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GÖDEL, LEIBNIZ AND “RUSSELL’S MATHEMATICAL LOGIC”

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1. Some known facts about Gödel’s interest in Leibniz

Kurt Gödel explicitly mentioned Leibniz in only one paper, “Russell’s mathematical logic”, which appeared in 1944 in the volume of the *Library of Living Philosophers* devoted to Bertrand Russell and edited by A. Schilpp.¹ Nevertheless, the tribute paid by Gödel to Leibniz in this text is so important that this alone suffices in attesting to the role that Leibniz’s work played in Gödel’s thought.

Besides this text, which will be extensively analysed in the following sections, evidence of Gödel’s deep interest in Leibniz can be adduced through Gödel’s own more or less public declarations, as reported by his friends or colleagues, and through the unpublished texts of the Gödel archives in the Princeton Firestone Library.

Among the most widely known evidence of the first kind there is what is reported by Karl Menger. According to him,² Gödel began to be interested in Leibniz’s work in the early thirties. Giving an account of Gödel’s trip to the USA in 1933, Menger remembers that Gödel was particularly worried about the fate of the Leibniz Archives, because of the political situation in Germany at that time. As Hao Wang first reported in his book *Beyond Analytic Philosophy: Doing Justice to what we know* (1987): “Menger asked Gödel, ‘Who could have an interest in destroying Leibniz’s writings?’ ‘Naturally those people who do not want men to become more intelligent’ Gödel replied. To Menger’s suggestion of Voltaire being a more likely target, Gödel answered ‘Who ever became more intelligent by reading Voltaire’s writings?’”³

¹[Gödel 1944], p. 119 and p.140 (we quote from the edition of [Gödel CWII]). This is the only reference considering exclusively the texts published during Gödel’s life. Apart from the letters and the still unpublished material from the Archives there is a further mention of Leibniz in the second preparatory draft of Gödel’s paper for Einstein ([Gödel 1949]) recently published in the III volume of the collected works. Gödel mentions Leibniz in these terms: “That time and space have no existence independent of and besides the things was asserted already by Leibniz.” [Gödel 46] p. 238.

² [Menger 1994]. Menger says that Gödel had already begun to concentrate on Leibniz in around 1932. A library slip contained in the Archives [Gödel Nachlass] folder 5/54, 050173 attests that Gödel asked for the Gerhard edition of Leibniz’s mathematical writings in 1929. See [van Atten Kennedy 2003].

³ [Wang 1987] p.103-4. The passage from Wang continue as follows: “Later (perhaps in or after the 1950s) Menger discussed Gödel’s ideas about the destruction of Leibniz’s writings with O. Morgenstern, who described how Gödel, to supply evidence for his belief, ‘took him one day into the Princeton University library and gathered together an abundance of real astonishing material’. The material consisted of books and articles with exact references to published writings of Leibniz on the one hand, and the very series of collections referred to on the other. Yet the cited writings are all missing in one strange manner or another. ‘This material was really highly astonishing’ said Morgenstern.”

If Gödel's level of interest in Leibniz was high in the thirties, his systematic study of Leibniz's work began in earnest in the forties. We know this fact directly from Gödel himself. In 1974 and 1975 Gödel received letters from Burke Grandjean asking him to answer a few questions about his intellectual and educational background. Gödel never answered these letters but replies were written by him and found in the archives.⁴

To the two questions:

"5. When, if at all, did you first study any of the works of the following:

a) Ludwig Boltzmann b) Jan Brouwer c) Paul Finsler d) Immanuel Kant e) Karl Kraus f) Fritz Mauthner g) Jules Richard h) Ludwig Wittgenstein

6. How much importance, if any, do you attribute to each of the scholars in the above list, in the development of your interests?"

Gödel answered:

"As to 5 I would like to say that only Kant had some infl. on my phil. thinking in gen (& that I got acq. with him about 1922), that I knew Wittgen. very superficially, that I read only two papers by Finsler (in or after 32) finally that the greatest phil. infl. on me came from Leibniz which I studied (about) 1943-46."⁵

There are two facts confirming Gödel's assertion:

a) the incredible amount of notes handwritten by Gödel on primary and secondary literature on Leibniz found in the Gödel Archives attesting to the depth of Gödel's study⁶; and b) the most significant account of Gödel's unpublished philosophical work, (represented by the so-called "Max Phil" manuscripts written between 1938 and at the earliest 1955), which contain, between 1943 and 1946⁷ an increasing amount of references to Leibniz.

The end of this intensive study on Leibniz in 1946 did not signify a turning point in Gödel's interests. Different evidence shows just how true are Gödel's replies to Grandjean's questionnaire in 1974 about Leibniz's great philosophical influence on him.

First of all one of the most significant manuscripts of the "Max Phil", Max Phil XIV, written over a long period between 1946 and 1955, shows how at that time Gödel was trying to develop a really general philosophical system based on Leibniz's monadology where, as in Leibniz's work, questions of physics, logic, mathematics and rational theology interact together in the aim of the construction of a metaphysical system.

Secondly at least two of Gödel's letters found in his archives show how, till the end of the fifties and the beginning of the sixties, Gödel was still trying to improve his knowledge of Leibniz and was searching for such a metaphysical system largely inspired by Leibniz's monadology.

⁴ [Wang 1987] p. 16-7. [Gödel CW IV] p. 441-50.

⁵ [Gödel CW IV] p. 449-50. This version comes from an undated draft of Gödel's replies.

⁶ The notes are contained in [Gödel Nachlass] series 5, boxes 24-38.

⁷ The Max-Phil is a series of 15 notebooks containing philosophical remarks which Gödel called "Max 0-XV". One of them, the XIII, was lost by Gödel himself. Up to now there is only a fragmentary transcription of the Max-Phil, made by Cheryl Dawson between 1992 and 1993, which covers around 750 pages out of 1500. This transcription circulates between scholars. It was in my possession recently, thanks to the kindness of the Dawsons. An increasing number of references to Leibniz are made especially in Max-Phil X and XI. Max Phil VII, written between July '42 and Sept. '42, contains no explicit reference to Leibniz. Max Phil VIII written between the 15th Sept. '42 and the 18th of Nov. '42 contains one explicit reference to Leibniz. Max Phil IX written between the 18th of November '42 and the 11th of March 1943, contains no explicit reference to Leibniz. Max Phil X, from the 12th of March '43 and the 27th of January 1944 contains fourteen references to Leibniz. Max Phil XI from the 28th of January '44 to the November 1944 contains seven references to Leibniz. Max Phil XII, from the 15th November 1944 and the 5th June 1945, is not transcribed. Max Phil XIII, as we said before, was lost by Gödel.

The first is a letter to Walter Pitts, the well-known co-author of *A Logical Calculus of Ideas Immanent in Nervous Activity* [McCulloch and Pitts 1943], one of those mathematicians who were among the fathers of cybernetics. The letter, dated September 23, 1958, is very eloquent on Gödel's lasting conviction about Leibniz's topicality for modern science. Gödel writes to Pitts:

"I was very much interested to hear that you are considering applications of Leibnizian ideas to modern physics [...]. I do think that the possible applications of Leibniz's work to modern science are far from being exhausted. In particular the unpublished manuscripts of Hanover may contain invaluable ideas as to the systematic solution of mathematical, as well as other scientific, problems. In fact this must be so, if Leibniz ever put down on paper what he definitively claimed to have discovered, and if the manuscripts concerned were not lost in the subsequent centuries."⁸

The second is one of the letters written by Gödel to his mother in 1961, about the afterlife. Having explained his philosophical convictions to her, Gödel (mentioning Leibniz) says:

"Of course, today we are far from being able to justify the theological world-view scientifically, but I think already today it may be possible purely rationally (without the support of faith and any sort of religion) to apprehend that the theological world-view is thoroughly compatible with all known facts (including the conditions that prevail on our earth). Two hundred fifty years ago the famous philosopher and mathematician Leibniz already tried to do that, and that is also what I have attempted in my last letter. What I called the theological world-view is the idea that the world and everything in it has meaning and reason to it and in fact a good and indubitable meaning. From that it follows directly that our earthly existence, since it in itself has a very doubtful meaning, can only be a means toward the goals of another existence. The idea that everything in the world has meaning is, after all, precisely analogous to the principle that everything has a cause, on which the whole of science rests"⁹

The third evidence of Gödel's lasting interest in Leibniz's work after 1946 and up until his death is taken from Hao Wang's conversations with Gödel from 1967 to 1972 [Wang 1996].

The image Wang offers of Gödel is that of a mathematician deeply convinced that mathematics describes a non-sensual reality, which exists independently both of the acts and the disposition of the human mind and therefore¹⁰ of a philosopher trying to explore and explain the objective reality of concepts and their relations. According to Wang, as time went by this mathematician and philosopher was more and more engaged in the attempt to cultivate a philosophy as a system in the old, classical sense, i.e. as a guide for scientific research and as a means of investigating the meaning of the world.¹¹ Wang affirms that from 1943 to 1958, under the influence of the Vienna Circle, Gödel approached philosophy by way

⁸ [Gödel CW V] p. 159.

⁹ [Gödel CW IV] p. 439. The content of these letters on the afterlife is often presented as one of the most clear manifestations of Gödel's fear of death, possibly related to a more deep mental fragility. A brief passage from [Wang 1996] (see below about this text) proves on the contrary a certain "political" or at least ethical ground for such a rational theology, which again relates him deeply to Leibniz. Gödel says to Wang in October 1972: "The rulers find it hard to manipulate the population: so they use materialism to manipulate the intellectuals and use religion to manipulate the workers. Before the communists can conquer the world, they will have some rational religion. The present ideal is not a sufficiently strong motive. Can't reform the world with a wrong philosophy. The founders of science were not atheists or materialists. Materialism began to appear in the second half of the eighteenth century". (p. 146, transcription n. 4.3.15)

¹⁰ The exact explication of this "therefore" depends on the period of Gödel's investigations, but it is a constant feature of his philosophy that mathematical objects depend on concepts.

¹¹ Cf. [Wang 1996] p. 309 and the letter to his mother mentioned above.

of its relation to science (logic and mathematics in the first place although from 1947 to 1950 Gödel worked on physics). Wang says that the Leibnizian project of a *Characteristica Universalis*, posing definitions concerning the primitive concepts of all science and developing deductively knowledge in all domains, was alive for Gödel up to the end of the fifties.¹² Nevertheless, Wang adds, the difficulties he encountered in preparing the Carnap paper,¹³ and the impossibility he felt of having a satisfactory account of the content of mathematics and of their concepts pushed him to a change of direction. Wang says that by 1959 Gödel had concluded that philosophy required a new method different from that of science and thought he had found such a method in Husserl's phenomenology. He adds that in around 1972, Gödel affirmed that he had not found what he was looking for.¹⁴

What also appears from the transcriptions of Gödel's conversations with Wang is that even though in around 1959 Gödel considered the necessity of changing his method,¹⁵ many logical and metaphysical aspects of Leibniz's thought were still at the centre of Gödel's work after the fifties. One of these aspects is certainly the search for a general type-free intensional logic of concepts with simple primitives from which all the other concepts can be composed, and the consequent questioning on the difference between concepts and objects. His ideas on this last subject in Wang's account strongly recall Leibniz's discussion about the difference between abstract and concrete, and complete and incomplete concepts. The second aspect concerns the importance of monadology especially for physics and biology. It is interesting in this sense to mention four passages reported by Wang, the first two because they are very explicit on the Leibnizian structure of Gödel's own monadology; the last two for their enigmatic, striking assertion of the importance of monads for modern science.

"0.2.1 My theory is a monadology with a central monad [namely God]. It is like the monadology by Leibniz in its general structure."¹⁶

0.2.2 My philosophy is rationalistic, idealistic, optimistic, and theological."¹⁷

"9.1.8 It is an idea of Leibniz that monads are spiritual in the sense that they have consciousness, experience, and drive on the active side and contain representations (*Vorstellungen*) on the passive side. Matter is also composed of such monads. We have the emotional idea that we should avoid inflicting pain on living things, but an electron or a piece

¹² [Wang 1987] p.174.

¹³ The one he prepared for the volume in honour of Carnap for the Schilpp's collection and that he never published. There are different drafts of this paper, under the title "Is mathematics syntax of language?" Two of them are published in the third volume of the *Collected Works*, [Gödel 1953].

¹⁴ [Wang 1996], chap. 2, especially pp.76-81 and p.88.

¹⁵ The conference of 1961 is the one where Gödel affirms explicitly his interest for Husserl and phenomenology [Gödel 1961] in [Gödel CW III] p. 383-4.

¹⁶ [Wang 1996] p. 8 (in the sequel we will quote Wang's transcription using only his own numbering.) That Gödel specifies in his conversations that his monadology has a general Leibnizian structure seems to us a clear indication that he was aware of other kind of monadology, such as that of Mahnke or Renouvier, but that he did not consider them interesting from the point of view of his own philosophical project; (two slips of paper from the archives prove that Gödel asked for *Eine neue Monadologie* by Dietrich Mahnke (1917) and *La nouvelle monadologie* de Charles Renouvier et Pratt (1899). See, nevertheless, [van Atten Kennedy 2003] p. 457 for another evaluation of Gödel's interest in Mahnke's phenomenology.

