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AN ECONOMIC VIEW OF CARBON ALLOWANCES MARKET

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ABSTRACT. The aim of this work is to bring an econometric approach upon the CO2 market. We identify the specificities of this market, and regarding the carbon as a commodity. We investigate the econometric particularities of CO2 prices behavior and their result of the calibration. We apprehend and explain the reasons of the non-Gaussian behavior of this market focusing mainly upon jump diffusion and generalized hyperbolic distributions. We test these results for the risk modeling of a structured product specific to the carbon market, the swap between two carbon instruments: the European Union Allowances and the Certified Emission Reductions. We estimate the counterparty risk for this kind of transaction and evaluate the impact of different models upon the risk measure and the allocated capital.

1. INTRODUCTION

Human activities, in particular the population growth and the development of industry over the last 200 years, have caused an increase in the emission and atmospheric concentration of certain gases, called "greenhouse gases" - primarily carbon dioxide and methane. These gases intensify the natural greenhouse effect that occurs on Earth, which in itself allows life to exist. The man-induced enhanced greenhouse effect leads to an increase in the average temperature of the planet that, would potentially cause increasingly severe and perhaps even more extreme disruptions to the Earth's climate, and consequently human activity.

Key words and phrases. Carbon, Normal Inverse Gaussian, CER, EUA, swap.
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As a consequence several governments, firms and individuals have taken steps to reduce their greenhouse gas (GHG) emissions either voluntarily, or, increasingly, because of current or expected regulatory constraints.

With its ratification by Russia in early 2005 the Kyoto protocol became an international law. According to its provisions, the industrialized countries have to reduce in the period 2008-2012 the greenhouse gas emissions by 5 percent with respect to the 1990 year levels.

The protocol dictates the trading of emission allowances as one of the primary mechanisms through which greenhouse gas emission reduction should be achieved. Thus, the right to pollute is considered to be a tradable asset, with its price determined by the market forces of supply and demand.

The European Union has agreed under the Kyoto Protocol to reduce its greenhouse gas emissions in the period 2008 - 2012 by 8 percent with respect to the 1990 year levels. The adopted strategy for meeting this target is the establishment of a European wide emission allowance market, the European Union Emission Trading Scheme (EU ETS) that was initiated on January 2005. The EU ETS is considered to be the largest single market for emission allowance trading, representing in 2007 approximatively 45 billion euros.

The carbon market encompasses both project-based emission reduction transactions and emissions trading of GHG emission allowances. The first one concerns the purchase of emission reductions (ERs) from a project which reduces greenhouse gases emissions compared with what would have happened otherwise. The second one concerns the allowances that are allocated under existing or upcoming cap-and-trade regimes.

In this paper, we define carbon transactions as contracts whereby one party pays another party in exchange for a given quantity of GHG emissions credits that the buyer can use to meet its objectives vis-à-vis climate mitigation. Carbon transactions can be grouped in two main categories:

- Trades of emission allowances, such as, for example, Assigned Amount Units (AAUs) under the Kyoto Protocol, or allowances under the EU Trading Scheme (EUAs). These allowances are created and allocated by a regulator, usually under a cap-and-trade regime;
- Project-based transactions, that is, transactions in which the buyer participates in the financing of a project which reduces GHG emissions compared which what would have happened otherwise, and get emission credits in return. Unlike allowance trading, a project-based transactions can occur even in the absence of a regulatory regime: an agreement between a buyer and a seller is sufficient.

In some recent works, few authors Benz [2], Daskalakis and Markellos [5],[6], Homburg [16] and Paolella [13] focused on the econometrical modeling of the GHG prices, underlying the particularities of this market like the non-Gaussian behavior, the auto-regressive phenomena and the presence of the convenience yield. In the present paper we consider a new class of models based on generalized hyperbolic distributions and we apply the results of prices calibration to a financial product specific to the CO2 market, the carbon arbitrage.

In order to have an econometric view of the GHG market we calibrated EUAS prices behavior using both Brownian and generalized hyperbolic models. We tested also the changing regimes hypothesis by integrating jumps in our diffusion models.

In the first section of this paper we focus mainly on the main features of the GHG market: the efficient market hypothesis, the resemblance to commodities, the convenience yield and the non-Gaussian behavior. The second section concerns the calibration of the econometric models based on Gaussian diffusion (Geometric Brownian diffusion, Mean reverse Brownian Diffusion, Brownian Jump diffusion and Mean reverse Brownian diffusion with jumps) and Generalized Hyperbolic (GH) distributions (Normal Inverse Gaussian distribution and Normal Inverse Gaussian-Brownian mixture). Section three emphasizes the applications of the previous sections on the risk measure of a structured product typical to carbon market the swap between two carbon instruments: the EUAs and the CERs. We benchmark the different models in order to identify the strengths and the weaknesses of each model. In section four we underline the main conclusions of our study.

2. Overview of GHG trading prices

As a direct consequence of the Kyoto protocol implementation spot quotations were launched on Powernext in June 2005, but was interrupted at the end of December 2007 because of transferring the CO2 trading activities on the Nyse-Euronext platform. Today (at the end of December 2008), spot transactions have began on the BlueNext market, only since early 2008. This means that we had to reconstruct spot prices using "futures" contracts quoted on ECX, for the two previous years. Moreover, long-term ECX futures contracts are not liquid enough, which means that we had to check the relevance of quotations and adjust data if necessary. Based on these facts we develop some reflections about the carbon around four topics: efficient market hypothesis (EMH), commodity likeliness, convenience yield and non-Gaussian behavior.

2.1. **The efficient market hypothesis.** The EMH is a common assumption in traditional financial economics. In its weak form EMH implies that the changes in financial time series (e.g., equity prices, interest rates, exchange rates) are white noise processes consisting of independent, identically distributed random variables. These assumptions imply that the time series follow random walks. One of the main empirical characteristics of financial assets' yields is the dynamic evolution of their volatilities.

The European GHG emission allowances market (EUA) is a direct consequence of a regulatory system commonly accepted by the market actors. Some sources of market inefficiency do exist:

- the new information is unequally diffused amongst market players. In fact some of the companies submitted to Kyoto protocol (ie. Vatenfall, Mittal...) or funds involved in quota/credit trading (ie. European Carbon Fund) are in same time investors and clients. Given their positions these players possess more information about the current status of the market than their peers. This informational dissymmetry weakens the efficiency hypothesis of GHG allowance market ;

- the EUA is perceived mainly as a financial liability and has not any intrinsic value capable of generating economic added value for an investor;
- both price and volatility are driven with a high impact by regulatory announces. Regulatory changes can modify the behavior of the market and bring the exchange system in a completely different regime (ie. may 2006);
- the EUA system is a dealer market. The European Carbon Exchange provides liquidity to investors by trading shares for themselves. Therefore we observe periods were the price is given by the market even if no trade exist. The advantage of having a continuous quotation has an obvious downside of irrelevant values over periods of low liquidity.

As a new market with only three years of history, the GHG emission allowances show a particular behavior due to small trading volumes, relative small number of market actors and regulatory pressure.

For example EUA 2008 and 2009 historical prices (Figure1) exhibit high variability regimes and discontinuities in offer/demand equilibrium. For a significant number of trading days the exchanged volumes of contracts are very small or even zero. In those particular days the prices are marked by the auction trading systems. Nevertheless those data are not relevant for statistical estimations due to the fact that they do not reflect a market equilibrium state.

A way to test efficiency of securities market is to search for autocorrelation and implicitly for persistence in yield and volatility levels. In order to deal with a possible lack of efficiency on the GHG allowance market we will search of persistence evidence and correlated phenomena.

