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MIGRANT WAGES, REMITTANCES AND RECIPIENT LABOUR SUPPLY IN A MORAL HAZARD MODEL

Claire Naiditch* and Radu Vranceanu†

Abstract
This paper analyzes the interaction between migrants’ income and remittances and between remittances and the labor supply of residents. The model is cast as a two-period game with imperfect information about the residents’ real economic situation. Residents subject to a good economic situation may behave as if they were in a poor economic situation only in order to manipulate remitters’ expectations. The latter, being aware of this risk, reduce the remitted amount accordingly. Therefore, in the equilibrium, residents who really are victims of the bad economic outlook, are penalized as compared to the perfect information set-up. In some circumstances, they can signal their type by drastically cutting working hours, thus further enhancing their precarity right when their economic situation is the worst.

JEL Classification: D82, F22, J22, O15.

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1 Introduction

Often the decision to migrate from a developing to a developed country is guided by economic considerations; in general, migrants are able to get better economic opportunities in the host country than at home. The left home relatives may also benefit from the migrants’ successful integration. Indeed, once they found a job abroad, migrants tend to send a significant part of their income to their families back home. Over the past fifteen years, international migrant remittances have become an increasingly prominent channel of financing the developing world, exceeding official aid and getting close to the FDI flow (Ratha, 2005). According to a recent study by the International Fund for Agricultural Development and the Inter-American Development Bank, in 2006 the flow of remittances towards the developing countries reached 301 billion US dollars.\(^1\)

Such a substantial amount of external funding must have an impact on the macroeconomic equilibrium of developing countries. Several authors studied the impact of remittances on inequalities and poverty in receiving countries (Adams, 2006; Adams and Page, 2005; Lopez-Cordoba, 2006; Adams, 2004). They show that remittances contributed to fighting poverty (measured by the account index) and especially to reducing the “depth of poverty” (measured by the poverty gap index) and the “severity of poverty” (measured by the squared poverty gap).

While this positive effect on poverty reduction should not be underemphasized, remittances may also bring about some unpleasant consequences. In particular, by increasing the recipients’ wealth, remittances can undermine their incentives to work, which, in turn, would slow down economic growth. Several studies have investigated the effect of remittances on labor supply in an empirical set-up without reaching a clear-cut conclusion. For instance, Rodriguez and Tiongson (2001) show that Filipino households with temporary overseas migrants tend to reduce their labor participation and hours worked. Airola (2005) observes a negative elasticity between remittances and labor supply in Mexico. Analyses by Cox-Edwards et al. (2007) and Amuedo-Dorantes and Pozo (2006), also based on Mexican data, observe a negative relationship between remittances and

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\(^1\) The study provides estimate for both official and unofficial flows of remittances. See http://www.ifad.org/events/remittances/index.htm. Official flows only were estimated to 207 billion US dollars by the World Bank (2007).
labor supply only in narrow segments of the population. Drinkwater et al. (2006) build a standard labor market matching model where remittances on the one hand decrease the opportunity cost of unemployment, and on the other hand, support job creation through capital accumulation; their empirical analysis suggests that the latter effect takes over the former.

Some authors have emphasized that the asymmetric information between the remitter and the recipient may provide another transmission channel for the negative effect of remittances on residents’ effort. For instance, Chami et al. (2005) analyze the impact of remittances when the resident, who gets these resources, is able to hide his effort to the remitter. In their model, the migrant is altruistic: his utility depends on the utility of his left home family. They show that remittances bring about two contradictory effects: on the one hand, an increase in remittances will reduce recipients’ work effort because they become less concerned about the risk of getting a small income from work; on the other hand, firms react to additional opportunism by increasing the dispersion of wages in order to stimulate work effort. Since the feed-back effect cannot offset the direct one, remittances have a negative net impact on output. Azam and Gubert (2005) analyze the migration of a family member as part of a diversification strategy that seeks to protect households from income uncertainty specific to agricultural production. Residents are assumed to get remittances only if their income falls below a given threshold. The authors highlight a moral hazard problem: households that can receive remittances tend to decrease their work effort, thus the probability that the output falls below the critical threshold increases.

This paper analyses the relationships between migrants’ income, amount of remittances and recipients’ labor supply in the presence of moral hazard. The analysis focuses on the case of international migrants, but would apply as well to inland migrations, for instance from rural areas to industrial cities, with the remark that the informational gap should be stronger in the case of international migrations.