¹⁷ *Ibid.* Without entering into an analysis which would be out of context here, we can add, on the basis of [Gödel 1951] that Gödel's rationalism can be summarised as the affirmation that for clear questions posed by reason, reason can also find a clear answer; and that idealism and theology have in common that they "see sense, purpose and reason in everything". Idealism can be interpreted in a general way as the doctrine which affirms that space-temporal perceived reality is the result of the relation between subject and objects because in a particular time they have no existence independently from this relation cf. [Gödel 1946] quoted in note 1 and [Gödel CW IV] p.527.

of rock also has experiences. We experience drives, pains and so on ourselves. The task is to discover universal laws of the interactions of monads, including people, electrons and so forth. For example attraction and repulsion are the drives of electrons and they contain representations of other elementary particles.

9.1.9 Monads [bions] are not another kind of material particles; they are not in fixed parts of space, they are nowhere and therefore not material objects. Matter will be spiritualized when the true theory of physics is found. Monads only act *into* space; they are not *in* space. They have an inner life or consciousness; in addition to relations to other particles (clear in Newtonian physics, where we know the relationship between the particles), they also have something inside; in quantum physics the electrons are objectively distributed in space, not at a fixed place at a fixed moment, but at a ring. Hence it is impossible for electrons to have different inner states, only different distributions.”¹⁸

2. *Open problems and methodological decisions*

In spite of the huge amount of evidence for Leibniz’s influence on Gödel’s philosophical project, it is hard to evaluate precisely how Gödel, in his work, reconciles this influence with other ones, which are also clearly attested to (for example Kant, Husserl, German idealism and especially Hegel whose name is repeatedly mentioned by Gödel to Wang). The exact reasons for the “turn” in around 1959 (as mentioned by Wang), the precise nature of the difficulties with concepts which prevented Gödel from publishing his Carnap paper, the way his interest in Husserl can be reconciled with his “Platonism”, the relation between his work in physics and his monadology are at the centre of modern investigations into Gödel’s philosophical thinking . Wang’s transcriptions of his conversations with Gödel are precious but not precise enough to develop a clear picture of Gödel’s philosophical project and indeed are too general to give an answer to the question which would be central for us: what is the exact distance from Leibniz that Gödel thought necessary to take account of modern science within a philosophical system?

What is clearly stated, on the basis of Wang’s account, is the metaphysical project which is strongly inspired by Leibniz: concepts as “forms” or structures of reality which cause existence of spatiotemporal objects as well as mathematical objects, the latter being the limiting case of spatiotemporal objects. ¹⁹

¹⁸ [Wang 1996] p. 291-2. In the lecture of 1951 mentioned in the previous note, Gödel affirms that the development of the *Zeitgeist* from the Renaissance to modern times has led particularly in physics to the situation that “the possibility of knowledge of the objectivizable states of affairs is denied, and it is asserted that we must be content to predict results of observations. This is really the end of all theoretical science in the usual sense (although the predicting can be completely sufficient for practical purposes such as making television sets or atomic bombs).” Gödel considered his own contribution to cosmology as an attempt to complete the modern theory of gravitation, which he considered as deeply unaccomplished, even from the mathematical point of view (see [Audureau 2004] pp.145-6). Gödel’s image of gravitation in the Leibnizian terms of monadic attraction and repulsion, as it is suggested in these two passages, is a way of refusing to acknowledge, in very imaginative terms, this *Zeitgeist*’s diktat. Especially the second passage (“matter will be spiritualized when the true theory of physics is found”) suggests that the difficulties in unifying modern physics do not depend on the question of finding the right mathematical model, but on the courage to deepen our hypothesis on the reasons for gravitation. Cf. also [Wang 1996] p. 58.

¹⁹ Cf. [Wang 1996] 0.02 and also “5.3.11: The beginning of physics was Newton’s work of 1687, which needs only very simple primitives: force, mass law. I look for a similar theory for philosophy or metaphysics. Metaphysicians believe it possible to find out what the objective reality is; there are only a few primitive entities causing the existence of other entities. Form (So-Sein) should be distinguished from existence (Da-Sein): the forms- though not the existence of the objects- were, in the middle age, thought to be within us.”

What is not clear enough, from Wang's account, however is the structure of the system, the method, the difficulties experienced by Gödel on applying it, and his precise attitude toward the philosophers of the past.

It is a fact that Gödel considered Leibniz's search for the primitive concepts of our knowledge and Husserl's "intuition of essences" as similar in their tasks²⁰, but it is not clear what exact place Gödel gave to Husserl in his search for an "updating" of Leibniz's project.

Gödel probably thought that Husserl's method could be an alternative to the Leibnizian project of the *Characteristica* intended as a (non-formal) deductive system giving the primitives by definition (through explicit axioms),²¹ although he told Wang that he could not arrive at a satisfactory solution. He probably considered and explored both approaches till his death. Some passages of Wang's transcriptions seem to go in this direction, such as the following:

5.3.7 Phenomenology is not the only approach. Another approach is to find a list of the main categories (e.g. causation, substance, action) and their interrelation, which however, are to be arrived at phenomenologically. The task must be done in the right manner.

9.3.10 Philosophy aims at a theory. Phenomenology does not give a theory. In a theory concepts and axioms must be combined, and the concepts must be precise ones. [...]

Even if this hypothesis is viable, it is still unclear what exactly are these primitive precise concepts, from which philosophy could be derived and what kind of logic he intended to apply to them –in view of arriving to a satisfactory system implementing his ideal of informal rigour. This is not a mere technical problem, as logical questions are fundamental in the construction of Gödel's system being at the heart of physical and metaphysical problems.²²

The problem is particularly pressing in relation to a very basic question: in what way are Gödelian concepts tied to Leibnizian ideas and what does Gödel actually mean by concepts? What does Gödel have in mind when, in 1972 during his conversation with Wang, he speaks of concepts in intension, as opposed to concepts in extensions? Answers to these questions are required in order to understand his dissatisfaction with his Carnap paper and his "turn" toward Husserl after 1959. There are very interesting analyses which really assimilate Gödel's concepts in intension with Husserl's notion of intension and that consider the

²⁰ Cf. [Wang 1996] "5.3.19 Leibniz believed in the idea of seeing the primitive concepts clearly and distinctly. When Husserl affirmed our ability to "intuit essences," he had in mind something like what Leibniz believed. Even Schelling adhered to this ideal, but Hegel moved away from it. True metaphysics is constantly going away. Kant was a skeptic [sic], or at least believed that skepticism [sic] is necessary for the transition to true philosophy."

²¹ Gödel spoke about the (non-formal) logic of concepts, which would give a new life to the *Characteristica* project, in the Russell paper (see further) and in the conversation with Wang [Wang 1996] chap. 8. One of the ways to think of such a system of logic is the system of [Church 1932], [Church 1933], mentioned in [Gödel 1944] and in [Wang 1996] p. 279. The informal aspect of this system is that the logical rules cannot be applied on the simple inspection of the symbolic structure of a linguistic expression but in order to apply them we have to take into consideration its meaning.

²² One of the clearest cases of such a connection between logical and metaphysical problems is the example of possibility, which seems to have an unacceptable deterministic nature in Leibniz's system: "9.4.2 With regard to the structure of the real world, Leibniz did not go nearly as far as Hegel, but merely gave some 'preparatory polemics'; some of the concepts, such as that of possibility are not clear in the work of Leibniz; Leibniz had in mind a build up of the world that has to be so determined as to lead to the best possible world [...]. See also about possibility the interesting 9.4.10 which mentions Hegel's first three categories - being, non-being and becoming: "Independently of Hegel's (particular choice of) primitive terms, the process is not in time, even less an analogy with history. It is right to begin with being, because we have something to talk about. But becoming should not come immediately after being and non-being; this is taking time too seriously and taking it objectively. It is very clear that possibility is the synthesis between being and non-being. It is an essential and natural definition of possibility to take it as the synthesis between being and non-being. Possibility is a "weakened form of being" [Wang 1996]

“phenomenological turn” as a definitive abandonment of a primitive naïve Platonism.²³ There are other interpretations which stress how such identification is problematic on the basis of Gödel’s famous assertion of the reflexivity of concepts, which is very clearly stated in the conversations with Wang and which seems to be in contradiction with a strict Husserlian frame.²⁴

Interpretative conjectures about Gödel’s philosophical project can and have been made²⁵ but they will remain mere conjectures without a serious and complete analysis of the philosophical material of the Archives, which is far from being completed.²⁶ What is totally missing is an understanding of Gödel’s philosophical positions in relation to his scientific work. Contrary to a quite general opinion Gödel continued to be interested in logic, physics and mathematics after the forties, as it appears from the Archives and from Wang’s account. We do not know how his philosophical work interacted with and guided his reflections on science.

Such a situation invites a methodological choice, which I think is the wisest option considering the present state of our knowledge. The painstaking, almost maniacal attention that Gödel always paid to the editing and publication of his writing commands our respect. We need to reconstruct Gödel’s thought-process, taking as a starting point his published work, which is very often misunderstood because of Gödel’s style (concise and sometimes enigmatic) but also because of the content of his writings, which are generally contrary to what the *Zeitgeist* considers as standard. We need to go from the published works to the unpublished materials and then back again. I would like in the following to apply such a method to the issue of Leibniz’s influence on Gödel, trying to use Wang’s material and the available transcriptions of the Max-Phil manuscripts in order to clarify and deepen some of Gödel’s explicit doctrines presented in the published work.

When we inquire about Leibniz’s influence on Gödel it is certainly from the Russell paper that we have to start, i.e. the text of 1944 that, up to his death, Gödel always considered as one of his most important publications in terms of the explanation of his philosophical ideas concerning logic. Making use of passages from the archives and from the conversations with Wang, we will try to solve some of the interpretative difficulties concerning this paper and to reconstruct some central Leibnizian ideas, which Gödel continued to elaborate till the end of his life. Certainly the Russell paper is essentially concerned with questions of logic and the

²³ Cf [van Atten Kennedy 2003], [Tieszen 98]. The idea of a progressive abandonment of Platonism is quite generally accepted (see for example [Parsons 1995]). Many of the commentators quote generally a passage from [Gödel 1933] p. 50, where Gödel, commenting on his own results on the foundations of mathematics, says: “The result of the preceding discussion is that our axioms if interpreted as meaningful statements, necessarily presuppose a kind of Platonism, which cannot satisfy any critical mind”. The passage is presented as an early symptom of Gödel turning away from Platonism, which should have then been accomplished completely in the fifties. There is another natural interpretation of this text without any negative weight against Platonism, if we consider that the critical minds are all those people who believe that the Kantian Copernican revolution is a definite progression in philosophy and that in this sense, accept his critical philosophy (see [Audureau 2004] p. 62). That Platonism concerning ideas is compatible with Leibniz’s conception of Ideas *in mente Dei* is a well known result of Leibnizian studies see for example [Mugnai 2001] chapter 2, or [Nachtomy 2007] chapter 1. See also below.

²⁴ [Crocco 2006] p.190 and in particular note 50.

²⁵ See also [Hintikka 1998], [Buldt 2002].

²⁶ Most of Gödel’s manuscripts and notes in the Archives are written in Gabelsberger. The French ANR “Agence nationale pour l’évaluation de la recherche” financed a four-year project under my direction, to revise and pursue the work of transcription. After one year of work we have a complete transcription of three of the Max Phil (X, XI and XIV) thanks to the work of Robin Rollinger and a group of people including Mark van Atten, Paola Cantù and Eva-Maria Engelen.

philosophy of mathematics, but Gödel's systematic approach to philosophy allows us, starting with logical questions, to reach some central metaphysical ones. The Russell paper is therefore an essential step on the way to understanding Gödel's philosophical thinking and also a fundamental step in order to understand his critical consideration of the Leibnizian project in the light of modern science.

3. The structure of Gödel's "Russell mathematical logic": problems of interpretation

Enigmatic, unfair with regard to Russell and disjointed: this is the judgement very often given about this paper; beginning with Russell himself, who, in his reply to contributors, avoided any answer to the questions evoked by Gödel; pursuing with Hermann Weyl, who in his 1946 review of the Schilpp volume says (concerning Gödel's contribution): "it is the work of a pointillist: a delicate pattern of partly disconnected, partly interrelated, critical remarks and suggestions"²⁷; ending with the judgement of Charles Parsons, who says in his introduction to the re-publication of the paper in the second volume of Gödel's Collected Works : "the organization of the paper is difficult for the present commentator to analyze". Parsons proposes a division of the paper in eight parts, as follows:

1. Introductory remarks (125-128)
2. Russell's theory of descriptions (128-31)
3. The paradoxes and the vicious circle principle (131-37)
4. Gödel's own realistic view of classes and "concepts" (137-41)
5. Contrast with Russell's "no-classes theory" and the ramified theory of types; limitations of the latter (141-47).
6. The simple theory of types (147-50)
7. The analyticity of the axioms of Principia (150-52)
8. Concluding remarks on mathematical logic and Leibniz's project of a universal characteristic (152-53).

This division does not completely fit with that which is suggested by Gödel through very explicit formulations as "I pass now to..." (131), "I now come in somewhat more details to..." 147, "In conclusion I want to say..." (150). These formulations suggest a division into four parts to which we can add the introduction and the conclusion as follows:

0. the introductory remarks (125-127) containing a definition of mathematical logic as the science underlying all other, a homage to Leibniz, considered as the father of this point of view, and a sketchy comparative analysis of the respective contributions of Peano, Frege and Russell in "put[ting] into effect" the Leibnizian project.