We used the most recent 990 daily negative log return data of EUA08 (Figure1) to plot the sample autocorrelation function (ACF). From Figure 2, we can see that

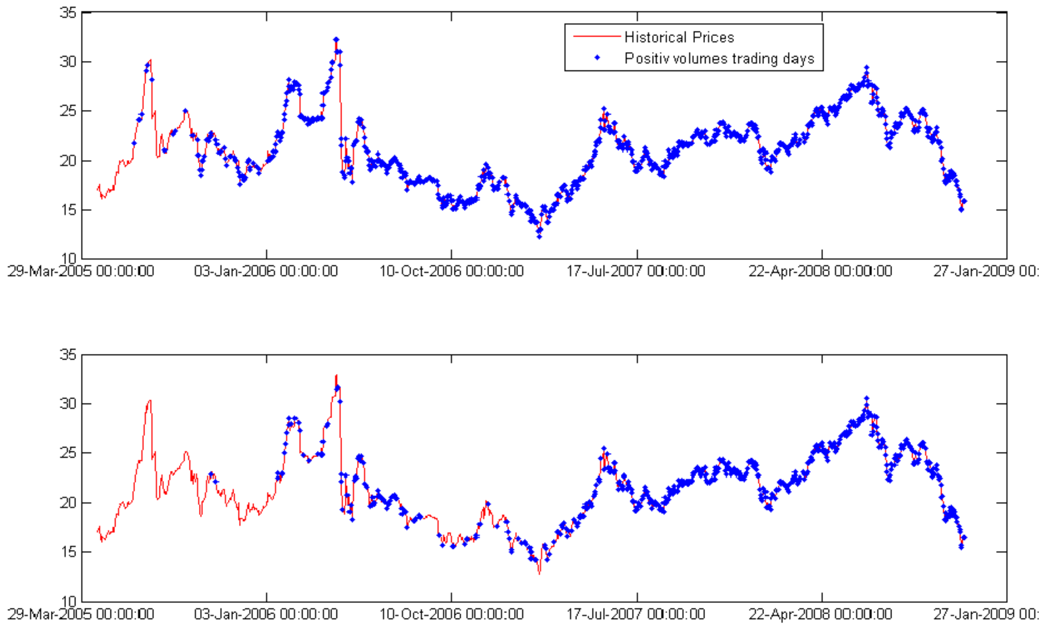

FIGURE 1. EUA08 and EUA09 prices history

the ACF of the negative log return series shows little evidence of serial correlation, while the ACF of squared return series does show evidence of serial dependence.

We observed also some relevant lags are shown for the first, sixth and thirteenth days. Nevertheless we remarked a strong persistence phenomena on the squared innovation time series before and after applying the linear filters.

2.2. **Commodity likeliness: Between physical reality and metaphysical concept.** The term "commodity" is commonly apprehended as a physical good, such as food, oil, and metals, which is interchangeable with another good of the same type, which is bought or sold by market players, usually through futures contracts. More generally, a "commodity" is a product which trades on a commodity exchange; this would also include energy contracts.

On one hand, we could look at GHG allowance prices as a commodity representing the right to pollute the environment. This right is underwritten by governments and given to different industries depending of their profile. In this case the price of an allowance would represent the marginal cost of reducing the GHG emissions. In

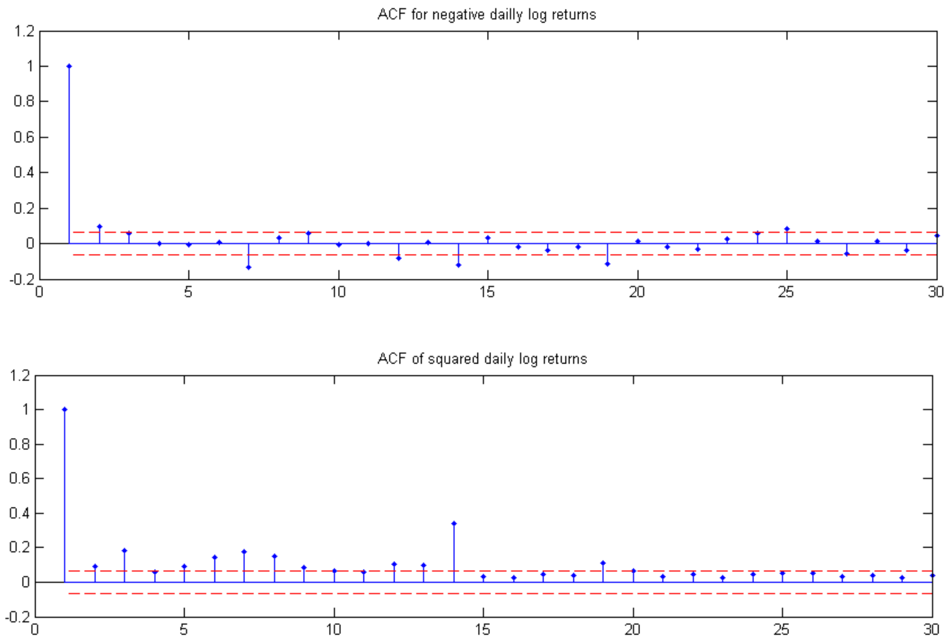

FIGURE 2. Autocorrelation for EUA08 negative daily returns

an efficient market environment this cost could be traded between industries submitted to emission constraints. The price would be established depending on the offer and the demand of the market. Thus, an independent investor would perceive her money as working in a physical mechanism that reduces the emissions.

On the other hand, the allowances are imposed by governments in order to stimulate the industries to reduce their emissions. If an industry has more allowances than actual emissions it will cash them out on the market. If it is short of allowances it will fill the need by buying it on the market. This kind of perception would push an independent perception to perceive her money as being held in a regulatory paper traded between industries without having behind any physical mechanism. Thereby the price of this security will be determined by the difference between allowed and real emissions of an industry.

2.3. **Convenience yield.** Commonly the convenience yield is a measure of the added value or premium associated with holding an underlying product or physical good, compared to the detention of the term contract (i.e. future, forward).

Sometimes, due to irregular market movements such as an inverted market, the holding of an underlying good or security may become more profitable than owning the contract or derivative instrument, due to its relative scarcity versus high demand.

An example would be purchasing physical bales of wheat rather than future contracts. In the hypothesis of a sudden drought and the demand for wheat increases, the difference between the first purchase price of the wheat versus the price after the shock would be the convenience yield.

If we apprehend the GHG allowances as a classic commodity like oil, gas or gold we should find similarities in the economic interpretation.

On the one hand, the agent has the option of flexibility with regards to consumption (no risk of commodity shortage). On the other hand, the decision to postpone consumption implies storage expenses. The net cost of these services per unit of time is termed the convenience yield δ. Intuitively, the convenience yield corresponds to dividend yield for stocks, thereby the price of a forward contract is given by:

$$F_{t,T} = S_t \cdot \exp((r_{t,T} - \delta_{t,T}) \cdot (T - t)), \tag{2.3.1}$$

where $F_{t,T}$ is the value at the moment t of the future contract for the maturity T, S_t is the spot value at the moment t, $r_{t,T}$ and $\delta_{t,T}$ are respectively the values of the rate and convenience yield for the maturity T.

One point of focus would be the convenience yield perceived as the amount of benefit that is associated with physically owning a particular good, rather than owning a futures contract for that good. When a good is easy to come by, an investor does not have need to own the actual good at that time, and can buy or sell as he pleases. When there is a shortage of a particular good, however, it is better to already own the good than to have to purchase it during the shortage because it is likely to be at a higher price due to the demand. The convenience yield is the benefit derived in the second scenario.

For example the quotas holders would not sell their quotas to realize an arbitrage opportunity (by selling the quota and buying futures contracts). Consequently they "value" their owner-right and the convenience yield is, then a major element while modeling the GHG price.

Since its introduction until the beginning of 2008 the second period market quoted only futures contract, with no price for the spot value of allowances. In order to find the implied convenience yield and spot price we used historical values of both futures contracts and interest rates. The scarcity of relevant data for longer maturities obliged us to consider only the 2008 and 2009 horizons and to suppose that the convenience yield curve is flat for those horizons. Nevertheless the impact of the uncertainty of the convenience yield on the estimation of the spot are not relevant.

The futures contracts can be valued using the expression (2.3.1), resulting in a system of two equations with two unknown variables: the implied spot and the convenience yield.

In order to solve the above equations at each trading day we take in account the bootstrapped value of interest rates. The results are showed in Figures 3 and 4.

