*$, (and vice versa). For the proof of uniqueness see the proof section. ■

The intuition for this result is that with a return probability of one both countries have the same labor force wherever the workers have been educated, and are concerned by efficiency considerations only, as illustrated below.

We can summarize. When all foreign students return to their home country, which is equivalent to countries taking account of their natives only, there are strong forces to differentiate the educational quality. Either countries decide to provide high quality at high cost and educate the high ability students or they decide to keep cost low with lower educational quality and to educate the low ability students. Whatever the decision may be the result in terms of welfare is the same for the country because the students return home and both countries share the same labor force with the same ability composition. The resulting equilibrium differentiated educational levels are unique and optimal.

3.3.1 Illustration

This section illustrates the response functions for $\pi = 1$ in the case of the following specifications. The cost function is quadratic: $c(e) = (e)^2$, $\overline{y} = 10$; $w_s = 2$ and $w_u = 1$. For $e^A \in [0; 10]$ and $e^B \in [0; 10]$ the restriction holds that $\overline{y} \geq y^{AB}$. Figure 2 displays the response function and the Nash equilibrium.

Response functions are first increasing in the educational level of the other country. The reason is the following. Starting with a low educational level in, let us say, country B all the upper ability types of students in the range $\left[y^{AB}, \overline{y}\right]$ study in country A. For increasing values of the educational level in country B, the threshold ability y^{AB} goes up. This implies that the average ability level of the more able students in country A rises. Maximizing their lifetime income country A increases in response also its educational level. The point is that both countries in setting their educational levels are only concerned with maximizing the lifetime income of their ability types of students. Since we know that students choose the country in an optimal way whatever the educational levels, countries have no incentive to attract students by setting their educational levels. Thus efficiency remains the only aim to be achieved by determining educational levels.

At some point where the educational level of country B is high enough - in Figure 2 the jump downwards of the response function of country A - the welfare gain of educating high ability students becomes smaller than the welfare gain of education low ability students. Now country A chooses to be the country educating low ability types and the roles of both countries are interchanged.

In the Nash equilibria where response functions intersect the optimal differentiation of the educational levels is achieved. In our example, the levels of the high and the low optimal education in the Nash equilibrium are given by:

$$\overline{e}^* = 8.1, \underline{e}^* = 4.3$$

The analysis changes when foreign students do not all return home because countries get an incentive to compete for students, as we examine now. This may result in a differentiation of educational qualities less than optimal, or even in no differentiation at all.

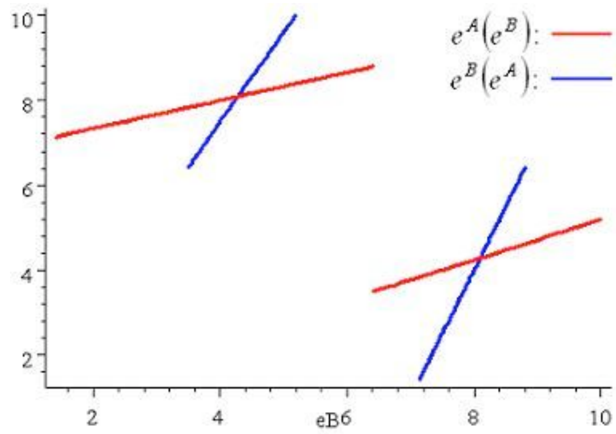

Figure 2: Response function in the case $\pi = 1$.

3.4 Graduates stay where they were educated ($\pi = 0$)

This section considers the other extreme scenario in which all students who study abroad stay there after graduation.

With distinct education levels $e^A > e^B$, income Y^A and Y^B are given by (22) and (23). Taking π equal to 0 in welfare criteria (20) and (21) yields :

$$W^A_{|\pi=0} = w_s \left(\overline{y} - y^{AB}\right) (\overline{y} + y^{AB})e^A + 2y^u w_u - 2c(e^A) \left(\overline{y} - y^{AB}\right) \tag{31}$$

$$W^B_{|\pi=0} = w_s \left(y^{AB} - y^u\right) (y^{AB} + y^u)e^B + 2y^u w_u - 2c(e^B)(y^{AB} - y^u), \tag{32}$$

and in the symmetric case where both countries choose the same level e, welfare in each country amounts to

$$W^A_{|\pi=0} = W^B_{|\pi=0} = w_s \left(\overline{y} - y^u\right) (\overline{y} + y^u)e/2 + 2y^u w_u - c(e)(\overline{y} - y^u). \tag{33}$$

Now the competition for students, who become skilled workers in the country of education, plays an important role in the choices of the education levels. This competition is especially harsh when the educational levels of the two countries approach one another. The massive reallocation of students around symmetric educational levels generates discontinuities in the welfare levels of the countries and incentives to differentiation. This is not true however at a level denoted by $\hat{e}$, which will play an important role.

To illustrate this, we first analyze the situation where countries choose the optimal educational level of a closed economy and then open up their borders for mobile students. We then analyze Nash-equilibria.

Incentives to differentiation Let each country choose the single-constrained optimal level e^* of a closed economy given by (8). We show that a marginal increase is profitable to a country. The following reasoning is illustrated in Figure 3. At the optimum e^* the marginal gain from a change in the educational level on the average student, $w_s \frac{\overline{y}+y^*}{2}$, is equal to the marginal cost, $c'(e^*)$. This implies that e^* is 'too high' for the marginal student with ability y^* and 'too low'

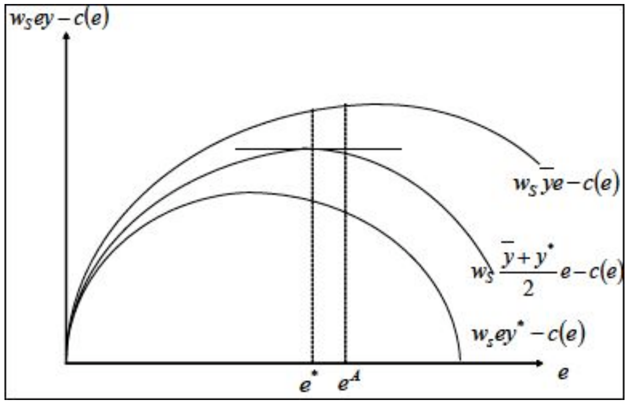

Figure 3: The sorting of ability types to countries with a single-constrained educational level.

for the top ability student.[7] It follows that if country A increases slightly its educational level, the set of students split into two parts with A attracting the top ability students. The set of individuals who decide to study is unchanged, $y^u = y^*$.

To assess the possible benefits for A, consider the type of an individual who is indifferent between studying in A and B, y^{AB}, satisfying $y^{AB} e^A w_s - c\left(e^A\right) = y^{AB} e^* w_s - c\left(e^*\right)$. Observe that by convexity of $c\left(e\right)$ we have that

$$y^{AB} = \frac{c\left(e^A\right) - c\left(e^*\right)}{w_s(e^A - e^*)} \geq \frac{c'(e^*)}{w_s} = \frac{\overline{y} + y^*}{2}.$$

where the last equation follows from the optimality conditions (see also Figure 3). Hence, country A, by providing a higher educational level than B not only attracts the best students but also deters half of the bottom students at least. This is true whatever the level e^A strictly larger than e^*. Taking the limit of y^{AB} when e^A tends to e^* gives

$$y^{lim}(e^*) = \frac{c'(e^*)}{w_s} = \frac{\overline{y} + y^*}{2}. \tag{34}$$

Thus if A increases slightly its educational level, the overall set of individuals who decide to study is unchanged, given by the individuals whose ability is larger than y^*. Individuals with ability larger than the average over the students, those with ability in $(\frac{\overline{y}+y^*}{2}, \overline{y})$ study in A, and individuals with ability lower than the average, those with ability in $(y^*, \frac{\overline{y}+y^*}{2})$, study in B. In words, for e^A arbitrarily close but larger than e^*, A has the same number of students as at the initial situation, but the ability composition has increased. This results in an improvement of welfare in country A. Simple computation gives that welfare is increased by

$$w_s \left(\frac{\overline{y} - y^*}{2}\right)^2 e^*.$$

[7]Specifically, consider the lifetime income of a young individual with ability y, $yew_s - c\left(e\right)$, as a function of e. It is concave in e with a derivative given by $yw_s - c'\left(e\right)$. For $y = y^*$ this derivative is negative at $e = e^*$, $y^* w_s - c'\left(e^*\right) < 0$, since $c'(e^*)$ is equal to $w_s \frac{\overline{y}+y^*}{2}$. Thus the lifetime income $y^* ew_s - c\left(e\right)$ decreases with e at $e = e^*$: a student with ability y^* prefers a (slightly) lower educational level than e^*. At the opposite, $\overline{y} w_s - c'\left(e^*\right) > 0$, and a similar argument gives that students with large enough ability strictly prefer a larger educational level than e^*.

