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The Meaning of πλασματικόν in Diophantus' Arithmetica

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1 Introduction

Diophantus' *Arithmetica* is an idiosyncratic treatise: it almost calls for interpretations that force conceptions into the text that are extraneous to the by-and-large ineffable aims of its author. A case in point is constituted by a short clause found in three problems of book I.[1] It is not a harmless clause: it contains the elusive adjective πλασματικόν as the crucial word, the translation and meaning of which sparked a long-lasting controversy that has become a non-negligible aspect of the debate about the possibility of interpreting Diophantus' approach and, more generally, Greek mathematics in algebraic terms.[2] The whole interpretative controversy is in fact entangled. On the one hand, it involves a "natural" algebraic interpretation of some of Diophantus' problems. On the other hand, it envisages the adaptation of geometric results to number theory,[3] since the controversial clause appears to qualify an implicit reference to some theorems in *Elements*, book II. For this reason, the debate was strictly linked with, and to a large extent preceded, Tannery's first statement of the interpretative frawework called «geometrical algebra».[4] The latter looks for geometrical (*Elements* II) or arithmetical (Diophantus' *Arithmetica*) approaches to the solution of equations of the second degree – conversely licensing the legitimacy of an algebraic reading of those very texts. I shall argue that the controversy about the meaning of the clause was grounded on a misunderstanding of the Greek text, and evaporates once the proper meaning of πλασματικόν is restored. The correct interpretation of the word, a technical term in the Greek rhetorical tradition that perfectly fits the context in

[1] As we shall see, a similar clause features also in three problems preserved only in the Arabic tradition of the *Arithmetica*. Most of the interpreters mentioned below did not know the books attested only in the Arabic translation by Qusṭā ibn Lūqā (second half of 9th century), as this came to scholarly attention in the late 60s of 20th century.
[2] Apparently, the *Arithmetica* raised such kind of interpretative problems very soon, if Qusṭā ibn Lūqā was eager to give an algebraic bent to his translation; see Diophante 1984, tome III, pp. l-lvi.
[3] In my discussion, I shall occasionally refer to Diophantus' approach as number-theoretic; at least, this seems to me the best translation of the syntagma ἀριθμητικὴ θεωρία by which Diophantus designates his own domain of research (*Diophanti Alexandrini Opera Omnia*, vol. I, p. 4.14 – I shall henceforth shorten this title as *OO*). Here and elsewhere I checked the text on the unpublished edition Diophante d'Alexandrie 1980.
[4] Tannery 1882.

which it is inserted in the *Arithmetica*, entails that neither Diophantus' text nor the controversial clause contained any (implicit) reference to Euclid's *Elements*. The clause is in fact a later annotation, that comments on a puzzling feature of some Diophantine problems and that got inserted into the text. In this perspective, the clause can even be read as a very *naïve* and rough form of protestation against an embryonic interpretation of some problems of the *Arithmetica* in geometrical or algebraic terms. As a side result, the present investigation will lead to a (admittedly very circumscribed) reappraisal of Byzantine scholarship.

The Greek textual tradition of the *Arithmetica* is rather complex; its relationships with the Arabic tradition raise problems that still remain unsettled.[5] The *Arithmetica* was originally redacted in 13 books.[6] A large collection of problems arranged in 6 books is preserved in Greek. Four books of problems are transmitted in Arabic translation, referred to in the titles and subscriptions of the very Arabic text as books IV to VII of Diophantus' treatise.[7] The two sets of problems are almost completely disjoint.[8] It is not clear whether such partitions into «books» exactly reflects the original Diophantine partition, or rearrangments of the material have occurred at some stages of the transmission.[9] It happens, then, that we read sections of the *Arithmetica* called «books IV-VI» both in Greek and in Arabic, but they are completely different: it appears that the Arabic books IV-VII should immediately follow the Greek I-III,[10] whereas the original place of the Greek IV-VI is still unclear.

In Section 2 I shall present the relevant text; after an exposition of what is a determination in Diophantine number theory (Section 3), past interpretations of the clause are discussed in

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[5] For a recent assessment of the Greek tradition see Allard 1982-3. The Arabic tradition is surveyed in the introductions of Sesiano 1982 and Diophante 1984, where one finds critical editions of what remains of the Arabic translation of the *Arithmetica* (preserved in one single manuscript).

[6] So Diophantus himself in his introduction; see *OO*, vol. I, p. 16.6-7.

[7] See Diophante 1984, tome III, pp. 1.2, 98.14, tome IV, pp. 1.2, 34.6, 35.2, 80.5, 81.2, 120.1.

[8] A few exceptions will be mentioned below.

[9] The criterion by which the material of the Greek *Arithmetica* is organized into books is not always perspicuous, if indeed any general criterion is at work (local criteria may apply; for instance, the sixth book contains problems of quite a different kind). Not every manuscript of the *Arithmetica* has it organized in 6 books; 3 of them divide book IV into two books, at least other 2 manuscripts divide book I into two books (see the descriptions of the Diophantine mss. in Allard 1982-3, pp. 58-72). Even if the involved manuscripts are late copies, it is by no means assured that a similar phenomenon could not have occurred in very early copies. Other phenomena might have blurred the distinction between the books. For instance, in the *Vat. gr.* 191 the books are simply separated by a blank line; the titles of the first four books are supplied by a different hand, the fifth and the sixth book carry no title. Moreover, neither in the *Matritensis Bibl. Nat.* 4678 nor in the *Vat. gr.* 191 (namely, the oldest mss.) the problems are numbered: they are simply separated by a blank space. Such features show that the organization of the *Arithmetica* into books was easily exposed to scholarly or even to scribal modifications. Recall that Tannery deemed problems II.1-7 and III.1-4 as spurious, on account of the fact that they contain material more properly pertaining to the last parts of books I and II, respectively, or that they repeat other propositions. Tannery suggested that those probles were *marginalia*, coming from a commentary to books I and II, that got inserted into the text (*OO*, vol. I, pp. 83 and 139 *in app.*, and Tannery 1884, pp. 80-82 of the reprint – Tannery suspected also problems II.17-18 and III.20-21). After comparison with the Arabic text, where problems VI.1-11 are likely interpolations, also problems IV.1-2 of the Greek text should be regarded as spurious (see Sesiano 1982, p. 53).

Section 4, where I shall also present my proposal in detail. In Section 5 the evidence coming from the Arabic translation will be taken into account. Section 6 is devoted to an analysis of Maximus Planudes' contribution to the issue, who appears to have given the clause the same meaning as the one proposed in the present paper. An overall assessment is finally offered in Section 7; in particular, I surmise that the clauses we find in the Arabic text of the *Arithmetica*, though almost identical in structure, do not translate the πλασματικόν-clause of the Greek text.

2 πλασματικόν in the Greek text of Diophantus' *Arithmetica*

Problem I.27 of Diophantus' *Arithmetica* has a διορισμός, that is, a «determination» of the conditions of resolubility of the problem. The sentence stating the determination can be easily recognized as such, since it immediately follows the complete enunciation of the problem, it is not instantiated, and it begins with the standard expression δεῖ δή «thus it must be that».[11] The determination is followed by a short clause, apparently commenting on some feature of what has just been stated. I shall henceforth refer to it as «the metamathematical clause». Here is the text:

> Εὑρεῖν δύο ἀριθμοὺς ὅπως ἡ σύνθεσις αὐτῶν καὶ ὁ πολλαπλασιασμὸς ποιῇ δοθέντας ἀριθμούς.
>
> Δεῖ δὴ τῶν εὑρισκομένων τὸν ἀπὸ τοῦ ἡμίσεος τοῦ συναμφοτέρου τετράγωνον τοῦ ὑπ' αὐτῶν ὑπερέχειν τετραγώνῳ. ἔστι δὲ τοῦτο πλασματικόν.[12]

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[10] See Sesiano 1982, pp. 4-8.

[11] The instantiated statement of what is to be constructed in a (geometric) problem also begins with the same standard expression; maybe this is the reason why Proclus, in his commentary on *Elements* I, calls this kind of statement «determination» (see also Tannery 1887, note 2 on p. 149, who, however, ascribes the terminological point to Geminus). In fact, Proclus discusses at some length only this meaning of «determination»; he mentions the other, almost surely original, meaning only *en passant* (*In primum Euclidis*, pp. 66.22-67.1 and 202.5-8 Friedlein, the latter a passage very likely lifted from Geminus, the former a short indication in the «catalogue of geometers» – the characterizations of the determinations in these passages are identical and they must be ascribed to a common source). Remarkably enough, Proclus does not offer any further elucidation when he is commenting on I.22, the first theorem in the *Elements* having a determination. A curious feature of Proclus' discussion is that, contrary to what is usually assumed, he never makes reference to the statement following the «setting out» of a *theorem* and beginning by the standard expression λέγω, ὅτι «I say, that» as a διορισμός. In fact, his discussion at *In primum Euclidis*, pp. 203-208 Friedlein, especially pp. 204.20-205.12, seems even to suggest that both «setting out» and «determination» exist only as parts of *problems*. The issue of the double meaning of «determination» is briefly discussed by Eutocius in his commentary on Apollonius' *Conica* (*Apollonii Pergaei quae graece exstant*, vol. II, p. 178.4-15), in order to explain the reference to determinations in Apollonius' description of book II of his *Conica* (*ibid.*, vol. I, p. 4.5-8). Eutocius there quotes the complete enunciation of *Elements* I.22 as an example of determination, a move that deluded Heiberg (see note 15 below). Given the peculiar character of the problems in the *Arithmetica*, Diophantus does not give the enunciation of a problem followed by an instantiated version of the enunciation itself, as is usual in geometric propositions (setting out + determination according to the Proclean scheme). Diophantus occasionally resumes, in the middle of a proof, what is sought for in a problem: such statements are sometimes introduced by δεῖ δή «thus it must be that» (*OO*, vol. I, pp. 46.2, 60.17, 232.2, 298.1) or by δεῖ δέ «and it must be that» (*ibid.*, pp. 116.8-9, 136.4-5, 220.23-24, 238.21-22, 244.22-23).