2.4. **The non-Gaussian behavior (fat tails and asymmetry).** This feature is the main particularity of the carbon market and is the main topic of this paper. The summary of statistics for EUA on the 2008-2012 horizons are provided in the Table 1. Those for the CER are given in Table 2. The preliminary tests reject the normality hypothesis of EUA08 daily returns. As we can remark from Figure 5 historical series show negative skewness mainly due to sudden market turndown in May 2005 and fat tails also reveled by the QQplot diagram.

Since its very beginning the European Carbon Exchange systems shows high volatility (around 50 percent) with sudden changes in prices levels. These facts make us think that the Gaussian yield behaviour applied to classic securities may

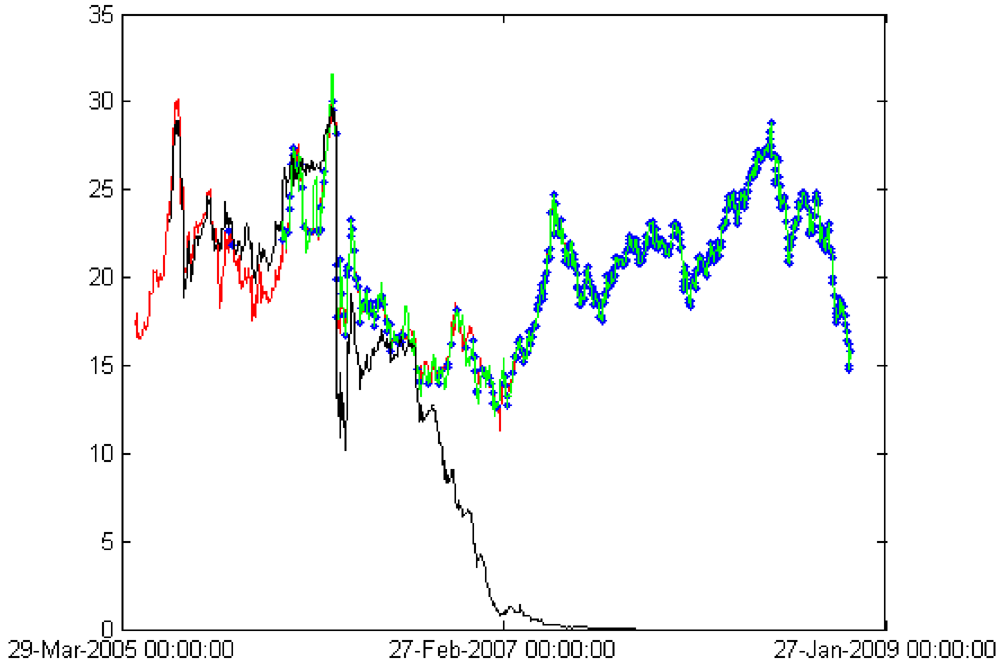

FIGURE 3. First period and second period CO2 spot prices between 2005 and 2009

not be an adequate model. Under these circumstances in this paper we shall explore two types of solutions:

- modelling the mean reversion and jumps in prices diffusions. These models were previously employed for describing commodities prices;
- using the generalized hyperbolic distributions (GH). This family includes and generalizes the familiar Gaussian and Student t distributions, and the so-called skewed t distributions, among many others. The NIG distribution, that has two semi-heavy tails, models skewness rather well, but only in cases where the tails are not too heavy. On the other hand, the skew Student t-distributions presented in the literature have two polynomial tails. Hence, they fit heavy-tailed data well, but they do not handle substantial skewness.

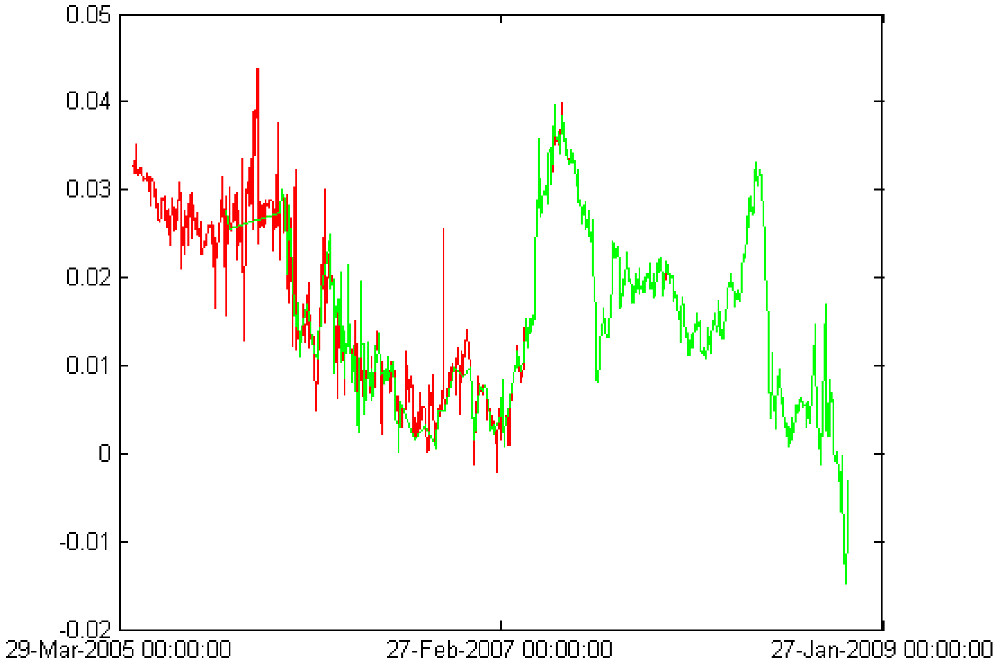

FIGURE 4. Second period convenience yield values between 2005 and 2009

Contract	EUA08	EUA09	EUA10	EUA11	EUA12
Average (10e-3)	-0.368	-0.311	-0.273	-0.239	-0.239
Annualized average	-0.093	-0.079	-0.069	-0.061	-0.061
Volatility	0.028	0.027	0.027	0.027	0.027
Annualized volatility	0.451	0.445	0.439	0.437	0.437
Skewness	-1.285	-1.228	-1.221	-1.221	-1.221
Kurtosis	20.882	20.893	20.458	19.592	19.592
Jarque-Bera test	Reject	Reject	Reject	Reject	Reject
Lillifors test	Reject	Reject	Reject	Reject	Reject

