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Deficit, Seigniorage and the Growth Laffer Curve
in developing countries

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Abstract: The endogenous growth literature has established the existence of an inverted-U curve between taxes and economic growth, namely a Growth Laffer Curve (GLC). We develop a growth model with public investment as the engine of perpetual growth, and look for the effect of deficit, tax and money financing on economic growth. We study in particular the way fiscal and monetary policies (through deficit and seigniorage respectively) deform the GLC. An empirical section based on a panel of developing countries provides GMM-system estimators that support our theoretical conclusions.

Keywords: Growth Laffer Curve, deficit, seigniorage, developing countries, GMM, panel data

JEL codes: E52, E63, H62
I. Introduction

The relationship between taxes and economic growth is one of the oldest and most studied topics in economics. However, as recently acknowledged by Slemrod (2003), despite an impressive amount of contributions the existing literature has failed to establish a clear-cut link between taxes and economic growth.

The absence of a straightforward effect is connected to the existence of the so-called Laffer Curve, which describes a non-linear (inverted-U curve) relationship between taxes and fiscal revenues. However, the theoretical literature provides conflicting conclusions concerning the existence of such a curve. For example, Fullerton (1982) shows that the Laffer Curve may not be continuous or lack a maximum, while Novales & Ruiz (2002) argue that it may have several maxima. The empirical evidence for developed countries is also far from conclusive. Hsing (1996) finds a hump-shaped relation between taxes and fiscal revenues, but the robustness of his approach is criticized by Dalamagas (1998). In a recent paper, Trabandt & Uhlig (2006) emphasize that the link between taxes and revenues depends on the type of taxes (on capital, labor or consumption) or the use of the marginal or average tax rate.\footnote{The difficulties related to the mere existence of a Laffer Curve are amplified when estimating its turning point. For developed countries, the optimal tax rate is apparently located somewhere between 35\% (Hsing, 1996) and 60\% (Trabandt & Uhlig, 2006). Despite this heterogeneity, one result that emerges from the literature (see, among others, Floden & Linden, 2001, or Jonsson & Klein, 2003) is that taxes in the US are below their optimal level, while above it in France, Germany or Northern Europe. This raises of course the question of why Governments would over/under estimate their taxes. Among several explanations, Buchanan & Lee (1982) consider the presence of two Laffer Curves, in the short-run and long-run respectively. If Governments focus on the short-run, they may fix a tax rate that is different from the one that is optimal in the long-run.}

The relative absence of sound proof in favor of a Laffer Curve may be due to the fact that, when modifying taxes, Governments privilege economic growth, instead of fiscal revenues (Mitchell, 2002\footnote{“Putting revenue maximization ahead of sound tax policy is therefore a misguided approach and should be discarded” (page 8).}). In this paper we embrace this approach and focus on the presence of a “Growth Laffer Curve” (GLC) between economic growth and taxes. In an endogenous growth model with public investment, Barro (1990) shows that the presence of a GLC reflects the arbitrage between two conflicting effects. On the one hand, the increasing side of this GLC is a consequence of the fact that higher taxes provide more resources for public investment, which is growth-enhancing. On the other hand, higher taxes also generate more distortion on private capital accumulation and consequently on economic growth. Once the tax rate is above a threshold value, the economy reaches the slippery side of the GLC, and taxes and economic growth are negatively correlated.
If the theoretical existence of a GLC is a rather well established result, the empirical evidence on developed countries is still weak (see, for example, Bleaney, Gemmel & Kneller, 2001, or Alesina, Ardagna, Perotti & Schiantarelli, 2002), and virtually non existent for developing countries. Concerning the latter, in one of the very few contributions on the topic, Easterly & Rebelo (1993) assert that “the evidence that tax rates matter for economic growth is disturbingly fragile” (page 442).

Several explanations are put forward in order to elucidate the absence of a GLC in developing countries. For example, Becsi (2000) considers that the profile of the GLC is sensitive to the use of collected revenues (public consumption or public investment). Since public spending efficiency is difficult to assess, particularly in developing countries, measurement errors may induce a bias in the estimation of the GLC. Alternatively, Heijman & van Ophen (2005) find that the “black market” distorts the GLC. The presence of a substantial tax evasion in developing countries may therefore be an impediment to finding a GLC.

In order to establish the presence of a GLC in developing countries, we propose in this paper a different strategy, based on the interaction between the GLC and the two other methods of Government finance, namely fiscal deficits and seigniorage. Indeed, developing countries typically rely on these two financing methods; it is therefore appealing to explore the relationship between taxes and growth when accounting for public deficits and seigniorage.³

Consequently, we develop an endogenous growth model in which public investment may be financed by taxes, seigniorage or public debt. We emphasize a GLC between taxes and growth and study the way deficits and seigniorage deform the GLC. First, we find that cutting deficits or raising seigniorage always lowers the GLC-maximizing tax-rate. Second, a higher deficit is always growth-reducing and thus moves the GLC downwards, while the effect of seigniorage on the GLC may be subject to nonlinearities.

Subsequently, we test these conclusions of the theoretical model by using the Generalized Method of Moments (GMM) technique. We find a GLC between taxes and growth for a panel of developing countries, when accounting for its interaction with fiscal deficits or seigniorage. Cutting deficits or raising seigniorage reduces the GLC-maximizing tax-rate, while an increase in deficits or seigniorage decreases economic growth, as in our theoretical model. In addition, to check for the robustness of the influence of seigniorage on the GLC, we distinguish between developing countries with restricted (fixed or quasi fixed) and unrestricted exchange rate regimes. The results are unchanged for countries with unrestricted monetary

³ In addition, as these resources are meant to finance public investment, our analysis comes close to a recent developed concept, namely the “fiscal space”, which depicts the way a given amount (or an increase) of public spending may be optimally financed through different means of financing (see Roy & Heuty, 2009).
policies. However, we find no evidence of a GLC indexed by seigniorage for countries with restricted monetary policy, suggesting that seigniorage does not significantly distort the relationship between taxes and growth in these countries, most likely because it is not an important way of Government finance when the exchange rate regime is restricted.

The rest of the paper is organized as follows. Section two develops the model and exhibits the way deficits and seigniorage impact the GLC, while in section three we test our theoretical results on a panel of developing countries using the GMM technique, and section four concludes.

II. The model

In this section we introduce the theoretical mechanism that we want to test in section three. Consider a closed economy with an infinitely lived representative agent and two authorities: a Government and a Central Bank.

