ENDOGENOUS INFORMATION AND SELF-INSURANCE IN INSURANCE MARKETS: A WELFARE ANALYSIS

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Abstract

We develop a model where consumers do not have ex-ante private information on their risk but can decide to acquire such information before insurance policy purchase. Adverse selection can arise endogenously in the insurance market. We focus on the case where information has decision-making value: information allows consumers to optimally choose a self-insurance action. We analyze insurance market response to endogenous information and consumers’ incentive to search for such information. Welfare costs caused by the lack of coverage against the risk to be a high risk are analyzed. The case of genetic testing serves as an illustration.

1 Introduction

The standard assumption in insurance models is that consumers are perfectly informed about their probability to incur a loss. In other words, individuals perfectly observe their risk (type), while insures do not. In insurance markets characterized by adverse selection, insurance firms offer self-selecting contracts: the well-known Rothschild/Stiglitz equilibrium allows insurers to separate the high- from the low-type consumers.

In many situations, however, consumers have only a vague perception of their probability of incurring a loss: they do not have ex-ante superior information. This is the case, for example, of health related risk. Nevertheless, recent developments in medical science makes genetic tests for many diseases available to consumers: whenever consumers choose to undertake a test, they decide to acquire more precise information about their risk. This means that individuals can learn information about their risk of illness before purchasing the insurance contract: information is endogenous.

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Consumers’ decision to learn information on their risk is influenced by the reaction of insurance market to such information. Market response to endogenous information is clearly essential in understanding consumers’ incentives to search for information and crucially depends on whether consumers’ information status is observable by insurers. Despite its importance, very few papers investigate the issue. Crocker and Snow (1992) show that, if insurers can observe whether or not consumers are informed and if consumers have no private prior information, then the private value of information is negative and consumers prefer to remain ignorant. The reason is that, when not informed, consumers have access to a full insurance contract based on the average probability of loss in the population. On the contrary, if consumers decide to acquire information on their types, insurers can write contracts that depend on the consumers’ information status: they offer different policies to the informed and uninformed (whether or not the test result is observed by the insurer). Ex-ante, risk-averse consumers obviously prefer the first scenario.

In the same vein Doherty and Thistle (1996) show that information has positive private value only when insurers cannot observe consumers’ information status, or if consumers can conceal that they performed the test. In this case all consumers learn their type (at zero cost) and the outcome is again the self-selecting Rothschild/Stiglitz equilibrium: high-risk consumers hide the test results and receive a full-insurance contract, low-risk ones show the test result and receive partial insurance. Doherty and Thistle (1996) examine the existence and characterization of equilibria under different configurations of information costs and benefits; however they focus on the case in which information has no decision-making value. In other words, consumers only choose whether to become perfectly informed on their risk or to stay ignorant: information does not create new opportunity and no (preventative) action can be taken.

However, in the case of genetic tests as in many other situations, precise information on morbidity allows consumers to take more efficient decisions: primary and secondary prevention measures are often available and these measures are more effective the higher is the precision of information about the individuals’ characteristics. As an example, let us consider the BRCA1 genetic mutation which is implicated in many hereditary breast cancer cases, and carries with it a very high risk of ovarian cancer. A woman who is positive to the BRCA1 test can undertake effective preventive measures to detect the illness at an early stage.

When information has decision-making value, consumers choose whether to become informed not only evaluating the consequences of information on the insurance premium but also taking into account the benefit of information in terms of more efficient actions. We interpret secondary prevention as a self-insurance action, that is an action that reduces the loss when the negative outcome occurs. Self-insurance has been defined in opposition to self-protection; the latter being an action that reduces the probability of the loss. As it is well known, when self-insurance or self-protection are considered, the standard trade-off between incentives and optimal risk sharing arises.

In this paper we analyze endogenous adverse selection in insurance markets.
where information has decision-making value. We focus on self-insurance (secondary prevention) for which the assumption of observable action is plausible. We analyze both the case where insurers observe consumers’ information status and the case where consumers’ information status is not observable. A crucial ingredient of our analysis is in that consumers face two different risks: the first risk is standard and is related to the monetary loss in the bad-outcome. The second one is associated to the risk of being a high-risk and, thus, it corresponds to the risk of paying a high premium. Despite they would obviously increase consumers’ welfare, the market seems not able to provide insurance policies that cover the "premium risk". Thus, it is worthwhile to analyze the reasons and the possible solutions for the market failure related to the lack of contingent market for insurance when information is endogenous.

The model more closely related to our is Doherty and Posey (1998). Also in their paper information has decision-making value, however the authors analyze the case of self-protection, we instead consider the case of self-insurance. A second important difference between the two papers is in that our simple model allows welfare analysis to be performed: we are able to investigate the welfare losses due to endogenous information asymmetry and the lack of insurance against the "premium risk".

The paper is organized as follows. Section 2 describes our assumptions and analyses the decision-maker’s problem. Section 4 shows the first-best of the model and discuss how to decentralize such allocation in the market. Section 3 describes how (second-best) insurance affects consumers’ choice of prevention. Finally, in section 5, the equilibrium in the insurance market is obtained allowing for different informational structures: first the case where information is symmetric and then the case where insurers do not observe decision-makers’ information status are analyzed. Section 6 provides some final remarks.