1. the description of Russell's early realistic attitudes and, as an example of it, his theory of definite descriptions. (127-131). The end of the introduction and the beginning of this part is marked by the following sentence: "I do not want, however, to go into any more details about either the formalism or the mathematical content of the *Principia*, but want to devote the subsequent portion of this essay to Russell's work concerning the analysis of the concepts and axioms underlying mathematical logic. In this field Russell had produced a great number of interesting ideas some of which are presented more clearly (or are contained only) in his earlier writings. I shall therefore frequently refer to these earlier writings, although their content may partly disagree with Russell's present standpoint." p.127

²⁷ [Weyl 1946].

2. the discussion of the paradoxes and the critical analysis of Russell's solution of them through the ramified theory of types (the theory of orders as Gödel calls it) and the "constructivistic" vicious circle principle which, according to Gödel, inspired it (pp.131-147). This part is introduced by the following passage "I pass now to the most important of Russell's investigations in the field of the analysis of the concepts of formal logic, namely those concerning the logical paradoxes and their solutions." (p.131)

3. the analysis of the simple theory of types (147-150). This part is introduced by the following passage: "I pass now in somewhat more detail to theory of simple types which appears in *Principia* as combined with the theory of orders; the former is however, (as remarked above) quite independent of the latter, since mixed types evidently do not contradict the vicious circle principle in any way." (p.147).

4. the question of the analyticity of the axioms of mathematics (150-152). This part is introduced by the sentence " In conclusion I want to say a few words about the question of whether (and in which sense) the axioms of Principia can be considered to be analytic."

5. the conclusion which comes back to Leibniz and his project. (p. 152-3)

There is no other so explicit transition sentence in the paper.

This question of the organisation of the paper is a not a minor issue but a central one for our analysis. The Russell paper was solicited from Gödel by Schilpp on the 18th of November 1942 and Gödel sent the final version at the end of September 1943. Gödel says that his intensive study of Leibniz began in 1943. The evidence given in section 1, about Gödel's concern about the fate of the Leibniz Archives in 1933, and the existence in the Gödel Archives of a sheet dated 1941, with a first version of his very Leibnizian proof on the existence of God, are sufficient to exclude that Gödel's work on Leibniz began in 1943. It is just its *intensive* form, which started in 1943, as the increasing amount of direct mentions of Leibniz in Max-Phil X and XI²⁸ proves. A plausible conjecture is therefore that the Russell paper has been the occasion of this intensive study. Having expressed in it what he considered to be the current situation of logical research and the reasons and difficulties associated with the Leibnizian project, Gödel could have then decided to deepen his knowledge of Leibniz's work in order to propose more clearly his own logical and ontological system. The interpretative key of the Russell paper could therefore lie in its structure. It opens and ends with Leibniz and his project of the *Characteristica*. This could mean that the way the arguments follow each other is related to a certain way of interpreting Leibniz at that time when Gödel was convincing himself of the necessity of an intensive study on Leibniz's work. The Russell paper could be interpreted as proposing a sort of programmatic list of the central questions that Gödel intended to explore more deeply, and which, from his point of view, had been treated by Russell in an unsatisfactory way. Before coming to the question of the content of this programmatic list, let's begin with this tribute to Leibniz, which comes in the form of both the introduction and the conclusion to the paper.

In the introduction Gödel defines , mathematical logic as a mathematical science, treating specific objects (such as classes, relations and combinations of symbols) and mathematical logic as a science "prior to all others, which contains the ideas and principles underlying all sciences" . It was in this second sense, he says, that Logic was considered by Leibniz in his *Characteristica Universalis*, and in this same second sense it was developed by Peano and Frege with the aim of furnishing "a logical calculus really sufficient for the kind of reasoning occurring in the exact sciences" . Gödel considers Russell's work as part of the movement initiated by Peano and Frege and he interprets and judges their works as attempts toward the realisation of the Leibnizian project of the *Characteristica*. As a matter of fact, Frege's project runs against the paradoxes and Russell (as Gödel says later) "by analysing the

²⁸ See note 7 before.

paradoxes to which Cantor's set theory had led, freed them from all mathematical technicalities, thus bringing to light the amazing fact that our logical intuitions (i.e. intuitions concerning such notions as truth, concept being, class) are self-contradictory”(p. 124). The criticism Gödel develops in the paper shows how, in order to accomplish the Leibnizian project and in spite of Russell's work, the fundamental notion of class and concept “need further elucidation” (p. 152). Gödel stresses, in the end, that the accomplishment of this project, if correctly realised, should really improve progress in science and especially in mathematics and reject any kind of interpretation of the paradoxes in terms of the utopian character of the Leibnizian project. He says:

“It seems reasonable to suspect that it is this incomplete understanding of the foundations [of classes and concepts] which is responsible for the fact that mathematical logic has up to now remained so far behind the high expectations of Peano and others who (in accordance to Leibniz's claims) had hoped that it would facilitate theoretical mathematics to the same extent as the decimal system of numbers has facilitated numerical computations. For how can one expect to solve mathematical problems systematically by mere analysis of the concepts occurring if our analysis so far does not suffice to set up the axioms? But there is no need to give up hope. Leibniz did not in his writings about the *Characteristica Universalis* speak of a utopian project [...].”²⁹

In the course of his introduction to the Russell paper in the third volume of Gödel's Collected Works, Charles Parsons considering these lines as “a sort of coda of this intricate paper” expresses his astonishment concerning these Gödelian statements: “His suggestions that the hopes expressed by Leibniz for his *Characteristica Universalis* might after all be realistic” he says “is one of the most striking and enigmatic utterances”. Parsons judges Gödel's assertions in the light of the development of set theory and considers this theory as having definitively broken up the dream of a *Grande logique*, able to directly give an account by itself (through the notion of extension) of the logical nature of the concept of set. There is clear evidence that Gödel never accepted this verdict and that he was, for all his life, directly committed to the Leibnizian idea of logic as the science of all sciences. Moreover his diagnosis of the difficulties of Frege's and of Russell's attempted solutions suggests the necessity of a return to Leibniz, beyond Frege's and especially Russell's works, compatible with the constraints of a foundation of modern mathematics. The “disconnected” structure of Gödel's analysis can probably disappear through a reconstruction of the Leibnizian background of this paper. Gödel seems to point out, implicitly, that some of what everyone at the time considers as “progress” accomplished by contemporary logic is perhaps nothing of the sort. Beyond the specific solutions or suggestions that Gödel formulates at that time, what is at stake is the rehabilitation of the great Leibnizian project as Gödel understood it.

Let's now come again to the four parts in which Gödel divides his paper.

Charles Parsons recognizes explicitly, beyond the introductory and conclusive remarks on Leibniz, the first, the second, the third (corresponding to 6 in his list) and the fourth (corresponding to 7 in his list) parts that we mentioned before. He divides the second part into three distinct sections (3-5 in his list), which is perfectly justified from an expository point of view. Nevertheless it seems to us that, without recognising the unity of part two and without searching for the general relation between the four parts, something of Gödel's intention is lost. What can we conjecture about this intention?

The first part from § 5 to §9 is introduced, as we saw before, by a sentence (§5) where, ending the introduction, Gödel affirms that he wants to devote the rest of the paper to Russell's work concerning the analysis of the concepts and axioms underlying mathematical logic and it is clear from the context that he intended mathematical logic in the sense which

²⁹ [Gödel 1944] p.140.

he explained it in the introduction (the science “which contains the ideas and principles underlying all sciences”). After some consideration of Russell’s early realistic attitude, contrasted with his further constructivism or fictionalism³⁰, the rest of this part is devoted to an example. Russell’s analysis of descriptive phrases is given as an exemplary case of this realistic attitude in the analysis of “fundamental logical concepts” (beginning of §7, at the end of p. 128). Russell’s and Frege’s positions are comparatively analysed and criticised but Gödel holds a quite enigmatic position on what he considers the real solution to the problem. What we will suggest in this paper on the basis of some of the Archive texts, is the fact that in the same period Gödel’s reflections on the problem of descriptive phrases is abundant and in a constant dialogue with the problem of individual concepts in Leibniz. This material shows how this problem was seen by Gödel in this period as a crossroads of semantic, epistemological and metaphysical questions, which involves in particular the relation between extension and intension, the status of mathematical objects and the nature of concepts, which are the subject matter of the rest of the paper.

The long second section runs from §10 to §36. This section is essentially concerned with the notion of concepts and classes and the difficulties to which they give rise. The theory of orders is presented as Russell’s attempt to clarify their nature and their relationship. A very deep analysis of the vicious circle principle on which it rests is proposed. This part is very coherent in its content and proposes a deep, critical analysis of Russell’s position in *Principia*.

§37 opens the third part of the paper. It begins with an analysis of Frege and Russell’s doctrine about concepts and propositional functions as unsaturated or incomplete entities, which is at the basis of the theory of simple types. In a general way Gödel criticises it (§37). Nevertheless he considers it as containing the fundamental strategy of the limited ranges of significance, which is clearly, according to him, the most promising one for the solution of the paradoxes through a type-free strategy (§38).

In section 5, we will suggest how the analysis of the second and third part of the paper, considered with regard to Wang’s conversations with Gödel, hints at a very peculiar doctrine of concepts, which in many regards brings to mind Leibniz’s analysis. In opposing Frege and Russell Gödel seems to come back to Leibniz in order to reject both unsaturation and (consequently) distinction of types for concepts.

On the one hand, Gödel’s criticism of the ramified theory of types shows, from his point of view, how wrong Russell’s treatment of intensions is, through the vicious circle principle and how it obscures the relations between the linguistic terms, their intensions and their extensions, i.e. (according to Gödel) predicates or notions, concepts and classes. From Gödel’s point of view, treating correctly the difference between extension and intension, concepts and classes, is a fundamental task which logic has to solve in order to find the correct solution to paradoxes. From Gödel’s point of view this is the very Leibnizian problem of the distinction between possible and real, eternal ideas and spatiotemporal (or quasi-spatiotemporal i.e. mathematical) objects

On the other hand, the simple theory of types, which is the most coherent realisation of an intensional calculus of concepts, gives Gödel the occasion to radically criticise the Fregean-Russellian doctrine of the incomplete nature of concepts considered as unsaturated. It seems that, according to Gödel, we couldn’t find the true logic of concept without “coming back” to

³⁰ See in [Gödel 1944] p. 125 the *Author addition of 1964 expanded in 1972* where Gödel explicitly uses the term “fictionalism”. This attitude is so described in the text of 1944: “When he started on a concrete problem, the objects to be analysed (e.g. the classes or propositions) soon for the most part turned into “logical fictions”. Though perhaps this needs not necessarily mean (according to the sense in which Russell uses this term) that these things do not exist, but only that we have no direct perception of them”.

this fundamental prejudice of contemporary logic, as it derives from an unaccomplished realism of concepts.

In the fourth and last part (§41, 42) Gödel distinguishes two senses of analytic, one purely formal (i.e. an axiom is analytic if, by virtue of the definition of the terms occurring in it, it can be defined in such a way that it reduces in a finite number of steps to the law of identity) and a second sense concerning the content of the terms not their definitions. In this second sense, an axiom is analytic if it is “true in virtue of the meaning of the concepts occurring in it”. He adds in note 47 that this view also permits a reduction to a special case of $a=a$, if the reduction is effected not in virtue of the definitions of the terms occurring, but in virtue of their meaning, which can never be completely expressed in a set of formal rules. The question of analyticity allows Gödel to show that the Leibnizian idea that truth of reasons are in a certain sense identities is still very much alive. The correct interpretation of the notion of analyticity is essential, from Gödel’s point of view, in order to overcome the empiricist-sensualist prejudices of contemporary theory of knowledge, as he argues in the Carnap paper.

All of the above suggests a kind of hidden interconnection between these four parts, which could be more clearly appreciated if we focused on the Leibnizian background of Gödel’s work. Beyond the Fregean-Russellian systematization of modern logic, Gödel seems to be claiming the necessity of coming back to some logical issues re-evaluating Leibniz’s ideas: 1) the notion of individual concept and the relations between possibility and existence that it presupposes; 2) the way to look at the opposition between extensions and intensions; and 3) the opposition between concepts, ideas in *mente homini* and ideas in *mente dei*, that is, the opposition between the subjective and objective sides of concepts. On some of these issues, Gödel expresses his disagreement with Russell’s solution to some of Frege’s difficulties, on others he rejects both Frege’s and Russell’s analysis of questions on which their agreement was explicit. This hypothesis could be satisfactorily confirmed when the Max Phil has been completely transcribed.³¹ Nevertheless with our current state of knowledge, it is possible using some of the texts already transcribed to formulate some important conclusions about this background. We will try in this paper to underline some problems with the four parts of the Russell paper, and to give some hints about the Leibnizian background of Gödel’s reflection.

4. Gödel’s discussion of descriptive phrases and the Leibnizian metaphysical problem of the individual concepts

If there were only one subject on which Russell’s analysis was quite universally celebrated, then it would certainly be that of definite descriptions. Russell’s solution for them was one of the favourite examples of analytic philosophers (perhaps together with the famous

³¹ There are substantial mentions on Frege in Max-Phil VII, VIII and IX, (respectively 7, 6 and 8 mentions) and some on Russell in Max Phil IV, V, VII, and IX and XIV (respectively 4, 4, 2, 2, 2, 9 and 3 mentions). But many of them have not yet been transcribed, in particular those from Max-Phil IX, which was written between 18th of November ’42 and the 11th of March 1943. We will use some of the other mentions, in the sequel.