[12] *OO*, vol. I, pp. 60.23-62.2.

To find two numbers in such a way that their sum and product make given numbers.

Thus it must be that, of the <numbers> found, the square on the half of both of them together exceeds the <rectangle contained> by them by a square. But this is *plasmatikon*.

Identical structures (enunciation-determination-metamathematical clause) can be found in *Arithmetica* I.28 and I.30 and nowhere else in the surviving Greek text of the *Arithmetica*.[13] The *crux* is the meaning of the word πλασματικόν, a controversial point since Xylander's first translation in 1575.

3 Determinations in the *Arithmetica*

In Greek geometry, a determination spells out the necessary condition under which a problem is soluble. The condition is always expressed in terms of the givens of the problem. Very elementary examples can be found in the *Elements*. In it, the necessary condition to be fulfilled is normally proved as valid in a theorem immediately preceding the problem needing the condition as a determination. The necessary condition of I.22 is proved in I.20, the condition of VI.28 in VI.27, the two conditions of XI.23 in XI.20-21. Let us read XI.23, where one of the determinations, the one referring to XI.20 and in italics in the translation, is embodied in the text of the enunciation, a move that is quite unusual:[14]

Ἐκ τριῶν γωνιῶν ἐπιπέδων, ὧν αἱ δύο τῆς λοιπῆς μείζονές εἰσι πάντῃ μεταλαμβανόμεναι, στερεὰν γωνίαν συστήσασθαι·

δεῖ δὴ τὰς τρεῖς τεσσάρων ὀρθῶν ἐλάσσονας εἶναι.[15]

To construct a solid angle out of three plane angles, *two of which, substituted in any manner,*[16] *are greater than the remaining one*;

thus it must be that the three <angles> are less than four rights.

[13] *OO*, vol. I, pp. 62.20-25 and 66.2-6. The metamathematical clauses are in these cases «But this too is *plasmatikon*», apparently on account of the fact that they immediately follow the first occurrence in I.27. Three further occurrences of very similar clauses in the Arabic translation will be discussed in Section 5.

[14] But the same happens in *Elements* IV.1, whose determination is an immediate consequence of I.20.

[15] *Euclidis Elementa*, vol. IV, p. 33.7-10. Actually, in the main text of Heiberg's critical edition of the *Elements*, only the determination of XI.23 begins with the "standard expression" δεῖ δή. However, this was Heiberg' mistake. Against the readings of all manuscripts, he decided to emend δή into δέ, following Proclus' lemma and a wrong reading in Eutocius' quotation of I.22 (that Heiberg was reading in a debased text), an emendation by August for VI.28. Heiberg corrected himself in a note contained in his edition of book XI (*Euclidis Elementa*, vol. IV, p. 33.9 *in app. I*), as he had established that Eutocius too read δή in I.22.

[16] The standard translation «taken together in any manner» is wrong; see Federspiel 2006.

Problems in Diophantine number theory are always submitted to two general requirements restricting the range of possible solutions. Diophantus admits in fact only positive quantities, exprimable in numbers, as solutions; these are positive rationals numbers in modern terms. This entails that a sort of presumptive determination is at work in any problem of the *Arithmetica*. It is in a sense an accidental feature if many problems (actually the overwhelming majority) do not need a determination. As a consequence, determinations in the *Arithmetica* can take either the form of an inequality, when positivity of the solution is at issue, or the form of an identification of species, when rationality of the solution is secured, or, in the same case, the form of a (negative) requirement of "congruence".[17] By «identification of species» I mean that the determinations of rationality identify the εἶδος «species» of the involved expression of the givens. In fact, the condition typically is formulated as the requirement that a well-defined expression of the givens be a square, or a cube, or a single power. Therefore, such conditions guarantee that the involved expression of the givens produces a (rational) number when an operation of taking a square, cube, … root is performed on it, as is actually required by the procedure of solution. It must be stressed that the determinations of rationality are never explicitly formulated as equalities, even if it is immediate to write them down in that form, and Diophantus himself might well have done that.[18] Determinations of positivity are instead stated as inequalities.[19] The following table lists the problems having determinations in the extant *Arithmetica*.[20] A few problems lack determinations, even if they would require one.[21]

Determinations/Books	Greek	Arabic
rationality	I.27-28, 30, IV.34-35	IV.17-22, V.7-12, VII.6
positivity	I.5-6, 8-9, 14, 16-17, 19, 21, 21 *aliter*, II.6-7	V.13
congruence	V.9, 11	

[17] The latter case occurs in V.9, 11 only. The determination of V.11 states that the given number cannot be of the form $8n+2$. The text of the determination of V.9 is corrupt but the needed condition can be reconstructed: see, e.g., Heath 1910, pp. 107-8.

[18] For instance, the verb «to make», which we have read above in the enunciation of I.27, is typical of the enunciations. During the proof, however, actual species are set out, and the verb is very often replaced, when statements corresponding to the enunciation are made, by «to be equal to» or simply by «to be». The identity of the three formulations is evident from such passages as, e.g., the last inference in I.29, where a deduction by transitivity displays in succession «to make», «to be», and «to be equal to» (*OO*, vol. I, p. 64.22-24).

[19] Absence of equalities as counterparts of inequalities can be found also in general proportion theory, where ratios can be terms of an inequality, but they are never said to be equal (they are said to be identical).

[20] Tannery deemed II.6-7 interpolations.

[21] The Greek IV.1-2 are attested also as Arabic V.7-8 but with differences that make the former almost certainly spurious. I.7 should have a determination of positivity (identical with the one of I.8), IV.15 a very complicated determination of rationality. V.20 is reduced to another problem (not attested in the preserved portion of the *Arithmetica*) that has a determination. In all instances where a determination is lacking the actual numbers by means of which the problem is instantiated and worked out are of course chosen *ad hoc*. V.10 is discussed just below.

lacking	I.7, IV.1-2, 15, V.10, 20	

First, I briefly discuss a terminological feature that might be misleading. Diophantus seems to call a determination προσδιορισμός, an intensive form of the canonical term διορισμός, which in its turn does not appear in the *Arithmetica*. The attested form occurs only in I.14 and V.10,[22] both problems being unrelated to those where πλασματικόν appears. In I.14 two numbers are sought such that the ratio of their product to their sum is given. The solution assumes one of the two solutions is fixed, and gives a precise value to it, making it, as a matter of fact, a new given.[23] Accordingly, the determination is expressed in terms of that solution (now a given) and the given ratio, and amounts to requiring that the former is greater than the latter.[24] Such a condition can rightly be termed a «further determination» since the givens of the original problem get modified in the course of the solution. The case of V.10 is more complicated. The problem has been suspected of authenticity since, uniquely in the *Arithmetica*, it sets out geometrical objects denoted by letters.[25] It is also framed in part as a canonical analysis, a short deduction being formulated as a chain of «data». Finally, it lacks an explicit determination.[26] The condition called προσδιορισμός in the text is instead a consequence of the one imposed on the numbers sought by the very enunciation of the problem.[27] It is thus the final step of a procedure of reduction of the original problem to another,[28] a fact that is proved in the first part of the proof on the basis of particular given numbers. The condition is then quite different from the one set out for I.14.[29] It is difficult to draw clear-cut conclusions from such a

[22] At *OO*, vol. I, pp. 36.6 and 340.9-10, respectively.
[23] This is in fact already recognized in the very formulation of the determination (*OO*, vol. I, p. 34.28), where the assigned solution is referred to as the «the *supposed* [ὑποτιθέμενον] multiplicity of units of one of the numbers».
[24] The actual formulation is worded in a way that is more complicated than that, but this is here immaterial (see *OO*, vol. I, p. 34.28-36.2).
[25] Hankel 1874, note marked * on p. 159. But it might well be that only the portion ranging on pp. 336.17-338.10 (actually, only the first half of the last line) is inauthentic, plus some further passages scattered in the ensuing reduction. The diagram attached to the proposition was added by the 17th century editor Bachet de Méziriac: he warns the reader that «in codice manu exarato deesse diagramma descriptionis Diophantææ quod nos restituimus, cùm absque illo non possint intelligi Græca authoris verba» (*Diophanti Alexandrini Arithmeticorum libri sex*, p. 307); cf. also *OO*, vol. I, p. 366.11 *in app*. Bachet de Méziriac made his edition after the ms. *Par. gr*. 2379.
[26] It should be a determination of rationality: the sum of the two given numbers plus a unit must be the sum of two squares.
[27] The problem requires to divide a unit into two parts such that, if different given numbers are added to each, the results will be squares. The two given numbers are taken to be 2 and 6; therefore, one of the two squares must be comprised between 2 and 3 (= the lesser given number 2 plus the unit set out). Two squares are then selected comprised between 2 and 3, namely 289/144 and 361/144. This entails that one of the numbers sought must be comprised between 17/12 and 19/12. After what we would call a "change of variable", this entails that a certain expression involving one of the numbers sought must be comprised between 17/12 and 19/12. This is the condition called προσδιορισμός in the text.
[28] That this is a true «reduction» is clear also to the redactor of this part of the proof, as he employs the verb ἀπάγειν to denote his own procedure (*OO*, vol. I, p. 338.3).
[29] With the crucial difference that the condition in V.10 imposes constraints on the solution, and not on the givens.

restricted sample. Maybe the term προσδιορισμός is not originally Diophantine, or maybe it is, but Diophantus employs it to designate «additional determinations» arising within sub-problems, and he would have regularly called the "main" determinations διορισμοί.[30] The occurrences in the *Arithmetica* are the first ones in the whole ancient *corpus*, even if this fact may well be irrelevant. Elsewhere in the *corpus*, the intensive form is found in Eutocius, *In De sphaera et cylindro* II.4,[31] and in Proclus,[32] in both authors in the sense of «additional determination». The use they make of the term does not suggest that it belonged firmly in the metamathematical lexicon, even if the evidence is too scanty to rule out this possibility.