The model is cast as a two-period game between a migrant who makes a transfer and a resident who benefits from the transfer, given asymmetric information about the real situation

\[^2\text{This formalization is much in line with those used in models of altruistic transfers within families (Barro, 1974; Becker, 1974; La Ferrère and Wolff, 2006; and especially Gatti, 2005).}\]
of the recipient. Both the migrant and the resident maximize their intertemporal utility. The model builds on the classical signaling methodology developed by Spence (1973, 2002).\(^3\) As in the paper by Chami et al. (2005), migrants are altruistic: their utility depends to some extent on the resident’s utility. By contrast, in our model income from work is endogenous: residents and migrants are subject to an elementary leisure/consumption trade-off that determines their hour supply.\(^4\) The optimal working time depends on their wages and other autonomous gains, including remittances. The migrant’s wage is common knowledge: he is supposed to be paid the same wage as other (migrant) workers in the same sector, which is public information in developed countries. On the other hand, the resident’s wage is private information. The migrant observes the resident’s working hours during the first period; he can use this information to upgrade his expectations about the recipient’s wage. This sequence of decisions opens the door for manipulating information: the resident subject to a good economic situation can behave as if he were in a bad situation only in order to make the donor believe that he is doing badly, and get more remittances. In an equilibrium with manipulation, when the resident works only a small amount of hours, the migrant cannot tell without ambiguity whether he made this choice because he gets a small wage or because he is trying to manipulate him. Given this uncertainty, the migrant will choose a smaller amount of remittances as compared to the perfect information set-up. As an upshot of all these, imperfect information imposes a real cost on recipients who really are victims of a poor economic outlook. To avoid this outcome, they can choose to signal their type by strongly reducing their working hours during the first period. Consequently, their income precarity edges up right when their economic situation is the worst.

One interesting feature of this model is its ability to describe the complex relationship between the level of remittances and the altruistic migrant’s wage in case of asymmetric information: on the one hand, a raise in the migrant’s wage implies an increase in the amount of remittances along a traditional wealth effect, and, on the other hand, the more acute risk of opportunistic

\(^3\) The logic of strategic signaling draws on Vickers (1986). See also Besancenot and Vranceanu (2005) for a related game.

\(^4\) Migrants can remit to their families and communities still in their origin country for several other reasons. Rapoport and Docquier (2006) list a series of motivations explaining the existence of remittances: altruism, exchange (purchase of various types of services, repayments of loans...), strategic motive (positive selection among migrants), insurance (risks diversification) and investment.
behavior calls for a reduction in the amount of remittances, what we refer to as the moral hazard effect. So far this link between remittances and the migrant’s wage (not the resident’s) has not been emphasized by existing theoretical analyses. Empirical evidence is rather scarce; it draws on the analysis of migrations from rural to urban areas of the same country. For instance, using data from a survey of African households in Nairobi, Kenya conducted in 1971, Johnson and Whitelaw (1974) show that the elasticity of remittances with respect to income is positive and ranges from 0.6 to 0.9, but that the fraction of the wage bill remitted decreases with the urban wage level. Mohammad et al. (1973) find the same result using data from a survey in nine urban centres in Punjab and Sind, Pakistan, conducted in 1971-72. However, Rempel and Lobdell (1978), using data from a survey conducted in 1968 in eight large urban centres in Kenya, find that the proportion of income remitted varies directly with the amount of income earned. Lucas and Stark (1985) and Hoddinott (1994) find that remittances are an increasing function of migrants’ earnings, using data respectively from Botswana (1978-79) and Kenya (1988). More recent papers studied the link between remittances and the education level of migrants, which can be seen as a proxy of their income. Rodriguez and Horton (1994) show that in the Philippines, the education level of migrants does not have a significant impact on the level of remittances they send. In an econometric study on a large panel of developing countries, Faini (2006) show that a higher skilled content of migration is found to be associated with a lower flow of remittances. Such contrasting facts are consistent with our results: depending on their relative strengths, the wealth effect can take over the moral hazard effect or vice-versa.

