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To cite this version:
Eleni Iliopulos. External imbalances and collateral constraints in a two-country world. 2009. halshs-00429600

HAL Id: halshs-00429600
https://halshs.archives-ouvertes.fr/halshs-00429600
Submitted on 3 Nov 2009

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in a two-country world

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2009.65
External imbalances and collateral constraints in a two-country world\textsuperscript{1,2}

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Version: October 14, 2009

\textsuperscript{1}I am very grateful to Guido Ascari, Stefano Bosi, and Tommaso Monacelli for their guidance and advice. I thank Giancarlo Corsetti, Michel Guillaud and Perhat Mihoubi for useful suggestions; I also thank Gaetano Bloise, Nicola Branzoli, Michel Juillard, Thomas Seegmuller, Tareq Sadeq, Thepthida Sopraseuth, the EPEE and the participants of ZEI and of the Dynare conference at the FED of Boston for interesting discussion. All errors are mine.

\textsuperscript{2}This manuscript is a revision of a chapter of my PhD thesis, (Iliopulos, 2008b).

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Résumé

Cet article étudie la dynamique du compte courant dans le cadre d’une grande économie ouverte, caractérisée par la présence d’agents hétérogènes soumis à des contraintes de crédits ainsi qu’une politique monétaire endogène. Nous incorporons trois caractéristiques-clé largement utilisées dans la littérature concernant la Nouvelle Macroéconomie Ouverte : i) un biais domestique intervenant dans le commerce extérieur, ii) des rigidités de prix, et iii) des biens durables (logements). Afin de limiter la tendance des agents à trop consommer et afin d’assurer (partiellement) les créanciers contre le risque de défaut, nous incluons des contraintes de collatéral.

Nous montrons que le degré d’impatience des agents soumis à des contraintes de collatéral peuvent être à l’origine de déséquilibres extérieurs permanents, notre modèle ayant un unique équilibre caractérisé par un niveau de dette extérieure positif.

Notre modèle nous permet d’analyser le lien existant entre les taux de change, les actifs réels et les flux de capitaux internationaux ce qui nous permettra d’analyser la transmission des chocs ainsi que le rôle de la politique monétaire. Par ailleurs, nous analyserons dans quelle mesure l’évolution du marché de l’immobilier peut affecter le compte courant et la dynamique des taux de change.

Codes JEL: E52, F32, F37, F41.

Mots-clés : Dynamique de la dette extérieure ; dynamique du taux de change réel ; biens durables ; contraintes financières ; rigidités de prix ; règles de Taylor.

Abstract

In this article, we focus on current account dynamics in large open economies characterized by debt-constrained heterogeneous agents and endogenous monetary policies. We incorporate three key features that have bulked large in the New Open Macroeconomics literature: i) home bias in trade ii) price rigidities and iii) durable goods (real properties). In order to limit agents’ willingness to consume and to (partially) insure creditors against the risk of default, we incorporate collateral constraints. We show that the impatience of collateral-constrained agents can be at the roots of permanent external imbalances. Indeed our model has a unique and dynamically determinate steady state, which is characterized by a positive level of debt. Our framework allows us to analyze the linkage between exchange rates, real assets and international capital flows. We focus on this mechanism so as to track the (international) transmission of shocks and the implications for the monetary policy. We show how developments hitting the house market can affect current account and exchange rate dynamics.

JEL classification codes: E52, F32, F37, F41.

Keywords: open economy, durable goods, collateral constraints, sticky prices, simple monetary rules.
1. INTRODUCTION

In recent years, agents’ access to credit has been increasingly limited by collateral constraints. The boom of collateral constraints is associated to the development of housing finance in several countries. Indeed, during the last 30 years, housing finance experienced a process of liberalization that introduced more competition and eventually allowed agents to borrow against collaterals. Collateral constraints (partially) insure that in case of default, the creditor can repossess at least (part of) the asset. This is why they are generally associated to better credit conditions for borrowers.

At the same time, the increase in house prices in several countries (together with the development of financial intermediation) has encouraged agents to extract equity from the revaluation of their real assets and further borrow against capital gains. This has eventually entailed the expansion of consumption and household debt.

Collateral constraints create a link between the value of real assets and aggregate domestic debt; in turn, they create a direct link between fluctuations of real asset prices and debt levels. Notice however that spillovers deriving from housing finance are not only a purely domestic issue. Indeed, thanks to the globalized structure of financial markets, intermediaries can convert mortgages into international assets. Both the development of financial intermediation and the boom of collateral constraints have thus reinforced also the link between real assets and the dynamics of international assets. Moreover, since household decisions on housing purchases are affected by interest rate swings, on the one hand, and international financial flows by international returns, on the other, collateral constraints have eventually tightened the mechanism linking real wealth, international capital flows and exchange rates.

The linkage between agents’ wealth, exchange rates and the direction of international capital flows has proved a very powerful mechanism at the roots of financial crises in the emerging world. Krugman (1999) studies the link between currency depreciation and the wealth of collateral-constrained firms in the context of the Asian crisis. He shows that if firms are collateral constrained and a large share of their debt is denominated in foreign currency, depreciation has a negative (and dramatic) impact on the wealth of firms. In turn, as implied by Bernanke and Gertler (1989), the fall in firms’ wealth, together with debt limits, can lead to a collapse in investments and in aggregate demand.

In our analysis we extend Krugman’s (1999) open-economy "Bernanke-Gertler" effect to a DSGE framework for the analysis of current account dynamics. This mechanism is at the roots of the transmission of various stochastic shocks. We focus in particular on developments affecting agents’ wealth (i.e., affecting the housing sector), and their impact on current account and exchange rate dynamics. We track the response of the economy to demand shocks and to the financial liberalization

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4 The share of collateral-constrained agents have increasingly boomed. In US, mortgage debt has increased from about 60% at the half of the last century to about 90% – including vehicles.

5 Indeed, housing finance was a highly regulated sector. For a survey on the developments of housing finance, see IMF (2008).

6 In Figure 1b we show the trends of current account and house prices for a panel of industrial countries characterized by significant external imbalances. For those countries, current account deficits show a co-movement with house prices, in the last decade: increasing current account deficits (surpluses) are indeed associated to rising (decreasing) house prices.

7 Aghion et al. (2000) provide a supply-driven story with analogous implications.
in housing finance. Finally, following Ferrero et al. (2008), we analyze the response of our two-country world to technology shocks; this will provide a deeper understanding of the mechanisms driving the dynamics of our model.\footnote{Iacoviello and Neri (2008) show that technology shocks and housing demand shocks count for one quarter each of the cyclical volatility of US housing investment and prices. Moreover, they show how in recent decades demand factors may have played a more prominent role.}

The role of collateral constraints has been widely analyzed in a closed economy framework. In presence of durable goods, Kiyotaki and Moore (1997) extend the seminal result of Becker (1980) and Becker and Foias (1987) with discounting heterogeneity to the case of collateral constraints. Their analysis shows that the steady-state level of debt (wealth) of impatient (patient) agents is defined by the debt constraint itself; moreover the collateral constraint plays an important role in transmitting the effects of various shocks to other sectors. Indeed, the "financial accelerator mechanism" (see Bernanke et al., 1999) amplifies endogenous developments hitting the credit market. Notice however that when debt contracts are in nominal terms and monetary policy controls interest rates, inflation dynamics amplify demand shocks but dampen supply shocks – working thus as a "financial decelerator" (see Iacoviello 2005). The reason is that, with nominal debt contracts, an increase in inflation has a positive effect on debtors’ wealth and a negative one of the wealth of borrowers – with important implications in light of an optimal monetary policy (see Monacelli, 2007).

In an open economy framework, Callegari (2007), Punzi (2007) and Matsuyama (1990) study the linkage between housing and current account dynamics. Since they do not incorporate nominal prices and exchange rates, there is no scope for them to analyze the role of monetary policy and price rigidities. Analogously, Iacoviello and Minetti (2006) study the interaction between domestic versus foreign borrowing in presence of collateral constraints. None of these works analyzes the above mentioned linkage between exchange rate dynamics, real asset prices and international capital flows.\footnote{Matsuyama (1990) analyzes the effects of fiscal policy shocks on the current account. In his small-economy setting, an increase in government spending represents a negative income shock for households; it thus dampens their consumption of house services and improves the current account balance. See also Iscan (2002) for an empirical analysis on current account and durables.}

In this essay, we extend Iacoviello (2005) and Monacelli (2007, 2008) New-Keynesian framework to a two country-world. This framework is very useful to analyze situations where agents are indebted in equilibrium. We show that, as in Kiyotaki and Moore (1997), discounting heterogeneity and debt limits insure that the collateral constraint is always binding. Moreover, our New-Keynesian approach allows analyzing trends of nominal variables and the role of monetary policy. Price rigidities and the monetary policy stance could indeed play a role in current account and exchange rate dynamics.\footnote{Caballero and Krishnamurthy (2001) build a model on emerging market crises where they analyze the interactions between domestic and international collateral constraints on firms with limited borrowing capacity.}

Eventually, our world economy is characterized by a positive level of external debt: the stricter collateral constraints, the lower the steady-state level of debt. Clearly, all shocks that affect house prices require adjusting external debt – and thus, a current account adjustment process. Terms of trade play an important role...
as a transmission channel of country and sector specific shocks.

The essay is organized as follows. In Section 2 we introduce our model and in Section 3 we analyze the steady state. In Section 4 we focus on the dynamics following shocks hitting the housing market while in Section 5 we analyze the behavior of our model in response to productivity shocks. Section 6 comments the main results of our analysis while the Appendix provides analytical details and Figures.

2. THE MODEL

This model is built on Monacelli (2007, 2008) and Iacoviello (2005) closed economies but is developed in a two-country setting. We consider Home and Foreign respectively (denoted by $H$ and $F$ for simplicity). Both countries are open in every ways but labor. The inhabitants of both countries have same preferences but are heterogeneous in their degree of impatience. More precisely, we assume that the representative inhabitant of country $H$ is more impatient than the one of country $F$. S/he is not a consumption smoother but her/his desire to consume is limited by a collateral constraint. For simplicity, we will denote the inhabitant of country $H$ as the borrower and the one of country $F$ as the saver.\textsuperscript{12}

Durable goods (real properties) and tradable goods are produced in a monopolistic competition framework by domestic and foreign firms; real properties are non-tradable goods and can be used as collateral. Think for instance of houses: leaving tourism a part, houses are generally owned by and sold to residents.\textsuperscript{13} Goods are then purchased and sold to final consumers by domestic retailers, in a competitive environment. The representative retailer in the housing sector in country $H$ (in country $F$), buys Home (Foreign)-produced durables and sell them to final consumers in country $H$ (country $F$) only. Analogously, the representative retailer in the tradable sector, buys both Home and Foreign-produced goods to sell them to final consumers in the Home (Foreign) country only. Final consumers enjoy services deriving from durables and consume tradable goods. Finally, agents can smooth their consumption by exchanging securities on international incomplete markets; debt in country $H$ is subject to a collateral constraint.

2.1. Retailers

We suppose that intermediate goods are sold in both countries to final consumers by an infinite set of retailers operating in a competitive environment. Goods markets in each country are segmented into the tradable and real properties sector. We will denote by $j = T$, the representative retailer operating in the tradable sector; $j = n$, the representative retailer operating in the durable sector (real properties). The retailer $j = T$ in country $H$ (in country $F$) buys Home and Foreign-produced tradables and sell them to the Home (Foreign) market. The retailer $j = n$ buys real properties produced in country $H$ (country $F$) and sell them to the Home (Foreign) market.

2.1.1. Tradables

Consider first the case of the retailer operating in the tradable sector in country $H$. S/he has access to both domestic and foreign-produced goods and sell them

\textsuperscript{12} Notice however that this will be an endogenous result of the model.

\textsuperscript{13} See also Engel and Wang (2008) for some discussion.
to $H$ final consumers only. In order to reflect consumer’s preferences, we assume that the behavior of retailers is affected by home bias.\textsuperscript{14} Retailers operate in a perfectly competitive environment and their basket of production is the following CES bundle:

$$Y_{T,t} = \left[ \alpha^\frac{1}{\eta} Y_{h,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^\frac{1}{\eta} Y_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{1}{\eta-1}}$$

where $\alpha$ represents the weight of Home produced goods in consumers’ bundles (in presence of home bias, $\alpha > 0.5$) and $\eta > 0$ is the elasticity of substitution between Home and Foreign produced goods. For simplicity, from now on we denote all variables referred to Home (Foreign) goods with the index $h$ (index $f$). Retailers’ demand for respectively Home and Foreign produced goods is the result of an optimization problem. The CES-related price index of tradables is:

$$P_{T,t} = \left[ \alpha P_{h,t}^{1-\eta} + (1-\alpha) P_{f,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} (1)$$

Retailers’ intermediate demand for Home produced and Foreign produced goods ($Y_{h,t}$ and $Y_{f,t}$, respectively) is:

$$Y_{h,t} = \alpha Y_{T,t} \left( \frac{P_{h,t}}{P_{T,t}} \right)^{-\eta} \quad (2)$$

$$Y_{f,t} = (1-\alpha) Y_{T,t} \left( \frac{P_{f,t}}{P_{T,t}} \right)^{-\eta} \quad (3)$$

In country $F$, retailers in the tradable sector behave symmetrically. This implies that the weight of Foreign produced goods on country $F$ CES production bundle is the same as the one of Home produced goods in country $H$\textsuperscript{15}, i.e.:

$$Y_{T,t}^* = \left[ (1-\alpha)^{\frac{1}{\eta}} Y_{h,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} Y_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{1}{\eta-1}} \quad (4)$$

Profit maximization implies that retailers’ intermediate demand for foreign and domestic goods respectively in country $F$ is:

$$Y_{h,t}^* = (1-\alpha) Y_{T,t}^* \left( \frac{P_{h,t}^*}{P_{T,t}^*} \right)^{-\eta} \quad (5)$$

$$Y_{f,t}^* = \alpha Y_{T,t}^* \left( \frac{P_{f,t}^*}{P_{T,t}^*} \right)^{-\eta} \quad (6)$$

Notice however that retailers also need to choose amongst the different (infinite) varieties of domestic and foreign goods, $i$. We suppose that the Home (Foreign)-produced basket of the representative retailer is in turn a CES bundle of a continuum of infinite varieties of goods, $i$. Retailers’ intermediate demand for a single variety of Home produced (Foreign) good is thus:

\textsuperscript{14}An alternative way to introduce home bias in our model would be to leave the choice between domestically-produced versus foreign-produced goods to consumers.