2 The model

Decision-makers are endowed with a fixed amount of wealth $w$, and are characterized by the von Neumann-Morgenstern utility function $u(w)$, increasing and concave. With probability $p_i$, $i = L, H$, the decision-maker faces the monetary loss $L(a)$, where $0 < L(a) < w$. The action $a$ is a self-insurance measure affecting the monetary loss. When the loss $L(.)$ is interpreted as the monetary equivalent of a negative health shock, the action $a$ refers to secondary prevention or early detection of disease. The action can take only two values, 0 and 1, and makes the loss decrease such that $L(1) = l < L(0) = L$. Moreover, the action $a$ is taken before the realization of the risk and implies a utility cost $\Psi(a)$, with $\Psi(0) = 0$ and $\Psi(1) = \Psi$.

We consider two consumers’ types, the high- and the low-risk ones, respectively characterized by the probabilities $p_L$ and $p_H$, with $0 < p_L < p_H < 1$. We assume that the probabilities $p_L$ and $p_H$ are fixed, so that no ex-ante moral hazard problem exists. The population proportions of high- and low-risk types are $\lambda$ and $(1 - \lambda)$ respectively. These parameters are assumed to be common
knowledge.
Consumers do not know their type *ex-ante*. The loss probability of uninformed individuals is $p_U = \lambda p_H + (1-\lambda)p_L$. Information can be gathered without cost by performing a diagnostic test (i.e. a genetic test). Risk neutral insurance companies can propose insurance contracts to consumers.

As it was mentioned in the introduction, consumers face two different risks: the risk related to the loss $L(a)$ and the risk of being a high-risk and paying a high premium. While insurance contracts designed to cover the risk of monetary losses of different nature and entities are really common, in the real world we do not observe *premium insurance*. These policies have been called *genetic insurance* by Tabarrok (1994). We will discuss such policies in section 4.1.

A crucial element in our analysis is the beneficial effect of information in terms of more efficient actions: when informed about his probability loss, the decision-maker is able to target his preventative effort. As will be clear in the next section, the optimal action is contingent on the loss probability. Ignorance can lead to under- or over-prevention. On the other hand, from an *ex-ante* perspective and since information leads to the premium risk, consumers can be worse off because of information.

### 2.1 The decision-maker’s problem without insurance

Let us first examine the case where no insurance is available. Here we focus on the decision whether to acquire information or not when neither the premium risk nor the risk of the loss $L(a)$ are covered in the market. The decision-maker chooses whether to take the test or not by anticipating that, in the subsequent stage, he will choose the optimal action given the information possibly acquired.

Proceeding backward, let us consider first the second stage, that is the choice of the preventive action. An individual characterized by loss probability $p \in \{p_L,p_U,p_H\}$ and choosing action $a$, achieves the following expected utility level:

$$V_0(p,a) = pu(w - L(a)) + (1 - p)u(w) - \Psi(a)$$

The individual chooses a positive amount of prevention if $V_0(p,1) \geq V_0(p,0)$, that is if $pu(w-l) + (1 - p)u(w) - \Psi \geq pu(w-L) + (1 - p)u(w)$, or:

$$p \geq \frac{\Psi}{u(w-l) - u(w-L)} = \frac{\Psi}{\Delta_0}$$  \hspace{1cm} (1)

The term $\Delta_0$ is positive and measures the benefit from prevention. Obviously, when the benefit from prevention is large and its cost $\Psi$ is low, inequality (1) is easily verified. For our purpose, inequality (1) is important because it shows that consumers choose prevention only when their loss probability is sufficiently high.

**Remark 1** Riskier types perform prevention more often.

**Definition 1** $\hat{a}(p)$ is the action chosen by an individual characterized by probability of loss $p$. $\hat{V}_0(p)$ is the individual’s indirect expected utility when the probability is $p$ and the chosen action $\hat{a}(p)$.
In the first stage, the uninformed decision-maker deciding whether to acquire information on his type should compare expected utility when informed and when uninformed, that is \( \bar{V}(p_U) \) to \( \lambda \bar{V}(p_H) + (1 - \lambda) \bar{V}(p_L) \). Note that, in general, the individual faces a trade-off: on the one hand, by learning his type he faces the premium-risk, on the other hand he is able to target his preventive effort to his personal characteristics.

Suppose that the optimal action for the low-type decision-maker corresponds to a positive level of prevention: \( \bar{\alpha}(p_L) = 1 \). Given Remark 1, this implies \( \bar{\alpha}(p_U) = \bar{\alpha}(p_H) = 1 \), and \( \bar{V}(p_U) = \lambda \bar{V}(p_H) + (1 - \lambda) \bar{V}(p_L) \). In words: a positive level of prevention is optimal whatever is the decision-maker's type and the individual is indifferent between acquiring and not acquiring information. We assume that, when indifferent, the individual chooses no-information gathering. Thus, here information is not acquired and the preventive action is always taken.

Similarly, if \( \bar{\alpha}(p_H) = 0 \), \( \bar{\alpha}(p_U) = \bar{\alpha}(p_L) = 0 \); no prevention is optimal whatever the decision-maker's type and the individual remains uninformed and doesn’t take preventive action. These two cases are summarized in the remark below.

**Remark 2**

(i) If prevention is optimal for the high-type, the uninformed decision-maker does not gather information and chooses positive prevention.

(ii) If no-prevention is optimal for the low-type, the uninformed decision-maker does not gather information and chooses no-prevention.