Table 1. Statistics for the EUA contracts 2008-2012

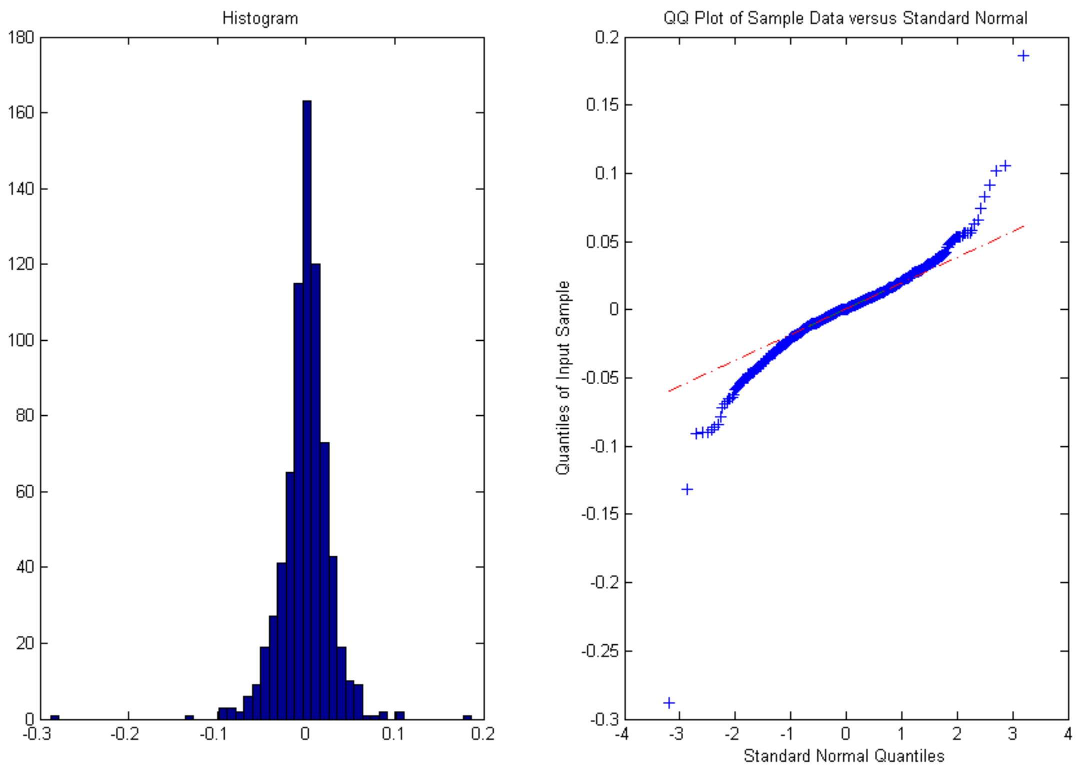

FIGURE 5. Distributions of EUA08 daily yields and QQ Plots

Contract	CER08	CER09	CER10	CER11	CER12
Average (10e-3)	-0.368	-0.311	-0.273	-0.239	-0.239
Annualized average	-0.093	-0.079	-0.069	-0.061	-0.061
Volatility	0.028	0.027	0.027	0.027	0.027
Annualized volatility	0.451	0.445	0.439	0.437	0.437
Skewness	-1.285	-1.228	-1.221	-1.221	-1.221
Kurtosis	20.882	20.893	20.458	19.592	19.592
Jarque-Bera test	Reject	Reject	Reject	Reject	Reject
Lillifors test	Reject	Reject	Reject	Reject	Reject

Table 2. Statistics for the CER contracts 2008-2012

3. Model calibration and analysis

The aim of the work developed in this section is not to find the "true" model that would explain the behavior of the carbon market but to propose a benchmark of different models usually used to describe the financial assets. Based on the historical time series we calibrate some models from the classical Brownian diffusion to more sophisticated models based on generalized hyperbolic distributions. We use the maximum likelihood as the main criteria to discriminate the fitness of the different models.

3.1. **Classic commodities models.** In this subsection we search for a model based on Gaussian yield distribution that could fit to CO2 prices behavior. We test different hypothesis like mean reversion and jumps in order to find the factors that could explain the information contained by the historical time series.

3.1.1. *Brownian diffusion.* A Geometric Brownian Motion (GBM) or Generalized Wiener Process (GWP) is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion. This model quarantines that prices levels are always positive. In this model proportional changes in the asset prices, denoted by S, are assumed to have constant instantaneous drift μ, and volatility σ. The mathematical description of this property is given by the following stochastic differential equation:

$$\frac{dS}{S} = \mu \cdot dt + \sigma \cdot dB. \tag{3.1.1}$$

Here dS represents the increment in the asset price process during a small interval of time dt, and dB is the underlying uncertainly driving the model and represents an increment in a Wiener process during dt. Under the transformation x = ln(S) and using Ito's lemma we obtain the following process for the natural logarithm of the spot prices:

$$dx = (\mu - \sigma^2) \cdot dt + \sigma \cdot dt. \tag{3.1.2}$$

Generally using a GBM to model the commodity spot prices yields a distribution of future spot prices that has a variance that increases without bound as the horizon increases. In an equilibrium setting we would expect that when prices are relatively high, supply will increase since higher cost producers of the commodity will enter the market putting a downward pressure on prices. Conversely, when prices are relatively low, supply will decrease since some of the higher cost producers will exit the market, putting upward pressure on prices. The impact of relative prices on the supply of the commodity will induce mean reversion in commodity prices.

In particular over the last few years the GHG market prices evolved in quasi-stable interval between 10 Euros and 40 Euros. Economically the lower limit represents the marginal cost that an industry would pay to depollute. The upper limit is nothing less that the relative penalty that a non-compliant industry is supposed to pay to government. Hence the GBM model that implies allowance prices over 100 Euros should be enriched with mean-reversion hypothesis.

Parameter	**Value**	**Confidence interval**
μ	-0.295	[-0.819,0.225]
σ	0.467	[0.445,0.492]
Log Likelihood	1672.90	-

Table 3. Results of the Brownian diffusion (GBM) calibration on EUA 2009

The results of the GBM calibration (Table 3) show that the historical volatility of the CO2 market is around 47 $\div$ and that the null hypothesis of the drift cannot be rejected.

3.1.2. *Mean reverse Brownian diffusion.* The mean-reverting stochastic behavior (GBMMR) of commodity spot prices can be well understood by looking at the one factor model developed by Schwartz [9] and developed for energy prices by Knittel [12] given by the following equation:

$$\frac{dS}{S} = \alpha \cdot (ln(S) - m) \cdot dt + \sigma \cdot dB. \tag{3.1.3}$$

In this model the spot price mean reverts to the long-term level $\overline{S} = \exp(\mu)$ at a speed given by the mean reversion rate, α, $\sigma > 0$. The meaning of mean reversion can be understood by looking at the first term of the equation (3.1.3). If the spot price S is above the long-term level S then the drift of the spot price will be negative and the price will tend to revert back towards the long-term level. Similarly, if the spot price is below the long-term level then the drift will be positive and the price will tend to move back towards the long-term level as the random change in the spot price may be of the opposite sign and greater in magnitude than the drift component.

Defining x = lnS and applying Ito's Lemma in equation (3.1.3), the natural logarithm of the spot price can be characterized by an Ornstein-Uhlenbeck (OU) stochastic process:

$$dx = (m - x) \cdot dt + \varsigma \cdot dB, \tag{3.1.4}$$

where m = μ -$\sigma^2/2\alpha$, $\varsigma > 0$. The speed of mean reversion, α>0, measures the degree of mean reversion to the long term mean log price m. The dynamics of the OU process under the equivalent martingale measure can be written as:

$$dx = (m^* - x) \cdot dt + \varsigma \cdot dB^*. \tag{3.1.5}$$

where m* = m - λ and λ is the market price of risk. The market price of risk can be calculated by taking into account futures term structure of the commodity. For further considerations, market price of risk will be assumed zero. By integrating equation (3.1.5) we get an expression for the logarithm of future spot prices:

$$x(t) = m^* \cdot (1 - e^{-\alpha(t-t_0)} + x(t_0)e^{-\alpha(t-t_0)}) + \sigma e^{-\alpha(t)} \cdot \int e^{-\alpha(u)} dB^*(u), \tag{3.1.6}$$

where t_0 represents the initial moment of the diffusion. In a discrete version the above equation becomes approximatively as follows:

$$x_t = c + \beta \cdot x_{t-1} + \epsilon_t, \tag{3.1.7}$$

$$where\ c = m^*(1 - e^{-\alpha}),\ \beta = e^{-\alpha},\ and\ \epsilon_t = \sigma \cdot \int e^{\alpha(u-t)} dB^*(u).$$

The error term $(\varepsilon_t)_t$ is a Gaussian white noise with variance ${\sigma_\varepsilon}^2$ equal to $\sigma^2(1 - e^{-2\alpha})/2\alpha$. In other words the conditional distribution of $x_t \mid x_{t-1}$ is given by the following expression:

$$x_t | x_{t-1} \sim N(c + \beta \cdot x_{t-1}, \sigma_\epsilon^2), \tag{3.1.8}$$

and N denotes the Gaussian law.

Parameter	**Value**	**Confidence interval**
c	-0.009	[-0.016, 0.036]
β	0.996	[0.987, 1.005]
σ_t	0.0029	[0.027, 0.030]
α	0.937	-
m	16.73	-
σ	0.468	-
Log Likelihood	1673.84	-

Table 4. Results of the Mean reverse Brownian motion (BGMMR) calibration on EUA 2009

As shown in the Table 4, the mean reverse Brownian diffusion model does not capture significantly more information about the allowance prices than the previous Brownian motion. The mean reversion hypothesis is common in the commodities analysis due to the fact that there are production and consumption cycles. As the carbon market is driven by the annual environmental compliance obligation, the existence of a cycle could be an underlying hypothesis. The other factors that determinate the market inefficiency have a strong influence on the EUA price and erode the mean reversion behavior, thereby showing an informational level lower than expected.