\textsuperscript{15}The corresponding price index is: $P_{T,t}^* = \left[ (1-\alpha) P_{h,t}^{*1-\eta} + \alpha P_{f,t}^{*1-\eta} \right]^{\frac{1}{1-\eta}}$
\[ Y_{h,t}(i) = Y_{h,t} \left( \frac{P_{h,t}(i)}{P_{h,t}} \right)^{-\varepsilon} \quad (7) \]

\[ Y_{f,t}(i) = Y_{f,t} \left( \frac{P_{f,t}(i)}{P_{h,t}} \right)^{-\varepsilon} \quad (8) \]

where \( \varepsilon > 1 \). Analogously, in the rest of the world:

\[ Y_{h,t}^*(i) = Y_{h,t}^* \left( \frac{P_{h,t}^*(i)}{P_{h,t}^*} \right)^{-\varepsilon} \quad (9) \]

\[ Y_{f,t}^*(i) = Y_{f,t}^* \left( \frac{P_{f,t}^*(i)}{P_{f,t}^*} \right)^{-\varepsilon} \quad (10) \]

2.1.2. Durable

Consider now the housing sector. The representative retailer in country \( H \) chooses a set of houses amongst an infinite continuum of domestically produced real properties. Her/his demand for each differentiated good, \( i \), is the result of profit maximization in a competitive environment \( 17 \), i.e.:

\[ Y_{n,t}(i) = Y_{n,t} \left( \frac{P_{n,t}(i)}{P_{n,t}} \right)^{-\varepsilon} \quad (11) \]

where \( \varepsilon \) is the elasticity of substitution between single goods, \( i \). Analogously, in the Foreign country, the intermediate demand for real properties of the representative retailer is:

\[ Y_{n,t}^*(i) = Y_{n,t}^* \left( \frac{P_{n,t}^*(i)}{P_{n,t}^*} \right)^{-\varepsilon} \quad (12) \]

2.2. Optimal consumption in country \( H \)

Consider now the representative inhabitant of country \( H \). His/her utility is a positive function of her/his basket of consumption, \( C_t \) and a negative function of her/his labor supply, \( N_t \), i.e. \( 19 \):

\[ \max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\} \]

\[ 16 \text{In order to keep countries } H \text{ and } F \text{ as symmetric as possible, we assume identical elasticities of substitution across countries.} \]

\[ 17 \text{Where the CES production bundle of the retailer is: } Y_{n,t} \equiv \left( \int Y_{n,t}(i)^{\frac{\varepsilon-1}{\varepsilon-1}} d_i \right)^{\frac{1}{\varepsilon-1}}. \text{ The associated price index is: } P_{n,t} \equiv \left( \int P_{n,t}(i)^{1-\varepsilon} d_i \right)^{\frac{1}{1-\varepsilon}} \]

\[ 18 \text{For simplicity, we assume that the elasticity of substitution between the infinite varieties of Home (Foreign) produced goods is the same for both sectors.} \]

\[ 19 \text{In our economy all agents have same preferences and maximize a generic utility function. In the numerical simulations we will consider the following functional form: } U_t = \ln C_t - \left( \frac{N_t}{1+\varphi} \right)^{1+\varphi} \]
where $\beta$ is the borrower’s discount factor. We do not introduce explicitly money in the utility function and we use it as the **numéraire** of our cashless economy à la Woodford.

The representative borrower consumes a bundle that is a CES composite of tradables and services deriving from the stock of real properties. For simplicity, we assume that agents start enjoying services deriving from durables in the same period they purchase them and are proportional to the stock of houses. We also assume that agents cannot rent/lend houses. The borrower consumption basket is thus:

$$C_t = \left[ \gamma^\frac{n-1}{2} C_{T,t}^\frac{n-1}{2} + (1 - \gamma)^\frac{n-1}{2} C_{n,t}^\frac{n-1}{2} \right]^{\frac{2}{n-1}}$$

where $\gamma$ is the weight of tradables in the basket and $\theta \geq 0$ is the elasticity of substitution between durable services and tradables. The following assumption on preferences always needs to hold in both countries:

**Assumption 1 (preferences)** $U_{k2} > 0$, $U_{kN} < 0$, $U_{kCC} < 0$, $U_{kCC} > U_{kNN}$ for every $(C_k, N_k)$ such that $C_k, N_k > 0, k = \text{borrower, saver}$.

Also, Inada conditions for consumption hold.

The individual budget constraint in real terms of tradable consumption is:\n
$$C_{T,t} + x_t (C_{n,t} - (1 - \delta) C_{n,t-1}) + R_{t-1} \frac{b_{t-1}}{\pi_{T,t}} \leq b_t + \frac{W_t N_t}{P_{T,t}} + \sum \Gamma \frac{P_{T,t}}{P_{T,t}} (13)$$

where $C_{T,t}$ represents tradable consumption, $P_{n,t} (C_{n,t} - (1 - \delta) C_{n,t-1})$ is the cost of durable expenditure in period $t$; $b$ are net external liabilities in real terms of tradable consumption\(^{22}\), where $B \equiv D - qD^*$ are net external liabilities in nominal terms, $D$ are home-currency domestic securities, $q$ is the nominal price of Foreign currency in terms of Home currency and $D^*$ are foreign-currency Foreign securities\(^{23}\). Notice also that $x_t \equiv \frac{P_{n,t}}{P_{T,t}}$ is the relative price of real properties and $\pi_{T,t} \equiv \frac{P_{T,t}}{P_{T,t-1}}$ is the aggregate inflation rate in the tradable sector\(^{24}\). In practice, in each period the borrower buys tradables, $C_T$, and real properties; s/he pays the debt service on her/his debt, where $R$ is the gross nominal interest rate factor. S/he enjoys resources coming from foreign borrowing, $B$, labor income, $WN$ and profits, $\Gamma$ coming from their firms (operating in a monopolistic competition framework). Labor is assumed mobile across sectors but not across countries; therefore, the wage is the same in each sector but not necessarily in each country.

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\(^{20}\)This implies that CPI inflation corresponds to aggregate inflation in the tradable goods sector.

\(^{21}\)The individual budget constraint in nominal terms is:

$$P_{T,t} C_{T,t} + P_{n,t} (C_{n,t} - (1 - \delta) C_{n,t-1}) + R_{t-1} b_{t-1} \leq B_t + W_t N_t + \sum \Gamma$$

The budget constraint is assumed to hold with equality around the deterministic steady state.

\(^{22}\)Notice that $b_{t-1} = \frac{B_{t-1}}{P_{T,t-1}}$.

\(^{23}\)See the Appendix for all details concerning the optimization program of the consumer.

\(^{24}\)The inflation rate in the durable sector is defined as $\pi_{n,t} \equiv \frac{P_{n,t}}{P_{n,t-1}}$, while $\pi_{h,t} \equiv \frac{P_{h,t}}{P_{h,t-1}}$. 

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[8]
Borrowers’ capacity to obtain credit is limited by a collateral constraint. We suppose that households’ debt is constrained to be a share of the value of their durables (real properties), and debt contracts are issued in nominal terms, i.e.:

\[ b_t \leq (1 - \chi) C_n,t x_t \]  

(14)

where \( \chi \) is the fraction of durables that cannot be used as a collateral and can be interpreted as the inverse of the loan-to-value ratio: the larger \( \chi \), the more stringent the constraint. For simplicity, we assume \( \chi \) to be an exogenous parameter of our model. The role of collateral constraints and the implications of their structure has been recently analyzed in a New-Keynesian framework by Calza et al. (2006) in a closed economy framework.\(^{25}\) Eventually, collateral constraints allow agents a better access to credit. Indeed, they partially ensure the creditor against the risk of default: in case of default, the creditor can always repossess (part of) the asset.\(^{26}\)

In our two-country world, constraint (14) reduces to a limit on international borrowing. This should not surprise the reader. Since we aim at analyzing current account dynamics, we are interested in the behavior of aggregate variables, and in the dynamics of flows (of goods and financial capital) between countries. In aggregate, the sum of domestically traded assets and liabilities in each country is equal to zero. Thus, if indebted, our representative agent of each country cannot but be indebted towards his foreign counterpart only. Indeed, thanks to the globalized structure of financial systems, mortgages can be easily converted into international assets. Our representative agent in each country can thus act as a financial intermediary and sold her/his (collateralized) debt abroad. In this vein, collateral constraints (and their impact) are transferred to an international dimension.

Notice finally that (14) implies also that an increase in the relative price of real properties allows agents to increase their level of debt.

The first order conditions of borrowers’ optimization program are:

\[
\begin{align*}
-\frac{U_{N,t}}{U_{T,t}} & = \frac{W_t}{P_{T,t}} \\
x_t U_{T,t} & = U_{n,t} + \beta (1 - \delta) E_t \left\{ U_{T,t+1} x_{t+1} \right\} + U_{T,t} \psi_t (1 - \chi) x_t \\
\psi_t & = 1 - \beta E_t \left\{ \frac{R_t}{\pi_{T,t+1}} \frac{U_{T,t+1}}{U_{T,t}} \right\}
\end{align*}
\]

(15) \hspace{2cm} (16) \hspace{2cm} (17)

Equation (15) represents a standard consumption/leisure arbitrage. Equation (16) represents the intertemporal demand for tradable consumption relatively to durables. In equilibrium, the value of the utility deriving to the borrower from present consumption of tradables needs to equal the value of direct utility deriving from the direct housing services plus the value of their indirect utility, i.e.: i) the utility deriving from the possibility of selling real properties and buying durable consumption in future, \( \beta (1 - \delta) E_t \left\{ U_{T,t+1} x_{t+1} \right\} \) and ii) the marginal utility stemming from relaxing the collateral constraint, and consuming additional non-durable goods, \( U_{T,t} \psi_t (1 - \chi) x_t \).

Equation (17) represents a modified Euler equation for country \( H \) – where \( \lambda_t \psi_t \) is the Lagrangian multiplier associated to the collateral constraint and \( \lambda_t \) the one

\(^{25}\)For more discussion see also Monacelli (2007, 2008), Iacoviello (2005) and Campbell and Hercowitz (2006).

\(^{26}\)In Bernanke and Gertler (1989) they are justified by the presence of private information and limited liability. In Kiyotaki and Moore (1997) they are the response to problems of enforcing contracts.
associated to the budget constraint. Clearly, it reduces to the standard Euler equation whenever \( \psi_t = 0 \). If \( \psi_t > 0 \), the marginal utility of current consumption exceeds the gain of shifting intertemporally one unit of consumption (savings):

\[
U_{T,t} > \beta E_t \left\{ \frac{R_t}{\pi_{T,t+1}} U_{T,t+1} \right\}.
\]

If this is the case in each period, the representative agent in country \( H \) is not a consumption smoother and finances current consumption with as much credit as s/he can have access to (i.e., the collateral constraint is binding). Clearly, \( \psi_t \) also represents the marginal value of additional debt\(^{27}\): by marginally relaxing the collateral constraint one can have access to more current consumption.

Finally, one can rewrite (16) as:

\[
\frac{U_{n,t}}{U_{T,t}} = x_t (1 - \psi_t (1 - \chi)) - \beta (1 - \delta) E_t \left\{ \frac{U_{T,t+1}}{U_{T,t}} x_{t+1} \right\}
\]  

(18)

Notice that the RHS of equation (18) represents the user cost of durables in terms of non-durables at time \( t \). It represents the price you pay for the flow of consumption services from durables during the period; the cost is obviously a positive function of the interest rate and the relative price of durables (for \( \psi_t \) fixed). By substituting (17) in (18) and log-linearizing (18), it is possible to isolate the effect of \( \psi \) on the user cost, during the dynamics around the steady state, as shown in the Appendix (its impact on the steady state is also discussed in the Appendix). The impact of a variation in \( \psi \) on the user cost crucially depends both on the structure of the collateral constraint and the parametrization. In our framework, it is generally negative. This implies that a decrease in the marginal utility of borrowing makes houses less useful and entails a substitution effect in favour of tradables.\(^{28}\)

### 2.3. Exchange rates and terms of trade

In presence of home bias, the law on one price only holds for the single basket of Foreign produced and Home produced tradables, separately. In practice:

\[
P_{h,t} = q_t P_{h,t}^*
\]

and

\[
P_{f,t} = q_t P_{f,t}^*
\]

where \( q \) is the nominal exchange rate (the Home-currency price of Foreign currency) and \( \alpha > \frac{1}{2} \) implies \( P_{T,t} \neq q_t P_{T,t}^* \).

In addition, in equilibrium, the following relationship between Home and Foreign net external liabilities always needs to hold:

\[
B_t = q_t B_t^*
\]  

(19)

where \( B \) is Home-currency net external debt in nominal terms and \( B^* \)\(^{29}\) are lenders’ net external assets in Foreign currency. Obviously, if \( B > 0 \) borrowers are net debtor and savers are net lenders.

\(^{27}\) \( \psi \) can also be interpreted as the price of an asset; indeed, it is tied to a payoff that depends on the deviation from the Euler equation – see Monacelli (2007).

\(^{28}\) Having said that, for very small depreciation rates, we recover Monacelli (2008) result in presence of a (slightly) different collateral constraint. See the Appendix.

\(^{29}\) Clearly, \( B^* = \frac{D}{q} - D^* \)
In our two-country simple world, houses cannot be rent. This implies that the CPI price index coincides with the aggregate price index of tradable goods, $P_T$.\footnote{Indeed, the CPI basket does not include the price of houses.} It is thus straightforward to calculate the CPI real exchange rate of country $H$ as:

\[
\epsilon_t = q_t \frac{P^*_t}{P_T^t}.
\]

For simplicity, from now on when referring to the real exchange rate of $H$, we will consider the CPI based real exchange rate.\footnote{Analogously, in presence of durable goods, Engel and Wang (2008) build a non-utility based consumption price index. This index is calculated as a weighted average of different prices subindexes and is used to compute the real exchange rate of their economy.} Notice finally that in absence of home bias, the CPI-based real exchange rate is constant in all periods; this would certainly be at odds with reality (see Figure 1a).

We finally introduce country $H$ terms of trade and we define them in the following way:

\[
S_t = \frac{P_{f,t}}{P_{h,t}} = \frac{P^*_{t}}{P^*_t}.
\]

Symmetrically, country $F$ terms of trade are thus:

\[
S^*_t = \frac{P_{h,t}}{P_{f,t}} = \frac{P^*_{t}}{P^*_t} = S_t^{-1}.
\]

The bond market-clearing condition (19) can be rewritten as a function of terms of trade, i.e.:

\[
b^*_t = b_t \left( \frac{(1 - \alpha) S_t^{1 - \eta} + \alpha}{\alpha S_t^{1 - \eta} + 1 - \alpha} \right)^{\frac{1}{1-\eta}}
\]

**2.4. Optimal consumption in country F**

We consider now the representative agent of country $F$. We suppose that agents in country $F$ are more patient than agents in country $H$. Thus, the discount rate of the borrower is strictly lower than the one of country $F$ representative agent (the saver). Savers maximizes the following utility function:

\[
\max E_0 \left\{ \sum_{t=0}^\infty \mu^t U^* (C^*_t, N^*_t) \right\}
\]

where $\mu$ is their discount factor, $N^*_t$ is labor effort and $C^*_t$ is a CES composite good of tradables and services arising from durables. For simplicity, from now on, all variables referring to the foreign country (and currency) will be indexed by an asterisk.

