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This paper examines simultaneous versus sequential choice of effort in a two-player contest with a general contest success function. The timing of moves, determined in a pre-play stage prior to the contest-subgame, as well as the value of the prize is allowed to be endogenous. Contrary to endogenous timing models with an exogenously fixed prize the present paper finds the following. (1) Players may decide to choose their effort simultaneously in the subgame perfect equilibrium (SPE) of the extended game, (2) the SPE does not need to be unique, (3) in particular, there is no unique SPE with sequential moves if costs of effort are exclusively endogenously determined, (4) if the unique SPE is sequential play, the win probability in the NE is in no way crucial for the determination of an endogenous leadership, (5) and symmetry among players does not rule out incentives for precommitment to effort locally away from the Nash-Cournot level

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JEL classification: C72, D23, D30.

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1 Introduction

By providing a framework for analyzing contests with endogenous timing, an endogenously determined prize and a general contest success function (CSF) the present paper strives to merge two strands of literature. The first group of papers focusses on the distinction between Cournot-Nash equilibria (NE) and Stackelberg equilibria in contest models with an exogenously fixed prize. The second group of papers is broadly concerned with the impact of an endogenously determined prize on the NE of a contest.

Strategic behavior in a two-player contest over a prize of fixed and common value was first explored by Dixit (1987), who uses a logit as well as a probit form of the CSF.[1] He finds that in a symmetric two-player contest there is no local incentive to precommit effort away from the Nash-Cournot level. Moreover, he shows that if two unevenly matched players compete in a sequential manner, it is the favorite (underdog) who has an incentive to overcommit (undercommit) effort compared to the NE.[2] Two decisive factors are responsible for this finding. First, the underdog's (favorite's) effort is a strategic complement (substitute) to that of the favorite (underdog), i.e. the underdog's best response function is downward sloping in the NE of the game while the favorite's is upward sloping.[3] Second, efforts exhibit negative externalities, i.e. each player's payoff is a decreasing function of the competitor's effort.[4] An important implication of this finding is that sequential play may increase or decrease social costs (compared to the NE) contingent on the leader's win probability in the NE.

In seminal contributions Baik and Shogren (1992) and Leininger (1993) independently extend the Dixit-framework by introducing a preplay stage in which the two players determine the order of their moves prior to the actual choice of effort. They show that in the unique SPE of the extended game the favorite (underdog) will never

[1] The logit form of the CSF expresses the probability of winning as a function of the *relative* effort of players (see Loury (1979) and Tullock (1980)). The probit form CSF is used when players experience some noise components regarding their effective effort (see Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983)).

[2] According to Dixit (1987) the favorite (underdog) is the player whose odds of victory in a two-player contest exceed (fall below) one-half at the NE.

[3] For the case of an oligopoly, the issue of strategic complementarity and substitutability has first been examined by Bulow et al. (1985) and Gal-Or (1985).

[4] The issue of positive vs. negative externalities is imminently important for the analysis of the leader's behavior in a Stackelberg game. See, for instance, Amir (1995) and Eaton (2004).

(always) move first. Hence, players' voluntary choice of timing leads unambiguously to a sequential move game which contradicts the rational explanation of a contest as a simultaneous move game as originated by Tullock (1980). Moreover, because of the particular order of moves, the unique SPE Pareto-dominates any other sequence of moves.

A limitation of the previous analysis is the fact that it does not address the consequences of an endogenous prize in a contest, a fact which has attracted increasing attention over the last two decades. Basically, there are two ways of endogenizing the value of a prize in a contest. Either (1) the prize itself is a control variable of the players or (2) the players' effort indirectly affects the value of the prize.[5]

An example for the first approach is Konrad (2002), where subsequently to the realization of a project, an incumbent decides about his investment in a project as well as about his effort in a contest in which he has to defend his project returns against a challenger. Epstein and Nitzan (2004) analyze in a political competition game the endogenous formation of policies prior to a lobbying contest.[6]

As opposed to this, we provide a framework which uses the second approach, i.e. a framework in which the effort exerted by a player affects the distribution as well as the value of the prize. Depending on whether the costs of effort are assumed to be exclusively or only partially endogenously determined, we distinguish between general and partial equilibrium models, or synonymously, between conflict models and rent-seeking models.[7]

A Cournot-Nash type example of a conflict model is Hirshleifer (1991), where, in a state-of-nature, two agents are endowed with an inalienable resource which can be used as an input in a valuable prize (production) or for appropriation. Since effective property rights are absent, the contestants face a trade-off between production and appropriation. He finds that in the NE the poorer player, defined with respect

[5] We do not address the issue of artificially created contest, where the contest designer selects the value of the prize awarded to fulfill a specific goal. See for example Moldovanu and Sela (2001), and Che and Gale (2003).

[6] See also Leidy (1994), who argues that a monopolist whose right is contested in a political market will spend lobbying effort and lower his price to defuse reformist opposition, and Hoffmann (2010), who shows in a two-player conflict model that the anticipation of potential appropriation forces agents to engage in trade, since this mutually reduces the gains from appropriation.

[7] A recent survey on rent-seeking models is provided by Congleton et al., eds (2008), whereas Garfinkel and Skaperdas (2007) present a systematic and comprehensive review of conflict models. See also Neary (1997) for a discussion on both concepts.

to the value of the initial resource, will catch up to the richer player due to the fact that each player uses his comparative advantage. In a comparable framework Skaperdas (1992) finds that contingent on the properties of the CSF, cooperation is not incommensurate with the lack of exogenously enforced property rights in a one-shot contest. In a related model Beviá and Corchón (2010) show that cooperation can be achieved by compensating the poorer player in order to avoid open conflict.[8]

An example of a rent-seeking model with an endogenous prize is Baye et al. (2005), who use an all-pay auction framework in order to compare different litigation systems. Here, different legal systems are based on different fee-shifting rules, which determine the value of the net-prize of the contest winner and looser contingent on their expenditures on legal representation. Another example is Shaffer (2006) who discusses positive and negative externalities of effort on the value of the prize. An example for the latter are territorial disputes, an example for the former are labor tournaments.[9]

The question we pose is whether the findings of Baik and Shogren (1992) and Leininger (1993) are generalizable beyond fixed prizes. Therefore, in order to unite contests with endogenous timing and with an endogenous prize, we provide a framework of a two-player contest under complete information, given a general production technology of the prize, and a general CSF. The extended game consists of a contest subgame and a pre-play stage in which players decide whether to exert effort as soon as or as late as possible. Subsequently, agents choose effort in the contest subgame according to their previous decision. Thus, the timing game matches the *extended game with observable delay* by Hamilton and Slutsky (1990) frequently used in games of endogenous timing.[10] No matter when exerted, the players' effort influences not only the win probability of both players but also the value of the prize. We will assume throughout the analysis that effort has a negative impact on the value of

[8]See also Anbarci et al. (2002), who compare various bargaining solutions. Here, bargaining takes place in the shadow of conflict, i.e. players have to make irreversible outlays before the bargain procedure. These investments not only alter a player's disagreement payoff but also the output subject to bargain. Dynamic conflict games are provided by Hirshleifer (1995), Grossman and Kim (1995), Hafer (2006), and Gonzales and Neary (2008).

[9]Early examples are provided, *inter alia*, by Alexeev and Leitzel (1996) who present a rent-seeking model of hostile take-overs of public companies. Here, anti-takeover strategies, such as the poison pill, diminish the target's stock (the prize). Chung (1996) shows that promotional effort increases the market share of a firm as well as the size of the whole market. Thus, effort-spending does have a positive externality on the combatant.

[10]See for example Amir and Grilo (1999), Normann (2002), and Amir and Stepanova (2006).

the prize and allow the costs of effort to be exclusively or only partially endogenously determined. Based on these assumptions we are able to provide solutions for rent-seeking and conflict games.[11] We examine how the endogeneity of the prize will influence the players' timing decision. In particular, we provide a taxonomy of endogenous timing based on the properties of the players' best response functions as well as on the characteristics of the prize-production technology. Hence, in a methodological sense, the paper is close to Kempf and Rota Graziosi (2009) who develop an endogenous timing game in which two countries provide public goods with spillovers. Here, a taxonomy is proposed depending on the sign of spillovers among countries and the nature of the strategic interaction between various public goods.

It is found, in line with Baik and Shogren (1992) and Leininger (1993), that the SPE of the extended game is Pareto dominated by no other sequential or simultaneous play payoff; and that, if sequential play emerges in equilibrium, the leader commits lower effort than in the NE. However, unlike the aforementioned literature, the present paper finds the following. (1) In the SPE of the extended game, players may decide to choose effort simultaneously, which partly reinforces the argument put forth by Tullock (1980) regarding the rational of a contest as a simultaneous move game. (2) The SPE of the extended game does not need to be unique. (3) In particular, there is no unique SPE with sequential moves if costs are exclusively endogenously determined. Hence, in a general equilibrium setting it is impossible to replicate the findings of Baik and Shogren (1992) and Leininger (1993). (4) If the unique SPE is sequential play, the win probability in the NE is in no way crucial for the determination of an endogenous leadership. This shows that in our setting it is possible for the favorite to become a Stackelberg-leader and the underdog to become a Stackelberg-follower. (5) Finally, contrary to Dixit (1987), we prove that in a symmetric game Cournot-Nash and Stackelberg equilibria typically do not coincide, i.e. there are local commitment incentives for players.