We have seen that under these assumptions the conditional distribution of the logarithm of the commodity spot prices is normal. This formulation rather underestimates the likelihood of large movements in the commodity spot price. As an attempt to capture the leptokurtosis that appears in the prices series we have to extend the mean-reverting model to accommodate large movements (jumps) in the spot prices. Such a popular extension of the standard mean-reverting diffusion process is the mean-reverting jump-diffusion process.

3.1.3. *Jump diffusion - Gaussian Mixture.* Before exploring the mean-reverting jump-diffusion process we will make a turnaround to the switching regime models, like an overture to the jumps models. A special case of the regime switching models is the Gaussian mixture - Gaussian movement with jumps (GBMJ) distributions [10]. Let the state (regime) that an unobserved process is, at time t, be noted as s_t, where there are m possible regimes (s_t = 1, 2,... , m). When the unobserved process is at the state j, i.e. s_t=j, the observed sample x_t is presumed to have been drawn from a N(μ_j ;${\sigma^2}_j$) distribution. Hence, the density of x_t conditional on the state variable s_t taking on the value j is

$$f(x_t/s_t = j; \theta) = \frac{1}{\sqrt{2\pi}\sigma_j} \cdot e^{-\frac{(x_t-\mu_j)^2}{2{\sigma_j}^2}}, \tag{3.1.9}$$

for j = 1, 2,...,m. Here θ is a vector of parameters that includes μ_1,μ_2 ... , μ_m and ${\sigma_1}^2$,${\sigma_2}^2$ ... , ${\sigma_m}^2$. The unobserved regime s_t is presumed to have been generated by some probability distribution, for which the unconditional probability that s_t takes on the value j is π_j defined as P(s_t = j;θ)=π_j for j = 1, 2,... m.

The unconditional density of x_t can be found by summing over all possible values that the state variable can take on:

$$f(x_t; \theta) = \sum_{j=1}^{m} P(x_t, s_t = j; \theta) = \sum_{j=1}^{m} \frac{\pi_j}{\sqrt{2\pi}\sigma_j} \cdot e^{-\frac{(x_t-\mu_j)^2}{2{\sigma_j}^2}}. \tag{3.1.10}$$

For example for a Mixture of two Gaussian distributions the unconditional density is :

$$f(x_t; \theta) = \frac{\pi_1}{\sqrt{2\pi}\sigma_1} \cdot e^{-\frac{(x_t-\mu_1)^2}{2{\sigma_1}^2}} + \frac{\pi_2}{\sqrt{2\pi}\sigma_2} \cdot e^{-\frac{(x_t-\mu_2)^2}{2{\sigma_2}^2}}. \tag{3.1.11}$$

Since the regime s_t is unobserved, the expression by equation (3.1.11) is the relevant density describing the actually observed data (x_t). If the state variable s_t is distributed across different times t, then the log likelihood for the observed data can be calculated from equation (3.1.11) as

$$L(\theta) = \sum_{t=1}^{m} log f(x_t; \theta). \tag{3.1.12}$$

The maximum likelihood estimate of θ is obtained by maximizing the expression (3.1.12) subject to constrains that $\pi_1 + \pi_2 + ... + \pi_m = 1$ and $\pi_j \geq 0$ for j = 1, 2,...,m. One way this can be achieved using numerical procedures or using the Expectation-Maximization algorithm developed by Dempster [7].

If jumps in prices correspond to the arrival of "abnormal" information, the number of such information arrivals ought not to be very large. For practical considerations, if t corresponds to one trading day, no more than one "abnormal" information arrival is to be expected on average. Furthermore, if returns were computed for finer time intervals, the Bernoulli model would converge to the Poisson model.

$$\frac{dS}{S} = \mu \cdot dt + \sigma \cdot dB + \kappa \cdot dQ. \tag{3.1.13}$$

At each time period if we do not have an arrival of abnormal information (event with probability (1-λ)) the next logprice is drawn by a conditional normal distribution with mean μ and variance σ^2. If we do have an arrival of "abnormal" information, a jump occurs and the log-price is drawn from a conditional normal distribution with mean $\mu + \mu_\kappa$ and variance $\sigma^2 + \sigma_\kappa^2$. Hence a Bernoulli jump-diffusion process can be written as a Gaussian mixture:

$$x_t \sim (1 - \lambda) \cdot N(\mu, \sigma^2) + \lambda \cdot N(\mu_\kappa, \sigma_\kappa^2), \tag{3.1.14}$$

where μ_κ is average size of a jump and σ_κ^2 is the variance of the jumps.

Parameter	Value	Confidence interval
λ	0.8042	[0.705, 0.903]
μ	0.121	[-0.315, 0.557]
σ	0.280	[0.240, 0.321]
μ_κ	-2.284	[-4.921,0.352]
σ_κ	0.82	[0.656,0.993]
Log Likelihood	1777.50	-

Table 5. Results of the Jump diffusion-Brownian mixture calibration on EUA 2009

The Gaussian mixtures model describes better the allowances prices evolution by including a switching regime hypothesis consecutive to market disruptions or to arrivals of new information (ie. regulatory). As shown in the Table 5, this model captures very well the asymmetric distribution of the information in the market between the big and small players.

A such informational broken symmetry was observed on the first three months of 2009 when the market was long due to an overallocation of some major industries. This event confined with a general turbulence on the energy market drove the allowances to the lowest historical price around 7-8, well below the economical limit of 10 representing the marginal cost of depoluting one tone of CO2.

Still under these circumstances the price level is free to fluctuate over one year horizon anywhere between zero and 150 euros. As we explained previously from an economic perspective the allowances price level should fall in a narrow band, hence emphasizing the mean reversion hypothesis. We consider now this case.

3.1.4. *Mean reverse Brownian diffusion with jumps.* A relatively simple mean-reverting jump-diffusion model for spot prices is described by the following equation:

$$\frac{dS}{S} = \alpha \cdot (ln(S) - m)dt + \sigma \cdot dB + \kappa \cdot dQ, \tag{3.1.15}$$

where the parameters are the same as in the simple mean-reverting model (3.1.3), κ represents the jump frequency, and dQ the jump metric. Due to the introduction of jumps we have some extra parameters that come into our model. If we have an arrival of abnormal information, a jump occurs and the log-price is drawn from a conditional normal distribution with mean $c + \beta \cdot x_{t-1} + \mu_\kappa$ and variance $\sigma_\varepsilon^2 + \sigma_\kappa^2$. Hence a mean-reverting Bernoulli jump-diffusion process can be written as a Gaussian mixture:

$$x_t \sim (1-\lambda) \cdot N(c + \beta \cdot x_{t-1}, \sigma_\epsilon^2) + \lambda \cdot N(\mu_\kappa, \sigma_\kappa^2), \tag{3.1.16}$$

where μ_κ is average size of a jump and σ_κ^2 is the variance of the jumps.

Parameter	**Value**	**Confidence interval**
λ	0.199	[0.097, 0.302]
c	-0.007	[-0.018, 0.031]
β	0.998	[0.989, 1.006]
σ_t	0.017	[0.015, 0.020]
α	0.512	-
m	29.678	-
σ	0.280	-
μ_κ	-2.230	[-4.870, 0.650]
σ_κ	0.823	[0.410, 0.995]
Log Likelihood	1778.25	-

Table 6. Results of the Mean reverse Brownian diffusion with jumps (GBMMRJ) calibration on EUA 2009

The calibration results (Table 6) show that the mean-reversion jump diffusion (GBMMRJ) model captures a little more information than the other models. Nevertheless the marginal improvement is minimal, given the large number of parameters and does not provide a robust choice. Once again the mean reversion model does not capture the suitable information. As a heavy tendency of the classic models calibration we underline the necessity of considering the jumps as an accurate approximation of markets regime variations.