Assumption 2 always holds:

**Assumption 2 (discounting)** $\beta, \mu \in (0, 1); \beta < \mu.$
Assumption 2 is crucial in explaining external debt dynamics in our model; it is not new in international finance.\textsuperscript{32} Ghironi et al. (2005) introduce heterogeneous discounting in an overlapping generation framework\textsuperscript{33}. More recently, Choi et al. (2008) track the dynamics of US current account introducing endogenous heterogeneous discounting.

Savers’ consumption basket is composed as the one of borrowers, i.e.:

\[ C_t^s = \left[ \gamma^\frac{1}{\nu} C^*_T, t^{\frac{\theta - 1}{\nu}} + (1 - \gamma)^{\frac{1}{\nu}} C^*_{n, t}^{\frac{\theta - 1}{\nu}} \right]^{\frac{1}{\theta - 1}} \]

where \( C^*_n \) are services deriving from real properties in the Foreign country and \( C^*_T \) is a basket of tradables. The budget constraint of the \( F \)-agent in real terms of tradable consumption is\textsuperscript{34}:

\[ C^*_T, t + x^*_t (C^*_n, t - (1 - \delta) C^*_n, t - 1) - R^*_t \frac{b^*_t}{\pi^*_T, t} \leq -b^*_t + \frac{W^*_t N^*_t}{P^*_T, t} + \sum_i \Gamma^*_i \]  \hspace{1cm} (22)

where \( W^*_t \) are foreign-currency wages in the Foreign country, \( R^*_t \) are nominal interest rates in the Foreign country and \( \Gamma^*_i \) are profits. Finally, \( B^* \equiv \frac{D}{q} - D^* \) are Foreign net external assets in Foreign currency and \( b^*_t \) are net external assets in real terms of tradables. Finally, we introduce a no-Ponzi game condition on net international assets\textsuperscript{35}:

\[ \lim_{i \to \infty} E_t \left( \frac{b^*_{t+i}}{R^*_{z}} \right) \leq 0 \]  \hspace{1cm} (23)

The first order conditions of savers’ optimization program are:

\[-U^*_N, t = \frac{W^*_t}{P^*_T, t} \]  \hspace{1cm} (24)

\[ U^*_t, t x^*_t = U^*_n, t + \mu (1 - \delta) E_t \left\{ x^*_t+1 U^*_T, t+1 \right\} \]  \hspace{1cm} (25)

\[ U^*_t, t = \mu E_t \left\{ U^*_T, t+1 \frac{R^*_t}{\pi^*_T, t+1} \right\} \]  \hspace{1cm} (26)

Equation (24) states the standard arbitrage condition between leisure and consumption and equation (25) is the intertemporal demand equation for durables versus nondurable goods. Equation (26) is a standard Euler equation.

Finally, we need to introduce the relation linking interest rates in country \( H \) and in country \( F \). In order to obtain it, we substitute for gross external assets and liabilities in the budget constraint of country \( H \) or \( F \) (remember that \( B \equiv D - qD^* \)). Rewriting the budget constraint in real terms of tradable consumption and recalculating the first order conditions for the borrower and/or the lender, one can easily find the needed condition. It is possible to show (see the Appendix for all details) that the following modified uncovered interest parity condition needs to hold:

\textsuperscript{32}Our hypothesis is also consistent with Masson et al. (1994) and Henriksen (2002) who relate current account dynamics with demographic factors. Analogously, Chinn and Prasad (2000) find that demographic factors are significant determinant of the current account balance.

\textsuperscript{33}For more discussion, see also Buiter (1981) and Weil (1989).

\textsuperscript{34}It holds with equality around a deterministic steady state.

\textsuperscript{35}This condition does not bind. Analogously, given lenders’ relative patience, any collateral constraint imposed on their borrowing would not bind (see the discussion in the following sections).
\[ R_t = E_t \left\{ \frac{q_{t+1}}{q_t} \right\} R_t^{*} \]  

(27)

The absence of arbitrage possibilities between domestic and foreign assets implies that the marginal utility of investing in Home assets is equal to the one agents obtain by investing in Foreign assets. Notice finally that given the stochastic setup of our framework and the assumption of incomplete markets, the uncovered interest parity condition only holds in expectations.

2.5. Production

We now set the structure of production in our two-country world. For simplicity we suppose that labor is homogeneous and mobile across sectors in the same country – but not around the world. We assume also that the representative agent in each country is also the owner of representative firms in each country. Markets in each country are segmented into tradables and durables (real properties). Firms in both sectors operate in a monopolistic competition environment and are characterized by the good they produce.

Real properties’ producers at Home (in the Foreign country) only sell their goods to Home (Foreign) markets while tradables’ producers sell them to retailers of both countries. We suppose that there are \( i \) firms producing \( i \) non perfectly substitutable durables (tradable goods). Each firm is characterized by a production function \( F \), which depends on labor, \( N_l \) and a productivity shifter \( A_l \), which is common for all firms within the same sector, \( l = h, n \ (f, n) \). The following proposition needs to hold:

Assumption 3 (technology): \( F \) is homogeneous of degree 1 with \( F \in C^2, F_N > 0, F_{NN} \leq 0 \). Moreover \( F (0) = 0, \lim_{N \rightarrow 0} F' (N) = +\infty, \lim_{N \rightarrow 0} F' (N) = 0 \).

2.5.1. Tradables

We first focus the attention on the tradable sector in country \( H \). Firm \( i \) production function is consistent with Assumption 3 and is defined for simplicity as:

\[ F_{i,t}(N_{h,t} (i)) = A_{h,t} N_{h,t} (i) \]

Firms maximize the profit function:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \lambda_{0,t} \left( F_{i,t}(N_{h,t} (i))P_{h,t} (i) - W_t N_{h,t} (i) - \omega T_{P_{h,t}} \left( \frac{P_{h,t} (i)}{P_{h,t-1} (i)} - 1 \right)^2 P_{h,t} \right) \right\}
\]

given retailers’ demand functions. \( \lambda \) is the borrower’s stochastic discount factor and:

\[ \lambda_{t,t+1} = \frac{\lambda_{0,t+1}}{\lambda_{0,t}} \equiv \beta E_t \left\{ \frac{1}{1 - \psi_t} \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{T,t}}{P_{T,t+1}} \right\} \]

where \( \lambda \) is the borrower’s Lagrangian multiplier (i.e., the marginal utility of income) of the representative consumer in country \( H \) and
are the firm’s costs associated to adjusting prices (menu costs); following Rotemberg and Woodford (1997), we assume that adjustment costs are quadratic. In practice, each period firms need to optimally balance the costs arising from resetting prices and the costs associated to deviating from optimality.

Analogously, the stochastic discount factor of firms in country $F$ is:

$$\Lambda_{t,t+1}^* = \frac{\Lambda_{t,t+1}^*}{\Lambda_{0,t}^*} = \mu E_t \left\{ \frac{\lambda_{t+1}^*}{\lambda_t^*} \left( \frac{P_{T,t+1}^*}{P_{T,t}^*} \right) \right\},$$

where $\Lambda^*$ is lenders’ stochastic discount factor and $\lambda^*$ is the Lagrangian multiplier associated to the saver’s budget constraint.

In both countries firms choose their optimal sequence $\{N_{h,t} (i), P_{h,t} (i)\}$, $\left\{ \{N_{f,t}^* (i), P_{f,t}^* (i)\} \right\}$. Nominal and real marginal costs in $H$ ($MC$ and $mc$, respectively) are:

$$MC_{h,t} = \frac{W_t}{A_{h,t}}$$
$$mc_{h,t} = \frac{W_t}{P_{h,t} A_{h,t}}$$

(28)

In a symmetric equilibrium: $P_{h,t} (i) = P_{h,t}$. We can thus simply solve for optimal prices. We obtain the following New Keynesian Phillips Curve (NKPC):

$$\omega_T (\pi_{h,t} - 1) \pi_{h,t} = F_{h,t} (N_{h,t}) \varepsilon \left[ \frac{(1 - \varepsilon)}{\varepsilon} + mc_{h,t} \right]$$

$$+ \omega_T \beta E_t \left\{ \left( \frac{1}{1 - \psi_t} \right) \frac{U_{T,t+1}}{U_{T,t}} \left[ \frac{\alpha + (1 - \alpha) S_t^{1-\eta}}{\alpha + (1 - \alpha) S_t^{1+\eta}} \right]^{1-\eta} (\pi_{h,t+1} - 1) \pi_{h,t+1} \right\}$$

The standard optimization program of the representative agent implies that in equilibrium there cannot be gains in exchanging leisure with consumption; the non-arbitrage condition leisure/consumption (15) needs to hold. Condition (15) can be here rewritten as:

$$-\frac{U_{N,t}}{U_{T,t}} = \frac{W_t}{P_{h,t} A_{h,t}} \left[ \frac{\alpha + (1 - \alpha) S_t^{1-\eta}}{\alpha + (1 - \alpha) S_t^{1+\eta}} \right]^{1-\eta}$$

therefore, when we substitute it in (28), we obtain:

$$mc_{h,t} = \frac{1}{A_{h,t}} \frac{-U_{N,t}}{U_{T,t}} \left[ \frac{\alpha + (1 - \alpha) S_t^{1-\eta}}{\alpha + (1 - \alpha) S_t^{1+\eta}} \right]^{1-\eta}$$

(29)

Clearly, terms of trade affects the Phillips curve both in the form of the marginal cost, and through the discount factor. By incorporating (29) and the relation $F_{h,t} (N_{h,t}) = A_{h,t} N_{h,t}$ in the above Phillips curve, we obtain:

$$\omega_T (\pi_{h,t} - 1) \pi_{h,t} = A_{h,t} N_{h,t} \varepsilon \left[ \frac{(1 - \varepsilon)}{\varepsilon} + \frac{1}{A_{h,t}} \frac{-U_{N,t}}{U_{T,t}} \left( \frac{\alpha + (1 - \alpha) S_t^{1-\eta}}{\alpha + (1 - \alpha) S_t^{1+\eta}} \right)^{1-\eta} \right]$$

(30)
Without price rigidities, the real marginal cost is constant at the mark-up. Notice also that in the tradable sector, terms of trade create a wedge between the rate of substitution between consumption and leisure on the one hand, and the marginal product of labor on the other. The real marginal cost is directly affected by movements in terms of trade. This creates a scope for policy intervention whenever the policy-maker aims at optimal policies (for some discussion see Faia and Monacelli, 2008).

Analogous considerations apply for country $F$. Marginal costs are:

\[ mc^*_{f,t} = \frac{1}{A^*_{f,t}} \frac{-U^*_{N,t}}{U^*_{T,t}} \left[ \alpha + (1 - \alpha) S^{-1+\eta}_t \right] \frac{1}{1-\eta} \]  

and the NKPC is:

\[ \omega_T (\pi^*_f - 1) \pi^*_f = A^*_f, t N^*_{f,t} \left[ \frac{(1 - \varepsilon)}{\varepsilon} + \left( \alpha + (1 - \alpha) S^{-1+\eta}_t \right) \frac{1}{1-\eta} \frac{-U^*_{N,t}}{A^*_f, t U^*_{T,t}} \right] \]

\[ + \omega_T \mu E_t \left\{ \frac{U^*_{T,t+1}}{U^*_{T,t}} \left[ \alpha + (1 - \alpha) S^{-1+\eta}_{t+1} \right] \frac{1}{1-\eta} (\pi^*_{f,t+1} - 1) \pi^*_f \right\} \]  

2.5.2. Durables

The price dynamics in the housing sector can be easily individuated by following the same lines of the previous section. The New Keynesian Phillips curve for durables (real properties) is:

\[ \omega_n (\pi_{n,t} - 1) \pi_{n,t} = A_{n,t} N_{n,t} \left[ \frac{(1 - \varepsilon)}{\varepsilon} + mc_{n,t} \right] + \omega_n \beta E_t \left\{ \frac{1}{1 - \psi_t} \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} (\pi_{n,t+1} - 1) \pi_{n,t+1} \frac{P_{n,t+1}}{P_{n,t}} \right\} \]

where, again:

\[ \Lambda_{t,t+1} = \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} = \beta E_t \left\{ \frac{1}{1 - \psi_t} \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{T,t}}{P_{T,t+1}} \right\} \]

and thus:

\[ \omega_n (\pi_{n,t} - 1) \pi_{n,t} = A_{n,t} N_{n,t} \left[ \frac{(1 - \varepsilon)}{\varepsilon} + mc_{n,t} \right] + \omega_n \beta E_t \left\{ \frac{1}{1 - \psi_t} \frac{U^*_{T,t+1} x_{t+1}}{U^*_{T,t} x_t} (\pi_{n,t+1} - 1) \pi_{n,t+1} \right\} \]

Also, given the arbitrage consumption-leisure, we can rewrite firms’ marginal costs in the housing sector as:

\[ mc_{n,t} = \frac{1}{A_{n,t} x_t} \frac{-U_{N,t}}{U^*_{T,t}} \]
Notice that there is a wedge between the ratio of marginal utilities and the marginal cost. This wedge is created by a variation in the relative price, $x$. When the monetary authority aims at implementing an optimal policy, the presence of this wedge leaves the policy maker a scope for policy intervention (for some discussion see Monacelli, 2007).