4 Interpretations of πλασματικόν

There are no occurrences of πλασματικόν in the Greek mathematical *corpus* other than the three recorded in the *Arithmetica*. I set out in a table the translations or paraphrases of the metamathematical clause proposed by a number of scholars; after that I shall discuss the interpretations leading to or stemming from such translations.[33]

Xylander[34]	hoc autem est effictum aliunde
Bachet[35]	id a quo aliud quippiam effingi et plasmari potest
Cossali[36]	cosa altronde […] formata
Nesselmann[37]	das lässt sich aber bewerkstelligen
Wertheim[38]	und mann kann immer solche Zahlen als gegeben annhemen, dass diese Bedingung erfüllt ist
Tannery[39]	hoc est formativum

[30] The verb διορίζεσθαι is in fact attested in the Greek Diophantus (in VI.14-15, at *OO*, vol. I, pp. 424.14 and 428.21); the meaning is the canonical one of «setting a determination». The verb προσδιορίζεσθαι does not appear.

[31] At *Archimedis Opera Omnia*, vol. III, p. 150.15. Eutocius is referring to the determination of the famous problem, arising in the solution of *De sphaera et cylindro* II.4, that Archimedes declared he had deferred to an appendix and that Eutocius claims he had recovered in an «old roll». Archimedes states the problem twice; in the second formulation the determination is included in the hypotheses, very much as in Elements XI.23 seen above, so as to produce a problem that is always resoluble. The Eutocean mention of the «additional determination» stresses this move.

[32] At *In primum Euclidis*, pp. 240.27 and 349.21 Friedlein. In Proclus one records also the occurrences at *In Rem publicam*, vol. I, p. 29.1 Kroll and at *In Cratylum*, p. 53.28 Pasquali, not in mathematical contexts.

[33] Add to these the translation in modern Greek by E.S. Stamatis, sounding in English as «and this is formal» (Stamatis 1963; I have access to this information only through Christianidis 1995, note 1).

[34] *Diophanti Alexandrini Rerum Arithmeticarum libri sex*, p. 36.

[35] *Diophanti Alexandrini Arithmeticorum libri sex*, p. 56. The proper translation (*ibid*., p. 51) is an uncompromising «est autem hoc Plasmaticum».

[36] Cossali 1797, p. 95. Cossali simply provides an Italian translation of Xylander's reading. See below for the skipped clause.

[37] Nesselmann 1842, p. 326.

[38] Wertheim 1890, p. 35.

[39] *OO*, vol. I, pp. 63 and 67.

Heath[40]	this is of the nature of a formula
ver Eecke[41]	chose qui est d'ailleurs figurative
Gandz[42]	this may be demonstrated by a geometric figure
Allard[43]	apte à rendre le problème convenablement déterminé
Caveing[44]	la condition nécessaire que nous allons énoncer est lisible sur une figuration graphique
Vitrac[45]	et ceci est une condition formelle

A preliminary issue to be discussed is whether the demonstrative pronoun τοῦτο in the metamathematical clause refers to the (condition expressed by) the determination or to the whole problem. As is clear from the table above, almost all interpreters more or less tacitly assume that the former is the case. The very position of the clause immediately after the determination, the presence of the coordinative particle δέ, and the fact that normally the pronoun τοῦτο refers to the last mentioned linguistic unit recommend this as the most natural assumption. As we shall see in Sect. 5, the Arabic translation appears to refer an analogous metamathematical clause to a «problem» that might be either the whole problem or a «problem» formulated in or alluded to by the determination. Both options have been endorsed in the available editions of the Arabic text, and Allard's paraphrasis above is in line with the interpretation of the clause advocated in one of these. Planudes' reading of the Greek text, to be discussed in Sect. 6, refers each metamathematical clause to the corresponding problem *because* of the peculiarities of the determination. Therefore, it must be ranged with those referring the metamathematical clauses to the determinations. Planudes' attitude shows that inquiring whether the metamathematical clause refers to the determination or to the whole problem may possibly entail no substantial changes in their interpretation.

A first fact bearing on the interpretation of πλασματικόν is that the related verb πλάσσειν «to form» is a technical term in the *Arithmetica*: it means «constructing» a plane or solid number (such as a square, a right-angled triangle, a cube) out of one or many numbers.[46]

[40] Heath 1910, note 1 on pp. 140-1.
[41] Diophante d'Alexandrie 1926, p. 36. Ver Eecke explains «figurative» as something «susceptible d'une représentation géométrique par transformation d'aires» (*ibid.*, note 4 on pp. 38-9).
[42] Gandz 1937, p. 465. Gandz simply endorses ver Eecke's interpretation without reservations.
[43] Allard 1983, pp. 675 and 728, referring to πλασματικόν only. See also Diophante d'Alexandrie 1980, pp. 454-456. Allard's paraphrasis corresponds to the translation in Diophante 1984 (see the next section).
[44] Caveing 1997, p. 392. The author provides only this paraphrasis.
[45] Vitrac 2005, p. 19.
[46] See, e.g., *OO*, vol. I, pp. 90.14, 392.6, 242.16, respectively. Cf. the similar usage in Nicomachus, *Introductio arithmetica* II.12.6 and II.14.1. The verb is there applied to the formation of higher-order means from the basic ones at II.28.4, with the variant ἀναπλάσσειν at II.28.2. As a consequence, the verb occurs several times in Iamblichus' "commentary" on the *Introductio* (see Pistelli's *index verborum*, p. 178 *ad vocem*). The terminology for the construction of means is endorsed by Pappus, *Collectio* III.28, p. 70.3 Hultsch, where it is said that the harmonic mean «can be formed» from three lines that are in geometric proportion. According to Theon Smyrnaeus, *Expositio*

Tannery finds a geometric analogon in the verb πορίζειν «to provide»;[47] maybe συνιστάναι «to construct» would be preferable though not exact.[48]

The debate has been further diverted by the obvious (to modern eyes) connection of *Arithmetica* I.27-28 and 30 with the problem of solving an equation of the second degree.[49] In fact, the three problems ask to find two numbers of which are given respectively the sum and the product (27), the sum and the sum of the squares (28), or the difference and the product (30). If we form the equations of the second degree "naturally" associated with problems 27 and 30, the determinations of these problems simply give the conditions for the discriminant of the associated equations to be a perfect square. Interpreters since Bachet de Méziriac have stressed such a connection, proposed that the problems and the determinations were already seen in this perspective by the ancients, and read the metamathematical clauses accordingly.

This taken for granted, the controversial point has become whether πλασματικόν is active or passive, and of course which is the exact meaning. Here the similarity of the determinations with the conditions of *Elements* II.5, 9-10 and 8,[50] respectively, comes into play.[51] Let us read, for instance, the enunciation of II.5:

> If a straight line be cut into equal and unequal <segments>, the rectangle contained by the unequal segments of the whole together with the square on the <straight line> between the cuts is equal to the square on the half.

The two «unequal segments» in this statement would correspond to the two «numbers found» in the determination of *Arithmetica* I.27 read above.[52] Xylander's original translation of πλασματικόν as «effictum aliunde» stresses the point that the determinations are «formed» looking at the theorems of *Elements* II as models – and therefore conveys a passive meaning. Cossali, Nesselmann, Heath, Gandz endorsed such an interpretation; Cossali specified it as

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rerum ad legendum Platonem utilium, pp. 107.15-111.9 Hiller, any ratio can be «formed» from simpler ratios and proportions such as the one of equality. Theon mentions Adrastus as his source, but the reported proofs are very likely the ones Eratosthenes gave.

[47] Tannery 1882, note 1 on p. 278 of the reprint.

[48] The geometric lexicon employs the special verb ἀναγράφειν «to describe» for the construction of squares.

[49] But recall the perceptive remarks in Mahoney 1971, especially p. 376: «computationally one cannot distinguish between solution of the pair of simultaneous equations *via* the identity and solution of a quadratic equation by the quadratic formula». The «identity» alluded to by Mahoney is exactly the one involved in the determination of I.27.

[50] The «conditions» are the consequents of the conditionals that constitute the enunciations of *Elements* II.5, 9-10 and 8. Very early statements of the connection with *Elements*, book II, are to be found in scholia to *Arithmetica* I.27 redacted in the late 13th century (see Sects. 6 and 7 below).

[51] Actually, both *Elements* II.9 and 10 fit the determination in *Arithmetica* I.28. Such an indeterminacy is already a mark of the improper character of the connection.