The underlying reason for the differences in the strategic incentives in our model compared to the Dixit-framework is that in the latter costs of effort are exclusively

[11] Note that if in a general equilibrium model the prize reacted positive to effort, the resulting Cournot-Nash equilibrium would be a corner solution, since the win probability as well as the prize increases in effort for both players and exogenously given costs of effort are, by definition, absent.

private costs, i.e. apart from the CSF, there is no additional negative externality stemming from the use of effort. Thus, the marginal payoff of a player does not depend on the marginal costs of his competitor. On the contrary, costs of effort in the present model are at least partially common costs, meaning that they have to be borne by both players. These additional negative externalities arise as a result of the endogenous prize assumption and may represent the opportunity costs of effort measured in terms of foregone production possibilities in a conflict framework. Or, in a rent-seeking framework, they may represent the negative responsiveness of the prize at hand to the effort exerted. Accordingly, common costs reshape the strategic incentives in the NE, compared to the private cost scenario.

Before introducing our model, it should certainly be emphasized that we are not the first to undertake the program of generalizing the findings of Baik and Shogren (1992) and Leininger (1993). However, almost all papers make the assumption of an exogenous prize. For example Yildirim (2005) prescinds from the feature that each player can only move once. Endogenous timing in contests with asymmetric information and a lottery CSF is studied by Fu (2006). Konrad and Leininger (2007) study endogenous sequencing in a n-player all-pay contest with complete information. Finally Kolmar (2008) analyzes the emergence of perfectly secure property rights in a stylized two-player conflict model. Although, as in the present paper, the prize is allowed to be endogenous, its value is contingent only on the effort of one player. Moreover, he does not address the question of endogenous timing in a rent-seeking framework and does not provide a taxonomy of endogenous leadership for the case of a general CSF and a general production technology.

The paper proceeds as follows. Section 2 presents the basic model and explores the nature of strategic substitutes vs. complements in our setting and its influence on the players' first-mover and second-mover advantages. Furthermore, it describes the equilibrium concepts used in the paper. Section 3 provides the equilibria in the full game and the taxonomy of endogenous leadership; we conclude in section 4.

2 The model

Consider a situation in which each of two players exert effort ($x_i \in \mathbb{R}^+$) in order to win a prize of common value, with $i = 1, 2$. The prize is allowed to be endogenous, i.e.

its value is contingent on the vector $\mathbf{x} = (x_1, x_2)$. The prize-production technology $V(\mathbf{x})$ has the following properties.[12]

Assumption 1 (Prize-production technology)

$$V_i(\mathbf{x}) \equiv \frac{\partial V(\mathbf{x})}{\partial x_i} < 0, \tag{1a}$$

$$V_{ii}(\mathbf{x}) \equiv \frac{\partial^2 V(\mathbf{x})}{\partial x_i^2} < 0. \tag{1b}$$

Assumptions (1a) and (1b) state that an increase in effort decreases the prize and that this negative effect increases in x_i. Note that the marginal productivity with respect to x_i might differ for the two players, i.e. $V_1(\mathbf{x}) \gtreqless V_2(\mathbf{x})$. Moreover, note that we allow for $q-$substitutes and $q-$complements, i.e., we do not restrict the sign of the cross derivatives of the prize-production technology $\left(V_{12}(\mathbf{x}) \equiv \frac{\partial^2 V(\mathbf{x})}{\partial x_1 \partial x_2} \gtreqless 0\right)$.[13] Thus, if $V_{12}(\mathbf{x}) > 0$, then an increase in player i's effort will decrease the (negative) marginal effect of player j's effort on the prize.

Example 1 (A conflict framework)

For example in Skaperdas (1992) each of two players possesses R_i units of an inalienable primary resource which can be used to produce one-to-one two kinds of inputs, x_i and y_i, where the latter will be used in the joint production of a single consumption good representing the prize while the former will be used as an input in the appropriative competition. Implementing the individual budget-constraint ($R_i = x_i + y_i$) and assuming a Cobb-Douglas type of production function, we get $V(\mathbf{x}) = (R_1 - x_1)^{\alpha}(R_2 - x_2)^{1-\alpha}$, with $R_i \in \mathbb{R}^+$ and $\alpha \in (0,1)$. This leads to $V_{12}(\mathbf{x}) > 0$.

Next, we turn to the CSF, $p^i : x_i \times x_j \to [0,1]$, which determines for any given value of the vector $\mathbf{x}$ player i's probability of winning the prize.[14] As a notational simplification we introduce $p(\mathbf{x})$ as the win probability of player 1 and $1 - p(\mathbf{x})$ as player 2's win probability. The function $p(\mathbf{x})$ exhibits the following properties.

[12] The subscript i (j) denotes the partial derivative with respect to x_i (x_j).

[13] The terms $q-$substitutes and $q-$complements have been suggested by Hicks (1956, p. 156). In the contest literature several specifications have been proposed with respect to the prize: Dixit (1987), Baik and Shogren (1992) and Grossman (2001) consider exogenous rents ($V(\mathbf{x}) = K$), Skaperdas and Syropoulos (1998) consider an endogenous rent, with $V_{12}(\mathbf{x}) = 0$, whereas Hirshleifer (1991) and Skaperdas (1992) assume $q-$complements ($V_{12}(\mathbf{x}) > 0$).

[14] To avoid repetition, we use $i, j = 1, 2$ and $i \neq j$ when it's obvious.

Assumption 2 (Contest success function)

$$p_1(\mathbf{x}) \equiv \frac{\partial p(\mathbf{x})}{\partial x_1} > 0 \ \textit{and}\ p_2(\mathbf{x}) \equiv \frac{\partial p(\mathbf{x})}{\partial x_2} < 0, \tag{2a}$$

$$p_{11}(\mathbf{x}) \equiv \frac{\partial^2 p(\mathbf{x})}{\partial x_1^2} < 0 \ \textit{and}\ p_{22}(\mathbf{x}) \equiv \frac{\partial^2 p(\mathbf{x})}{\partial x_2^2} > 0, \tag{2b}$$

$$p_{12}(\mathbf{x})(1-p(\mathbf{x}))\,p(\mathbf{x}) - p_2(\mathbf{x})\,p_1(\mathbf{x})(1-2p(\mathbf{x})) = 0. \tag{2c}$$

Assumptions (2a) and (2b) show that each players win probability is an increasing (decreasing) and concave (convex) function of his own (his competitor's) effort. Assumption (2c) is a technical one which allows us to simplify the analysis for the proof of the uniqueness of the NE.[15]

The payoff function of player 1 and 2 are given by

$$\Pi^1(\mathbf{x}) = p(\mathbf{x})\,V(\mathbf{x}) - C^1(x_1), \tag{3.1}$$

$$\Pi^2(\mathbf{x}) = (1-p(\mathbf{x}))\,V(\mathbf{x}) - C^2(x_2), \tag{3.2}$$

where $C_i^i(x_i) \geq 0$, and $C_{ii}^i(x_i) \geq 0$. Each agent maximizes his expected payoff which equals the prize that goes to the sole winner, weighted by the probability that he wins the contest minus the sure effort cost. These effort costs are allowed to be zero.[16] We remark that the agents' objective functions have two kinds of properties. First, these functions exhibit *plain substitutes* as defined by Eaton (2004). Therefore, the sign of the cross derivatives of the payoff function is negative, i.e., we have negative

[15] It is similar to assumption (3) in (Skaperdas, 1992, p. 725). It is worth noting that assumption (2) is fulfilled by any logit form CSF represented by the function

$$p(\mathbf{x}) = \begin{cases} \frac{f_1(x_1)}{f_1(x_1)+f_2(x_2)}, & \text{if } \mathbf{x} \neq \mathbf{0}, \\ \frac{1}{2}, & \text{if } \mathbf{x} = \mathbf{0}, \end{cases}$$

as long as each player's impact function $f : \mathbb{R}^+ \to \mathbb{R}^+$ is a twice differentiable, increasing and concave function. The same holds for any probit form CSF,

$$p(\mathbf{x}) = G\left(f_1(x_1) - f_2(x_2)\right),$$

where G represents the cumulative density function of the difference in the noise components $(\epsilon_1 - \epsilon_2)$.