More generally, there is a discontinuity when the education levels become equalized. The reason comes from the students' mobility which results in a change in the skilled labor force. Thus the forces towards differentiation are strong. We now study how this affects equilibrium.

Nash-equilibrium By the same computation as in (34), a country that increases marginally its level attracts all students with ability larger than the value $y^{lim} = c'(e)/w_s$. Similarly, a country that decreases marginally its level attracts all students with ability lower than this value. There are overall the same number of students as at the symmetric situation (y^u changes marginally and only if a country decreases its level) but the ability composition of those who study in A or in B is affected. When the net benefit from educating high ability students, those with ability larger than y^{lim} is strictly larger than the net benefit from educating low ability students, those with ability between y^{lim} and y^u, increasing marginally the educational level above that of the other country so as to attract the high ability students is surely beneficial. This was shown to be the case at the optimal single level e^*. Similarly, when the net benefit from educating low ability students is larger than that from educating high ability students, a country surely benefits from choosing its education level slightly below that of the other country. There is a (unique) level, denoted by $\hat{e}$, for which these benefits are equalized. Such a level is larger than e^* and is the only possible candidate for a symmetric equilibrium in pure strategies. We make this precise.

Next lemma analyzes the the welfare of a country as it educational level is close to the other country's level. Consider A for example. Keeping the education level e^B fixed at e, start with a lower level in country A and increase it. When e^A reaches e, A and B share the students equally, and by symmetry A's welfare $W^A_{|\pi=0}(e,e)$ equals half the total welfare $\frac{1}{2}TW(e,e)$. This may generate a jump in country A's welfare, measured by $\frac{1}{2}TW(e,e) - \lim_{e^A \to e^-} W^A_{|\pi=0}(e^A,e)$. When e^A becomes larger than e, the roles of A and B are exchanged with A now attracting the high ability students. Hence, there may be another jump. According to the next lemma, these two jumps are equal, in particular they are of the same sign.

Lemma 3 *The jumps in A welfare when e^A approaches e from below or from above are equal:*

$$\lim_{e^A \to e^+} W^A_{|\pi=0}(e^A,e) - \frac{1}{2}TW(e,e) = \frac{1}{2}TW(e,e) - \lim_{e^A \to e^-} W^A_{|\pi=0}(e^A,e) \tag{35}$$

At a symmetric equilibrium, the jumps must be null.

Proof. See proof section. ■

The term on the left hand side measures the jump in A's welfare as A increases its level up to B's level and the term on the right hand side is the jump in A's welfare as A improves upon B's level.

The intuition behind the lemma is the following one. The sum of the two countries welfare depends on the levels e^A and e^B in a continuous way, which implies that around symmetric levels (e,e), countries are playing approximately a constant two-person game. The ability composition of those who study in A or in B, however, depends on which level is larger and affects the share of the total welfare received by each country. The jump in A welfare when e^A is decreased from above towards e is exactly compensated by the jump in B welfare. Exchanging the role of A and B gives the result.

Let $\hat{e}$ be a value for which the jumps are null. At this level, the net benefit from educating high ability students, those with ability larger than y^{AB}, is exactly equal to the net benefit from

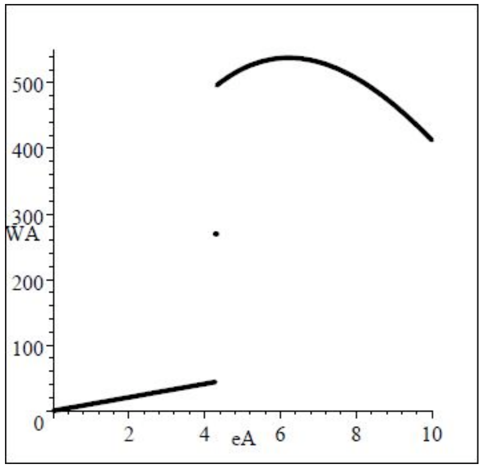

Figure 4: Welfare of country A at $e^B = \underline{e} = 4.3$ and $\pi = 0$.

educating low ability students, those with ability between y^{AB} and y^u. By definition the value of $W^A_{|\pi=0}(e^A, \hat{e})$ is close to $TW(\hat{e}, \hat{e})$ for e^A close to $\hat{e}$, and the same for B by symmetry.

Under some conditions on the cost function, level $\hat{e}$ gives rise to an equilibrium, which is furthermore the unique equilibrium.

Proposition 2 *Assume that all graduates stay where they were educated ($\pi = 0$). If countries start with the optimal educational level in a closed economy and open up their borders, students' mobility induces countries to increase the educational level above this optimal level.*
With a quadratic cost function, an equilibrium in pure strategies is symmetric. Countries choose both the same educational level $\hat{e}$, larger than e^*.

Proof. See the proof section. ■

Thus in the case where an equilibrium is necessarily symmetric, as for a quadratic cost function, the outcome is worse than in the closed economy case. When no differentiation occurs at equilibrium, opening the borders and introducing competition for students can only impair welfare if the starting situation was the optimal one in a closed economy. Competition for students ends up with a too high quality of education and too few students.

3.4.1 Illustration

This section illustrates the competition for students and the induced discontinuities of the welfare functions as established above. We use again our example of a quadratic cost function with the specification of parameters as in section 3.3.1. In Figure 4 the welfare of country A is represented for the fixed value of $e^B = \underline{e}^*$.

The welfare function is piecewise defined with a cutoff point when A chooses the same level as B. It is concave in each domain but because of the discontinuity when $e^A = e^B$, welfare is not overall concave in e^A. There is a jump when e^A reaches e^B because A and B now share the students equally. The other jump, which is in the same direction equal to the first one by Lemma 3, occurs when e^A becomes larger than e^B. The roles of A and B are exchanged with A now attracting the high ability students.

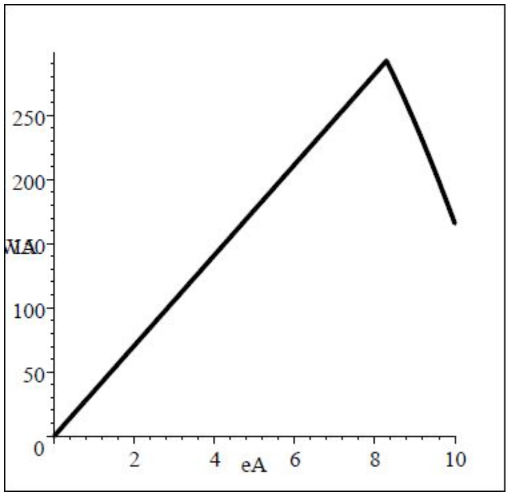

Figure 5: Welfare of country A at $e^B = \hat{e} = 8.285$ and $\pi = 0$.

Which educational level country A will choose depends on a comparison of the maximum welfare levels on each part, e^A strictly less, equal to or strictly larger than $e^B = \underline{e}^* = 4.3$. In the first part, for $e^A < \underline{e}^*$, welfare increases linear with e^A and the maximum is achieved by approaching the educational level $\underline{e}^*$ from below. The maximal welfare is $\lim_{e^A \to \underline{e}^*} W^A\left(e^A < \underline{e}^*\right) \approx 44.05$. In the second part, country A chooses the same level as country B, $e^A = \underline{e}^*$, welfare is 269.5 from (33). In the third part, $e^A > \underline{e}^*$, the welfare function is strictly concave and has a maximum value $W^A\left(e^A > \underline{e}^*\right) = 537.8$ reached at $e^A = 6.2$. Hence, country A maximizes its welfare by choosing an educational level which is higher than the optimal level in country B, $e^A = 6.2 > \underline{e}^*$.