«cosa altronde, cioè dai teoremi 5.° e 9.° del libro II di Euclide, formata».[53] Reacting to Xylander's reading, Bachet de Méziriac envisaged an active meaning: he took Diophantus as suggesting that from the solution of the problems or from the added determinations rules could be «plasmari et formari» for solving certain normal forms of equations of the second degree. Bachet de Méziriac even maintained that the clause in I.28 was interpolated since only 27 and 30 can properly give rise to such rules.[54] Tannery sided with Bachet de Méziriac, and, pushing his comparison with the use of πορίζειν to an extreme, ventured into proposing three corollaries (πορίσματα) in which the normal forms of the equations of the second degree are solved in a Diophantine style.[55] Caveing offers a rather detailed discussion of the issue,[56] presenting first a morphological and lexical *status quaestionis*. Caveing's interpretation stresses the reference to the graphic component inherent in the "proofs" of the determinations as we find them in *Elements* II. By this he validates and further qualifies ver Eecke's thesis, that the reference was in general to the possibility of representing the problems in the *Arithmetica* in a geometric setting.[57]

The problem with some of the above interpretations is that they move inside the perspective that the determinations attached to *Arithmetica* I.27-28 and 30 have a prominent mathematical status, and therefore deserve to be qualified by such an elusive adjective as πλασματικόν. Such a special status becomes hardly perceptible once the anachronistic reference to the equations of the second degree is dropped: what remains is only a patent reference to the theorems of *Elements* II. On the other hand, the interpretations that are satisfied on stressing such a reference simply forget the role a determination plays in a problem in number theory. This is definitely not something that can be «demonstrated by a geometric figure»: a determination is a constraint on a problem, and as such is neither demonstrable nor refutable without considering the actual

[52] Cf. the totally misleading algebraic transcription of the enunciation of *Elements* II.5 at *Euclidis Elementa*, vol. I, p. 73, *in app. 1*.
[53] Cossali 1797, p. 95; the entire discussion, where Cossali refutes Bachet de Méziriac's position, extends on pp. 91-95. Cossali is more eager to read *Data* 58-59 as providing rules for solving equations of the second degree.
[54] «Quamobrem cum a trigesima prima nulla formetur hujusmodi regula, non dubito eadem verba (et hoc quoque Plasmaticum est) ibi temere inculcata esse, ab ipso scilicet scholiasta, vel imperito amanuensi ex aliis quaestionibus eo translata», at *Diophanti Alexandrini Arithmeticorum libri sex*, pp. 56-57. The ordinal «trigesima prima» refers to the reckoning of Diophantus' problems according to Bachet de Méziriac's edition, where each of the alternative proofs in I.18, 19 and 21 is given a separate number.
[55] Tannery 1882, pp. 276-9 of the reprint. These corollaries were placed after I.27 and I.30. This is in keeping with Tannery's contention that the *Porisms* referred to by Diophantus himself were not a separate treatise of his, but simply a series of corollaries interspersed among the problems of the *Arithmetica* and now lost. Tannery was led astray by his conviction that the normal forms of the equations of the second degree were the items referred to by Diophantus when he says in his preliminaries that he will show «later also how, in the case where two species were left out equal to one <species> only, such <a problem> is solved» (*OO*, vol. I, p. 14.23-24).
[56] Caveing 1997, pp. 389-392.
[57] Diophante d'Alexandrie 1926, note 4 on pp. 38-9.

data of the problem. Nor can it be «lisible sur une figuration graphique», as this entails that the numbers involved in the constraint are allowed to assume arbitrary values.

What makes all these interpretations inadequate is simply the fact that those qualified by the metamathematical clauses are not determinations at all. First, they are expressed in terms of the solutions sought. This is explicit in I.27 and implicit in 28 and 30; these are the only determinations in the Greek *Arithmetica* formulated *uniquely* in terms of the solutions.[58] But this is not the way a determination should be formulated:[59] a determination sets limitations on the *givens* of a problem in order that this is soluble.[60] Second, and most importantly, the determinations of I.27-28, 30 are always fulfilled *insofar as they refer to the solutions*: they are identities and therefore cannot impose any limitations on the solubility of the problem. Of course, had the determinations been formulated in terms of the given numbers, they would not have been empty, but then they would have no longer preserved any connections with the theorems in *Elements* II. Hence, in the form they are written, the determinations do not really determine anything, or better said any of them is «fictitious» – in Greek πλασματικόν (by the way, this confirms that the meaning of the adjective is passive). The adjective is formed from the noun πλάσμα, whose meaning, according to the LSJ lexicon, is either *i*) «anything formed or moulded, image, figure», or *ii*) «formed style»,[61] «affected execution» of a musical piece, or *iii*) «forgery, fiction». From the latter sense, e.g., the adjective πλασματώδης «fictitious», found several times in the Aristotelian *corpus*, stems, and the noun πλασματίας, that is a forgery or someone addicted to forging. A πλασματογράφος is a writer of fictitious speeches for possible but not real occasions. In fact, the noun πλάσμα «fiction» and other terms derived from it, such as the adjectives πλασματικός and πλαστός and the verb πλάσσειν, were very soon raised to the status of technical terms in the late Hellenistic classification of διήγησις «narration», the criterion being adherence of its subject-matter to truthfulness.[62] In an earlier

[58] One might object that references to the solutions can be found in I.14, I.21, and II.6-7. In fact, this is not the case. For I.14, this has been discussed above. The determination in I.21 appears to be a limitation arising from the particular procedure of solution adopted (and in fact the determination in I.21 *aliter* sets out another condition). The references to «them» (*scil.* the numbers sought) in the determinations of II.6-7 are a way to distinguish the two «excesses» at issue, the excess of the numbers sought being in fact among the givens of the problem (the «excess» in II.7 is assigned even if it is not among the givens identified in the enunciation, very much as is done in I.14).

[59] This is particularly striking, if compared with the careful formulation of other determinations. See, for instance, the enunciation + determination of I.5 (*OO*, vol. I, p. 20.10-16), where two different verbs denote the given numbers: one is said to be «assigned», the other being «given». Clearly, the aim is to make the reference to the two given numbers to be made in the determination unambiguous.

[60] The determinations of problems I.27-28, 30 are of course written in terms of the givens, e.g., in Heath 1921, p. 487, but the received text of the *Arithmetica* is thereby obviously falsified.

[61] Or «style» *tout court*, a meaning that can be traced back at least to Tauriscus, a disciple of Crates (2nd century B.C.; the latter is a contemporary of the great Alexandrine scholar Aristarchus); see Sextus Empiricus, *Adversus mathematicos* I.248-249.

[62] See the syntheses in Meijering 1987, pp. 72-90, especially 84-87, and Papadopoulou 1999.

phase, the above terms had a meaning that was not associated with verisimilitude, carrying only negative overtones.[63] However, starting at least from Asclepiades of Myrlea's (1st century B.C. – maybe a pupil of the renowned grammarian Dionysius Thrax) tripartite classification of narratives,[64] πλάσμα and akin terms acquired a further, more specific, connotation:[65] they denote a piece of literary composition whose subject-matter is untrue, but nevertheless realistic (ὡς ἀληθὴς ἱστορία «like-true story» in Asclepiades' words). This is opposed, on the one side, to μῦθος, or ψευδὴς ἱστορία «false story», that denotes untrue and unrealistic stories, insofar as they include elements that are obviously false, and, on the other side, to (ἀληθὴς) ἱστορία «true story». Of the former alternative genre «there is one kind only, the genealogical», namely, generation by means of impossible births or metamorphoses and, more generally, what pertains to evidently impossible features of mythical narrations.[66] Of the latter, «there are three parts: one is that about the persons of gods and heroes and notable men,[67] another about places and times, another about actions». Literary genres attached to πλάσμα are asserted to be comedy and mime, whose subject-matter may give the impression of truth, while being pure fiction. Many occurrences of the lexical galaxy centred on πλάσμα found in scholia, that actually make up the bulk of the sources at our disposal about ancient literary theory, confirm that the denomination was a canonical one.

Sectorial branches of ancient literary criticism preferred another term to Asclepiades' «fiction», seemingly in order to cope with the peculiar elements of realistic fiction found in tragedy. Apparently, meaning I of πλασματικόν in the LSJ lexicon, namely, «imitative, dramatic» is to be traced back to a misunderstanding of this identification. The adduced passage is in (pseudo-)Hermogenes' *Progymnasmata*,[68] where in fact it is only said that some call δραματικόν «dramatic» the πλασματικόν genus of tales. It is clear that the author of the *Progymnasmata* is mentioning and identifying two pre-existing technical terms, and hence we should not assume that the meaning of one of them is being explained in terms of the other. As a consequence, all occurrences of the adjective πλασματικόν must actually be related to «fictitious» as the sole meaning. This is an easy check since very few occurrences can be

[63] Cf. Xenophanes' opinion about the tales of Titans, Giants and Centaurs as πλάσματα (Atheneus, *Deipnosophistae* XI.7, 462F = lines 21-22 of fr. 21 B 1 Diels-Kranz), the Aristotelian fragment reported by Strabo in *Geographica* XIII.1.36 (= fr. 402 Gigon = fr. 162 Rose), and some scholia reporting assessments of Megaclides, a Homeric critic of the second half of the 4th century B.C. (for a discussion of the scholia see Papadopoulou 1999, pp. 204-206).
[64] See Sextus Empiricus, *Adversus mathematicos* I.252-253. The text must be corrected according to Kaibel's transposition.
[65] The tension between the two connotations persisted in later authors.
[66] This is clear from Sextus' resumption of the classification at *Adversus mathematicos* I.263-265.
[67] The presence of gods and heroes should not surprise us.

recorded in the entire ancient Greek *corpus*. All of them are found in sources later than Sextus, with the unique notable exception of an Eudemian fragment preserved by Simplicius.[69] Whoever wrote the metamethematical clauses, then, must have been a scholar both able to look at the text of the *Arithmetica* with quite an insightful mathematical eye and well acquainted with rhetorical terminology.

To be sure, the only attested semantic area attached to πλασματικόν might count for nothing, since in the *Arithmetica* the term might well have been employed in an outlandish, technical sense. The value of such an objection is greatly diminished by the fact that in the *Arithmetica* the term is introduced without a word of explanation and at the beginning of the treatise: postulating for it a technical meaning must be a secondary option once the non-technical one provides an acceptable reading.

The issue gets complicated by the testimony of the Arabic version of the *Arithmetica*.