[16] For $C_i^i(x_i) = 0$ the present model describes a conflict model, i.e. a model in which the marginal costs of effort are exclusively endogenously determined.

spillovers with respect to the effort invested:

$$\Pi_2^1(\mathbf{x}) \equiv \frac{\partial \Pi^1(\mathbf{x})}{\partial x_2} = p_2(\mathbf{x})\, V(\mathbf{x}) + p(\mathbf{x})\, V_2(\mathbf{x}) < 0, \tag{4.1}$$

$$\Pi_1^2(\mathbf{x}) \equiv \frac{\partial \Pi_2(\mathbf{x})}{\partial x_1} = -p_1(\mathbf{x}) V(\mathbf{x}) + (1 - p(\mathbf{x})) V_2(\mathbf{x}) < 0. \tag{4.2}$$

A second property concerns the strategic interactions among agents' efforts. Following Bulow et al. (1985), we will say that efforts are strategic substitutes (SS) for agent i if his marginal payoff decreases in the effort of player j, and they are strategic complements (SC) if agent i's marginal payoff increases in agent j's effort. Due to the properties of the CSF, the players' marginal payoff depend in a non-monotonic way on the competitor's effort. Following Dixit (1987) we thus define SS and SC in the neighborhood of the NE.

2.1 Efforts in the three basic games

Now, we consider the three basic games; the Cournot-Nash game (Γ^N) and the two Stackelberg games, depending on whether agent 1 or agent 2 leads (Γ^{S_1} or Γ^{S_2}). The NE of the contest subgame $\left(\Gamma^N\right)$ is defined by the following system of maximization programs

$$\begin{cases} x_i^N \equiv \underset{x_i}{\operatorname{argmax}} \quad \Pi^i(\mathbf{x}), \quad x_j^N \text{ given}, \\ x_j^N \equiv \underset{x_j}{\operatorname{argmax}} \quad \Pi^j(\mathbf{x}), \quad x_i^N \text{ given}. \end{cases} \tag{5}$$

The FOCs for players 1 and 2 are therefore evaluated at $\mathbf{x}^N$, which denotes the NE values $\left(\mathbf{x}^N \equiv \left(x_1^N, x_2^N\right)\right)$.[17] The FOCs for player 1 and 2 are therefore

$$p_1\left(\mathbf{x}^N\right) V\left(\mathbf{x}^N\right) + p\left(\mathbf{x}^N\right) V_1\left(\mathbf{x}^N\right) - C_1^1(x_1^N) = 0, \tag{6.1}$$

$$-p_2\left(\mathbf{x}^N\right) V\left(\mathbf{x}^N\right) + \left(1 - p\left(\mathbf{x}^N\right)\right) V_2\left(\mathbf{x}^N\right) - C_2^2(x_2^N) = 0. \tag{6.2}$$

In order to establish the uniqueness of the NE we need the following assumption.[18]

[17] In a similar way, we will note $\mathbf{x}^{S_i} \equiv \left(x_i^L, x_j^F\left(x_i^L\right)\right)$ the levels of effort at the Stackelberg equilibrium in which player i leads.

[18] It is worth noting that a less strict assumption is needed if the uniqueness of the NE is to be proven, when $C_i^i(x_i) = 0$ for both players. Then the assumption needed is given by

$$V_{11}\left(\mathbf{x}^N\right) V_{22}\left(\mathbf{x}^N\right) \geq \left(V_{12}\left(\mathbf{x}^N\right)\right)^2,$$

Assumption 3

$$\left(\underline{p}\left(\mathbf{x}^N\right)\right)^2 V_{11}\left(\mathbf{x}^N\right) V_{22}\left(\mathbf{x}^N\right) \geq \left(\bar{p}\left(\mathbf{x}^N\right) V_{12}\left(\mathbf{x}^N\right)\right)^2 \tag{7}$$

with $\bar{p}\left(\mathbf{x}^N\right) = \max\left\{p\left(\mathbf{x}^N\right), 1 - p\left(\mathbf{x}^N\right)\right\}$ *and* $\underline{p}\left(\mathbf{x}^N\right) = \min\left\{p\left(\mathbf{x}^N\right), 1 - p\left(\mathbf{x}^N\right)\right\}$.

Now we can establish the following lemma.

Lemma 1

Under assumptions (1), (2), *and* (3) *the Nash equilibrium of the Cournot-Nash game* (Γ^N) *exists and is unique.*

We now turn to the issue of strategic incentives in the NE of the contest subgame. Applying the Envelope theorem to (6), it is easy to show that

$$\frac{dx_j}{dx_i} = -\frac{\Pi^j_{ij}(\mathbf{x})}{\Pi^j_{jj}(\mathbf{x})} \lesseqqgtr 0 \Leftrightarrow \Pi^i_{ij}(\mathbf{x}) \lesseqqgtr 0,$$

i.e., the sign of the slope of agent i's best response function at a point in the strategy space is solely determined by the cross effect on the marginal payoff function which - as was said earlier - may vary. However, uniqueness of the NE implicates that our definition of strategic interaction (SS or SC) is unique for each player. Implementing the FOC in each player's cross derivative of the payoff function yields:

$$\Pi^1_{12}\left(\mathbf{x}^N\right) = p\, V_{12}(\mathbf{x}^N) + \Omega(\mathbf{x}^N), \tag{8.1}$$

$$\Pi^2_{12}\left(\mathbf{x}^N\right) = (1-p) V_{12}(\mathbf{x}^N) - \Omega(\mathbf{x}^N), \tag{8.2}$$

with

$$\Omega(\mathbf{x}^N) = \frac{p_1(\mathbf{x}^N)\, C^2_2(x^N_2)}{1 - p(\mathbf{x}^N)} + \frac{p_2(\mathbf{x}^N)\, C^1_1(x^N_1)}{p(\mathbf{x}^N)}. \tag{9}$$

These conditions state that the sum of the cross effects on the marginal payoff function equals the cross derivatives of the production function, i.e. $\Pi^1_{12}(\mathbf{x}^N) + \Pi^2_{12}(\mathbf{x}^N) \equiv V_{12}(\mathbf{x}^N)$. Since $V_{12}(\mathbf{x}) \gtreqqless 0$, we have three different cases. A game of SC ($\Pi^i_{12}\left(\mathbf{x}^N\right) \in \mathbb{R}^+$), which is only consistent with q-complements ($V_{12}\left(\mathbf{x}^N\right) > 0$) and a game of SS ($\Pi^i_{12}\left(\mathbf{x}^N\right) \in \mathbb{R}^-$), which is only consistent with q-substitutes ($V_{12}\left(\mathbf{x}^N\right) < 0$). The mixed case ($\Pi^j_{12}\left(\mathbf{x}^N\right) > 0 > \Pi^i_{12}\left(\mathbf{x}^N\right)$) emerges with either

which corresponds to the assumption of non-increasing returns to scale in production by Skaperdas (1992).

q-substitutes or q-complements and it is also the only case that is consistent with $V_{12}\left(\mathbf{x}^N\right) = 0$. Note that $\Omega\left(\mathbf{x}^N\right) = 0$ if $C_i^i(x_i) = 0$, i.e. in a conflict model the strategic incentives of both players are always aligned and depend solely on $V_{12}(\mathbf{x}^N)$. Hence, given a symmetric game and $V_{12}\left(\mathbf{x}^N\right) \neq 0$, there are local commitment incentives, which contradicts (Dixit, 1987, p. 893).[19]

Example 1 (A conflict framework - continued)

Let us assume that the game is symmetric. In particular, let us assume that we have a specific type of logit form CSF, a Tullock CSF, with $p(x) = \dfrac{x_1^r}{x_1^r + x_2^r}$ *and* $r \in (0, 1]$. *The prize-production technology exhibits equal marginal productivity of both factors* ($\alpha = \frac{1}{2}$), *and* $R_1 = R_2 = R$. *The symmetric NE gives us* $x_1^N = x_2^N = \frac{r\,R}{1+r}$ *and* $\Pi_{12}^1(\mathbf{x}^N) = \Pi_{12}^2(\mathbf{x}^N) = \frac{1+r}{8\,R}$. *Hence, in this symmetric conflict model both players regard effort as SC.*

Next, we would like to compare our result to the findings associated with the Dixit-framework. If the prize is assumed to be exogenously fixed and equal to $K > 0$, then

$$\Pi_{12}^1(\mathbf{x}^N) = -\Pi_{12}^2 = p_{12}(\mathbf{x}^N)\,K. \tag{10}$$

Accordingly, either $p_{12}(\mathbf{x}^N) = 0$ and the strategic incentives are aligned and equal to zero, or $p_{12}(\mathbf{x}^N) \neq 0$, then the strategic incentives are directly opposed. Moreover, given a logit or probit form of the CSF

$$p\left(\mathbf{x}^N\right) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \frac{1}{2} \Leftrightarrow p_{12}\left(\mathbf{x}^N\right) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0. \tag{11}$$

Hence, as stated by Dixit (1987), the favorite's (underdog's) effort is SS (SC) to the underdog's (favorite's) effort.[20] This no longer holds if we introduce an endogenous prize. Then, the probability of winning the contest has to be distinguished from

[19] We do not examine the case where $\Pi_{12}^1(\mathbf{x}) = \Pi_{12}^2(\mathbf{x}) = 0$ as further restrictions on the third derivatives of the CSF would be required to establish whether there are local commitment incentives in this case, which is beyond the scope of this paper (see Dixit (1999) and Baye and Shin (1999)).

