Is there a link between fractal dimensions and other indicators of the built-up environment at a regional level
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Is there a link between fractal dimension and residential environment at a regional level?
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Is there a link between fractal dimension and residential environment at a regional level?

Introduction

It is well known that there is a great diversity of morphologies of built-up areas at a regional level, and that the patterns are influenced by a large variety of factors and processes such as natural site conditions, agricultural traditions, historical context, social development, technological context, accessibility as well as economic forces and/or land-use planning rules, which intertwine in a complex system (see e.g. Milne, 1991; McGarigal and Marks, 1995; Antrop, 1997; 2000; 2006; Antrop and Van Eetvelde, 2000; Allain, 2005). Built-up patterns also result from the evolution over time of these components. Historically the most important change imposed by humans on the landscape in a country like Belgium was the clearing of forest for agricultural purposes. Hence, villages were traditionally very compact. Subsequently, commerce, industrialisation and transportation, together with the growing population, have increased human’s need and ability to shape the environment according to our requirements. Belgian geographers traditionally define four types of urban level, mainly by density values and commuting patterns: urban centre, suburban fringe, rural countryside around the urban network, and remote rural countryside (Van der Haegen et al. 1996). At each of these levels the driving forces in terms of accessibility, urbanisation and globalisation are active in different ways (Antrop and Van Eetvelde, 2000).

Our aim here is to analyse the spatial arrangement of individual buildings (built-up patterns) and to see how they vary within the region being studied. We are also interested in how far the form of the built environment corresponds to the history of urbanisation and to residential tastes. We suspect that common life styles, planning rules, architectural choices, etc. may lead to similar built-up patterns. More specifically, the paper aims at testing two main hypotheses:

- Fractal dimensions allow a synthetic description of the built environment of each commune. By considering this hypothesis, we ask questions about the specificity of the fractal indices: are the computed fractal indices unique, or are they highly correlated to other measures commonly used in urban geography and economics for describing the built environment at this level of analysis? Four variables are considered, each of them throwing light on an aspect of the built environment of each commune: population density, distance to the closest CBD (commercial and business district), type of housing, and the date of construction.

- Fractal dimensions can be used to evaluate the quality of the built environment of each commune. More precisely, we hypothesise that the multi-scale organisation of a built-up area reveals the good qualities of that environment from a functional (not an aesthetic) point of view. This hypothesis stems from the argument developed by Cavailhès et al. (2004), which suggests that a set of fractal forms allows the optimisation of the accessibility of various amenities, including green and rural amenities as well as urban amenities (retail and services). Even if not all fractal forms allow optimal access to all types of amenities, it seems realistic to suppose that the multiscale organisation of a pattern favours good access to various amenities, which can be used as an indicator of a good built environment from a functional point of view.

In order to test this hypothesis, we explore the statistical relationships between three selected fractal indices and two socio-economic indicators of the quality of the built environment: land rent (on the assumption that the higher the quality of the built environment, the higher the
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The built environment can also be considered from an aesthetic point of view. In each commune, the evaluation of the quality of the built environment from a functional perspective is consistent with the mean evaluation of the aesthetic quality of the residential environment of each building. There are two points of view on the question of landscape evaluation: one considers the landscape as it is seen (and thus perceived) from each building in a studied area (see e.g. Paterson and Boyle, 2002; Cavailhès and Joly 2006); the other considers landscape quality as a global characteristic of an area (e.g. Geoghegan et al., 1997; Kestens et al., 2004). Our goal here is to find an argument that could help to link these two points of view together.

In order to position this paper in a more general scientific context, two points should be considered. First, this paper is related to research in geography, urban economy and urban planning, that aim to link urban forms and other socio-economic variables such as social equity, quality of life, and access to facilities, jobs and green spaces. This field of research has been explored especially in the context of the debate about the compact city. To give only two examples, statistical relationships between form indicators and social/environmental variables can be found in Burton (2000), and the relationship between form and the perception of their environment by individuals is explored in Garcia and Riera (2003). Second, this paper represents a contribution to the use of spatial metrics for the analysis of urban environments, as developed for example by Alberti & Waddell (2000), Parker & Meretsky (2004) and Herold et al. (2002). This paper hopefully takes one more step towards the demise of the use of spatial units that are composite from a morphological point of view.

The data used in this paper only covers the southern part of Belgium (Wallonia) and is different from that used in companion papers. Thomas et al. (2007b) studied the southern periphery of Brussels (Nuts3 region) with a particular concentration on the effect of the size and shape of the units as well as the meaning of fractality compared to density. A first, very exploratory, attempt at linking fractal dimension to socio-economic variables was made by De Keersmaecker et al. (2004). Another paper by Thomas et al. (2008) studies the same set of Walloon communes as those analysed in the present paper, but it focuses on a new technique for clustering fractal dimensions. Empirically, it shows that fractal indices partition the region into six sub-areas that do not correspond to the “natural landscapes”; no other variables are considered. These earlier papers help to orient the empirical and methodological choices which inform this article.

This paper is organised as follows. Section 2 gives a step-by-step explanation of the method used; variables are selected and justified, and the area being studied is roughly described. Section 3 gives the results in terms of exploratory data analysis, bivariate and multivariate analysis. Section 4 concludes the paper.

