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Derivative Pricing and Hedging on Carbon Market

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Abstract- **The aim of this work is to bring an econometric approach upon the CO2 market. We identify the specificities of this market, and analyze the carbon as a commodity. We investigate the econometric particularities of CO2 prices behavior and their result of the calibration. We apprehend and explain the reasons of the non-Gaussian behavior of this market focusing mainly upon jump diffusions and generalized hyperbolic distributions. These models are used for pricing and hedging of carbon options. We estimate the pricing accuracy of each model and the capacity to provide an efficient dynamic hedging.**

Keywords- carbon; EUA; options; estimation; pricing; hedging; NIG; jumps.

I. INTRODUCTION

Human activities, in particular the population growth and the development of industry over the last 200 years, have caused an increase in the emission and atmospheric concentration of certain gases, called "greenhouse gases" (GHG) - primarily carbon dioxide and methane.

With its ratification by Russia in early 2005 the Kyoto protocol became an international law. According to its provisions, the industrialized countries have to reduce in the period 2008-2012 the greenhouse gas emissions by 5 percent with respect to the 1990 year levels. The protocol dictates the trading of emission allowances as one of the primary mechanisms through which greenhouse gas emission reduction should be achieved. Thus, the right to pollute is considered to be a tradable asset, with its price determined by the market forces of supply and demand.

In some recent works, few authors Benz [2], Daskalakis and Markellos [5],[6], Homburg [16] and Paolella [13] focused on the econometrical modeling of the GHG prices, underlying the particularities of this market like the non-Gaussian behavior, the auto-regressive phenomena and the presence of the convenience yield. In order to have an econometric view of the GHG market we calibrated European Union Allowances (EUAs) prices behavior using both Brownian and generalized hyperbolic models. We tested also the changing regimes hypothesis by integrating jumps in our diffusion models.

In this work we show some results concerning the economy of the carbon market from an analytical perspective, completely new for this market. Thus we will discuss:

- the econometric analysis of the carbon market; We show that generalizedhyperbolic family fit the best the carbon behavior,
- the theory of pricing under NIG distribution; We underline the importance of probability space change from the historical to risk neutral odds,
- the calibration of model for vanilla option pricing; We demonstrate that among few models benchmarked the NIG gives the best pricing,
- the dynamic hedging of carbon derivatives; We show the behavior ofdifferent dynamic delta hedging models.

II. CALIBRATION

We calibrated over the historical data for EUA prices (January 06-March 09) the following models:

- The geometric brownian motion

$$dS = S\mu\, dt + S\sigma\, dB, \tag{1}$$

- The geometric brownian motion with mean reversion

$$dS = S\alpha\,(ln(S)-\mu)\, dt + S\sigma\, dB, \tag{2}$$

- The geometric brownian motion with jumps

$$dS = S\mu dt + S\,\sigma\, dB \tag{3}$$

with probability $(1-p)$ and

$$dS = S(\mu + \mu_{saut})dt + S\,\sigma\, dB1 + S\,\sigma_{saut}\, dB2 \tag{4}$$

with probability p,

- The normal-inverse-gaussian process

$$dS = S\, dNIG(\alpha,\beta,\mu,\delta). \tag{5}$$

The results are summarized in Table I and they obviously lead to the conclusion that the NIG process is among the models estimated the best one for the dynamic of Log returns (the log likelihood is the greatest, the higher moments: Skewness and Kurtosis are the closest from those of the historical distribution). We also put the QQplot graphics in Fig. 1 for geometric brownian motion (left) and NIG (right) to underline the difference: the NIG process gives a fairly better estimation especially for huge variation values.

TABLE I. CALIBRATION RESULTS

	Parameters					LL	Skew	Kurt
GBM	μ	σ	-	-	-	1672	0	3
	-0.29	0.46						
	[-0.82 , 0.25]	[0.43 , 0.48]						
GBMMR	μ	σ	α	-	-	1673	0.011	2.99
	0.99	0.46	-0.009					
	[0.98 , 1.005]	[0.44 , 0.49]	[-0.01 , 0.03]					
GBMJ	μ	σ	μ_{saut}	σ_{saut}	*freq*	1775	-0.63	8.04
	0.121	0.28	-2.28	0.82	0.20			
	[-0.31 , 0.55]	[0.24 , 0.32]	[-4.92 , 0.35]	[0.66 , 0.99]	[0.09 0.29]			
NIG	α	β	μ	δ	-	1788	-0.41	9.35
	21.97	-2.42	0.0011	0.0182				
	[21.41 , 22.54]	[-2.74 , -2.09]	[0.0009 , 0.0013]	[0.018 ,0.0186]				
Historical data	-	-	-	-	-	-	-0.87	17.12