[20] For the case of a logit type CSF, we get

$$p_{12}\left(\mathbf{x}^N\right) = \frac{f_1'\left(x_1^N\right) f_2'\left(x_2^N\right)}{\left(f_1\left(x_1^N\right) + f_2\left(x_2^N\right)\right)^3} \left(f_1\left(x_1^N\right) - f_2\left(x_2^N\right)\right),$$

for the case of a probit type CSF, we get

$$p_{12}(\mathbf{x}^N) = -g'\big(f_1(x_1^N) - f_2(x_2^N)\big)\big(f_1'(x_1^N)\, f_2'(x_2^N)\big),$$

where g is the density associated with G.

local commitment incentives.

Example 2 (A rent-seeking framework)
Assume that $V(\mathbf{x}) = 10 - (4\,x_1)^2 - (x_2)^2$ *so that* $V_{12}(\mathbf{x}) = 0$. *Moreover, we use a CSF of the logit type and assume that* $C^i(x_i) = x_i$, *which leads, given eq.* (8), *to* $\Pi^1(\mathbf{x}^N) = -\Pi^2(\mathbf{x}^N) = \Omega(\mathbf{x}^N) = \frac{f_1'(x_1)-f_2'(x_2)}{f_1(x_1)+f_2(x_2)}$. *In particular assume that* $f_1(x_1) = 2\,x_1$, $f_2(x_2) = x_2$, *so that* $\Omega(\mathbf{x}) > 0$, $\forall$ $\mathbf{x} > 0$. *Hence, efforts are SC (SS) for player* 1 (2), *which solely depends on player* 1*'s advantage with respect to the impact function* $f_i(x_i)$. *The effort levels in the NE are* $x_1^N \approx 0.3309$ *and* $x_2^N \approx 0.8959$, *hence* $p_{12}(\mathbf{x}^N) \approx -0.1239$, *showing that player* 1 *is the underdog, which contradicts Dixit (1987). The reason for this is that although player* 1 *is more effective with regards to the impact factor this advantage is overcompensated with respect to the value of the CSF in the NE by the fact that* $V_1(x) \ll V_2(x)$. *Thus, player* 1*'s deeper impact on the prize leads to* $f_2(x_2^N) > f_1(x_1^N)$ *and consequently to* $p_{12}(\mathbf{x}^N) < 0$.

Next, we turn to the sequential move games.[21] The subgame perfect equilibrium of the contest subgame (the Stackelberg equilibrium) is determined by applying backward induction. Thus, in the game where agent i leads (Γ^{S_i}), we first focus on the follower's (F) maximization program which is $x_j^F(x_i) \equiv \underset{x_j}{\operatorname{argmax}}\ \Pi^j(\mathbf{x})$. This yields

$$\Pi_j^j\left(x_j^F(x_i), x_i\right) = 0. \tag{12}$$

We assume that the second order condition of the leader's maximization program holds. In particular, we assume that

Assumption 4

$$\frac{d^2\Pi^i\left(x_i, x_j^F(x_i)\right)}{dx_i^2} < 0.$$

This assumption is crucial, since it assures the existence and uniqueness of the Stackelberg equilibrium, where the latter property guarantees that the sign of the slope of a player's best response function at the NE is equal to the sign of the slope of the same player's best response function, once he becomes a Stackelberg follower in a sequential move game.

2.2 Effort ranking

Given the optimizing behavior in the basic games, we are now in the position to establish the rankings of the levels of effort in the different equilibria. We distinguish

[21] We are aware of the fact that given assumptions (1) and (2) we cannot rule out corner solutions for the sequential move games. This topic has been analyzed, for example, by Grossman and Kim (1995). However, we will assume only interior solutions for the sequential move games.

two lemmas since each relies on a specific characteristic of our framework. Lemma (2) is a direct consequence of the plain substitutes of efforts (cf. equations 4), lemma (3) results from the strict concavity of the leaders payoff function (assumption 4).

Lemma 2

Under assumptions (1), (2), (3), *and* (4) *the levels of efforts for the Nash and Stackelberg games are such that*

$$\forall i \in \{1, 2\}, \quad x_i^N > x_i^F.$$

Lemma (2) states that, independent of the strategic interaction (SS or SC), the follower will always choose a level of effort which is lower, compared to the effort exerted by the same player in the NE.[22]

Lemma 3

Under assumptions (1), (2), (3), *and* (4) *the level of effort for the Nash and Stackelberg games are such that*

$$\Pi_{ij}^j\left(\mathbf{x}^N\right) > 0 \Leftrightarrow x_i^N > x_i^L,$$
$$\Pi_{ij}^j\left(\mathbf{x}^N\right) < 0 \Leftrightarrow x_i^N < x_i^L.$$

Our second result compares the effort exerted by the Stackelberg leader with the one exerted in the NE of the contest subgame. If efforts of player i are SC (SS) for player j, the Stackelberg-leader i reduces (increases) his effort compared to the one in the NE.

To resume, we find that the equilibrium levels of effort in both Stackelberg games are lower than the one obtained at the NE ($x_i^N > x_i^L$ and $x_j^N > x_j^F$) if efforts are SC for both players. Here, the leader, say agent 1, undercommits his effort relative to his effort in the NE, which induces the follower, agent 2, to decrease his own effort because of the SC property. In turn, this increases the leader's payoff because of the plain substitutes property. In the case of SS for both players, the ranking of effort is unambiguously corresponding to $x_i^L > x_i^N > x_i^F$. The leader overcommits his effort compared to the NE, which induces the follower to decrease his effort because of the SS property. Finally, in the mixed game ($\Pi_{ij}^j\left(\mathbf{x}^N\right) > 0 > \Pi_{ij}^i\left(\mathbf{x}^N\right)$), we obtain that player j overcommits effort compared to the NE ($x_j^L > x_j^N$), whereas player i

[22]For the rest of the paper, we pose $x_j^F \equiv x_j^F(x_i^L)$.

undercommits effort ($x_i^L < x_i^N$).

2.3 First-mover/Second-mover advantage and incentive

Given these rankings, we can now compare the payoffs in the three basic games (Γ^N, Γ^{S_1} and Γ^{S_2}), which will give us the opportunity of detecting potential first-mover (second-mover) advantages or first-mover (second-mover) incentives, which we need for our last lemma. We define these concepts as follows:

Definition 1 (First-mover (second-mover) advantage)
Player i has a

$$\left\{ \begin{array}{l} \textit{first-mover advantage} \\ \textit{second-mover advantage} \end{array} \right\} \Leftrightarrow \Pi^i\left(\mathbf{x}^{S_i}\right) \left\{ \begin{array}{c} > \\ < \end{array} \right\} \Pi^i\left(\mathbf{x}^{S_j}\right).$$

Next, we compare the payoffs in the NE to the one obtained in the Stackelberg equilibrium.

Definition 2 (First-mover (second-mover) incentive)
Player i has a

$$\left\{ \begin{array}{l} \textit{first-mover incentive} \\ \textit{second-mover incentive} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} \Pi^i\left(\mathbf{x}^{S_i}\right) \\ \Pi^i\left(\mathbf{x}^{S_j}\right) \end{array} \right\} \geq \Pi^i\left(\mathbf{x}^N\right).$$

It is worth noting that whatever the nature of strategic interactions (SC or SS) might be, players always have a first-mover incentive, that is they weakly prefer their leader payoff over their payoff in the NE $\left(\Pi^i\left(\mathbf{x}^{S_i}\right) \geq \Pi^i\left(\mathbf{x}^N\right)\right)$. This result holds for a continuous strategy spaces and follows from the definition of the leader's maximization program. From lemmas (2) and (3) follows the last lemma.

Lemma 4
Under assumptions (1), (2), (3), *and* (4) *we have:*

1. *If efforts of player j are strategic complements for player i at the Nash equilibrium $\left(\Pi_{ij}^i(\mathbf{x}^N) > 0\right)$, then player i has a second-mover incentive:*

$$\Pi^i\left(\mathbf{x}^{S_j}\right) > \Pi^i\left(\mathbf{x}^N\right).$$

2. *If efforts of player j are strategic substitutes for player i at the Nash equilibrium* $\left(\Pi^i_{ij}(\mathbf{x}^N) < 0\right)$*, then player i has a first-mover advantage:*

$$\Pi^i\left(\mathbf{x}^{S_i}\right) > \Pi^i\left(\mathbf{x}^{S_j}\right).$$

If efforts are SC for player i, player j reduces his level of effort at the Stackelberg equilibrium in which he leads, compared to the NE (see lemma (3)). This increases the payoff of player i due to the property of plain substitute and induces the second-mover incentive. If efforts are SS for player i, we unambiguously have $x_j^L > x_j^N$, and then player i prefers leading over following due to the negative externality of player j's effort.

An interesting point of the preceding lemma is that we establish a second-mover incentive or a first-mover advantage for player i depending only on the concept of strategic complementarity or, respectively, strategic substitutability of efforts for player j at the NE; that is without assuming monotonicity of the best response function. It is actually natural that the slope of the best response function of the opponent determines *in fine* the existence of a first-mover/second-mover incentive or advantage. For instance, SC of efforts for player j at the NE transforms a decrease of player i's effort into an incentive for player j to reduce his efforts, which benefits player i since efforts are plain substitutes.