Figure 5 illustrates that the welfare function of country A becomes continuous at the symmetric equilibrium $e^A = e^B = \hat{e}$. Calculation yield the optimal single level, e^*, and the value $\hat{e}$ as:

$$e^* = 6.765, \hat{e} = 8.285$$

As expected, the educational level at the symmetric equilibrium, $\hat{e}$, is larger than the optimal level in a closed economy, e^*.

The best response functions of both countries are depicted in Figure 6.

The intuition for the shape of the response function of country A (and vice versa for country B) is the following. Starting with a low educational level in country B an increase of this level attracts high ability students from country A. As a response country A decreases its educational level in order to regain the students lost. The essential point in the case of $\pi = 0$ is that the students do not come back once they have decided to study in a foreign country. Thus countries compete for the students in the first place by setting their educational levels appropriately. If country A's educational level gets close to the level of country B the number of students in both countries is nearly the same as with a symmetric provision of levels but the composition of abilities depends on which level is higher. Country A starts to keep its educational level slightly higher than country B in order to keep the high ability students. Hence the response function increases with the level in the other country. At some point the welfare gain of educating high ability students in A is equal to the welfare gain of educating the low ability types. This

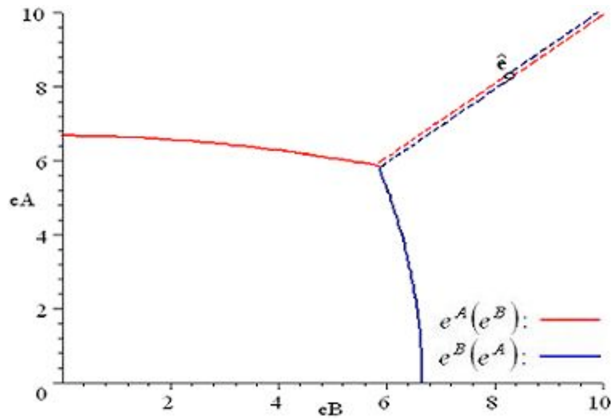

Figure 6: Response functions in the case of $\pi = 0$.

point where both educational levels are symmetric - marked by a circle - is the only one where the welfare functions in both countries are well defined and therefore the only candidate for a Nash equilibrium. If country B increases its educational level further country A benefits from educating the lower ability students by keeping its educational level slightly below the level of country B.

Now we analyze the case of a partial return migration of foreign students.

3.5 Graduates only partially return to their home country ($\pi < 1$)

In the intermediate case, we have seen that the welfare of each country is a combination of the welfare obtained in each extreme case, as given by (26):

$$W^I(e^A, e^B) = (1-\pi)W^I_{|\pi=0} + \pi W^I_{|\pi=1}(e^A, e^B) \; I = A, B. \tag{36}$$

Hence, the analysis of the two extreme cases will be helpful.

If some foreign students stay in the country after graduation the competition for students is beneficial. First we show that this competition induces a force towards less differentiation than is optimal. In particular, the optimal differentiation levels $\overline{e}^*, \underline{e}^*$ do not form a Nash equilibrium. Hence, a return probability falling below unity implies that the differentiation shrinks inefficiently. Second, if the return probability of foreign students is small enough symmetric educational levels are a Nash-equilibrium in pure strategies.

Asymmetric Nash equilibrium and optimal differentiation Consider the optimal distinct levels, with for instance A choosing the highest level: $e^A = \overline{e}^*$ and $e^B = \underline{e}^*$. Let country A contemplate changing its educational level. From section 3.3 we know that for $\pi = 1$ the optimally differentiated educational levels are chosen in the Nash-equilibrium. Hence, the marginal change of welfare for $\pi = 1$ by changing e^A is zero at the point of optimal differentiation. From the representation of welfare as convex combination of the two extreme cases in (36) we can now infer that the change in welfare at the point of optimal differentiation only depends on the change of welfare for $\pi = 0$.

The forces towards more convergence or more differentiation can be analyzed more generally by considering marginal changes starting at unequal levels, say $e^A > e^B$ (assuming that each country educates some students, i.e. $0 < y^u < y^{AB} < \overline{y}$). As long as we consider variations in educational levels that are small enough so that the educational level in A is still higher than in B, A continues to attract the students with the highest ability. Hence, a marginal change in e^A or in e^B modifies the allocation of the students at the margin only through the modifications of the thresholds y^{AB} and y^u. From the convexity of c, increasing e^A or increasing e^B increases the threshold value y^{AB}, meaning here that the number of students in B increases. As for y^u, it is independent of e^A.

A marginal change in e^A yields the marginal change in $W^A_{|\pi=0}$

$$\frac{\partial W^A_{|\pi=0}}{\partial e^A}(e^A, e^B) = 2\left(\overline{y} - y^{AB}\right)\left[w_s \frac{\overline{y} + y^{AB}}{2} - c'(e^A)\right] + 2[y^{AB}e^A w_s - c\left(e^A\right)](-\frac{\partial y^{AB}}{\partial e^A}). \quad (37)$$

The marginal change is composed of two terms. The first term reflects the efficiency gains (possibly negative) on the current population of students in A that result from changing the educational level. The second term reflects the migration effect that results from changes in that population through the modification of the threshold. It is equal to the change in the number of students, $-2\frac{\partial y^{AB}}{\partial e^A}$, multiplied by the lifetime income per such student, $[y^{AB}e^A w_s - c\left(e^A\right)]$. Observe that this lifetime income is surely positive, hence the second term in (37) is always negative. As a result, country A prefers a lower educational level than the one that would maximize the efficiency gains (given the level e^B) so as to attract more students. This readily explains why the optimal educational values do not form an equilibrium: given that B chooses $\underline{e}^*$, country A prefers a lower educational level than the optimal one $\overline{e}^*$.

Similarly, a marginal change in the lower educational level e^B yields the marginal change in W^B

$$\begin{aligned}\frac{\partial W^B_{|\pi=0}}{\partial e^B}(e^A, e^B) &= 2\left(y^{AB} - y^u\right)\left[w_s \frac{y^{AB} + y^u}{2} - c'(e^B)\right] + 2[y^{AB}e^B w_s - c\left(e^B\right)]\frac{\partial y^{AB}}{\partial e^B} \\ &\quad + 2[y^u e^B w_s - w_u - c\left(e^B\right)](-\frac{\partial y^u}{\partial e^B}). \end{aligned} \quad (38)$$

The marginal change in welfare is composed of three terms : the efficiency gains on the students in B (the first term), a migration effect due to students moving from B to A (the second term), and a change in the incentives to become skilled (the third term).

The efficiency gains and the migration effects are interpreted similarly as for A. Observe that the migration effect is exactly the opposite of the one for A : the impact of the change in the population of the students between A or B results in a simple transfer of welfare between the two countries at the margin. The reason is that the attracted students who move from a country to another one have roughly the same lifetime income in both countries. This can be checked by noticing that $y^{AB}e^B w_s - c\left(e^B\right) = y^{AB}e^A w_s - c\left(e^A\right)$ (by the arbitrage condition (18) for the marginal student y^{AB}) and $\frac{\partial y^{AB}}{\partial e^A} = \frac{\partial y^{AB}}{\partial e^B}$. Thus, the second term is always positive and provides an incentive to increase the educational level above the efficient one (given e^A).