**Data and methodology**

**Built-up areas**

Empirical analyses are conducted here on the southern part of Belgium, which corresponds to a Nuts1 region: Wallonia. It covers 16,845 sq. km., has 3.4 million inhabitants and is administratively divided into 262 communes (or townships). Each commune is made up of several villages: before 1977, these villages formed communes, which were then merged into larger administrative units. Hence, the built-up area of a commune often consists, nowadays, of a juxtaposition of several independent morphological sets, which has to be taken into account when interpreting the results of the analyses. We here decided to work at the communal level mainly because (1) in periurban areas sprawl has made the former villages to merge, and (2) some data are not available by former villages (land rent, etc). We are aware that the choice of the scale of analysis influences the results and that the size and shape of the communes vary.
The PLI (Plan de Localisation Informatique) developed by MRW-DGATLP (2004) was used to compute fractality; it is based on the digitised topographical maps at a scale of 1:10000 (IGN, Institut Géographique National) and the land register (cadastre). This database is updated annually and is available by commune. The built-up areas correspond to a vectorial layer of information where buildings are individually identifiable. It was rasterised in order to obtain a binary image in which each built-up pixel represents a 21 by 21 metres zone that is fully or partially occupied by a building or part of a building. No information is provided about the function of the pixels (residence, public service, industry, service or farm), the height of the buildings, their architectural characteristics or their occupants (i.e. the number of inhabitants or the number of jobs located within a pixel).

Socio-economic data

As well as the morphological data, census data were collected for each commune (Data produced by the I.N.S., Institut National de Statistique). The choice of census data was based on models commonly used in spatial analysis (geography, economics) as well as on the hypotheses discussed in Section 1. Appendix 1 gives the main statistical descriptors of these variables for the area being studied. Hence, the values of the fractal dimensions will be compared to:

- Population density (here called Density) is expressed as the number of inhabitants divided by the total area of the commune. It has already been demonstrated that fractal dimension is not synonymous with density (Batty and Xie, 1996; Thomas et al., 2007b, 2008), but we expect the correlation to be positive. This hypothesis stems from two observations: on the one hand, high densities are often synonymous of quite regular built-up morphologies; on the other hand, density is often pretty high in the some communes of Wallonia. Previous research has already discussed the subtle link between fractal dimension and density (Thomas et al., 2007b; Burton, 2000). Density will also here be considered as a control variable: we analyse the fractal dimension for some ranges of density values.

- It is common knowledge in urban analysis that the further we go from the CBD, the larger the residential plots become and the lower the density. History has led to very dense and homogeneous city centres where no or very few plots of land are still available for construction, and to sprawling peripheries (see e.g. Burton, 2000; Allain, 2005). Urban sprawl is characterised by a loss of multiscale characteristics of the built-up areas at a local analysis level. But, at a more global analysis level, the peri-urban built-up patterns in Wallonia remain very contrasted. Hence, we expect that the higher the distance from a city-centre, the lower the value of $D_1$. Heterogeneity will increase with distance from the CBD. Distance is here measured as the straight-line distance between the centroid of each commune and its closest CBD. The road network is so tight that a straight-line distances is a sustainable assumption (Vandenbulcke et al., 2007).

- Some characteristics of the morphology of dwellings are also available in the 2001 Population and Housing Census. The type of housing is of major interest for us. For each commune we computed: the percentage of households living in an apartment (Apartments), in terraced houses (attached on both sides, called Terraced), in detached houses (Detached), and in semi-detached houses (attached on one side, Semis). A priori we expect that a large percentage of terraced housing corresponds to a ribbon-like morphology, while a large percentage of fully detached houses leads to a diversity of morphologies depending on the land-use planning rules or fashion. Indeed, detached houses can be the result of strong planning rules and hence homogeneous plots, or, on the contrary, to loose planning rules and hence quite heterogeneous housing styles.

- It also seems logical to assume that the morphology of the buildings will depend upon the year of development/peri-urbanisation (i.e. on the history of urbanisation). Hence, census information on the date of construction of the buildings was used. Percentages are computed
without taking missing values into account (Thomas and Vanneste, 2007). \(H_{1919}\) represents the percentage of dwellings built before 1919, \(H_{1946-1970}\) the percentage built between 1946 and 1970, \(H_{1971-1980}\) the percentage built between 1971 and 1980, \(H_{1981-1990}\) the percentage built between 1981 and 1990, and \(H_{1991-2000}\) the percentage of dwellings built between 1991 and 2000. We expected recent construction to characterise new suburbs, and hence more heterogeneous built-up areas. High percentages of old buildings are often typical of historical centres (Vanneste et al., 2007).

From a socio-economic perspective, we also tested whether people pay more to live in a more heterogeneous built environments (see Cavailhès and Joly, 2006; Burton, 2000). The multi-scale aspect of residential environment has an impact on land prices. We first considered the price of a square meter of land (\(\text{Landprice}\)), which was computed using the Stadim data. These data are privately owned and do not include the highest and lowest quintile; the average value of the remaining three quintiles was computed for three years (1999, 2000 and 2001). Land is limited to “building land” that was sold within that period. Another variable used for evaluating land prices was the average price of plots of land between 360 and 720 sq. m large that were sold during the same period of time (\(\text{PricePlot}\)) (same data source). Average house prices are difficult to consider, because we have no information on the size and quality of the house that was sold: the average value can include very small houses on a large plots, as well as luxury mansions on small plots. We know that in Belgium, land prices and house rentals are not well correlated (Thomas and Vanneste, 2007): land prices depend on the pressure on land, while rents are dominated by Brussels which is the largest city. Hence, we here also consider the percentage of dwellings with a monthly rent greater than 750 euros, whatever the type of housing (\(\text{Highrental}\)) (Source: 2001 Census). Last but not least, the household’s median income (\(\text{Income}\)) was also considered. This is a measure of welfare, but also a proxy of “human capital” (Thissse and Thomas, 2007). In Belgium rich people prefer to live in communes where built-up and other areas are multi-scaled.

Last but not least, one of the original features of the 2001 Census, is that one question covered the perception of the environment. Each head of household had to judge his/her environment as pleasant, decent or unpleasant in several respects. Here we only consider the aesthetic of the buildings around the house and created the variable “percentage of households that were not pleased with their built environment” (\(\text{Unpleasant}\)). We wanted to test the quality of life, the happiness of the inhabitants with their neighbouring built environment. This taps into the interaction between nature/landscape, cultural history and inhabitants. The scenic quality is considered as an important characteristic (Antrop, 2006; Burton, 2000). This is perhaps the most interesting/original part of the analysis. The underlying question is the relationship between the assessment of the environment by the survey, and by the fractal indices. How far do they reveal the same trend?

