Toward waste management contracts

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Abstract

This paper deals with the cost of treatment of the ultimate waste, that is waste which cannot, in the absence of recycling opportunities, be reduced by a suitable taxation scheme. We propose a new way to handle this waste based on a Waste Management Contracts (WMC) which largely implicates the households in the cost reduction process. Within a set of feasible, i.e. budget balancing, incentive compatible and acceptable, contracts we characterize the optimal WMC and compare this system to a more standard one based on an Advanced and a Disposal Fee

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1 Introduction

For the last few years, environmentalists and policy makers have focused on increasing attention to the question of waste management (see for instance Jenkins [11], Dinan [5] or Fullerton and Kinnanan [7]) and it is now largely recognized that consumption generates an increasing amount of garbage, the handling of which induces a growing social cost. From that point of view, it becomes obvious that the households, acting as Citizens, should participate in the waste management programs or at least have enough incentives to do so. Their contributions can be twice : they can try to reduce the amount of waste induced by her consumption and, perhaps more generally, furnish a targeted effort which reduces the waste treatment cost.

Most of the literature, in the best of our knowledge, essentially focus on the reduction of the amount of waste. These policies for green design often combine upstream and downstream taxation schemes in order to modify the design of the product or to encourage recycling (see for instance Fullerton and Wu [9], Walls and Palmer [17], Calcott and Walls [1], [2]). Even if these policies are more or less efficient according to the commodity under consideration (see Palmer and al [14] or Jenkins and al [12]), it is however widely acknowledged that a *two-part taxation* mechanism based on a generalized deposit-refund system (see Palmer and Walls [13] or Fullerton and Wolverton [10]) is able to implement a social optimal waste reduction policy. But the transmission of these incentives rely on the existence of markets for recyclable, and on the ability to use theses goods in a reversed production system. So, even these potentials exist, it is quite difficult to assume that no residual waste remains or that its collection and destruction is free of cost.

Our paper essentially deals with this ultimate waste and our object is to design a contract which largely implicates the households in the reduction of its treatment cost. One well-known option consists in implementing a per bag pricing policy. This pecular contract provides some incentives to reduce the amount of waste, hence it global treatement cost. In fact, the agency in charge of the waste destruction decentralizes a part of the its activity at the consumer level because their effort reduces the amount of waste and, in compensation, implictely transfert to these agents a amount of money measured by the reduction in their global waste treatment charge.

It is nevertheless largely acknowledged that this effort is not directly observable and that the proxy which consists in measuring the amount of waste induces illegal dumping (see Fullerton and Kinnanan [7], [8]). This drawback gives rise to inefficiency and leads to second best policies since the implementation of a Disposal Fee (DF for short) policy requires a costly incentive scheme in order to prevent midnight dumping (see Choe and Fraser[3])[1]. This is why par bag pricing policy are often based on a monitoring technology associated to a *two-part tariff*, that is a DF which prevents illegal dumping and an Advanced DF (ADF for short) which is directly includes (by a tax) in the price of the goods such that both the monitoring and the waste treatment costs are covered[2].

Shinkuma[15] even argues that in the presence of both illegal dumping and a transaction cost associated to a recycling subsidy, this *two-part tariff* can challenge a deposit-refund system, i.e. a *two-part taxation*, especially if the marginal transaction cost is high enough. To be more precise, if the waste reduction effort conduct to recycling, it may happen that a recycling subsidy net of the transaction cost induces a lower level of effort than an incentive compatible *two-part tariff*.

But our objectif is not to introduce a new mechanism which challenges a *two-part taxation* system. We favour complementarity since the first attempt to reduce the amount of waste by encouraging reprocessing while the second should motivate the households to provide a specific effort in order to reduce the cost of treatment of the ultimate waste. This is typically not the case with a *two-part tariff* policy since the DF induces some incentives to reduce the total amount of waste including the recyclable. One instrument, the per bag pricing policy, targets therfore two objectives : recycling and ultimate waste treatment cost. But this also imply that the household's effort should be more targeted. He should at least provide two kinds of efforts : one motivated by standard market mechanisms which induces recycling and the second being rewarded by the social gains obtained by the reduction of the ultimate waste treatment cost.

Our propostion consist in adding targeted contract. This gives the opportunity to the households to freely act for the presevation of environment by participating to the waste treatment reduction cost and to obtain, if they accept the contract, a financial compensation. This seems, *a priori*, rather abstract. But a policy maker can easily identify some specific activities belonging to the waste pretreatment operation before destruction and centralize these processes at the household's level. It includes, for instance, sorting by using different bins, the destruction of a certain kind of material, composting and so on. The idea is simply to give to the waste producers, the opportunity to realize some pretreatment activities or, if not, to pay for it. This is, for instance, the motivation of the new regulation which came into force in the UK in October

[1] It is even argued by Choe and Fraser [4] that the introduction of a specific household effort reduces the flexibility of a deposit and refund policy and leads to a second best.

[2] For the more general class of durable goods, Shinkuma [16] even argues that an ADF policy is less efficient than a DF one. In this case, an ADF policy reduces the incentives to repair the commodity and depresses the second hand market. This induces an excessive consumption and a larger disposal cost in respect to the social optimum.

2007 (for more detail the reader is referred to *Treatment of non-hazardous waste for landfill : your waste-your responsibility* Environment Agency UK [6]).

The mechanism beyond this contract is quite simple. The households pre-pay the waste treatment cost as in a standard full ADF system and are not charged for their waste disposal. But they have the opportunity to sign a contract which specifies an activity reducing the waste treatment cost, for instance the reception of several bins associated to a commitment to sort, and which stipulates a monetary compensation for this job. Households may however cheat by taking the money and by making no effort at all. This is why we also introduce a monitoring mechanism in order to depiste this offending behavior But this one is quite different from the one associated to a DF system. It does not penalize illegal dumping, but an infringement to the contract. The control, therefore, only bear on the people who accept the proposal and not on all the population ,like in a DF system.

In this paper we however do not want to enter the specific nature of these waste management contracts (WMC for short). The only thing which is important from the point of view of the policy maker is that it reduces the treatment cost of the ultimate waste in some proportion. This is why we assume that such a contract specifies (i) a proportion of cost reduction to reach (ii) a payment in compensation and (iii) a probability of being controlled associated to a fine.

We however restrict the set of all contract in the following way. We first require that the public agency in charge of the waste destruction works under an *ex post budget balancing constraint,* i.e. the effective waste treatment cost and the monitoring cost must be covered by the ADF net of the payment to the contractors and perhaps augmented of the collected fine. This mean that we do not want to levy a specific (lump sum) tax in order to cover the loss of this agency. Secondly, we do not want to consider contracts that nobody wants to sign. This is why we also introduce a *participation constraint* which states that at least some agents are ready to accept it without cheating. Finally, and since the efforts provided by the households are not observable, we also restrict our attention to WMC which are enforceable, i.e. satisfying an *incentive compatible constraint.*

These contracts are implemented in a model constructed in the line of Choe and Fraser [3]. But we depart from this one in several respect. We first assume that the policy maker has no opportunity to modify the waste content of a good by taxing the producers. By doing so, we only deal with ultimate waste produced by the consumers. In counter-part we consider heterogeneous agents with respect to their willingness to pay for the commodity and their ability to provide an effort. These characteristics are distributed over a continum of agent but no peculiar assumptions are imposed on this distribution.

In this setting we characterize the set of WMC which satisfies the budget, the participation and the incentive compatible constraints. In fact we show that we can impose restrictions on the required waste treatment rate and on the probability of control which are equivalent to the previous conditions. This characterization gives us the opportunity to study, in a second step, the welfare properties of these contracts. In this second best world, we essentially show that, within this set of constraints, a public waste cleaning agency always has an incentive to push the required waste treatment rate as high as possible and to lower the probability of control in order to limit the monitoring cost. The first result is essentially linked to the entrance of "poorer and environment friendlier" consumers into the market. In fact a higher waste treatment rate requires a higher subsidy, this typically gives the opportunity to poorer consumers to buy the goods as long as they are not to much affect by the required effort. The second effect works in the opposite way : a higher rate of control strengthens the budget constraint and induces a lower subsidy. These preliminary results allow us in a third step to characterize an optimal WMC from a welfare maximisation point of view. We show that A optimal WMC always contributes to the reduction of the waste management cost independently from the monitoring cost and that the optimal policy follows from an arbitrage between the welfare gain relative to a rise in the required waste treatment rate and the increasing monitoring costs due to its implementation . The optimal policy is also related to the average cost of the effort in the population. We finally compare our WMC's to an *two-part tariff policy* consisting in setting both an ADF and a DF. We essentially show that a WMC is more efficient especially when the total amount of fees is not smaller then the waste treatment cost.

The paper proceeds as follows : the next section depicts our basic assumptions and describes a WMC. In section 3, we present the restrictions on the set of these contracts imposed by the incentive, the participation and the budget balancing constraints. In section 4, we associate to each feasible contract its level of welfare and give some basic properties of this function. Section 5 is devoted to construction of the optimal contract. In section 6 we compare a WMC to an two part tariff based on a advanced and a disposal fee. Finally, the last section contains some concluding remarks. The proofs of the different results are relegated to an appendix.

2 The basic assumptions and the WMC

We consider a commodity produced by a representative firm and sold to a continuum of consumers. Consumption produces ultimate waste, i.e. which cannot be reduced by any kind of market mechanism like a deposit-refund system nor recycled even partially. For simplicity we assume that one unit of consumption generates one unit of waste. Its destruction is not free.