The representative agent has two sources of revenue, from production and from interest-bearing public debt. At each period $t$, output $Y_t$ is generated using private capital $K_t$ and productive public expenditures $G_t$, with $0 < \varepsilon < 1$ the elasticity of output with respect to private capital:

$$Y_t = F(K_t, G_t) = K_t^\varepsilon G_t^{1-\varepsilon}$$

(1)

Population is normalized to unity and all variables may be interpreted as per capita. Public spending stands for public investment (it has a flow dimension, but considering public investment that has a stock dimension, as in Futagami, Morita & Shibata, 1992, would leave our results unchanged) and we abstract from congestion effects, so the production function is close to Barro (1990).4

The representative agent may hold Government bonds ($B_t$), which return the real interest rate ($r_t$). Interest revenues and output may be used for (private) investment ($\dot{K}_t + \delta K_t$, with $\delta$ the private capital depreciation, and $\dot{K}_t \equiv dK_t / dt$), for buying new bonds ($\dot{B}_t$), for (private) consumption ($C_t$) and for paying flat rate taxes on output ($\tau Y_t$).6 Households also

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4 Under the condition $0 < \varepsilon < 1$, the production function has decreasing returns to scale which enables the existence of a competitive equilibrium (remark that $G_t$ is exogenous for households). However, in equilibrium $G_t$ will be endogenously determined and production will exhibit constant returns (a necessary condition for the presence of an endogenous growth long-run path).

5 We could easily extend the model to the presence of private bonds. However, in our representative agent model, private bonds are not held in equilibrium, thus Government is the only debtor.

6 Taxing interests on public debt would not change the model, except for the shadow variables in the maximization program.
hold real money $M_r$ (with $M_r \equiv \tilde{M}_r / P_r$ the real money stock, $\tilde{M}_r$ the nominal money stock and $P_r$ the price level) and $\pi_r M_r$ stands for the real money stock depreciation per unit of time (with $\pi_r \equiv \tilde{P}_r / P_r$ the inflation rate and $R_r = \pi_r + r_r$ the nominal interest rate). Consequently, the representative agent budget constraint is:

$$r_r B_r + (1 - \pi_r) Y_r = C_r + (\dot{K}_r + \delta K_r) + \dot{B}_r + (\dot{M}_r + \pi_r M_r)$$

(2)

Under the budget constraint, the representative household maximizes the present value of discounted intertemporal utility of the flow of consumption, with $\beta > 0$ the discount rate:

$$W = \int_0^\infty U(C_t) \exp(-\beta t) dt$$

(3)

To obtain an endogenous growth path, we consider an isoelastic instantaneous utility function, with constant elasticity of substitution $1/\sigma = -U_{cc} C_t / U_c$ ($U_c \equiv dU / dC$):

$$U(C_t) = \begin{cases} 
\frac{C_t^{1-\sigma} - 1}{1-\sigma}, & \text{for } \sigma \neq 1 \\
\log(C_t), & \text{for } \sigma = 1 
\end{cases}$$

(4)

To conceive the existence of an optimum for welfare maximization, the intertemporal utility $U$ should be bounded, which leads to $(1 - \sigma) \gamma_c < \beta$, with $\gamma_c$ the long-run growth rate of variable $x$.

To motivate a money demand, we suppose that the household is money constrained on his consumption and investment, via a cash-in-advance (CIA) constraint. To simplify calculations, we suppose, with no qualitative impact on results, that public spending is also subject to the CIA constraint:

$$\phi [C_t + (\dot{K}_r + \delta K_r) + G_t] = M_r$$

(5)

In standard CIA models, coefficient $\phi$ is a constant parameter. In this paper, we suppose that $\phi$ may negatively depend on nominal interest rate: if the nominal interest rate increases, households attempt to save real balances and adopt more efficient means of payment (such as credit cards, for example). In other words, money velocity $(1/\phi)$ will increase in periods being characterized by high interest rates (a higher interest rate increases the

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7 The latter condition corresponds to a no-Ponzi game constraint, $\gamma_c < r$.

8 Assuming a CIA constraint on consumption and private investment only (a case first studied by Stockman, 1981, in an exogenous growth setup) does not qualitatively change the model. The presence of public investment in the
opportunity cost of money; consequently, the real-money demand is lower, and money velocity is higher, see, e.g., Rodriguez-Mendizabal, 2006).

In what follows, we assert that: \( \phi = \phi(R_t) \), with \( \phi(R_t) \equiv d\phi(R_t)/dR_t < 0 \). This procedure allows generalizing the CIA technology, which can be compared to more general “transactions cost” technologies (see Minea & Villieu, 2009a, for the micro-foundations of the underlying “transaction cost” model). In equilibrium \( Y_t = C_t + (K_t + \delta K_t) + G_t \), such a specification provides a quite general and usual demand for real balances, depending both on real income and, if \( \phi(R_t) < 0 \), on nominal interest rate: \( M_t = \phi(R_t)Y_t \).\(^9\) We find the CIA technology strictly speaking when \( \phi(R_t) = 0 \), so that \( \phi \) becomes a constant parameter.

*The Central Bank and the Government*

We focus next on the monetary block of the model. The banking system generates the nominal stock of money \( \tilde{M}_t \) (since (5) yields a transaction money demand, one might consider that \( \tilde{M}_t \) includes all means of payment: cash balances and deposit accounts). The Central Bank fixes the nominal stock of high-powered money \( \tilde{H}_t \), which is linked to the nominal money stock by a standard “money multiplier” \( (1/\eta > 1) \) such that: \( \tilde{M}_t = (1/\eta)\tilde{H}_t \). The multiplier may depend on the ratio of banknotes to the stock of money and on bank reserve requirements, which we do not model explicitly.\(^10\) Money market equilibrium will define the price level \( P_t = \tilde{M}_t/M_t = \tilde{H}_t/\eta M_t \). Thus, in our model, “high-powered money”, which provides seigniorage for government finance, must be distinguished from money used in transactions. The higher the multiplier, the more important the share of money generated by private banks, and the greater the discrepancy between the stock of money used for transactions (the one in the CIA constraint) and the stock of high-powered money. Consequently, only part of the seigniorage on the total money stock is retrieved by the Central Bank.

We suppose that the monetary policy of the Central Bank involves an exogenous growth rate on the nominal money stock \( \tilde{M}_t/\tilde{M}_t = \omega \) and consequently on the base money

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CIA yields a simple money demand, \( M_t = \phi Y_t \) in equilibrium, which simplifies calculations. We assume a strictly positive interest rate, so that the CIA constraint holds as equality.

\(^9\) Such a functional form in which money demand depends on the interest rate appears in the CIA model with credit goods and cash goods of Lucas & Stokey (1987), for example.

\(^10\) Some micro-foundations of the money multiplier \( \eta \) can be found in Englund & Svensson (1988) or Hartley (1988), in cash-in-advance models including two types of goods: “cash goods”, to be paid with cash reserves and “check goods” which need bank deposits.
(\(\omega = \hat{H}_i / \tilde{H}_i\)), since the multiplier is constant. The Central Bank collects seigniorage on the (real) monetary base, namely \(\omega \hat{H}_i / P_i = \omega \eta M_i\), and transfers it at no cost to Government.