**Methodology**

In this section we explain some general fractal concepts practically and intuitively, and show how they fit with our objectives. For extensive mathematical formulations, we refer the reader to Mandelbrot (1982), Batty and Longley (1994), Frankhauser (1994; 1998), Lam and de Cola (2002) and Mattila (1995).

**Computing fractal indices**

Let us take the example of a commune located in the southern part of Wallonia:Arlon, 119 square kilometres and 26 000 inhabitants. The built-up area in Figure 1 (top) shows quite an agglomerated city in the centre of the image, surrounded by several star-shaped villages with urbanisation along the transportation axes.
Figure 1: Arlon: Original built-up areas (top), dilated built-up areas (bottom left) and borders (bottom right)

Source: Plan d’Information Informatique (P.I.I.), 2002

Three fractal dimensions can be computed: the fractal dimension of the built-up areas (denoted $D_{Surf-dil0}$), the fractal dimension of the dilated built-up areas ($D_{Surf-dil3}$), and the fractal dimension of the border of the dilated built-up areas ($D_{Bord-dil3}$). From experience we know that the combination of these three indices yields a good characterisation of the built-up form of each commune (see Thomas et al., 2008). All the dimensions were computed by means of correlation analysis: each pixel belonging to the built-up area is surrounded by a small square window of size $\epsilon$. The number of built-up pixels within each window is enumerated. This allows the mean number of pair correlations $N(\epsilon)$ per window of that size to be computed. The operation is repeated for windows of increasing size $\epsilon$. This results in a series of points that can be represented on a Cartesian graph where the X-axis represents the size of the window $\epsilon = (2i+1)$, where $i$ is the iteration step, and the Y-axis represents the mean number of points counted per window.
The next step consists of fitting this empirical curve to a theoretical curve that corresponds to a fractal law, i.e. a power law which links the number of correlations to the size of the window (Figure 2):

\[ N = \varepsilon^D \] \[1\]

The exponent \( D \) is the fractal dimension, or, in this case, the correlation dimension.

However, real world patterns may not strictly follow a fractal law. Hence it is useful to introduce a generalised fractal law, which contains to two additional parameters:

\[ N = \alpha \varepsilon^D + c \] \[2\]

where \( \alpha \) summarises different kinds of deviations of the fractal law (Gouyet, 1996; Frankhauser, 1998; Thomas et al., 2007a; Thomas et al., 2007b). In order to ensure a correct estimation, it is necessary to introduce the constant \( c \) (Frankhauser, 1998). Parameter \( c \) of the estimated function corresponds to the point of origin on the \( Y \)-axis. Parameter \( \alpha \) is called the "pre-factor of shape". It gives a synthetic indication of local deviations from the estimated fractal law. For a mathematical fractal structure, \( \alpha \) should be equal to 1, in accordance with [1]. Experience shows that when \( \alpha \) goes above 4 or below 0.1, the fractality of the studied pattern is not confirmed (see Thomas et al., 2007a). The value of \( \alpha \) estimated for Arlon is 1.39. We have already demonstrated that the built-up pattern of each commune of Wallonia

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X-axis = size of the analysis window (number of pixels)

Y-axis = mean number of occupied pixels per window

The empirical curve is in blue and the estimated curve is in red.

**Figure 2: Screen print of the DSurf-dill0 estimation by Fractalyse**
consists of a juxtaposition of several independent morphological sets (i.e. the former villages) (Thomas et al., 2007a). This clearly influences the value of the prefactor $a$, but does not affect the fractality of each morphological set, or the value of the fractal dimension $D$.

A non-linear regression was used to estimate the parameters $a$, $D$ and $c$ that best fit the empirical curve. All fractal analyses and estimations were carried out by using the software Fractalyse 2.12 (Vuidel et al., 2002). The fractal dimension of a built-up area ($D_{\text{surf}}$) can take any value between 0 and 2. When $D_{\text{surf}}$ is equal to 2, the built-up pattern is quite uniformly distributed following a one-scale logic. $D_{\text{surf}} = 0$ corresponds to a limiting case in which the pattern is made up of a single point (e.g. a single farm building surrounded by fields). $D_{\text{surf}} < 1$ corresponds to a pattern made up of unconnected elements (a high number of built-up clusters separated one from another). $D_{\text{surf}} > 1$ corresponds to a mix of connected elements forming large clusters, connected elements forming small clusters, and isolated elements. The nearer $D_{\text{surf}}$ is to 2, the more the elements are connected to each other and belong to a single large cluster. For Arlon, $D_{\text{surf-dil0}}$ (dimension of the undilated built-up pattern) takes the value of 1.59, which characterises a fairly well connected multi-scale pattern with numerous free spaces of different sizes, some of which are very large.

The quality of the estimation of the fractal dimension is controlled by using a correlation coefficient $R$. We often consider that an estimation is acceptable when $R$ exceeds 0.9999. If the fit between the two curves (empirical and estimated) is poor, we can either conclude that the pattern being studied is not fractal, or that it is multi-fractal (Tannier and Pumain, 2005). Here we here only consider built-up patterns with a good fit between the empirical and the estimated curves.