3 Selecting a leader through a timing game

The issue of endogenous timing is examined according to the concept proposed by Hamilton and Slutsky (1990) in their *extended game with observable delay*. This extended game $\tilde{\Gamma}$ allows players to choose non-cooperatively and simultaneously their effort in a preplay stage either as soon as or as late as possible. The set of possible pure strategies of player i is $a_i \in \{\mathbf{e}, \mathbf{l}\}$, where $\mathbf{e} \equiv$ early and $\mathbf{l} \equiv$ late. Their decision is announced by the players subsequently. In the consecutive *basic game* (Γ^k, with $k = \{N, S_1, S_2\}$) the players choose their effort according to their timing decision to which they are committed. Thus, the *basic game* consists of the different constituent games: Γ^N if the strategy profile $a = \{a_1, a_2\} = \{\mathbf{l}, \mathbf{l}\}$ or $a = \{\mathbf{e}, \mathbf{e}\}$, Γ^{S_1} for $a = \{\mathbf{e}, \mathbf{l}\}$, and Γ^{S_2} for $a = \{\mathbf{l}, \mathbf{e}\}$. Thus, if players decide to choose effort at different times, the player who chooses to move late observes the effort exerted

by the player who chose to move *early* and acts accordingly.[23] It is worth noting that the order of moves does not affect the payoffs which are conditional only on the players' strategies.

The normal form representation of the pre-contest stage is shown in table 1.[24] Here, $\mathbf{x}^N = (x_1^N, x_2^N)$ and $\mathbf{x}^{\mathbf{S_i}} = (x_i^L, x_j^F)$.

Player 2

		e	**l**
Player 1	**e**	$\Pi^1(\mathbf{x}^N), \Pi^2(\mathbf{x}^N)$	$\Pi^1(\mathbf{x}^{\mathbf{S_1}}), \Pi^2(\mathbf{x}^{\mathbf{S_1}})$
	l	$\Pi^1(\mathbf{x}^{\mathbf{S_2}}), \Pi^2(\mathbf{x}^{\mathbf{S_2}})$	$\Pi^1(\mathbf{x}^N), \Pi^2(\mathbf{x}^N)$

Table 1
Normal form representation of $\tilde{\Gamma}$

3.1 Solutions to the leadership problem

The solution to this reduced form game is equivalent to characterizing the solution to the leadership problem. There is no leader if both players choose the same action; a leader emerges when they choose complementary roles. We obtain the following proposition:

Proposition 5

Under assumptions (1), (2), (3), *and* (4) *we have:*

1. *If efforts are strategic complements for both players* $\left(\Pi_{ij}^i\left(\mathbf{x}^N\right) > 0\right)$, *the subgame perfect equilibria are the two Stackelberg equilibria,*

2. *if efforts are strategic substitutes for both players* $\left(\Pi_{ij}^i\left(\mathbf{x}^N\right) < 0\right)$, *the unique subgame perfect equilibrium is the Cournot-Nash equilibrium,*

3. *if* $\Pi_{ij}^j\left(\mathbf{x}^N\right) > 0 > \Pi_{ij}^i\left(\mathbf{x}^N\right)$, *the unique subgame perfect equilibrium is the Stackelberg equilibrium with player* i *as Stackelberg-leader and player* j *as Stackelberg-follower.*

[23] Following Hamilton and Slutsky (1990) and Amir and Stepanova (2006), we restrict our attention to the SPE of $\tilde{\Gamma}$.

[24] We remark that the literature on endogenous timing remains divided about how to qualify the situation where both players choose to lead. Indeed, Dowrick (1986) and more recently Damme and Hurkens (1999) consider Stackelberg warfare where both countries apply their action as a leader. In contrast, Hamilton and Slutsky (1990) or Amir and Stepanova (2006) apprehend this situation as the static Nash game. They emphasize that Stackelberg warfare can occur only through error, since the underlying strategy of one player is inconsistent with the other player's strategy (Hamilton and Slutsky, 1990, p. 42).

In the first case $\left(\Pi^i_{ij}(\mathbf{x}^N) > 0\right)$, both players have a second-mover incentive. Given the fact that the leader's payoff is always higher than the payoff in the NE, a coordination game results with two pure strategy Nash equilibria, Γ^{S_1} and Γ^{S_2}. To solve this issue we utilize the equilibrium selection concept of *risk dominance* of Harsanyi and Selten (1988). It allows us to select an equilibrium as long as the game is not symmetric. This criterion consists into a minimization of the risk of a coordination failure due to strategic uncertainty.[25] As stressed by Amir and Stepanova (2006), a resolution for risk-dominance is not possible without using a precise specification of the problem. Hence, an example follows.

Example 3 (Another conflict framework)

As before, we will use a Skaperdas-type of conflict model, i.e. each player is endowed R_i units of a resource which can be used for production or appropriation. Moreover, we assume that the CSF is of the logit type with $f_i(x_i) = x_i$, and the prize-production technology is given by $V(x) = \sqrt{(R_1 - 2\,x_1)}\sqrt{(R_2 - x_2)}$. If, additionally, $R_1 = R_2 = 10$ then the level of efforts in NE are given by $x_1^N \approx 2.7539$ and $x_2^N = 4.3628$, so that $p_{12}\left(\mathbf{x}^N\right) < 0$. Since $V_{12}(\mathbf{x}) > 0$ we thus have a game of SC where player 2 is the favorite. According to proposition (5) the SPEs of the extended game are the two Stackelberg equilibria, so that we have a game of coordination in the preplay stage.

	e	**l**
e	1.9473, 3.0849	2.0034, 3.8099
l	2.4030, 3.1343	1.9473, 3.0849

Table 2
Payoffs in the 3rd example

Utilizing the risk-dominance concept allows us to select an equilibrium. In our framework, the SPE Γ^{S_1} risk-dominates Γ^{S_2} if the former is associated with the larger product of deviation losses, denoted by Δ. More formally, $\Gamma^{S_1} \underset{risk}{\succ} \Gamma^{S_2} \Leftrightarrow \Delta > 0$, with

$$\Delta \equiv \left(\Pi^1\left(\mathbf{x}^{S_1}\right) - \Pi^1\left(\mathbf{x}^N\right)\right)\left(\Pi^2\left(\mathbf{x}^{S_1}\right) - \Pi^2\left(\mathbf{x}^N\right)\right) - \left(\Pi^1\left(\mathbf{x}^{S_2}\right) - \Pi^1\left(\mathbf{x}^N\right)\right)\left(\Pi^2\left(\mathbf{x}^{S_2}\right) - \Pi^2\left(\mathbf{x}^N\right)\right).$$

Given the payoffs in the three different games (cf. table (2)) we thus obtain $\Delta \approx 0.0182 > 0$, or equivalently, we find that the SPE in which player 1 leads risk-dominates the other SPE.

In the second part of proposition (5) $\left(\Pi^i_{ij}(\mathbf{x}^N) < 0\right)$ both players have a first-mover advantage and prefer their simultaneous play payoff over their Stackelberg-follower

[25] This uncertainty comes from the fact that a player is always unsure of the other player's move because of the multiplicity of solutions. Harsanyi and Selten (1988) defined the risk-dominance as follows: An equilibrium risk-dominates another equilibrium when the former is less risky than the latter, that is the risk-dominant equilibrium is the one for which the product of the deviation losses is the largest.

payoff. Thus, both players have the dominant strategy *early* which leads to a Cournot-Nash game (Γ^N).

In the third case ($\Pi_{ij}^j\left(\mathbf{x}^N\right) > 0 > \Pi_{ij}^i\left(\mathbf{x}^N\right)$) player i, who has a first-mover advantage, prefers his NE payoff over his follower payoff and has therefore a dominant strategy (*early*). Player j, on the other hand, has a second-mover incentive, that

$\Pi_{12}^1\left(\mathbf{x}^N\right)$	$V_{12}(\mathbf{x}^N) > 0$	$V_{12}(\mathbf{x}^N) < 0$	$V_{12}(\mathbf{x}^N) = 0$
> 0	Both players may lead if $V_{12}(\mathbf{x}^N) < \Pi_{12}^1(\mathbf{x}^N)$ 1 follows 2 leads if $V_{12}(\mathbf{x}^N) > \Pi_{12}^1(\mathbf{x}^N)$	1 follows, 2 leads	1 follows 2 leads
< 0	1 leads, 2 follows	no player leads if $V_{12}(\mathbf{x}^N) > \Pi_{12}^1(\mathbf{x}^N)$ 1 leads, 2 follows if $V_{12}(\mathbf{x}^N) < \Pi_{12}^1(\mathbf{x}^N)$	1 leads 2 follows

Table 3
A taxonomy of endogenous leadership

is he prefers his follower payoff over his NE payoff. Therefore, given the dominant strategy of his opponent, player j chooses *late*, so that the unique solution to the timing game is Γ^{S_i}.