³⁶ Cf. for example [Quine 1941].

adage “to be is to be the value of a variable”) to show the possibility of making “progress” in philosophy.³⁶

Gödel provides two explicit reasons for considering the issue. The first reason (p. 127) is that it gives a clear example of the fact that, for Russell, questions of logic are questions concerning reality, which require definite answers, and are not matters of convention. The second reason (*ibid.* and p. 143) is that it shows how Russell’s realistic attitude was stronger in theory than in practice because his logical analysis often obeys a constructivist order of ideas.³⁷ Nevertheless, this illustrative power of the theory of definite descriptions in Russell’s philosophy does not exhaust Gödel’s interest. Its treatment by Gödel is quite enigmatic: in no way does Gödel subscribe to Russell’s solution,³⁸ but at the same time he seems hesitant to give a precise diagnosis as to what is wrong with it. After having presented Frege’s and Russell’s analysis, Gödel affirms: “As to the question in the logical sense, I cannot help the feeling that the problem raised by Frege’s puzzling conclusion has only been evaded by Russell’s theory of descriptions and that there is something behind it which is not completely understood.” The fact is that the logical analysis of the descriptive phrases directly involves semantic and metaphysical problems, which Gödel was working on at that time.

Let’s first recall the problem, as Gödel presents it. When we ask: ‘what is the meaning of descriptive phrases?’ Gödel says that there is an apparently obvious answer that Frege adopted. An expression such as “the author of Waverly” denotes or signifies Walter Scott and in a general way a descriptive phrase denotes the object it describes, if it exists. Nevertheless, this apparently obvious answer, in conjunction with a further apparently obvious axiom stating that the signification of a composite expression, containing constituents which have themselves a signification, depends only on the signification of these constituents (not on the manner on which this signification is expressed) implies “almost inevitably”³⁹ the unexpected consequence that all true sentences have the same signification, “the True” as it is called by Frege, and that all false sentences signify “the False”.⁴⁰ The other principles, which lead “almost inevitably” to this consequence are established by Gödel as follows :

1) “F(a) and the “a is the object having the property F and which is identical with a” have the same meaning;

2) Every proposition “speaks about something” i.e. can be brought in the form $\phi(a)$; and

3) For every object a and b, there exists a true proposition of the form $\phi(a, b)$ (as $a \neq b$ or $a = a \wedge b = b$);

From these three principles and from the two implied by the obvious axioms mentioned before, i.e.,

4) The possibility to substitute co-denotative expressions, independently of the way they are denoted and

5) The fact that descriptive phrases denote the objects they describe,
then we can derive Frege’s conclusion.

³⁷ [Gödel 1944] p. 143. See also the passage at p.127 already quoted in note 30.

³⁸ See especially the last sentence of this part: “There seems to be only one purely formal respect in which one may give preference to Russell’s theory of descriptions. By defining the meaning of sentences involving descriptions in the above manner, he avoids in his logical system any axiom about the particle “the” [...] Closer examination, however shows that this advantage of Russell’s theory over Frege’s subsists only as one interprets definitions as mere typographical abbreviations, not as introducing names for objects described by the definitions, a feature which is common to Frege and Russell.” [Gödel 1944] p. 130-1.

³⁹ *Ibid.*, p. 129.

⁴⁰ It’s important to stress that Gödel speaks about the “almost metaphysical sense” in which Frege intended this doctrine of the truth. Beyond the question of the correctness of Frege’s assertion lies the fact that for Gödel such a problem does have definitively metaphysical consequences.

Let's do that explicitly, just to fix terms, following Alonzo Church's suggestion⁴¹.

Church restricts Gödel's argument to sentences containing descriptive phrases and show how passing from the sentence "Sir Walter Scott is the author of Waverley" to the sentence "The number of counties in Utah is 29" through the equivalent sentences: "29 is the number of novels on Waverley that Walter Scott wrote altogether" and "29 is the number of counties in Utah". We will use this suggestion to generalize Church's argument.⁴²

Suppose that "F(a)" and "G(b)" are two true sentences which "speak about" two different objects a and b, through two different properties F and G. Using our five principles we can derive the second from the first, the only semantic aspect common to them being their truth.

i) F(a)	By hypothesis
ii) $a = (\exists x)(F(x) \wedge x = a)$	From i), by principles (1), (5), using Peano's definition of 'the' (reversed iota).
iii) $a = (\exists x)(F(x) \wedge x = a \wedge b = b)$	From ii) by principles (3), (4), supposing that the two predicates " $(\exists x)(F(x) \wedge x = a)$ " and " $(\exists x)(F(x) \wedge x = a \wedge b = b)$ " are co-denotative because they have the same extension.
iv) $b = (\exists y)(y = b \wedge a = (\exists x)(F(x) \wedge x = a))$	From iii) by the principle (2)
v) $b = (\exists y)(y = b \wedge G(y))$	From iv) by principles (3), (4), supposing again that the two predicates, having the same extension, are co-denotative
vi) G(b)	From v) by principles (1) and (5)

Therefore, from a true sentence we can derive a true one, whatever its content is. And from this, it follows that, if sentences have a denotation at all, this can be only their truth-value, which (according to the doctrine of saturated and unsaturated expressions) are objects.

In contrast with this Fregean doctrine of the denotation of sentences, there is the Russellian one, apparently more obvious from Gödel's point of view. According to the Russellian doctrine the things which correspond to true sentences, in the real world, are facts, whilst false sentences correspond to nothing. Nevertheless, because of the "almost inevitable" conclusion from the fact that descriptive phrases indicate objects to the fact that sentences indicate truth-values, Russell gives a different, less obvious interpretation of descriptive phrases. A descriptive phrase, according to Russell, denotes nothing, but has a meaning only in the context of a sentence. To be short and using the famous Russellian example, "The

⁴¹ The argument in its informal structure was presented by Alonzo Church in his *Introduction to mathematical logic* (1956) p.24-25, but it was clearly anticipated by Gödel at pages 128-129 of his paper on Russell. There is a very well-known modern version of this argument given by Davidson ("Truth and Meaning" 1967) which refers in a note to Church (1956), without mentioning Gödel. Nevertheless Davidson's formulation hides the question of extension and intension using the general extra-hypothesis that logically equivalent terms have the same reference, which never was a Gödelian thesis. Moreover the structure of Davidson's argument is more closely related to [Church 1943], which, as Parsons also remarks in his introduction to the Russell paper, depends "on somewhat different assumptions" ([Gödel CW II] p.104). Actually [Church 1943] is addressed to Carnap's *Introduction to semantics*, and is therefore different from [Church 1956], which on the contrary focuses on Frege's analysis.

⁴² "[...] the sentence "Sir Walter Scott is the author of Waverley" must have the same denotation as the sentence "Sir Walter Scott is the man who wrote twenty-nine Waverley novels altogether", since the name "the author of Waverley" is replaced by another name of the same person; the latter sentence, it is plausible to suppose, if it is not synonymous with "The number such that Sir Walter Scott is the man who wrote that many Waverley novels altogether is twenty nine", is at least so nearly so as to ensure its having the same denotation; and from this last sentence in turn, replacing the complete subject by another name of the same number, we obtain, as still having the same denotation, the sentence "The number of counties in Utah is twenty-nine" [Church 1956] p. 24-25.

author of Waverley” says nothing about Scott, but is only a “roundabout way of asserting something about the concepts occurring in the descriptive phrase.” (p. 130).

Gödel adds that Russell adduces two arguments in favour of this view, namely:

“that a descriptive phrase can be meaningfully employed even if the object described does not exist (e.g., in the sentence “The present king of France does not exist”); that one may very well understand a sentence containing a descriptive phrase without being acquainted with the object described, whereas it seems impossible to understand a sentence without being acquainted with the objects about which something is being asserted.”⁴³

This means that from Russell’s point of view there are semantic and epistemological reasons, which justify his deflationist analysis of descriptive phrases. Does Gödel accept such arguments?

A short text found in the Nachlass with item number 040269, and designated by the editor of Gödel’s Collected Works as Reprint E,⁴⁴ concerns some of Gödel’s commentaries on Bernays’s review, in 1946, of Gödel’s Russell paper. To Bernays’s general remark of a sort of a lack of clarity in Gödel’s paper about the distinction between Sense and Denotation⁴⁵ Gödel answers with two interesting remarks, which are the following:

(4) *Das Probl. der beschreibung ist durch “Sinn” und “Bedeutung” in befriedigender Weise gelöst.*

5) *Das Extents. axiom gilt nicht für Begriffe.*⁴⁶

On the basis of this text and the content of the paper let’s give a first interpretation of Gödel’s analysis in the following terms. Frege’s conclusion is unacceptable from Gödel’s point of view. Truth-values are not objects denoted by sentences. But how is it possible to avoid such a conclusion? It suffices to look at the theses, which lead to Frege’s puzzling conclusion. It seems plausible that principles (1), (2) and (3) are quite deep, rational requirements from Gödel’s point of view, just consequences of the meaning of identity and of predication. This leaves (4), and (5) which can be doubted.

Principle (4) can be doubted at least if we assume that predicates denote concepts⁴⁷ and that the principle of extensionality does not hold for concepts. This principle is evoked, in a very hypothetical form, some pages further on in the paper. Gödel says at §22, speaking of concepts:

"It may even be that the axiom of extensionality or at least something near to it holds for concepts".

But the pages about Bernays’s review definitively reject this hypothesis.⁴⁸ What then does it mean to reject principle (4) ?

Considering Church’s example, we cannot substitute the fact that “29 is the number of the counties in Utah” for “29 is the number of novels on Waverley that Walter Scott wrote

⁴³ In note 7 of his text Gödel explicitly mentions Frege’s doctrine of sense and contrasts it with Russell’s analysis in these terms “From the indication (*Bedeutung*) of a sentence has to be distinguished what Frege called its meaning (*Sinn*) which is the conceptual correlate of the objective existing fact (or “the True”). This one should expect to be in Russell’s theory a possible fact (or rather the possibility of a fact, which would exist also in the case of a false proposition. But Russell, as he says, could never believe that such “curious shadowy” things really exist.”

⁴⁴ [Gödel CW II] p. 321-2.

⁴⁵ [Bernays 1946]. See also [Crocco 2006] section 3.

⁴⁶ 4) The problem of description is solved in a satisfactory way by “sense” and “denotation”. 5) The axiom of extensionality does not hold for concepts.

⁴⁷ I argued in favour of such a thesis in [Crocco 2006] giving as the most important evidence a passage from [Gödel 1951] in [Gödel CW1995] p. 320.

⁴⁸ See [Wang 1987], p.307.

altogether”, because “to be the number of the counties in Utah” and “to be the number of the novels on Waverly written by Walter Scott” do not denote the same concepts, even if these concepts have the same extensions. In the same way we cannot pass from “to be equal to the object which has the property F and is equal to a” to “to be equal to the object which has the property F and is equal to a and $b=b$ ” because these predicates denote different concepts even if they have the same extensions. Now, if we reject the principle that two concepts which have the same extension are co-denotative, the derivation from $F(a)$ to $G(b)$ is stopped, and we can assume that sentences indicate facts, but descriptive phrases, if they have *Bedeutungen* at all, indicate individuals.

This interpretation seems quite natural. Nevertheless, Gödel closes the analysis with the enigmatic assertion we quoted before: “As to the question in the logical sense, I cannot help the feeling that the problem raised by Frege's puzzling conclusion has only been evaded by Russell's theory of descriptions and that there is something behind it which is not completely understood.” Is that because he doubts (5) in place of or together with (4)? What really are the consequences of the semantic interpretation of descriptive phrases from a logical-metaphysical point of view? What are the questions raised by Frege's puzzling conclusion that Russell's analysis evaded? What does Gödel not clearly understand about definite description? Is it just his hesitation concerning the extensional principle for concepts that bothers him? In what precise sense is the Fregean Sinn-Bedeutung distinction interpreted by Gödel? How did Gödel think he could solve the problem of descriptive phrases without *Bedeutungen*? Only a complete analysis of the material in the Archives and especially of the Max-Phil can give us a definite answer to these questions.

Nevertheless with regard to the accessible content of the Max-Phil there are some important facts which strongly corroborate the interpretative hypothesis stated in section 3, , and they are as follows:

1) The problem of descriptive phrases is discussed by Gödel in several pages of the Max-Phil before 18th November 1942, the date of Schilpp's first letter to Gödel, asking for the Russell paper. The context of Gödel's discussion of this subject matter, before November 1942, is the analysis of the paradoxes and in particular of the problem of the existence of the paradoxical classes which are mere pluralities. The philosophical discussion concerns more generally the kind of unity, which is presupposed by an object and in particular by a mathematical object.

In Max-Phil IV⁴⁹ Gödel says at p. 195 in a *Bemerkung Grundlagen* :

“Das Wesentliche der Verstandestätigkeit scheint also zu sein, Mehrheiten als neue Einheiten zu betrachten und das Nichts als Einheit zu betrachten. Die Sinngebung von anfangs Sinnlosem (∞) geht immer weiter, aber kann prinzipiell nie vollständig zu Ende geführt werden (auch die im Lebesgueschen Sinn messbaren Mengen können noch erweitert werden) oder es kann durchgeführt werden bis (im wesentlichen) auf einen einzigen Fall (vgl. Division).”