We denote by c the cost of the destruction of one unit of waste.

Since we mainly focus on the consumer behavior, we also largely simplify the behavior of the representative firm by assuming that (i) the commodity is sold on a competitive market, (ii) there is no way to reduce the intrinsic waste content at the production level and (iii) the unit production cost is zero. This means, in other words, that the competitive price p reflects the part of the waste treatment cost which is prepaid by the households and which, in our WMC, coincides to the unit treatment cost c.

2.1 The demand side

We introduce a continuum of heterogeneous consumers[3] who decide to buy or not the good, i.e. $x \in \{0,1\}$ and suffer from the intensity of the effort $e \in [0,1]$ dedicated to the reduction of the waste treatment cost. They share the same utility function $u(x,e,\mu) = \alpha x - \theta e - \mu$ where μ denotes the monetary spending. They are nevrtheless heterogeneous in respect to their willingness to pay $\alpha \in [0,A]$ and to their marginal cost $\theta \in [0,\Theta]$ of the waste treatment effort. However, in order to make sure that at least one agent is able to consume when the waste management cost is prepaid, we assume that $A > c$, and, in the same vein, we say that $\Theta > c$ otherwise all consumers are willing to provide an effort.

The distribution of these two characteristics across the population is summarized by a probability distribution over $[0,A] \times [0,\Theta]$ whose cumulated distribution function (c.d.f. for short) is denoted $F(\alpha,\theta)$. This one is assumed to be absolutely continuous with a strictly positive density $f(\alpha,\theta) := \partial^2_{\alpha,\theta} F(\alpha,\theta) > 0$. Moreover we denote by $f(\alpha,\cdot) := \int_0^\Theta f(\alpha,\theta)d\theta$ the marginal density of α and by $f(a/\theta) := \frac{f(\alpha,\theta)}{f(\cdot,\theta)}$ the conditional density of α given θ. A symmetric interpretation holds for $f(\cdot,\theta)$ and $f(\theta/\alpha)$.

Now remember that the effort contributes to the reduction of the waste treatment cost. We measure the outcome of this activity by the proportion $r(e)$ by which the unit waste treatment cost is reduced[4]. We however assume that there is an upper bound $\bar{r} \leq 1$ to this proportion and that this relation is linear, i.e. $r(e) = \bar{r} \cdot e$ for $e \in [0,1]$. The largest waste management cost reduction is obtained when the intensity of the effort is maximal. From that point of view we

[3] The selection of a discrete choice model can perhaps seems restrictive. But in most of the literature, the authors essentially choose a representative agent economy with continuum of choices. We have the conviction that our approach is as general as this one since no restrictions are put on the distribution of the characteristics. From that point of view, we even not transfer to the aggregated behavior the properties related to the optimization problem of a single agent.

[4] This assumption particulary fits well in our discrete choice model since a consumer buys at most one unit and therefore induces a waste treatment cost of at least c. Otherwise one should take into account the amount of good consumed.

can say that $e(r) = \frac{r}{\bar{r}}$ denotes the level of effort required to reduce the waste management cost in a proportion of $r \in [0, \bar{r}]$.

2.2 The Waste Management Contract

The mechanism beyond the WMC is the following. We first implement a full ADF system in the sense that the waste treatment cost is included in the price of the commodity and the waste disposal is free. But we give to each household the opportunity to sign a contract which contributes to the reduction of the waste treatment cost. If she accepts, she has to provide some effort in order to reach an assigned target, is randomly controlled and receives a reward. For instance, she can accept the delivering of several bins, commits herself to sort and being reward proportionally to the amount of sorted waste. This contract therefore tranfers some costly waste treatment and/or destruction activities from the agency in charge of this task to the consumers and gives the opportunity to this agency to motivate the household by the gain obtained from the reduction of the global waste treatment cost.

The simplest way to introduce such a contract in our model is to assume that the waste treatment agency chooses as target a proportion $r \in [0, \bar{r}]$ of cost reduction per unit of treated waste. She then specifies, within a contract, a set of task which induces this cost cut, and proposes a payment s per unit of transformed waste. We even assume for simplicity that the household can only transform her own waste[5].

A monitoring system is however required in order to make sure that the terms of the contract are fulfilled or more generally if the cost reduction target r is achieved. So let us denote by π the probability that a consumer who accepts the contract is controlled and let us introduce a cost $m(\pi)$ per realized control. We assume that this cost is increasing and convex (i.e. $m'(\pi) > 0$ and $m"(\pi) > 0$) and that the absence of monitoring is free (i.e. $m(0) = 0$) while perfect motoring is very expensive (i.e. $m(1) > c$). It remains to define the fine f paid by the offenders. This one cannot be too disproportionate to the fault but nevertheless not too low. This is why we decide to set the fine at the waste treatment cost, i.e. $f = c$ since in our discrete choice model each consumer only produces one unit of waste.

To summarize, we say that a WMC is described by a triple (r, s, π) given by a cost reduction target, a subsidy, and a probability of control. Moreover it is important to notice that this probability of control only applies to the households who accept the contract.

[5]In a more general setting, one can even imagine that the environmental friendly consumers, i.e. with a low θ, want to transform the waste of their neighborhoods who do not take the contract or even that these contracts induces a market for waste.

2.3 The choice of a consumer

In our discrete choice model, if a consumer of type (α, θ) buys nothing, her utility is nought. If she purchases the good, she can refuse (utility indexed by 0) the waste reduction contract and stay in a standard ADF. In this case, she pre-pays the cost c of waste disposal and has no incentive to make an effort. Under our zero marginal production cost assumption, she pays $p = c$ for the good and her utility is given by :

$$u_0^{(\alpha,\theta)} = \alpha - c$$

If she accepts the contract, she obtains a subsidy of s, but she always has the opportunity either to execute or not (indexed by e or $\bar{e}$) the terms of the contract. In the first case she makes the required effort and delivers the transformed waste. Her utility is therefore given by :

$$u_e^{(\alpha,\theta)}(r, s) = (\alpha - c) + s - \theta \frac{r}{\bar{r}}$$

Otherwise, she makes no effort but takes the risk of being caught with probability π and of being charged of a fine of c. We even assume that there is no cost to mask her infringement[6] contrary to the DF litterature which introduces an illegal dumping cost. She therefore obtains :

$$u_{\bar{e}}^{(\alpha,\theta)}(s, \pi) = (\alpha - c) + s - \pi c$$

From that point of view, the best strategy of a consumer of type (α, θ) is the one which gives her the highest payoff, i.e. which satisfies

$$\max \left\{ 0, u_0^{(\alpha,\theta)}, u_e^{(\alpha,\theta)}(r, s), u_{\bar{e}}^{(\alpha,\theta)}(s, \pi) \right\}$$

3 The set of feasible contracts

The main purpose of this section is to construct the contracts which can be proposed by the policy maker and which have the property that (i) all agents who accept the contract have enough incentives to realize the required cost reduction target (ii) at least some agents are willing to participate in the program, and (iii) the waste treatment agency covers both the waste management and the monitoring costs.

[6] It is always possible to introduce such a cost into the model but this does not really change the results. It gives to the waste treatment agency the opportunity to increase the subsidy, to require a higher effort and/or to reduce the probability of control without breaking the incentive constraint.

The first condition can be defined quite easily. We simply require that cheating is for everybody the worsts choice.

Definition 1 *The Incentive Constraint (IC) is satisfied iff*

$$\forall (\alpha,\theta) \in [0,A] \times [0,\Theta], \qquad max\left\{u_e^{(\alpha,\theta)}(r,s), u_0^{(\alpha,\theta)}, 0\right\} \geq u_{\bar{e}}^{(\alpha,\theta)}(s,\pi) \tag{IC}$$

In order to define the participation constraint, we need to define what we means by "some agents" accept the contract. Since we work with a continuum of agents, we simply require that there must be a subset E of agents with non zero measure which accepts the contract and perform the effort. So if $P(E)$ denotes the proportion of these households, we say that :

Definition 2 *The Participation Constraint (PC) is verified iff*
$\exists E \subset [0,A] \times [0,\Theta]$ *,and* $P(E) > 0$ *such that :*

$$\forall(\alpha,\theta) \in E, \quad u_e^{(\alpha,\theta)}(r,s) = \max\left\{0, u_0^{(\alpha,\theta)}, u_e^{(\alpha,\theta)}(r,s), u_{\bar{e}}^{(\alpha,\theta)}(r,\pi)\right\} \tag{PC}$$

The construction of the budget constraint requires some additional notations. So let us denote by $P(0)$ and $P(\bar{E})$ the proportion of households buying the good who respectively refuse the contract, and cheat[7]. Moreover let us remember that in our discrete choice model each consumer only produces one unit of waste, it follows that the advance disposal fees collected per unit of waste is described by $\left[(P(\bar{E}) + P(E) + P(0)) \cdot c\right]$ while the waste treatment cost is given by $\left[(P(\bar{E}) + P(0)) \cdot c + P(E) \cdot (1-r) \cdot c\right]$ since a proportion $P(\bar{E})$ of the households do not respect the contract. The subsidies paid to the agents are nevertheless of $\left[(P(\bar{E}) + P(E)) \cdot s\right]$. Concerning the monitoring activity, the controls at a rate of π only apply to the population of contractor and costs therefore $\left[\pi \cdot (P(\bar{E}) + P(E)) \cdot m(\pi)\right]$ but they generate fines which render $\left[\pi \cdot P(\bar{E}) \cdot c\right]$. It follows that the budget constraint is given by :

$$P(E) \cdot (s - r \cdot c + \pi \cdot m(\pi)) + P(\bar{E}) \cdot (s - \pi \cdot c + \pi \cdot m(\pi)) \leq 0$$

But, we can even go a step further. Let us remember that the subsidy s acts, for each consumer, like a discount on the price of the good. From that point of view and whatever π and r are, any waste treatment agency which seeks for a contract which maximize the total surplus always exhaust this constraint. This is why we can say :

[7] These probabilities of course depend of all the parameters of the model. We omit them in order to keep the notations rather simple.