Whenever Government resources (from taxes and seigniorage) are insufficient to finance public investment, it may run into deficits, in which case the Government must pay interest on the issued debt, so that its budget constraint is (in real terms):

\[
G_i - (\pi Y_i + \eta \omega M_i) + r_i B_i = \tilde{B}_i
\]

(6)

With respect to Barro (1990) balanced budget constraint, we allow here for an unbalanced budget and for money financing. Indeed, in an endogenous growth setup, one may study the long-run effects of permanent deficits. On the one hand, since all variables grow at the same rate in the steady-state, the public debt growth rate should be equal to this balanced economic growth rate, that we denote by \(\gamma^*\). Consequently, we can introduce permanent (long-run) deficits. On the other hand, the intertemporal government budget constraint does not restrain public debt to be constant in the long run, but only its growth rate to be lower than the real interest rate (or, the no-Ponzi game condition). Therefore, deficits cannot be “too high” in the long-run.

One simple manner to fulfill these conditions is to assume that the Government targets a long-run debt to GDP ratio, namely \((B/Y)^* = \theta\) (a star stands for steady-state values).\(^{11}\) This assumption is compatible with the existence of permanent deficits in the long-run, and the deficit ratio associated with the target \(\theta\) is simply: \(d \equiv (\dot{B}/Y)^* = \gamma^* \theta\). Remark also that the solvability condition \((\dot{B}/B)^* = \gamma^* < r^*\) holds if \(d < r^* \theta\) in the long-run.

Steady-state equilibrium

The household maximizes (3) subject to (1)-(2)-(4)-(5), given \(k_0\) and a standard transversality condition (see Appendix 1 for details). As usual in growth models, we define intensive variables \(c \equiv C/K\), \(m \equiv M/K\), \(g \equiv G/K\), \(b \equiv B/K\), and derive the long-run growth solution by setting \(\dot{c} = \dot{m} = \dot{g} = \dot{b} = 0\). By so doing, we find constant \(c\), \(m\), \(g\) and \(b\) values, so that initial variables \((C, M, G, B, K)\) grow at the same constant rate \(\gamma\). Appendix 1 shows that the steady-state may be depicted by two relations between \(\gamma\) and \(g\):

\(^{11}\) In what follows, \(\theta\) informs us about the way public debt distorts the relation between taxes and long-run growth. Thus, \(\theta\) may be equally interpreted as describing the fiscal stance of Government.
\[ \gamma^* = \left[ \frac{\varepsilon (1-\tau) g^{1-\varepsilon}}{1 + R^* \phi(R^*)} - \delta - \beta \right] / \sigma \]  

(7)

\[ g^* = \left[ \varepsilon + \eta \omega \phi(R^*) - \theta S(\gamma^*) \right]^{1/\varepsilon} \]  

(8)

We define \( S(\gamma^*) \equiv \beta - (1-\sigma)\gamma^* \) and the nominal interest rate is \( R^* = \omega + S(\gamma^*) \) in the long-run.

The first relation is simply the Keynes-Ramsey relation \( \dot{C}/C = \gamma = (r - \beta)/\sigma \) in the long-run. If we abstract from the transaction cost constraint by setting \( \phi = 0 \), the real interest rate would match the net return of private investment: \( r = (1-\tau)F_k - \delta = \varepsilon (1-\tau)g^{1-\varepsilon} - \delta \).

However, with transaction costs \( \phi > 0 \), the return on capital is deflated by the transaction cost on new capital goods \( (1+R\phi(R)) \), with the nominal interest rate: \( R = \omega + r - \dot{M}/M = \omega + r - \gamma = \omega + S(\gamma) \). Equation (8) reproduces the government constraint (6): \( (\gamma - r)b = g - g^{1-\varepsilon} - \eta \omega m \), with \( b = \theta g^{1-a} \) in steady-state, and the real money demand coming from the CIA constraint (5): \( m = \phi(R)g^{1-\varepsilon} \), with \( c = g^{1-\varepsilon} - g - \gamma - \delta \) in equilibrium.

Using (7)-(8), we find an implicit equation for the long-run economic growth:

\[ \gamma^* = \left[ \frac{\varepsilon (1-\tau) [\varepsilon + \eta \omega \phi(R^*) - \theta S(\gamma^*)]^{(1-\varepsilon)/\varepsilon}}{1 + R^* \phi(R^*)} - \delta - \beta \right] / \sigma \]  

(9)

with \( R^* = \omega + \beta - (1-\sigma)\gamma^* \) and \( S(\gamma^*) = \beta - (1-\sigma)\gamma^* \).

**Taxes and economic growth: the augmented Growth Laffer Curve**

As developed in the *Introduction*, the Laffer Curve describes an inverted-U relation between taxes and fiscal revenues. However, since fiscal revenues are constantly growing in steady-state in endogenous growth models, the Laffer Curve is replaced by a Growth Laffer Curve (GLC) between taxes and long-run economic growth.

Similarly to Barro (1990), our model exhibits a hump-shaped curve that links economic growth and taxes. We can see in (9) that a higher tax rate lowers growth through the term \( (1-\tau) \): this negative influence captures the distortion of a higher flat-rate tax on private capital accumulation and consequently on long-run growth. Simultaneously, higher taxes allow for more resources that Government may use to finance public investment, which stimulates private capital accumulation and economic growth. This favorable effect of taxes is described

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12 We propose below an explicit value of \( \gamma^* \) for the case \( \sigma = 1 \) (a logarithmic instantaneous utility function).
by the second term of the numerator in (9). Using the implicit-function theorem, we may find the tax rate that maximizes long-run growth (namely, $\hat{\tau}$):

$$\hat{\tau} = (1 - \varepsilon) - \varepsilon [\eta \alpha \phi (R') - S(\gamma') \theta]$$

with $R' = \omega + S(\gamma')$ (10)

Equations (9)-(10) show that both the public debt $\theta$ and the money growth rate $\omega$ influence not only the GLC between taxes and economic growth, but also the GLC-maximizing tax rate $\hat{\tau}$. We explore below the direction of these effects.

With respect to public debt, we notice that, ceteris paribus, any increase in the long-run debt-to-income ratio ($\theta$) weakens long-run economic growth: $d\gamma' / d\theta < 0$, for the solvability condition $S(\gamma') = \beta - (1 - \sigma)\gamma' < 0$ to hold. Since Governments are forbidden to play Ponzi games, new resources from deficits in the steady state ($\dot{B} = \gamma B = \gamma' B$) are always falling short with respect to the debt cost (the debt burden $rB$). Consequently, the permanent net flow of resources provided by public debt financing in steady-state is negative. Thus, ceteris paribus, productive spending must fall or distorsive taxes must go up, explaining why long-run growth is reduced. Our result still holds when considering an increase in the deficit ratio instead of the public debt ratio (see Figure 1 below and Minea & Villieu, 2009b).

This finding exerts two effects on the GLC. First, since the negative effect of public debt holds for any value of the tax rate, a raise in $\theta$ moves the GLC downwards (for any value of the tax rate $\tau$). Second, the GLC-maximizing tax rate is increasing in $\theta$. The intuitive explanation of the second result is that, compared to a no-debt no-money case ($\theta = \phi = 0$, $\hat{\tau}^B = 1 - \varepsilon$ as in Barro, 1990), an upward jump in $\theta$ lowers long-run growth, since the public-debt burden crowds out productive expenditures. To restore (part of) productive expenditures, government must increase the tax rate beyond the Barro value $\hat{\tau}^B$.