A change in the lowest educational level has an additional impact on the incentives to become skilled, as reflected by the third term in (38). Observe that a marginal change in e^B has a null overall marginal impact on the welfare of the marginal students by the arbitrage condition for students with ability y^u. This is however not the case for B because this arbitrage condition (19) yields $\left[y^u e^B w_s - w_u - c\left(e^B\right)\right] = w^u$. Hence the third term is not null. The reason is that unskilled individuals are immobile. Hence an increase in the number of students has a positive impact on B due to the migration from unskilled citizens of A who become students in B. The welfare benefit to B is equal to the lifetime gain of these marginal students, $2w^u$, multiplied by the marginal increase of students (coming from A), $-\frac{\partial y^u}{\partial e^B}$. This explains the third term. Thus, country B has an additional incentive to increase its educational level when this increases the incentive to study (i.e., when $-\frac{\partial y^u}{\partial e^B} > 0$). In that case, increasing the education level above the efficient one allows B to attract students both at the bottom and at the top of its students' population. When instead $\frac{\partial y^u}{\partial e^B} > 0$, the third term is negative and diminishes the force to convergence. How the value y^u changes is ambiguous. In the proof of the following Proposition 3 we derive a condition under which the country with the lower educational level has an incentive to increase its level above the optimal one.

To sum up, competition on the marginal students who are close to being indifferent between the two countries is a force towards less differentiation. However, attracting additional workers by inciting them to become skilled may be a countervailing effect on a country which chooses a low educational level.

We can derive the following proposition:

Proposition 3 *Assume that $\pi < 1$. Optimally differentiated educational levels do not constitute an equilibrium. Supposing for example $(e^A, e^B) = (\overline{e}^*, \underline{e}^*)$, country A has an incentive to choose an educational level less than $\overline{e}^*$. Furthermore, if the optimal differentiation is not too large, i.e. $\underline{e}^* > \frac{\overline{e}^*}{2}$, then country B has an incentive to choose an educational level higher than $\underline{e}^*$.*

Proof. See proof section. ■

The intuition for this result is that a country takes into account the impact of its educational level on foreign students who only partially return to their home country. Since the attracted students are indifferent between the two educational levels, there is no welfare loss on the aggregate. Those foreign students who stay in the country where they have been educated have at the margin the same lifetime income which results in a transfer of welfare from their home country to the country of education. The country with the higher educational level has an incentive to decrease this level in order to attract higher ability types of students in country B. The country with the lower educational level has an incentive to increase this level so that the threshold ability level and the number of students increase.

The welfare implications can be explained by externalities due to students' mobility. If country A with the higher educational level increases its educational level the lower ability range of its students moves to country B. This creates a positive externality on country B because it increases the number of students and improves the ability composition of students in country B. As a result country A chooses an educational level which is lower than optimal. If country B with the lower educational level increases this level it attracts the lower range of ability types of country A. This imposes a negative externality on country A which loses students. Hence, country B chooses a higher educational level than the optimal one.

As the forces toward competition depend on π equilibrium choices do as well. We can safely conjecture that, under the conditions stated in Proposition 3, there is an equilibrium with differentiated educational levels for π close enough to 1, and that furthermore differentiation is lower than the efficient one in the sense that $\underline{e}^* < e^B < e^A < \overline{e}^*$, assuming that A provides the larger level.

Symmetric Nash equilibrium Now we turn to the analysis of symmetric educational levels. Recall that a country's welfare, say A's welfare, keeping the other country's level fixed at e_B, is continuous for $\pi = 1$. Instead, for $\pi = 0$, the welfare is discontinuous when the education levels become equalized as stated in Lemma 3 except if e_B is set equal to $\hat{e}$. Hence, the welfare in the intermediate case $0 < \pi < 1$ is also discontinuous and Lemma 3 is valid for any π smaller than 1. The following proposition shows that the stronger the competition for students (the smaller π), the stronger the forces to the equalization of the education levels.

Proposition 4 *Whatever the value of π, $\pi < 1$, $(\hat{e}, \hat{e})$ is the unique candidate for a symmetric equilibrium,*

It is not an equilibrium when the return probability is too large, close to 1.

If $(\hat{e}, \hat{e})$ is an equilibrium for π, then it is also an equilibrium when graduates return probability is lower, that is for any value smaller than π.

Proof. See proof section. ■

Summing up these results show that, for return probabilities close to unity, any Nash equilibrium is asymmetric and the degree of differentiation is smaller than would be optimal. The reason are externalities of students' mobility. However, for low return probabilities a Nash equilibrium may be symmetric and the educational level in both countries is smaller than the optimal level in closed economies. In this case countries forgo the possible gain in welfare due to differentiation because the low return migration of students prompts countries to compete for students by equalizing their educational levels.

3.5.1 Illustration

Figure 4 from section 3.4.1 which represents the welfare function of A for $e^B = \underline{e}^*$ and $\pi = 0$ shows that even in a case where the choice of the educational level is restricted to $\underline{e}^*$ for country B, the level chosen by country A falls short of the optimally differentiated level, $\hat{e}^A < \overline{e}^*$.

Furthermore, we illustrate for $e^B = \hat{e} = 8.285$ that the welfare function of country A is continuous independently of all return probabilities in the range of $0 \leq \pi < 1$. The welfare functions of country A for various return probabilities is shown in Figure 7.

Only in the first two cases of $\pi = 0$ and $\pi = 2/3$, $\hat{e}$ yields a symmetric equilibrim. As expected, the return probability π must be small enough to guarantee the existence of the symmetric equilibrium.

4 Conclusion

We have examined competition in fee-financed quality levels of higher education. The mobility of students affects educational quality in countries in a very different way depending on the degree of return migration. In the extreme case in which all foreign students return to their home country educational levels are differentiated optimally. Hence, opening up borders for mobile students results in a clear-cut overall welfare gain since the various ability types of students are matched more appropriately by differing education levels than in a closed economy with just one educational level for all ability types. However, in the more relevant case in which some foreign students stay in the country where they have been educated the differentiation of educational levels is less than optimal. The reason is that both countries compete to attract foreign students. In particular, at the optimal differentiation levels, the country with the largest education level has an incentive to lower its education level to attract the best students of the other country, which is harmful for its own students. Similarly under some condition the country with the lowest education level has an incentive to raise its education level above the optimal one which

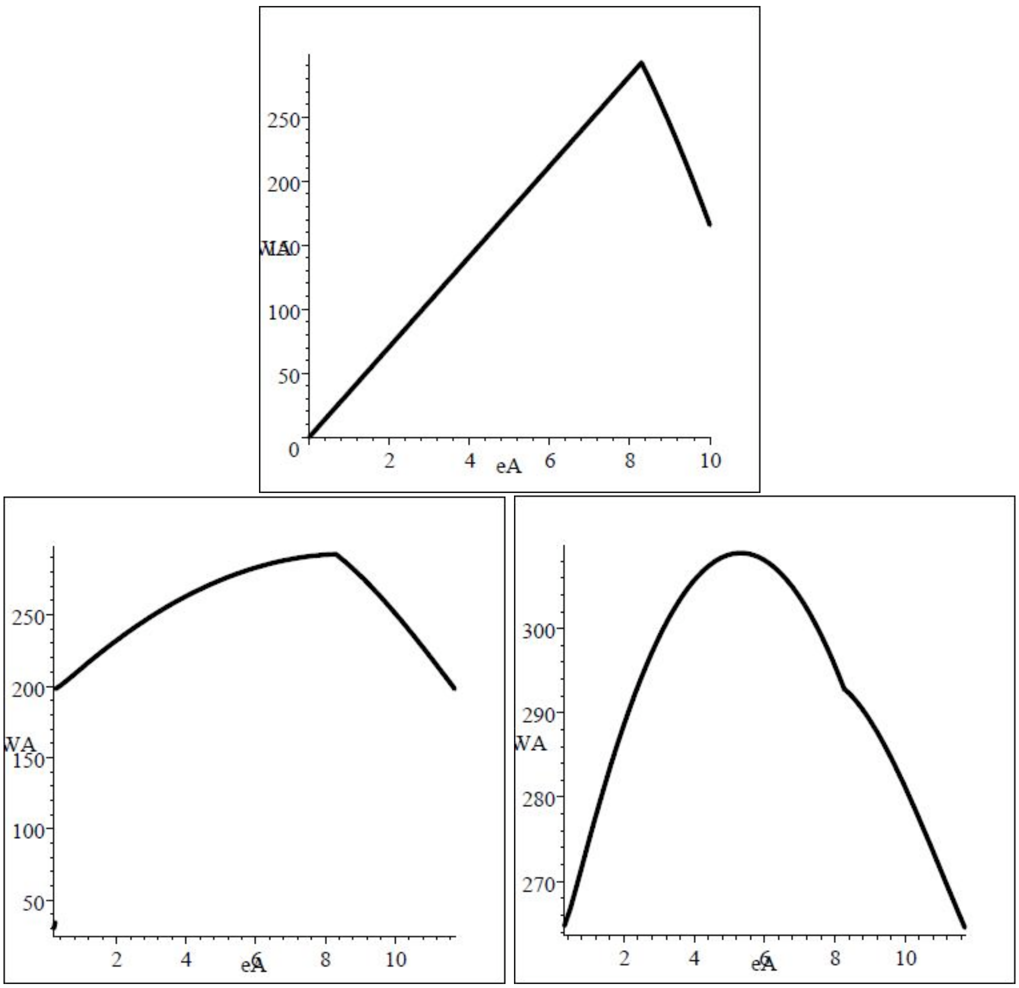

Figure 7: Welfare of country A at $e^B = \hat{e} = 8.285$ and $\pi = 0, 2/3, 0.9$.