Incorporating (33) in the above New-Keynesian Phillips curve, we obtain:

$$\omega_n (\pi_{n,t} - 1) \pi_{n,t} = A_{n,t} N_{n,t} \varepsilon \left[ \frac{(1 - \varepsilon)}{\varepsilon} + \frac{1}{x_t} \frac{-U_{N,t}}{U_{T,t} A_{n,t}} \right] + \omega_n \beta E_t \left\{ \left( \frac{1}{} \right) \frac{U_{T,t+1}}{U_{T,t}} \frac{x_{t+1}}{x_t} \left( \pi_{n,t+1} - 1 \right) \pi_{n,t+1} \right\}$$

(34)

Analogously, the NKPC in country $F$ is:

$$\omega_n (\pi^*_{n,t} - 1) \pi^*_{n,t} = A^*_{n,t} N^*_{n,t} \varepsilon \left[ \frac{(1 - \varepsilon)}{\varepsilon} + \frac{1}{x_t^*} \frac{-U^*_{N,t}}{U^*_{T,t} A^*_{n,t}} \right] + \omega_n \beta E_t \left\{ \left( \frac{}{} \right) \frac{U^*_{T,t+1}}{U^*_{T,t}} \frac{x^*_{t+1}}{x_t} \left( \pi^*_{n,t+1} - 1 \right) \pi^*_{n,t+1} \right\}$$

(35)

where real marginal costs are:

$$m c^*_{n,t} = \frac{1}{x_t^*} \frac{-U^*_{N,t}}{U^*_{T,t} A^*_{n,t}}$$

(36)

Notice that we have a priori allowed for the possibility of price rigidities in the housing sector. Having said that, we will reasonably assume in our benchmark simulations that house prices are not rigid. Indeed, as Iacoviello and Neri (2008)\textsuperscript{36} remark, most houses are priced for the first time when they are sold. Moreover, since each house is a very expensive good, in case menu costs had a significant fixed component, the incentive to renegotiate its price would predominate. \textsuperscript{37}

### 2.6. Market clearing

#### 2.6.1. Home country

For markets to be cleared in country $H$, total purchases of real properties need to equal the total domestic production; they also need to account for the costs of price rigidities. We remind the reader that in this model real properties are non-tradable goods. Thus:

$$A_{n,t} N_{n,t} = C_{n,t} - (1 - \delta) C_{n,t-1} + \frac{\omega_n}{2} (\pi_{n,t} - 1)^2$$

Given that labor is not mobile across countries, labor market clearing implies:

$$N_{n,t} + N_{h,t} = N_t$$

(37)

Therefore:

$$A_{n,t} N_{n,t} = C_{n,t} - (1 - \delta) C_{n,t-1} + \frac{\omega_n}{2} (\pi_{n,t} - 1)^2$$

(38)

\textsuperscript{36}See also Barsky, House and Kimball (2007).

\textsuperscript{37}Our simplified framework does not allow to model phenomena related to asset prices bubbles. Further research should focus on possible ways to introduce price bubbles in the housing sector.
Focus now on the Home sector of tradables. Market clearing requires:

$$A_{h,t} N_{h,t} = C_{h,t} + C^*_h + \frac{\omega}{2} (\pi_{h,t} - 1)^2$$

where $C_h$ and $C^*_h$ are consumption levels of Home tradables at Home and in the Foreign country, respectively. Notice that local retailers of tradables in country $H$ operate in a perfectly competitive environment and only sell their products to Home inhabitants (in practice, they simply act as aggregators). Therefore, the market of Home-produced tradables clears when the total amount of produced goods equals the aggregate demand of retailers. In practice, $C_{h,t} = Y_{h,t}$, $C^*_t = Y^*_{h,t}$, $C_{T,t} = Y_{T,t}$. Recalling that retailers’ demand for domestically and foreign produced goods are respectively, $Y_{h,t} = \alpha Y_{T,t} \left( \frac{P_{h,t}}{P_{T,t}} \right)^{-\eta}$ and $Y_{f,t} = (1 - \alpha) Y_{T,t} \left( \frac{P_{f,t}}{P_{T,t}} \right)^{-\eta}$ the market clearing condition for the tradable sector can be rewritten first as:

$$A_{h,t} N_{h,t} = \alpha Y_{T,t} \left( \frac{P_{h,t}}{P_{T,t}} \right)^{-\eta} + (1 - \alpha) Y^*_{T,t} \left( \frac{P^*_{h,t}}{P^*_{T,t}} \right)^{-\eta} + \frac{\omega}{2} (\pi_{h,t} - 1)^2$$

and then as a function of terms of trade, i.e.:

$$A_{h,t} N_{h,t} = \alpha C_{t,t} \left[ \alpha + (1 - \alpha) S^{1-\eta} \right]^{1-\eta} + (1 - \alpha) C^*_t \left[ (1 - \alpha) + \alpha S^{1-\eta} \right]^{1-\eta} + \frac{\omega T}{2} (\pi_{h,t} - 1)^2$$

(39)

2.6.2. Country $F$

Given the symmetric structure of our world economy, market clearing conditions for country $F$ have a symmetric structure to that of country $H$. Market clearing conditions for country $F$ are listed in the following.

Labor market clearing requires:

$$N^*_n,t + N^*_f,t = N^*_t$$

(40)

Market clearing in the non-tradable sector requires:

$$A^*_n,t N^*_n,t = C^*_n,t - (1 - \delta) C^*_n,t-1 + \frac{\omega_n}{2} (\pi^*_n,t - 1)^2$$

(41)

Finally, market clearing in the tradable sector implies:

$$A^*_f,t N^*_f,t = (1 - \alpha) C_{T,t} \left[ \alpha S^{q-1}_t + (1 - \alpha) \right]^{\eta} + \alpha C^*_t \left[ S^{q-1}_t (1 - \alpha) + \alpha \right]^{\frac{\eta}{2}} + \frac{\omega}{2} (\pi^*_f,t - 1)^2$$

(42)

2.6.3. Budget constraints and current account

If monopolistic firms are owned by the inhabitants of the country in which they are located, the resources-expenditure balance of the borrower is given by the budget constraint (13), holding with equality around a deterministic steady state. Therefore, by substituting for real profits and for (39) and (38) in the borrower’s
budget constraint, the resource constraint of the representative agent in country \( H \) is:

\[
(1 - \alpha) \frac{C_{T,t} S_t^{1-\eta}}{\alpha + (1 - \alpha) S_t^{1-\eta}} - (1 - \alpha) C^*_T \left[ \frac{1 - \alpha + \alpha S_t^{1-\eta}}{\alpha + (1 - \alpha) S_t^{1-\eta}} \right] = b_t - R_{t-1} \frac{b_{t-1}}{\pi_{T,t}}
\]

Equation (43) shows that the inflows of foreign resources net of interest payments (the RHS) needs to equalize the consumption of tradables at Home, net of Foreign consumption of tradables (weighted for terms of trade). Equation (43) is also the current account equation for country \( H \). More explicitly, we define the current account of country \( H \) (in real terms of home tradable consumption) as the variation of home-currency assets (in real terms of tradable consumption)\(^{38}\), i.e.:

\[
ca_t = \left( \frac{b_{t-1}}{\pi_{T,t}} - b_t \right)
\]

Equation (46) can be also interpreted as a current account equation of country \( F \).

2.7. Monetary policy

The recent house prices boom and the prospect of a global downturn as a consequence of sharp softening in housing sectors have triggered a debate on whether policy makers should respond to house prices. Conventional mainstreams agree that central bankers should respond to asset price changes only when they affect inflation, output and expectations (Mishkin, 2007). However, there are benefits to be derived from leaning against the wind...etc. increases(e) interest rates to stem

\(^{38}\)In our numerical simulations, we will focus on the ratio of the current account over GDP.
the growth of house price bubbles and help restrain the building of financial imbalances" (IMF, 2008).

There is also a general agreement on the desiderability to target inflation only in the sectors where prices are stickiest. In this light, by letting prices free to move in the flexible-prices sectors, the monetary authority avoids output swings in the sticky-price sectors to stabilize the overall price index.

While refraining from normative issues, we limit our analysis to the effects of stochastic shocks in presence of alternative policy stances. In our framework, we assume that exchange rates are completely flexible and policy makers do not engage in any specific exchange rate policy. This leaves the central bank three possible targets: durable goods inflation, tradable goods inflation and/or domestically-produced tradables' inflation. In the following, when focusing on the response of the economy to shocks, we will track the adjustment dynamics with different Taylor rules.

In our simplified exercise, we assume that the policy maker does not aim to stabilize output. Clearly, as remarked by Iacoviello (2005) in a similar framework, output targeting may be a source of possible policy trade-offs. For the moment we suppose that each policy maker react both to durables' and home-produced tradables' inflation according to the following Taylor rules:

\[
R_t - R = \left( \frac{\pi_{h,t}}{\pi_h} \right)^{\Phi_{1,h}} \left( \frac{\pi_{n,t}}{\pi_n} \right)^{\Phi_{2,h}} \tag{47}
\]

\[
R^*_t - R^* = \left( \frac{\pi_{f,t}}{\pi_f} \right)^{\Phi_{1,f}} \left( \frac{\pi_{n,t}}{\pi_n} \right)^{\Phi_{2,f}} \tag{48}
\]

In a two country setup, nominal determinacy requires \( \Phi_1 \) and/or \( \Phi_2 \) to be sufficiently large; we assume that the monetary policy is set so as to ensure sufficient conditions for nominal determinacy (see Benigno and Benigno, 2000). Notice finally that the above simple rules are not efficient: any change affecting the natural interest rates will likely entail an inflationary/deflationary bias.

2.8. Equilibrium conditions

For each monetary policy in country \( H \) and \( F \), the equilibrium of our world economy is defined by (13) and (14) holding with equality around the deterministic steady state (see discussion in the following section), (15), (16), (25), (24), (26), (17), (27) and the no-Ponzi game condition, (23). In the tradable production sector, (30) and (32) need to hold while in the durables production sector (34) and (35). Market clearing is insured by (37)-(43) and (19). Finally purchasing parity conditions need to hold.

3. STEADY STATE

In this section we focus on the qualitative features and the dynamic properties of the steady state. Once we have proved the existence of a "well behaving

\textsuperscript{39}For more discussion on house prices and monetary policy targets see Borio and White (2004), Bordo and Jeanne (2002).

\textsuperscript{40}See Aoki (2001) and Benigno (2004).

\textsuperscript{41}Given the collateral constraint, the no-Ponzi game condition is not binding.
"equilibrium", it is possible compute it analytically (see the Appendix).

3.1. Dynamic properties of the steady state

In order to show the existence of a determinate steady state and on its features we first shift our attention to the (modified) steady-state Euler equations of our model.

Consider first equations (26) and (58) – see the Appendix – at the steady state. Notice also that in steady state, $\pi = \nu \pi^*_T$ (where $\nu$ is the steady-state depreciation rate of the nominal exchange rate). Indeed terms of trade are fixed in steady state. Also, long-run values of tradable inflation coincide with the target of the monetary policy for tradables in the Home and in the Foreign country, respectively. We can thus pin down the long-run value of $\psi$, i.e.:

$$\psi = 1 - \frac{\beta}{\mu}$$ (49)

implying that

$$1 > \psi > 0$$ (50)

whenever $0 < \frac{\beta}{\mu} < 1$. Notice however that since $1 > \mu > \beta > 0$, inequality (50) always holds. Eventually, Assumption 2 reduces to the Becker (1980) and Becker and Foias (1987, 1994) condition (see the following Proposition).

**Proposition 1.** Under assumptions 1-3 and a monetary policy that insures nominal determinacy, if the system is sufficiently close to the deterministic steady state, then $b_t = (1 - \chi) C_{n,t} x_t$ for every $t \geq 0$.

*Proof.* The formal proof is in Becker (1980) and Becker and Foias (1987, 1994) with zero-borrowing constraints. In order to ensure the existence of a "dominant consumer" in this model, we need to focus on (modified) Euler equations. Given that the saver is a consumption smoother, the ratio of her/his steady-state marginal utilities is equal to one, i.e.:

$$\frac{U_{T,t}}{U_{T,t+1}} = R_t^* \frac{\mu}{\pi_{T,t+1}^*} = 1$$ (51)

at the contrary, equation (17) shows that whenever at the steady state $0 < \psi < 1$,

$$0 < 1 - \beta \frac{R_t}{\pi_{T,t+1}} \frac{U_{T,t+1}}{U_{T,t}}$$

and the borrower is thus the "dominated consumer". Indeed, Assumption 2 ensures that $0 < \psi < 1$.

Proposition 1 implies that in our framework the borrower is always debt-constrained. Indeed, given that the Lagrangian multiplier $\psi$ is positive, the constraint must be binding.\footnote{See also Iacoviello (2005) for additional discussion.}
In addition, Proposition 1 states that there exists an unique steady state, which is characterized by a non-zero level of external liabilities. The steady state is also dynamically determinate. It follows that the steady state of our system is not characterized by unit roots. Indeed, as in the standard model of Becker (1980), the steady state is determined by the Euler equation of the patient agent and does not depend on initial conditions. Also, external debt is uniquely pinned down by the collateral constraint. Proposition 1 allows thus to introduce the following corollary:

**Corollary 1.** If Proposition 1 holds, the dynamics of our two-country economy with heterogeneous agents and imperfect financial markets are not characterized by unit roots.

The above Corollary needs some comments. Indeed, the pioneer analysis of Obstfeld and Rogoff (1995) shows that when markets are incomplete, the steady state of an open economy is generally subject to unit roots. This means that the steady state depends on initial values (i.e., initial external assets/liabilities) and transient shocks have long-run effects.

In a two-country framework, Cole and Obstfeld (1991) rule out unit roots under particular functional forms for utility. Analogously, Corsetti and Pesenti (2001) introduce an unitary elasticity of substitution between domestic and foreign goods.\(^{43}\) This specification provides full risk sharing, renders securities redundant and implies a zero current-account balance for every period, also in presence of home bias. In turn, a zero current-account balance refrains transitory shocks from having long-run effects.

Alternatively, the non-stationarity of the steady state is ruled out by Cavallo and Ghironi (2002) and Ghironi (2006) in a framework of overlapping generations; in their model, zero steady-state liabilities are an endogenous result. Still in a framework of overlapping generations, Ghironi et al. (2005) introduce heterogeneous discounting and extend this result to the case of non-zero long-run external liabilities; in this setting, the possibility for the (relatively more) patient agent to hold all capital in steady state is ruled out by the absence of intergenerational bequest. This result should not surprise the reader; empirical evidence (see Lane and Milesi-Ferretti, 2001, 2002) show that non-zero long-run external assets/liabilities are a common phenomenon.

Schmitt-Grohe and Uribe (2003) provide a detailed analysis on stationarity-inducing methods to rule out unit roots in a small-economy framework.\(^{44}\) The proposed modifications to the standard model aim at inducing stationarity of the equilibrium dynamics: they make the steady-state independent from initial conditions. However, when stationarity is induced by portfolio adjustment costs or interest rate premia, long-run external assets that need to be set exogenously. Indeed there is an exogenous level of debt around which the adjustment function is centered. When stationarity is induced by endogenous discounting, the discount factor function is centered around an exogenous steady-state level for consumption.

In Becker (1980) seminal article, the steady state is endogenously defined by the Euler equation of the "dominant consumer" (i.e., the patient agent) and it is independent from initial conditions. By extending Becker (1980) seminal result to

\(^{43}\)For a literature review see Lane (2001).