**Computing fractal indices on dilated realities**

Let us now consider the dilated built-up area ($D_{\text{surf-dil3}}$) of Arlon (Figure 1). The principle of dilation consists of surrounding each occupied point (pixel) by a black border, the size of which increases at each iteration step. We decided to measure the fractal dimension after three dilations. Hence all the details referring to distances of less than 63 m. were obscured. This means that the smallest free spaces separating each building, which mainly correspond to the road network and the gardens of houses, disappear (Figure 1). In other words, after 3 dilation steps, the pattern is fully connected in a fractal manner and the fractal dimension of the dilated built-up area ($D_{\text{surf-dil3}}$) should always be higher than that of the undilated area; $D_{\text{surf-dil3}}$ should also always be greater than 1. Previous experience has shown that the fractal dimension of a pattern can vary a lot in the first few steps of the dilation. In the current research, we wanted to determine whether these large variations in the fractal dimension resulted from the transition from one fractal organisation to another, or from a non-fractal organisation of the smallest free spaces separating each building. If $D_{\text{surf-dil3}}$ differs substantially from $D_{\text{surf-dil0}}$, it means that the way of combining the smallest free spaces with the surrounding built-up pixels follows a fractal organisation very different from the spatial organisation of the built-up pattern at larger scales. On the contrary, if $D_{\text{surf-dil3}}$ is not very different from $D_{\text{surf-dil0}}$, it means that the smallest free spaces do not follow a multi-scale logic: they only influence the value of $D_{\text{surf-dil0}}$ slightly by introducing noise into the measure. For Arlon, $D_{\text{surf-dil3}}$ equals 1.71. Hence by dilation, the pattern becomes more uniform. We conclude that the spatial distribution of the houses is rather irregular on a scale of less than 63 m., i.e. the interstitial space between houses varies. However, this effect disappears for $D_{\text{surf-dil3}}$.

The third (and last) step of the morphological characterisation of the built-up pattern of Arlon is to measure the fractal dimension of its dilated borders ($D_{\text{bord-dil3}}$). We decided to work on the dilated borders in order to eliminate the smallest free spaces, whose spatial organisation is not well known and tends to be rather irregular. Thus, we only consider the connected part of the built-up pattern and avoid any possible measurement artefacts that could arise from the
is there a link between fractal dimension and residential environment at a regional level

Is there a link between fractal dimension and residential environment at a regional level? This value is close to that of $D_{\text{surf-dil3}}$. This corresponds to the rather dendrite-like aspect of many Walloon communes, where built-up areas follow the street network. For more compact patterns $D_{\text{surf-dil3}}$ tends to be higher than $D_{\text{bord-dil3}}$.

Each commune is analysed separately. Because the size of the communes varies, the size of the window of analysis used for calculating the fractal dimensions also varies. This may lead to slight differences in the absolute values of the fractal dimensions, but does not affect the general conclusions of the present research. Each window is centred on the administrative centre of the corresponding commune. It is worth noting that $D$ does not vary significantly with the choice of the centre of the window (see Thomas et al., 2007b for further comments).

Results

The results of our research are organised as follows: Section 3.1 gives a general description of the results, Section 3.2 comments the bivariate relationships between $D$ and the variables described in Section 2.2 and, finally, Section 3.3 describes a multivariate analysis conducted to explore whether $D$ is complementary to or repetitive of more traditional socio-economic variables.

Exploratory data analysis

For the entire set of Walloon communes ($n=262$), the fractal dimension of the built-up areas $D_{\text{surf}}$ is larger than that of the borders ($D_{\text{bord}}$), and the observed variation is also larger for $D_{\text{surf}}$ (see Appendix 1). Hence $D$ is far from revealing a compact structure of the buildings (compactness would yield $D_{\text{surf}} \approx 2$ and $D_{\text{bord}} \approx 1$); the history of the urbanisation of Wallonia has led to a large variety of built-up morphologies (see Thomas et al., 2008).

Figure 3: Frequency distribution of $D_{\text{surf-dil0}}$, $D_{\text{bord-dil3}}$ and $D_{\text{surf-dil3}}$ for the 262 communes of Wallonia

Note: $D$ values obtained by Fractalyse are here analysed by means of SAS JMP 6.0.2

Some Walloon fractal dimensions on undilated areas ($D_{\text{surf-dil0}}$) are below 1.0 (Figure 3); these values correspond to structures close to Fourier dusts, i.e. small dispersed villages or isolated hamlets in sparsely populated areas. The significance of these small values is low compared to other measurements ($R < 0.9999$). Note that for slightly dilated areas, the fractal dimension is — as expected — always larger than 1.0 for areas ($D_{\text{surf-dil3}}$) as well as for the corresponding borders ($D_{\text{bord-dil3}}$). Let us here remember that values of $D_{\text{surf-dil0}}$ close to 2.0 correspond to perfect homogeneity, and that $D_{\text{surf-dil0}}$ values equal to 1.0 correspond to street/ribbon-villages. As expected, the 262 values of $D_{\text{surf-dil0}}$ are highly correlated to those of $D_{\text{bord-dil3}}$ (Pearson correlation coefficient $r = 0.94$). This high correlation is the reason why $D_{\text{bord-dil3}}$ was later dropped from our bivariate analyses.
$D_{\text{Surf-dil3}}$ and $D_{\text{Bord-dil3}}$ are positively and significantly, but not perfectly, correlated ($r = 0.81$) (Figure 4). They do not reflect exactly the same reality: the morphology of the border is different from that of the area, even though not independent. The less the two values differ, the more the urban pattern will be tentacular and sprawling\(^9\). On the contrary high values of $D_{\text{Surf}}$ and values of $D_{\text{Bord}}$ close to 1.0 correspond to compact patterns with smooth borders\(^10\). Hence, both $D_{\text{Surf}}$ and $D_{\text{Bord}}$ are involved in characterising the built-up morphology. This is also true for $D_{\text{Surf-dil0}}$ and $D_{\text{Bord-dil3}}$ ($r = 0.64$). If we restrict ourselves to $D_{\text{Surf-dil0}}$ values greater than 1.0 (due to significance levels) then the two correlation coefficients given above become respectively 0.73 ($D_{\text{Surf-dil3}}$ and $D_{\text{Bord-dil3}}$) and 0.51 ($D_{\text{Surf-dil0}}$ and $D_{\text{Bord-dil3}}$). (see Thomas et al. (2008) and Figure 7 (left) below for further comments on the maps).