More interesting are the cases where \( \bar{\alpha}(p_L) = 0 \) and \( \bar{\alpha}(p_H) = 1 \), that is \( p_L \leq \frac{\Psi}{\Delta_0} \leq p_H \). Here positive prevention is optimal for high-risks, whereas no-prevention is the optimal choice for low-risks. Thus, when informed, decision-makers' expected utility becomes:

\[
\lambda \bar{V}(p_H) + (1 - \lambda) \bar{V}(p_L) = \lambda (p_H u(w - l) + (1 - p_H) u(w) - \Psi) \\
+ (1 - \lambda) (p_L u(w - L) + (1 - p_L) u(w)) \\
= \lambda p_H u(w - l) + (1 - \lambda) p_L u(w - L) + (1 - p_U) u(w) - \lambda \Psi
\]

Let us consider the following assumption:

Assumption 1: \( p_L \leq p_U \leq \frac{\Psi}{\Delta_0} \leq p_H \)

Without insurance and under Assumption 1, when they are uninformed, individuals do not undertake prevention; whereas, when they are informed, only high-types choose a positive level of prevention.

**Remark 3** Without insurance and under Assumption 1, uninformed decision-makers acquire information on their risk-type: information has a positive value.

**Proof.** Under assumption 1, \( \bar{V}(p_U) = p_U u(w - L) + (1 - p_U) u(w) \) whereas \( \lambda \bar{V}(p_H) + (1 - \lambda) \bar{V}(p_L) = \lambda p_H u(w - l) + (1 - \lambda) p_L u(w - L) + (1 - p_U) u(w) - \lambda \Psi \). It is easy to verify that \( \bar{V}(p_U) < \lambda \bar{V}(p_H) + (1 - \lambda) \bar{V}(p_L) \). ■
The previous remark shows that, without insurance and under Assumption 1, the benefit of information in terms of more efficient prevention choice prevails over its cost in terms of increased risk: uninformed consumers undertake the test.\footnote{Such result is robust to the introduction of a cost for the test, provided the cost is sufficiently low.}

3 \textit{Ex-post} optimal insurance

We analyze here the optimal insurance contract from an \textit{ex-post} perspective, that is when decision-makers are informed and their type and preventative action are observable by the social planner. For each type the social planner maximizes:

$$
\begin{align*}
\max_{P_i, I_i, a_i} & p_i u(w - P_i - L(a_i) + I_i) + (1 - p_i) u(w - P_i) - \Psi(a_i) \\
\text{s.t.:} & P_i = p_i I_i
\end{align*}
$$

where \(i = L, H\). Obviously the optimal contract provides full-insurance: \(I_i = L(a_i)\). Under full actuarial insurance the level \(W_i(a)\) of expected utility achieved by a consumer with risk \(p_i\) and action \(a\) is:

$$
W_i(a) = u(w - p_i L(a)) - \Psi(a)
$$

Prevention is positive if:

$$
u(w - p_i l) - \Psi \geq u(w - p_i L)$$

or:

$$
\Delta(p_i) = u(w - p_i l) - u(w - p_i L) \geq \Psi
$$

\textbf{Remark 4} (i) Higher risks are more likely to perform prevention. (ii) Under full-insurance, the optimal level of prevention is lower than the optimal level without insurance.

\textbf{Proof.} (i) We have to prove that \(\Delta(p_i)\) is an increasing function. In fact, 
\[
\frac{\partial \Delta(p_i)}{\partial p_i} = -p_i u'(w - p_i l) + p_i u'(w - p_i L) = p_i \left[u'(w - p_i L) - u'(w - p_i l)\right] > 0.
\]
(ii) Recall that inequality (1) indicates the threshold value for positive prevention choice without insurance. We have to prove that \(\Delta(p_i) \leq p_i \Delta_0\). This inequality can be rewritten as 
\[
u(w - p_i l) - u(w - p_i L) \leq p_i \left[u(w - l) - u(w - L)\right] = p_i \left[u(w - l) - u(w - L) + (1 - p_i) [u(w) - u(w)]\right] \leq [p_i u(w - l) + (1 - p_i) u(w)] - [p_i u(w - L) + (1 - p_i) u(w)]
\]
which is true given that \(l < L\) and \(u(\cdot)\) concave. \(\blacksquare\)

\textbf{Remark 5} Insurance discourages prevention for a given risk: when \(\Delta(p_i) < \Psi \leq p_i \Delta_0\), the fully insured decision-maker does not prevent although the uninsured one does.
Let us consider social welfare in the *ex-post* optimal allocation:

**Definition 2** The *ex-post* optimal allocation is such that all decision-makers are informed and fully insured. No redistribution between types arises and social welfare is:

\[
W^*_T = \max_{a_H, a_L} W_T(a_H, a_L) = \lambda u(w - p_H L(a_H)) + (1 - \lambda) u(w - p_L L(a_L)) - \lambda \Psi(a_H) - (1 - \lambda) \Psi(a_L)
\]

As we will show in section 5, the *ex-post* optimal allocation cannot be decentralized thought an insurance market where insurers could observe information status, risk type and decision-makers' action; that is in a market with symmetric information. In fact, when the *ex-ante* choice to acquire information is taken into account, an insurance market with symmetric information does not provide enough incentives to gather information such that inefficient prevention choices are taken. On the contrary, when the information status is not observable by insurers, all decision-makers acquire information irrespective of the value of prevention cost \(\Psi\). This means that, with adverse selection, the insurance market provides good incentives for information acquisition.

### 4 Ex-ante optimal insurance (the first-best)

We now define the "*ex-ante* optimal allocation" as the one that maximizes *ex-ante* expected utility under feasibility constraint. Both coverages for the premium-risk and for the risk of the loss are available. It is as if the social planner designs the contract "under the veil of ignorance". Decision-makers perform the test *after* the contract is offered. Everything is observable and contractible. However, since it is defined in utility terms, the cost of the action \(a_i\) is not insurable.