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13 Rewriting (9) as an implicit function $\Omega(\gamma, \omega, \tau, \theta) = 0$, the total differential is:

$$\frac{d\Omega}{d\gamma} d\gamma + \frac{d\Omega}{d\tau} d\tau + \frac{d\Omega}{d\omega} d\omega + \frac{d\Omega}{d\theta} d\theta = 0$$

and the tax-ceiling is obtained from $\frac{d\gamma}{d\tau} = -\frac{d\Omega}{d\tau} / \frac{d\gamma}{d\tau} = 0$.

14 In percentage of income, the term $(r - \gamma)B$ corresponds precisely to $S(\gamma') \theta = [\beta - (1 - \sigma)\gamma'] \theta$ in (9).

15 The effect of an increase in the public debt (deficit) ratio on economic growth may be different if we allow for the public debt not to be paid back (for example, public debt in developing countries is often external and is subjected to periodical alleviations). However, the analysis of debt alleviation policies at international level, which involves several specific problems (aid or debt alleviation is accorded conditioning to macroeconomic objectives, the donators may have other goals than public investment,…) is beyond the scope of this paper. Differentiating between domestic (internal) public debt (as in our model) and external public debt leaves our results unchanged, but complicates the model (we should allow for an open economy setup). On the contrary, if we consider the possibility that Government may not repay public debt, a “free lunch” problem may appear, namely the Government may increase public investment and economic growth at no cost. We may introduce an exogenous parameter to account for the share of the public debt which is not repaid, but we consider instead that $\theta$ is the share of public debt that must be paid back.
Contrary to the monotonous influence of public debt on long-run growth, seigniorage describes an inverted-U curve with respect to economic growth.\(^{16}\) This curve reflects the trade-off between the positive influence of an increase in the money growth rate \(\omega\) (according to the numerator of (9), a higher money growth rate yields extra resources for growth-enhancing public investment) and its negative impact through higher transaction costs (denominator of relation (9)). The economic growth-maximizing money growth rate is:\(^{17}\)

\[
\hat{\omega} = \frac{\alpha(R)(\tau - R\theta) - 1}{\phi + (\eta\phi + \theta)\alpha(R)}
\]

where: \(\alpha(R) = \frac{e(\phi + \phi R)}{(1 - e)\eta\phi(1 + \phi R)}\), with \(\phi_{\omega} \equiv \frac{d\phi(R)}{dR}\) (11)

Consequently, the effect of \(\omega\) on the GLC depends on the relative position of \(\omega\) with respect to \(\hat{\omega}\). Raising \(\omega\) increases economic growth if \(\omega < \hat{\omega}\), but decreases it if \(\omega > \hat{\omega}\). However, raising the money growth rate always decreases the growth-maximizing tax rate \(d\hat{\tau}/d\omega < 0\): since seigniorage resources are more important, the Government may cut the optimal tax \(\hat{\tau}\) (or to put it differently, seigniorage and taxes are substitute instruments for Government financing).

**Analytical results and some simulations**

To find explicit results, suppose a logarithmic instantaneous utility function \((\sigma = 1)\) and a strict CIA technology \((\phi(R) = \phi)\). The long-run economic growth rate \(\gamma'\) and growth-maximizing values \(\hat{\tau}\) and \(\hat{\omega}\) simplify to:

\[
\gamma' = \frac{e(1 - \tau)(\tau + \eta\omega\phi(\omega + \beta) - \beta\theta)^{(1 - \epsilon)/\epsilon}}{1 + (\omega + \beta)\phi(\omega + \beta)} - \delta - \beta
\]

(9')

\[
\hat{\tau} = (1 - \epsilon) - e(\eta\omega\phi - \rho\theta)
\]

(10')

\[
\hat{\omega} = \frac{\eta(1 - \epsilon)(1 + \phi\beta) - e(\tau - \beta\theta)}{\eta\phi(2\epsilon - 1)}
\]

(11')

One can easily verify in (9')-(10') our previous findings in the general case. An increase in the debt ratio decreases economic growth in (9'), moves downwards the GLC, and increases the optimal tax rate \(\hat{\tau}\) in (10'). The effect on the GLC of an increase in the money-growth rate \(\omega\) depends on the \(\hat{\omega}\) value (see (9')), but raising \(\omega\) always decreases \(\hat{\tau}\) in (10').

\(^{16}\) Our model is compatible with extensive empirical evidence supporting the presence of non linearities between money (inflation) and economic growth (see, among others, the recent contributions of Arai, Kinnwall & Skogman, 2004, and Burdekin, Denzau, Keil, Sitthiyot & Willett, 2004).
Results in the general case are also confirmed by our simulations below, performed for the functional from \( \phi(R_t) = 2\phi R_t^{-\alpha} \) and usual values for parameters.\(^\text{18}\)

a) the effect of public debt and deficit on the GLC

In Figure 1, we represent the change in the growth Laffer curve following a rise in the long-run debt target \( (\theta) \) or in the long-run deficit ratio \( (d) \). According to the left chart of Figure 1, raising the public debt to GDP ratio exerts two effects on the relation between taxes (horizontal axis) and long-run economic growth (vertical axis). First, the GLC moves downwards. Second, the GLC-maximizing tax rate increases in response to an increase in \( \theta \), as we have seen.

Considering deficit, instead of public debt, has no qualitative effect on these results, as shown in the right chart of Figure 1. Since the deficit to GDP ratio is \( d \equiv (\bar{B}/\bar{Y}) = \gamma \theta \), we replace \( \theta \) with \( d/\gamma \) in (9) and obtain similar partial derivatives. Thus, the GLC still moves downwards and its maximum still moves towards the right, in response to a rise in the deficit ratio. In the empirical section below, we take advantage of this correspondence between a change in public debt and a change in the deficit ratio.

Figure 1 – The simulated effect of public debt and deficit on the GLC

\[ \sigma = 2, \; \beta = 0.05, \; \delta = 0.1, \varepsilon = 0.8, \; \omega = 0.05, \; \eta = 0.2, \; \phi = 0.5 \]

\(^{17}\)From the total differential of \( \Omega(\gamma, \omega, \tau, \theta) = 0 \), namely \( \frac{\partial \Omega}{\partial \gamma} d\gamma + \frac{\partial \Omega}{\partial \tau} d\tau + \frac{\partial \Omega}{\partial \omega} d\omega + \frac{\partial \Omega}{\partial \theta} d\theta = 0 \), we find the growth maximizing money-growth rate from \( \frac{d\gamma}{d\omega} = -\frac{\frac{\partial \Omega}{\partial \omega}}{\frac{\partial \Omega}{\partial \gamma}} = 0 \).