**Figure 4: DBord-dil3 versus DSurf-dil3**

Let us simply recall here that the map clearly reveals the urbanisation process: $D_{\text{Surf-dil0}}$ is high in the cities along the 19\(^{th}\) century Sambre–Meuse industrial axis but also in some communes on to the southern edge of Brussels, in the northern part of the region. In these areas, the built-up area is more uniformly distributed than in other areas. Indeed in the former industrialised communes (Charleroi, Liège, etc.), we can assume that the initially empty interstitial sites have been progressively filled up with time; this also holds for the areas in the southern part of Brussels where peri-urbanisation is very significant. The southern part of Wallonia (Province of Luxembourg) is less urbanised, and two types of morphology coexist in accordance with more traditional landscape analyses (Dussart, 1961; Feltz, 2004): compact ($D_{\text{Surf-dil0}} = 1.23$ to 1.39) and elongated villages ($D_{\text{Surf-dil0}} = 1.01$ to 1.23). In the elongated villages, tentacular concentrations of buildings along transportation axes lead to patterns with a high degree of contrast: almost empty corridors exist between the built-up branches. Finally, values of $D_{\text{Surf-dil0}}$ smaller than 1.0 are observed in some communes mainly composed of isolated settlements (villages, hamlets and large farms) that are highly dispersed and consist of detached splashes rather than large clusters; in this respect they resemble Fournier dusts.

It seems likely that the shape of the built-up areas within the Walloon communes is the result not only of physical characteristics (soil, relief, initial vegetation, afforestation, etc.), but also of human activities and land use (suburbanisation, etc.). Hence the older and more densely populated communes are more uniformly built up and have high values of $D_{\text{Surf}}$. 

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Medium values are observed in communes with more tentacular patterns, which were more recently affected by peri-urbanisation. More rural areas are dominated by small isolated “splashes” ($D_{\text{surf}} < 1.0$). This corresponds to the different stages of peri-urbanisation (see Thomas et al., 2008, for further details).

### Bivariate relationships

**Morphology and density**

Theoretical expectations suggest a strong difference between $D$ and density (see e.g. Batty and Xie, 1996 or Thomas et al., 2007b). Empirically, if fractal dimension was synonymous with density, the Pearson correlation between these two variables would be close to 1.0. In fact the correlations between population density ($\text{Density}$) and the various measures of $D$ are $0.63$ ($D_{\text{surf-dil0}}$), $0.58$ ($D_{\text{surf-dil3}}$) and $0.28$ ($D_{\text{surf-dil3}}$) (see Table 1). These coefficients are almost the same if the analysis is restricted to communes with a population density of less than 1 000 inhabitants/sq. km ($n = 243$) or those with a population density below 305 inhabitants/sq. km. This kind of restriction limits the variation in density and hence produces a more homogeneous sub-sample. With the exception of $D_{\text{surf-dil1}}$ no substantial difference is observed between the sub-samples.

![Table 1: Pearson correlation coefficient between $D$ and population density (log)](image)

<table>
<thead>
<tr>
<th></th>
<th>$N$</th>
<th>$D_{\text{surf-dil0}}$</th>
<th>$D_{\text{surf-dil3}}$</th>
<th>$D_{\text{surf-dil3}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All</strong></td>
<td>262</td>
<td>0.626</td>
<td>0.584</td>
<td>0.275</td>
</tr>
<tr>
<td>$\text{Density} &lt; 1000$ inhab/sq km</td>
<td>243</td>
<td>0.682</td>
<td>0.627</td>
<td>0.412</td>
</tr>
<tr>
<td>$\text{Density} &lt; 305^*$ inhab/sq km</td>
<td>197</td>
<td>0.691</td>
<td>0.614</td>
<td>0.516</td>
</tr>
</tbody>
</table>

$^*$ = third quartiles

Whatever the subset of communes used when considering density, the same kind of results is obtained: as density increases, built-up areas become more homogeneous (less irregular). The positive correlation coefficients indicate that the higher the population density, the more numerous are the built-up elements (i.e. the pixels) and the more they are connected to each other following a multiscale logic. This confirms the results previously obtained by Batty and Kim (1992), Longley and Mesev (2002) and, for a subset of Wallonia, by Thomas et al., 2007b. It is possible that the imperfect correlation between population density and the fractal dimension of built-up area is explained by the fact that $D_{\text{surf-dil0}}$ is computed on a two dimensional image, while for high densities, the third dimension (the height of the buildings) should be taken into consideration. This supposition has yet to be verified. Let us however emphasise that uniform patterns may also correspond to low density housing. Indeed the dimension of a ward consisting of individual houses with the same lot size would be close to two. This is confirmed in Table 3 (below), with the percentage of households living in apartments that is positively related to $D$.

The fractal dimension of dilated borders ($D_{\text{surf-dil3}}$) is less highly correlated with the population density than the fractal dimension of built-up areas ($D_{\text{surf-dil0}}$). Theoretically, a given population density can correspond to several types of spatial patterns. This is not captured by the fractal dimension of the built-up area ($D_{\text{surf-dil0}}$), but appears through the fractal dimension of its dilated border ($D_{\text{surf-dil3}}$). Hence, a very hierarchical pattern with free spaces of different sizes, characterised by a ($D_{\text{surf-dil3}}$) value slightly above 1, can correspond either to a fairly high...
population density or to a lower density population. On the contrary, a uniform built-up pattern characterised by a high degree of space filling and thus a value of $D_{\text{Bord-di3}}$ close to 2 always corresponds to a high population density. However, the administrative division into communes disrupt this regularity in Wallonia (see Thomas et al., 2007b for a discussion).