Let us define \(P_i\) and \(I_i\), \(i = L, H\), the premium and the indemnity respectively. The social planner maximizes:

\[
\begin{align*}
\max_{P_H, I_H, P_L, I_L, a_H, a_L} & \quad \lambda (p_H u(w - P_H - L(a_H) + I_H) + (1 - p_H) u(w - P_H) - \Psi(a_H)) + (1 - \lambda) (p_L u(w - P_L - L(a_L) + I_L) + (1 - p_L) u(w - P_L) - \Psi(a_L)) \\
\text{s.t.:} & \quad \lambda P_H + (1 - \lambda) P_L = \lambda p_H I_H + (1 - \lambda) p_L I_L
\end{align*}
\]

Note that the previous program also corresponds to the utilitarian optimum: the utility functions of the two decision-makers' types are summed up and weighted by the proportion of each type in the whole population. What is crucial is the timing: in first-best expected utility is maximized under the veil of ignorance, in the utilitarian optimum expected utility is maximized *interim*, that is after the information on the type is revealed to decision-makers.

Obviously the first-best implies full insurance: \(I_i = L(a_i), i = L, H\). Moreover, the optimal premium is uniform and equal to \(P^* = \lambda p_H L(a_L) + (1 - \lambda) p_L L(a_L)\).
Since both types pay the same premium irrespective of their loss \( L(a_i) \) and get utility \( u(w - P^*) \), whenever the action chosen by the two types is different, the social planner attributes different utility levels to the two groups. In particular, the type performing more prevention suffers the higher disutility and, thus, is characterized by the lower utility.

We saw from Remark 1 that high-types are more likely to perform prevention than the low-types. This is because their probability to benefit from prevention is higher. As a consequence we expect that, whenever the two groups act differently, the high-risks are worse off.

The optimal values of \( a_i \) is the solutions of:

\[
\max_{a_H, a_L} W^*(a_H, a_L) = u(w - \lambda p_H L(a_H) - (1 - \lambda) p_L L(a_L)) - \lambda \Psi(a_H) - (1 - \lambda) \Psi(a_L)
\]

Note that, according to who performs prevention, four possible values for the welfare function are possible:

\[
\begin{align*}
W_1^* &= u(w - p_H) - \Psi \\
W_2^* &= u(w - \lambda p_H L - (1 - \lambda) p_L L) - \lambda \Psi \\
W_3^* &= u(w - \lambda p_H L - (1 - \lambda) p_L L) - (1 - \lambda) \Psi \\
W_4^* &= u(w - p_L L)
\end{align*}
\]

Welfare is \( W_1^* \) (\( W_4^* \)) when both types (no type) perform prevention. \( W_2^* \) and \( W_3^* \) correspond to the case where only high-types and only low-types respectively choose positive prevention.

As it was discussed before, when only one decision-makers’ type performs prevention, the most natural case to analyze is the one where prevention is performed by high-types. As a consequence we assume that, \( \forall \Psi, W_2^* \geq W_3^* \).

It can be easily checked that such inequality is always verified if the following assumption holds:

\[
\text{Assumption 2:} \quad \begin{cases} 
\lambda p_H \geq (1 - \lambda) p_L \\
\lambda \leq (1 - \lambda)
\end{cases}
\]

Inequalities 2a and 2b are sufficient conditions such that it is socially optimal that only high-type decision-makers perform prevention (thus, later on we exclude the case expressed by the welfare function \( W_4^* \)). Note that, according to assumption 2b, high-risks must be less likely than low-risk decision-makers: \( \lambda \leq 1/2 \). Assumption 2a and 2b together indicate that the loss probability \( p_H \) must be sufficiently higher than \( p_L \), in particular \( p_H \geq \frac{1-\lambda}{\lambda} p_L \) where \( \frac{1-\lambda}{\lambda} \geq 1 \).

**Proposition 1** Under assumption 2, first-best is such that:

\footnote{When the decision-makers choose among a continuum of possible actions, no assumption 2 is required. The optimal action always increases with the loss probability \( p_i \). In other words, provided that \( p_H > p_L \), the high-types always choose a higher amount of prevention than the low-types.}
Proposition 1 shows that, when the cost of prevention is low, it is optimal to have both types performing prevention. As the cost of prevention increases, only high-types choose positive prevention. Finally, when the cost is sufficiently high, no prevention is performed. Figure 1 below describes social welfare in first-best as a function of the cost of prevention $\Psi$ and offers a graphical representation of Proposition 1.

Note that, for $\Psi_2 \leq \Psi \leq \Psi_1$, the high-types reach utility $u(w - P^*) - \Psi$, whereas the low-types gain $u(w - P^*)$, where $P^* = \lambda p_H l - (1 - \lambda)p_L L$.

**Remark 6** When $\Psi_2 \leq \Psi \leq \Psi_1$, in first-best high-types decision-makers are worse off.

When they are fully insured, decision-makers benefit from prevention only because prevention allows to pay a lower premium (whereas, without insurance, we showed that the benefit from prevention was in terms of decreased loss). However, here the premium is uniform, thus it can be that cross subsidization arises between different types. In fact, on the one hand the high-types are characterized by the higher loss probability, on the other hand they are the only ones performing prevention. It is interesting to ask whether the first-best does redistribute resources from the high- to the low-types when $\Psi_2 \leq$
\[ \Psi \leq \Psi_1. \] To see that, let us consider the premium high-types would pay for fair insurance when \( \Psi_2 \leq \Psi \leq \Psi_1 \), that is \( P_H = p_H l \). Comparing \( P_H \) and \( P^* = \lambda p_H l + (1 - \lambda) p_L L \) we see that \( P^* \geq P_H \) implies \( p_H l \leq p_L L \). Thus, when \( \Psi_2 \leq \Psi \leq \Psi_1 \) high-types pay more than their fair premium in the first-best if prevention leads to a large fall in the monetary loss or if \( l \) is sufficiently low (which is compatible with assumption 2: \( p_H \) sufficiently higher than \( p_L \)). We can state the following remark:

**Remark 7** When \( \Psi_2 \leq \Psi \leq \Psi_1 \) and the benefit from prevention is sufficiently high \( \left( l \leq \frac{p_H L}{p_L} \right) \), first-best redistributes resources from the high- to the low-types.