\(^{44}\)Bodestein (2006) provides an analogous analysis applied to a two-country world.
the case of a two-country framework, we also rule out unit roots. Indeed, in steady state, the Euler equation only depends on the parameters of the model. This result implies that (sufficiently small) stochastic shocks do not have long-run effects and the steady state is not subject to unit roots dynamics.\footnote{The steady state is globally unique. Having said that, the dynamic properties of the steady state hold only around the steady state itself. The reason is that the dynamic properties cannot but be calculated by inspecting the Jacobian matrix (and thus, after linearization). Some considerations apply for the stability properties of the model (see Becker and Fois 1997).}

Notice finally that these results do not depend neither on nominal rigidities nor on the introduction of durables.

4. CURRENT ACCOUNT DYNAMICS AND HOUSING MARKET SPILLOVERS

The recent swings in house prices and their dramatic consequences have triggered a debate on the implications for the global economy. Indeed, the current degree of financial globalization has likely enhanced the international transmission of shocks. In a recent article on the sustainability of US current account and the dollar, Krugman (2007) discusses the linkage between the value of the dollar and the housing sector. He provides a stylized analysis on the effects of a fall of both housing prices and the value of the dollar; finally, he evaluates the implications of the monetary policy stance.

In our analysis we are not interested in evaluating the sustainability of current trends. Instead, we incorporate Krugman’s (2007) message so as to explore the transmission mechanisms of shocks arising from the housing market. Indeed, as mentioned in the Introduction, the linkage between international capital flows, exchange rate and the wealth of collateral-constrained agents has proved a fundamental transmission mechanism in the context of currency crises in emerging countries. While extending the focus to fluctuations in housing wealth, we extend the application of the open-economy "Bernanke-Gertler" effect to the analysis of current account dynamics.

In this section, we study the impact of developments affecting the housing market and their implications for the dynamics of international variables. We will first analyze the effect of likely stochastic shocks affecting consumption. As in Ferrero et al. (2008), preference shocks are here meant to capture structural factors entailing changes in consumption patterns.\footnote{Iacoviello (2005) introduces preference shocks so as to proxy temporary tax advantages or sudden increases in demand for houses fuelled by optimistic consumer expectations. In the same vein, Krugman (2007) considers a shock in investors’ expectations.}

Finally, in order to proxy the recent financial liberalization, we introduce a (very) persistent stochastic shock affecting the tightness of the collateral constraint and we see how this affect current account and exchange rate dynamics.

4.1. Calibration\footnote{For more details on IRFs, see also the Appendix (Section 7.4.1).}
Quarterly discount factors are set respectively to $\mu = 0.99$ and $\beta = 0.98$.\footnote{Carroll and Samwick (1997) estimate discount factors to be in a range between 0.91 and 0.99.} The structure of the model implies that the real interest rate of the $F$ country is pinned down by the discount factor of the dominant consumer, and thus is equal to $\frac{1}{\beta}$.

We stick on Monacelli (2007) benchmark value for houses depreciation rate and we set $\delta = (0.025)^{1/4}$, which is consistent an annual depreciation between 1.5% and 3%. Analogously, we adopt the average loan-to-value ratio on home mortgages for the period 1952-2005 in U.S. and let $\chi = 0.25$.\footnote{It is also consistent with current trends in industrial countries, see IMF (2008).}

We calibrate $\gamma$ by assuming that the share of durable spending on total spending in Home, $\left(\frac{\delta C_n}{\delta C_n + C_T}\right)$, is equal to 0.2. This is consistent with U.S. data on spending.

We calibrate $v$ by assuming that the steady state level of labor in $F$ is one third of one unit of time, so as to proxy European trends; the inverse elasticity of labor supply is assumed to be equal to 3.

The elasticity of substitution amongst single varieties (for each sector in each country) is set equal to 8. This implies a mark-up of about 15%. The elasticity of substitution between the basket of home and foreign goods is reasonably assumed to be lower than 8 and is set equal to 2. Moreover, the share of Home (Foreign) good consumption in the Home (Foreign) country, $\alpha$, is set equal to 0.7.

In our benchmark parametrization, we follow the standard literature on sticky prices adjustment and we assume a frequency of four quarters for tradable goods.\footnote{See Bils and Klenow (2004) and Monacelli (2007) for some recent discussion on the frequency of price adjustment in U.S.}

### 4.2. Demand shocks

We now focus the attention on the possible effects of two different demand shocks: 1) a positive shock in preferences for housing in the debtor country, $H$ (so as to proxy the demand shock for houses in countries characterized by great amount of net external liabilities); a positive shock on preferences for tradables in country $F$ (so as to proxy a generalized inflation boom). Indeed, as suggested by Iacoviello and Neri (2008), demand shocks have played a major role in explaining the cyclical volatility of houses and prices in recent years.

#### 4.2.1. Demand shock in the housing sector

Suppose now that country $H$ is affected by a positive shock in consumers’ preferences such that its inhabitants desire to buy more houses. For simplicity, we suppose that the shock has a log-normal distribution such that\footnote{The utility function is thus: $\max E_0 \left\{ \sum_{t=0}^\infty \beta^t U_t (e^{\rho_n \cdot t} C_n, C_T, N_t) \right\}$}:

\[
\begin{align*}
 p_{n,t} &= \rho_n p_{n,t-1} + u_t \\
u_t &\sim (iid)
\end{align*}
\]

where we let $\rho_n = 0.85$, following the calibration of Iacoviello (2005).

We see that in response to an increase in households’ appetite for real properties, the financial accelerator is at work. The increase in the stock of real assets – and thus, in housing relative prices – entails a better access to credit.\footnote{Indeed, the marginal value of borrowing, $\psi$, decreases.} Since agents...
in country $H$ are impatient, they use all their collateral to obtain credit. In turn, this allows them to consume tradables and to further increase their stock of houses: external debt increases on impact and continues to gradually accumulate before the shock is absorbed. The accumulation of collateral is accommodated by a gradual decrease in the user cost of durables. By inspecting equation (18) – see also the Appendix –, one can see that the user cost of durables is a positive function of both the interest rate and the relative price of durables. Both relative prices and interest rates jump on impact and decrease gradually.

Given the expansion of debt, interest rates increase; notice in particular that in absence of nominal rigidities, the interest rate and the level of inflation are determined by the Taylor rule together with the modified Euler equations of the borrower. Indeed, the central bank reacts to the expansionary effects of the shocks by tightening interest rates.

Notice in particular that in our two-country world, developments affecting housing prices are transmitted internationally through terms of trades. The transmission channel is evident in absence of prices rigidities. If prices are flexible, real marginal costs need to be constant in each period. In our case, given an identical mark-up for all sectors and mobile labor across sectors, it always needs to hold:

$$\frac{1}{x_t} \frac{-U_{N,t}}{U_{T,t}A_{n,t}} = \frac{(\varepsilon - 1)}{\varepsilon} = \frac{1}{A_{h,t}} \frac{-U_{N,t}}{U_{T,t}} \left[ \alpha + (1 - \alpha) S_{t}^{1-\eta} \right]^{1/\eta}$$

implying that the increase in housing relative prices entails an improvement in terms of trade. Eventually this will transmit the effect of the shock to country $F$.

Borrowers enjoy thus a positive income effect. As you can see in (52), the decrease in $S$ enhances the consumption of tradables. Moreover, the decrease in the relative price of foreign goods prompts borrowers to substitute Home with Foreign tradables consumption; this switching effect entails a trade deficit. The trade deficit together with a (quantitatively small) increase in interest payments triggers the insurgence of a current account deficit.

Thanks to the income effect, aggregate labor supply does not increase on impact in country $H$ (as expected, it does increase in country $F$): indeed, the positive income effect deriving from the improvement in terms of trade offsets the increase in aggregate consumption in $H$. Aggregate labor gradually increases as soon as terms of trade start deteriorating, before of the absorption of the shock.

Notice in particular that we have assumed that debt contracts are stipulated in nominal terms. In a closed economy framework, Iacoviello (2005) and Monacelli (2007) show that an increase in inflation implies a shift of resources in favour of the borrower. In our framework, the impact of shocks on inflation is affected by terms of trade. In turn, the impact of terms of trade on the inflation index of tradables is generally not uniquely signed. Indeed, terms of trade affect both the Home-produced tradables inflation and the CPI index of inflation, according to definition (1) and (20).

Clearly, if CPI inflation rises, real interest payments decrease.

53 For a focus on the role of $\psi$, see the above discussion. See also Monacelli (2008) with a similar (but not identical) collateral constraint.

54 Clearly, qualitative results would hold also with different mark-up.

55 For a more detailed discussion on the trasmission of shocks between countries, see the following section.

56 Marginal costs in the Home tradable sector are a positive functon of terms of trade

57 In response to the preference shock for housing $\pi_T < \pi_h$, if prices are flexible. However, if prices of tradable goods are sticky, the opposite is true and $\pi_T > \pi_h$. 


The fall in terms of trade plays a double role in response to the shock: on the one hand, by making Foreign consumption cheaper, it accommodates borrowers’ impatience and willingness to increase current consumption. On the other hand, it enhances lenders’ accumulation of Foreign currency assets and thus, their future investment incomes – we remind the readers that lenders are consumption smoother. Notice indeed that the amount of Foreign external assets is a negative function of terms of trade, according to (21). This is due to a revaluation effect of lenders’ assets. Notice that the structure of our two-country world is characterized by the fact that borrowers do not lend; therefore, borrowers’ net external liabilities coincide with their gross external liabilities. In turn, given that borrowers’ debt is denominated in Home currency, assets revaluation effects associated to exchange rate swings are ruled out.58 Conversely, given that lenders are not indebted and their holdings of gross and net external assets coincide, their wealth is subject to revaluation effects associated to exchange-rate fluctuations. As you can see in Figure 2, the fall in $S$ has a positive impact on Foreign external assets, which is associated to the initial appreciation of the nominal exchange rate. Notice also that given home bias, the improvement of terms of trade is associated to a real appreciation in the first period; indeed, the (nominal and real) exchange rate undershoots on impact so as to depreciate during the adjustment process. At the end of the adjustment, the nominal exchange rate is eventually more depreciated than its pre-shock value.

The current account deficit is eventually absorbed as a consequence of the gradual decrease in borrowers’ tradable consumption on the one hand; and the decrease in borrowers’ ability to access to credit, on the other.

To conclude, the above results show that in response to a preference shock for durables, the increase in house relative prices is transmitted internationally through an improvement in terms of trade: house prices and terms of trade move in opposite directions. Borrowers experience a current account deficit; the exchange rate appreciates on impact and depreciates throughout the adjustment. Notice interestingly that these trends are consistent with US data on current account, house prices and exchange rates during the last years. Around 2005, the US have experienced a steeper increase in house prices, a temporary undershooting of the exchange rate and a deterioration of the current account (see Figure 1a, 1b).59 Our results are consistent with Iacoviello and Neri (2008). According to their empirical analysis, during the 2000-2005 period, housing preference shocks played a major role in explaining the boom in housing investments and prices in US.

Nominal rigidities In Figure 3, we show the effect of the above shock in presence of different types of price rigidities. We compare the dynamics of variables when tradables have a 4 quarter frequency in price adjustment (case 1 in Figure 3), with the case of price flexibility (case 2) and the case where only house prices have a 4 quarter frequency in price adjustment (case 3).

Figure 3 shows that the scenario characterized by price flexibility is intermediate between the other two. The main effect of price rigidities lies in their impact on the relative price durables-tradables.

58 In presence of Home-denominated debt and foreign denominated assets, revaluation effects on foreign assets can have a significant impact on the dynamics of external debt. For some discussion, see, among many others, Gourinchas and Rey (2007), Iliopoulos and Miller (2007), Obstfeld and Rogoff (2004), IMF (2005a).

59 Clearly, to drive any conclusion more empirical work would be needed.
If tradables are sticky, the relative price of durables jumps higher than when prices are flexible because the price of tradables does not immediately adjust to the expansionary effect of the shock. Notice that since the prices of durables are here kept flexible, it still needs to hold: \( \frac{1}{\pi_t} \frac{U_{N,t}}{U_{T,t}} = \frac{(\varepsilon - 1)}{\varepsilon} \). In the tradable-goods sector inflation is now pinned down by the New Keynesian Phillips curve, (30). Therefore, terms of trade and the relative price of durables are no more directly linked; still, they continue to move inversely. Following the jump in \( x \), terms of trade improve but in a smaller extent (indeed, the price of Home produced goods increase more slowly).

Following the stronger jump in relative prices, the user cost of durables jumps higher; thus, agents afford a smaller amount of real properties. Having said that, the increase in the value of houses allows them to continue borrowing so as to increase consumption; there is thus a switching effect in favour of tradable consumption. Notice however that the dynamics of the current account are not affected by price rigidities; indeed, the switching effect in favour of tradable consumption is offset by the smaller extent of the switching effect in favour of Foreign tradables (due to the weaker improvement of terms of trade). Finally the effects of price rigidities are not quantitatively relevant for the dynamics of the real exchange rate. As in Ferrero et al. (2008), the behavior of real-international variables do not differ significantly from the flexible-case scenario: they depend mainly on real factors.

**Monetary policy** We now focus on the role of monetary policy. We choose to assume the perspective of the Home central bank; we aim at investigating the effects of different targets on the dynamics of our collateral-constrained open economy. Given the structure of our two-country two-sector economy, policy makers in each country can choose alternative targets: tradable goods inflation, Home-produced goods inflation and durable-goods inflation.

We consider the following alternative simple monetary rules:

\[
\frac{R_t}{R} = \left( \frac{\pi_{h,t}}{\pi_h} \right)^{\Phi_{1,h}}, \Phi_{1,h} \rightarrow \infty \tag{53}
\]

\[
\frac{R_t}{R} = \left( \frac{\pi_{T,t}}{\pi_T} \right)^{\Phi_{1,h}} \left( \frac{\pi_{n,t}}{\pi_n} \right)^{\Phi_{2,h}} \tag{54}
\]

\[
\frac{R_t}{R} = \left( \frac{\pi_{T,t}}{\pi_T} \right)^{\Phi_{1,h}}, \Phi_{1,h} \rightarrow \infty \tag{55}
\]

together with the benchmark rule, (47). According to rule (53) (i.e., scenario 2 in the simulations of Figure 4), the monetary authority stabilizes inflation of Home produced goods only (i.e., \( \Phi_{2,h} = 0 \) and \( \Phi_{1,h} \) is large). According to rule (54), scenario 3, the central bank targets both CPI-inflation and inflation in the housing sector. Notice however that this specification implies that the monetary authority directly targets also the inflation of imported goods; in this way, s/he directly responds to shocks that may be imported from abroad. Finally, when the central bank follows rule (55), scenario 4, s/he responds to tradable (CPI) inflation but disregards the trends of house prices.

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\( ^{60} \) However, price rigidities in the tradable sector introduce larger swings in the nominal exchange rate.