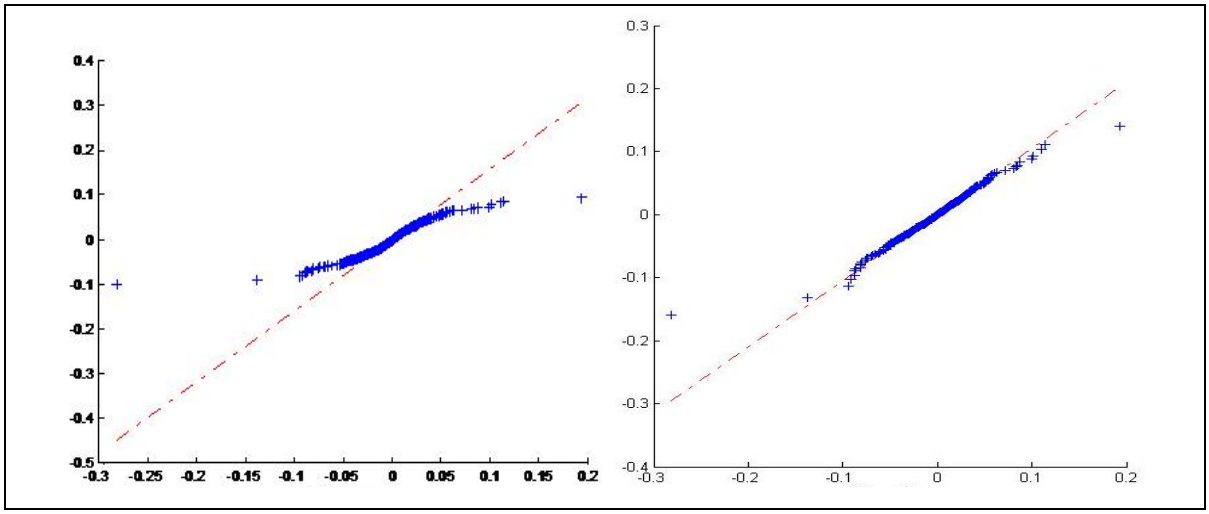

Figure 1. QQplot for geometric brownian motion (left) and NIG (right)

III. PRICING VANILLA OPTIONS

We could now price European options thanks to Monte Carlo methods. We compare two dynamics (the BM and the NIG). We have added a drift to get the risk neutral probability (Bu [18])

$$dX= (r- 0.5\sigma^2)dt+\sigma dB \quad (5)$$

$$dX=(r-\mu+\delta((\alpha^2-(\beta+1)^2)^{0.5}-(\alpha^2-\beta^2)^{0.5}))dt +dNIG \quad (6)$$

With the few market prices for options that appeared in 2009, we compute different error statistics:

$$ARPE = \frac{1}{N}\sum_i \frac{\left|C_{Theoretic}(T_i,K_i) - C_{Observed,i}\right|}{C_{Observed,i}}$$
$$RMSE = \sqrt{\sum_i \frac{\left(C_{Theoretic}(T_i,K_i) - C_{Observed,i}\right)^2}{N}} \quad (7)$$

TABLE II. PRICING ERROR RESULTS

	Brownian Motion	NIG
ARPE	0.5439	0.5235
RMSE	0.925	0.893

The error results shown in Table II on pricing are lower for the NIG process. This model gives better result for pricing.

IV. HEDGING

The NIG process gives better results for option pricing on carbon market, but while dealing with options it is also important to take a look at hedging. We consider a call option on EUA during the beginning of 2009 (January to mid-June) with strike 15 € and an initial spot value around 15 € . Here the results for the option price (Fig. 2) and delta (Fig.3) for the two models (geometric Brownian motion in green and exponential NIG process in blue).

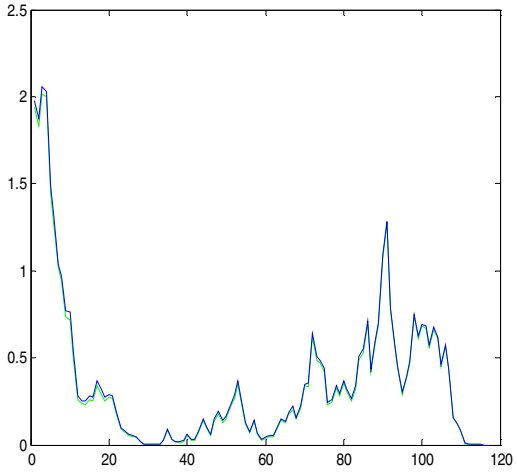

Figure 2. Option price evolution

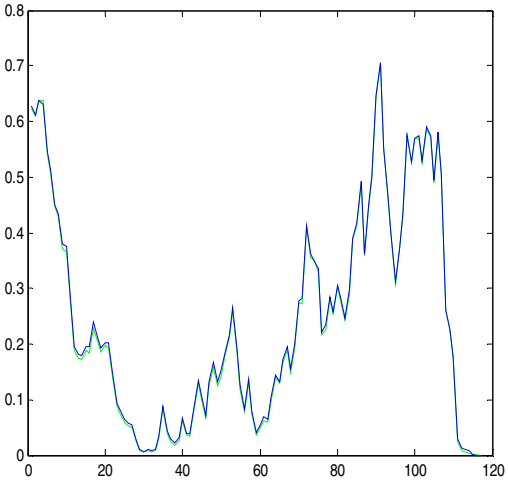

Figure 3. Option delta evolution

We assume that the seller of the option adopts a delta neutral hedging strategy and we compute the P&L of this option for our two models. And for a one million allowances position we get a result of -442 000 € for the geometric Brownian motion and -389 000 € for the exponential NIG. The results are negative because we took the estimated parameters (volatility) over the whole period instead of local implicit volatility in order to compare the results (NIG parameters are not local too). Once more, results are better for the NIG process.

V. CONCLUSIONS

We presented in this work some new results on the carbon market. We calibrated different models for the carbon price dynamic over the historical data and the generalized hyperbolic family (here the specific case of NIG process) lead to the best estimation even for the tail values. With a probability switch by adding a drift we got the risk neutral dynamic. Hence, we could price European options on CO2 with Monte Carlo methods. We compared the results with the market prices through the computation of error statistics and they support the NIG process that gives lower pricing errors than the Brownian motion. The hedging analysis of the two dynamics gives a best P&L for the NIG process even if the differences in deltas and prices are quite small (green and blue lines almost merged). All these results lead to consider the NIG process to be a really efficient way to model the EUA dynamic. Hence pricing derivatives with Monte Carlo methods on carbon market considering this dynamic is quite interesting.

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