In such a case, not only the high-types pay a disutility cost because of prevention, they also pay a higher premium than they would pay with fair insurance. In particular, while only high-types pay the cost of prevention, both types receive its benefit. Interestingly, here, the general result that low-risks subsidize high-risks does not hold anymore.

### 4.1 Premium insurance

Tabarrok (1994), considering and discussing the case of genetic testing, proposes to decentralize the optimal allocation by creating an explicit market for insurance against the possibility to be a high-risk ("genetic insurance"). The insurance policy should be mandatory: information acquisition is possible only after premium insurance has been purchased.\(^3\) This is necessary to avoid adverse-selection problems (we will discuss this point more in details later on).

Let us consider our model. Suppose, as before, that the decision-maker’s action is observable, such that full insurance can be implemented in a competitive insurance market. If premium insurance is available and all decision-makers purchase it, they pay the premium \( P_{PI} = \lambda p_H L (a_H) + (1 - \lambda) p_L L (a_L) \). After premium insurance has been bought, decision-makers acquire information performing the test and exhibit their test result to insurers in the competitive market. Those who learn that they type is high receive \( p_H L (a_H) \) and, with that amount, purchase fair insurance in the market; those who learn that they type is low receive \( p_L L (a_L) \) and purchase fair insurance as well.

Decision-makers purchase premium insurance if their utility having performed the test is higher than their utility without information:

\[ u (w - P_{PI}) - \Psi (a_{PI}) \geq u (w - p_U L (a)) - \Psi (a_U) \quad (9) \]

Under Assumption 2 the l.h.s. of (9) can be \( W_1^+, W_2^+ \) or \( W_2^+ \). Note that, when both types choose the same preventative action, then \( P_{PI} = p_U L (a) \). Thus, the left- and the right-hand side of (9) are different only for \( \Psi_2 \leq \Psi \leq \Psi_1 \). However, in such a case, since with information acquisition the action \( a \) is targeted on the decision-makers’ loss probability, utility under premium insurance weakly

\(^3\)This can be enforced by making it illegal for physicians and laboratories to run tests without proof that genetic insurance has been bought.
dominates utility when decision-makers are uninformed. We can conclude that premium insurance does allow the utilitarian optimum to be decentralized.

Note that premium insurance presents some similarities with Cochrane’s (1995) "time-consistent insurance". As Cochrane writes, time-consistent insurance provides premium insurance as well as insurance against the uncertain component of one period health expenditures. Moreover, the key feature for time-consistent insurance contracts is a severance payment: a person whose premium increases (for example because a long-term illness is diagnosed) receives a lump sum equals to the increased present value of his premium. The severance payment compensates for changes in premium and every consumer always purchases insurance at his actuarially fair premium.

What is different with respect to severance payments in time-consistent insurance is that, in our context, decision-makers face the problem of endogenous information acquisition and the insurance market must anticipate consumers’ choice when designing insurance policies. Adverse selection can be a crucial issue.

5 The insurance market

Ex-ante all decision-makers are uninformed; they can remain uninformed or perform a test. As in the real world, premium insurance is not available here; insurance firms offer coverage only for the monetary loss $L(a)$. The insurance market is assumed to be competitive. The timing of actions is the following: first, insurance companies propose contracts which can depend on decision-makers’ information status, type and level of prevention according to their observability; then insurees choose whether to perform the test, accept a contract and decide their level of prevention.

We first consider the case where information is symmetric and then the case where it is asymmetric.

5.1 Endogenous choice of information acquisition with symmetric information

In this subsection insurance firms know the test result. Thus, insurers observe decision-makers’ risk as well as their action. Full-insurance is provided. Insurance firms can offer three different types of contract: the full coverage contract for uninformed, for high-type and for low-type decision-makers.

If the decision-maker chooses to remain uninformed, he obtains with certainty the full coverage contract for uninformed and achieves the following level of

\[^4\text{In this model self-insurance imposes a utility cost that is not insurable. However, secondary prevention is generally (also) characterized by monetary costs not explicitly modeled here. Since the informational structure of the present and the following subsection allows contracts to provide full insurance for the monetary loss, we think that it is plausible to consider such lack of coverage for the disutility costs of prevention.}^\]
utility:

\[ W_U^* = \max_{a_U} W_U(a_U) = u(w - p_U L(a_U)) - \Psi(a_U) \]

Whereas, if he chooses to perform the test he can obtain either the full coverage contract for high-types or the full coverage contract for low-types. Thus, he obtains the following expected utility:

\[ W_T^* = \max_{a_H, a_L} W_T(a_H, a_L) \]

\[ = \lambda u(w - p_H L(a_H)) + (1 - \lambda)u(w - p_L L(a_L)) - \lambda \Psi(a_H) - (1 - \lambda)\Psi(a_L) \]

\[ = \lambda \max_{a_H} W_H(a_H) + (1 - \lambda)\max_{a_L} W_L(a_L) \]

\[ = \lambda W_H^* + (1 - \lambda)W_L^* \]

Let us denote \( a_T^*, a_L^* \equiv \arg \max W_T(a_H, a_L). \)

> From (2), when \( \Psi \leq \Delta(p_U) \) uninformed decision-makers choose \( a = 1 \) and expected utility is \( W_U^*(a = 1) = u(w - p_U L) - \Psi. \) When \( \Psi \geq \Delta(p_U) \) uninformed decision-makers choose \( a = 0 \) and expected utility is \( W_U^*(a = 0) = u(w - p_U L). \)

We can state the following lemma:

**Lemma 1** Under full information, when \( \Psi \leq \Delta(p_U) \) and \( \Psi \geq \Delta(p_H) \), expected utility without the test dominates expected utility with the test: decision-makers prefer to stay uninformed.