Applying proposition (5) we now provide a taxonomy of endogenous leadership based on the properties of the prize-production technology (in particular, the sign of $V_{12}\left(\mathbf{x}^N\right)$) as well as on the sign of the slope of both players' best response functions in the NE, presented in table 3. It shows that we might find in equilibrium that either *the favorite chooses effort simultaneously with his competitor*, or that *the favorite may lead*. The former case will emerge if $\{\Pi_{12}^1\left(\mathbf{x}^N\right), V_{12}\left(\mathbf{x}^N\right)\} \in \mathbb{R}^-$ and $V_{12}\left(\mathbf{x}^N\right) > \Pi_{12}^1\left(\mathbf{x}^N\right)$. The latter case may emerge if, for example, $\{\Pi_{12}^1\left(\mathbf{x}^N\right), V_{12}\left(\mathbf{x}^N\right)\} \in \mathbb{R}^+$ and $V_{12}\left(\mathbf{x}^N\right) > \Pi_{12}^1\left(\mathbf{x}^N\right)$.[26]

From proposition (5), we may deduce the following corollary.

[26] Without specification of, for instance, the prize-production technology, it is unclear whether the favorite leads or follows in this case.

Corollary 6 (Hamilton and Slutsky (1990) and rent dissipation)
Every SPE of the extended game $\widetilde{\Gamma}$ is Pareto undominated, although rent dissipation might be higher than in non-SPE. More precisely,

(1) if both players regard their effort as SC $\left(\Pi^i_{ij}(\mathbf{x}^N) > 0\right)$, both subgame perfect equilibria (Γ^{S_1} and Γ^{S_2}) Pareto-dominate the NE. Moreover, the levels of effort for the Nash and Stackelberg games are such that $x_i^N + x_j^N > x_i^L + x_j^F$.

(2) If both players regard their effort as SS $\left(\Pi^i_{ij}(\mathbf{x}^N) < 0\right)$, the payoffs in the Cournot-Nash equilibrium and in the two Stackelberg equilibria are not Pareto-rankable. Moreover, the levels of effort are such that $x_i^N + x_j^N \lesseqqgtr x_i^L + x_j^F$.

(3) If $\Pi^j_{ij}(\mathbf{x}^N) > 0 > \Pi^i_{ij}(\mathbf{x}^N)$, the subgame perfect equilibrium (Γ^{S_i}) Pareto-dominates Γ^{S_j} as well as Γ^N. Moreover, the levels of effort are such that $x_i^N + x_j^N > x_j^L + x_i^F$.

Thus, as in the fixed-prize framework, players' voluntary choice of timing leads to a second-best efficient outcome. These findings are based on the following facts: If we observe sequential play in the SPE, the leader always *undercommits* effort compared to the NE. If we observe simultaneous play in equilibrium, both players' efforts are - ceteris paribus - lower than their Stackelberg leader effort.

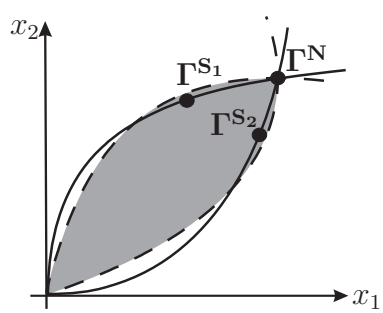

Figure 1
A game of strategic complements

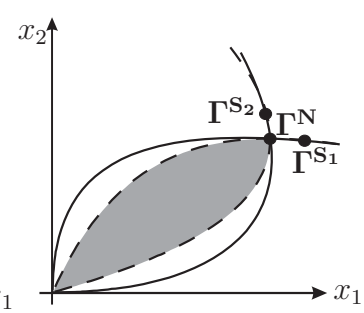

Figure 2
A game of strategic substitutes

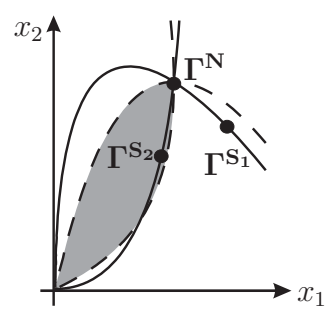

Figure 3
A mixed game

In the first case $\left(\Pi^i_{ij}(\mathbf{x}^N) > 0\right)$ both players' best response functions enter the Pareto superior set if we specify it at the NE and which is represented by the grey surface in figure (1). The solid convex (concave) curve represents the best response function of player 1 (2), the dashed concave (convex) curve the iso-payoff curve of player 1 (2) in the NE of the game. The leader in this case undercommits effort compared to the NE (cf. lemma (2)) and therefore both Stackelberg equilibria Pareto dominate the

NE. Moreover, the rent dissipation in Γ^{S_1} and Γ^{S_2} falls short compared to Γ^N. In the second case $\left(\Pi^i_{ij}(\mathbf{x}^N) < 0\right)$ both players prefer their NE payoff over their follower payoff and therefore neither of the best response functions enters the Pareto-superior set (cf. figure (2)). That is why Γ^N, Γ^{S_1} and Γ^{S_2} cannot be ranked in a Pareto sense in this case. Note that the difference in the rent dissipation between the Cournot-Nash game and the two sequential move games is indeterminate in this case. Thus, choosing effort sequentially might lead to social improvement in terms of resources spent in the contest compared to the SPE of the game.

In the third case $(\Pi^1_{12}(\mathbf{x}^N) > 0 > \Pi^2_{12}(\mathbf{x}^N))$ only player 1 prefers his NE payoff over his follower payoff (cf. figure (3)). Thus, only player 1's best response function enters the Pareto superior set. Moreover, player 2 undercommits effort. Since in this case the SPE of $\widetilde{\Gamma}$ is the Stackelberg equilibrium with player 2 being the leader, Γ^{S_2} Pareto dominates Γ^N as well as Γ^{S_1}.

4 Conclusion

Based on the endogenous timing game by Hamilton and Slutsky (1990), we have provided a framework for the analysis of endogenous leadership in contests with an endogenously determined prize. In a stage prior to the contest, the players decided whether they will exert effort as soon as or as late as possible; and their decision, to which they are committed, is announced to the other player subsequently. In this model we have provided a taxonomy of endogenous leadership, based on the properties of the players' best response functions as well as on the characteristics of the prize-production technology. Thus, we were able to generalize the findings of Baik and Shogren (1992) and Leininger (1993) regarding the behavior of the Stackelberg-leader as well as the fact that the SPE of the extended game is always Pareto-undominated. However, there are differences compared to the aforementioned literature. In particular, we were able to establish that the SPE of the extended game may be represented by a simultaneous move game, and that in a sequential move SPE the leader might be the favourite of the Cournot-Nash game.

Our work can be extended in various ways:

Regarding the previous work of Yildirim (2005) and Romano and Yildirim (2005) it would be interesting to establish in which way the findings of the present paper would be modified if one abstains from the assumption that each player is allowed

to exert effort only once. For instance, in the case were players are evenly matched, Yildirim (2005) finds that the outcome of the game is equivalent to a game where players move simultaneously, although effort might be exerted *early* and *late*. Therefore, allowing the players in our framework to exert effort twice might eliminate the coordination issue in a game of strategic complements.

Finally, in a rent-seeking framework one may allow for a prize which *in*creases in the effort of the players. Previous papers dealing with this topic include Cohen et al. (2008) and Gershkov et al. (2009). Although the prize is assumed to depend in a positive manner on the effort exerted, the issue of endogenous timing has not yet been analyzed. Contingent on the properties of the prize-production technology, this might lead to a game in which the payoff of a player does not react in a monotonic manner on the effort of his competitor. Hence, one might find in the NE that the effort of each player has a positive effect on each player's payoff, which would reshape the commitment incentives in the sequential move games.

These extensions are the subject of current research.

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A Appendix - Proofs

A.1 Proof of lemma 1

Here, we prove the existence and uniqueness of the Nash equilibrium.

A.1.1 Existence of the Nash equilibrium

Given assumptions (1) and (2), the payoff function $\Pi^i(\mathbf{x})$, given by equations (3), is continuous in (x_i, x_j). We now show that each player's payoff function is strictly concave in his own strategy. The second derivative of the payoff function yields

$$\begin{aligned} \Pi^1_{11}(\mathbf{x}) &= p_{11}(\mathbf{x})\,V(\mathbf{x}) + 2p_1(\mathbf{x})\,V_1(\mathbf{x}) + p(\mathbf{x})\,V_{11}(\mathbf{x}) - C^1_{11}(x_1), & \text{(A.1)} \\ \Pi^2_{22}(\mathbf{x}) &= -p_{22}(\mathbf{x})\,V(\mathbf{x}) - 2p_2(\mathbf{x})\,V_2(\mathbf{x}) + (1-p(\mathbf{x}))\,V_{22}(\mathbf{x}) - C^2_{22}(x_2). & \text{(A.2)} \end{aligned}$$

Assumptions (1) and (2) together with $C^i_{ii}(x_i) \geq 0$ imply that

$$\Pi^1_{11}(\mathbf{x}) < 0 \quad \text{and} \quad \Pi^2_{22}(\mathbf{x}) < 0. \tag{A.3}$$

Therefore the solution to the maximization problem (cf. eq. 5) is unique. Since the payoff is continuous we can conclude that best response function $BR_i(x_j)$ is single-valued and continuous.