\( ^{61} \) One could think of commodities such as oil. For instance, by targeting core inflation, the Fed does not directly respond to the increase in oil prices.
In Figure 4, we show the adjustment dynamics following a preference shock for housing under the likely scenario where the prices of tradables are sticky in both countries.

When the policy maker implements rule (55) aggregate tradable inflation is best stabilized, together with terms of trade. Therefore, most of the weight of the adjustment of the shock is carried by the relative price of houses, \( x \) (on impact it jumps higher). This implies that, on impact, agents afford a smaller amount of durables; having said that, they can afford more tradable consumption – thanks to a stronger wealth effect related to the increase of value of their assets and a switching effect in favour of tradable consumption. Notice however that the impact of the shock on the current account is not significantly different from the one in the other scenarios. Indeed, given the smaller improvement in terms of trade, the stronger switching effect in favour of tradable consumption is offset by a weaker switching effect in favour of Foreign consumption.

Notice finally that when the central bank targets housing prices, inflation is generally less stabilized in all baskets of goods.\(^{62}\) Given that we have assumed flexible house prices, this result confirms the desirability of targeting stickiest prices only. The monetary policy is indeed less effective in stabilizing the effects of preference shocks even if interest rates rise higher. In addition, as in Iacoviello (2005) and Monacelli (2007), targeting house prices does not significantly improve the adjustment of real (international) variables; the current account (in real terms of tradable consumption) is not significantly affected by the introduction of house prices in the Taylor rule. Moreover, when the central bank targets also housing inflation, the nominal and real exchange rate experience larger swings.

4.2.2. Demand shock for tradables in country F

The two-country structure of our model prompts us to analyze the transmission of shocks from one country to the other. In this section we focus on the transmission of a positive demand shock from country \( F \) to \( H \).

Suppose that country \( F \) is affected by positive demand shock for tradables. For simplicity, we suppose that the shock has a log-normal distribution such that:

\[
PT,t = \rho_PT,t-1 + u_t
\]

\[u_t \sim (iid)\]

where we let \( \rho_T = 0.85 \), as above.\(^{63}\)

Focusing first on \( F \), we see that, as expected, the positive shock triggers a strong increase in tradable consumption.

If prices are flexible, real marginal costs in each sector need to be equal to the mark up. This needs to hold in each period. We have assumed for simplicity the same mark-up for all sectors\(^ {64}\), therefore:

\[\beta_t U_t \left( \sum_{t=0}^T \beta^{t} U_t \left( C_{n,t}^{x}, e^{PT,t} C_{T,t}, N_t^F \right) \right)\]

\(^{62}\)Moreover, when the central bank implements rule (53), aggregate tradable inflation is more volatile but the single baskets of durable inflation and Home tradables are better stabilized. This implies that the relative price of houses is (slightly) better stabilized. The nominal interest rate increases less, leading in turn to smaller swings in the nominal exchange rate.

\(^{63}\)Lenders’ utility function is thus: \( max E_D \left\{ \sum_{t=0}^T \beta^t U_t \left( C_{n,t}^{x}, e^{PT,t} C_{T,t}, N_t^F \right) \right\} \)

\(^{64}\)If mark up are not identical, results wouldn’t qualitatively change.
In a closed economy, the term \( \alpha + (1 - \alpha) S_t^{\eta-1} \) would be equal to 1 and the relative price \( x^* \) would be fixed.\(^{65}\) In our framework, both \( x^* \) and \( S \) represent a wedge and are allowed to accommodate the shock by moving proportionally.

Figure 5 shows that the preference shock for tradables makes relative prices, \( x^* \) decrease; this entails a proportional fall in \( S \).\(^{66}\) Notice that the decrease of both variables leaves consumption and labor less scope for jumping – dampening thus the expansionary effect of the shock on tradable consumption. Eventually, a decrease in \( x^* \) also dampens the substitution effect between durables and non-durables following the shock in preferences. Indeed, given that lenders are not collateral constrained, all shocks affecting the real price of durables can entail a strong substitution between goods – see the arbitrage equation, (25).

The variation in terms of trade transmits the shock to country \( H \). Focusing now on country \( H \), we see that the variation of \( S \) implies a variation of \( x \) in the opposite direction with respect to \( x^* \):  

\[
\frac{1}{x_t} \frac{-U_{N,t}^*}{e^{P_t} U_{T,t}^* A_{n,t}} = \frac{(\varepsilon - 1)}{\varepsilon} \left[ \alpha + (1 - \alpha) S_t^{\eta-1} \right]^{\frac{1}{\eta}} \frac{-U_{N,t}^*}{A_{n,t}^* U_{T,t}^* e^{P_t}}
\]

The decrease in \( S \) entails a positive income effect for borrowers and triggers an increase in tradable consumption, \( C_T \). Eventually, the shock is also transmitted to the relative price, \( x \), which moves opposite to \( S \). This adds a positive wealth effect on borrowers’ collateral. Borrowers can thus obtain more credit and further increase their level of consumption. Surprisingly, the effect of the preference shock in country \( F \) has thus an expansionary effect on borrowers’ consumption. Thanks to the strong income effect, tradable consumption in country \( H \) (slightly) increases more than aggregate labor in the tradable sector.\(^{67}\) However, country \( H \) experiences a temporary (small) trade surplus – price effect. The Home central bank reacts thus promptly to keep this expansionary effect under control; then, interest payments together with the increase in terms of trade entail a (small) current account deficit which is slowly absorbed.\(^{68}\)

If prices are flexible, the dynamics of interest rates and inflation can be pinned down by combining the modified Euler equations with the Taylor rules. Given that the simple monetary rules used by policy makers are not efficient, inflation is allowed to jump. The nominal exchange rate accommodates the stance of both policies and the real exchange rate co-moves with terms of trade: it (slightly) appreciates on impact and depreciates during the adjustment. In steady state, the nominal exchange rate is more depreciated with respect to its the pre-shock level.

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\(^{65}\)See Monacelli (2007).

\(^{66}\)In country \( F \), the Taylor rule triggers a jump in interest rates. This makes current consumption more expensive and gives the lenders an incentive to accumulate more assets (partly offsetting the effect of the shock). The accumulation of assets is in turn enhanced by the revaluation effects associated to the change in \( S \).

\(^{67}\)Indeed, the decrease in terms of trade has a negative income effect for lenders; this prompts them to supply more labor.

\(^{68}\)Even if tradable consumption decreases more than labor in the tradable sector. Clearly, the quantitative impact of the shock on both \( U_{T,t} \) (and thus, \( C_T \)) and \( U_{N,t} \) (and thus \( N \)) depends on the parameters of the utility function.
The shock on preferences makes lenders increase their tradable consumption on impact but deteriorates their terms of trade – contributing to a (small) current account deficit for country $F$. Then, the increase in interest rate payments, together with the gradual increase in terms of trade entails a current account deficit for country $H$. The current account deficit is eventually balanced when the effect of the variation in interest incomes and the increase of country $H$ tradable consumption are absorbed.

**Nominal rigidities** In order to investigate the transmission of the shock in presence of nominal price rigidities we consider the following scenarios: flexible prices (case 1 in Figure 6a, 6b); nominal rigidities for Home tradables only (case 2); nominal rigidities for Foreign tradables only (case 3); nominal rigidities for both Home and Foreign tradables (case 4).

Figure 6a and 6b shows that at a qualitative level, the impact of the shock in country $F$ depends on the existence of nominal rigidities in country $F$: the dynamics of case 1 (3) are indeed analogous to those of case 2 (4) but the impact of rigidities is quantitatively very small. If prices are rigid in country $F$, the dynamics of the relative price of durables, $x^*$ and of terms of trade, $S$ are still linked. However, agents expectations and sectorial inflation create a wedge between the dynamics of the two variables – following the New Keynesian Phillips curve, (32). In the tradable sector, $\pi_f^T$ cannot jump and most of the pressure of the expansion falls on the non-tradable sector. As a consequence, relative prices, $x^*$ experience larger fluctuations – and terms of trade decrease less. There is a quantitatively small substitution effect in favour of durables when the relative price of houses is lower.

As expected, the effect of the shock on the relative price of durables in country $H$ depends on price rigidities in country $H$ (see Figure 6b). The shock will be eventually transmitted on tradable and durable consumption through the relative price of durables, $x$: the higher the relative price, the smaller the amount of purchased houses.

We notice that the introduction of nominal rigidities has a very small quantitative effect on current account and exchange rate dynamics.

**Monetary policy** We now focus on the role of the monetary policy. We continue assuming the perspective of the Home policy maker and we analyze the effects of a demand shock for tradables when the policy maker implements rules (47), (53)-(55), respectively. Notice however that given our focus on shocks affecting country $F$, we allow the central bank in country $F$ to modify her/his benchmark Taylor rule.\(^{70}\)

Figure 7 shows the response of variables to a preference shock for tradables in country $F$, when the central bank follows different monetary rules. Notice that the above transmission mechanisms apply also here; however, changing the monetary

\[^{69}\text{In country } H \text{, the dynamics of case 1 (2) are indeed analogous to those of case 3 (4) and the impact of rigidities is quantitatively very small.}\]

\[^{70}\text{In particular, whenever country } H \text{ does not target house prices (this correspond to case 2 and 4 in the numerical simulations), the central bank in country } F \text{ follows the simple benchmark rule:}\]

\[
\frac{R^*_t}{R^*} = \left(\frac{\pi^*_t}{\pi^*_f}\right)^{\Phi_{1,t}}, \quad \Phi_{1,t} \rightarrow \infty
\]

\[^{71}\text{Notice however that trends in } H \text{ would not be significantly affected if country } F \text{ continued using the benchmark rule (47).}\]
rule has quantitatively smaller implications. Rule (47) and (54) have very similar implications. When instead rule (55) and (56) are implemented, terms of trade are better stabilized and large part of the adjustment is carried by a jump in the relative price of houses, \( x \). This entails a stronger increase in tradable consumption and a stronger fall in the consumption of durables. This effect is offset by a smaller switching effect in favour of Foreign consumption. Smaller amounts of interest payments entail a smaller current account deficit during the adjustment. The current account follows the line of these trends when the central bank implements rules (53) and (56).

Notice finally that when the policy maker targets house prices also, interest rates react more strongly, entailing larger nominal exchange rate swings (this is consistent with the above results\(^{71}\)). In turn, higher interest rates entail larger interest payments and thus, a (slightly) larger current account deficit during the adjustment.

4.3. Financial liberalization

In this Section we focus on the effects of financial liberalization. We aim at proxying the liberalization process that characterized housing finance during the last decades.\(^{72}\)

In our simplified framework, we proxy the process of financial liberalization as a loosening of collateral requirements. In particular, we introduce a stochastic (very persistent) shock hitting the share of real properties that can be used as a collateral, \( (1 - \chi) \). Increasing this share allows agents to access larger amounts of credit. Following the same lines of the previous sections, we assume that the shock of financial liberalization has a log normal distribution such that:

\[
l_t = \rho_l l_{t-1} + u_t \\
u_t \sim (iid)
\]

where we let \( \rho_T = 0.99 \). In Figure 8a we show the adjustment dynamics when benchmark Taylor rules are implemented and prices of tradables are rigid. If the collateral constraint is loosened, the marginal value of borrowing, \( \psi \), decreases. To the contrary, the marginal utility of durable consumption increase together with their relative price, \( x \); thus, the user cost of durables increase so that agents afford smaller amounts of houses and substitute durable with tradable consumption.\(^{74}\)

The positive wealth effect on borrowers’ assets adds on the loosening of the collateral constraint; therefore, borrowers can further increase their level of debt (remember that borrowers are not consumption smoother). In turn, this boosts consumption.

The above transmission channels imply that the increase in house relative prices translates into an improvement in terms of trade\(^{75}\). Borrowers enjoy thus also a

\(^{71}\)Analogously, targeting house price also is not efficient: on impact, inflation jumps higher in all baskets – except for house prices.

\(^{72}\)The liberalization of both international financial markets and housing finance took different forms and followed diverse patterns in different countries. IMF (2008) provides an index proxying the degree of liberalization of mortgages markets in a panel of countries.

\(^{73}\)Notice however that prices increase both in the tradable and in the nontradable sector.

\(^{74}\)See the above discussion on the impact of \( \psi \) on the user cost of durables; see also the Appendix.

\(^{75}\)In response to the shock, if all prices are flexible, \( \pi_T < \pi_h \); if tradables prices are sticky, terms of trade dampen tradable inflation on impact (i.e., \( \pi_T < \pi_h \)) but enhance inflation during
positive income effect that further enhances their level of (tradable) consumption.76 Country $H$ experiences the insurgence of a trade deficit that is at the roots of a current account deficit. Moreover, interest payments increase and add on the deterioration of the current account. Indeed, the reaction of the central bank to the expansionary effect of the shock triggers an increase in interest rates in country $H$. The exchange rate undershoots on impact and depreciates during the adjustment. In steady state, the nominal exchange rate is more depreciated with respect to the initial situation. Notice finally that the dynamics of the current account, exchange rates and house prices are qualitatively similar to the dynamics following a preference shock for housing. Having said that, the current account deficit implied by a preference shock for tradables is quantitatively more significant (comparison not shown in the simulations).

As in the previous section, price rigidities do not have an important effect on international real variables77. Indeed, the current account deficit is not significantly affected by price rigidities; as in the previous section, price rigidities entail a larger jump in the relative price of houses, associated with a smaller improvement in terms of trade. Therefore, the stronger switching effect in favour of tradable consumption is offset by the weaker switching effect in favour of foreign tradables (see Figure 8b).

Finally, in Figure 8c we show the impact of different simple monetary rules; as you can notice, the effects follow the same logic of the previous sections; as for the preference shock for houses, the current account is not significantly affected by the monetary policy stance (but the nominal and the real exchange rate are).

5. QUANTITATIVE INSIGHTS AND DYNAMICS OF THE MODEL

In this Section we will continue exploring the dynamics characterizing our model. We will first analyze the effect of an aggregate technology shock in country $F$ and its transmission to country $H$. We will then focus on an aggregate shock in country $H$ so as to compare the adjustment dynamics of the two countries.

5.1. Aggregate technology shock in country $F$

Suppose that country $F$ is affected by a positive aggregate technology shock; the shock we consider hits both housing and the tradable production sector so that productivity coefficients $A_f^*, A_n^*$ increase in the same way. As in the previous sections, we suppose that the productivity shock has a lognormal distribution, such that:

$$a_{f,t} = \rho a_{f,t-1} + u_t$$
$$u_t \sim (iid)$$

the rest of the adjustment (i.e., $\pi_T > \pi_h$) – dampen in turn real interest rates (in terms of tradable consumption).