**Proof.** For \( \Psi \leq \Delta(p_U) \) and \( \Psi \geq \Delta(p_H) \), with full coverage high-, low-types and uninformed decision-makers choose the same action: \( a_T^* = a_L^* = a_U. \) Since decision-makers are risk-averse, this implies \( W_U^* > W_T^* \). \( \blacksquare \)

The previous lemma shows that, when information disclosed by the test has no decision-making value, the test is not performed since it increases decision-makers’ risk.

When the test is performed three cases can arise: for \( \Psi \leq \Delta(p_L) \) both types choose \( a = 1 \) and expected utility becomes \( W_T^* = \lambda u(w - p_H L) + (1 - \lambda)u(w - p_L L) - \Psi. \) For \( \Psi \geq \Delta(p_H) \) both types choose \( a = 0 \) and expected utility becomes \( W_T^* = \lambda u(w - p_H L) + (1 - \lambda)u(w - p_L L). \) Finally, when \( \Delta(p_L) \leq \Psi \leq \Delta(p_H) \), only high-types choose positive prevention and expected utility is \( W_T^* = \lambda u(w - p_H L) + (1 - \lambda)u(w - p_L L) - \lambda \Psi. \)

As it was stated in Lemma 1, for \( \Psi \leq \Delta(p_L) \) and \( \Psi \geq \Delta(p_H) \), utility without the test dominates expected utility with the test: \( W_U^* > W_T^* \). Whereas, for \( \Delta(p_L) \leq \Psi \leq \Delta(p_H) \), it can be that utility without the test dominates expected utility with the test, or the opposite. In particular, according to the level of decision-makers’ risk-aversion, two possible cases arise, as it can be seen in the following graphs where the levels of (expected) utility is a function of \( \Psi. \)

In figure 2, whatever the value of \( \Psi \), utility without the test always dominates expected utility with the test.
On the contrary, in figure 3, since risk-aversion is low, the intercept \((0, u(w - p_H))\) is close to \((0, \lambda u(w - p_H) + (1 - \lambda)u(w - p_L))\). Thus, in the interval \([\Delta(p_L), \Delta(p_H)]\) values of \(\Psi\) such that expected utility with the test dominates utility without it exist.

![Figure 2: decision-maker’s utility with and without the test when risk-aversion is high.](image)

![Figure 3: decision-maker’s utility with and without the test when risk-aversion is low.](image)

It can be easily verified that:

**Lemma 2** Under full information, when \(\Delta(p_L) \leq \Psi \leq \Delta(p_H)\), expected utility with the test can be higher or lower than utility without the test. In particular,
when the following sufficient condition is satisfied:

\[ \lambda u(w - p_Ul) + (1 - \lambda)u(w - p_Ul) \leq \lambda u(w - p_Hl) + (1 - \lambda)u(w - p_LL) \]  

\[ (10) \]

expected utility with the test dominates utility without the test in the interval \( \Psi_3 \leq \Psi \leq \Psi_4 \), where:

\[ \Psi_3 = \frac{1}{1 - \lambda} [u(w - p_Ul) - \lambda u(w - p_Hl) - (1 - \lambda)u(w - p_LL)] \]

\[ \Psi_4 = \frac{1}{\lambda} [\lambda u(w - p_Hl) + (1 - \lambda)u(w - p_LL) - u(w - p_Ul)] \]

**Proof.** For \( \Delta(p_L) \leq \Delta(p_U) \leq \Psi \leq \Delta(p_H) \), only high-types choose positive prevention under full coverage. Expected utility with the test dominates utility without the test if \( \lambda u(w - p_Hl) + (1 - \lambda)u(w - p_LL) - \lambda \Psi \geq u(w - p_Ul) \). For \( \Delta(p_L) \leq \Psi \leq \Delta(p_U) \leq \Delta(p_H) \), both the uninformed and the high-type decision-makers choose positive prevention. Expected utility with the test dominates utility without the test if \( \lambda u(w - p_Hl) + (1 - \lambda)u(w - p_LL) - \lambda \Psi \geq u(w - p_Ul) - \Psi \). Putting together the previous inequalities, expected utility with the test dominates utility without the test if:

\[ \frac{1}{1 - \lambda} [u(w - p_Ul) - \lambda u(w - p_Hl) - (1 - \lambda)u(w - p_LL)] = \Psi_3 \leq \Psi \]

\[ \leq \frac{1}{\lambda} [\lambda u(w - p_Hl) + (1 - \lambda)u(w - p_LL) - u(w - p_Ul)] = \Psi_4 \]

which gives inequality 10.  

>From Lemma 1 and Lemma 2:

**Proposition 2** Under full information, (i) When the opposite of inequality (10) holds, decision-makers always remain uninformed. (ii) When inequality (10) holds, decision-makers only perform the test for \( \Psi_3 \leq \Psi \leq \Psi_4 \).