\(^{18}\)The elasticity of output with respect to public capital is 20% \((1 - \varepsilon = 0.20)\), private capital, the discount rate and the money growth rate are between 5% and 10%, financial development is set to 0.5 (on a scale from 0 to 1), and the consumption elasticity of substitution \((1/\sigma)\) is between 0.5 and 1.
b) the effect of seigniorage on the GLC

Figure 2 below depicts the impact of monetary policy on the GLC. According to (9), the effect of a change in \( \omega \) depends on its relative value with respect to \( \hat{\omega} \). Figure 2 illustrates the only two possible cases: raising \( \omega \) increases economic growth when \( \omega < \hat{\omega} \) (the left hand side picture), but decreases economic growth if \( \omega > \hat{\omega} \) (the right hand side picture). However, the effect of \( \omega \) on the GLC-maximizing tax rate \( \hat{\tau} \) does not depend on the \( \hat{\omega} \) value: in both pictures, raising \( \omega \) always decreases \( \hat{\tau} \). Of course, the threshold \( \hat{\omega} \) depends on all parameters of the model in our simulations, and will depend on structural characteristics of countries in the econometric estimations.

Figure 2 – The simulated effect of the seigniorage on the GLC

\[
\sigma = 1.5, \quad \beta = \delta = 0.1, \quad \varepsilon = 0.8, \quad \theta = 0.75, \quad \eta = 0.9, \quad \phi = 0.25
\]

\[
\sigma = 2, \quad \beta = 0.05, \quad \delta = 0.1, \quad \varepsilon = 0.8, \quad \theta = 0.5, \quad \eta = 0.1, \quad \phi = 0.5
\]

In the following section we attempt to confirm empirically our results concerning the shifts of the GLC in response to jumps in the deficit ratio and seigniorage, using a panel of developing countries.
III. The empirical evidence of the interaction between deficit, seigniorage, and the GLC

The econometric model and data presentation

The goal of this section is to test empirically the conclusions of our theoretical model. We focus on a panel of 48 developing countries (Appendix 2 presents the list of countries included in our sample) over the period 1980-2006. In order to reduce the potential effects of unaccounted short-term fluctuations, all variables are computed as five-year averages, for the following five sub-periods 1980-1985, 1986-1990, 1991-1995, 1996-2000 and 2001-2006.

Considering the lack of availability of data on public debt, in this section we use data on deficit (instead of public debt). Since both variables provide the same qualitative conclusions in the theoretical model, as illustrated by Figure 1, using deficits does not raise any specific issue. Adopting the notations from the theoretical model, we aim to estimate the following two equations (with \(i\) and \(t\) country and time period indicators respectively):

\[
\gamma_i = \alpha_\gamma + \alpha_\gamma \tau_i + \alpha_\gamma \tau_i^2 + \alpha_\gamma \tau_i d_i + \alpha_\gamma \tau_i^2 d_i + \alpha_\gamma X_i + \mu_i + \lambda_i + u_i \tag{12}
\]

\[
\gamma_i = \beta_\gamma + \beta_\gamma \tau_i + \beta_\gamma \tau_i^2 + \beta_\gamma \tau_i \omega_i + \beta_\gamma \tau_i^2 \omega_i + \beta_\gamma X_i + \mu_i + \lambda_i + u_i \tag{13}
\]

The dependent variable is the rate of growth of per capita GDP \(\gamma_i\), the vector \(X_i\) captures different control variables (to be discussed below), \(\mu_i\) stands for country fixed effects, \(\lambda_i\) is a time-specific effect that controls for unaccounted common time-varying factors and \(u_i\) is a standard error term. According to the theoretical model, the terms in tax \(\tau_i\) and square tax \(\tau_i^2\) capture the existence of a GLC, while their alternative interaction with fiscal deficits \(d_i\) in equation (12), respectively seigniorage \(\omega_i\) in equation (13), is intended to search for the way fiscal or monetary policies deform the GLC. To put it differently, we test for the existence of a set of GLC indexed by either deficit (in (12)) or seigniorage (in (13)).

We split variables in two categories, namely interest and control variables. Within the first group, we define the tax rate as the Government revenue, excluding grants, in % of GDP (source GFS and IMF Article IV). We compute the deficit as the difference between spending (source World Bank, World Development Indicators (WDI), and African WDI for countries with too many missing values) and revenues (including grants), both in % of GDP. Finally, we measure seigniorage as the change in reserve money (line 14a of IFS, IMF) as % of nominal GDP (line 99b in IFS, IMF).

---

19 We consider all low-income and lower-middle-income economies in the World Bank classification for which data are available (in particular, there are many missing values mainly in the deficit and education variables).
We establish the control variables (the vector $X_\alpha$) by drawing on the recent growth literature (Easterly & Rebelo, 1993, Temple, 1999, or Adam & Bevan, 2005) and focusing on traditional explanatory variables for economic growth. First, we include the log of initial per capita GDP in order to search for a catching-up effect. Second, one of the most widely used determinants of growth is the investment-to-GDP ratio (it includes both private and public investment), which is measured as the gross capital formation in % of GDP. The third control variable is the human capital (approximated by the percentage of secondary school attained in total population). Finally, the two remaining controls are the degree of trade openness (exports plus imports in % of GDP) and the population growth rate. With the exception of human capital (from the Barro-Lee dataset), all control variables are extracted from WDI (see Appendix 3 for several descriptive statistics).

**Empirical evidence of the existence of a GLC in developing countries**

It is widely asserted that growth regressions are fraught with many concerns (see, among others, Islam, 1995, Caselli, Esquivel & Lefort, 1996, or Temple, 1999). As discussed by Caselli et al. (1996), there exist mainly two sources of inconsistency in the empirical work on economic growth, both in cross-section and panel analysis. First, the incorrect treatment of country-specific effects, representing differences in technology or tastes, gives rise to the omitted variables bias. Second, most regressors might be endogenous to economic growth, and the presence of simultaneous or reversed causality can generate a bias in the estimation.

In addition to these problems, our analysis based on panel data produces a third issue that must be dealt with. Indeed, the presence of the initial level of per capita GDP (equivalent to the lagged dependent variable) as explanatory variable may produce biased coefficient estimates, since initial per capita GDP is by construction correlated with the error-term (Nickell, 1981).

One way to handle (i) the unobserved country-specific effects and (ii) the endogeneity of explanatory variables and of the lagged dependent variable, is to use the Generalized Method of Moments (GMM) technique. The first-differenced GMM estimator, proposed by Arellano & Bond (1991), instruments the right-hand-side variables in the first-difference equations using levels of the series in lag (of one period or more). However, subsequent evidence (Arellano & Bover, 1995, and Blundell & Bond, 1998) highlights that when the explanatory variables are persistent over time, the lagged levels of these variables are weak instruments for the equations in differences and suggests an estimator that reduces potential biases and imprecision associated with the difference estimator. This GMM-system estimator
combines in a system the previous regressions in differences instrumented by lagged values, with an additional set of equations in levels, by using lagged first differences as instruments.

The results of the GMM-System estimation for the GLC conditional to deficit (equation (12)) and seigniorage (equation (13)) are reported in Table 1. Remark first that for each of the four regressions [1]-[4], the Sargan test of over-identification and the test of second-order autocorrelation (AR(2)) support the validity of our GMM-system estimations.20

Regressions [1]-[2] test the existence of a GLC indexed by the deficit ratio, as explained in equation (12). Before discussing the impact of variables on interest, note that a certain number of regularities emerge among the control variables. First, a higher (private and public) investment to GDP ratio significantly increases economic growth (as in our theoretical model). Second, the initial level of per capita GDP is significantly inversely correlated with economic growth, which reflects the conditional convergence among the developing countries of our sample. Third, we find some evidence supporting the positive impact of human capital in developing countries. Finally, in regressions [2] and [4] we control for population growth and the degree of openness. However, since coefficients are not significant, their effect on economic growth is uncertain.