Definition 3 *The Balancing Budget Constraint (BBC) is satisfied iff*

$$P(E)\,(s - r \cdot c + \pi \cdot m(\pi)) + P(\bar{E})\,(s - \pi \cdot c + \pi \cdot m(\pi)) = 0 \tag{BBC}$$

It becomes now important to identify the set of contracts which satisfies these three conditions. If IC and PC are verified, we knows respectively that $P(E) > 0$ and $P(\bar{E}) = 0$, this means under BBC that the subsidy $s = r \cdot c - \pi \cdot m(\pi)$. We can even go a step further. At least from an intuitive point of view we can guess that if this subsidy is negative, surely nobody wants to participate to the waste management program. On the other hand, if this one is too high, especially if it is higher than the expected cost of the fine, i.e. $s > \pi \cdot c$, the households would have an incentive accept the contract and to cheat. This clearly imposes an upper an a lower bound on the subsidy. We can even say that the lowez bound must be strictly positive unless $r = \pi = 0$, otherwise it would be impossible to find an open set[8] E on which PC is satisfied. This is why we can say that :

Lemma 1 *If IC, PC, and BBC are satisfied then (i) $s = r \cdot c - \pi \cdot m(\pi)$, (ii) $r \cdot c - \pi \cdot m(\pi) > 0$ except for $r = \pi = 0$ and (iii) $r \cdot c - \pi \cdot m(\pi) \leq \pi \cdot c$*

But what is more interesting for us is that these conditions are not only necessary but also sufficient. In fact we can say that :

Proposition 1 *The set of feasible contracts (i.e. satisfying BBC, PC, and IC) is fully characterized by the three previous conditions. In other words the subsidy is given by $s = r \cdot c - \pi \cdot m(\pi)$ and the required rate r of reduction of the waste treatment cost and the probability π of control belong to :*

$$\mathcal{F} = \left\{ (r, \pi) \in [0, \bar{r}] \times [0, 1] : \frac{\pi \cdot m(\pi)}{c} < r \leq \frac{\pi \cdot (m(\pi) + c)}{c} \right\} \cup \{(0, 0)\}$$

Finally and since we restrict, up to now, our attention to feasible contracts, let us observe that $u_e^{(\alpha,\theta)}(r, s)$ and $u_{\bar{e}}^{(\alpha,\theta)}(s, \pi)$ can be, in this case, written as respectively :

$$\begin{cases} u_e^{(\alpha,\theta)}(r, \pi) = (\alpha - c) + (r \cdot c - \pi \cdot m(\pi)) - \theta \frac{r}{\bar{r}} \\ u_{\bar{e}}^{(\alpha,\theta)}(r, \pi) = (\alpha - c) + (r \cdot c - \pi \cdot m(\pi)) - \pi c \end{cases}$$

[8]Since our distribution of probability is absolutely continous, only sets containing an open set have non-zero probability.

4 The surplus and its basic properties

Since we have assumed that there is a public agency which is in charge of the destruction of this ultimate waste, she will propose a feasible contract which maximises the surplus of the consumers. So let us now derive the surplus associated to each feasible contract.

In order to compute the one, we essentially concentrate our attention on non-trivial feasible contracts, i.e. with $(r, \pi) \neq (0,0)$. This last case simply corresponds to a full ADF system in which the average surplus is given by $\int_0^{\Theta} \int_c^A u_0^{(\alpha,\theta)} dF$. Moreover, to perform this computation, it becomes also important to distinguish the set of people who buy the commodity and accept the contract from those who refuse it. This first set of households is given by :

$$C(r,\pi) = \left\{ (\alpha,\theta) \in [0,A] \times [0,\Theta] : u_e^{(\alpha,\theta)}(r,\pi) \underset{(1)}{\geq} u_0^{(\alpha,\theta)} \underset{(2)}{\geq} 0 \right\}$$

A simple computation shows that condition (1) is equivalent to $\theta \frac{r}{\bar{r}} \leq c \cdot r - \pi \cdot m(\pi)$ or in other words that :

$$\theta \leq \theta(r,\pi) := \bar{r} \left(c - \frac{\pi \cdot m(\pi)}{r} \right)$$

Since condition (2) must be satisfied, we also observe that :

$$\forall \theta \leq \theta(r,\pi), \alpha \geq \alpha(r,\pi,\theta) := \theta \frac{r}{\bar{r}} + \pi \cdot m(\pi) + (1-r) \cdot c$$

Moreover, it is immediate that $\alpha(r,\pi,\theta) \in [\pi \cdot m(\pi) + (1-r) \cdot c, c] \subset [0,A]$ and $\theta(r,\pi) \in [0,\Theta]$ since $c < \Theta$ and $\bar{r} \leq 1$. We can therefore say that :

$$C(r,\pi) = \{(\alpha,\theta) \in [0,A] \times [0,\Theta] : 0 \leq \theta \leq \theta(r,\pi) \text{ and } \alpha \geq \alpha(r,\pi,\theta)\}$$

It follows that the surplus computed on the population which buy the good and accept the contract is given by[9] :

$$S_C(r,\pi) = \int_{C(r,\pi)} u_e^{(\alpha,\theta)}(r,\pi) dF = \int_0^{\theta(r,\pi)} \left(\int_{\alpha(r,\pi,\theta)}^A u_e^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta) d\alpha \right) f(\cdot,\theta) d\theta$$

Let us now move to the surplus of the set of households who buy the commodity but do not

[9] In order to prevent any confusion, let us notice that this quantity is not the average surplus of the consumers who buy the good and execute the contract. If the reader is interesting in this quantity, he must divide this surplus by the probability of being in this set, that is $P[C(r,\pi)]$.

accept the contract. The set of these agents is described by[10] :

$$\bar{C}(r,\pi) = \left\{ (\alpha,\theta) \in [0,A] \times [0,\Theta] : u_0^{(\alpha,\theta)} \underset{(1)}{\geq} u_e^{(\alpha,\theta)}(r,\pi) \underset{(2)}{\geq} 0 \right\}$$

By (1) we can say that $\theta \geq \theta(r,\pi)$ and by (2) that $\alpha \geq c$. Their surplus therefore corresponds to :

$$S_{\bar{C}}(r,\pi) = \int_{\bar{C}(r,\pi)} u_0^{(\alpha,\theta)} dF = \int_{\theta(r,\pi)}^{\Theta} \left(\int_c^A u_0^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta) d\alpha \right) f(\cdot,\theta) d\theta$$

It follows that the total surplus is simply given by

$$S(r,\pi) := S_C(r,\pi) + S_{\bar{C}}(r,\pi)$$

We even observe that :

Remark 1 *Since the total surplus under a full ADF system (i.e.$(r,\pi) = (0,0)$) is given $\int_0^{\Theta} \int_c^A u_0^{(\alpha,\theta)} dF$, we can say $S(r,\pi) \geq \int_0^{\Theta} \int_c^A u_0^{(\alpha,\theta)} dF$ and if the budget balancing subsidy $s = c \cdot r - \pi \cdot m(\pi)$ is strictly positive then this inequality holds strictly. A full ADF system is therefore always at least weakly dominated by a non trivial and efficient waste treatment contract.*

Let us now spell out comparative static properties of a feasible WMC. First, if the monitoring probability π increases, we know that the budget balancing subsidy $s = c \cdot r - \pi \cdot c(\pi)$ automatically decreases. We can thus conclude, at least from an intuitive point of view, that the welfare of the consumers who have adopted the waste management contract decreases, the same being true for the total surplus.

The effect of a change of the required waste treatment rate r is however less obvious. On the one hand, an increase in r contributes to a higher subsidy s. This provides, for the households who buy the good, more incentives to accept the contract and gives the opportunity to new consumers to enter the market. Yet, on the other hand, this also implies that the consumers who accept the contract provide a higher level of effort. We nevertheless show that this increase of the effort is offset by the increase of the price cut. More formally, we say that :

Proposition 2 *Let us denote by $P(r,\pi) := \int_0^{\theta(r,\pi)} \int_{\alpha(r,\pi,\theta)}^A dF$ the proportion of households who accept the waste management contract and by $\bar{\Theta}(r,\pi) := \int_0^{\theta(r,\pi)} \int_{\alpha(r,\pi,\theta)}^A \theta \frac{dF}{P(r,\pi)}$ their average marginal desutility of the effort. We observe that :*

[10] Since we have assumed that our measure is absolutely continuous, we decide by convention to only use weak inequalities.