Proposition 2 shows that, under symmetric information, even if insurance against the premium risk is not available, decision-makers may prefer to acquire information. This can be the case when aversion to risk is sufficiently low, such that decision-makers do not suffer too much because of increased risk. Moreover, this is possible for intermediate values of prevention cost \( \Psi \), that is when it is efficient for the high-risk to perform prevention. In particular, when prevention cost \( \Psi \) is close to \( \Delta(p_U) \), ignorance can impose excessive costs to uninformed decision-makers: for \( \Psi_3 \leq \Psi \leq \Delta(p_U) \) uninformed low-types perform prevention even if its cost is too high, for \( \Delta(p_U) \leq \Psi \leq \Psi_4 \) uninformed high-types do not perform prevention even if its cost is sufficiently low.

It is interesting to compare the first-best to the allocations described in Proposition 2. Note that \( u(w - \lambda p_Hl - (1 - \lambda)p_LL) \geq \lambda u(w - p_Hl) + (1 - \lambda)u(w - p_LL) \), thus, the intercept of the line describing expected utility when the test is performed and high-types choose positive prevention lies below the point \((0, u(w - \lambda p_Hl - (1 - \lambda)p_LL))\). We can state the following proposition:
Proposition 3: When premium insurance is not available and information is symmetric: (i) if risk-aversion is high such that decision-makers always prefer to remain uninformed, over-prevention arises for $\Psi < \Psi \leq \Delta(p_U)$ whereas under-prevention arises for $\Delta(p_U) < \Psi < \Psi_1$. (ii) if risk-aversion is low such that uninformed decision-makers perform the test for $\Psi_3 \leq \Psi \leq \Psi_4$, prevention choice is optimal in such an interval, whereas over-prevention arises for $\Psi_2 < \Psi < \Psi_3$ and under-prevention arises for $\Psi_4 < \Psi < \Psi_1$. Welfare losses are lower than in the previous case. (iii) First-best is always reached for $\Psi \leq \Psi_2$ and $\Psi \geq \Psi_1$. (iv) The allocation is ex-post efficient only for low risk-aversion and $\Psi_3 \leq \Psi \leq \Psi_4$.

Proof. See figure 4.

Take the case where the opposite of inequality (10) holds such that decision-makers always prefer to stay uninformed, Proposition 3 shows that the lack of coverage for the premium risk leads to a welfare cost also when information in the market is symmetric. In particular, for $\Psi_2 \leq \Psi \leq \Delta(p_U)$ uninformed low-types perform prevention even if its cost is too high, for $\Delta(p_U) \leq \Psi \leq \Psi_1$ uninformed high-types do not perform prevention even if its cost is sufficiently low. Prevention choices are optimal only for $\Psi \leq \Psi_2$ and for $\Psi \geq \Psi_1$, in such cases the first-best is reached.

Let us consider now the case where inequality (10) holds, this corresponds to the situation in which decision-makers prefer to acquire information in the interval $\Psi_3 \leq \Psi \leq \Psi_4$. In such an interval prevention choices are now optimal, however, since no coverage against premium-risk exists, decision-makers’ utility is lower than in first-best. Note that here welfare losses are lower than in the case before. Thus, decision-makers are better off when they are characterized...
by low risk-aversion because they can benefit of more efficient choices at least for certain values of prevention cost $\Psi$.

5.2 Endogenous choice of information acquisition when decision-makers’ information status is not observable

In the previous subsection information was symmetric. Here, we assume that decision-makers can secretly take the test before insurance purchase and are then free to show the test result or to conceal it. As in the previous subsection, if decision-makers show the test result to insurers, the latter can offer contracts contingent on such information. Moreover, as before prevention is observable such that insurance contracts can be contingent on decision-makers’ action too.

The following proposition can be derived:

**Proposition 4** When the information status is not observable and the insuree can conceal the test result, at the equilibrium decision-makers perform the test and show the test-result to the insurer when they learn to be low-risk. The equilibrium is ex-post efficient and the level of expected utility achieved is as in (3).

**Proof.** (i) Full-insurance contracts. Suppose first that companies are constrained to offer full-insurance contracts. If prevention is contractible, insurance companies propose *ex-ante* 6 full-insurance contracts contingent on the test result (possibly observed) and on the decision-maker’s action. The insurance premiums are:

<table>
<thead>
<tr>
<th>Decision</th>
<th>Prevent</th>
<th>Don’t Prevent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Show L</td>
<td>$\pi_{L1} = p_{LL}$</td>
<td>$\pi_{L0} = p_{LL}$</td>
</tr>
<tr>
<td>Show H</td>
<td>$\pi_{H1} = p_{HL}$</td>
<td>$\pi_{H0} = p_{HL}$</td>
</tr>
<tr>
<td>Don’t show</td>
<td>$\pi_{N1}$</td>
<td>$\pi_{N0}$</td>
</tr>
</tbody>
</table>
The uninformed consumer faces the following decision-tree:

At equilibrium, we have necessarily $p_H l \geq \pi_{N1} \geq p_L l$ and $p_H L \geq \pi_{N0} \geq p_L L$. So that, when the test result is $L$ (respectively $H$), it is optimal to show (respectively conceal) it.

When deciding whether to perform the test or not, the consumer must compare:

$$
\lambda \max \{ u(w - \pi_{N1}) - \Psi, u(w - \pi_{N0}) \} + (1 - \lambda) \max_{a_L} (u(w - p_L L(a_L)) - \Psi(a_L))
$$

with:

$$
\max(u(w - \pi_{N1}) - \Psi, u(w - \pi_{N0}))
$$

where (12) is expected utility when the test is performed: with probability $\lambda$ the decision maker is high-risk, does not show the test, and chooses the maximum between full-insurance with prevention and full-insurance without prevention; with probability $1 - \lambda$ the decision maker is low-risk, shows the test, and maximizes his (full-insurance) utility with respect to the action.