5 The Arabic metamathematical clauses

In the portion of the *Arithmetica* preserved only in Arabic we read three metamathematical clauses strictly analogous to those found in book I: they follow the determinations in IV.17, IV.19, and V.7. I provide the translations of the metamathematical clause in IV.19 proposed by the two editors of the Arabic Diophantus:[70]

Sesiano[71]	this belongs again to the constructible problems
Rashed[72]	c'est là aussi un des problèmes convenablement déterminés

[68] *Progymnasmata* 2.3, p. 4.16 Rabe = p. 183 Patillon.
[69] Simplicius, *In Physicam*, p. 48.24 Diels. This is inserted in a long extract from Eudemus (the whole quotation, extending on p. 48.8-26, is fr. 34 Wehrli). The issue is whether there is a science dealing with its own principles or not. Eudemus reduces the latter hypothesis to an infinite regress. In the former hypothesis, says Eudemus, additional arguments shall be needed in order to explain why a particular science is allowed to deal with its own principles, whereas the others are not, for otherwise the characterization of that science would resemble something fictitious (πλασματικῷ γὰρ ἔοικε τὸ ἴδιον). Torstrik's emendation to πλάσματι is unnecessary, and is rightly confined in the apparatus by Diels.
[70] I have not chosen the first occurrence in IV.17 since it does not have a perspicuous text: it does not seem to refer to the determination or to the problem but to the square excess. Rashed (Diophante 1984, tome III, p. 27) translates «qui est dit convenablement déterminé». Sesiano modified the text of IV.17 in order to put it in agreement with the others.
[71] Sesiano 1982, lines 495-6 on p. 100 of his translation. I have skipped Sesiano's integrations. As shall be clear from the discussion below, Sesiano takes the «constructible problems» to be the ones formulated by the determinations, and hence refers the metamathematical clause to the determination itself.
[72] Diophante 1984, tome III, p. 30. Cf. also the same statement in Allard 1983, pp. 675 and 728.

The structure of the sentence is identical with the ones already seen in book I. One is led to assume that the Greek πλασματικόν corresponds to Arabic *muhayya'* and that the metamathematical clauses comment on analogous features of the determinations.[73] This might suggest that all of them belong to a Greek textual layer earlier than the intervention of the editors that made the styles of the Greek and the Arabic texts, as well as the format of the proofs, appreciably different. However, the similarities stop there. In fact, the determinations in the Arabic tradition affected by the metamathematical clauses are rather different from the corresponding ones in book I. The most important difference is that they are well-formed because they impose real constraints on the givens: the glossed determinations in book IV state that the two numbers assigned as given in the enunciation contain a square; the one in book V requires that a more complicated expression of the givens is a square. A further feature of the Arabic Diophantus is that the metamathematical clause is absent in many problems with apparently similar determinations. Since such problems immediately follow the three problems that do have metamathematical clauses, this is rather surprising. Sesiano provides a list of such "similar" problems, according to the following argument.[74] Following Wertheim's reading read above, he takes the rationality condition imposed on the solutions as a «problem» (namely, the one alluded to in the metamathematical clauses) to be solved, not as a constraint on the numbers given. In his view, then, the question naturally arises «of how to find acceptable values» for these numbers, i.e., how to find values of the givens such that the problem admits solutions. This problem, in Sesiano's view, is dealt with in the determinations, and it «is easily overcome since the condition represents a "constructible" (*muhayya'* = πλασματικόν) problem». The following common feature of the glossed determinations makes them «constructible problems»: they are of the form $f(k,l) = r^n$, with k, l the given numbers, n a known natural number, (at least) one of the two given numbers involved appearing to the first power. In this way, one can always select pairs of rational givens. To sum up, the metamathematical clause has the function of pinpointing exactly this feature of the problems, namely, of admitting a determination that is formulated in such a well-defined way as to permit the selection of pairs of rational givens. The problems containing determinations that are «constructible» in this sense are I.27-28, 30 (Greek), IV.17, 19-22 (Arabic), and V.7-12 (Arabic). Actually, only three other problems in the

[73] Notice that the Arabic term is a past participle. The presence of the noun «problem» in the Arabic metamathematical clauses only entails that the translator probably interpreted them as referring to the whole problem and added the term, but says nothing about the original reference of the clause he was translating. Nothing can be inferred on this count from the linguistic expression of the metamathematical clause in the Arabic translation.

[74] Sesiano 1982, pp. 192-3. Quotations in what follows are from the same pages.

Arabic books have determinations: those in IV.18 and VII.6 concern rationality,[75] V.13 positivity. The former two are not in Sesiano's list since they involve one given only.

It is clear that Sesiano's classification is quite artificial and to a great extent *ad hoc*. First, finding givens that make a problem determinate is a metamathematical issue confronting the redactor of a problem, hardly pertaining to the problem itself. To what purpose should one make this feature explicit? Second, all determinations in Sesiano's list share *also* the feature that they state conditions of rationality by imposing that some expression is a single power. This alternative criterion, to which I do not see on what grounds Sesiano's should be preferred, would require including in the list IV.18 and VII.6 (Arabic) and again IV.34-35 (Greek).[76] In fact, a quick census of the determinations in both the Greek and the Arabic text of the *Arithmetica* shows that those listed by Sesiano and the four additions just mentioned are the only problems in which the attested determination is an identification of species. Moreover, there is no reason why V.7 should have the metamathematical clause and V.8-12 should not, since they too require that a certain combination of the givens is a square, or why, conversely, not to mark with the clause problems IV.20-22, where the expression of the givens involved is different from that of IV.17 and 19. Nor can a criterion alternative to Sesiano's be the complexity of the condition involved in the determination, since those of book IV are all equally simple and those of book V are all equally complex. Finally, in the whole tradition there are no determinations of rationality concerning at least two givens and that are non-constructible in Sesiano's sense: singling out the group of constructible determinations against a non-existent background makes the identification empty.

Vitrac recalls the use of πλάσσειν as a technical term, in this case to «form» squares, and proposes that the metamathematical clauses refer to the fact that the expression of the givens is identified with a square. This would explain why problems IV.18, 20-22 in the Arabic text do not have the metamathematical clause (they do not involve squares), why V.7 does have but V.8-12 do not (it is pointless to repeat the clause in a string of consecutive propositions), but leaves it unexplained why restating the clause in I.28 and 30, just after I.27, and in IV.19, just after IV.17. What is more, the verb πλάσσειν features also in the expressions where a cube is

[75] The determination of IV.18 says that the given number must be a cube, that of VII.6 that the given ratio must be a square. Notice, however, that the formulation of the latter «determination» is not canonical; it is a postpositive explanation and looks very much as a (utterly trivial) marginal commentary that got inserted into the text.

[76] The conditions in IV.34-35 are stronger than necessary, but the real ones are hardly perspicuous and difficult to handle in order to set the right givens.

«formed»: one would then expect to find the metamathematical clause also in problems such as the Arabic IV.18 and 20.[77] The geometric connotation, however present, is a minor feature.[78]

The widest possible characterization of some mathematical relevance, namely, the general requirement of rationality of the solutions, was proposed to explain the metamathematical clauses by Rashed.[79] Rashed, who insists on the fact that the condition must be explicitly formulated if the problem has to be regarded as «convenablement déterminé», further qualifies his contention by remarking that only the very first determinations of rationality in each book are affected by the metamathematical clauses,[80] and that this feature authenticates these clauses as originally Diophantine. I wonder whether such a proposal can count as an explanation at all. In this way, in fact, any semantic and linguistic connotation of the term is treated as a secondary issue (to be sure, the semantical range of πλάσσειν is rather wide), and what drives the interpretation is simply the mathematical content of a general class of statements that *might* have been affected by the same metamathematical clauses. Admittedly, other adjectives might have worked as well, and what makes the term πλασματικόν particularly suitable to qualify determination of rationality remains unexplained – to the point where it is even difficult to figure out why such a general feature of the determinations should have deserved a mention.[81] Finally, all interpretations must suppose that many determinations of the same kind, whichever

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[77] See, e.g., *OO*, vol. I, pp. 242.16, 246.25, 258.15.

[78] Vitrac 2005, p. 20.

[79] Diophante 1984, tome III, pp. 133-138. Rashed took his inspiration from the rationality-clause the Arabic mathematician al-Karajī attaches to the determinations of the paraphrases of *Arithmetica* IV.17 and 19 contained as problems V.16 and 18 in his treatise *al-Fakhrī* – the numbering is as in Woepcke 1853; Rashed, who provides translations of al-Karajī's propositions at Diophante 1984, tome III, pp. 132 and 138-139, shifts the problems by one unit, referring to them as V.17 and 19. Al-Karajī (beginning of 11th century) epitomized some Arabic Diophantus, but he never mentions him, and we cannot determine from which kind of sources he drew the many diophantine problems he presents in the *Fakhrī*: the excerpts amount to nearly all of book I of the *Aritmetica*, most of book II, the whole book III one problem excepted, and almost all of the Arabic book IV (see Woepcke 1853, pp. 18-24). Al-Karajī's rationality-clause is, in Rashed's French translation, «sinon le problème ne donnera pas une solution rationnelle» and Rashed takes it as a paraphrase of the the *muhayya'*-clause. However, al-Karajī has exactly the same rationality-clause in problem V.18 of his *Fakhrī*; this is a paraphrase of IV.18 in the Arabic *Arithmetica*, a proposition that has not the *muhayya'*-clause as IV.17 and 19 do have. Conversely, the same rationality-clause follows the determination of problem V.19 in the *Fakhrī*, a problem that is not attested in the Arabic *Arithmetica* (see Diophante 1984, tome III, p. 139). What is more, problems III.8 and I.36 of the *Fakhrī*, corresponding to *Arithmetica* I.28 and 30, have no determinations, as they were directly set out by assigning particular values to the given numbers (see Woepcke 1853, p. 19; A. Djebbar kindly checked this on *Al-Fakhrī*, ms. Istanbul, Laleli 2714, ff. 73v-74r and 61v-62r, respectively). For the same reason, several determinations of positivity are omitted by al-Karajī when he presents problems whose correlatives in the *Arithmetica* do have such determinations. Finally, no problem in the *Fakhrī* corresponds to *Arithmetica* I.27. Therefore, one is not entitled to assume that there is any correlation between the *muhayya'*-clause (and *a fortiori* the metamathematical clause) and al-Karajī's rationality-clause, that appears, when present, to be simply a way to pinpoint the fact that a problem has a determination of rationality.