(i) *$\forall (r,\pi) \in \mathcal{F}$ and $(r,\pi) \neq (0,0)$, $\partial_\pi S(r,\pi) = -(\pi m'(\pi) + m(\pi))P(r,\pi) < 0$, i.e. when the probability of control increases, the consumers surplus decreases.*

(ii) *$\forall (r,\pi) \in \mathcal{F}$ and $(r,\pi) \neq (0,0)$, $\partial_r S(r,\pi) = \left(c - \frac{\bar{\Theta}(r,\pi)}{\bar{r}}\right) P(r,\pi) > 0$ i.e. the surplus increases with the required waste treatment rate r since for all consumers who accept the contract, the subsidy, $c \cdot r - \pi \cdot m(\pi)$ is always greater than the monetary evaluation of the effort $\theta \frac{r}{\bar{r}}$. This implies that $c > \frac{\theta}{\bar{r}}$ for all these consumers, and it follows that $c > \frac{\bar{\Theta}(r,\pi)}{\bar{r}}$.*

5 The optimal WMC

Let us now try to characterize the optimal WMC. As usual in a second best situation, the social planner tries to implement the feasible contract which maximizes the total surplus, i.e. she chooses :

$$(r^*, \pi^*) \in \arg\max_{(r,\pi)\in\mathcal{F}} \underbrace{\int_{C(r,\pi)} u_e^{(\alpha,\theta)}(r,\pi) dF + \int_{\bar{C}(r,\pi)} u_0^{(\alpha,\theta)} dF}_{S(r,\pi)} \tag{1}$$

Moreover by the early definition of feasible contracts (see proposition 1) and the fact that $S(r,\pi)$ is increasing in r (see proposition 2), it is obvious that the upper constraint on r wil by binding. We can therefore reduce the set $\mathcal{F}$ to

$$\mathcal{F}' = \left\{ (r,\pi) \in [0,\bar{r}] \times [0,1] : r = \min\left\{ \frac{\pi \cdot (m(\pi) + c)}{c}, \bar{r} \right\} \right\}$$

This is clearly a closed subset of the compact set $[0,\bar{r}] \times [0,1]$, hence a compact set. Since $S(r,\pi)$ is also a continuous function, we can say without ambiguity that :

Lemma 2 *There always exists an optimal WMC which solves the previous program*

But it could also be interesting to spell out the different properties of this optimal VMC. So let us first concentrate our attention to the constraint $\mathcal{F}'$. Since the surplus is decreasing in π, an optimal WMC can never by such that $\frac{\pi \cdot (m(\pi^*)+c)}{c} > \bar{r}$, because, in this case, it possible to reduce the monitoring cost without modifying the cost reduction target r. We can therefore "forget" the min in the definition on $\mathcal{F}'$. Since we have also assumed that monitoring cost are large when everybody is controled, i.e. $m(1) > c$, we can say because $\frac{\pi \cdot (m(\pi)+c)}{c}$ is increasing that

Proposition 3 *The following properties hold :*
(i) The probability of control is bounded from above by $\pi_{\sup} < 1$ which solves $\frac{\pi_{\sup} \cdot (m(\pi_{\sup})+c)}{c} = \bar{r}$

(ii) The optimal strategy corresponds to a situation in which the subsidy is equal to the cost of a cheating strategy i.e. $s = r \cdot c - \pi \cdot m(\pi) = \pi \cdot c$.
(iii) we can associate to each $r \in [0, \bar{r}]$ *a unique probability of control* $\pi(r)$ *with the property that the subsidy corresponds to the cost of cheating.*

From that point of view, especially by using (iii), we can replace our constrained optimization problem by an unconstrained one by replacing the probability of control by $\pi(r)$, which solves $r \cdot c - \pi \cdot m(\pi) = \pi \cdot c$. In other words, we can say that the optimal cost reduction target is given by :

$$r^* \in \arg\max_{r \in [0,\bar{r}]} \underbrace{\int_{C(r,\pi)} u_e^{(\alpha,\theta)}(r, \pi(r)) dF + \int_{\bar{C}(r,\pi)} u_0^{(\alpha,\theta)} dF}_{S(r,\pi(r))}$$

This brings us to the conclusion that :

Proposition 4 *At an optimal WMC* (r^*, π^*)*, on observes that :*
(i) both the target r^* *and the probability of control* π^* *are strictly positive.*
(ii) the household who accept the contract receives a strictly positive subsidy $s^* = r \cdot c - \pi^* \cdot m(\pi^*) > 0$
(iii) the following marginal condition is satisfied :

$$\left(c - \frac{\bar{\Theta}(r^*, \pi^*)}{\bar{r}} \right) - (\pi^* \cdot m'(\pi^*) + m(\pi^*)) \cdot \left. \frac{d\pi}{dr} \right|_{r^*} \geq 0 \text{ with equality when } r^* < \bar{r}$$

In order to comment this last condition let us first remember that $\bar{\Theta}(r^*, \pi^*)$ is the average of the repartion of the marginal disutility of the effort across the households who accept the WMC, so that $\frac{\bar{\Theta}(r^*,\pi^*)}{\bar{r}}$ stands for the average marginal cost of an increase in the rate r of waste treatment. But we also notice that $c - (\pi^* \cdot m'(\pi^*) + m(\pi^*)) \cdot \left.\frac{d\pi}{dr}\right|_{r^*}$ is nothing else then marginal benefit from accepting a WMC since this quantity is the derivative of the subsidy, i.e. $\frac{ds(r,\pi(r))}{dr}$, this marginal benefit being the same for all households who accept the contract. From that point of view, the previous marginal condition can be written as

$$\frac{ds(r, \pi(r))}{dr} \geq \frac{\bar{\Theta}(r^*, \pi^*)}{\bar{r}} \text{with equality when } r^* < \bar{r}$$

and simply states that on average the marginal gain must be equal to the marginal costs when an optimum is reached.

If we now come back to $\theta(r, \pi) := \bar{r}\left(c - \frac{\pi \cdot m(\pi)}{r}\right)$ which was defined as the highest disutility of the effort for which a WMC can be accepted, we can note that $\theta(r, \pi) \leq \bar{r}c$. As a consequence, we can say that the average value of this disutility $\bar{\Theta}(r, \pi) < \bar{r}c$, or, in other words, $\forall r$, $\frac{\bar{\Theta}(r,\pi(r))}{\bar{r}} < c$.

If one now has in mind, under our assumptions, that the marginal monitoring costs are small when π is close to 0, i.e. $(\pi^* \cdot m'(\pi^*) + m(\pi^*)) \cdot \frac{d\pi}{dr}\Big|_{r^*} \simeq 0$ for r close to 0, we can say the marginal benefit $\frac{ds(r,\pi(r))}{dr} \simeq c$. This is why r^* is always strictly positive. From that point of view, any WMC is better in term of welfare then a ADF since this contract coincides to a trivial WMC for which $r = \pi = 0$

6 A WMC versus a Two Part Tariff

At that point, we know that a WMC challenges a pure ADF. But is this contract better then a two part tariff (TPT) build on both an ADF and a DF ?

The answer to this question is less obvious because these two mechanisms are quite different. On the one hand, the first enforces a level of effort since the contract explicitly spells the cost reducing activities while a DF system leaves this choice to the household. But, one the other hand, this contractual agreement simplifies the monitoring activity since the controller only look for infringements to the contract within the population who signed it, while in the other case, each consumer has an incentive to illegally dump their waste. So even, if a TPT provides more freedom for the household, its implementation is probably more expensive in terms of monitoring.

Even is this issue is important, we nevertheless assume, for simplicity, that the monitoring cost $m(\pi)$ per control is the same in both systems. In the context, we basically that an optimal WMC always dominated a TPT as long as the total amount of per unit fees[11] (i.e. ADF+DF) is not smaller then the unit waste treatment cost. Moreover, if this last condition is not satisfied, we even show that we can replicates the welfare effect of a TPT by a suitable WMC which is not necessary incentive compatible but which reduces the total waste treatment cost (i.e. including the global monitoring cost). We even argue that a TPT is this case not really fair since it induces a transfer of wealth from the household who provide an effort to those who do nothing at all.

In order to illustrate this point, let us first introduce a TPT in our model. This one will be characterized by a triple (a, d, π) which specifies the ADF, noted by a, included in the price of the commodity, the DF, noted by d, charged for unit of waste, and the probability π that a household will be controlled for illegal dumping. The utility function of a household of type (α, θ) remains unchanged and is always given by :

$$u(x, e, \mu) = \alpha x - \theta e - \mu \text{ with } x = \{0, 1\} \text{ and } e \in [0, 1]$$

[11] Remember that we have assumed that one unit of good produces one unit of waste. From that point of view, the ADF which is basically charged on consumption can be compared with a DF charged on the amount of waste.

But, the effort and therefore level of waste reduction is now endogenous. This is why we explicitly have to pay attention to illegal dumping, to introduce a individual waste production function and to compute the cost of consumption. Concerning the first point, we note by f the proportion of waste which is illegally disposed, the monitoring cost $m(\pi)$ per control as well as the fine remain the same. The waste production function must however be consistent with the cost reduction function introduced in section 2, this one is therefore given by $\min\{x - \bar{r} \cdot e, 0\}$. From that point of view, the expected cost of consumption is for each agent given by :

$$C(x, e, f) = a \cdot x + ((1 - f) \cdot d + f \cdot c \cdot \pi) \cdot \min\{x - \bar{r} \cdot e, 0\}$$
$$\text{with } x = \{0, 1\},\ e \in [0, 1], \text{ and } f \in [0, 1]$$

and each household chooses (x, e, f) in order to maximize $u(x, e, C(x, e, f))$.