The strategy space $X \in \mathbb{R}^+$ is convex. Next, we show that the strategy space is also compact, i.e. there exists an upper bound on the strategy space. Define $\bar{x}_1 > 0$ such that

$$\Pi^1(\bar{x}_1, 0) = 0.$$

Thus, since $\Pi^1_2(\mathbf{x}) < 0$

$$\Pi^1(\bar{x}_1, x_2) > \Pi^1(x_1, x_2),$$

for any $x_1 > \bar{x}_1$ and for all $x_2 \in \mathbb{R}^+$. Therefore for all $x_1 > \bar{x}_1$, $\bar{x}_1$ strictly dominates x_1. Hence, after elimination of strictly dominated strategies the strategy space of player 1 becomes $[0, \bar{x}_1]$ which is a compact, convex and non-empty set. By symmetry the same argument can be applied to player 2.

A Nash-equilibrium satisfies the following equations:

$$\begin{aligned} BR_1(x_2) &= x_1, & \text{(A.4)} \\ BR_2(x_1) &= x_2. & \text{(A.5)} \end{aligned}$$

By substituting (A.5) into (A.4) we see that the NE is given by a fixed point of the composite function $\mathcal{BR}(x_1) \equiv BR_1 \circ BR_2 : \mathbb{R}^+ \to \mathbb{R}^+$, whereas the composite function $\mathcal{BR}(x_1)$ is a continuous mapping of a nonempty, convex and compact set into itself. Hence, the existence of a fixed point directly follows from Brouwer's Fixed Point Theorem.

A.1.2 Uniqueness of the Nash equilibrium

We now prove the uniqueness of the NE, i.e. we prove that $\mathcal{BR}(x_1)$ has a unique fixed point. Turning to the best response function, we know that

$$BR'_i(x_j) = -\frac{\Pi^i_{12}(\mathbf{x})}{\Pi^i_{ii}(\mathbf{x})}. \tag{A.6}$$

Because of (A.3), we know that the slope of player i's best response function is determined by the sign of $\Pi^i_{12}(\mathbf{x})$ (cf. eq. 8). Combining $\Pi^i_{12}(\mathbf{x})$ with the FOC of player i (cf. eq. 6) yields

$$\Pi^1_{12}\left(\mathbf{x}^N\right) = p\left(\mathbf{x}^N\right) V_{12} + \Omega\left(\mathbf{x}^N\right), \tag{A.7}$$

$$\Pi^2_{12}\left(\mathbf{x}^N\right) = (1 - p\left(\mathbf{x}^N\right))V_{12} - \Omega\left(\mathbf{x}^N\right), \tag{A.8}$$

with

$$\Omega\left(\mathbf{x}^N\right) \equiv \frac{p_1\left(\mathbf{x}^N\right)\, C^2_2(x_2)}{1 - p\left(\mathbf{x}^N\right)} + \frac{p_2\left(\mathbf{x}^N\right)\, C^1_1(x_1)}{p\left(\mathbf{x}^N\right)}, \tag{A.9}$$

which is obviously equal to zero if $C^i_i(x_i) = 0$. Now, we will split cases.

- **<u>Case 1</u>: Efforts are strategic complements (substitutes) for both players**
  We first explore the case where efforts are strategic complements (substitutes) for both players, i.e., we have $sign\left(\Pi^1_{12}\left(\mathbf{x}^N\right)\right) = sign\left(\Pi^2_{12}\left(\mathbf{x}^N\right)\right)$. In order to do this we will first show that in this case any NE is locally stable[27], i.e.,

$$\left|\frac{\Pi^1_{12}\left(\mathbf{x}^N\right)}{\Pi^1_{11}\left(\mathbf{x}^N\right)}\right| \left|\frac{\Pi^2_{12}\left(\mathbf{x}^N\right)}{\Pi^2_{22}\left(\mathbf{x}^N\right)}\right| < 1.$$

  Since $sign\left(\Pi^1_{12}\left(\mathbf{x}^N\right)\right) = sign\left(\Pi^2_{12}\left(\mathbf{x}^N\right)\right)$, we have

$$\begin{aligned}\left|\Pi^1_{12}\left(\mathbf{x}^N\right)\right| \left|\Pi^2_{12}\left(\mathbf{x}^N\right)\right| &= \Pi^1_{12}\left(\mathbf{x}\right) \Pi^2_{12}\left(\mathbf{x}^N\right) \\ &= \left(\Omega\left(\mathbf{x}^N\right) + p\left(\mathbf{x}^N\right) V_{12}\left(\mathbf{x}^N\right)\right)\left(-\Omega\left(\mathbf{x}^N\right) + \left(1 - p\left(\mathbf{x}^N\right)\right) V_{12}\left(\mathbf{x}^N\right)\right)\end{aligned}$$

  Implementing $\bar{p}\left(\mathbf{x}^N\right) = \max\left\{p\left(\mathbf{x}^N\right), 1 - p\left(\mathbf{x}^N\right)\right\}$ leads to

$$\begin{aligned}\Pi^1_{12}\left(\mathbf{x}^N\right)\, \Pi^2_{12}\left(\mathbf{x}^N\right) &\leq \left(\Omega\left(\mathbf{x}^N\right) + \bar{p}\left(\mathbf{x}^N\right) V_{12}\left(\mathbf{x}^N\right)\right)\left(-\Omega\left(\mathbf{x}^N\right) + \bar{p}\left(\mathbf{x}^N\right) V_{12}\left(\mathbf{x}^N\right)\right) \\ &= \left(\bar{p}\left(\mathbf{x}^N\right)\, V_{12}\left(\mathbf{x}^N\right)\right)^2 - \left(\Omega\left(\mathbf{x}^N\right)\right)^2 \\ &< \left(\bar{p}\left(\mathbf{x}^N\right)\, V_{12}\left(\mathbf{x}^N\right)\right)^2.\end{aligned}$$

  Using (A.1) and (A.2), and implementing $\underline{p}\left(\mathbf{x}^N\right) = \min\left\{p\left(\mathbf{x}^N\right), 1 - p\left(\mathbf{x}^N\right)\right\}$, we deduce

$$\Pi^1_{11}\left(\mathbf{x}^N\right) < p\left(\mathbf{x}^N\right) V_{11}\left(\mathbf{x}^N\right) \leq \underline{p}\left(\mathbf{x}^N\right)\, V_{11}\left(\mathbf{x}^N\right) < 0,$$

  and

$$\Pi^2_{22}\left(\mathbf{x}^N\right) < \left(1 - p\left(\mathbf{x}^N\right)\right) V_{22}\left(\mathbf{x}^N\right) \leq \underline{p}\left(\mathbf{x}^N\right)\, V_{22}\left(\mathbf{x}^N\right) < 0.$$

  Thus, combining the preceding inequalities and assumption (3) yields local stability of the NE, i.e.

$$\left|\frac{\Pi^1_{12}\left(\mathbf{x}^N\right)}{\Pi^1_{11}\left(\mathbf{x}^N\right)} \frac{\Pi^2_{12}\left(\mathbf{x}^N\right)}{\Pi^1_{22}\left(\mathbf{x}^N\right)}\right| = \frac{\Pi^1_{12}\left(\mathbf{x}^N\right)}{\Pi^1_{11}\left(\mathbf{x}^N\right)} \frac{\Pi^2_{12}\left(\mathbf{x}^N\right)}{\Pi^1_{22}\left(\mathbf{x}^N\right)} < \frac{\left(\bar{p}\left(\mathbf{x}^N\right)\, V_{12}\left(\mathbf{x}^N\right)\right)^2}{\left(\underline{p}\left(\mathbf{x}^N\right)\right)^2\, V_{11}\left(\mathbf{x}^N\right) V_{22}\left(\mathbf{x}^N\right)} \leq 1.$$

  Subsequently we will show that local stability implies uniqueness of the NE. Following the work of Skaperdas (1992, p. 737) we know that local stability rules out the existence of equilibria which are limit points of other equilibria. That is, there are finitely many equilibria which are isolated. Hence, if (x^N_1, x^N_2) is an equilibrium then there is an $\varepsilon \equiv \varepsilon(x^N_1, x^N_2)$ such

[27] See Vives (2001), p. 51 for a discussion on local stability.

that for all $x_i^* \in [x_i^N - \varepsilon, x_i^N + \varepsilon]$, with $i = 1, 2$, (x_1^*, x_2^*) is not an equilibrium. Since

$$\begin{aligned} \mathcal{BR}'(x_1^N) \equiv (BR_1 \circ BR_2)'(x_1^N) &= BR_1'\left(BR_2(x_1^N)\right) BR_2'(x_1^N) \\ \Leftrightarrow \quad \mathcal{BR}'(x_1^N) &= \frac{\Pi_{12}^1\left(\mathbf{x}^N\right)}{\Pi_{11}^1\left(\mathbf{x}^N\right)} \frac{\Pi_{12}^2\left(\mathbf{x}^N\right)}{\Pi_{22}^2\left(\mathbf{x}^N\right)} \\ \Leftrightarrow \quad 0 &< \mathcal{BR}'(x_1^N) < 1. \end{aligned} \tag{A.10}$$