76 The shock has a negative impact on aggregate labor in country $H$ and a positive effect on aggregate labor in country $F$.

77 As expected, price rigidities entail relatively larger swings in the nominal exchange rate.
and we suppose for simplicity $A_{f,t}^* = A_{n,t}^* = e^{\rho f t}$.78

In response to a positive technology shock in country $F$, lenders increase savings (i.e., net external assets), consumption79 and work less in the tradable sector.80

At the same time, the decrease in marginal costs makes Foreign goods cheaper; therefore, $S$ decreases81.

When prices are flexible, $\frac{1}{\pi_t} \frac{U_{N,t}}{U_{T,t} A_{n,t}} = \left[ \alpha + (1 - \alpha) S_t^{\eta - 1} \right]^{-\frac{1}{\eta - 1}} \frac{U_{N,t}}{A_{f,t} U_{T,t}}$; thus, house relative prices decrease.

The fall in terms of trade has a negative effect on savers’ income (and a positive effect on their net external assets) and makes Home consumption more expensive. Terms of trade, labor supply and consumption show an hump-shaped trend. This depends on the following offsetting effects: i) the direct effects of the shock on the variables (entailing an increase in $F$-tradable consumption, a decrease in $S$ and in $F$-labor supply) and ii) the feedbacks associated the negative income effect implied by the fall of terms of trade in the first period (See also equation (42)).

The shock is thus transmitted to country $H$ entailing a positive income effect for borrowers; tradable production in country $H$ decreases but tradable consumption is sustained by imports. Since $\frac{1}{\pi_t} \frac{U_{N,t}}{U_{T,t} A_{n,t}} = \frac{1}{A_{h,t} U_{T,t}} \left[ \alpha + (1 - \alpha) S_t^{1 - \eta} \right]^{-\frac{1}{\eta - 1}}$, the effect of terms of trade is transmitted to relative house prices82 and enhances borrowers’ collateral. The financial accelerator is thus at work. The hump-shaped trend of consumption follows both the trends of terms of trade and of house relative prices (the latter, through the financial accelerator channel).83

In country $F$, the productivity shock triggers a fall in inflation; there is thus scope for a monetary loosening. Notice also that the shock in $F$ entails a (smaller) fall in inflation also in country $H$; indeed, marginal costs in $H$ are a negative function of relative house prices and a positive function of terms of trade. Thus, the effects associated to the changes in relative prices more than offsets the quantitatively (very) small increase in aggregate labor. Interest rates in country $H$ are eventually determined by combining the Taylor rule with the Euler equations. Focusing now on the international transmission of the shock, notice that the improvement of terms of trade triggers a trade deficit for country $H$ (see Figure 9c). On impact, country $H$ also experiences a current account deficit. Having said that, the decrease in interest payments eventually allows country $H$ to carry a current account surplus throughout the adjustment. The nominal exchange rate appreciates on impact and depreciates throughout the adjustment. Same considerations apply for the real exchange rate. In the long run, the nominal exchange rate is more depreciated than in the pre-shock situation.

The positive technology shock in country $F$ entails thus an improvement of borrowers’ terms of trade and an increase in house prices. During the whole ad-

---

78 We assume that $\rho_f = 0.85$. Engel and Wang (2008) estimate the coefficient related to a productivity shock in the non-durable sector only to be equal to 0.87 and the one in the durable sector as 0.9.

79 Moreover, the associated loosening of the monetary policy prompts agents to consume more – see the following.

80 Clearly, quantitative results depend on the parameters of the model.

81 The effect is eventually enhanced by an appreciation of country-$H$ nominal exchange rate.

82 Notice that house prices do not increase; housing relative prices increase because the price of the aggregate basket of tradables decreases more.

83 Terms of trade trigger the transmission of the shock. The transmission channel depends on a positive net effect on borrowers’ income. Indeed, aggregate consumption increase in country $H$ and labor swings are quantitatively insignificant.
justment process, borrowers experience a trade deficit. However, while on impact they also experience a (quantitatively not significant) current account deficit, decreasing interest payments help sustaining a current account surplus throughout the adjustment (see Figure 9c).

Finally, in Figure 9a and 9b we show the effect of nominal rigidities for tradables on the transmission of the shock. Price rigidities do not seem to significantly affect the international transmission of technology shocks. As in the above analysis, price rigidities introduce a wedge on relative prices entailing quantitatively small consumption switching effects.84

5.2. Aggregate technology shock in country H

Suppose now that country $H$ is affected by a positive aggregate technology shock; as in the previous section, the shock we consider hits both the housing and the tradable production sector so that productivity terms $A_{h,t}, A_{n,t}$ increase in the same way. As in the previous sections, we suppose that the shock has a lognormal distribution, such that:

$$a_{h,t} = \rho_a a_{h,t-1} + u_t$$
$$u_t \sim (iid)$$

and we suppose for simplicity $A_{h,t} = A_{n,t} = e^{a_{h,t}}$.85 In response to a positive productivity shock, borrowers increase their level of consumption and decrease their labor effort. In Figure 10a we compare the response of output and labor when either country $H$ or $F$ are subject to a positive productivity shock. Given that borrowers do not save and are impatient, the effect on consumption and on labor effort is stronger in country $H$; for the same reason, aggregate output in $H$ increases in a smaller extent.

In response to a productivity shock in $H$, domestic goods become cheaper than Foreign ones. This entails an increase in terms of trade and borrowers experience thus a negative income effect. This effect is transmitted to house prices; if prices are flexible, $\frac{1}{x_t} \frac{-U_{x,t}}{U_{T,t} A_{n,t}} = \frac{1}{A_{h,t}} \frac{-U_{x,t}}{U_{T,t}} \left[ \alpha + (1 - \alpha) S_{1-t}^{1-\eta} \right]^{1-\eta}$. Therefore, the increase in terms of trade entails also a negative wealth effect on borrowers’ real assets. These two effects partly offset the impact of the productivity shock on labor. Thus, labor effort show an hump-shaped response (see Figure 10a) and terms of trade respond in turn following an analogous trend – see equation (39). The decrease in houses relative prices makes houses less expensive and allow a better access to real properties.86 The access to foreign credit allows borrowers to increase consumption above output (see Figure 10b) so that interest payments accumulate. There is thus a strong upward pressure on prices due to the increase in terms of trade and the decrease of house relative prices; for this reason (together with the strong upward

84 If central banks do not target house prices, the current account initially falls stronger. Moreover, during the adjustment country $H$ experiences a smaller current account surplus due to a smaller fall in interest rates and interest payments (see Figure 9d).

85 As above, we assume that $\rho_a = 0.85$

86 Having said that, the marginal value of borrowing, $\psi$, decreases together with increasing external borrowing.
pressure on consumption) inflation increase in all sectors.\textsuperscript{87} Indeed, the downward pressure on prices due to the shock is more than offset by these effects.\textsuperscript{88} The central bank needs thus to (slightly) tighten interest rates.

On impact, country $H$ experiences a trade deficit that is at the root of a current account deficit; during the adjustment, the trade deficit translates into a trade surplus (at the roots of a current account surplus). There is a positive correlation between terms of trade and the real exchange rate; the (real and nominal) exchange rate depreciates on impact and makes Home goods cheaper; it appreciates during the adjustment. At the end of the adjustment, the nominal exchange rate is appreciated with respect to the initial level.

Finally, as expected, price rigidities do not affect the current account. While affecting the relative price of houses, they induce a small switching effect in favour of tradable consumption; however, since terms of trades rise relatively higher, there is a switching effect in favour of Home-produced tradables (see Figure 10c).\textsuperscript{89}

6. CONCLUDING REMARKS

We have focused on the current account dynamics of a two-country world populated by heterogeneous agents in their degree of impatience. We have shown that if the inhabitants of country $H$ are more impatient than the ones of the Foreign country, and their willingness to consume is limited by a collateral constraint, we can extend Becker (1980) and Becker and Foias (1987) seminal result to an open-economy dimension. Indeed, given that the $H$-inhabitants are not consumption smoother, they always prefer to borrow as much as possible and the collateral constraint is binding in each period. In the long run, the collateral-constrained country is characterized by a positive amount of external debt and a balanced current account; non-zero liabilities are thus an endogenous result of our model and are consistent with a dynamically determinate steady state.

We have then analyzed the effect of shocks and their international transmission. While considering the developments of housing wealth, we have in practice extended the application of Krugman (1999) open-economy "Bernanke-Gertler" effect to the analysis of current account dynamics. The dynamics of our model have shown the transmission channel that links developments affecting the housing sector to current account and exchange rate dynamics. House prices and terms of trade are linked; therefore, all shocks affecting house prices are transmitted internationally. Conversely, all shocks entailing a change in terms of trade affect the housing sector. Interestingly, the co-movements that we have individuated in response to likely shocks – emphasized by Iacoviello and Neri (2008) – seem to be in line with US recent trends. Further research should focus on an empirical validation.

\textsuperscript{87}Both $P_h$ and $P_n$ increase in country $H$. However, since terms of trade entail a stronger increase in $P_T$, house relative prices fall. Notice that the increase of Home currency foreign prices is mostly due to the depreciation of the nominal exchange rate.

\textsuperscript{88}Notice that, everything else fixed, terms of trade keep aggregate tradable inflation higher ($\pi_T > \pi_h$). This keeps interest payments lower and partly offsets the negative income effects.

\textsuperscript{89}The effect of different monetary stances are analogous. Having said that, when the monetary policy implements rule (55), the nominal interest rate jumps less, entailing a smaller nominal depreciation. Terms of trade are better stabilized and the consumption of Foreign tradables is thus enhanced. However, this effect on trade is offset by a switching effect in favour of durable consumption; the latter is triggered by a decrease in relative house prices.
We have also studied the role of price rigidities. As in Ferrero et al. (2008), price rigidities do not significantly affect the trends of real international variables: while entailing larger swings in the nominal interest rates, they don’t significantly affect the current account.

Finally, we have focused on the monetary policy stance. Our study extends the results of Iacoviello (2005) and Monacelli (2007) in an open-economy framework; indeed, targeting house prices does not improve the adjustment dynamics of (international) real variables such as the current account. However, consistently with Ferrero et al. (2008) our results show that targeting flexible prices (i.e., house prices) can induce larger swings in nominal interest rates and in the nominal exchange rate.

REFERENCES


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7. APPENDIX

7.1. The detailed optimization program of the borrower.

Utility function:

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\}$$

Complete budget constraint, in nominal terms:

$$P_{T,t} C_{T,t} + P_{n,t} (C_{n,t} - (1 - \delta) C_{n,t-1}) + R_{t-1} D_{t-1} - q_t R_{t-1}^* D_{t-1}^* \leq D_t - q_t D_t^* + W_t N_t + \sum \Gamma$$

where $D$ are the bonds issued at Home in Home currency and $D^*$ are bonds issued in the Foreign country in Foreign currency. The individual budget constraint in real terms of tradable consumption is:

$$C_{T,t} + x_t (C_{n,t} - (1 - \delta) C_{n,t-1}) + R_{t-1} \frac{d_{t-1}}{\pi_{T,t}} - q_t R_{t-1} d_{t-1}^* \frac{P_{T,t-1}^*}{P_{T,t}}$$

$$\leq d_t - q_t d_t^* \frac{P_{T,t}^*}{P_{T,t}} + W_t N_t + \sum \frac{\Gamma}{P_{T,t}}$$

Using price index definitions, and the law of one price, (57) can be rewritten as:

The collateral constraint is:

$$d_t - q_t d_t^* \frac{P_{T,t}^*}{P_{T,t}} \leq (1 - \chi) C_{n,t} x_t$$

First order conditions:

a) Arbitrage leisure/consumption:

$$U_{T,t} = \lambda_t$$

b) Arbitrage tradable consumption/durable services:

$$x_t U_{T,t} = U_{n,t} + \beta (1 - \delta) E_t \{ U_{T,t+1} x_{t+1} \} + U_{T,t} \psi_t (1 - \chi) x_t$$

c) Modified Euler equation

$$1 = E_t \left\{ \frac{U_{T,t}}{U_{T,t+1}} \frac{\pi_{T,t+1}}{R_t} \right\} \frac{(1 - \psi_t)}{\beta}$$

b) Optimal condition for foreign securities:

$$1 = E_t \left\{ \frac{U_{T,t}}{U_{T,t+1}} \frac{q_t}{q_{t+1}} \frac{\pi_{T,t+1}}{R_t^*} \right\} \frac{(1 - \psi_t)}{\beta}$$

Equations (57) and (58) implies that the following non-arbitrage condition needs to hold, i.e.:

$$R_t = E_t \left\{ \frac{q_{t+1}}{q_t} \right\} R_t^*$$
7.2. The user cost of durables during the transition

We now aim at disentangling the impact of each component of the RHS of (18) on the user cost of durables, during the transition toward the steady state. We thus log linearize equation (18) so as to solve for the log deviations of variables. Let the log deviation of the user cost be denominated as $Z_t$. Then:

$$
Z_t = (x + \Omega) \frac{\hat{x}_t}{Z} + (A + \Omega) \frac{\hat{\psi}_t}{Z} + (\Gamma + A) \frac{E_t \{ \hat{\pi}_{T,t+1} \}}{Z} + (A + \Gamma) \frac{E_t \{ \hat{x}_{t+1} \}}{Z} - \frac{R_t}{Z} (A + \Gamma)
$$

where we define:

$$
A \equiv \psi (1 - \delta) \frac{\pi_{T,X}}{R} \\
\Omega \equiv -\psi (1 - \chi) x \\
\Gamma \equiv -(1 - \delta) \frac{\pi_{T,X}}{R}
$$

Moreover, variables without time index are steady-state values and hatted variables are variables in log deviation from the deterministic steady state. We also notice that in steady state: $\psi = 1 - \frac{\beta}{\bar{\mu}}$.

Results show that:

i) An increase in $x$ has a positive effect on the user cost (as expected) as long as:

$$
x [1 - \psi (1 - \chi)] > 0
$$

which is comfortably satisfied with our benchmark parametrization.

ii) An increase in both $E_t \{ \hat{\pi}_{T,t+1} \}$ and $E_t \{ \hat{x}_{t+1} \}$ has a negative effect on the user cost as long as

$$
-x \frac{\pi_{T}}{R} (1 - \delta) (1 - \psi) < 0
$$

which is comfortably satisfied with our benchmark parametrization (and for any reasonable ones).

iii) An increase in the nominal interest rate entail an increase in the user cost as long as

$$
-x \frac{\pi_{T}}{R} (1 - \delta) (\psi - 1) > 0
$$

which is comfortably satisfied in our benchmark parametrization (and for any reasonable ones).

iv) An increase in $\psi$ has a negative effect on the user cost as long as:

$$
\psi x \left[ (1 - \delta) \frac{\pi_{T}}{R} - 1 + \chi \right] < 0
$$

which is satisfied with our calibration. This implies that an increase in the marginal value of borrowing makes agents substitute tradables in favor of houses. Relative prices and $\psi$ have thus opposite effects on the user cost (for everything else fixed).