Understanding (probably opposed here to reason) is essentially the activity of unifying the manifold and creating from nothing new unities. The attribution of a Sinn (Sinngebung) to expressions of unities initially deprived of any interpretation (as some descriptive phrases) can always be improved, but this extension is essentially indefinite and incomplete. Exceptions and limiting cases, where the expression becomes “sinnlos”, will always subsist, as far as incompleteness is the essential feature of understanding. The allusion to the operation of dividing by zero as a limiting case of division, and the “singular points” or limiting points” of significance, is reminiscent of the discussion of the strategy of the Limited ranges of significance in the Russell paper where Gödel, who is very sympathetic with the

⁴⁹ Max-Phil IV was written between the first of May 1941 and the 30th of April 1942.

diagnosis of the paradoxes guiding such a strategy, says that through it, “paradoxes will appear as something analogous as dividing by zero” ([Gödel 1944] p. 150).⁵⁰

Peano’s analysis of descriptive phrases and of the reversed iota operator is analysed in detail (Max Phil IV, p 191-3 and Max Phil VII p. 509) but also comparatively in respect with Russell’s and Frege’s solutions (Max-Phil VII p. 528-29)⁵¹.

2) More striking, there is a passage in Max-Phil VIII, (p. 563)⁵² which lists the possible solutions of the problem and indicates what for Gödel is the ‘right one’. This latter has a clear connection with the Leibnizian notion of individual concepts conceived in the logical space, that is, for Leibniz in God’s understanding, although Leibniz is not here explicitly mentioned. This *Bemerkung* constitutes a grammatical remark such as those that the Max-Phil usually opposes to the philosophical (or foundational or mathematical or physical or theological ones). What is at stake in these grammatical remarks is the analysis of what is said or affirmed through language, i.e. what is the most obvious and natural content of our sentences in respect to and in agreement with the metaphysical and logical structure of reality.

“Bemerkung (Grammatik): Die Bedeutung einer Zeichenverwendung ist der Gegenstand über den im Satz gesprochen wird (dessen Vorstellung im Zuhörer erweckt werden will und im Redner vorschwebt). Die wahre Bedeutung der Zeichenverwendungen der Form $(\exists x) \phi(x)$ <ist>:

1. Nach Frege der betreffende Gegenstand (widerlegt durch Scotts Argument).

2. Nach Russell nichts. (Es ist eine zweckmäßige „façon de parler“, aber was zweckmäßig <ist> und was grammatisch wie ein Eigenname behandelt wird, bedeutet wahrscheinlich auch etwas. Auch der fruchtbaren Aufgabe diese zu finden wird durch Russellsche „Ausflucht“ gerade ausgedichtet.)

3. Es könnte den Begriff ϕ bedeuten. Aber was ist dann der Bedeutungsunterschied von „Sohn des A“ und „der Sohn des A“? Außerdem hat man das Gefühl, dass es etwas „wirklich“ bedeutet. (Das ist der Fregesche Sinn).

4. Es bedeutet eine Seite (oder einen Teil) des Gegenstandes (Scott ist nicht nur <der> Autor des Wav<erly>* (*Nap.<oleon>, der die Gefahr für seine Flanke erkannte“ ist gewissermaßen ein anderer <Teil>). Es bedeutet Scott, insofern er der Aut.<or> des Wav.<erly> ist.

5. (Richtige Lösung): Es bedeutet: „einen Ort im logischen Raum“ (*oder besser „das was sich an diesem Ort befindet“. Das ist aber nicht der ganze Scott, sondern ein Teil), d.h. etwas, was sich zur abstrakten Begriffswelt so verhält wie der Raum (und die Punkte) zur Welt der Sinnbegriffe. Die Dinge überdecken jedes einen bestimmten Teil dieses logischen Raums. Eine nicht eindeutige Kennzeichnung bedeutet einen endlichen Teil des logischen Raums. Diese Gegenstände (Mathematik und logischen Raum und ihre Punkte) bilden eine dritte Art von Entitäten zwischen Dingen und Begriffen. Der logische Raum sieht verschieden aus je nachdem, was für einen „Möglichkeitsbegriff“ man zugrunde legt. Der Sinn verhält sich zur Bedeutung in diesem Sinn wie die Def<inition> eines Punktes zum Punkt. Diese Bedeutung liegt also, was „Vielfältigkeit“ oder „Grad der Identifizierung“ oder „Nähe zum Ding und Ferne von der Erscheinung“ betreffen in der Mitte zwischen Sinn und Bedeutung.“⁵³

What Russell considers (wrongly) as an incomplete symbol, indicates according to Gödel a very precise *Bedeutung*, which is what is presupposed in the understanding of the linguistic

⁵⁰ See later section 5.2 about this point.

⁵¹ Max-Phil VII was written between the 15th of July and the 10th of September 1942.

⁵² Max Phil VIII was written between the 15th of September 1942 and the 18th of November 1942.

⁵³ The passage continues some pages after (p. 592) as follows “Fort<setzung von S. 563> 6. Eine Kennzeichnung bedeutet nicht einen bestimmten Gegenstand, sondern eine Funktion, welche jeder 'Welt' einen bestimmten Gegenstand zuordnet. (Daher bedeuten Scott und Verfasser des Wav<erly> nicht dasselbe.”

exchange. Nevertheless, Gödel says, different solutions can be proposed for the interpretation of this *Bedeutung*. The first is Frege's interpretation, but the Scott argument shows the implausibility of its consequences. The second is Russell's solution, which evades (*Ausflucht*) the important problem to find a (metaphysically) fecund solution to this semantic problem.

The third and fourth solutions were considered by Gödel at p. 529 of Max-Phil VII. They agree with the principle of compositionality and with the hypothesis that the *Bedeutungen* of sentences are states of affairs (or propositions) and not truth-values. The "Scott argument" does not apply to them because they give a purely intensional interpretation of language (nouns, descriptive phrases, and concepts). Both these solutions are therefore quite Russellian except for the fact that they consider concepts as real entities. The third assimilates the *Bedeutung* of a descriptive phrase with a concept, so eliminating the opposition between concepts and objects and therefore between what is independent from space and time and what it is not so. The fourth is a generalisation of the third, inasmuch as it considers that the concept, *Bedeutung* of the descriptive phrase, is a complex one, composed of concepts each of which representing an aspect, a part of it (the *Bedeutung* in quantum such and such).

Finally Gödel presents the right solution, which saves the extensional (Fregean) and intensional (Russellian) interpretation, by way of a very Leibnizian doctrine about the relationship between possible and real. A place in the logical space (in the "region des vérités éternelles" as Leibniz calls it, evoking Augustine in the 11th chapter of the 4th book of the *Nouveaux essais*) is a well-determined portion of it, representing a third kind of entities between things and concepts. Things are space-temporal. Concepts are independent from space and time, but also incomplete, in the Leibnizian sense that they can be applied to different subjects. Well-determined portions of the logical space are individual concepts, which are neither things nor concepts, but are related to both of them.

Can this third kind of entities be assimilated to Senses (Sinn)? In a way yes, because they are a perspective on a possible *Bedeutungen*. Are they *Bedeutungen*? In another way yes, because they are what the descriptive phrase refers to. Therefore they are intermediate ("*in der Mitte zwischen Sinn und Bedeutung*").

3) In Max Phil X partially written during the redaction of the Russell paper (March 1943-July 1944), Gödel comes back to the question on several pages (11, 17, 21, 26, 30, 32, 33, 37, 51, 63, 66-7, 70-1). The context of the discussion here mentions Frege's distinction between *Sinn und Bedeutung* and his analysis of concepts (pp. 9, 44, 92); Leibniz's analysis of existence (in space and time) and logical possibility (p. 57), predication (pp.56 and 65) and the relationship between what is simple and what is complex (pp.74-5).

There is also a *Bemerkung Philosophie* (p. 66-7) confirming the idea expressed in Max-Phil VIII about the right solution for the analysis of descriptive phrases:

Bemerkung (Philosophie): Eine leere Beschreibung (der König von Frankreich) ist etwas Ähnliches wie ein Raumpunkt, an dem sich kein Körper befindet (nämlich im logischen Raum). Der Raumpunkt ist sogar ein Spezialfall davon, da er bedeutet: derjenige Körper, welcher in so und so einer Lage zu gewissen gegebenen Körpern sich befindet. Es gibt wahrscheinlich außer dem körperlichen Raum (welcher das Schema der räumlichen Relationen ist, d.h. einen Überblick über die hinsichtlich der räumlichen Relationen bestehenden Möglichkeiten gewährt) auch Raum für andere Relationsgenera (sobald dieser Raum bekannt ist, <ist> es wahrscheinlich ebenso naheliegend Theorien aufzustellen wie die Newtonsche Physik). Ein Beispiel <ist> der Verwandtschaftsraum: Der Existenz von Isomorphismen im körperlichen Raum entspricht im allgemeinen wahrscheinlich die durch<wegi...>ge Erfüllbarkeit der „philosophischen Proposition<ortion>“ $x : a = b : c$. Das gibt auch die Möglichkeit, dass, obwohl z.B. die Allklasse nicht existiert, es sehr fruchtbar sein könnte, mit ihr zu operieren, denn es existieren: 1.) der Begriff der Allklasse

(intens<ional>) 2.) die entsprechenden Punkte des logischen Raums. Die Raumpunkte sind gewissermaßen ein Mittelding zwischen Nichts und Etwas.

4) In Max Phil XIV, written from 1946 onwards, Gödel explicitly comes back to the connection between existence and possibility invoked by descriptive phrases and insists on the metaphysical dimension of the logical problem. Leibniz is this time mentioned explicitly. Bem<erkung> (Gram<matik>): Der objektive Gehalt der Behauptungen „Der König ist der kleinste Mann“ und „Der kleinste Mann ist der König“ ist derselbe, und sie unterscheiden sich nur dadurch, dass die „Mitteilung“ sich auf einen anderen Teil des ganzen Sachverhalts bezieht (und ein anderer Teil vorge setzt wird). Im ersten Fall wird auf die Existenz der Verbindung der betreffenden Person mit „kleinstem Mann“, das andere Mal mit „König von Frankreich“ aufmerksam gemacht. Das heißt, im ersten Fall wird (impliciter) mitbehauptet, dass der König nicht 170 groß ist etc., im zweiten Fall, dass der kleinste Mann nicht Schuster ist. Ganz ebenso verhalten sich Darstellungen durch Aktiv und Passiv, obwohl aber in diesen Fällen bloß verschiedene Aspekte derselben Sache dargestellt werden. So gibt es doch wieder „natürliche“ Aspekte, nämlich wo das Wichtige behauptet (d.h. *betont*) wird und wo die Existenz der Gegenstände der Beschreibung dem Hörer bekannt ist. Bei einem natürlichen Aufbau der Gesamtwissenschaft sollen also offenbar prädikative Aussagen (d.h. Allsätze) mit Existenzsätzen (aufgrund der vorhergehenden Allsätze) abwechseln. Zwischen \in und $=$ besteht kein so wesentlicher Unterschied wie zwischen \exists und diesen beiden. Eigentlich sollten daher zwei Arten von Urteilen unterschieden werden: solche, die etwas zuschreiben (d.h. die Existenz von Verbindungen behaupten), und solche, welche <eine> andere Existenz behaupten. Aber es besteht die Tendenz in der Sprache, auch das auf die Existenz einer Verbindung zurückzuführen, nämlich Verbindung mit der Welt (es ist „in der Welt“, nicht „abgeschieden“ oder mit Gott), d.h. „Verbindung schlechthin“, (= est) soviel wie Existenz (oder heißt es: Verbindung mit etwas). Verbindung sollte vielleicht lieber als ein „Innewohnen“ (Teilsein) aufgefasst werden (Leibniz) oder als „Einssein“. „Etwas kennen“ bedeutet nicht bloß: ein Bild (das es von anderen zu unterscheiden gestattet) in sich haben, sondern ein das Wesen ausdrückendes Bild. Zum Beispiel: Man kann die Verbindung zwischen A und B nur dann <kennen>, wenn man sie als „Verbindung zwischen a und b“ kennt (oder insofern <man sie als „Verbindung zwischen a und b“ kennt>).⁵⁴

The beginning is very Fregean in the “mood” of the *Begriffsschrift* by the reference to the difference of the *Betonen* to the aspect of a fact, and the choice between active and passive in the presentation of the same content. The second part is more Leibnizian, as it concerns the relation between predication and existence.

The two first examples (considering how they are rephrased by Gödel : *Im ersten Fall wird auf die Existenz der Verbindung der betreffenden Person mit „kleinstem Mann“, das andere Mal mit „König von Frankreich“ aufmerksam gemacht*) seem to concern the current King of France, i.e. a non-existent individual, where existence here means existence in space and time.⁵⁵ What these two sentences express is that the *betreffenden Person*, presented by the descriptive phrase in the grammatical position of subject, that is the person who should be known by the hearer according to the way we use descriptive phrases, is the same as the person described by the descriptive phrase in the position of the nominal predicate. How can

⁵⁴ [Gödel Nachlass] Series III, Box 6b, Folder 72, Max Phil XIV, pp 036-038.

⁵⁵ It could be possible that *König von Frankreich* here does not mean the present king of France, and that the phrases have to be interpreted as talking of, for example, Pepin the Short. It seems to us just a little disturbing that such a “famous” example, as that of “the King of France”, echoing such a multitude of logical and metaphysical discussions could have been mentioned in such a “naïve” way by Gödel. Anyway the substance of our analysis does not change if we consider that the descriptive phrases refer “in extension” to a real individual.

it be so if the person in question does not exist? How can it be so if we are talking about the present King of France? There are two possibilities. The Fregean interpretation is that in this case the sentence has a content, a sense, but lacks a denotation, and therefore has no truth-value. The second one, the Leibnizian way of considering such a problem, is that the true denotation of the descriptive phrases are individual concepts in the Leibnizian sense, that is complete concepts of individuals. Some of these complete concepts correspond to individual substances. Others correspond to nothing in the spatiotemporal world. They nevertheless exist in a logical sense.