If we now take the point of view of a public agency in charge of the waste treatment, it is immediate that the DF must verify $d \leq c \cdot \pi$ because in this case the households depose her waste legally, i.e. choose $f = 0$. But this agency has also an incentive to minimize her monitoring cost and therefore to set the probability of control at $\pi = \frac{d}{c}$. This is why we concentrate our attention on TPTs given by $(a, d, \frac{d}{c})$. Moreover, as for a WMC, we also require that this agency is subject to a budget constraint in the sense that the collected fees must cover the monitoring and the waste treatment costs. This last cost can even be evaluated easily since the utility of each buyer is linear in effort. In fact she sets $e = 1$ or $e = 0$ respectively when $\theta \geq d \cdot \bar{r}$ or $\theta < d \cdot \bar{r}$. From that point of we can summarize all our observations in the following lemma :

Lemma 3 *Let (a, d, π) be a TPT, we can say that :*
(i) When $\pi = \frac{d}{c}$, this TPT prevents from illegal dumping and minimizes for a given couple (a, d) the monitoring cost.
(ii) The indirect expected utility of a consumer of type $(\alpha, \theta) \in [0, A] \times [0, \Theta]$ is given by $V_{TPT}^{(\alpha,\theta)} = \max\{0, \alpha - (a + d) + \max\{d \cdot \bar{r} - \theta, 0\}\}$
(iii) If $p(a, d)$ denotes the proportion of the effective buyers who perform a waste reduction effort, the budget constraint per effective buyer is satisfied when $a + d \geq c + \frac{d}{c} m\left(\frac{d}{c}\right) - \bar{r}(c - d)\, p(a, d)$

Let us now compare both systems. For that purpose, we associate to a given TPT $(a, d, \frac{d}{c})$ a WMC characterized by $(r, s, \pi) = (\bar{r}, c - (a + d) + \bar{r}d, \bar{r}\frac{d}{c})$, i.e. with the property that the target of the reduction of the cost is $\bar{r}$, the subsidy is given by $s = c - (a + d) + \bar{r}d$, and the probability of control is $\pi = \bar{r}\frac{d}{c}$. Under this WMC, the consumer has, as usually, only four options : he can do nothing, only buy the good, buy it and accept WMC, and finally buy it, accept the contract and cheat. Moreover, if we refer to section 3.3, the indirect utility of the household of type (α, θ)

is, in this case, given by :

$$\begin{aligned} V_{WMC}^{(\alpha,\theta)} &= \max\left\{0, u_0^{(\alpha,\theta)}, u_e^{(\alpha,\theta)}(r,s), u_{\bar{e}}^{(\alpha,\theta)}(s,\pi)\right\} \\ &= \max\{0, \alpha - c, \alpha - \theta - (a+d) + \bar{r}d, \alpha - (a+d)\} \\ &= \max\{0, \alpha - \min\{c, a+d\} + \max\{\bar{r}d - \theta, 0\}\} \end{aligned}$$

From this last equation, it is immediate that this WMC improves the utility of all agents as long as the total amount of per unit fees (i.e. ADF+DF) is not smaller then the waste treatment cost, i.e. when $a + d \geq c$. Under this condition, we can even show that this WMC is in our feasible set. But we can associates such a WMC to each TPT. This therefore means that an optimal WMC always beats, from a welfare point of view, an optimal TPT as long as for this last one $a + d \geq c$. The results are however less clear cut in the other case. In fact we can say :

Proposition 5 *Let $(a, d, \frac{d}{c})$ be a TPT which satisfies the budget constraint and let $(\bar{r}, c - (a + d) + \bar{r}d, \bar{r}\frac{d}{c})$ be an associated WMC. We can say :*
(i) If $a + d \geq c$, this WMC dominates in term of welfare the TPT. This contract satisfies both (PC) and (IC) and spares money since the budget of the waste treatment agency is in excess.
(ii) If a welfare maximizing TPT verifies $a + d \geq c$, then this one is strictly dominated by the optimal WMC.
(iii) If, $a + d < c$ the WMC associated to a TPT allocates the utilities in the same way. This contract does not verify (IC) but nevertheless satisfy (PC) and spare money with respect to the TPT.

But let us quickly come back to the case where a TPT verifies $a + d < c$. In this situation, each household of type $\alpha > a + d$ and $\theta \geq \bar{r}d$ supplies no effort at all and support a total fee of $(a + d)$ which is strictly smaller the cost c of the treatment of her waste. This is why our WMC which replicates the TPT is not able to guaranty that the terms of the contract are always fullfilled because these agents, under a WMC, always accept the contract and cheat, i.e. $V_{WMC}^{(\alpha,\theta)} = u_{\bar{e}}^{(\alpha,\theta)}(s,\pi)$. But this also means that these consumers free rides their obligations in a TPT because they benefit from a reduced waste treatment cost by doing nothing. In other words, they benefits from the effort of the more environmental friendly agent which contributes to a reduction of the total waste treatment cost. In this case, a TPT is therefore not really fair. By contrast, this situation never occurs when a Incentive Compatible WMC is implemented since the people who refuse the contract support the total waste treatment cost.

7 Concluding remarks

In the paper, we essentially addressed the question of reduction of the cost induced by the treatment of the ultimate waste produced by the consumers. But ultimate waste, we mean the residual garbage for which there exists no recycling opportunities stimulated by a suitable taxation scheme. This onr, even if it cannot be totally destroyed, often requires additional costly transformations before being, say, reintroduced in our environment. Since these costs are supported by the society, and especially the consumers, the idea of this paper was essentially to look at a mechanism which implicates these agents in the reduction of this treatment cost (instead of reduction of the amount of waste) by providing some voluntary effort. More precisely, we introduced what we called a Waste Management Contract. In this setting, the households are charged of an ADF which covers the waste treatment cost but have nevertheless the opportunity to accept a WMC which specifies a set of cost reducing activities which can be performed at their level and for which they earn a subsidy. This contract was also coupled with a monitoring scheme in order to prevent infringements. In this context, we first identified the set of feasible contracts, i.e. those satisfying an Incentive, a Participation and a Budget Balancing constraints, then we characterized an optimal contract from a welfare point of view, and finally compare this kind of agreement to a more standard system which couples an ADF and a DF.

This paper however remains particular in several respects. First, even if our argument requires no specific assumptions on the distribution of the characteristics of agents and remains quite general from that point of view, we however assumed that (i) the effect of the effort on the cost reduction rate is linear, and (ii) that the preferences of each agent remain linear. It could perhaps be interesting to relax these assumptions by introducing a more general relation between the effort and its effect on the waste treatment cost, or even to leave our discrete choice setting.

From a less technical point of view, the reader surely also notices that we basically concentrate our attention to waste management policies which handle the treatment of "end-of-pipe" pollution.. Recyclable and incentives which reduces the waste content of a good are not explicitly taken into consideration. This of course requires a more global model, but is not without consequences especially if the households have to allocate a limited effort between the cost reducing activities prescribed by a WMC and a standard recycling behavior motivated by a deposit-refund system. We however leave this point to future works.

Finally we have also assumed, as usual in this literature, that the market of the good works competitively. If this assumption is relaxed, the optimal design of the contract not only takes into account the waste management issue but also its effects on market power. In this case, we are typically in he situation in which one instrument, i.e. the WMC, tries to regulates two inefficencies : the imperfect observability of the effort and the existence of a behavior of the firms which try to extract the rent of imperfect competition.

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APPENDIX

A Proof of lemma 1

Step 1 $BBC, IC, PC \Rightarrow (i)\ s = r \cdot c - \pi \cdot m(\pi)$

Since IC and PC respectively induces that $P(E) > 0$, i.e. at least some consumers accect the contract, and $P(\bar{E}) = 0$, i.e. nobody cheats, the result directly follows from BBC.

Step 2 $BBC, IC, PC \Rightarrow (ii)\ r \cdot c - \pi \cdot m(\pi) > 0$ except for $r = \pi = 0$

Let us first verify that $((i)$ *and non*$(ii)) \Rightarrow$ *non*(PC). In fact if (i) is true, $u_e^{(\alpha,\theta)}(r,s)$ and $u_{\bar{e}}^{(\alpha,\theta)}(s,\pi)$ can be written as respectively :

$$\begin{cases} u_e^{(\alpha,\theta)}(r,\pi) = (\alpha - c) + (r \cdot c - \pi \cdot m(\pi)) - \theta \frac{r}{\bar{r}} \\ u_{\bar{e}}^{(\alpha,\theta)}(r,\pi) = (\alpha - c) + (r \cdot c - \pi \cdot m(\pi)) - \pi c \end{cases} \tag{2}$$

So if $r \cdot c - \pi \cdot m(\pi) \leq 0$ and $(r,\pi) \neq (0,0)$ then $\forall(\alpha,\theta) \in [0,A] \times]0,\Theta]$, $u_0^{(\alpha,\theta)} = \alpha - c > u_e^{(\alpha,\theta)}(r,\pi)$. Since this implies that $\max\left\{0, u_0^{(\alpha,\theta)}, u_{\bar{e}}^{(\alpha,\theta)}(r,\pi)\right\} > u_e^{(\alpha,\theta)}(r,s)$, PC can only be verified for a subset of $[0,A] \times \{0\}$, a set which contains no open subsets. But our probability distribution is absolutely continuous (i.e. only sets containing open sets has a strictly positive probability), it follows that PC cannot be true. But this preliminary observation leads us also to the conclusion (by contraposition) that $PC \Rightarrow (non(i)\ or\ (ii))$. But by step 1, $(BBC, IC, PC) \Rightarrow (i)$, hence we can say that $BBC, IC, PC \Rightarrow (ii)$