In this paper, the interaction between the migrant and the resident is assumed to last only two periods. This is the most stylized framework where we can analyze Bayesian learning and strategic signaling. Such a framework is best suited for analyzing temporary separations, characterized by relatively short migration spells such as those who characterize nowadays East - West European migration. When permanent migration is considered, a repeated game would provide a better analytical framework. Since the transfer game is finite, the results of the repeated game would not be fundamentally different from the two-period game; it would however allow to put forward
the critical moment when the manipulating resident should reveal his type.\footnote{The outcome would be different in an infinite horizon game with punishment for "manipulation". In this alternative set-up the incentive to cooperate is very strong.}

In order to focus on the economic interpretation of the model, in this paper we will only consider equilibria featuring interior solutions to the various optimization problems. Hence, in order to rule out all corner solutions, we will impose from the outset several acceptable restrictions on parameters values. For instance, we want to make sure that parameters are such that the optimal transfer is strictly positive. For a broader range of parameters, the optimal transfer to "rich residents" might well be zero, while it would be positive for "poor" residents. The existence of the zero lower bound on the optimal transfer would reduce the scope for manipulation, without fully eliminating it.

The paper is organized as follows. The next section introduces the basic assumptions and the rule of the game. Section 3 analyses the equilibrium when an explicit signaling strategy cannot be implemented. Section 4 studies the equilibrium when the resident subject to a poor economic situation is able to signal his type by drastically cutting his working hours. The final section presents the conclusion.

\section{Main assumptions}

The problem is cast as a, imperfect information game between the migrant (or remitter) and the resident (or recipient). The two agents live over two periods:\footnote{The migrant and the resident can for example be a couple, with one emigrating and the other left at home.} the first period starts at time $t = 1$ and ends at time $t = 2$, the second starts at time $t = 2$ and ends at time $t = 3$. Thereafter, the two periods will be denoted by index $t$, which represents the beginning of each period ($t \in \{1, 2\}$). To keep formalization as simple as possible, we assume that during each time period, the two agents consume all of their available resources (i.e. they do not save). Both the migrant and the resident have a job: their income from work depends on their wages and working hours. In addition, the migrant is altruistic: at the beginning of the second period, he commits to remitting part of his income to the resident, depending on his own income and on his perceived economic situation of the resident. We also assume that during the first period, the resident receives an exogenously
given public aid $A$; in developing countries, such transfers are in general low, but cannot be zero, if only foreign official aid is taken into account. Many governments also give transfers with a poverty alleviation purpose, often financed by revenues from exports of natural resources. We assume that $A$ is such that the resident’s optimal hour supply during the first period is strictly positive.

Let $s$ denote the migrant’s wage and $w^i$ denote the resident’s wage. Total working time will be normalized to unity, hence $s$ and $w^i$ should be interpreted as the one-period wage income to be obtained by a worker who would work the maximum working time. While the migrant’s wage is public information, the resident’s wage is private information. Residents’ income in a developing country depends on many factors, such as terms of trade, technology changes, weather, armed conflicts, diseases, and so on. Some of these factors can be observed by migrants living thousands miles away, some cannot. In this paper, the emphasis is placed on the latter set of factors.

In order to introduce the assumption of imperfect information in the simplest way, we assume that there are only two possible states of nature: the resident’s economic situation can be either good or bad, where he gets accordingly either a (H)igh wage, $w^i = w^H$, or a (L)ow wage, $w^i = w^L$, with $w^L < w^H$. Let $\Pr[w^H]$ denote the objective probability that the good state of the world occurs, and $\Pr[w^L] = 1 - \Pr[w^H]$ the probability that the bad state of the world occurs. In order to keep the problem simple, we assume thereafter that $\Pr[w^H] = 0.5$.

Figure 1 presents the sequence of decisions of the players along a time line.
resident is good (he is then paid the high wage \( w^H \)) or bad (he is then paid the low wage \( w^L \)). The migrant does not know the recipient’s real economic situation, but knows the objective probability of occurrence of the good (or the bad) economic situation.\(^7\) The resident receives a public aid \( A \) and chooses his first-period working hours \( h_1 \). The migrant also chooses his first-period working hours \( \tau_1 \).