reduces also the lifetime income of its home students. Furthermore, if the probability of return migration is sufficiently low, countries do not differentiate quality at all and the symmetric educational level is inefficiently high.

Our analysis confirms an important insight of Justman and Thisse (2000). They showed that their underinvestment result critically depends on the government's objective of residential welfare. If in contrast governments take into account the welfare of native-born highly educated this may lead to overinvestment. In our model government's concern exclusively about natives whereever they work results in the optimal differentiation in educational levels whereas a stronger emphasis on foreign-born highly educated who work in the country leads to suboptimal differentiaiton. Hence, statements about efficiency outcomes of higher education systems as a result of increasing human capital mobility depend strongly on the imputed goals of governments.

What can be said about the return migration of foreign students? The extent of return migration varies considerably between countries and depends on several factors, among others the family status, immigration policy and the comparative employment opportunities in the origin and destination countries (Lu et al., 2009; Tremblay, 2002, 2005). There are some empirical studies showing that the fraction of foreign students who stay in their host countries of higher education upon graduation is substantial. The stay rates of foreign students in the United States are estimated to lie between 20 percent and 30 percent, (Rosenzweig, 2006, Lowell, Bump and Martin, 2007).up to about 65 percent for those who earned a doctorate in the United States (Finn, 2003; Lowell, Bump and Martin, 2007). Foreign students who participated in a special scholarship program in Germany have been estimated to stay in the country at a rate of 35 percent (Hein and Plesch, 2008). Those return rates are partially a result of political intervention. Lowell et al. (2007) report that e.g. the visa application process is a strong policy tool to attract and to keep foreign students. Hence, there seems to be scope for competition for students and many OECD countries are aware of this option (Chaloff and Lemaitre, 2009). A policy implication of our analysis is this that education policies which try to lock-in foreign students are detrimental for efficiency.

There are several ways to extend the analysis. Our paper focuses on the part of educational quality that is fee-financed, neglecting the impact that it might have on the basic tax-fianced system. In the presence of perfect credit markets and the absence of externalities this way of financing is optimal (see Demange et al., 2008a). However, tax financing in most countries are a matter of fact and can be seen as a means to redistribute resources in favor of students. It would be worthwhile to allow for mixed financing by fees and taxes on wage income. With partial tax financing countries may have a weaker incentive to compete for students since they generate a cost factor in the government's budget which has to be financed by skilled and unskilled workers. This may result in a differentiation more closely to the optimal one when not all foreign students return home. Finally, another line of extensions is the introduction of heterogeneity in the countries. One could distinguish small and large countries by incorporating cost functions with differing fixed cost components, the smaller country with a higher fix cost in setting up higher education. Another type of heterogeneity between countries stems from differences in their school sectors. An improvement in school education enlarges the range of abilities. In terms of our model this means that countries differ in their top ability type of students. The heterogeneity of countries presumably modifies the outcome of competition for students. We leave those extensions for future research.

5 Proofs

Proof of Lemma 2 Let $e^A > e^B$. We have

$$\begin{aligned} TW(e^A, e^B) &= w_s \left[2\left(\overline{y} - y^{AB}\right) \frac{\overline{y} + y^{AB}}{2} e^A + 2\left(y^{AB} - y^u\right) \frac{y^{AB} + y^u}{2} e^B \right] \\ &\quad + 4 w_u y^u - 2\left(\overline{y} - y^{AB}\right) c\left(e^A\right) - 2\left(y^{AB} - y^u\right) c\left(e^B\right). \end{aligned}$$

The derivative of this expression with respect to e^A is the sum of a direct effect on the current students, $\left(\overline{y} - y^{AB}\right)\left[w_s\left(\overline{y} + y^{AB}\right) - 2c'(e^A)\right]$, and an indirect effect due to the change in the threshold y^{AB}. The indirect effect is null because there is no distortion in individuals' choices. This can be checked as follows. The indirect effect is equal to

$$2\frac{\partial y^{AB}}{\partial e}[-w_s y^{AB} e^A + c(e^A) + w_s y^{AB} e^B - c(e^B)].$$

The term in brackets is null because individuals split voluntarily across countries according to $w_s y^{AB} e^A - c(e^A) = w_s y^{AB} e^B - c(e^B)$ whatever the educational levels are. We are left with the direct effect. Taking the limit when e^A tends to $e^B = e$, y^{AB} tends to $y^{lim} = \frac{c'(e)}{w_s}$. Thus the direct effect $\left(\overline{y} - y^{AB}\right)\left[w_s\left(\overline{y} + y^{AB}\right) - 2c'(e^A)\right]$ tends to $w_s\left(\overline{y} - \frac{c'(e)}{w_s}\right)^2$, which is equal to $\lim_{e^A \to e^+} \frac{\partial TW}{\partial e^A}(e^A, e)$, as given in (29).

Consider now the derivative of TW with respect to e^B with $e^B < e^A$. It is the sum of the direct effect, $\left(y^{AB} - y^u\right)\left[w_s\left(y^{AB} + y^u\right) - 2c'\left(e^B\right)\right]$ and indirect effects due to the changes in the thresholds y^{AB} and y^u. The indirect effect due to the change in y^{AB} vanishes by the same argument as above. Similarly, the marginal indirect effect due to the change in y^u is null because there is no distortion in individuals' choices. This is checked since the indirect effect is

$$\frac{\partial y^u}{\partial e}[-w_s y^u e^B + c(e^B) + 2w_u]$$

and the term in square brackets is null. Again we are left with the direct effect only. Taking the limit as e^B tends to $e^A = e$, the direct effect $\left(y^{AB} - y^u\right)\left[w_s\left(y^{AB} + y^u\right) - 2c'\left(e^B\right)\right]$ tends to $\left(y^{lim} - y^u\right)\left(w_s\left(y^{lim} + y^u\right) - 2c'(e)\right)$ and using $w_s y^{lim} = c'(e)$ gives $\lim_{e^B \to < e} \frac{\partial TW}{\partial e^B}(e, e^B) = -w_s\left(\frac{c'(e)}{w_s} - y^u\right)^2$ as given in (29).

Furthermore, setting the direct effects equal to zero a comparison with optimality conditions in (16) shows that maximal total welfare is achieved with $e^A = \overline{e}^*$ and $e^B = \underline{e}^*$. ■.