Table 1 – The GLC conditional to deficit and seigniorage

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>COEFFICIENT</th>
<th>Growth GMM-System</th>
<th>Growth GMM-System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>[1]</td>
<td>[2]</td>
</tr>
<tr>
<td>Tax</td>
<td>$\alpha_1$</td>
<td>0.496*</td>
<td>0.589**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.287)</td>
<td>(0.291)</td>
</tr>
<tr>
<td>Tax$^2$</td>
<td>$\alpha_2$</td>
<td>-0.0113**</td>
<td>-0.0120**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0054)</td>
<td>(0.0051)</td>
</tr>
<tr>
<td>Tax*Deficit</td>
<td>$\alpha_3$</td>
<td>-0.0445**</td>
<td>-0.0532**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0220)</td>
<td>(0.0238)</td>
</tr>
<tr>
<td>Tax$^2$*Deficit</td>
<td>$\alpha_4$</td>
<td>0.00108***</td>
<td>0.00125***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00043)</td>
<td>(0.00045)</td>
</tr>
<tr>
<td>Tax*Seigniorage</td>
<td>$\beta_1$</td>
<td>-0.302**</td>
<td>-0.280**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.135)</td>
<td>(0.118)</td>
</tr>
<tr>
<td>Tax$^2$*Seigniorage</td>
<td>$\beta_4$</td>
<td>0.00804***</td>
<td>0.00746***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00288)</td>
<td>(0.00259)</td>
</tr>
<tr>
<td>Deficit</td>
<td></td>
<td>0.328</td>
<td>0.426</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.283)</td>
<td>(0.282)</td>
</tr>
<tr>
<td>Seigniorage</td>
<td></td>
<td>-0.202</td>
<td>-0.290**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.156)</td>
<td>(0.127)</td>
</tr>
</tbody>
</table>

20 The Sargan test of over-identification tests the null hypothesis that the instruments are valid, since not correlated with the residual. The AR(2) test assesses the absence of second-order serial correlation ($H_0$).
We now turn our attention to our main goal, namely the interaction between the GLC and fiscal deficits. In both regressions [1]-[2], all terms that include taxes are significant. The coefficients of the squared terms in taxes support the existence of an inverted-U curve, irrespective of the value of deficit in our sample (since $\alpha_2 + \alpha_4 d_n < 0, \forall d_n$). Furthermore, the estimated tax rate that maximizes this curve is positive, since $\alpha_1 + \alpha_3 d_n > 0, \forall d_n$. Consequently, the econometric evidence supports the existence of the GLC curve indexed by the deficit ratio, as described by Figure 3 (for estimated coefficients from regression [1]).

*Figure 3 – GLC reaction following a change in the deficit-to-GDP ratio (regression [1])*
According to Figure 3, a higher deficit generates two effects on the relation between taxes and economic growth. First, it moves the GLC downwards for all tax values in our sample: at a given tax rate, raising deficits lowers economic growth. Second, a higher deficit increases the GLC-maximizing estimated tax rate, which is around 22%. These two empirical results corroborate our theoretical findings and confirm that, in developing countries, there exists a hump-shaped curve between taxes and economic growth, and that this GLC changes in response to a change in the deficit to GDP ratio. Finally, note that the estimated GLC-maximizing tax rate (\( \hat{\tau} \)) is comparable to its value in the theoretical model, namely slightly superior to \( 1 - \varepsilon = 20\% \).

We focus next on the relationship between the GLC and seigniorage. Regressions [3]-[4] support the presence of a significant GLC indexed by the seigniorage value for the developing countries in our sample. Figure 4 below plots the family of GLC curves with respect to several seigniorage values.

*Figure 4 – GLC reaction following a change in the seigniorage-to-GDP ratio (regression [3])*

Estimations show that both the GLC-maximizing tax rate and economic growth decrease following an increase in seigniorage. According to our model, this result may reproduce the fact that, given their structural parameters, the countries in our sample are located above the seigniorage threshold, namely on the slippery side of the seigniorage hump-shaped curve. While the negative link between seigniorage and growth in developing countries is well documented (see Edwards, 1994, for Latin America, or Fouda, 1997, for evidence on Cameroun), our estimations focus on the way seigniorage distorts the relationship between taxes and economic growth in developing countries.
**Robustness analysis**

The relation between taxes, seigniorage and growth is likely to be different according to the extent to which it is possible for the Government to finance public expenditures with seigniorage. One can expect that the existence of a GLC conditioned by the level of seigniorage might be less pertinent in countries where the use of seigniorage, as an alternative financing method, is highly limited. We investigate this issue in Table 2 by dividing our sample into two sub-samples of developing countries, according to their degree of rigidity in their exchange rate arrangements.\(^{21}\) The first category regroups countries whose monetary policy is restrained, because of a fixed exchange rate arrangement, and the second one consists of countries whose monetary policy is subject to fewer constraints, partly because of a more flexible exchange rate regime.

**Table 2 – GLC conditional to seigniorage with respect to the monetary policy regime**

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>Fixed exchange rate regime</th>
<th>Flexible exchange rate regime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[5A]</td>
<td>[5B]</td>
</tr>
<tr>
<td>Tax</td>
<td>0.0375</td>
<td>0.837*</td>
</tr>
<tr>
<td></td>
<td>(0.239)</td>
<td>(0.508)</td>
</tr>
<tr>
<td>(\text{Tax}^2)</td>
<td>-0.00237</td>
<td>-0.0224**</td>
</tr>
<tr>
<td></td>
<td>(0.00574)</td>
<td>(0.0102)</td>
</tr>
<tr>
<td>(\text{Tax}\times\text{Seigniorage})</td>
<td>0.263</td>
<td>-0.399**</td>
</tr>
<tr>
<td></td>
<td>(0.421)</td>
<td>(0.195)</td>
</tr>
<tr>
<td>(\text{Tax}^2\times\text{Seigniorage})</td>
<td>-0.00356</td>
<td>0.0106**</td>
</tr>
<tr>
<td></td>
<td>(0.00874)</td>
<td>(0.0050)</td>
</tr>
<tr>
<td>Seigniorage</td>
<td>-1.346</td>
<td>2.434</td>
</tr>
<tr>
<td></td>
<td>(3.582)</td>
<td>(1.701)</td>
</tr>
<tr>
<td>Seigniorage^2</td>
<td>-1.185**</td>
<td>-0.0487</td>
</tr>
<tr>
<td></td>
<td>(0.428)</td>
<td>(0.0424)</td>
</tr>
<tr>
<td>Deficit</td>
<td>-0.277**</td>
<td>0.0368</td>
</tr>
<tr>
<td></td>
<td>(0.093)</td>
<td>(0.0775)</td>
</tr>
<tr>
<td>Initial GDP per capita (log)</td>
<td>-0.813</td>
<td>-1.058</td>
</tr>
<tr>
<td></td>
<td>(1.589)</td>
<td>(0.922)</td>
</tr>
<tr>
<td>Investment</td>
<td>0.213***</td>
<td>0.202**</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.087)</td>
</tr>
<tr>
<td>Education</td>
<td>0.091</td>
<td>0.076**</td>
</tr>
<tr>
<td></td>
<td>(0.141)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>Population Growth</td>
<td>-0.173</td>
<td>0.399</td>
</tr>
<tr>
<td></td>
<td>(0.631)</td>
<td>(0.347)</td>
</tr>
<tr>
<td>Openness</td>
<td>-0.0266</td>
<td>-0.0408</td>
</tr>
<tr>
<td></td>
<td>(0.0249)</td>
<td>(0.0320)</td>
</tr>
<tr>
<td>Observations</td>
<td>53</td>
<td>110</td>
</tr>
<tr>
<td>Number of countries</td>
<td>14</td>
<td>34</td>
</tr>
<tr>
<td>AR(2) p-value</td>
<td>0.043</td>
<td>0.692</td>
</tr>
<tr>
<td>Sargan (p-value)</td>
<td>1.000</td>
<td>0.965</td>
</tr>
</tbody>
</table>