3.2. **Generalized Hyperbolic models.** Mandelbrot was the first that challenged the idea that the Gaussian behavior of financial yields in the early 50's. His suggestion that the normal distribution is to "gentle" regarding the extreme events was fully proved by the past financial downturns as well as the actual crisis. More by challenging the normality of the yields we touch indirectly to the validity of the EMH. As the carbon market is characterized by a lack of transparency and by a quasi-regulated status it is obvious that the completeness of the information contained in the market price is relative. Even if the classic commodities models seem to give sufficient results, from the conceptual perspective it is necessary to challenge the normality of the carbon assets yields and to search for other classes of distributions that give more priority to extreme events.

As some recent research of Eberlein and Prause [8], Brandorff-Nielsen [3] showed the distributions of many financial quantities are well-known to have heavy tails, exhibit skewness, and have other non-Gaussian characteristics. The empirical distribution of daily returns from financial market variables such as exchange rates, equity prices, and interest rates, is often skewed, having one heavy, and one semi-heavy, or more Gaussian-like tail.

In this section we will make a quick review of the generalized hyperbolic functions and focus on the Normal Inverse Gaussian, one of the more used. Then we will compare the fitness of the NIG with that of the other members of this family and with the classic models. We will also add the mean reversion hypothesis into a discrete form.

As quoted in a lot of works in the literature [17], the generic form of a generalized hyperbolic model is given by :

$$f(x;\lambda;\chi;\psi;\mu;\sigma;\gamma) = \frac{(\sqrt{\psi\chi})^{-\lambda}\psi^{\lambda}(\psi+\frac{\gamma^2}{\sigma^2})^{0.5-\lambda}}{\sqrt{2\pi}\sigma K_\lambda(\sqrt{\psi\chi})} \times \frac{K_{\lambda-0.5}(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})e^{\frac{\gamma(x-\mu)}{\sigma^2}}}{(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})^{\lambda-0.5}},$$

where $K_\lambda(x)$ is the modified Bessel function of the third kind as described in the following formula [17]

$$K_\lambda(x) = \frac{1}{2}\int_0^\infty y^{\lambda-1}e^{-\frac{x}{2}(y+y^{-1})}dy. \tag{3.2.1}$$

3.2.1. *NIG distribution.* Among the generalized hyperbolic family the Normal Inverse Gaussian and the skew t-Student are by far the most quoted in the specific literature. From a pure analytic perspective we can apprehend the NIG distribution by setting $\lambda = -\frac{1}{2}$ in the previous equation.

$$f(x;-\frac{1}{2};\chi;\psi;\mu;\sigma;\gamma) = \frac{\chi^{\frac{1}{2}}(\psi+\frac{\gamma^2}{\sigma^2})}{\pi\sigma e^{\sqrt{-\psi\chi}}} \times \frac{K_1(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})e^{\frac{\gamma(x-\mu)}{\sigma^2}}}{(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})}.$$

By changing the variables of the previous equation $c = \frac{1}{\sigma^2}$; $\beta = \frac{\gamma}{\sigma^2}$; $\delta = \sqrt{\frac{\chi}{c}}$; $\alpha = \sqrt{\frac{\psi}{\sigma^2}+\beta^2}$ we obtain a more popular representation in the scientific literature.

From a more intuitive perspective the NIG distribution is in fact a normal function where the variance follows the Inverse Gaussian distribution.

$$X|Y = y \sim N(\mu+\beta\cdot y, y) \tag{3.2.2}$$

$$Y \sim IG(\delta\gamma,\gamma^2) \quad with \quad \gamma = \sqrt{\alpha^2-\beta^2}. \tag{3.2.3}$$

After few functional transformations that are not made to be digested in the present work we obtain that the density of a random variable X following the $NIG(\alpha,\beta,\mu,\ \delta)$ distribution is given by [4] [11]:

$$f_{NIG}(x;\alpha;\beta;\mu;\delta) = \frac{\delta\alpha\cdot exp(\delta\gamma+\beta(x-\mu))}{\pi\cdot\sqrt{\delta^2+(x-\mu)^2}}K_1(\alpha\sqrt{\delta^2+(x-\mu)^2}).$$

Hence it is obvious that the Normal Inverse Gaussian distribution is a mixture of normal and inverse Gaussian distributions.

It is also important to emphasize the moments (mean, variance, skewness and kurtosis) of a random variable X that follows a $NIG(\alpha,\beta,\mu,\delta)$:

$$\mathtt{E}(X) = \mu + \delta\frac{\beta}{\gamma} \tag{3.2.4}$$

$$\mathtt{V}(X) = \delta\frac{\alpha^2}{\gamma^3} \tag{3.2.5}$$

$$\mathtt{S}(X) = 3\frac{\beta}{\alpha \cdot \sqrt{\delta\gamma}} \tag{3.2.6}$$

$$\mathtt{E}(K) = 3 + 3(1 + 4(\frac{\beta}{\alpha})^2)\frac{1}{\delta\gamma}. \tag{3.2.7}$$

It is obvious that the NIG distribution allows behavior characterized by heavy tail and strong asymmetries, depending on the parameters α, β and δ.

The Figure 6 shows the fitted NIG distribution versus daily EUA08 yields.

Parameter	**Value**	**Confidence interval**
α	22.8921	[22.3260, 23.4570]
β	-2.318	[-2.6170, -2.0180]
μ	0.00113	[0.0009, 0.0012]
δ	0.0189	[0.0186, 0.0191]
Log Likelihood	1790.71	-

Table 7. Results of the NIG calibration on EUA 2009

In terms of calibration (Table 7) the NIG distribution captures from far more information than the classic models described in the previous section. The main reason is the ability of the GH models to be customized in same time to different skews and tails forms. In our case the carbon market is far from being Gaussian and the Gaussian mixture evoked previously fill in partially for this handicap but is still keeping the behavior in a normal universe. The NIG distribution brings

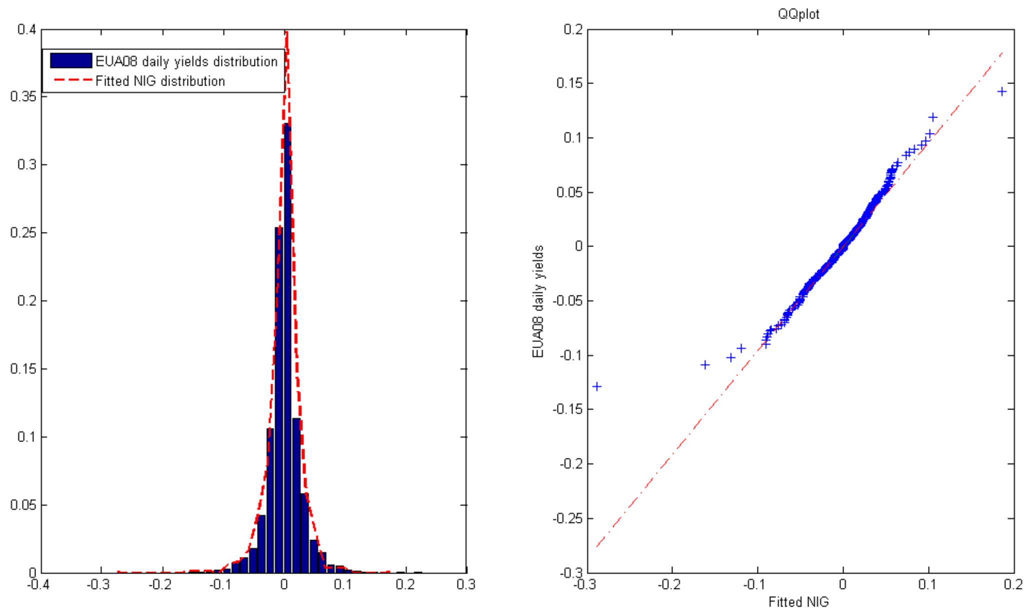

FIGURE 6. NIG Distribution adapted on EUA08: on the left yields histogram and the fitted NIG; on the right QQ plot.

another dimension with more parameters that are less intuitive than the Brownian process, but that are more suitable to asymmetric distributions with fat tails.