- <u>Case 2</u>: **Efforts are strategic complements (substitutes) for player i and strategic substitutes (complements) for player j.**
  Next, we turn to the case where efforts are strategic complements for one player and strategic substitutes for the other player. In this case we can rule out a dense set of equilibria due to the fact that $sign(BR_1'(x_2)) \neq sign(BR_2'(x_1))$. Moreover, in this case

$$\begin{aligned} \mathcal{BR}'(x_1^N) &= \frac{\Pi_{12}^1\left(\mathbf{x}^N\right)}{\Pi_{11}^1\left(\mathbf{x}^N\right)} \frac{\Pi_{12}^2\left(\mathbf{x}^N\right)}{\Pi_{22}^2\left(\mathbf{x}^N\right)} \\ \Leftrightarrow \quad \mathcal{BR}'(x_1^N) &< 0. \end{aligned} \tag{A.11}$$

The final step is to show that in either case we can rule out the existence of another equilibrium. Suppose that (x_1^a, x_2^a) and (x_1^b, x_2^b) are two isolated equilibria, with $x_1^a < x_1^b$, and no equilibrium exists for $x_1 \in (x_1^a, x_1^b)$. Using (A.10) and (A.11) and starting from (x_1^a, x_2^a) leads to

$$x_1 > \mathcal{BR}(x_1) \quad \forall\, x_1 \in (x_1^a, x_1^b). \tag{A.12}$$

Consequently, starting from (x_1^b, x_2^b),

$$x_1 < \mathcal{BR}(x_1) \quad \forall\, x_1 \in (x_1^a, x_1^b), \tag{A.13}$$

which contradicts (A.12). Thus, there exists a unique NE.

□

A.2 Proof of lemma 2 (Comparison of the levels of effort I)

By definitions of the Stackelberg and the Nash equilibriums, we have

$$\Pi^i\left(x_i^L, x_j^F\left(x_i^L\right)\right) \geq \Pi^i\left(x_i^N, x_j^N\right). \tag{A.14}$$

The leader of the Stackelberg game always has a utility level superior or equal to the utility level obtained at the Nash equilibrium. The definition of the Nash equilibrium induces

$$\Pi^i\left(x_i^N, x_j^N\right) = \max_{x_i} \Pi^i\left(x_i, x_j^N\right) \geq \Pi^i\left(x_i^L, x_j^N\right).$$

Assuming that $x_j^N < x_j^F$ then involves

$$\Pi^i\left(x_i^N, x_j^N\right) \geq \Pi^i\left(x_i^L, x_j^N\right) > \Pi^i\left(x_i^L, x_j^F\right),$$

since efforts are plain substitutes $\left(\Pi_j^i\left(\mathbf{x}^N\right) < 0\right)$. But this contradicts the relation (A.14). Therefore, we deduce that

$$\Pi_j^i(\mathbf{x}) < 0 \Leftrightarrow x_j^N > x_j^F. \tag{A.15}$$

□

A.3 Proof of lemma 3 (Comparison of the levels of effort II)

Let the function $\Psi_i(x_i)$ be

$$\Psi_i(x_i) = \Pi_i^i\left(x_i, x_j^F(x_i)\right) + \Pi_j^i\left(x_i, x_j^F(x_i)\right)\frac{dx_j^F(x_i)}{dx_i}. \tag{A.16}$$

This function corresponds to the first derivative of the leader payoff function. For x_i^L, we obtain the FOC of the leader, that is $\Psi_i\left(x_i^L\right) = 0$. Since $\Psi_i'(x_i) < 0$ we find that the Stackelberg equilibrium exists and is unique. Next, we split cases.

- If $\Pi_{ij}^j\left(\mathbf{x}^N\right) > 0$ efforts are strategic complements for the player j at the Nash equilibrium. We deduce that $\frac{dx_j^F(x_i)}{dx_i} > 0$ at x_i^N, i.e., the best response function of player j is increasing at the Nash equilibrium. We thus have

$$\begin{aligned}\Psi_i\left(x_i^N\right) &= \Pi_i^i\left(x_i^N, x_j^F(x_i^N)\right) + \Pi_j^i\left(x_i^N, x_j^F(x_i^N)\right)\frac{dx_j^F(x_i)}{dx_i} \\ &= \Pi_j^i\left(x_i^N, x_j^F(x_i^N)\right)\frac{dx_j^F(x_i)}{dx_i} < 0 = \Psi_i\left(x_i^L\right),\end{aligned}$$

  since by definition $\Pi_i^i\left(x_i^N, x_j^F(x_i^N)\right) = 0$, $\Pi_j^i\left(\mathbf{x}^N\right) < 0$ and $\frac{dx_j^F(x_i)}{dx_i} > 0$. The decreasing of $\Psi_i\left(\mathbf{x}^N\right)$ in x_i involves

$$\Psi_i\left(x_i^N\right) < \Psi_i\left(x_i^L\right) \Leftrightarrow x_i^N > x_i^L. \tag{A.17}$$

- If $\Pi_{ij}^j\left(\mathbf{x}^N\right) < 0$, then we have $\frac{dx_j^F(x_i)}{dx_i} < 0$ at the Nash equilibrium, and consequently

$$\Psi_i\left(x_i^N\right) > 0 \Leftrightarrow x_i^N < x_i^L. \tag{A.18}$$

□

A.4 Proof of lemma 4 (First-mover advantage and second-mover incentive)

We consider two cases:

- If $\Pi_j^i\left(\mathbf{x}^N\right) < 0$ and $\Pi_{ij}^i\left(\mathbf{x}^N\right) > 0$, the rankings are: $x_i^F > x_i^N$ and $x_j^N > x_j^L$. We have

$$\begin{aligned}\Pi^i\left(x_i^F, x_j^L\right) &= \max_{x_i}\Pi^i\left(x_i, x_j^L\right) \geq \Pi^i\left(x_i^N, x_j^L\right) \\ &> \Pi^i\left(x_i^N, x_j^N\right),\end{aligned}$$

  where the first inequality results from the follower's maximization program, and the second from the fact that $x_j^L < x_j^N$ and $\Pi_j^i(\mathbf{x}) < 0$. Player i then has a second-mover incentive.

- If $\Pi_j^i\left(\mathbf{x}^N\right) < 0$ and $\Pi_{ij}^i\left(\mathbf{x}^N\right) < 0$, the ranking are: $x_i^F > x_i^N$ and $x_j^N < x_j^L$. We have

$$\begin{aligned}\Pi^i\left(x_i^L, x_j^F\right) &\geq \Pi^i\left(x_i^N, x_j^N\right) \\ &= \max_{x_i}\Pi^i\left(x_i, x_j^N\right) \geq \Pi^i\left(x_i^F, x_j^N\right) \\ &> \Pi^i\left(x_i^F, x_j^L\right),\end{aligned}$$

  where the first inequality results from (A.14), the second from the definition of the Nash maximization program, and the third from the fact that $x_j^N < x_j^L$ and $\Pi_j^i(\mathbf{x}) < 0$. Player i then has a first-mover advantage and a second-mover *dis*-incentive, since $\Pi^i\left(x_i^N, x_j^N\right) > \Pi^i\left(x_i^F, x_j^L\right)$.□

A.5 Proof of proposition 5 (SPE)

Each player has a first-mover incentive, i.e. $\Pi^i\left(x_i^L, x_j^F\right) > \Pi^i\left(x_i^N, x_j^N\right)$ (cf. eq. A.14). In order to determine the SPE, we will utilize our previous findings.

- If we have a game of SC ($\Pi_{ij}^i\left(\mathbf{x}^N\right) > 0$ for both players), then given lemma 4, both players have a second-mover and first-mover incentive. Consequently Γ^{S_i} and Γ^{S_j} are SPE of the extended game.
- If we have a game of SS ($\Pi_{ij}^i\left(\mathbf{x}^N\right) < 0$ for both players), then given lemma 4, both players have a first-mover advantage and a second-mover incentive. Consequently Γ^N is the unique SPE of the extended game.
- If we have a mixed game ($\Pi_{ij}^j\left(\mathbf{x}^N\right) > 0 > \Pi_{ij}^i\left(\mathbf{x}^N\right)$), we have from preceding computations

$$\begin{cases} \Pi^j\left(x_j^F, x_i^L\right) > \Pi^j\left(x_j^N, x_i^N\right) \\ \Pi^i\left(x_i^L, x_j^F\right) > \Pi^i\left(x_i^N, x_j^N\right) > \Pi^i\left(x_i^F, x_j^L\right). \end{cases}$$

  To move early is a dominant strategy for player i since he has a first-mover advantage and a second-mover *dis*-incentive. Player j has a second-mover advantage. Therefore, given the dominant strategy of player i, player j will choose to move late. Consequently Γ^{S_i} is the unique SPE of the extended game.