Proof of Proposition 1 Let $e^A > e^B$. Country A chooses its educational level by maximizing the natives' welfare from (30). Differentiating the concave welfare function with respect to e^A yields the first-order condition:

$$\begin{aligned} \frac{\partial W^A_{|\pi=1}}{\partial e^A} &= \left(\overline{y} - y^{AB}\right)\left[w_s \frac{\overline{y} + y^{AB}}{2} - c'\left(e^A\right)\right] \\ &\quad + \frac{\partial y^{AB}}{\partial e^A}\left[w_s e^B y^{AB} - c\left(e^B\right) - \left(w_s e^A y^{AB} - c\left(e^A\right)\right)\right] \\ &= 0 \end{aligned} \tag{39}$$

From Lemma 1 we know that students partition themselves optimally across both countries so that $w_s e^B y^{AB} - c\left(e^B\right) = w_s e^A y^{AB} - c\left(e^A\right)$. Hence, country A's educational level is determined by the condition $w_s \frac{\overline{y} + y^{AB}}{2} = c'\left(e^A\right)$ which is the optimal educational level according to

(16). The welfare maximum of country B is given by

$$\begin{aligned}\frac{\partial W^{B}_{|\pi=1}}{\partial e^{B}} &= \left(y^{AB}-y^{u}\right)\left[w_s\frac{y^{AB}+y^{u}}{2}-c'\left(e^{B}\right)\right] \\ &\quad +\frac{\partial y^{AB}}{\partial e^{B}}\left[w_s e^{B}y^{AB}-c\left(e^{B}\right)-\left(w_s e^{A}y^{AB}-c\left(e^{A}\right)\right)\right] \\ &\quad -\frac{\partial y^{u}}{\partial e^{B}}\left[w_s e^{B}y^{u}-c\left(e^{B}\right)-2w_u\right] \\ &= 0.\end{aligned} \tag{40}$$

From Lemma 1 again, students split optimally across countries according to (18) and the lower ability types of individuals divide optimally in remaining unskilled and receiving higher education in country B: $w_s e^{B} y^{u} - c\left(e^{B}\right) = 2w_u$. Hence, the educational level chosen by country B is given by $w_s \frac{y^{AB}+y^{u}}{2} = c'\left(e^{B}\right)$ which is optimal according to condition (16).

Since the voluntary partition of both students between countries and unskilled and skilled labor is optimal whatever the educational level is, countries choose always the optimally differentiated educational levels and the Nash equilibrium is unique.

Proof of Lemma 3 The sum of the welfare in the two countries associated to two education levels, e^{A}, e^{B} is given by the total welfare function:

$$\begin{aligned}TW(e^{A},e^{B}) &= [W^{A}_{|\pi=0}+W^{B}_{|\pi=0}](e^{A},e^{B}) \\ &= w_s\left[2\left(\overline{y}-y^{AB}\right)\frac{\overline{y}+y^{AB}}{2}e^{A}+2\left(y^{AB}-y^{u}\right)\frac{y^{AB}+y^{u}}{2}e^{B}\right] \\ &\quad + 4w_u y^{u} - 2\left(\overline{y}-y^{AB}\right)c\left(e^{A}\right)-2\left(y^{AB}-y^{u}\right)c\left(e^{B}\right).\end{aligned}$$

TW is continuous. Taking the limit as e^{A} and e^{B} tend to e, we obtain

$$\lim_{e^{A}\to e^{-},e^{B}\to e^{+}} W^{A}_{|\pi=0}(e^{A},e^{B}) + \lim_{e^{A}\to e^{-},e^{B}\to e^{+}} W^{B}_{|\pi=0}(e^{A},e^{B}) = TW(e,e). \tag{41}$$

By symmetry $W^{A}_{|\pi=0}(e^{A},e^{B}) = W^{B}_{|\pi=0}(e^{B},e^{A})$ and $2W^{A}_{|\pi=0}(e,e) = 2W^{B}_{|\pi=0}(e,e) = TW(e,e)$. This gives

$$\lim_{e^{A}\to e^{-},e^{B}\to e^{+}} W^{B}_{|\pi=0}(e^{A},e^{B}) = \lim_{e^{A}\to e^{+},e^{B}\to e^{-}} W^{A}_{|\pi=0}(e^{A},e^{B}) \tag{42}$$

Plugging (42) into (41)

$$\lim_{e^{A}\to e^{-},e^{B}\to e^{+}} W^{A}_{|\pi=0}(e^{A},e^{B}) + \lim_{e^{A}\to e^{+},e^{B}\to e^{-}} W^{A}_{|\pi=0}(e^{A},e^{B}) = TW(e,e) \tag{43}$$

Taking $e^{B} = e$ in the above equation gives equation (35).

Since the jumps are in the same direction, jumps must be null at a symmetric equilibrium: if the jump is positive at (e,e), a country benefits by proposing a level higher than e and if it is negative it benefits by proposing a smaller one. ∎

Proof of Proposition 2 Let us determine the best response of a country, say A, to the educational level chosen by the other country. Given $e^{B} = e$, consider $W^{A}(e^{A}, e)$ as a function of e^{A}. We need to distinguish three cases depending on e^{A} being smaller than, equal to, or larger than e. Also we want both countries to have students, which requires $y^{u} < y^{AB} < \overline{y}$. The first inequality holds true if $\frac{c'(e)}{w_s} > \frac{2w_u + c(e))}{w_s e}$ (cf. (18) and (19)), i.e. for $c(e) = e^2$ if $e^2 > 2w_u$.

1) As long as $e^A < e^B = e$, country A attracts students with low ability, i.e., between $y^u(e^A)$ and y^{AB}. The derivative is (cf. (38) where the roles of A and B are exchanged and where we use $[-y^u e^A w_s + w_u + c\left(e^A\right)] = -w_u$)

$$\begin{aligned} \frac{\partial W^A}{\partial e^A} &= 2\left(y^{AB} - y^u\right)\left[w_s \frac{y^{AB} + y^u}{2} - c'(e^A)\right] \\ &\quad +2[y^{AB} e^A w_s - c\left(e^A\right)]\frac{\partial y^{AB}}{\partial e^A} - 2w_u \frac{\partial y^u}{\partial e^A} \\ \text{for } e^A &< e^B. \end{aligned} \tag{44}$$

Check that with a quadratic cost function, the derivative is linear in e^A and increasing: A prefers to be as close as possible to e in this zone.Therefore if the country prefers to be the one with the lowest level its best response is 'almost' to match the other country's level e. Furthermore, this implies that there is no asymmetric equilibrium since the country with the lower educational level increases its welfare by increasing its level.

2) Consider the zone with $e^A > e$. We have

$$\begin{aligned} \frac{\partial W^A}{\partial e^A} &= 2\left(\overline{y} - y^{AB}\right)\left[w_s \frac{\overline{y} + y^{AB}}{2} - c'(e^A)\right] - 2[y^{AB} e^A w_s - c\left(e^A\right)]\frac{\partial y^{AB}}{\partial e^A} \\ \text{for } e^A &> e^B. \end{aligned} \tag{45}$$

The derivative is null at an 'interior' best response, one that is indeed above e. Check that with a quadratic cost function the best response is decreasing with e (write $\frac{\partial W^A}{\partial e^A} = 0$ and impose the solution to be larger than e). The minimum ability level of those who decide to study y^u depends only on the minimum educational level. Thus it is continuous and stays constant for e^A larger than e.

3) We need to examine carefully the behavior when e^A is close to e because of discontinuities. We know that the limit of y^{AB} when e^A tends to e is $y^{lim}(e)$ given by (27):

Let us denote by $D(e)$ the jump on A's welfare as e^A approaches $e = e^B$ from above: $D(e) = \lim_{e^A \to e^+} W^A_{|\pi=0}(e^A, e) - \frac{1}{2} TW(e, e)$. Since $W^A_{|\pi=0} + W^B_{|\pi=0} = TW$, we have

$$[W^A_{|\pi=0} - \frac{1}{2} TW](e^A, e^B) = \frac{1}{2}[W^A_{|\pi=0} - W^B_{|\pi=0}](e^A, e^B).$$

¿From (31) and (32) the value $\frac{1}{2}[W^A_{|\pi=0} - W^B_{|\pi=0}](e^A, e^B)$ for $e^A > e^B$ is

$$\left(\overline{y} - y^{AB}\right)\left[w_s \frac{\overline{y} + y^{AB}}{2} e^A - c\left(e^A\right)\right] - \left(y^{AB} - y^u\right)\left[w_s \frac{y^{AB} + y^u}{2} e^B - c(e^B)\right]$$

where y^{AB} is function of e^A and e^B and y^u of e^B. $D(e)$ is obtained by taking the limit when e^A tends to $e = e^B$ from above. Since y^{AB} tends to $y^{lim}(e)$, and y^u is equal to $y^u(e) = \frac{2w_u + c(e)}{w_s e}$, we obtain

$$D(e) = \left(\overline{y} - y^{lim}(e)\right)\left[w_s \frac{\overline{y} + y^{lim}(e)}{2} e - c\left(e\right)\right] - \left(y^{lim}(e) - y^u(e)\right)\left[w_s \frac{y^{lim}(e) + y^u(e)}{2} e - c(e)\right]$$