Let us here add that, as expected, when the distance between a commune and the closest city increases ($Distance$), the fractal dimension decreases significantly: large values of $D$ are found within cities or very close to them. It is well-known that in Belgium, diversity is to be found and is appreciated by households in peri-urban and more rural areas ($r$ between $Distance$ and $D_{\text{Surf-di0}} = 0.53$, $D_{\text{Surf-di3}} = 0.47$ and $D_{\text{Surf-di3}} = 0.34$).

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Last but not least, we here limit ourselves to simple correlation between 2 variables and scatterplots. Some readers might have expected in depth regression analyses and residuals. At this stage of our knowledge, we here prefer to be careful about the use of metrics and statistical tests pertaining to fractals. Hence, we do not speak about “regression” nor about “explanatory variables”.

**Morphology and history**

One way of measuring the historical aspects of morphology is to consider the percentage of buildings constructed during a certain period of time, as given by the 2001 Census (Table 2). Note that, for the sake of clarity, the results associated with $D_{\text{Surf-di3}}$ are not reported here, as $D_{\text{Surf-di0}}$ and $D_{\text{Surf-di3}}$ are highly correlated over the 262 communes ($r = 0.93$) and Table 1 clearly illustrated the similarity of the results with the two measures.

![Table 2: Pearson correlation coefficient between $D$ and the age of the dwellings (% constructed in each period of time)](

<table>
<thead>
<tr>
<th>Year</th>
<th>$D_{\text{Surf-di0}}$</th>
<th>$D_{\text{Bord-di3}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hbef1919</td>
<td>-0.494</td>
<td>-0.344</td>
</tr>
<tr>
<td>H1919-1945</td>
<td>0.328</td>
<td>n.s.</td>
</tr>
<tr>
<td>H1946-1970</td>
<td>0.402</td>
<td>0.239</td>
</tr>
<tr>
<td>H1970-1980</td>
<td>n.s.</td>
<td>n.s.</td>
</tr>
<tr>
<td>H1981-1990</td>
<td>n.s.</td>
<td>n.s.</td>
</tr>
<tr>
<td>H1991-2001</td>
<td>n.s.</td>
<td>n.s.</td>
</tr>
</tbody>
</table>

**Table 2: Pearson correlation coefficient between $D$ and the age of the dwellings (% constructed in each period of time)**

Table 2 reveals that there are no significant relationships between the percentage of dwellings built after 1970 and $D$. This means that, after 1970, (peri-)urbanisation in Wallonia went in all directions; this confirms the results found by Vanneste et al. (2007) and De Keersmaecker et al. (2002). For earlier dates, the relationships are significant but with different signs. Between 1919 and 1970, the relationships are positive: $D$ increases as the proportion of housings built between 1919 and 1970 rises. These were periods of reconstruction after the two World Wars, and mainly characterise cities (Thomas et al., 2008; Vanneste et al., 2007) and hence more homogeneous built-up areas. However the correlation between the percentage of housings built before 1919 and $D$ is negative and significant: high percentages of this type of dwelling characterise deprived rural areas where heterogeneity is larger.

Hence, the observations made on Wallonia confirm our hypotheses: each period of construction has its own style and planning rules. $D$ helps us to measure the differences.
Fractal dimension and type of building

If $D$ measures the morphology of the housing stock, then it should be related to the very few variables pertaining to morphology which are available in the census. On average, housing in Belgium is quite diversified (Thomas et al., 2008; Vanneste et al., 2007): detached houses sprawl out of the cities and the peri-urbanisation; terraced houses characterise urban agglomerations and ribbon villages. Housing estates (in French: “lotissement”) are quite diverse (in some areas they are homogeneous, in others not). Peri-urbanisation in Belgium takes a variety of different forms, and these are captured by the $D$ values (Vanneste et al., 2007).

Table 3 shows that when the percentage of detached houses is high (on the periphery of cities), fractal dimensions are low ($r < 0$): it is well known that in Wallonia (and more generally in Belgium), the suburbs are heterogeneous and characterised by detached villas with gardens and varying plot size. However, as Figure 5 (below) suggests, this general trend is nuanced by the fact that there is considerable variation in $D$ values for a given value of Detached and vice versa. This means there is a variety of built-up areas and planning conditions across the communes.
The percentage of semi-detached houses is difficult to interpret (no significant relationships); this structure is rarer in Belgium and is observed all over the country (Vanneste et al., 2007). High percentages of terraced houses are related to high values of $D$: these characterise dense villages and urban communes (homogeneity of the built-up areas). Hence in Wallonia, $D$ increases when the percentage of detached houses decreases or when the percentage of terraced houses increases. This trend is similar for both borders and areas, but the correlation coefficients are higher for fractal dimensions measured on areas. This implies that there is a strong centre-periphery structure: at this scale of analysis, diversity is smaller within highly urbanised communes. However, the coefficients are below 0.5, suggesting that this relationship does not always hold.

**Morphology and rents**

Table 4 shows that $D$ is high where average land prices are high (positive $r$). It is well known in urban geography, that CBDs are more expensive than suburbs, and that they correspond to very densely built-up areas (see e.g. Burton, 2000; Allain, 2005). Land prices are particularly high in the northern part of Wallonia, due to the closeness of Brussels, and in the regional city-centres (Thomas and Vanneste, 2007). At this meso-scale of analysis city centres are quite homogeneous and land is expensive because it is rare (a plot is hard to find and even a very small one is expensive, often due to fixed taxes/costs).
Table 4: Pearson correlation coefficients between $D$ and the price of land

In Belgium, rents have a different spatial structure from land prices (see Thomas and Vanneste, 2007): Brussels dominates the market, and the quality of housing plays a determining role in the rent. Table 4 shows that there is no significant relationship between rents and the value of $D$.