Now, suppose that:

$$
\max(u(w - \pi_{N1}) - \Psi, u(w - \pi_{N0})) \geq \max_{a_L} (u(w - p_L L(a_L)) - \Psi(a_L))
$$

then nobody performs the test and $\pi_{N1} = p_U l$, $\pi_{N0} = p_U L$. This is impossible.
since:

$$\max_{a_L}(w - p_L L(a_L)) - \Psi(a_L) > \max_{a_U}(w - p_U L(a_U)) - \Psi(a_U)$$

We can conclude that the only possible equilibrium is such that uninformed decision-makers perform the test and show it to the insurer only when the result is $L$. Thus, all decision-maker not showing the test are high-risk and $\pi_{N1} p_H = \pi_N = p_H L$.

(ii) **Menu of partial-insurance contracts** Suppose now that Insurance companies can propose *ex-ante* self selective contracts with partial coverage. In this case, companies will propose full-insurance contracts if the insurees show the test result, and a menu of self-selective contracts for those who don’t. We do not model here the competition scenario that leads insurance companies to self-selective fair (actuarial) contracts, we simply suppose that competition is such that only "fair contracts" are sustainable. Assuming as before that prevention is observable, we obtain the set of contracts depicted in the following table.

<table>
<thead>
<tr>
<th></th>
<th>positive prevention</th>
<th>no prevention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Show L</td>
<td>$\pi_{L1} = p_L L$, full coverage</td>
<td>$\pi_{L0} = p_L L$, full coverage</td>
</tr>
<tr>
<td>Show H</td>
<td>$\pi_{H1} = p_H L$, full coverage</td>
<td>$\pi_{H0} = p_H L$, full coverage</td>
</tr>
<tr>
<td>Don’t show</td>
<td>$M_1$ partial/full coverage</td>
<td>$M_0$ partial/full coverage</td>
</tr>
</tbody>
</table>

where $M_i$ is a set of 3 self-selective contracts designed for the 3 possible types $H$, $L$, and $U$. These contracts correspond obviously to the Rothschild and Stiglitz allocation where $L$- and $U$-types are partially insured while $H$-types obtain full actuarial insurance.
The uninformed consumer now faces the following decision-tree:

When the test is taken it is optimal to show the result $L$. When the result is $H$, the insuree is indifferent since he obtains the same full fair insurance in the self-selective menu. Performing the test gives:

$$W_T^* = \max_{a_H, a_L} W_T(a_H, a_L) = \lambda u(w - p_H L(a_H)) + (1 - \lambda) u(w - p_L L(a_L)) - \lambda \Psi(a_H) - (1 - \lambda) \Psi(a_L)$$

Thus:

$$W_T^* = \max_{a_H, a_L} W_T(a_H, a_L) = \lambda \max(u(w - p_H L), u(w - p_H l) - \Psi) + (1 - \lambda) \max(u(w - p_L L), u(w - p_L l) - \Psi)$$

$$= \lambda W_H^* + (1 - \lambda) W_L^*$$

If, on the other hand, the insuree decides to remain uniformed, he obtains a partial insurance coverage ($y$ or $Y$ according to whether he chooses prevention or not) which correspond to the binding self-selective contract:

$$u(w - p_H L) = p_H u(w - p_U Y + Y - L) + (1 - p_H) u(w - p_U Y) = U_H(Y)$$
$$u(w - p_H I) = p_H u(w - p_U y + y - l) + (1 - p_H) u(w - p_U y) = U_H(y)$$

Self-selective constraints give:

$$W_H^* = \max(U_H(Y), U_H(y) - \Psi)$$
We also have:

\[
U_L(y) = p_L u(w - p_U y + y - l) + (1 - p_L) u(w - p_U l) \\
< u(p_L (w - p_U y + y - l) + (1 - p_L) (w - p_U l)) \\
= u(w - p_L l - (p_U - p_L) y) \\
< u(w - p_L l)
\]

and similarly \( U_L(Y) < u(w - p_L L) \)

This implies:

\[
W^*_L > \max(U_L(Y), U_L(y) - \Psi)
\]

Which finally gives:

\[
\lambda W^*_H + (1 - \lambda) W^*_L > \max(\lambda U_H(Y) + (1 - \lambda) U_L(Y), \lambda U_H(y) + (1 - \lambda) U_L(y) - \Psi) \\
\lambda W^*_H + (1 - \lambda) W^*_L > \max(U_U(Y), U_U(y) - \Psi)
\]

So that uninformed decision-makers strictly prefer to take the test. \( \blacksquare \)

Proposition 4 shows that, when the information status is not observable by insurers, all decision-makers acquire information irrespective of the value of prevention cost \( \Psi \). This proves that, with adverse selection, the insurance market provides good incentives for information acquisition. Since decision-makers learn their risk, here prevention choices are always optimal. In the equilibrium allocation welfare losses are exclusively due to the lack of premium insurance. First-best is not reached for any value of prevention cost \( \Psi \).

The following proposition compares social welfare in the allocation with symmetric and asymmetric information:

**Proposition 5** When premium insurance is not available, inequality (10) holds and \( \Psi_3 \leq \Psi \leq \Psi_4 \), decision-makers' utility is the same under symmetric and asymmetric information. In all the other cases, decision-makers are better off under symmetric information.

6 Conclusion
To be written....

References