Step 3 $BBC, IC, PC \Rightarrow (iii)\ r \cdot c - \pi \cdot m(\pi) \leq \pi \cdot c$

As in step 2, if we show that $((i)$ and $non(iii)) \Rightarrow non(IC)$ our result is obtained. It therefore remains to find an agent (α,θ) with the property that $u_{\bar{e}}^{(\alpha,\theta)}(r,\pi) > max\left\{u_e^{(\alpha,\theta)}(r,\pi), u_0^{(\alpha,\theta)}, 0\right\}$. So let us set $(\alpha,\theta) = (A,\Theta)$. Since (i) is true we can use (2) and because $A > c$ and $non(iii)$, we observe that $u_{\bar{e}}^{(A,\Theta)}(r,\pi) > u_0^{(A,\Theta)} > 0$. It therefore remains to verify that $u_{\bar{e}}^{(A,\Theta)}(r,\pi) > u_e^{(A,\Theta)}(r,\pi)$. So let observe that :

$$\begin{aligned}
u_{\bar{e}}^{(A,\Theta)}(r,\pi) &= (A-c) + (r \cdot c - \pi \cdot m(\pi)) - \pi \cdot c \\
&> (A-c) + (r \cdot c - \pi \cdot m(\pi)) - r \cdot c \quad \left\{\begin{array}{l} \text{since } non(iii) \text{ and } \pi \cdot m(\pi) > 0 \\ \text{imply that } r \cdot c > \pi \cdot c \end{array}\right\} \\
&= (A-c) + (r \cdot c - \pi \cdot m(\pi)) - \frac{r}{\bar{r}} \cdot (\bar{r} \cdot c) \\
&> (A-c) + (r \cdot c - \pi \cdot m(\pi)) - \frac{r}{\bar{r}} \cdot \Theta \quad \text{since } \Theta > c \text{ and } \bar{r} \leq 1 \\
&= u_e^{(A,\Theta)}(r,\pi)
\end{aligned}$$

B Proof of proposition 1

Remark : In this proof, (i), (ii) and (iii) referee to the property exhibited in lemma 1.

Step 1 : $((i)$ and $(iii)) \Rightarrow IC$

Let us first notice that under (i), $u_{\bar{e}}^{(\alpha,\theta)}(r,\pi)$ is defined by (2). So if (iii) holds, we have

$$\forall \alpha \in [0,A] \quad u_0^{(\alpha,\theta)} = \alpha - c \geq \alpha - c + r \cdot c - \pi \cdot m(\pi) - \pi \cdot c = u_{\bar{e}}^{(\alpha,\theta)}(r,\pi)$$

It follows, by the definition of a maximum, that :

$$\forall (\alpha,\theta) \in [0,A] \times [0,\Theta], \qquad \max\left\{u_e^{(\alpha,\theta)}(r,s), u_0^{(\alpha,\theta)}, 0\right\} \geq u_{\bar{e}}^{(\alpha,\theta)}(s,\pi) \tag{IC}$$

Step 2 : $((i)$ and $(iii)) \Rightarrow BBC$

By step 1, we know that IC is true. It follows that $P(\bar{E}) = 0$ and since (i) is verified we can write that

$$P(E)\,(s - (1-r)c + \pi \cdot m(\pi)) + P(\bar{E})\,(s - \pi \cdot c + \pi \cdot m(\pi)) = 0 \tag{BBC}$$

Step 3 : $((i)$, (ii) and $(iii)) \Rightarrow PC$

Let us first observe that under (i), $u_e^{(\alpha,\theta)}(r,\pi)$ and $u_{\bar{e}}^{(\alpha,\theta)}(r,\pi)$ are given by (2). Moreover by (ii) we typically have to sub-case one in which $r = \pi = 0$ and one in which $s = (r \cdot c - \pi \cdot m(\pi)) > 0$ and $(r,\pi) \neq (0,0)$, In the first one, (PC) is obviously satisfied since

$$\forall (\alpha,\theta) \in [0,A] \times [0,\Theta] \quad u_0^{(\alpha,\theta)} = u_e^{(\alpha,\theta)}(r,\pi) = u_{\bar{e}}^{(\alpha,\theta)}(r,\pi)$$

Now let us move to the second one. Since $A > c$, we can say that $\forall (\alpha,\theta) \in E =]c,A[\times]0, \frac{\bar{r}}{r}s[$

$$u_e^{(\alpha,\theta)}(r,\pi) = (\alpha - c) + s - \theta\frac{r}{\bar{r}} > \alpha - c = u_0^{(\alpha,\theta)} > 0$$

But we also know that $((i)$ and $(iii)) \Rightarrow IC$, or in other words that :

$$\forall\,(\alpha,\theta) \in [0,A]\times[0,\Theta]\,, \qquad max\left\{u_e^{(\alpha,\theta)}(r,s), u_0^{(\alpha,\theta)}, 0\right\} \geq u_{\bar{e}}^{(\alpha,\theta)}(s,\pi)$$

By using the previous equation we conclude that :

$$\forall\,(\alpha,\theta) \in E =]c,A[\times\left]0,\frac{\bar{r}}{r}s\right[,\; u_e^{(\alpha,\theta)}(r,s) > max\left\{u_{\bar{e}}^{(\alpha,\theta)}(s,\pi), u_0^{(\alpha,\theta)}, 0\right\}$$

C Proof of proposition 2

This computation is a tedious exercise since $S(r,\pi) = S_C(r,\pi) + S_{\bar{C}}(r,\pi)$ with

$$\begin{cases} S_C(r,\pi) = \int_0^{\theta(r,\pi)}\left(\int_{\alpha(r,\pi,\theta)}^A u_e^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta) d\alpha\right) f(\cdot,\theta) d\theta \\ S_{\bar{C}}(r,\pi) = \int_{\theta(r,\pi)}^{\Theta}\left(\int_c^A u_0^{(\alpha,\theta)} f(\alpha/\theta) d\alpha\right) f(\cdot,\theta) d\theta \end{cases}$$

But the reader observes that r and π work in a rather similar way. So if x stand either for r and π, we obtain that :

$$\begin{aligned} \partial_x S(r,\pi) = \partial_x S_C(r,\pi) + \partial_x S_{\bar{C}}(r,\pi) = \\ \left(\left(\int_{\alpha(r,\pi,\theta)}^A u_e^{(\alpha,\theta)}(r,\pi)\cdot f(\alpha/\theta) d\alpha\right) f(\cdot,\theta)\right)\Bigg|_{\theta=\theta(r,\pi)} \times \partial_x\theta(r,\pi) \quad (3a) \\ + \int_0^{\theta(r,\pi)}\left(\int_{\alpha(r,\pi,\theta)}^A \partial_x u_e^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta) d\alpha - \left(u_e^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta)\right)\Big|_{\alpha=\alpha(r,\pi,\theta)} \partial_x\alpha(r,\pi,\theta)\right) f(\cdot,\theta) d\theta \quad (3b) \\ - \left(\left(\int_c^A u_0^{(\alpha,\theta)}\cdot f(\alpha/\theta) d\alpha\right) f(\cdot,\theta)\right)\Bigg|_{\theta=\theta(r,\pi)} \times \partial_x\theta(r,\pi) \quad (3c) \end{aligned}$$

Since $\theta(r,\pi) = \bar{r}\left(c - \frac{\pi\cdot m(\pi)}{r}\right)$ and $\alpha(r,\pi,\theta) := \frac{\theta r}{\bar{r}} + c - (rc - m(\pi))$, we also remark that :

$$\begin{cases} \text{(i)}\;\; u_e^{(\alpha,\theta)}(r,\pi)\Big|_{\theta=\theta(r,\pi)} = \alpha - (1-r)\cdot c - \pi\cdot m(\pi) - \bar{r}\left(c - \frac{\pi\cdot m(\pi)}{r}\right)\frac{r}{\bar{r}} = \alpha - c = u_0^{(\alpha,\theta)}\Big|_{\theta=\theta(r,\pi)} \\ \text{(ii)}\;\; \alpha(r,\pi,\theta)|_{\theta=\theta(r,\pi)} = \bar{r}\left(c - \frac{\pi\cdot m(\pi)}{r}\right)\frac{r}{\bar{r}} + c - (rc - \pi m(\pi)) = c \\ \text{(iii)}\;\; u_e^{(\alpha,\theta)}(r,\pi)\Big|_{\alpha=\alpha(r,\pi,\theta)} = \theta\frac{r}{\bar{r}} + c - (rc - \pi m(\pi)) - (1-r)\cdot c - \pi\cdot c(\pi) - \theta\frac{r}{\bar{r}} = 0 \end{cases}$$

By (i) and (ii) the first (3a) and the third (3c) term in the preceding sum simplify, and (iii) reduces the second (3b). We can therefore say that :

$$\partial_x S(r,\pi) = \int_0^{\theta(r,\pi)}\left(\int_{\alpha(r,\pi,\theta)}^A \partial_x u_e^{(\alpha,\theta)}(r,\pi)\cdot f(\alpha/\theta) d\alpha\right) f(\cdot,\theta) d\theta$$

Now remember that $\partial_\pi u_e^{(\alpha,\theta)}(r,\pi) = -(\pi m'(\pi) + m(\pi)) < 0$ for all $\pi > 0$ and that the only feasible contract for which $\pi = 0$ is $(r,\pi) = (0,0)$, we can therefore say that :

$$\forall (r,\pi) \in \mathcal{F} \text{ and } (r,\pi) \neq (0,0), \quad \partial_\pi S(r,\pi) = -(\pi m'(\pi) + m(\pi))\int_0^{\theta(r,\pi)}\int_{\alpha(r,\pi,\theta)}^A dF < 0$$

where $\int_0^{\theta(r,\pi)}\int_{\alpha(r,\pi,\theta)}^A dF = P(r,\pi)$ the proportion of households who accept the waste management contract.