**Morphology and perception of the habitat**

Special attention is now put on the link between $D$ and the perception of the built environment by households. If $D$ measures the morphology of the built-up areas, we expect some spatial structures to be better perceived than others. For instance, high values of $D_{\text{Bord-dil3}}$ correspond to tentacular borders that can be associated with rather good accessibility of the edge of the urban area, and this should correspond to good access to green or natural areas.

In our data set, high values of $D$ are associated with environments that are perceived as unpleasant or merely satisfactory, and negatively associated with those perceived as pleasant (Table 5); hence, homogeneity of built-up areas is not considered pleasant. Note that the relationship is again stronger for $D_{\text{surf}}$ than for $D_{\text{Bord}}$: the morphology of the border is less important than that of the area. This may be partly due to the administrative border of the communes/towns not always corresponding to the functional/morphological border (see full discussion in Thomas et al., 2007).

All coefficients are significant at the 0.001 level except those marked * (which are significant between the 0.001 and 0.020 levels)

Table 5: Pearson correlation coefficients between $D$ and perceptions of the immediate surroundings of the accommodation

These correlation coefficients are illustrated by a scatter plot (Figure 6) and two maps (Figure 7). Communes where substantial proportions of households are dissatisfied with their immediate environment frequently have $D$ values close to 2: households do not like uniformity! Communes with large percentages of very satisfied households have a high variety of $D$ values (from 0.5 to 1.8). There is no simple relationship between residential satisfaction (taste) and built-up morphology in Wallonia; on the contrary there is a considerable diversity of housing environments and tastes.

The variation in %Unpleasant is larger than that in the values of $D$: when $D$ is close to 2 (homogeneity), more variation is observed in %Unpleasant because it covers different types of communes (Figure 7). These include cities and deprived areas with former heavy industries (such as Charleroi), but $D$ can also be high in peri-urban communes with a lot of housing estates (“lotissements”), and in rich suburban communes where people are happier with their environment. On the other hand, low values of %Unpleasant correspond to small $D$ values, with little variation. This result is quite interesting in terms of land-use planning.
Before drawing some conclusions, let us here try to summarise our bivariate results statistically by conducting factor analyses. This will enable us to detect resemblances in our preceding descriptions and to see to what extent the $D$-values reflect the more classical socio-economic variables used in this paper. Several principal component analyses were conducted on different sets of variables; two are reported here. A Varimax rotation was applied to detect the resemblances better. Table 6 refers to the analysis of all 262 communes, and Table 7 to the communes with a population density of less than 305 inhabitants per sq. km. (the first three quartiles). Both tables give the loadings for the components extracted under the condition that the eigenvalues are greater than 1; loadings of more than 0.4 are reported here. Quite interestingly, whatever the variables incorporated in the analysis, the same types of results appear (analyses not reported here).

The first component ($C_1$) refers to density: it corresponds to high population densities, a high proportion of terraced houses, low income (remember that in most Belgian cities, on average low incomes characterise the urban centres and high incomes the suburbs/peripheries (see e.g. Thomas and Zenou, 1999)). Component 2 ($C_2$) refers to the structure of income. High positive loadings correspond to high incomes and to houses built in the second phase of urbanisation (1970-1980). Hence $C_2$ is typical of the peri-urban communes, where people are generally satisfied with their environment: in Belgium, people like “green environments”. The third component ($C_3$) can be called age of construction as it characterises variables related to the age of the buildings. The fourth and last component ($C_4$) refers to real-estate: high prices for a plot of land, high rentals, and large average incomes.
Is there a link between fractal dimension and residential environment at a regional level?

Table 6: Principal component loadings (after Varimax rotation) for all communes

loadings higher that 0.4 are here reported; in bold : loading higher that 0.7

The fractal dimensions ($D_{\text{surf-dil0}}$, $D_{\text{bord-dil3}}$) appear together on the first component ($C_1$). Fractal dimension measures something different to the other variables. It comes closest to the type of houses (Detached) and the population density (Density). Hence, this first analysis confirms a strong centre-periphery structure in terms of density and distance to CBD, and the associated characteristics of the housing (type, rent, period of construction). This result led us to perform a similar analysis “controlling for Density”, that is to say limiting the variation in population density in order to better isolate the morphology of the built up area (Table 7).

Table 7: Principal component loadings (after Varimax rotation) for communes with a density of less than 305 inhabitants per sq. km. (first three quartiles)

loadings higher that 0.4 are here reported; in bold : loading higher that 0.7

The results are presented in Table 7 and confirm the preceding ones: the fractal dimension parameters only appear on one component ($C_1$). Two other variables (Density and Distance) also appear on $C_1$, but both of these also load on $C_4$. All the other variables appear on $C_2$ (small percentage of recently built housings, small proportion of detached houses, people unhappy with their environment), $C_3$ (age of the buildings) and $C_4$ (price and income). Fractal dimension gives complementary information; in Wallonia, density and distance to the nearest city are the variables that bear the greatest resemblance to $D$. When considering communes...
with homogeneous and lower densities (Table 7). Density and Distance to CBD appear on 2 components instead of 1; hey now also are related to Price and Income, which was not the case when including densely inhabited and urbanised communes (Table 6), giving a better image of the periurban Walloon realities: it is common knowledge the in Belgium, suburbs are better off (Dujardin et al., 2008; Thomas and Vanneste, 2007). Both analyses also put forward the importance of history, and more particularly the period of suburbanisation ($C3$).

**Conclusion**

This paper complements an earlier paper analysing the spatial distribution of $D$ values within Wallonia (Thomas et al., 2008) in which the reader can find interesting complementary descriptive analysis about the fractal dimensions. It explores the relationship between fractal dimension and some more classical socio-economic variables. On a methodological point of view, the analyses presented in this paper can be seen as an attempt to gauge three measures of fractal dimension through their confrontation with a series of more commonly used socio-economical indicators.