Notice however that for $\delta \to 0$, this result is reversed and we recover Monacelli (2008) result with a slightly different collateral constraint. Moreover, if we impose the following constraint:

$$
b_t = (1 - \delta) (1 - \chi) E_t \left\{ C_{n,t} x_{t+1} \frac{\pi_{T,t+1}}{R_t} \right\}
$$
as in Monacelli (2008), the impact of an increase in $\psi_t$ pushes up the user cost of durables as long as

$$x\psi_t (1 - \delta) [\pi_T - (1 - \chi) \pi_n] > 0$$

which is comfortably satisfied with our benchmark parametrization. This has important implications for consumption patterns, see Monacelli (2008).

7.3. Steady state: analytical solution

We now explicitly calculate the steady state of our model. Long term inflation levels are defined by the target of the monetary policy (we assume that $\bar{\pi}_n^* = \bar{\pi}_n = \bar{\pi}_T = \bar{\pi}_T = \pi_f = 1$) and the saver’s discount rate pins thus down both the real rate of return in $F$, $R R = \frac{1}{\rho}$ and $\psi = 1 - \frac{\beta}{\rho}$, as in Monacelli (2007).

In steady state, the price rigidities à la Rotemberg are no more at stake; the steady state of our framework coincides with the flexible prices steady state. Marginal costs are thus equal to the mark up. Assuming for simplicity the same mark-up for all sector in both countries,

$$mc_n = mc_h = mc_f^* = mc_n^* = \frac{\varepsilon - 1}{\varepsilon} \quad (59)$$

Supposing for analytical simplicity that the elasticity of substitution between tradables and houses $\theta$ is unitary\(^{90}\), the consumption aggregator assumes a Cobb-Douglas specification; conditions (59), (29) (31) (33), (25), (36) and (16) allow us to pin down the durable and non-durable level of consumption both at Home and in Foreign and relative prices $x$ and $x^*$:

$$C_n = \frac{(1 - \gamma)}{\nu N^* \theta} a_1 \quad (60)$$

$$C_T = \frac{\varepsilon - 1}{\varepsilon} \frac{\gamma}{\nu N^* \theta} \left[ \alpha + (1 - \alpha) S^{1-n} \right]^{\frac{1}{1-n}} \quad (61)$$

$$x = \left[ \alpha + (1 - \alpha) S^{1-n} \right]^{\frac{1}{1-n}} \quad (62)$$

$$C_n^* = \frac{(1 - \gamma)}{\nu N^* \theta} a_2 \quad (63)$$

$$x^* = \left[ \alpha + (1 - \alpha) S^{n-1} \right]^{\frac{1}{1-n}} \quad (64)$$

$$C_T^* = e_1 \frac{\gamma}{\nu N^* \theta} \left[ \alpha + (1 - \alpha) S^{n-1} \right]^{\frac{1}{1-n}} \quad (65)$$

where $e_1 = \frac{\varepsilon - 1}{\varepsilon}, a_1 = 1 - \beta (1 - \delta) - \psi (1 - \chi)$ and $a_2 = 1 - \mu (1 - \delta)$. Notice that in steady state the amount of borrowers’ real properties is a positive function of the marginal value of additional borrowing, $\psi$. Indeed, the greater the value of borrowing, the larger the amount of collateral agents are wiling to hold. Analogously, the smaller the inverse of the loan-to-value ratio, the larger the amount of steady-state durables. Given the higher service they provide as collateral, agents have a stronger incentive to hold them in the long run.

\(^{90}\)We will keep this simplification during the simulation of our model.
Substituting (60) and (62) in (14) we obtain the steady-state level for net external debt in Home:

\[ b = (1 - \gamma) \frac{(1 - \gamma) \beta_1 (1 - \alpha) S_{1-\eta}^{-\gamma}}{v N^\varphi} \left[ (1 - \alpha) + (1 - \alpha) S_{1-\eta}^{-\gamma} \right] \frac{1}{1-\eta} \]  

(66)

and the one in Foreign:

\[ b^* = b \left( \frac{(1 - \alpha) S_{1-\eta}^{-\gamma} + \alpha}{\alpha S_{1-\eta}^{-\gamma} + 1 - \alpha} \right) \frac{1}{1-\eta} \]

We pin down steady-state levels for \( N_h \), \( N_n \) and \( N \) by substituting (60) and (62) in (37),(38) and(39), i.e.:

\[ N_h = \alpha \frac{\gamma}{v N^\varphi} \left[ (1 - \gamma) + \alpha S_{1-\eta}^{-\gamma} \right]^{-1} \]

(67)

\[ + (1 - \alpha) \frac{\gamma}{v N^\varphi} \left[ (1 - \alpha) + \alpha S_{1-\eta}^{-\gamma} \right]^{\gamma} \frac{1}{1-\eta} \]

\[ N_n = \delta \frac{(1 - \gamma) \beta_1}{v N^\varphi} \frac{1}{a_1} \]

(68)

\[ N = N_h + N_n \]  

(69)

The terms of trade, \( S \), are pinned down by substituting all above steady-state values in (43).

An analogous procedure allows us to find all Foreign steady-state values, i.e.:

\[ N_n^* = \delta \frac{(1 - \gamma) \beta_1}{a_2} \frac{1}{v N^\varphi} \]

(70)

\[ N_j^* = (1 - \alpha) \frac{\gamma}{v N^\varphi} \left[ \frac{(1 - \gamma) S_{1-\eta}^{-\gamma} + (1 - \alpha)}{(1 - \alpha) S_{1-\eta}^{-\gamma} + (1 - \alpha)} \right] \frac{1}{1-\eta} \]

(71)

\[ + \alpha \frac{\gamma}{v N^\varphi} \left[ (1 - \gamma) S_{1-\eta}^{-\gamma} + (1 - \alpha) \right]^{-1} \]
7.4. Figures

*Figure 1a*: Trends in the nominal and effective CPI exchange rate (BIS database)
Figure 1b. House prices and current account
(Ecowin database)
Figure 2: Preference shock for houses in $H$, flexible prices.

Figure 3: Preference shock for houses in $H$, different price rigidities.
Figure 4: Preference shock for houses in $H$ and Taylor rules.

Figure 5: Preference shock for tradables in country $F$, flexible prices.
Figure 6a: Preference shock for tradables in country $F$, different price rigidities.

Figure 6b: Preference shock for tradables in country $F$, different price rigidities.
Figure 7: Preference shock for tradables in country $F$ and Taylor rules.

Figure 8a: Increase in financial liberalization.
Figure 8b: Increase in financial liberalization, different price rigidities.

Figure 8c: Increase in financial liberalization, different Taylor rules.
Figure 9a: Aggregate productivity shock in country $F$, price rigidities in country $F$.

Figure 9b: Aggregate productivity shock in country $F$, price rigidities in country $H$. 
Figure 9c: International transmission of an aggregate technology shock in country $F$.

Figure 9d: International transmission of an aggregate technology shock in country $F$ and Taylor rules.
Figure 10a: Aggregate productivity shocks in country $H$ and $F$, respectively.

Figure 10b: Aggregate productivity shock in country $H$. 
Figure 10c: Aggregate productivity shock in country $H$, price rigidities in country $H$.

Figure 10d: Aggregate productivity shocks in country $H$ and Taylor rules.
7.4.1. Notes on impulse response functions and calibration:

Numerical simulations represent the response of the economy following shocks of 1% standard deviation; time is in quarters. Impulse response functions represent percentage deviation from the steady state (if not otherwise specified). \( dq \) refers to the depreciation rate of nominal exchange rate while \( realdq \) to the depreciation rate of the real exchange rate. \( cagdp \) refers to the ratio CA/GDP; \( rb \) represents real interest payments in the Home country and \( rb^* \) represents interest incomes in country \( F \). \( y_h \) refers to the domestic production of Home tradables and \( y^*_f \) represents the Foreign production of Foreign tradables. All other symbols reflect the notation in the text.

Figure 2. Calibration: \( \Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1 \). All prices are here flexible.

Figure 3. Calibration: \( \Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1 \). Prices are flexible in country \( F \). Variables indexed with 1 refer to the case of price rigidities in the tradable sector, in country \( H \); variables indexed with 2 refer to the case of price flexibility in both sectors; variables indexed with 3 refer to the case of price rigidities in the housing sector, in country \( H \).

Figure 4. Calibration: \( \Phi_{1,f} = \Phi_{2,f} = 1 \) always. Nominal rigidities for tradables. Case 1 refers to \( \Phi_{1,h} = \Phi_{2,h} = 1 \), benchmark Taylor rule. Case 2 to \( \Phi_{1,h} = 10, \Phi_{2,h} = 0 \), where \( \Phi_{1,h} \) is associated to Home produced tradables. Case 3 to \( \Phi_{1,h} = \Phi_{2,h} = 1 \), where \( \Phi_{1,h} \) is associated to the aggregate inflation index of tradables; case 4 to \( \Phi_{1,h} = 10, \Phi_{2,h} = 0 \), where \( \Phi_{1,h} \) is associated to the aggregate inflation index of tradables.

Figure 5. Calibration: \( \Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1 \). Flexible prices in both countries.

Figure 6a. Calibration: \( \Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1 \). Case 1 refers to flexible prices; nominal rigidities for Home tradables only refer to case 2; nominal rigidities for Foreign tradables only refer to case 3; nominal rigidities for both Home and Foreign tradables refer to case 4.

Figure 6b. Calibration: \( \Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1 \). Case 1 refers to flexible prices; nominal rigidities for Home tradables only refer to case 2; nominal rigidities for Foreign tradables only refer to case 3; nominal rigidities for both Home and Foreign tradables refer to case 4.

Figure 7. Nominal rigidities for tradables. Case 1 refers to \( \Phi_{1,h} = \Phi_{2,h} = 1 \), benchmark Taylor rule. Case 2 to \( \Phi_{1,h} = 10, \Phi_{2,h} = 0 \), where \( \Phi_{1,h} \) is associated to Home produced goods and the Foreign central bank does not target house prices, \( \Phi_{2,f} = 0 \). Case 3 to \( \Phi_{1,h} = \Phi_{2,h} = 1 \), where \( \Phi_{1,h} \) is associated to the aggregate index for aggregate tradable inflation; case 4 to \( \Phi_{1,h} = 10, \Phi_{2,h} = 0 \), where \( \Phi_{1,h} \) is associated to aggregate inflation index for tradables and the Foreign central bank does not target house prices, \( \Phi_{2,f} = 0 \).

Figure 8a. Calibration: \( \Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1 \). Price rigidities in the tradable sector in both countries.

Figure 8b. Calibration: \( \Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1 \). Case 2 corresponds to price rigidities in the tradable sector; case 1 corresponds to the benchmark flexible prices in both countries.

Figure 8c. Nominal rigidities for tradables in both countries; \( \Phi_{1,f} = \Phi_{2,f} = 1 \) always. Case 1 refers to \( \Phi_{1,h} = \Phi_{2,h} = 1 \), benchmark Taylor rule. Case 2 to \( \Phi_{1,h} = 10, \Phi_{2,h} = 0 \), where \( \Phi_{1,h} \) is associated to Home produced goods. Case 3 to \( \Phi_{1,h} = \Phi_{2,h} = 1 \), where \( \Phi_{1,h} \) is associated to the aggregate index of inflation for
tradables; case 4 to $\Phi_{1,h}=10$, $\Phi_{2,h}=0$, where $\Phi_{1,h}$ is associated to aggregate tradables.

Figure 9a. Calibration: $\Phi_{1,h}=\Phi_{2,h}=\Phi_{1,f}=\Phi_{2,f}=1$. In case 2, prices are rigid in the tradable sector, in country $F$. In case 1, all prices are flexible.

Figure 9b. Calibration: $\Phi_{1,h}=\Phi_{2,h}=\Phi_{1,f}=\Phi_{2,f}=1$. In case 2, prices are rigid in the tradable sector, in country $H$. In case 1, all prices are flexible.

Figure 9c. Calibration: $\Phi_{1,h}=\Phi_{2,h}=\Phi_{1,f}=\Phi_{2,f}=1$. All prices are flexible.

Figure 9d. Nominal rigidities for all tradables. Case 1 refers to $\Phi_{1,h}=\Phi_{2,h}=1$, benchmark Taylor rule. Case 2 to $\Phi_{1,h}=10$, $\Phi_{2,h}=0$, where $\Phi_{1,h}$ is associated to Home produced goods and the Foreign central bank does not target house prices, $\Phi_{2,f}=0$. Case 3 to $\Phi_{1,h}=\Phi_{2,h}=1$, where $\Phi_{1,h}$ is associated to the aggregate inflation index of tradables; case 4 to $\Phi_{1,h}=10$, $\Phi_{2,h}=0$, where $\Phi_{1,h}$ is associated to the aggregate inflation index of tradables and the Foreign central bank does not target house prices, $\Phi_{2,f}=0$.

Figure 3.10a. Calibration: $\Phi_{1,h}=\Phi_{2,h}=\Phi_{1,f}=\Phi_{2,f}=1$. All prices are flexible. Variables referring to country $H$ show the response to a productivity shock in country $H$; variables referring to country $F$ refer to the response of a productivity shock in country $F$.

Figure 10b. Calibration: $\Phi_{1,h}=\Phi_{2,h}=\Phi_{1,f}=\Phi_{2,f}=1$. All prices are flexible.

Figure 10c. Calibration: $\Phi_{1,h}=\Phi_{2,h}=\Phi_{1,f}=\Phi_{2,f}=1$. In case 2, prices are rigid in the tradable sector, in country $H$. In case 1, all prices are flexible.

Figure 10d. Nominal rigidities for tradables; $\Phi_{1,f}=\Phi_{2,f}=1$. Case 1 refers to $\Phi_{1,h}=\Phi_{2,h}=1$, benchmark Taylor rule. Case 2 to $\Phi_{1,h}=10$, $\Phi_{2,h}=0$, where $\Phi_{1,h}$ is associated to Home produced goods and the Foreign central bank does not target house prices, $\Phi_{2,f}=0$. Case 3 to $\Phi_{1,h}=\Phi_{2,h}=1$, where $\Phi_{1,h}$ is associated to the aggregate inflation index of tradables; case 4 to $\Phi_{1,h}=10$, $\Phi_{2,h}=0$, where $\Phi_{1,h}$ is associated to the aggregate inflation index of tradables and the Foreign central bank does not target house prices, $\Phi_{2,f}=0$. 