3.2.2. *Other GH distributions.* Even if the NIG and the Skew-t Student distributions are the most reputed amongst the generalized hyperbolic family, a righteous approach would be to scan for the distribution that captures the highest level of information offered by the times series. By changing λ from -0.5 to other values we could search for the distribution providing the best fit. This topic will constitute the focus of a future work.

3.2.3. *NIG - Gaussian mixture.* As we already saw in the previous section the Gaussian mixture hypothesis captures a lot of information concerning carbon prices behavior. Beside the historical series the economical facts show as that the carbon prices fluctuates, this conducts us to intricate the heavy tail asymmetric behavior with the classic Gaussian hypothesis, and we consider the following model:

$$\frac{dS}{S} = \lambda \cdot \boldsymbol{dNIG}(\alpha, \beta, \delta, \mu) \cdot dt + (1 - \lambda) \cdot \sigma \boldsymbol{dB}. \tag{3.2.8}$$

Given the fact that the NIG has far more complex form than the Gaussian distribution the integration of the precedent equation to a continuous form is not obvious. Thus we choose to make the calibration under a discrete form, and provide the results in Table 8.

Parameter	**Value**	**Confidence interval**
λ	0.251	[-0.062, 0.565]
α	17.228	[6.679,27.749]
β	-2.743	[-5.370,-0.107]
μ	0.002	[0.000,0.012]
δ	0.015	[0.009,0.020]
σ	0.429	[0.303,0.565]
LogLikelihood	1792.37	-

Table 8. Results of the NIG-Gaussian (NIG-GBM) mixture calibration on EUA 2009

Evidently, in Table 8 we observe that this mixture (NIG-GBM) captures more information than the simple NIG, and indicate us that we might have a market with to different regimes:

- a normal regime with a volatility of 42 % close to that observed in the previous section and
- a NIG regime that appears with a frequency of 75 %, a value closed to the jumps frequency from the Gaussian mixtures. This regime would describe market disrupters and turbulent regimes.

Nevertheless the incremental information compared with the NIG calibration is still little and the confidence interval of the jump frequency show little relevance. We believe that this is due to the fact that the market is still in a birth stage and the pattern of behavior are not yet very accentuated.

The bottom line of this part is that carbon allowances need both four moments distributions and broken symmetry regimes in order to capture more of its behavior. Future developments would include switching regimes with generalized hyperbolic distributions and autoregressive volatility behavior.

4. Application to risk modeling of a CO2 derivative

At the dawn of the carbon market a wide range of specific financial products aroused in order to answer to the different needs and to profit from arbitrage opportunities. From forward contracts to exotic options and structured products the financial institutions cover all the spectrum of derivatives mainly by leveraging on their commodities market experience.

Amongst those products, one of the most popular is the EUA-CER arbitrage swap, a CO2 structured strategy developed by most of the financial institutions in the carbon finance. This product allows to generate riskless income, by taking the price difference between CER and EUA prices. It is important to mention that under the European Environmental Compliance directives a company can be compliant if owns the necessary amount of allowances. Nevertheless it is also allowed to own Kyoto credits (CER) instead of EUA in order to be compliant at a level between 10 and 20÷ depending on the local regulation.

4.1. **The Arbitrage Swap.** The difference between prices of the EUA and CER can vary over time. The carbon arbitrage swap creates profitability from the immobilized allowances and income from the prices difference between EUA and CER without any consequences associated with the price fluctuation. The figure 7 presents the flows generated by the product.

The carbon arbitrage swap can be adapted for time horizons between one and five years 2008-2012, until the end of the second Kyoto period. It is tailored in such

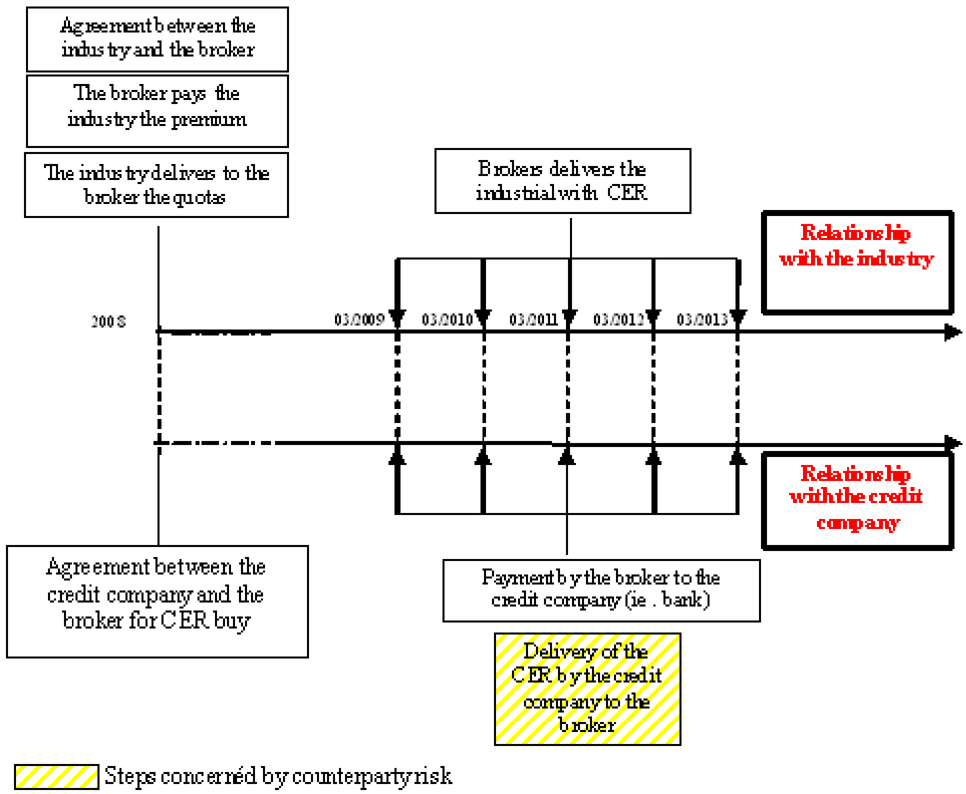

FIGURE 7. Carbon Arbitrage product

manner that the client is compliant at each regulatory deadline. Hence the client receives the credits each year before its compliance date.

At the beginning of the transaction the industrial company delivers to the broker the quotas that are cashed out via a financial institution (credit company) on the market. In the same time the broker locks by an agreement with the credit company, the prices for the futures deliveries of the credits. In the following years the credit company will deliver periodically before the equivalent credits for the received quotas.

For example let us consider an industrial company with a CER limitation of 10÷ and suppose that the company has 1 million allocated 2008 quotas. The industry can thus surrender 100,000 CERs per year. Over 5 years therefore it has recourse to 500,000 CERs. In March 2008 the company transfers 500,000 of the 1 millon

EUAs which it receives from the Commission into CERs. Since the CER trades at a lower price than the EUA, the difference in price in the exchange releases a premium. This difference in price fluctuates today between 4 and 5,5 euros. In our example, the premium would be fixed for example at 4 euros per ton of CO2 given up for conversion. The customer will receive 500,000 * 4 = 2 million Euros at the date of the signature of the contract. The delivery of the EUA between the broker and the industry can be done immediately.

4.2. **Economical capital allocated to an Arbitrage Swap product.** It is obvious that the financial counterparty could be under default any time between the date of the contract and the effective deliveries dates of the CERs. In this case the broker must replace the CER, by buying them at the market price. Under a scenario of a rising trend of CERs, the broker should fill the difference between the negotiated price at the beginning of the contract and the market price at the moment of default.