Fractal measures unequivocally characterise the spatial organisation of urban patterns. They can be used to measure the extent to which built-up areas are distributed in a uniform or spasmodic way. The fractal investigation of the borders of built-up areas has shown that it is possible to describe quantitatively how smooth or how dendritic they are. Applying the fractal approach to different kinds of urban patterns at various scales helps to link their morphology to the historical context in which they emerged and to establish relations between the values of the indices and certain planning concepts, indicating the practical intentions of urban planning. It should be emphasised that the same type of spatial organisation may correspond to strongly planned areas of town, but may also be found in less planned areas. Hence, these measures are suitable for characterising the form of urban patterns for planning purposes and allow more subtle orientations of urban development to be defined. This paper has shown how far $D$ values differ from other, more classical, socio-economic descriptors of the population of a commune or a built-up landscape.

When interpreting the results it becomes obvious that a fractal approach to urban patterns helps us to improve our knowledge of their spatial organisation, regardless of how planned they were. Obviously multi-scale pattern organisation is an interesting way to manage the consequences of new lifestyles, which tend to demand good access to both urban and rural amenities, and at the same time help to reduce the risks of a diffuse sprawl which tends to lessen the quality of the environment and to generate more and more traffic flows. A more detailed analysis, differentiating the types of land-use (housing, services, street network etc.) could help to increase our knowledge of the mix and proximity of different types of activities in an urban context. But unfortunately, the data are lacking.

To our first question (see Section 1) “do fractal dimensions allow a synthetic description of the built environment of each commune?” we can definitively give a positive answer. The analysis performed at a regional level completes and confirms former micro-level analyses on cities. The second hypothesis was that “fractal dimensions can be used to evaluate the quality of the built environment of each commune”. It was shown in this paper that people tend to appreciate morphological (fractal) diversity, defined as the existence of empty areas of different sizes. In terms of peri-urbanisation, this result really sets the cat among the pigeons. In Wallonia people still appreciate residential locations in green areas. Hence, a lot has still to be done if planners want to promote a compact city (Burton, 2000). Last but not least, we are aware that further in-depth fractal analyses have still to be conducted on wards and villages in order to better isolate some local environmental and planning effects.
Bibliographie


Dujardin C., Selod H., Thomas I. 2008. Unemployment and urban structure for young adults. The case of Brussels. Urban Studies , 45(1), 89-113


Annexe

Appendix 1

The main statistical descriptors of the census variables for Wallonia

| Source of most variables: Institut National de Statistique, Enquête Socio-Economique 2001 |

| Source of most variables: Institut National de Statistique, Enquête Socio-Economique 2001 |

Notes

1 See Section 2.3.1 below for the definition of D.

2 CBDs are the centres of the most important Belgian cities (see Querriau et al., 2005).

3 D is often estimated by using a double logarithmic representation of the power law. We preferred to minimise the least square deviations by means of a non-linear regression, since this avoids any implicit assumptions about local deviations from fractal law.

4 A cluster is defined as a set of built-up elements connected in a fractal manner. From a fractal point of view, elements are connected when either they are contiguous (i.e. directly connected by at least one point), or they are separated by free spaces whose spatial organisation follows a one-scale logic.

5 Connectivity is here always considered from a fractal point of view.

6 Experience showed that very high correlation values are usually observed. However it turns out that local deviations from the power law often do not affect the correlation value very much. This led us to consider only results with a very high quality of adjustment as valid.

7 If necessary, in order to ensure a good estimation of the fractal dimension, it is possible to restrict the range of values for the analysis. The scaling behaviour curve is often used for this (see Frankhauser, 2004). However this procedure was not performed here in order to ensure the comparability of the communes.

8 If Dsurf = DBord, the patterns correspond to Sierpinski carpets; if Dsurf > DBord they are tarragons.

9 This situation corresponds to a Sierpinski carpet.

10 Such forms are close to Euclidean objects such as circles or squares.
Pour citer cet article

Référence électronique

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Abstract / Résumé

Fractal dimension is used to measure the spatial arrangement (morphology) of built-up areas within a Nuts1 region (Wallonia, Belgium) and, more particularly, to test to what extent fractal dimension is related to some variables commonly used in urban economics/urban geography to characterise built environment, housings and residential choice (such as land price, housing rentals, history of urbanisation, type of housing). Special attention is put on the link with the perception of the built environment by households. A multivariate analysis concludes the paper. It is shown that fractal indices differ from other indicators and are very useful for characterising and understanding built landscapes, as well as for modelling and planning urban realities.

Keywords : Belgium, fractal dimension, built-up patterns, socio-economic indicators, landscape quality, Wallonia

Existe-t-il un lien entre dimension fractale et environnement résidentiel à l'échelle régionale ?

La dimension fractale est utilisée pour mesurer la répartition spatiale (morphologie) des espaces bâtis au sein d'une région Nuts1 (Wallonie, Belgique). Plus particulièrement, on cherche à tester l’intensité de la relation entre la dimension fractale et d'autres variables habituellement utilisées en économie et géographie urbaines, pour caractériser le choix résidentiel, les logements et l'espace bâti en général (prix foncier, rente foncière, histoire de l'urbanisation, types de logements...). Une attention particulière est portée sur la relation entre dimension fractale et perception, par les ménages, de la qualité de l'environnement bâti. L'article se termine avec une analyse multivariée qui montre que les indicateurs fractals utilisés diffèrent d'autres indicateurs et sont utiles pour caractériser et comprendre les paysages bâtis urbains, ainsi que pour modéliser et aménager les espaces urbains.

Mots clés : Belgique, dimension fractale, tissus bâtis, indicateurs socio-économiques, qualité paysagère, Wallonie