[80] Notice also that most problems in each book entailing determinations of rationality are attested as sequences of consecutive propositions.

[81] The Arabic term, from which Rashed primarily moves, carries connotations «de disposition, de préparation, de forme déterminée», that are hardly present in the Greek verb.

"natural kind" had been conjured up to capture the essence of the glossed determinations, were left unglossed. Rashed employs exactly this argument to reject the proposal that the metamathematical clauses refer to problems that can be reduced to equations of the second degree,[82] but then fails to observe that the same argument would work against his own proposal.

Maybe, grouping the determinations by means of a unifying mathematical feature that explains the glossed ones is simply unmethodical. It entails enlarging the range of determinations that *might* have the metamathematical clause – and then explaining away with *ad hoc* arguments those determinations in the range that do not have the clause.

6 Some scholia. Maximus Planudes

Among the *scholia vetera* to the *Arithmetica* we read the following annotation to problem I.27, intended to explain the meaning of πλασματικόν:

> Ἤτοι οὐκ ἐπιτηδεύσει τινὶ γινόμενον, ἀλλ' αὐτῇ τῇ πλάσει συναναφαινόμενον.[83]
>
> I.e. not resulting from a certain intellectual effort, but becoming manifest together with the formation itself.

The scholium seems to say that the determination is manifest if one follows the actual solution of the problem, and does not require much effort to be formulated. This is not very informative, and confirms that the exact meaning of πλασματικόν was not within easy reach, even for Greek-speaking interpreters of Diophantus. In the scholia that Maximus Planudes redacted at the very end of the 13th century as a sort of running commentary to books I and II, we read something more interesting:[84]

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[82] Diophante 1984, tome III, p. 136.

[83] First edited in *OO*, vol. II, p. 260.17-18. The best edition is in Allard 1983, pp. 692 (text, numbered as scholium 74) and 728 (French translation and commentary), Allard's translation, who overinterpretates the Greek text, is «ce qui veut dire que la condition n'est pas due à une quelquonque nécessité contraignante, mais qu'elle apparaît par la détermination même», whose meaning I am unable to discern. In Christianidis 1995, pp. 38-39, one finds a more perspicuous translation: «c'est-à-dire qui est fait non pas à dessein, mais qui se manifeste ensemble avec la formulation». As is clear from the proposed translations, the meaning of the term ἐπιτήδευσις in this sentence appears not to be immediately evident. The scholium is written by a late hand in the margins of the mss. *Vat. gr.* 191 and *Vat. gr.* 304. As the latter ms. was corrected using the former as a model, it is not said that the scholium was contained in the common subarchetype of the two manuscripts. I have checked on the ms. *Vat. gr.* 191, f. 365r: here the word πλασματικόν is marked by an interlinear sign, repeated in the margin before the scholium.

[84] I take the problem whether such scholia are really Planudean or not as settled in the affirmative; the scholia are assigned to Planudes in the autograph *Ambrosianus* & 157 sup. See in the first place Tannery's discussion at *OO*, vol. II, pp. xiv-xvi, and, more recently, Allard 1979. The same scholium has been studied in Christianidis 1995, but

Τὸ κζ^{ον} καί τινα τῶν μετ' αὐτὸ πλασματικόν φησιν ὁ Διόφαντος· οἶμαι δὲ τοῦτο λέγειν διὰ τοὺς ἐν αὐτοῖς προσδιορισμούς·[85] οὐ γάρ τισι μὲν ἔσται <τὰ> τῶν ἐν αὐτοῖς προσδιορισμῶν, τισὶ δ' οὐκ ἔσται, ἀλλὰ πᾶσιν ἁπλῶς ἀριθμοῖς ἁρμόσει, καὶ ἀνάγκη πάντας ἀριθμοὺς οὕτως ἔχειν· ὅθεν καὶ οὐδὲ δικαίως ἂν καλοῖντο προσδιορισμοὶ τὰ τοιαῦτα. ἔστι γε μὴν ὁ τοιοῦτος προσδιορισμὸς τοῦ κζ^{ου} ὁ αὐτὸς τῇ προτάσει τοῦ ε^{ου} τοῦ β^{ου} τῶν Στοιχείων, τῇ λεγούσῃ· [...][86]

Diophantus names the 27th and some among those following it *plasmatikon*; I think he says that because of the determinations in them:[87] for it is not the case that what <is stated> in the determinations in them will hold for some <numbers>, whereas for some it will not. On the contrary, it applies in general to all numbers, and it is necessary that all numbers stand in such a relation: it results from this that such things could not be properly called «determinations». And in truth, such a determination of the 27th is the same as the enunciation of the 5th of the 2nd of the *Elements*, saying: [...]

Planudes goes on by reporting the enunciation of *Elements* II.5, but then he is at a loss and proposes an emended determination that weakens the original one: «it must be that the <square> on half the sum becomes of more units than the <number> resulting from the multiplication of the two».[88] Planudes then proposes the same kind of restatement for I.28. The inequality is no longer an identity but it is of course identically true, as it is a weaker (and trivial) consequence of the original identity and as it is still not formulated in terms of the givens.[89] What Planudes did not realize is that it was enough to restate the original determinations in terms of the givens in order to make them work. Nevertheless, he had seen where a part of the problem with the received text lies: as they are stated, the determinations are identically true and hence they are not real determinations. In addition, what he writes about the character of the determination seems to me to lead to an indisputable conclusion: the meaning he assigns to the adjective

the author, whose aim was manifestly to confirm Rashed's interpretation by restating it in his own words, was unable to see the point made by Planudes.

[85] The use of the term by Planudes is likely to have been induced by the two occurrences in the *Arithmetica*; I translate «determination».

[86] *OO*, vol. II, p. 198.16-24. The scholium was reported also in Xylander's translation, where it is ascribed to an anonymous «scholiastes» (*Diophanti Alexandrini Rerum Arithmeticarum libri sex*, pp. 36-7). Neither Xylander nor other interpreters after him appear to have taken advantage of its presence.

[87] Hence Planudes takes the metamathematical clause as referring to the problem, but the explanation that follows clearly links the qualifier πλασματικόν to features of the determination.

[88] *OO*, vol. II, p. 199.3-5.

[89] Of little use is a scholium, written in a 14th century hand, in the *Matritensis Bibl. Nat.* 4678: it checks that the proposed solutions of the problem fulfill the condition of the determination. Edition in Allard 1983, pp. 707 (text,

πλασματικόν is «fictitious». Such a reading would be only natural to him, as this is the only meaning he could gather from the rhetorical tradition.[90] However, Planudes himself appears not to give credit to his own proposal, and just at the end of the scholium to I.27 he proposes another reading. This alternative interpretation is presented by the phrase «In another way to: but this is *plasmatikon*» and runs thus «He [*scil.* Diophantus] says that because of the determination; the determination is that the numbers found should not be equal (for neither the proof nor the determination will say the truth in these <cases>), but unequal: except that not only they should be different, but in addition also those of the other determination, the one *we* set out, should be retained <as such>».[91] Planudes is here compiling another scholium, containing the alternative interpretation. In the quotes above (see the words marked in italics) and in the commentary on I.28, where the other explanation is repeated,[92] he clearly opposes his own interpretation to the alternative one. The condition in the latter simply aims at excluding the degenerate case in which the excess square in the determination of I.27 is zero. Planudes extends the alternative interpretation to the mathematical clause in I.28, where in fact he could not compile a corresponding scholium, as is clear from the way of his reference. Such pointless remarks should not detain us any longer.

The Planudean scholia offer another, puzzling piece of information. When explaining problem I.30, that in the transmitted Greek text has a determination, Planudes remarks: «Not even this needs a determination»,[93] where the «not even» refers to problem I.29, that does not need a determination and in fact does not have it. If we are to believe Planudes, then, the scholium containing the determination + metamathematical clause to problem I.30 had not found its way into the text of one of his exemplars at the time he redacted the scholia.[94] What is puzzling is that the two main testimonies of the Planudean recension of the *Arithmetica* plainly contradict Planudes' statement.[95] As the apparatus of Tannery's edition is particularly poor – in

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numbered as scholium 182) and 755 (French translation and commentary). The scholium wrongly referes to the solutions as the «assigned numbers».

[90] A training that we must assume the anonymous scholiast quoted just above had not been exposed to.

[91] *OO*, vol. II, p. 200.4-9.

[92] *Ibid.*, p. 200.20-23.

[93] *Ibid.*, p. 202.8.

[94] From Planudes' correspondence we infer that he had very likely access to more than one copy of the *Arithmetica*: he writes to the protosebastes Theodorus Muzalon that he is sending him back his exemplar of Diophantus' book in a better status than when he received it; he asked Manuel Bryennius to send him his copy for checking his own (Treu 1890, p. 82, lines 31-36 of letter 67, and p. 53, lines 6-10 of letter 33 = pp. 99.24-29 and 66.13-17 Leone, respectively). The first reconstruction of the whole affair is in Wendel 1940, in particular pp. 414-417, and Wendel 1941, pp. 80-2; cf. also Allard 1979, pp. 226-8, and, better still, Allard 1983, pp. 669-672 and 681. In particular, Allard confutes Wendel's contention that Muzalon's exemplar was the *Matritensis Bibl. Nat.* 4678.