In the new heavy regulated environment each financial establishment should have enough capital to cover the extreme risk undertaken by its operations. In our case the broker should have put aside enough economical capital to cover the consequences of a probable default of the financial counterparty. A classic metric of the economical capital used by many financial institutions for a structured product is the value at risk (VaR). The VaR is a very intuitive measure but has some obvious failures as it shows no subaditivity (the value of the risk measure for two risks combined will not be greater than for the risks treated separately). Some remedy proposal are discussed by Artzner[1] that will not be applied in this paper in order to keep a basic metric and to compare only the adequacy of the underlying model. Mathematically, VaR is defined as follows, for α given:

$$VaR_\alpha(X) = -inf(x \mid \mathbf{P}[X \leq x > \alpha]). \tag{4.2.1}$$

The risk undertaken by the broker is the difference between the negotiated credit price and the observed price in case of a default of the credit company. Hence in order to evaluate the Value at Risk of the product three problems arise:

- estimate the default risk over the product time horizon ;
- estimate the spread behavior between the EUA and CER ;
- cumulate the credit and the market risks.

The default risk is estimated via the default probabilities and transition matrices given by the rating agencies. The behavior of the EUA - CER spread deserves a more detailed analysis and will not constitute the object of the present paper. We will model the spread by a fixed discount rate to the EUA prices. We will provide the analysis in a future work. The imbrication of both credit and market risk is made via a CreditMetrics-like approach. Otherwise we simulate the exposure scenarios in case of counterparty default, thereby obtaining the aggregated risk distribution.

As we mentioned previously the product exposure in case of default is the spread between the CER price at the default moment and the price at the moment of the deal on upward market. The following equation is nothing else but the price of an European call with a strike equivalent to the initial price of the CER.

$$Exposure(t) = \mathbf{E}(max(0, (S_{CER}(t) - S_{CER}(0)) \cdot e^{-r \cdot t}), \tag{4.2.2}$$

where r is the free risk rate and $S_{CER}(t)$ and $S_{CER}(0)$ are the CER prices at the moments t and 0;

Our approach to obtain the loss distribution consists in simulating via a Monte Carlo method a grand number of events with two potential consequences:

- No default observed and no loss occurred
- Default of the financial counterparty and an estimated loss as the average of the positive spreads between the future CER prices and the initial CER price,

then,

$$Loss(t) = [Zeros_{[NS\cdot(1-PD(t))]}, Exposure(t)_{[NS\cdot PD(t)]}], \tag{4.2.3}$$

where $Zeros_{[NS\cdot(1-PD(t))]}$ is a vector containing zeros with a length of $NS \cdot (1 - PD(t))$, Loss(t) is the simulated vector of potential losses, and PD(t) is the default probability at the moment t.

As we show, at a certain horizon t, the simulated loss vector is the result of the concatenation of a zero vector for cases of non-default and of the exposure vector for the default cases. In the environment of investment banking, the VaR measure is considered for a 10 working days horizon and at a 95% confidence level. This kind of risk metric is adapted for marketable and liquid securities, but once we enter in the area of structured products with long horizon and low liquidity, it is suitable to consider a VaR time horizon close to that of the product.

The classic view of risk universe is under a normal metric and a risk-neutral probability adapted for the efficient markets. In our case we considered a Monte Carlo VaR based on non-Gaussian distributions calibrated on the historical times series. We emphasize, in the next subsection, the main difference between the various models taken in account.

4.3. **Value at Risk: Classic versus GH models.** In the following, we compare the economical capital figures for the carbon arbitrage swap measured via different models calibrated in the previous sections. We apply the example given in the previous paragraph with 1 million quotas per year, for each delivery horizon on the product, thereby providing a Value at Risk figure for each year.

The product risk exposure depends on two factors: the nominal exposure of the swap and the volatility of the market. It is obvious that the nominal exposure is diminishing with the passage of time because periodic deliveries are made. On the other hand the passage of the time amplifies the volatility of the market and indirectly the product risk exposure. The two factors have different sensibilities depending on the time horizon and in consequence the maximum marginal exposure is at the half way horizon time. In table 10 we show the average loss for different horizons and in tables 11, 12, and 13 the results of Monte Carlo VaR for α values of 99.5%, 99.9% and 99.99%.

It appears that for a given α the VaR have heterogenous values depending of the models. The GH and jumps models give the most conservative results due to the fact that they contain more information about the tail behavior than the classic models, hence emphasizing the potential extreme events.

For different values of α the variations of the risk measures are also depending of the chosen model. Hence models with strong kurtosis tend to show bigger variations for different percentiles that classic models. This depends on the capacity of the model to enclose significant information that could characterized the extreme percentiles. In these cases, figures of GH and jump models consume more capital than Brownian diffusion models.

The horizon plays an essential role for this type of product. As we have already shown the risk horizon for the arbitrage swap is measured in years and goes far beyond the classic "10 days" used by the option desks. For longer horizon the VaR becomes bigger but the incremental VaR from one year to another has a maximum value between the second and the third year.

Model	**2008**	**2009**	**2010**	**2011**	**2012**
GBM	0.0148	0.0620	0.0820	0.0783	0.051
GBMMR	0.0174	0.0276	0.0315	0.0267	0.0156
GBMMRJ	0.0217	0.0362	0.0394	0.0324	0.0194
GBMJ	0.0242	0.0473	0.0554	0.0484	0.0272
NIG	0.0411	0.0991	0.1482	0.1511	0.1092
NIG-GBM	0.0461	0.1245	0.1921	0.1952	0.1402

Table 10. Average of the loss distributions

It appears that the averages of the lost distributions stay in the same range of values for all models for a given horizon. Nevertheless the models with fat tails show a bigger average due to the extreme losses that could appear.

Model	**2008**	**2009**	**2010**	**2011**	**2012**
GBM	0.2358	2.4458	4.3021	4.3385	2.9806
GBMMR	0.2156	1.3789	1.9544	1.7293	1.0188
GBMMRJ	0.2345	1.4850	2.2347	2.0264	1.2299
GBM	0.1553	1.3879	2.2468	2.2581	1.3059
NIG	0.5770	5.2909	8.9410	9.1141	6.3059
NIG-GBM	0.5937	6.2808	11.3493	11.4375	8.0929

Table 11. $VaR_{99.5}$

Model	**2008**	**2009**	**2010**	**2011**	**2012**
GBM	9.2113	16.7318	19.3572	17.4601	10.4790
GBMMR	5.5986	7.1933	7.0370	5.1112	2.7391
GBMMRJ	7.1489	10.1157	9.3211	7.0537	3.8999
GBMJ	8.0263	13.0906	14.0142	11.5880	6.4137
NIG	13.3518	24.7028	31.6558	28.0115	19.1064
NIG-GBM	15.3028	30.4945	39.6635	37.6635	24.6062

Table 12. $VaR_{99.9}$

Model	**2008**	**2009**	**2010**	**2011**	**2012**
GBM	25.6763	45.5230	51.7478	49.0629	28.6319
GBMMR	16.5090	16.0876	14.7093	9.9976	5.3056
GBMMRJ	19.4343	24.9377	21.6928	15.3175	8.4437
GBM	26.0841	41.3185	43.0824	35.0128	18.9109
NIG	33.0043	61.2307	75.3453	75.7261	52.8432
NIG-GBM	37.9612	84.4808	99.6631	97.9891	6.8784

Table 13. $VaR_{99.99}$

The VaR results are more heterogenous and show big differences between Gaussian and Generalized hyperbolic models. It appears also that jump models are more conservative in term of capital allocation.

5. Conclusions

Understanding the GHG market goes beyond the classic stochastic apprehension of the financial assets like commodities and enters in a more subjective area of the behavioral finance. Thus, the main topic of this paper is to search for an econometric model that could fit the best the historical time series. The discriminant factor to rank models relevance was the likelihood. The GHG prices show a pronounced non-Gaussian behavior with fat tails and negative skewness. The NIG distribution outperforms the classic Brownian models in term of quantity of information. It appears clearly that jumps are a necessary hypothesis for an accurate modeling of the CO2 prices.

We applied the results of the model calibration on the risk estimation of a financial product specific to the CO2 market, the EUA -CER swap. The economical capital allocated for carbon transaction are more conservative when we use Generalized hyperbolic models than with classic Brownian models.

Further developments will include the autoregressive models [15],[14] and memory effects in the model calibration, the econometric study of the EUA - CER spread and the macro-economical model of the CO2 market taking in account fundamental factors.

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