This proves (i) of proposition 2.

Let us move to (ii) of proposition 2. Since $e(r) = \frac{r}{\bar{r}}$, we observe that $\partial_r u_e^{(\alpha,\theta)}(r,\pi) = c - \frac{\theta}{\bar{r}}$. Moreover $\forall \theta < \theta(r,\pi) =: \bar{r}\left(c - \frac{\pi \cdot m(\pi)}{r}\right)$, we can say that :

$$c - \frac{\theta}{\bar{r}} > c - \bar{r}\left(c - \frac{\pi \cdot m(\pi)}{r}\right)\frac{1}{\bar{r}} = \frac{\pi \cdot c(\pi)}{r} \geq 0$$

This implies that :

$$\forall (r,\pi) \in \mathcal{F} \text{ and } (r,\pi) \neq (0,0), \quad \partial_r S(r,\pi) = \int_0^{\theta(r,\pi)} \left(c - \frac{\theta}{\bar{r}}\right)\left(\int_{\alpha(r,\pi,\theta)}^{A} f(\alpha/\theta)d\alpha\right) f(\cdot,\theta)d\theta > 0$$

Now let us denote by

$$\bar{\Theta}(r,\pi) := \int_0^{\theta(r,\pi)} \theta \frac{\left(\int_{\alpha(r,\pi,\theta)}^{A} f(\alpha/\theta)d\alpha\right)}{\int_0^{\theta(r,\pi)} \int_{\alpha(r,\pi,\theta)}^{A} dF} f(\cdot,\theta)d\theta$$

the average desutility of the effort for the households who accept the waste management contract. This yields :

$$\forall (r,\pi) \in \mathcal{F} \text{ and } (r,\pi) \neq (0,0), \quad \partial_r S(r,\pi) = \left(c - \frac{\bar{\Theta}(r,\pi)}{\bar{r}}\right) P(r,\pi)$$

D Proof of Lemma 2

The proof is obvious since the surplus $S(r,\pi)$ is continuous and the set $\mathcal{F}$ of feasible contracts is compact.

E Proof of proposition 3

Remember that $(r^*,\pi^*) \in \underset{(r,\pi)\in\mathcal{F}'}{\arg\max} S(r,\pi)$ with $\mathcal{F}' = \left\{(r,\pi) \in [0,\bar{r}] \times [0,1] : r = \min\left\{\frac{\pi\cdot(m(\pi)+c)}{c}, \bar{r}\right\}\right\}$

Point (i) : $\exists \pi_{\text{sup}} < 1$ given by $\frac{\pi_{\text{sup}}\cdot\left(m(\pi_{\text{sup}})+c\right)}{c} = \bar{r}$ such that $\pi^* \leq \pi_{\text{sup}}$

Let us first verify that π_{sup} exists and smaller then 1. To see this, let us observe that $f(\pi) = \frac{\pi\cdot(m(\pi)+c)}{c}$ is increasing in π and let us remember that we have assumed that $m(0) = 0$ and $m(1) > c$. The range of f is therefore given by $f([0,1]) = [0, f(1)]$ with $f(1) > 2$. Since $\bar{r} \in [0,1]$, there always exists a unique $\pi_{\text{sup}}(\bar{r}) < 1$ solving $\frac{\pi_{\text{sup}}\cdot\left(m(\pi_{\text{sup}})+c\right)}{c} = \bar{r}$

Now let us verify that $\pi^* \leq \pi_{\text{sup}}$. Assume the contrary. Since $f'(\pi) > 0$, it is immediate by the definition of $\mathcal{F}'$ that $r^* = \bar{r}$. But the same holds for $\pi' = \frac{\pi^*+\pi_{\text{sup}}}{2} < \pi^*$. Now remember by proposition 2 that $\partial_\pi S(r,\pi) < 0$, it follows that $S(\bar{r},\pi') > S(\bar{r},\pi^*)$ which contradicts the fact that (r^*,π^*) is an optimal solution.

Point (ii) : $s^* = r^* \cdot c - \pi^* \cdot m(\pi^*) = \pi^* \cdot c$.

Since $f'(\pi) > 0$, by step 1 we know that $f(\pi^*) \leq f(\pi_{\text{sup}}) = \bar{r}$. It follows by the definition of $\mathcal{F}'$, that $r^* = \frac{\pi^*\cdot\left(m(\pi^*)+c\right)}{c}$ or in other words that $r^* \cdot c - \pi^* \cdot m(\pi^*) = \pi^* \cdot c$

Point (iii) : $\exists \pi : [0,\bar{r}] \to [0,\pi_{\text{sup}}]$, with the property that $r = \frac{\pi(r)\cdot(m(\pi(r))+c)}{c}$.

The same arguments as in the first part of the proof of step 1 apply.

F Proof of proposition 4

Since the search of an optimal contract reduces to the computation of a waste reduction rate which satisfies

$$r^* \in \underset{0\le r\le \bar{r}}{\arg\max} S(r,\pi(r)).$$

Point (i) : $r^* \neq 0$

Let us compute $\lim_{r\to 0} \partial_r S(r,\pi(r))$. We observe by proposition 2 that

$$\lim_{r\to 0} \partial_r S(r,\pi(r)) = \lim_{r\to 0}\left(\partial_r S(r,\pi)|_{\pi=\pi(r)}\right) + \lim_{r\to 0}\left(\partial_\pi S(r,\pi)|_{\pi=\pi(r)}\right)\cdot \lim_{r\to 0}\frac{d\pi(r)}{dr}$$

with

$$\partial_r S(r,\pi) = \left(c - \frac{\bar{\Theta}(r,\pi)}{\bar{r}}\right) P(r,\pi) \quad \partial_\pi S(r,\pi) = -(\pi m'(\pi) + m(\pi))P(r,\pi)$$

$$\frac{d\pi}{dr} = \frac{c}{\pi(r)\cdot m'(\pi(r)) + m(\pi(r)) + c}$$

Now remember that $m(\pi)$ is increasing and convex, it follows that $\lim_{\pi\to 0} m'(\pi)$ is bounded. Since $P(r,\pi(r)) \in [0,1]$, $m(0) = 0$ and $\pi(0) = 0$, we can state that $\lim_{r\to 0} \partial_\pi S(r,\pi(r)) = 0$. But we can also observe that $\frac{d\pi}{dr} = \frac{c}{\pi(r)\cdot m'(\pi(r))+m(\pi(r))+c} \in [0,1]$, so, by continuity, the same holds for $\lim_{r\to 0}\dfrac{d\pi}{dr}$. We can therefore say (see proof of proposition 2) that :

$$\begin{aligned}\lim_{r\to 0} \partial_r S(r,\pi(r)) &= \lim_{r\to 0}\left(\partial_r S(r,\pi)|_{\pi=\pi(r)}\right)\\ &= \int_0^{\lim_{r\to 0}\theta(r,\pi(r))}\left(c - \frac{\theta}{\bar{r}}\right)\left(\int_{\lim_{r\to 0}\alpha(r,\pi(r),\theta)}^{A} f(\alpha/\theta)d\alpha\right) f(\cdot,\theta)d\theta\end{aligned}$$

Now let us observe that :

(i) $\lim_{r\to 0}\alpha(r,\pi(r),\theta) = \lim_{r\to 0}\left(\theta\frac{r}{\bar{r}} + \pi(r)\cdot m(\pi(r)) + (1-r)\cdot c\right) = c$

(ii) $\lim_{r\to 0}\theta(r,\pi(r)) = \lim_{r\to 0}\bar{r}\left(c - \frac{\pi(r)\cdot m(\pi(r))}{r}\right) = \bar{r}c$ since by L'hôpital's rule

$$\lim_{r\to 0}\frac{\pi(r)\cdot m(\pi(r))}{r} = \lim_{r\to 0}\left(m(\pi(r)) + \pi(r)m'(\pi(r))\right)\frac{d\pi}{dr} = 0$$

(remember that $\lim_{\pi\to 0} m'(\pi)$ is bounded and $\frac{d\pi}{dr}\in[0,1]$)

We can therefore say that

$$\lim_{r\to 0}\partial_r S(r,\pi(r)) = \int_0^{\bar{r}c}\left(c - \frac{\theta}{\bar{r}}\right)\left(\int_c^A f(\alpha/\theta)d\alpha\right) f(\cdot,\theta)d\theta$$