Robust standard errors in brackets. \(p<0.01, \quad p<0.05, \quad p<0.1\). Constant and time fixed effects included in all estimations. The population growth and the education rate are treated as exogenous whereas the other variables are considered as not strictly exogenous and are instrumented with their first-order to third-order lagged values.

\(^{21}\) The distinction between fixed and flexible exchange rate regimes is made according to the 2004 IMF classification system based on members’ actual arrangements as identified by IMF staff (see Appendix 4). An alternative option would be to use the degree of Central Bank independence to distinguish between countries.
For the 14 countries with a constrained exchange rate regime (regression [5A]), the presence of GLC conditioned by the level of seigniorage is not verified empirically. This may be due either to the small size of the sample (which may affect the robustness of the estimation results, as indicated by the AR(2) and Sargan tests), but also to the fact that in these countries, seigniorage is not an important financing mean of public spending (because they cannot (easily) resort to seigniorage) and will therefore not significantly interact with taxes in terms of economic growth effects.

Second, regression [5B] confirms that seigniorage had a significant influence on the relation between taxes and economic growth in the countries that were less constrained in their monetary policy over the period 1980-2006. Moreover, raising seigniorage lowers the GLC-maximizing tax rate in line with theoretical conclusion, and reduces, as expected, economic growth (see Figure 5).

Figure 5 – The effect of seigniorage on the GLC in countries with flexible exchange rate

IV. Conclusion

The link between taxes and economic growth has been very little explored in developing countries. However, developing countries are particularly concerned with collecting taxes in order to finance public goods that are crucial to reach the Millennium Development Goals. Since public spending is an important growth determinant and given the importance of the economic growth path for developing countries, one should carefully explore the different methods of Government finance for these countries.

Therefore, we propose an endogenous growth model in which public spending may be financed through taxes, deficits and seigniorage. We find first that, when accounting for public investment, there exists a GLC between taxes and economic growth. Second, this GLC depends on both fiscal deficits and seigniorage. On the one hand, a lower deficit-to-GDP-ratio
or a higher money-growth rate reduces the GLC-maximizing tax rate. This may explain why Governments have difficulties in defining a fiscal policy that perfectly matches the optimal tax rate. On the other hand, a higher deficit ratio always moves the GLC downwards, while the effect of seigniorage is subject to nonlinearities.

Empirical evidence using the GMM technique on a sample of developing countries supports our theoretical conclusions, namely the presence of a hump-shaped relation between taxes and economic growth, indexed by deficit and seigniorage. However, the empirical analysis provides three additional results. First, higher seigniorage reduces economic growth for the countries in our sample. According to our theoretical model, this negative relation between seigniorage and growth, also emphasized by Bose, Holman & Neanidis (2007), may be explained by the fact that countries are located above the threshold in seigniorage. If this is the case, these countries may probably increase economic growth by reducing seigniorage.

Second, there is no significant interaction between seigniorage and the GLC in countries with restricted monetary policy (with fixed or quasi-fixed exchange rate regimes, for example). Consequently, one must be careful in assessing the influence of policy mix on economic growth in such countries (for example, in the West African Economic and Monetary Union, WAEMU).

Third, the estimated GLC-maximizing tax-rate is between 15% and 25%, namely below the corresponding value for developed countries (located between 35% and 60%, see footnote 1 in the Introduction). On the one hand, such a low value may suggest that taxes are highly distorsive in developing countries, a result in accordance with microeconomic studies (see, for example, Gauthier & Reinikka, 2006, and Fisman & Svensson, 2007). However, on the other hand, our finding may describe a relatively low productivity of taxes, due to a poor allocation of Government resources. In this case, raising the share of productive public spending (public investment) may enhance both the efficiency of the tax system and economic growth.

Finally, our analysis may contribute to the debate concerning the “fiscal space” for developing countries, which explores the optimal way in which different financing methods may be used to finance Government spending. As acknowledged by the IMF or Heller (2005), a Government may enhance the efficiency within the fiscal space by either raising the productivity of public spending (allowing for a larger share of public investment or more productive public investment) or cutting fiscal deficits. In addition to this evidence, using money as a financing instrument should be carefully supervised, since high money growth rates may generate large inflation rates, which may reduce the real value of available seigniorage resources.
References

Appendix 1 : Resolution of the model

Defining $A_t = B_t + M_t$ and $Z_t = \dot{K}_t$, the current Hamiltonian becomes:

$$H_c = U(C_t) + \lambda_1 [r_t B_t + (1 - \tau)F(\dot{K}_t, G_t) - C_t - \pi_t M_t - Z_t]$$

$$+ \lambda_2 [Z_t - \delta K_t] + q_t (A_t - B_t - M_t) + \mu_t [M_t - \phi(C_t + Z_t + G_t)]$$

where $\lambda_1$ and $\lambda_2$ are the costate variables associated respectively with $A_t$ and $K_t$, and $q$ and $\mu$ are the multiplier of static constraints. First order conditions are (time indexes are henceforth omitted):

\[ I/M : \quad \mu = \pi \lambda_1 + q \quad \text{(A1a)} \]
\[ B : \quad q = r \lambda_1 \quad \text{(A1b)} \]
\[ C : \quad U_c(C) = \lambda_1 (1 + \phi \mu / \lambda_1) \quad \text{(A1c)} \]
\[ Z : \quad \lambda_2 / \lambda_1 = 1 + \phi \mu / \lambda_1 \quad \text{(A1d)} \]
\[ A : \quad \dot{\lambda}_1 = \beta \lambda_1 - q \quad \text{(A1e)} \]
\[ K : \quad \dot{\lambda}_2 / \lambda_2 = \beta + \delta - \lambda_1 (1 - \tau)F_K(K) / \lambda_2 \quad \text{(A1f)} \]