Gödel stresses that the content affirmed in *Der König ist der kleinste Mann* and *Der kleinste Mann ist der König* is the same. The *Sachverhalt* referred to is the same, only the message is different. The sentences only differ in their accent, in what is (negatively) implicated through the assertion, in what is excluded by it: *Das heißt, im ersten Fall wird (impliziter) mitbehauptet, dass der König nicht 170 groß ist etc., im zweiten Fall, dass der kleinste Mann nicht Schuster ist.* Again in a very Fregean way of speaking, Gödel draws the parallel between these two phrases and the ones, which differ from their verb mood, active or passive. Considering the Fregean example of the *Begriffsschrift*, the sentence “The Greeks defeated the Persians at the battle of Platea” and the phrase “The Persians were defeated by the Greeks at the battle of Platea” have the same content, express the same fact, even if with differences in emphasis, in the way the speaker wants to capture the attention of the hearer. While in the first case the question concerns aspects of the (possible) denotation, which are identified (being the King of France, being a little man, not-being a shoemaker), on the active/passive case, the question concerns only the insistence, the emphasis we put on some component of the fact.

What seems interesting here for Gödel is to remark how there are natural semantic and pragmatic aspects of both of these phrases implied by their use (*So gibt es doch wieder „natürliche“ Aspekte, nämlich wo das Wichtige behauptet (d.h. betont) wird und wo die Existenz der Gegenstände der Beschreibung dem Hörer bekannt ist*). When we use a descriptive phrase we normally presuppose the existence of the person described, nevertheless the assertion of this existence need not be considered as part of the objective content expressed by the phrase. In a similar way the accent, which is given through the active or passive form of a phrase, is not a part of its content, of its denotation. The criticism of Russell’s analysis seems here quite explicit. This passage clarifies Gödel’s remark about his Russell paper, found in the *Nachlass* before. The distinction between *Sinn und Bedeutung* does indeed solve in a satisfactory way the problem of the descriptive phrases. But what is existence and how should we interpret a descriptive phrase without *Bedeutung*?

The second part of the passage answers these questions in a very “Leibnizian mood.”

After having affirmed that when we (humans) try a general natural construction (*Aufbau*) of our knowledge, we have to switch from universal to existential assertions because our knowledge is primarily a knowledge of what is real and actually exists in space and time, Gödel makes a distinction between the assertion of the existence of a link between concepts (predication or identity) and the assertion of the existence of an object. The latter is also the assertion of a link but in the sense that an object is a part of the actual space-temporal world. A sentence, where a descriptive phrase does not refer to an existing individual (such as the ones concerning the King of France), asserts the link between two aspects, two parts of an individual concept, two conceptual components of it and it is therefore perfectly understandable, without considering or being acquainted with an actual individual. The transition from concepts to objects is expressed through the differentiation of two kinds of link. The first link (the one expressed by predication through \in for *est* or through identity by $=$) is a link between concepts in intension, a link independent of the world or “with God” as Gödel says; the second one (the one expressed by \exists) is a link of the objects, existing in space

and time, within the real world. “A is B” should express two kinds of judgements, says Gödel, one stressing a conceptual link between A and B through predication (A being or not being an individual concept and predication expressing a purely logical link) and the other expressing simple existence. Nevertheless it is possible to interpret the latter as expressing the actual link between a property B and an actual object A and therefore the link between A and the world to which it belongs (of which it is a part) because, as in Leibniz, the real world is just the maximal consistent set of concepts permitting the existence of the maximum number of individuals possible. In this sense existence can be considered as expressing a link with something that exists (within the real world) instead of a purely logical link.

What is important is that, in spite of this distinction of judgement, both of these links, since they express a connection between parts, would be better expressed through the Leibnizian *in esse*, (*innewohnen*). Existence and predication are therefore different logical concepts, although they can be considered as two aspects of the same logical relation of *in esse*.

A first conclusion can be drawn here, in the perspective of the Russell paper. A definite description as “the author of Waverley” is definitively about Walter Scott. Nevertheless it could be about him in two different ways. It could be about the individual complete concept of Walter Scott or about the spatiotemporally existing individual object, which corresponds extensionally to such a concept. In both cases, the concepts used to identify the reference (the complete concept or the real individual) express aspects of him and therefore cannot be substituted with other concepts only extensionally equivalent to them. From a logical point of view the concept of existence (expressed by the existential quantifier) can be used with the usual logical rules, when we talk of objects which are supposed to exist in space and time or closed to space and time (such as mathematical objects). When we talk of individual concepts or more generally of nonexistent objects, we should perhaps use another kind of rules, which a logic of concepts should provide.

Therefore, all these remarks confirm that in order to avoid Frege’s puzzling conclusion, Gödel considers the best solution to be the partial rejection of both principles (4) and (5).

Moreover, the remarks also confirm the fact that the treatment of the problem of descriptive phrases is not at all episodic in the Russell paper, but rather it runs through the entire paper, as it requires a better understanding of the opposition between extensions and intensions of concepts.

5 Concepts, classes and Gödel’s return to Leibniz

There is a deep link between the second and third parts of the Russell paper. Both have as a background the same questions. What is the correct definition of concepts and classes, which frees them from paradoxes? What is the correct characterisation of propositional functions⁵⁶ in intension (concepts as Gödel called them, underlining that concepts are different from the combination of symbols expressing them) and of propositional functions in extension (classes)?

On one side, the second part of the Russell paper explicitly considers Russell’s diagnosis of paradoxes (§10, p.131). According to Gödel, who mentions Russell’s paper of 1906, Russell came to the conclusion that “the erroneous axiom [generating the paradoxes] consists in assuming that for every propositional function there exists the class of objects satisfying it, or that every propositional function exists ‘as a separate entity’”.

On the basis of such a diagnosis there were two possible paths of analysis open to Russell:

⁵⁶ Gödel seems to consider a propositional function as what is simply abstracted from a proposition.

The first, rejecting the existence of classes and concepts in general, aimed at determining under what further hypothesis (concerning the propositional functions) these entities (i.e. concepts and classes) do exist (§11, p. 132).

Gödel mentions the two directions explored by Russell, according to this first possibility. The Zig-Zag theory (which he calls the intensional solution) makes the existence of a class or a concept depend on some constraints of simplicity of the content or of the meaning of the propositional function.⁵⁷ The limitation of size (which he calls the extensional solution) makes the existence of a class or of a concept depend on some constraints of the extension of the propositional function (requiring that it is not too big).⁵⁸

The second possibility, rejecting altogether the existence of classes and concepts, was based on the more radical “no-class theory”, according to which classes and concepts never exist as real entities. (14, p. 133).

According to Gödel, Russell’s work after 1906 was largely based on the second possibility, the no-class theory, although from *Principia* on Russell presents it under the auspices of the logical principle that justifies it, that is, the Vicious Circle Principle. The rest of the second part of the paper is devoted to the analysis of this principle, of its presuppositions and implications.

On the other side, the third part of the Russell paper is devoted to a second possible diagnosis of the paradoxes that Russell never formulated as such and that Gödel affirms to be specially suited for their intensional form: this is the Limited ranges of significance theory. “It consists in blaming the paradoxes not on the axiom that every propositional function defines a concept or class but on the assumption that every concept gives a meaningful proposition, if asserted of any arbitrary object or objects as arguments” (§38, p. 148).

Before going into more detail about these two parts of the Russell paper, it is important to underline a tenacious source of misunderstanding about Gödel, when speaking of concepts and intensions. The notion of intension used here by Gödel seems to conform to that used at the time by Russell, following a tradition coming from Leibniz and Peano.⁵⁹

To a ‘term’ (a sign of a function or a predicate), there corresponds two aspects, which cannot be dissociated from one another (like two sides of the same coin): the intension which was traditionally considered as the logical product of the simple primitive operations or concepts composing the function or the concept to which the predicate refers, and the extension which was the class of the individual falling under the intension. One of the classical places where Leibniz refers to this double interpretation (whose root is in Porphyry’s work) is in the *Nouveaux Essais, Livre IV De la connaissance*, chap. XVII, §8, where Leibniz opposes “*la manière vulgaire [qui] regarde plutôt les individus, [à] celle d’Aristote [qui] a plus d’égard aux idées ou aux universaux*”.

⁵⁷ This is not the only place (p.132) where Gödel seems to distinguish the content from the meaning of an expression (see for example p. 139 in this same paper: “it is true that such property ϕ (or such proposition $\phi(a)$) will have to contain themselves as constituents of their content (or of their meaning).” (the stress is mine). We will come back to this problem in section 6. One possible interpretation of this distinction is the following: a propositional function (considered as a linguistic element) expresses a content (a *Sinn* in Frege’s terms) and has a meaning (a *Bedeutung* in Frege’s terms), which can be considered as a class (extensionally) or a concept (intensionally). (cf. below our discussion of the notion of intension in this same section).

⁵⁸ “The second one would make the existence of a class or concept depend on the extension of the propositional function (requiring that it be not too big), the first one on its content or meaning (requiring a certain kind of “simplicity” the precise formulation of which would be the problem)” Ibid. p. 132. Axiomatic set theory in Zermelo’s or Von-Neumann’s version are given as examples of implementing the extensional solution, Quine’s stratification as implementing the intensional one.

⁵⁹ See [Crocco 2006] section 4, for more details.

This tradition has to be opposed to the one coming very plausible from Kant for whom concepts are ultimately rules of production of objects, our knowledge being essentially knowledge of objects and not of universals. According to this tradition, which was spread through the (wrong) interpretation of Frege given by Carnap, and which is common to Carnap, Weyl and Husserl, concepts in intension are assimilated to a way of grasping, forming or unifying a plurality of things, whereas concepts in extensions are those pluralities. This second sense of intension can be likened to Frege's *Sinn* of a predicate.

Can intensions in the first sense (intension₁) be likened to intensions in this second sense (intension₂)? The answer seems to be in the negative, inasmuch as the notion of a logical product of simple concepts doesn't necessarily have anything to do with the way we form or unify a plurality. Moreover nothing prevents an intension₁ to be grasped or given to us in a particular way (*in quantum* such-and-such) or through a given perspective (intension₂) i.e. through a specific sense, which has to be distinguished from the concept itself (intension₁). The first notion is ontological, the second mainly epistemological.⁶⁰

Therefore it seems that, at this time, Gödel distinguishes the level of language from the Fregean level of *Sinn*, (which he generally calls "content", and that we have identified with the notion of intension₂) and from the double level of reference (meaning, as he called it, or in the Fregean terms *Bedeutung*) which nevertheless can be considered from the double point of view of intension₁ and extension.

Gödel repeatedly uses the expression "classes and concepts" to refer to the entities which are referred to as propositional functions (intended as fragments of propositions). The words "classes and concepts" are used in a synonymous way with the words "intensions and extensions", and the deepest question at stake for Gödel is clearly to determine the correct nature of such entities.

5.1 The theory of orders

The second part of "Russell's mathematical logic" focuses on the theory of order, which is, according to Gödel, based on the Vicious Circle Principle (VCP). A double confusion seems to affect Russell's diagnosis of paradoxes through the VCP. The first confusion concerns language and what it expresses, which is in particular (according to Gödel) the confusion between predicates and concepts, made by Russell through the notion of propositional functions. The second one concerns the two sides of what is the reference of the linguistic expressions, that is the intensional and the extensional sides.

To clarify these confusions is an important step in the realisation of Leibniz's project, which is after all Gödel's ultimate aim.

The first move made by Gödel to this effect is to distinguish three forms of the VCP in the paper. The strongest states that: 'no totality can contain members definable only in terms of this totality'. The second weaker form of the VCP is obtained by replacing the term 'definable only in terms of' with 'involving'. The third form is obtained from the second one by replacing the term 'involving' with the term 'presupposing'. When the notion of 'presupposing' means 'presupposing for the existence', the third form, the weakest, becomes a rational exigency that every theory should satisfy, but the first and second forms can be denied, at least from a realistic point of view on the entities of logic and mathematics.

The second move made by Gödel is to carefully distinguish the intensional and extensional versions of these principles. For the intensional version, concerning propositional functions,

⁶⁰ This intension₂ can be what Gödel calls the content of a propositional function, distinguishing it from its meaning, cf. note 57 before.

in order to prevent intensional paradoxes, we have to add the principle that a propositional function presupposes the totality of its arguments and of its values (Gödel 1944, p. 126). It seems clear from Gödel's claims that we have, as a consequence, three corresponding forms of VCP for propositional functions. The first states that 'nothing defined in terms of a propositional function can be a possible argument of this function'. The second form is obtained by replacing 'defined in terms of' with 'involving' and the third with the term 'presupposing'.⁶¹

The first form of the VCP (in both extensional and intensional versions) makes impredicative definitions impossible. The ramified theory of types conforms to it. Gödel gives two arguments in order to explain that this first form is implausible. The first argument concerns the interpretation of the totality that it implies. Actually it implies the assimilation of "all" (expressing the totality) to an infinite conjunction i.e. that "reference to a totality implies reference to all single elements of it" (p. 136). There is no reason to accept such an interpretation and Gödel mentions Carnap and Langford's analysis of "all" in terms of analyticity.