But we have assume that F is absolutely continuous with strictly positive density, hence $\left(\int_c^A f(\alpha/\theta)d\alpha\right) > 0$ and $f(\cdot,\theta) > 0$. We deduce that

$$\lim_{r\to 0}\partial_r S(r,\pi(r)) > \int_0^{\bar{r}c}\left(c - \frac{\bar{r}c}{\bar{r}}\right)\left(\int_c^A f(\alpha/\theta)d\alpha\right) f(\cdot,\theta)d\theta = 0$$

From that point of view, it is impossible that $r^* = 0$

Point (ii) : $s^* = \pi(r^*)c > 0$

This follows directly from point (ii) of proposition 3 and the fact that $r^* > 0$

Point (iii) : the marginal condition

Since $r > 0$, we know from the Kuhn-Tucker first order conditions that :

$$\begin{cases} \partial_r S(r,\pi(r)) + \partial_\pi S(r,\pi(r))\dfrac{d\pi}{dr} - \lambda = 0 \\ \lambda\,(r-\bar{r}) = 0 \quad r-\bar{r} \leq 0 \quad \lambda \geq 0 \end{cases}$$

or equivalently that

$$\partial_r S(r,\pi(r)) + \partial_\pi S(r,\pi(r))\frac{d\pi}{dr} \geq 0 \text{ with equality if } r < \bar{r}$$

By replacing the different derivatives by their value we finally obtain :

$$\left(c - \frac{\bar{\Theta}(r^*,\pi^*)}{\bar{r}} - (\pi^* m'(\pi^*) + m(\pi^*))\frac{d\pi}{dr}\right) P(r^*,\pi^*) \geq 0 \text{ with equality if } r < \bar{r}$$

G Proof of Lemma 3

Let us remember that $\pi = \frac{d}{c}$ and that each consumer solves

$$\max_{(x,e,f)\in\{0,1\}\times[0,1]\times[0,1]} \alpha x - \theta e - \alpha x - ((1-f)\cdot d + f\cdot c\cdot\pi)\cdot \min\{x - \bar{r}\cdot e, 0\}$$

Point (i) : $\pi = \frac{d}{c}$

If $\pi = \frac{d}{c}$, the consumer is indifferent between all value of f, hence $f = 1$ is an optimal strategy.

Point (ii) : $\forall(\alpha,\theta)\in[0,A]\times[0,\Theta]$, $V_{TPT}^{(\alpha,\theta)} = \max\{0, \alpha - (a+d) + \max\{d\cdot\bar{r} - \theta, 0\}\}$

If $\pi = \frac{d}{c}$, the previous program becomes :

$$\max_{(x,e,f)\in\{0,1\}\times[0,1]} \alpha x - \theta e - \alpha x - d\cdot \min\{x - \bar{r}\cdot e, 0\}$$

So if $x = 0$, we can say that $e = 0$ is the optimal effort and the indirect utility is given $V^{(\alpha,\theta)}(0) = 0$. If $x = 1$, the optimal effort will be respectively $e = 1$ or $e = 0$ when $\theta \leq \bar{r}\cdot d$ or $\theta > \bar{r}\cdot d$. It follows that $V^{(\alpha,\theta)}(1) = \alpha - a - d + \max\{\bar{r}\cdot d - \theta, 0\}$. Since each consumer chooses the best solution between both we can conclude that :

$$V^{(\alpha,\theta)} = \{\alpha - a - d + \max\{\bar{r}\cdot d - \theta, 0\}, 0\}$$

Point (iii) : the budget constraint is $a + d \geq c + \frac{d}{c} m\left(\frac{d}{c}\right) - \bar{r}\,(c-d)\,p(a,d)$

The waste cleaning agency collects a ADF for each unit of good sold on the market, a complete disposal fee paid by the households who perform no effort and a reduced fee for those who set their effort at $e = 1$. Since $\pi = \frac{d}{c}$, no fines are collected. From that point of view, she obtains on average par effective buyer :

$$a + d\cdot(1 - p(a,d)) + d\cdot(1-\bar{r})\cdot p(a,d)$$

where $p(a,d)$ denote the proportion of effective buyers who provide an effort. But she needs to cover (i) the total waste management cost for the households who do no effort, (ii) a reduced cost for those who reduce her amount of waste, and (iiii) the monitoring cost by having in mind that $\pi = \frac{d}{c}$ and that the control applies to all the effective buyers. She therefore spends on average par effective buyer :

$$c\cdot(1-p(a,d)) + c\cdot(1-\bar{r})\cdot p(a,d) + \frac{d}{c} m\left(\frac{d}{c}\right)$$

If follows, after rearrangement, that the budget constraint is given by :

$$a+d \geq c+\frac{d}{c} m\left(\frac{d}{c}\right)-\bar{r}(c-d) p(a, d)$$

H Proof of proposition 5

Point (i) : $\forall(\alpha,\theta)$, $V_{WMC}^{(\alpha,\theta)} \geq V_{TPT}^{(\alpha,\theta)}$ and WMC satisfies (PC), (IC) and leave the budget in excess.

If $a+d \geq c$, the first result is obvious since :

$$\left\{\begin{array}{l} V_{TPT}^{(\alpha,\theta)}=\max \{0, \alpha-(a+d)+\max \{d \cdot \bar{r}-\theta, 0\}\} \\ V_{WMC}^{(\alpha,\theta)}=\max \{0, \alpha-\min \{c, a+d\}+\max \{\bar{r} d-\theta, 0\}\} \end{array}\right.$$

(PC) is also satisfied since each consumers characterized by $\theta \leq \bar{r}d$ is willing to sign the contract. Now observe that

$$u_{\bar{e}}^{(\alpha,\theta)}(s,\pi)=\alpha-(a+d) \leq \alpha-c=u_0^{(\alpha,\theta)}$$

It follows that cheating is always a dominated strategy, hence (IC) is true.

Let us now move to the budget constraint. The agency collects c per effective buyer and spends (i) the total waste treatment cost for buyers who refuse the contract, (ii) the subsidy which is paid to the agents who accept the WMC, (iii) the remaining waste treatment cost for these agents, and (iv) the monitoring cost of the households who accept the contract. So if we denote by $p_C(a,d)$ the proportion of buyers which refuse the WMC, we can say that the budget constraint, per effective buyer, is given by :

$$\begin{aligned} c \quad \geq \quad & c \cdot p_C(a, d)+(c-(a+d)+\bar{r} d) \cdot(1-p_C(a, d)) \\ & +(1-\bar{r}) \cdot c \cdot(1-p_C(a, d))+\bar{r} \frac{d}{c} m\left(\bar{r} \frac{d}{c}\right) \cdot(1-p_C(a, d)) \end{aligned}$$

By rearranging this expression, we obtain :

$$a+d \geq c+\bar{r} \frac{d}{c} m\left(\bar{r} \frac{d}{c}\right)-\bar{r}(c-d)$$

But we know that the related TPT satisfies the budget constraint. Hence by lemma 3, we can say that budget is in excess since :

$$\begin{aligned} a+d \quad & \geq \quad c+\frac{d}{c} m\left(\frac{d}{c}\right)-\bar{r}(c-d) p(a, d) \\ & \geq \quad c+\frac{d}{c} m\left(\frac{d}{c}\right)-\bar{r}(c-d) \text { since } p(a, d) \leq 1 \\ & > \quad c+\bar{r} \frac{d}{c} m\left(\bar{r} \frac{d}{c}\right)-\bar{r}(c-d) \text { since } \bar{r}<1 \text { and } m(\pi), m\prime(\pi)>0 \end{aligned}$$

Point (ii) : if $a+d \geq c$, the optimal WMC strictly dominates the optimal TPT

Obvious, if one distributes the budget in excess by increasing the subsidy.

Point (iii) : $a+d<c$

By the early definition of $V_{TPT}^{(\alpha,\theta)}$ and $V_{WMC}^{(\alpha,\theta)}$, we immediately $\forall(\alpha,\theta)$, $V_{WMC}^{(\alpha,\theta)}=V_{TPT}^{(\alpha,\theta)}$. The utility allocation is therefore the same under both mechanisms. A similar argument as is point (i) makes sure that (IC) is true. But (IC) is not satisfied since $\forall\,(\alpha,\theta)$ such that $\alpha>a+d$ and $\theta>\bar{r}d$ we have $V_{WMC}^{(\alpha,\theta)}=u_{\bar{e}}^{(\alpha,\theta)}(s,\pi)$, i.e. cheating is optimal. Moreover, since both mechanisms allocates the same utility, the proportion of the effective buyers who perform a waste reduction effort is same and is equal to $p(a,d)$. Let us now move to the budget constraint. Its computation is similar to point (i), we simply has to take into account (i) that a fine is collected and (ii) that every buyer is controlled and perceives a subsidy. We obtain the following constraint per unit of effective buyers.

$$c + (1 - p(a,d)) \cdot \left(\bar{r}\frac{d}{c}\right) \cdot c \geq c \cdot (1 - p(a,d)) + (c - (a+d) + \bar{r}d) + (1 - \bar{r}) \cdot c \cdot (p(a,d)) + \bar{r}\frac{d}{c}m\left(\bar{r}\frac{d}{c}\right)$$

This budget constraint is therefore equivalent to

$$a + d \geq c + \bar{r}\frac{d}{c}m\left(\bar{r}\frac{d}{c}\right) - \bar{r}\,(c-d)\,p(a,d)$$

It follows, by a similar argument as in point (i) that the budget is in excess