From (A1a) and (A1b) we obtain the nominal interest rate: $R = r + \pi = \mu / \lambda_1$. First order conditions (A1c-A1f) can be rewritten as:

\[ C : \quad U_c(C) = \lambda_1 (1 + \phi R) \quad \text{(A2a)} \]
\[ Z : \quad \lambda_2 / \lambda_1 = 1 + \phi R \quad \text{(A2b)} \]
\[ A : \quad \dot{\lambda}_1 / \lambda_1 = \beta - r \quad \text{(A2c)} \]
\[ K : \quad \dot{\lambda}_2 / \lambda_2 = \beta + \delta - (1 - \tau)F_K(K) / (1 + \phi R) \quad \text{(A2d)} \]

As usual with a CIA constraint on consumption, the nominal interest rate introduces a wedge between the marginal utility of consumption and the shadow price of wealth ($\lambda_1$) in (A2a). If $\phi > 0$, the rate of return of investment must also be deflated by the cost of financing $(1 + \phi R_t)$ in (A2d). In addition, the CIA constraint on investment introduces a wedge between the real return of wealth (the real interest rate $r$ in (A2c)) and the net return of investment $(1 - \tau)F_K(\dot{K}) / (1 + \phi R) - \delta$ in (A2d)), since wealth does not allow acquiring capital goods directly. Note that this wedge disappears if $\phi = 0$, as $\lambda_1 = \lambda_2$ in (A2d). Thus, the Keynes-Ramsey relation is \( \dot{C} / C = -[\dot{\lambda}_1 / \lambda_1 + (\phi R + \phi) R / (1 + \phi R)] / \sigma \).

Goods market equilibrium yields the IS curve $\dot{K} / K = (G / K)^{\tau} - (C / K) - (G / K) - \delta$. Money market equilibrium provides $\dot{M} / M = \omega - \pi$ and the money constraint is, in equilibrium, $M = \phi Y$. The deficit (long run public debt) to GDP ratio is $d = B / Y \quad (B / Y = \theta)$ and $\dot{B} / B = r + (G / K) / (B / K) - \tau (Y / K) / (B / K) - \eta \omega (M / K) / (B / K)$ is the Government budget constraint. Using intensive variables $c = C / K, \quad g = G / K, \quad m = M / K = \phi Y / K$ and $b = B / K$, we find:
\[
\dot{c}/c = \left[ r - \beta - (\phi R + \phi)\dot{R}/(1 + \phi R) \right] / \sigma - \left( g^{1-\varepsilon} - c - g - \delta \right)
\]

(A3a)

\[
\dot{R} = \left[ (1 + \phi R)(r + \delta) - c(1 - \tau)g^{1-\varepsilon} \right]/(\phi R + \phi)
\]

(A3b)

\[
\dot{b}/b = dg^{1-\varepsilon}/b - \left( g^{1-\varepsilon} - c - g - \delta \right)
\]

(A3c)

\[
\dot{m}/m = \omega + r - R - \left( g^{1-\varepsilon} - c - g - \delta \right)
\]

(A3d)

\[
m = \phi g^{1-\varepsilon}
\]

(A3e)

\[
g - (d + \tau)g^{1-\varepsilon} = \eta \omega m - rb
\]

(A3f)

In an Appendix available on request we show that the reduced form of our model is composed of three differential equations in \(c\), \(b\) and \(R\) and that the equilibrium is saddle-point stable. To find the steady-state endogenous growth solution, we impose \(\dot{c} = \dot{m} = \dot{b} = \dot{R} = 0\), thus initial variables \(c\), \(k\), \(b\), \(g\) and \(m\) grow at the same constant rate \(\gamma^*\), while \(R\), \(r\) and \(\pi\) are constant. We find (7) in the main text from (A3a), with the real interest rate from (A3b). Finally, we find (8) in the main text from (A3f) and (A3e) namely: \(g^\varepsilon = \tau + \eta \omega \phi - \theta(\gamma^* - r^*)\), with \(d = \gamma^* \theta\) in steady-state and \(\gamma^* - r^* = \beta - (1 - \sigma)\gamma^* = S(\gamma^*)\).

Appendix 2: List of Countries (48 countries in our sample)
Algeria, Bangladesh, Benin, Bolivia, Burundi, Cameroon, Central African Republic, China, Colombia, Congo Democratic Republic, Congo Republic, Dominican Republic, Egypt, Fiji, Gambia, Ghana, Guatemala, Honduras, India, Indonesia, Iran, Jamaica, Jordan, Kenya, Lesotho, Malawi, Mali, Mauritania, Mozambique, Nepal, Nicaragua, Niger, Pakistan, Papua New Guinea, Paraguay, Peru, Philippines, Rwanda, Senegal, Sierra Leone, Sri Lanka, Sudan, Thailand, Togo, Tunisia, Uganda, Zambia, Zimbabwe.

Appendix 3: Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per capita GDP growth</td>
<td>163</td>
<td>0.769</td>
<td>3.3</td>
<td>-9.932</td>
<td>9.408</td>
</tr>
<tr>
<td>Tax revenue (% GDP)</td>
<td>163</td>
<td>19.27</td>
<td>8.170</td>
<td>4.049</td>
<td>46.259</td>
</tr>
<tr>
<td>Deficit (% GDP)</td>
<td>163</td>
<td>4.895</td>
<td>5.168</td>
<td>-11.745</td>
<td>22.098</td>
</tr>
<tr>
<td>Seigniorage (% GDP)</td>
<td>163</td>
<td>1.094</td>
<td>1.696</td>
<td>-0.492</td>
<td>19.249</td>
</tr>
<tr>
<td>Population Growth</td>
<td>163</td>
<td>2.385</td>
<td>1.119</td>
<td>-5.128</td>
<td>7.413</td>
</tr>
<tr>
<td>Investment (% GDP)</td>
<td>163</td>
<td>20.967</td>
<td>7.928</td>
<td>5.629</td>
<td>60.496</td>
</tr>
<tr>
<td>Initial GDP per capita (log)</td>
<td>163</td>
<td>6.248</td>
<td>0.843</td>
<td>4.395</td>
<td>8.07</td>
</tr>
<tr>
<td>Education</td>
<td>163</td>
<td>12.84</td>
<td>9.479</td>
<td>0.5</td>
<td>42.2</td>
</tr>
</tbody>
</table>

Appendix 4: Classification according to the exchange rate arrangements
Fix or quasi fix exchange rate regime:

Flexible or quasi flexible exchange rate regime:
Algeria, Bangladesh, Bolivia, Burundi, Colombia, Congo Democratic Republic, Dominican Republic, Egypt, Gambia, Ghana, Guatemala, India, Indonesia, Iran, Jamaica, Kenya, Malawi, Mauritania, Mozambique, Nicaragua, Pakistan, Papua New Guinea, Paraguay, Peru, Philippines, Rwanda, Sierra Leone, Sri Lanka, Sudan, Thailand, Togo, Tunisia, Uganda, Zambia.