The second argument concerns the interpretation of the entities (concepts in the intensional version or classes in the extensional one) involved in it: "even if "all" means an infinite conjunction it seems that the vicious circle principle applies only if the entities involved are constructed by ourselves" (*ibid.*). He adds that there is nothing in the least absurd in the existence of totalities containing members that can be described (i.e. uniquely characterized) only by reference to this totality. Only the constructivist prejudice against concepts and classes as real entities can therefore allow the first form of the VCP. On the other hand, Gödel says that concepts are objective entities, i.e. properties and relations between objects (Gödel 1944, p. 128). As real entities, they have to be distinguished from predicates (which are linguistic entities) and from notions, where the term notion means a symbol of a predicate together with a rule of translation of sentences containing this symbol into sentences not containing it. Two different definitions determine different notions. On the contrary, says Gödel, the same concept can be expressed in different ways,

Concerning the second form of the VCP, Zermelo-Fraenkel set theory (which does not satisfy the first form of the VCP, admitting impredicative definitions) conforms indeed to the second one in its extensional version. Actually, it forbids that a set could belong to itself, that is, that a totality (a set) can contain (involve) itself as an element.

Finally the theory of simple types, if it is interpreted intensionally (with quantifier ranging over properties and relations considered as real entities), and if the universal quantifier is interpreted as an infinite conjunction (thus involving the properties on which it ranges over), violates the second form of the VCP in its version for propositional functions. This fact motivated Gödel's assertion on p. 127, according to which the theory of simple types 'has nothing to do with the vicious circle principle', which means that the theory of simple types contradicts the first form of the VCP and is independent from the second one.⁶²

By the analysis of the last two examples, Gödel in some way contrasts concepts with classes (as pluralities of objects). The difference between concepts and classes is grounded in self-reflexivity. It is perfectly coherent to think that concepts are able to support self-

⁶¹ Gödel specifies (p.134) that in the second edition of the Principia this intensional version of the VCP is dropped because Russell adopts the principle according to which functions can occur in propositions only through their values i.e. extensionally. In that case, he said, the paradoxes are avoided by the theory of simple types, interpreted extensionally.

⁶² Actually, the theory of simple types can conform to the second weaker form (when considered extensionally as in Zermelo-Fraenkel set theory, allowing mixed types) or not conform to it (when considered intensionally with universal quantification taken as an infinite conjunction and with or without mixed types) and so independently from the fact of allowing mixed types.

reflexivity, and that therefore the VCP in its second form does not also apply to concepts. Of concepts (as opposed to notions), it is possible to refer to their totality, to claim that some of them can be described only by reference to all of them (or at least all of a given type) and to say that a property can contain itself as a constituent of its meaning or content (p. 139, cf; note 57 above). Gödel adds that an approximation of this kind of self-reflexivity is in fact given in his theorems of incompleteness, where a proposition contains, as part of its meaning, the assertion of its own demonstrability, and where the demonstrability of a proposition (in the case where the axioms and the rules of inference are correct) implies the proposition in question. Concerning classes, on the contrary, we can consider the fact that the VCP applies in the second and third form as a plausible assumption, sufficient for the development of all contemporary mathematics. Impredicative definitions are allowed for classes, but it is impossible to say that $x \in y$ when x is not less than y . Therefore one is then led to something like the Zermelo set theory. Zermelo's theory is based on an iterative notion of set, where sets are split up into levels, obtained by the relation 'set of' on a specific level, and where mixture and transfinite types are allowed. Gödel clearly considers Zermelo's solution sufficient for the needs of mathematics. The last thesis clearly proposes that we search for different solutions for paradoxes of concepts and those of classes by opposing the needs of logic to the more restricted needs of mathematics, that is, by opposing the notion of structure in the abstract sense and the iterative notion of set. Gödel stresses that abstract structures can contain elements without presupposing them (ontologically), just as a sentence or a phrase belongs to a language, without having ontological primacy over it. On the contrary, sentences presuppose a language, even if they are involved in (belong to) it. In the same way, concepts (which are abstract structures) can contain concepts (can have other concepts as parts of them) without ontologically presupposing them.

Recognizing that the same logical relation of predication can be interpreted both in intensional and extensional terms, giving an explicit preference to the intensional form, opposing the needs of logic to the more peculiar needs of mathematics are all common aspects of a very profound return to Leibniz beyond Frege's analysis. Russell, contrary to Frege, had been very sympathetic towards the Leibnizian interpretation of intension and extension of terms⁶³ but the way he treats propositional functions in intension through the ramified theory of types is unacceptable for Gödel. As suggested in Wang 5.3.17, 8.6.18, the relation of predication consists, in intensional terms, in the application of a concept to an object or to another concept. On the contrary, extensionally, it can be interpreted in terms of an object belonging to a class of objects. Classes are paradoxical objects indeed, but we can describe an iterative process of engendering non-paradoxical multiplicities of objects, i.e. sets representing extensionally a concept. The confusion or the reduction between these two aspects of predication (the intensional and the extensional one) has been constant in modern logic since Frege and Russell's works. Therefore, Gödel stresses explicitly the difference

⁶³ The most explicit text on intensions is in [Russell Whitehead 1910] chapter III, p. 72 of the Introduction, where Russell presents classes as incomplete symbols, mere symbolic or linguistic conveniences, 'not genuine objects as their members are if they are individuals'. Then, he goes on to claim: "It is an old dispute whether formal logic should concern itself mainly with intensions or with extensions. In general logicians whose training was mainly philosophical have decided for intensions, while those whose training was mainly mathematical have decided for extensions. The fact seems to be that, while mathematical logic requires extensions, philosophical logic refuses to supply anything except intensions. Our theory of classes recognises and reconciles these two apparently opposite facts, by showing that an extension (which is the same as a class) is an incomplete symbol, whose use always acquires its meaning through a reference to intension." These intensions, for Russell, are nothing but the propositional functions, which are the constituent parts of propositions and presuppose them ontologically.

between them, asserting that the extensional predication cannot be reflexive and the intensional one is in general reflexive. This fact explains, according to him, the difference between set theory and logic. As Gödel puts it in his conversation with Wang:

8.6.15 For a long time there has been a confusion between logic and mathematics. Once we make and use a sharp distinction between sets and concepts, we have made several advances. We have a reasonably convincing foundation for ordinary mathematics according to the iterative concept of set. Going beyond sets becomes understandable and, in fact, a necessary step for a comprehensive conception of logic. We come back to the program of developing *a grand logic* [sic] except that we are no longer troubled by the consequences of the confusion between sets and concepts.

8.6.2 Mathematicians are primarily interested in extensions and we have a systematic study of extensions in set theory, which remains a mathematical subject except in its foundations. Mathematicians form and use concepts, but they do not investigate generally how concepts are formed, as is to be done in logic. We do not have an equally well-developed theory of concepts comparable with set theory.

6.1.11 Because of the unsolved intensional paradoxes for concepts like *concept*, *proposition*, *proof*, and so on, in their most general sense, no proof using the self-reflexivity of these concepts can be regarded as conclusive in the present stage of development of logic, although, after a satisfactory solution of these paradoxes, such an argument may turn out to be conclusive.

Nevertheless, Gödel stresses that recognising such a distinction between logic and mathematics in no way precludes the possibility of a “calculus of conversion” between the intensional and the extensional interpretation of concepts. It just means that in the actual state of our knowledge we have not found such a “calculus of conversion” and therefore, that the theory of sets is the only satisfying frame which we can use for the foundations of mathematics. Gödel seems to think that even if to some concept there would correspond no set, i.e. no object that could be obtained through the iterative idealised repetition of the “set of” operation, there is still the possibility to conjoin to each set its “defining concept”:

8.6.4 “It is not in the ideas (of *set* and *concept*) themselves that every set is the extension of a concept. Sets might exist which correspond to no concepts. The proposition “for every set, there is a [defining] concept” requires a proof. But I conjecture that it is true. If so, everything in logic and mathematics is a concept: a set, if extensional; and a concept (only) otherwise.”

Wang says that the opposition between concepts and objects (sets) increased in Gödel’s thoughts after the Russell paper. Nevertheless besides the logical property that we mentioned above (concepts can be applied to themselves, objects cannot belong to themselves) in the conversations with Wang, objects are contrasted with concepts also through a metaphysical characterisation, which closely resembles Leibniz’s analysis:

9.1.27 Monads are objects. Sets (of objects) are objects. A set is a unity (or whole) of which the elements are constituents. Objects are in space or closed to space. Sets are the limiting case of spatiotemporal objects and also of wholes. Among objects, there are physical objects and spatiotemporal objects. Pure sets are sets that do not involve non-set objects - so that the only Urelement in the universe of pure sets is the empty set. Pure sets are the mathematical objects and make up the world of mathematics.

8.2.4 Sets are the limiting case of spatiotemporal objects- either as an analogue of constructing a whole physical body as determined entirely by its parts (so that the interconnections of the parts play no role) or as an analogue of synthesizing various aspects to

get one object, with the difference that the interconnections of the aspects are disregarded. Sets are quasi-spatial.⁶⁴

What is the exact relation between spatiotemporal objects, monads and mathematical objects? Only a complete transcription of the Max-Phil can answer this question.

5.2 The theory of simple types

The second and the third parts of “Russell’s mathematical logic” contain a main thesis, which intimately correlates them. This main thesis states that the theory of simple types is independent from the theory of orders (p. 127, 147) and it is essential from Gödel’s point of view to show the fundamental difference between them, in order to present the new principle, which the theory of simple types brings in (i.e. the principle of the Limited ranges of significance) as totally independent from the constructive theory of orders.

The third part specifies two points of view from which the theory of simple types can be justified. The first one involves the principle of unsaturation (or typical ambiguity)⁶⁵ for propositional functions and is, according to Gödel, explicitly adduced by Russell: “The reason adduced (in addition to its “consonance with common sense”) is very similar to Frege’s who, in his system had already assumed the theory of simple types for functions, but failed to avoid the paradoxes, because he operated with classes, (or, rather, functions in extension) without any restrictions. This reason is that (owing to the variable it contains) a propositional function is something ambiguous or, as Frege says something unsaturated, wanting supplementation) and therefore can occur in a meaningful proposition only in such a way that this ambiguity is eliminated (e.g. by substituting a constant for the variable or applying quantification to it). The consequences are that a function cannot replace an individual in a proposition, because the latter has no ambiguity to be removed, and that functions with different kinds of arguments (i.e. different ambiguities) cannot replace each other; which is the essence of the theory of simple types.” ([Gödel 1944] p. 147-8). Propositional functions are in this sense “fragments” of propositions, “which have no meaning in themselves, but only insofar as one can use them for forming propositions by combining several of them, which is possible only if they “fit together”, i.e. they are of the appropriate types.”

The second point of view (called realistic by Gödel at p. 149 §2) is independent from this “fictionalist standpoint” because it considers (extensionally) classes as essentially formed by an infinite iteration of the process of forming classes out of infinitely many individuals. The same process seems applicable to form propositional functions in the intensional sense, if we consider predication of a concept to an argument x as expressing a proposition of infinite lengths of the form $x=a_1 \vee x=a_2 \vee \dots \vee x=a_n$ where a_i are individuals or classes obtained on a (logically) precedent level of iteration.

Generally speaking the theory of simple types is considered by Gödel as “a stepping stone” (end p. 148) towards a more satisfactory system. On the one hand it has the clear advantage of bringing in a new idea for the solution of paradoxes: “[i]t consists in blaming the paradoxes not on the axiom that every propositional function defines a concept or a class, but on the assumption that every concept gives a meaningful proposition, if asserted of every object or objects as argument”. On the other hand it is not completely plausible from a realistic point of view “since what one would expect to be a concept (such as, e.g. “transitivity” or the number

⁶⁴ See section 6 below about these assertions.

⁶⁵ As a matter of fact Russell calls ambiguity what Frege called unsaturation, or at least something very close to it. See [Russell Whitehead 1910] section IV of the introduction of the first edition. This notion of ambiguity has to be distinguished from the systematic ambiguity of truth and falsehood of section III.

two) would seem to be something behind its various “realizations” on the different levels and therefore does not exist according to this theory of types” (p. 148 end).

This example is particularly important to understand Gödel’s criticism of the theory of simple types: transitivity is a property of relations. A relation is called transitive if and only if when applied to arguments a, b, c , if it relates a to b and b to c it relates also a to c . (where a, b , and c can be objects, classes or concepts). Since, in the theory of simple types, types are mutually exclusive, it follows that transitivity should be defined for relations of types 1, 2, and so on. According to the theory of simple types, a concept (property or relation) exists insofar as it has a homogeneous range of arguments from the point of view of types. Therefore, the concept of transitivity which applies to relations of each type, does not exist, and speaking of it is a sort of *abus de langage*.