For $c(e) = \alpha e^2$, $w_s y^{lim}(e) = 2\alpha e$ hence $w_s \frac{y^{lim}}{2} e - c(e) = 0$. Thus

$$D(e) = \frac{w_s}{2} e[\left(\overline{y} - y^{lim}(e)\right)\overline{y} - \left(y^{lim}(e) - y^u(e)\right) y^u(e)] \tag{46}$$

$D(e)$ is of the same sign as the term in brackets. This term is positive at e^* (as we know from the text or directly using that $\left(\overline{y} - y^{lim}(e)\right) = \left(y^{lim}(e) - y^u(e)\right)$ at e^*). It is negative for e large enough so that $y^{lim}(e)$ equals to $\overline{y}$). Furthermore, the term decreases with e because $y^{lim}(e)$ increases; $\left(y^{lim} - y^u\right) = \frac{1}{w_s}(\alpha e - \frac{w_u}{e})$ increases, and $y^u(e) = \frac{2w_u + \alpha e^2}{w_s e}$ increases. Hence there is a unique value $\hat{e}$ for which $D(\hat{e}) = 0$, and furthermore this value is larger than e^*.

At $\widehat{e}$ level, the net benefit from educating high ability students, those with ability larger than y^{AB}, is exactly equal to the net benefit from educating low ability students, those with ability between y^{AB} and y^u.

Consider a value e larger than $\widehat{e}$. No country benefits by improving the educational level. Each benefits from choosing an educational level just below the other one. Similarly, for a value e smaller than $\widehat{e}$, a country benefits by improving the educational level above e. We are left with $(\widehat{e}, \widehat{e})$ as the only possibility for an equilibrium in pure strategies.

We determine conditions under which $(\widehat{e}, \widehat{e})$ is indeed an equilibrium. Take $e^B = \widehat{e}$, and consider the welfare of A, for example (by symmetry the same argument works for B). A's welfare is continuous at $\widehat{e}$. Furthermore it increases for $e^A < \widehat{e}$. Hence, if A's welfare decreases for $e^A > \widehat{e}$, $\widehat{e}$ is indeed a best response to $e^B = \widehat{e}$ and $(\widehat{e}, \widehat{e})$ is an equilibrium. Recall that a country's welfare is concave when it has the largest education level. Therefore, A's welfare decreases for $e > \widehat{e}$ if and only if the 'right' derivative $\lim_{e^A \to > e} \frac{\partial W^A}{\partial e^A}$ is negative (since, in that case, the concavity of W^A for $e > \widehat{e}$ implies that $\frac{\partial W^A}{\partial e^A}$ is negative for $e > \widehat{e}$ and, hence, A's welfare decreases). ■

Proof of Proposition 3 Countries maximize welfare

$$W^I(e^A, e^B) = (1 - \pi) W^I_{|\pi=0} + \pi W^I_{|\pi=1}(e^A, e^B) \;\; I = A, B. \qquad (26)$$

We evaluate the partial derivatives of the welfare functions W^I at $(\overline{e}^*, \underline{e}^*)$, that is assuming that countries provide the optimally differentiated educational levels $e^A = \overline{e}^*$ and $e^B = \underline{e}^*$. From (26), these partial derivatives are a combination of the partial derivatives of $W^I_{|\pi=0}$ and $W^I_{|\pi=1}$, $I = A, B$.

The derivatives $\frac{\partial W^A_{|\pi=1}}{\partial e^A}(\overline{e}^*, \underline{e}^*)$ and $\frac{\partial W^B_{|\pi=1}}{\partial e^B}(\overline{e}^*, \underline{e}^*)$ are null because optimal differentiation levels form a Nash equilibrium (Proposition 1). Thus

$$\frac{\partial W^A}{\partial e^A}(\overline{e}^*, \underline{e}^*) = (1 - \pi) \frac{\partial W^A_{\pi=0}}{\partial e^A}(\overline{e}^*, \underline{e}^*)$$

and similarly for B. Expressions (37) and (38) give

$$\frac{\partial W^A_{|\pi=0}}{\partial e^A}(e^A, e^B) = 2\left(\overline{y} - y^{AB}\right)\left[w_s \frac{\overline{y} + y^{AB}}{2} - c'(e^A)\right] + 2[y^{AB} e^A w_s - c\left(e^A\right)](-\frac{\partial y^{AB}}{\partial e^A})$$

$$\begin{aligned}\frac{\partial W^B_{|\pi=0}}{\partial e^B}(e^A, e^B) \;\; &= \;\; 2\left(y^{AB} - y^u\right)\left[w_s \frac{y^{AB} + y^u}{2} - c'(e^B)\right] + 2[y^{AB} e^B w_s - c\left(e^B\right)] \frac{\partial y^{AB}}{\partial e^B} \\ &\quad + 2[y^u e^B w_s - w_u - c\left(e^B\right)](-\frac{\partial y^u}{\partial e^B}).\end{aligned}$$

At $(\overline{e}^*, \underline{e}^*)$, the first terms in squared brackets in (37) and (38), which represent the efficiency gains, are zero.

The second term in (37) is negative since $[y^{AB}e^{A}w_s - c\left(e^{A}\right)] > 0$ and

$$\frac{\partial y^{AB}}{\partial e^{A}} = \frac{c'\left(e^{A}\right) - w_s y^{AB}}{w_s\left(e^{A} - e^{B}\right)} > 0.$$

Hence, country A's welfare increases if e^{A} decreases below $\overline{e}^{*}$.

The second term in (38) is positive since $[y^{AB}e^{B}w_s - c\left(e^{B}\right)] > 0$ and

$$\frac{\partial y^{AB}}{\partial e^{B}} = \frac{w_s y^{AB} - c'\left(e^{B}\right)}{w_s\left(e^{A} - e^{B}\right)} > 0.$$

The third term in (38) is negative since $[-y^{u}e^{B}w_s + w_u + c\left(e^{B}\right)] = -w_u$ by definition of y^{u} (see (19)). And at $e^{B} = \underline{e}^{*}$ we get:

$$\left.\frac{\partial y^{u}}{\partial e^{B}}\right|_{e^{B}=\underline{e}^{*}} = \frac{c'\left(e^{B}\right) - w_s y^{u}}{w_s e^{B}} > 0$$

because $c'\left(e^{B}\right) = w_s \frac{y^{AB}+y^{u}}{2} > w_s y^{u}$. An educational level e^{B} higher than $\underline{e}^{*}$ is welfare improving for country B if and only if

$$\left[[y^{AB}e^{B}w_s - c\left(e^{B}\right)]\frac{\partial y^{AB}}{\partial e^{B}} - w_u\frac{\partial y^{u}}{\partial e^{B}}\right]_{e^{B}=\underline{e}^{*}} > 0. \tag{47}$$

We compare both terms factorwise. First we have $y^{AB}e^{B}w_s - c\left(e^{B}\right) > y^{u}e^{B}w_s - c\left(e^{B}\right) = 2w_u > w_u$. Second we show that $\frac{\partial y^{AB}}{\partial e^{B}} > \frac{\partial y^{u}}{\partial e^{B}}$ if $e^{B} > \frac{e^{A}}{2}$ at the optimal differentiation. For the following calculation we use that the optimally differentiated educational level e^{B} satisfies $c'\left(e^{B}\right) = w_s \frac{y^{AB}+y^{u}}{2}$:

$$\begin{aligned} & \frac{w_s y^{AB} - c'\left(e^{B}\right)}{w_s\left(e^{A} - e^{B}\right)} - \frac{c'\left(e^{B}\right) - w_s y^{u}}{w_s e^{B}} \\ = \; & \frac{1}{\left(e^{A} - e^{B}\right) w_s e^{B}}\left[\left(e^{B} - \frac{e^{A}}{2}\right) w_s \left(y^{AB} - y^{u}\right)\right]. \end{aligned}$$