[95] Planudes appears also to contradict himself: at the beginning of the scholium transcribed above he says that «Diophantus names the 27th and some among those following it *plasmatikon*», but it turns out that the subsequent scholia actually point out only one such problem so named.

particular, almost no variant readings from manuscripts of the Planudean class are recorded – one has to check directly on the manuscripts. One of the fragments preserved in the *Ambrosianus* & 157 sup. (= *gr*. 780 Martini and Bassi), a Planudean authograph written around 1293,[96] regularly includes a part of the determination and the metamathematical clause in I.30.[97] The *Marcianus graecus* 308, a copy of the *Ambrosianus* as the surviving portions of the latter suffice to make it clear,[98] has both the determination and the metamathematical clause in all three problems of book I, at ff. 111r, 112v, and 114r, respectively. Even more puzzling is the fact that Tannery claims to have transcribed the Planudean scholia exactly from the *Marcianus gr*. 308; apparently, he did not notice the problem. Very likely, the reasons for Planudes' statement will remain unknown. Maybe he carelessly compiled a scholium that refers to a very early textual state of affairs.

A final, interesting clue comes from a scholium to *Arithmetica* I.27, written by a contemporary hand in the late 13th century manuscript *Matritensis Bibl. Nat*. 4678. Some of the scholia in the *Matritensis* bear conspicuous similarities to some of the Planudean scholia. They are not identical, however, and nowhere in the *Matritensis* can the hand of Planudes be found. One must conclude that the scholia were lifted from the same manuscript, in fact the common ancestor of the Planudean mss. and of the family of the *Matritensis*.[99] The scholiast does not refer to the metamathematical clause but comments on the determination:

> ’Αναγκαίως ὀφείλει ἡ ὑπεροχὴ ἣν ὑπερέχει ὁ ἀπὸ τοῦ ἡμίσεος ἀμφοτέρων τοῦ ὑπὸ ἀμφοτέρων εἶναι τετράγωνον. ’Εὰν γὰρ εὐθεῖα γραμμὴ ἢ ἀριθμός τμηθῇ εἰς ἴσα καὶ ἄνισα, <τὸ ἀπὸ> τοῦ ἡμίσεος ἴσον τῷ ὑπὸ τῶν <ἀνίσων μετὰ> τοῦ ἀπὸ τοῦ μεταξὺ τῶν τομῶν.[100]

> *Necessarily* the excess *must be* <such> that the <square> on the half of both of them exceeds the <rectangle contained> by both of them by a square. For if a straight line or a number be cut into equal and unequal <segments>, <the square on> the half is equal to the <rectangle contained> by the <unequal segments together with> the <square> on the one between the cuts.

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[96] The first identification of the manuscript as an autograph is in Turyn 1972, p. 79.
[97] The relevant fragment is on f. 20r-v.
[98] See Allard 1982-3, pp. 100-2.
[99] *Ibid*., pp. 680-681. Allard surmises that this ancestor coincides with the manuscript Planudes borrowed from Theodorus Muzalon.
[100] The scholium is edited in Allard 1983, pp. 691 (text, numbered as scholium 72) and 727-728 (French translation and commentary).

The scholiast, who apparently read the determination in the same formulation as the one we read, points out the connection with *Elements* II.5. Most importantly, he appear to recognize, through the double emphasis on necessity, that the stated determination is an identity. On account of the remarks just expounded, it is not clear whether this scholium and Planudes' commentaries could be taken as wholly independent elaborations. It might well be that Planudes shaped his own commentary on *Arithmetica* I.27 against the background of this scholium, lifting from it both the idea that the attested determination is an identity and the reference to *Elements* II.5. If this was the case, he added on his own, as a scholar trained in rhetorical matters, an explanation of the metamathematical clause in line with the proposal in the scholium. The absence of the explicative particle γάρ at the beginning of the scholium suggests that it was not preceded by other material (maybe commenting on the metamathematical clause) that got lost during the transcription.

7 Assessment

The interpretation proposed in the present study moves in a perspective that is somehow inverse to the one embraced by past and contemporary commentators. On the basis of the Greek occurrences I have argued that the technical meaning of the adjective πλασματικόν fixed in the rhetorical tradition is exactly what is needed also in the mathematical context of the *Arithmetica*. Such a meaning fits well a very peculiar feature of the determinations qualified by the term, a feature that only these determinations in the whole tradition of the *Arithmetica* do share. The next task is to try to explain the Arabic metamathematical clauses. Of course, the only practicable route is to explain them away. A well-known commodity can be invoked to this end.

If the meaning of πλασματικόν is «fictitious», I cannot see any reasons why Diophantus should have added the metamathematical clauses to the determinations: most likely, the Greek metamathematical clauses are marginal comments that crept into the text at a very early stage.[101] Their very compact and aphoristic form is typical of marginal annotations, and in fact some of the occurrences of the adjective in the Greek *corpus* are in scholia. Of some interest is for

[101] Ver Eecke already made this proposal (Diophante d'Alexandrie 1926, note 6 on pp. 36-7), but for entirely different reasons. He simply maintains that each metamathematical clause was inserted to point out the «figurative» character of the condition in the determination, and his argument ends here. Almost surely spurious metamathematical annotations in the Greek *Arithmetica* include explanations such as, e.g., that of the phrase ἐν τῇ ἀορίστῳ «in the indeterminate» at *OO*, vol. I, p. 278.10-12. Clauses of this kind must *a priori* raise suspicions of inauthenticity, as they are the typical outcome of late reflections on the text.

instance the clause ἔστι τοῦτο πλασματικόν in a scholium to *Odyssey* 1.8, found at f. 10r of the manuscript *Marcianus graecus* 613:[102]

> <Ὑπερίονος·> ἐπίθετον τοῦ Ἡλίου παρὰ τὸ ὑπεράνωθεν ἡμᾶς ἰέναι. ἔστι τοῦτο πλασματικόν. Ἡσίοδος δὲ Ὑπερίονος αὐτὸν γενεαλογεῖ.
>
> <Of Hyperion:> epithet of the Sun, from the fact of going above us. This is fictitious. But Hesiod ranges him among the descendants of Hyperion.

The way the same scholarly remarks contained in this scholium occur (with irrelevant lexical changes) in other manuscripts or collections of scholia shows that three layers of annotations have been conflated here, one proposing the paraetymology, two others unfavourably replying to it (apparently on the basis of Hesiod making Hyperion the Sun's father at *Theogonia* 371-374 and 1011) or to the original negative reaction. Of the three clauses that make up the scholium, in fact, both the first clause, proposing the paraetymology, and the third one, that shows it to be implausible (notice that the δέ cannot but be adversative if the third clause immediately follows the first one) make up a single annotation included in the *corpus* of the so-called V scholia.[103] As the etymology dates back at least to the founder of the Stoic school Zeno of Cizium, who maintained that the names of the Titans indicate the elements of the Cosmos; in particular, «Hyperion indicates the upper motion, from the fact of going above [ἀπὸ τοῦ ὑπεράνω ἰέναι]»,[104] the annotation in the V scholia is already the result of conflating two different layers of scholarly annotations. The first clause is attested in isolation in the same fol. 10r of the *Marc. gr.* 613, written in a late 13th century hand. This scholiast transcribed only a part of what he was lifting from the V scholia, as was usual with him.[105] A second hand in the same manuscript, later than the preceding one, copied instead the V scholium in its entirety, including the πλασματικόν-clause. This we have read above. The actual layout of the whole annotation makes it almost sure that the πλασματικόν-clause was added by the copyist after he transcribed the V scholium, but it is not clear whether he intented to pinpoint the fictitious character of the paraetymology or to stress the implausibility of the reply based on Hesiod's genealogy. If the

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[102] *Scholium* M^{a} *ad Odysseam* 1.8 Ludwich = *scholia ad Odysseam* α 8j_1-j_2 Pontani. The scholium is written a late 13th century-early 14th century hand.

[103] The *corpus* of the V scholia to the *Odyssey* (formerly referred to as «scholia Didymi»; now occasionally named «D scholia») is a "glossary" of sorts transmitted separately from the Homeric text; its main testimony is the 10th century ms. *Oxon. Bodl. Auct.* V.1.51.

[104] Cf. *Stoicorum Veterum Fragmenta*, vol. I, fr. 100. For other testimonies see *Scholia Graeca in Odysseam*, p. 16 *in app*.

[105] Pontani 2005, p. 252.

former were the case, the δέ in the final clause should have been replaced by an explicative γάρ, but clearly we cannot expect that the scholiast was eager to introduce such refinements.

A similar phenomenon of superposition of scholarly material might have occurred in the case of the metamathematical clauses in the Greek text of *Arithmetica*. In fact, I strongly doubt that the determinations themselves of I.27-28 and 30 are original, and would suggest assigning them too to some early scholiast. I would even suggest that, at least for book I, determinations and metamathematical clauses, even if the latter were in fact comments on the former, entered into the text together, before the Arabic translation. Of course, it is entirely possible that the attested determinations of I.27-28 and 30 did not squarely replace the original ones, but are simply the product of some local adjustements of the primitive formulation. It is difficult to have clear-cut opinions on the issue, but a small clue could come from considering the presence of the particle δέ in the methamatematical clause. If these are scholia that have been inserted in the text, the δέ might be no more than a particle added to make the insertion run smoothly with it. It could be debated whether «and» or «but» is the best rendering of the particle, depending on whether the writer thought that the reader would interpret the metamathematical clause from the context as an additional fact or an adversative one.[106] As δέ connects without implying *per se* a specific relation of thought, we have no elements to decide. On the other hand, in scholia successively reporting different opinions (for instance about the readings to be accepted in, or about the interpretation of, a certain Homeric passage), these are often marked with a δέ carrying a strong adversative force. The presence of δέ in the metamathematical clause, which would be an odd feature in an isolated scholium to a mathematical text,[107] might then receive a more satisfactory explanation exactly as a particle marking, within a scholium, an opposition to another scholium perceived as inadequate.