Considering the conversations with Wang, we see that this diagnosis of the theory of simple types relies on a deeper rejection of Frege’s and Russell’s analysis. Rejecting Frege’s and Russell’s idea of the priority of propositions over concepts and the identification of concepts with propositional functions is a way to come back to Leibniz on a very fundamental issue of modern logic.⁶⁶ The theory of unsaturation, which commands this priority in Frege’s work, seems to be, according to Gödel, completely false. Propositional functions are, through the doctrine of unsaturation, considered as “fragments” of propositions, which have no meaning in themselves, but only in so far as one can use them to form propositions. On the contrary, Leibniz’s ideas do not come from propositions but from simpler concepts. In the same sense, from Gödel’s point of view, unsaturation has nothing to do with concepts which are entities composed by primitive concepts:

9.1.26 *Concepts*. A concept is a whole – a conceptual whole – composed out of primitive concepts such as negation, existence, conjunction, universality, object, (the concept of) concept, whole, meaning and so on. We have no clear idea of the totality of all concepts. A concept is a whole in a stronger sense than sets; it is a more organic whole, as a human body is an organic whole of its parts.⁶⁷

The rejection of the notion of unsaturation coupled with the criticism about the lack of plausibility of the theory of simple types from a realistic point of view (see above) opens the problem of the notion of predication. The doctrine of unsaturation allows Frege and Russell to avoid an explicit relation of predication, that is: a relation connecting terms to form a proposition. There is, nevertheless, a price to pay for such an elimination of an *explicit* relation of predication: considering a (propositional) function as a fragment of a proposition implies distinguishing different types of concepts according to the different arguments they accept. Such a classification gives rise to an infinite *implicit*⁶⁸ hierarchy of logical relations of predication each one differing fundamentally from the others.⁶⁹ It is a well-known fact, remarked upon by Gödel, that in spite of this differentiation of types – a consequence of the distinction of an infinite⁷⁰ hierarchy of logical relations of predication – Frege failed to avoid paradoxes because he operated with classes (functions in extensions) without restrictions. Types for concepts and constraints of types for classes are fundamental for Russell too, both

⁶⁶ [Crocco 2006]

⁶⁷ Cf. also Carnap paper [Gödel 1953] p. 360.

⁶⁸ Implicit means here that the logical relation of predication is completely determined by the nature of the concepts to which it applies and of the arguments that they receive. See also [Angelelli 1967] pp. 92-101.

⁶⁹ Cf. the end of “Über Funktion und Begriff” [Frege 1891] and “Begriff und Gegenstand” [Frege 1892] §17.

⁷⁰ Actually Frege just needs three different levels because of its function \cap , which allows him to pass to classes, which are not typified.

in his first sketchy attempt with simple types, and in his second one with ramified types. As an infinite hierarchy of types means an infinite hierarchy of relations of predication, Gödel rejects types openly from 1944. For Gödel there is only one fundamental relation which is meaningful predication (in the double form of application or belonging)⁷¹ and which warrants the unity of logic:

5.3.17 The basis of every thing is meaningful predication, such as Px , x belongs to A , xRy , and so on. Husserl had this. Hegel did not have this; that is why his philosophy lacks clarity. [...]

8.6.18 [...] Logic studies only what a concept applies to. Application is the only primitive concept apart from the familiar concepts of predicate logic with which we define other concepts.

9.1.16 The significance of mathematical logic for philosophy lies in its power to make explicit by illustrating and providing a frame for axiomatic method. Mathematical logic makes explicit the central place of predication in the philosophical foundation of rational thought.

The search for a type-free logic, allowing self application of concepts, except for limiting cases is one of the main tasks of logic, necessary to implement Leibniz's project.

The existence of one unifying logical relation between terms is a central thesis in Leibniz's logical work. To reduce the connection between terms to the unique relation of "*inesse*" gives him the opportunity to unify the different kinds of syllogism. Modern logic rejects Leibniz's analysis on this issue. Frege and Russell both criticised what they considered a lack of distinction (between predication, subsumption and existence) resulting from this unification. Is there any evidence that Gödel was willing to rehabilitate Leibniz's notion of predication? And in this case how did he consider the relations between predication, subsumption and existence? Some hints have been given in section 4 in the passage from Max-Phil XIV talking about the Leibnizian *inesse*, but again, only a complete analysis of the Max-Phil could answer these questions.

6. Analyticity and truths of reason

Russell pleaded for the logical nature of mathematics at least since the *Principles* in 1903, mentioning Leibniz (through Couturat's work of 1901) as the one who strongly advocated that "all mathematics is deduction by logical principles from logical principles" (chap. 1, section 5). He added: "owing partly to a faulty logic, partly to belief in the logical necessity of Euclidian geometry, he [Leibniz] was left into hopeless errors, in the endeavour to carry out in detail a view which, in its general outline, is known to be correct" (ibid.).

Gödel does not mention Russell's (and Frege's) notion of analyticity (deduction by logical principles from logical principles) but refers directly to Leibniz's notion of truth of reason (truths which can be reduced in a finite numbers of steps to identities) and asks the question whether the axioms of Principia can be said to be analytic in this sense. He then distinguishes two means for accomplishing such a reduction: in virtue of explicit definitions (or by rules for eliminating them from sentences containing them); and in virtue of the meaning of the concept occurring in them, where this meaning may perhaps be indefinable.

In the first sense, his incompleteness theorem shows that even the theory of integers cannot be said to be analytic. But this is not the case of the second sense, and Gödel quite explicitly suggests that this is a way to revert to the very Leibnizian notion of truths of reason. Actually,

⁷¹ See below in this section, for the reasons of this distinction.

in this second sense “every mathematical proposition could perhaps be reduced to a special case of $a=a$, namely if the reduction is effected not in virtue of the definitions of the terms occurring, but in virtue of their meaning, which can never be completely expressed in a formal set of formal rules” (ibid. note 47).

Analyticity (in the second sense) can be considered as expressing some kind of identity of meaning. Gödel said that all the axioms of Principia (except the axiom of infinity) can be considered as analytic in this sense for certain interpretations of the primitive terms “namely if the term “predicative function” is replaced either by “class” (in the extensional sense) or (leaving out the axiom of choice) by “concepts” since nothing can express better the meaning of the term “class” than the axiom of classes [Zermelo’s *Aussonderungssaxiom*] and the axiom of choice and since, on the other hand, the meaning of the term “concept” seems to imply that every propositional function defines a concept.

The same idea is stated at the end of the 1951 paper ([Gödel CW III] pp 320 and 321). In 1951 Gödel gives an account of analytic truths (truths of reason) in terms of the logical relations between concepts and specifies how far this idea of analyticity is from that of the Vienna’s circle: mathematical truths are analytical but even if they are true whatever the situation of the world is, this fact does not imply that they are “void of content” because their content consists exactly in the relations between concepts or between concepts and abstract objects, which both exist in a strong sense independently of our sensations of the world of the things.

“ [...] it is correct that a mathematical proposition says nothing about the physical or psychical reality existing in space and time, because it is true already owing to the meaning of the terms occurring in it, irrespectively of the world of real things. What is wrong, however, is that the meaning of the terms (that is the concepts they denote) is asserted to be something man-made and consisting merely in semantical conventions. The truth, I believe, is that these concepts form an objective reality of their own, which we cannot create or change, but only perceive and describe.” (p. 320)

In the second passage the two adjectives “incomplete” and “indistinct” again remind us of Leibniz very closely.

“I wish to repeat that “analytic” here does not mean “true owing to our definitions” but rather “true owing to the nature of the concepts occurring [[therein]]” in contradistinction to the properties and behaviour of things”. This concept of analytic is so far from meaning “void of content” that it is perfectly possible that an analytic proposition might be undecidable (or decidable only with a [certain] probability). For our knowledge of the world of concepts may be as limited or as incomplete as that of the world of things. It is certainly undeniable that this knowledge, in certain cases, not only is incomplete but even indistinct.” (p. 321)

There are also two interesting passages in the Carnap paper, where the Fregean distinction between *Sinn* and *Bedeutung* is evoked as useful in order to clarify this notion of analyticity. Gödel, after reiterating the assertion that “analytic” cannot mean void of content affirms: “The neglect of the conceptual content (i.e. the “sense” according to Frege) as something objective (i.e. non-psychological) is also responsible for the wrong view that the conclusion in logical inference, objectively, contains no information beyond that contained in the premises [...] for the conclusion represents the empirical (or more generally the extra-logical) content of the premises, or part of it, in a conceptually different form and that the conclusion is implied by the premises is itself an objective fact concerning the primitive terms of logic occurring [in] and specific for these terms.”⁷² That means that we have to distinguish the conceptual content of a sentence from the factual one, and that this conceptual content plays a specific role in

⁷² [Gödel 1953] version III page 350 note 40

explaining the nature of mathematical and logical sentences, which are true irrespectively of the world of real things.

As it is explicitly stated in version V of the Carnap paper, this conceptual content is composed in the case of concepts by “primitive ones” (CW III end of p. 360) and in the case of propositions by the very concepts occurring in a sentence: “one may very well say that the proposition mentioned above [it will rain or it will not rain tomorrow] although it says nothing about rain, does express a property of “not” and “or”” ([Gödel CW III] p. 362)

An open problem has to be pointed out here, connected with the interpretation of the oppositions intension/extension, content/meaning mentioned in section 5. Frege’s distinction between *Sinn* and *Bedeutung* concerning predicates, prescribes that the *Bedeutung* of a predicate is the concept itself and its *Sinn* is the way of presenting it. Frege himself never pronounces on what these concepts such as *Bedeutungen* are supposed to be. In particular, although he considers that concepts are “composed” by their “characteristic marks” he never pronounced on the nature of this composition from characteristic marks. He never says explicitly if there are primitive concepts (as *Bedeutungen*) through which all the others can be composed and what they are. Nevertheless the principle of extensionality, in Frege’s system, guarantees an identity criterion. Two concepts (as *Bedeutungen*) are identical if and only if so are their extensions. This last principle cannot fit with Gödel’s expression of concepts, as we saw in section 4, and so we are sent back to the problem of understanding in what precise sense is the Fregean *Sinn-Bedeutung* distinction interpreted by Gödel? Does Gödel accept the idea that we have to distinguish the way we apprehend a concept from the concept itself?

Many passages in Max-Phil refer to Frege’s distinction between *Sinn* and *Bedeutung*. A definitive answer to our question can be only given when the transcriptions have been completed.

We conjectured (cf section 5 above) that Gödel distinguishes the three levels: language, sense (*Sinn*, content, intension₂) and meaning (*Bedeutung* as intension₁ or extensions).

In the present state of our knowledge, we can only say that there is a Leibnizian distinction, which could have inspired Gödel here. Leibniz draws a distinction between concepts and ideas *in mente homini*, which is paralleled by that between thoughts and truths.⁷³ Leibniz considered concepts as depending on acts of the mind, which consist in grasping ideas, (where the latter exist independently from this act and are a sort of disposition which are activated at the occasion of experience). It is not impossible that Gödel thought, at that time, that Frege’s distinction between *Sinn* and *Bedeutung* is an actualisation of the Leibnizian one, considering that the different ways to affirm a *Bedeutung* do not depend on a psychological act, but are deeply connected with their expressions in language. It is also possible that these same concepts (considered as senses of the ideas *in mente homini*) are what, Gödel says (in his paper of 1961) would be studied through phenomenology.

It remains that these concepts “*in mente homini*” are in a relation of “expression” with the ideas “*in mente dei*”, that is with the structures of the possible and of the real, which can be at most recognised by intuition and never constructed by us. Gödel speaks with Wang about *ideas*,⁷⁴ supposed to be different from concepts inasmuch as they are independent from languages. He gives as examples of ideas: *concept*, *absolute truth*, *absolute definability* ([Wang 1996], 6.1.11-16) and says that if the idea of absolute proof can be clarified, then we

⁷³ [Leibniz A] VI 4B, p. 1369, VI, 6, p. 106. See also [Mugnai 2001], chapter 1. As is well known, the difference is stated by Leibniz in order to contrast the Lockian anti-innatism and his “static” conception of knowledge.

⁷⁴ [Wang 1996] p. 268, and especially 8.4.20-22. Wang refers to Kantian ideas, opposed to concept, but 8.4.23 specifies that “Kant’s distinction between ideas and concepts is not clear. But it is helpful in trying to define precise concepts.

can solve the intensional paradoxes, and therefore prove the superiority of minds over computers. For Gödel, as for Frege, before 1959,⁷⁵ senses are probably those double-faced entities, looking both toward language and toward reference (the latter being human concepts in intension₁ or in extension). On the contrary Gödel, after 1959, probably did not consider language as being central for grasping concepts (intended as ideas *in mente homini* expressing ideas *in mente dei* that is structures of reality). He thinks that while the axiomatic method (as opposed to explicit definitions) is the essential way to clearly and precisely define objects and concepts, it is not the only method to grasp them. Gödel probably took it to be necessary to provide a definition of concepts in a way independent from language. The concept of calculability is the most striking example of a definition, which is independent of language. Gödel hoped that in the future the same kind of characterisation would be found in general for other concepts. To arrive at such a characterisation of concepts we should try to grasp simple and primitive concepts and clarify what their knowledge consists of. Husserl's phenomenology appeared to him to be a possible tool for clarifying the content of a concept, its sense, in other words for approaching as nearly as possible the right perspective on it. Following this interpretation, what Gödel seeks in Husserl is a method with which to grasp concepts, that is, a way to clarify senses *independently from language*. Nevertheless, the sense of a concept, the acts we have to perform to grasp it, need not necessarily be identified with the concept itself (neither with the idea *in mente homini*, nor with the idea *in mente Dei*, using Leibniz's way of speaking).

This interpretation is compatible with the letter of the conference of 1961 [Gödel CW III] page 382. Gödel speaks here of *Sinn*:

Nun gibt es ja heute den Beginn einer Wissenschaft, welche behauptet, eine systematische Methode für eine solche Sinnklärung zu haben, und das ist die von Husserl begründete Phänomenologie. Die Sinnklärung besteht hier darin, dass man die betreffenden Begriffschärfer ins Auge fasst, indem man die Aufmerksamkeit in einer bestimmen Weise dirigiert, nämlich auf unsere eigene Akte bei der Verwendungen dieser Begriffe, auf unsere Mächte bei der Vollführung unsere Akte, etc.

It is also compatible with the general Leibnizian inspiration of Gödel's analysis, which is the lasting background to his philosophical research.

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⁷⁵ Cf. the passage in [Gödel 1953] quoted in section 4, and its reference in note 38.

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