If $e^{B} > \frac{e^{A}}{2}$ we have $\frac{\partial y^{AB}}{\partial e^{B}} > \frac{\partial y^{u}}{\partial e^{B}}$ which proves the proposition. ■

Proof of Proposition 4 Consider the unique candidate for a symmetric equilibrium, $(\hat{e}, \hat{e})$. By definition of $\hat{e}$, a country's objective is continuous at $\hat{e}$ with respect to its educational level. However, there is typically a kink, that is the right and left derivatives do not coincide. Take $e^{B} = \hat{e}$, and consider A welfare for example (by symmetry the same argument works for B). An equilibrium is obtained at $(\hat{e}, \hat{e})$ only if the left derivative is nonnegative and the right one is non positive, i.e.

$$\lim_{e^{A}\to<\hat{e}} \frac{\partial W^{A}_{|\pi}}{\partial e^{A}}(e^{A}, \hat{e}) \geq 0 \text{ and } \lim_{e^{A}\to>\hat{e}} \frac{\partial W^{A}_{|\pi}}{\partial e^{A}}(e^{A}, \hat{e}) \leq 0. \tag{48}$$

Assume there is scope for differentiation at $\hat{e}$. Then conditions (48) are never satisfied at $\pi = 1$. More precisely, since $W^{A}_{|\pi=1}(e^{A}, e^{B})$ is half total welfare $TW(e^{A}, e^{B})$, we know from lemma 2 that

$$\lim_{e^{A}\to<\hat{e}} \frac{\partial W^{A}_{|\pi=1}}{\partial e^{A}}(e^{A}, \hat{e}) \leq 0, \text{ and } \lim_{e^{A}\to>\hat{e}} \frac{\partial W^{A}_{|\pi=1}}{\partial e^{A}}(e^{A}, \hat{e}) \geq 0.$$

Consider now a return probability $\pi < 1$. A country's objective criterion is a combination of the criterion that obtains in the two extreme cases. Thus we have for $e^A \neq \hat{e}$

$$\frac{\partial W^A_{|\pi}}{\partial e^A}(e^A, \hat{e}) = (1-\pi)\frac{\partial W^A_{|\pi=0}}{\partial e^A}(e^A, \hat{e}) + \pi \frac{\partial W^A_{|\pi=1}}{\partial e^A}(e^A, \hat{e}) \quad (49)$$

and the same convex combination applies at the limit when e^A tends to $\hat{e}$ alternatively for $e^A > \hat{e}$ and $e^A < \hat{e}$. Assume that the first inequality in (48) holds for $\pi < 1$. Since $\lim_{e^A \to <\hat{e}} \frac{\partial W^A_{|\pi=1}}{\partial e^A}(e^A, \hat{e}) \leq 0$ it must be that $\lim_{e^A \to <\hat{e}} \frac{\partial W^A_{|\pi=0}}{\partial e^A}(e^A, \hat{e})$ is positive. Decreasing π increases the weight on this positive term and decreases on the negative one: surely $\lim_{e^A \to <\hat{e}} \frac{\partial W^A_{|\pi=1}}{\partial e^A}(e^A, \hat{e}) > 0$ for smaller π. Similarly, if the second inequality (48) holds for $\pi < 1$ it holds for any smaller value for π. This proves Proposition 4. ▪

References

[1] Blackorby C., W. Bossert and D. Donaldson (2006). "Population Ethics", Handbook of Rational and Social Choice, edited by P. Anand, P. Pattanaik and C. Puppe, to be published by Oxford University Press.

[2] Chaloff, J. and G. Lemaitre (2009). "Managing Highly-skilled Labour Migration: A Comparative Analysis of Migration Policies and Challenges in OECD Countries", OECD Social, Employment and Migration Working Paper 79.

[3] Coniglio, N.D. and F. Prota (2003). "Human Capital Accumulation and Migration in a Peripheral EU Region: The Case of Basilicata", SNF Working Paper No. 43/2003, Bergen.

[4] Del Rey, E. (2001). "Economic Integration and Public Provision of Education," Empirica 28, 203-218.

[5] Demange, G., R. Fenge and S. Uebelmesser (2008a). "Financing Higher Education and Labor Mobility", CESifo Working Paper No. 2362, Munich.

[6] Demange, G., R. Fenge and S. Uebelmesser (2008b). "The Provision of Higher Education in a Global World - Analysis and Policy Implications", CESifo Economic Studies 54, 248-276.

[7] Ehrenberg , R.G. and R.S. Smith (1993). Modern Labor Economics: Theory and Public Policy, Harper Collins: New York.

[8] Eurostat (2009). Database for education and training 2009.

[9] Finn, M. (2003). "Stay Rates of Foreign Doctorate Recipients from U.S. Universities", Oak Ridge Institute for Science and Education, Oak Ridge.

[10] Gabszewicz J, and J. -F. Thisse, (1979) "Price competition, quality and income disparities" Journal of Economic Theory, Volume 20, Issue 3, 340-359.

[11] Gale, D. (1973). "Pure Exchange Equilibrium of Dynamic Economic Models", Journal of Economic Theory 6, 12-36.

[12] Haupt, A., T. Krieger and T. Lange (2010). "Competition for the International Pool of Talents: Education Policy with Student Mobility", CCES Discussion Paper No. 31, Hitotsubashi University.

[13] Hein, M. and J. Plesch (2008). "How Can Scholarship Institutions Foster the Return of Foreign Students?", DFG Research Group 'Heterogenous Labor' Discussion Paper 08/02.

[14] Hunt, J. (2006). "Staunching Emigration from East Germany: Age and the Determinants of Migration", Journal of the European Economic Association 4, 1014-1037.

[15] Jacobs, B. and R. van der Ploeg (2006). "Guide to reform of higher education: a European perspective", Economic Policy 47, 535-592.

[16] Justman, M. and J.-F. Thisse (2000). "Local Public Funding of Higher Education when Skilled Labor is Imperfectly Mobile," International Tax and Public Finance 7, 247-258.

[17] Kemnitz, A. (2007). "University Funding Reform Competition, and Teaching Quality", Journal of Institutional and Theoretical Economics 163, 356-378.

[18] Lange, T. (2009). "Public Funding of Higher Education when Students and Skilled Workers are Mobile", FinanzArchiv / Public Finance Analysis 65, 178-199.

[19] Lowell, L., Bump, M. and S. Martin (2007). "Foreign Students Coming to America: The Impact of Policy, Procedures, and Economic Competition", Project Summary Report, ISIM Georgetown University.

[20] Lu, Y., L. Zong and b. Schissel (2009), "To Stay or Return: Migration Intentions of Students from People's Republic of China in Saskatchewan, Canada", International Migration and Integration 10, 283-310.

[21] Mauro, P. and A. Spilimbergo (1999). "How do the Skilled and the Unskilled Respond to Regional Shocks? The Case of Spain", IMF Staff Papers 46, 1-17.

[22] Mechtenberg, L. and R. Strausz (2008). "The Bologna Process: How student mobility affects multi-cultural skills and educational quality", International Tax and Public Finance 15, 109-130.

[23] OECD (2009): "Education at a Glance 2009", OECD, Paris

[24] Rosenzweig, M. (2006). "Higher Education and International Migration in Asia: Brain Circulation", Paper prepared for the Regional Bank Conference on Development Economics in Beijing: Higher Education and Development, January 2007.

[25] Shaked, A. and J. Sutton (1982). "Relaxing Price Competition Through Product Differentiation", The Review of Economic Studies 49, 3-13.

[26] Tremblay, K. (2002), "Student Mobility between and towards OECD countries: A Comparative Analysis", in: OECD, International Mobility of the Highly Skilled, Paris, 39-67

[27] Tremblay, K. (2005): "Academic Mobility and Immigration", Journal of Studies in International Education 9, 196-228.