If one accepts that the actual determinations do not coincide with what Diophantus wrote in the first place, one must wonder which mechanism led to the misleading and ultimately erroneous form that we read. I regard it as unlikely that the problems were left without determinations by Diophantus, the extant ones being in this case an inept attempt at completing the text, even if we have seen that this remote possibility might have occurred at the end of the

[106] As we have seen in the table set out at the beginning of Sect. 4, Xylander («autem») and Nesselman («aber») emphasize the adversative shade.

[107] However, it is not difficult to find mathematical scholia with a δέ in second position, even if they are far less frequent than literary scholia with this peculiarity. I have reckoned 21 of them in the whole *corpus* of scholia to the *Elements* edited by Heiberg in vol. V of his edition. At least in two of these scholia the δέ conveys a rather clear adversative shade, as a reaction of sorts to the main text (*Euclidis Elementa*, vol. V,1, scholium 2 on p. 165 and 90 on p. 191). Recall, however, that the presence of δέ in a scholium that is an excerptum from a larger text can be due to the syntactical structure of the excerpted text. This is not the case with the two scholia just mentioned.

preceding section. The possibility I regard as more likely is that the actual form replaced the original one in the course of the tradition, maybe just because the new formulation nicely shows the connection with *Elements* II. In fact, if it is quite easy to rewrite the determinations in terms of the givens, e.g., as «the square on half the given sum exceeds the given product by a square» in the case of I.27,[108] the opposite process is of course easy as well. Some support to this hypothesis is provided by the lexical changes between the enunciations and the determinations of I.27-28, 30:[109] πολλαπλασιασμός «product» is replaced by τὸ ὑπ' αὐτῶν «the <rectangle contained> by them», σύνθεσις «sum» by συναμφότερος «both of them together», with the insertion of geometrical overtones that strengthen the link with *Elements* II.[110] As is natural, the modifications affected to a greater extent I.27, namely, the first proposition in the string, where in fact one finds the incongruous reference to the numbers found.[111] On the other hand, those in I.27-28 and 30 are the only determinations in the *Arithmetica* sharing two peculiarities: they *can* be formulated by making reference *only* to the solutions and they give rise to numerical identities when formulated in this way. This rather bewildering feature might well have induced some unease in ancient readers, or simply suggested to them that making the determinations fit the perceived model of *Elements* II could improve the deductive fabric of the treatise.

The issue of the origin of the scholarly material contained in the scholia to the *Arithmetica* deserves a short digression. Only one Greek commentary on the treatise is mentioned, namely the one credited to Hypatia (d. 415 A.D.), but the only source is far from trustworthy.[112] To explain the incompleteness of the Greek version of the *Arithmetica*, Tannery suggested that Hypatia's commentary reached only as far as the sixth book: as a consequence, the remaining seven books got lost and the surviving text derives from the one Hypatia commented on.[113] The

[108] Determinations where some linguistic device must be set up in order to distinguish between two givens are at work, e.g., in *Arithmetica* I.8-9. Cf. also the discussion above of the formulations of the determinations in II.6-7.

[109] The formulations of the "setting out" are completely in line with those of the enunciations.

[110] However, a word such as συναμφότερος to denote the sum is found elsewhere in the *Arithmetica*: see *OO*, vol. II, p. 283 *sub voce*.

[111] I suspect in fact that I.27-28 and 30 have been affected by other scholarly modifications. For instance, the rather idiosyncratic deductive sequence in I.27 at *OO*, vol. I, p. 62.5-11 (from καὶ ἐπεὶ to β), is in my opinion almost surely spurious.

[112] According to the *Suda*, here drawing from Hesychius (1st half of the 6th century), Hypatia «wrote a commentary on Diophantus, the astronomical table, a commentary on Apollonius' *Conics*» (*Suidae Lexicon* Υ 166, pars IV, p. 644.3-5 Adler). The Greek text as it stands has been regarded as not satisfactory, since no astronomical table is known that might possibly be ascribed to Hypatia. The accepted emendation (inserting εἰς «on» before «the astronomical table») was first proposed in Bernhardy's edition of the *Suda* (1834-1853) and then endorsed in Tannery 1880, pp. 76-77 of the reprint.

[113] Tannery 1884, pp. 78-79 of the reprint. In the *stemma codicum* presented in the *prolegomena* to his edition of the Greek text (*OO*, vol. II, p. xxiii), Tannery even postulates that a «deperditum antiquum exemplar Hypatianae recensionis» is the common ancestor of the whole Greek tradition. A number of conjectures of this kind, put forward by past and recent scholars, have created the myth of Hypatia the mathematician. For a more detailed discussion see Acerbi 2008.

discovery of the Arabic translation, containing four books that find their proper place after the first three Greek books, disproved such an hypothesis. In its turn, the Arabic version, appreciably different in its format from the Greek one, led Sesiano to conjecture that the former was the translation of the text commented by Hypatia.[114] A difficulty with this proposal is that the Arabic version is a new recension of the treatise, not a commentary as the *Suda* reports, and normally a commentary does not interfere with the text in such a way. Granted, the commentary might have been accompanied by a new recension, but this assumption is unsupported by our sources, and the modifications with respect to the format of the Greek text are extensive but almost invariably dull and of little mathematical relevance. It remains, though, that the mere existence of a *corpus* of scholia to the *Arithmetica*, of alternative proofs attested both in Greek and in Arabic, of problems that have been deemed interpolations by the interpreters, leave the possibility open that the common source of such scholarly material was in fact an ancient commentary, subsequently epitomized by Byzantine scholars. However, this is simply a possibility: one should bear in mind that the only testimony on a would-be Hypatian commentary on the *Arithmetica* is the single half-line in the *Suda* quoted in the preceding note; any further conjecture is nothing but scholarly romancing. It is not unlikely that her commentary is to be ranked among the fictitious entities the Byzantine lexicographers concocted in their work of cutting and pasting epitomes.

As for what happened with the metamathematical clauses attested outside book I, I am unable to propose anything better than the following reconstruction, just to suggest how the state of affairs we read in the present texts might have been produced. The Greek and the Arabic metamathematical clauses do not come from the same hand. A clue comes from the fact that all metamathematical clauses after the first one in I.27 should have included the adverb «also» – IV.19 has it, whereas IV.17 and V.7 do not – unless we suppose that glossing the first proposition in a new book was interpreted as making a fresh start. This might be the case if the Arabic metamathematical clauses were introduced by another scholar, be him Greek or Arabic, who attempted to complete the text having in his mind the *only* determinations of rationality preceding the Arabic IV.17, namely, those in I.27-28 and 30. In fact, since the Arabic book IV follows the Greek book III, it happens that the metamathematical clauses in I.27-28, 30 and IV.17, 19 (Arabic) are the first five determinations of rationality in the whole *Arithmetica* as we can reconstruct it from the extant sources. The scholiast excluded IV.18, and VII.6 after that,

[114] Sesiano 1982, pp. 68-75.

since they do not involve two givens but only one,[115] and refrained from marking all similar and consecutive determinations, resuming his job with book V. At this point two scenarios seem possible to me. 1) The second scholar misunderstood the sense of the metamathematical clauses attached to I.27-28 and 30 (I assume that the second scholar read all of them in the text and not in the margins). Maybe he interpreted πλασματικόν as referring to the fact that the determinations secured rationality, as the Arabic translation attests.[116] 2) The Arabic word *muhayya'* is not a translation of πλασματικόν. As the Arabic term carries connotations «de disposition, de préparation, de forme déterminée», that the verb πλάσσειν does not have, it (or the Greek term it translates) might have been inserted to mark the first «well-formed» determinations after the fictitious ones in I.27-28, 30, within a series of annotations coordinated to the ones in book I but carrying exactly the opposite meaning. Most notably, *muhayya'* has not the same root as the term *'amila*, that in the *Arithmetica* canonically translates πλάσσειν (that has, instead, the same root as πλασματικόν), even if this negative argument cannot be given too much weight.

It should be clear to the reader that any proposals about the origin of the Arabic metamathematical clauses will remain uncorroborated speculations unless further evidence is discovered. The aim of this paper has been restoring the actual meaning of a controversial clause, thereby providing sound arguments to recognize it, and the determinations to which it is attached, as a spurious product of reflection on the mathematical contents and organization of some problems. Besides solving a long-standing problem of translation and interpretation of the *Arithmetica*, the reading proposed in the present paper seems to me to have the merit of freeing the debate about the algebraic character of the *Arithmetica* of a spurious element. Conversely, it severs any link with the geometric lemmas in *Elements* II, a feature that has been a central element of past and recent historiography on the *Arithmetica*, and that eventually was taken to lend a strong support to the hard-to-die historiographical figment of the «geometrical algebra». If it is time to begin reading Diophantus in his own terms, we should preventively try to determine which is the meaning of such «terms».

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[115] The enunciations of IV.17 and IV.18 are nicely symmetrical: it is not clear to me whether this feature suggests that either of them is spurious.

concerning Arabic sources. I am also indebted to F. Schironi, F. Ademollo, A. Jones, and F. Pontani for their suggestions. B. Vitrac's remarks saved me from one serious blunder and greatly improved the argument.

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[116] But this meaning might well be an overinterpretation by Quṣṭā ibn Lūqā of the Greek term, as we have suggested above.

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