At the beginning of the second period \((t = 2)\), the migrant, knowing \( h_1 \), upgrades his beliefs over the resident’s wage, decides on the level of remittances \( T \) he will send to the resident to replace the public subsidy,\(^8\) and chooses his second-period working hours \( \tau_2 \). The resident then receives the remittances and reveals his real economic situation by choosing his working hours, \( h_2 \). We assume that the parameters of the model are such that the remitted amount is strictly positive.

Since at the end of the game the resident has revealed his true type, in the context of a strong interpersonal relationship typical of family ties, it may be reasonable to assume that shirkers must bear an additional psychological cost, related to the liar’s stigma, to (foregone) sympathy or merely to remorse. Converted into money units, such a psychological cost can be represented as an end-of-game penalty.\(^9\) Hence, at the end of the game \((t = 3)\), shirkers are subject to a penalty \( \theta \) (\( \theta \geq 0 \)). Of course, for a very large penalty, the problem would become trivial because in this case manipulation is a dominated strategy. The problem under scrutiny is interesting only if \( \theta \) is small or even zero.

Let us now introduce the players’ payoffs. At each period \( t \in \{1, 2\} \), the resident’s and the migrant’s single period preferences over consumption and leisure are respectively summarized by

\(^7\) Of course, in real life, migrants have some information about the residents’ economic situation, and would adjust their priors accordingly from the outset of the game. Yet this would not change the basic structure of the game, if they still can learn something about the resident’s situation from observing his hours supply during the first period.

\(^8\) In an alternative formulation, remittances could be added to the public subsidy. The structure of the problem would not change, but the formulas would be unnecessarily complicated.

\(^9\) Such a cost has the same effect on the scope for manipulation as a form of limited altruism on behalf of the resident with respect to the migrant.
the following utility functions:

\[
\begin{align*}
U_t &= U(c_t, h_t) = c_t(1 - h_t), \\
\text{and} \\
V_t &= V(x_t, \tau_t) = x_t(1 - \tau_t),
\end{align*}
\]

where \(c_t\) and \(x_t\) denote respectively the resident’s and the migrant’s consumption; the maximum duration of work is standardized to the unit and \(h_t\) and \(\tau_t\) denote respectively the resident’s and the migrant’s working hours.\(^{10}\)

The migrant’s single period utility depends to some extent on the resident’s utility since he is altruistic (he does not take into account the psychological cost incurred by a manipulating resident). Thus, we can define the migrant’s single period utility \(W_t\) by:

\[
W_t = W(x_t, \tau_t, c_t, h_t) = \left[ V(x_t, \tau_t) \right]^{(1-\beta)} \left[ U(c_t, h_t) \right]^{\beta}
\]

where \(\beta\) denotes the degree of altruism, with \(\beta \in [0, 1]\). When \(\beta = 0\), the migrant is selfish: the resident’s welfare does not matter to him. For \(\beta > 0\), the migrant can be said to be altruistic.

The resident’s and the migrant’s intertemporal utility can respectively be written simply using an additive form:

\[
\begin{align*}
Z &= U(c_1, h_1) + U(c_2, h_2) - 1_{[\text{shirk}]} t, \\
\Sigma &= W_1 + W_2 = [V(x_1, \tau_1)]^{(1-\beta)} [U(c_1, h_1)]^{\beta} + [V(x_2, \tau_2)]^{(1-\beta)} [U(c_2, h_2)]^{\beta}.
\end{align*}
\]

Here, the factor \(1_{[\text{shirk}]}\) takes the value one in the case of a manipulating resident (who earns \(w^H\) and supplies less than his optimal working hours in case of perfect information), and zero in all other cases.

Finally, at each period \(t \in \{1, 2\}\), the resident’s and the migrant’s budget constraints are respectively:

\[
\begin{align*}
c_t &= w^i h_t + R_t, \quad \text{with } i \in \{L, H\}, \\
\text{and} \\
x_t &= s \tau_t + B_t,
\end{align*}
\]

\(^{10}\) The Cobb-Douglas function conveys in a simple way the neoclassical assumptions about the convexity of leisure/consumption preferences.
where \( R_t \) and \( B_t \) denote respectively the resident’s and the migrant’s non-earned income at period \( t \). The resident’s non-earned income during the first period is the exogenous aid, \( R_1 = A \), and during the second period, it is the amount remitted by the migrant, \( R_2 = T \). During the first period, the migrant receives no exogenous income \( (B_1 = 0) \), and during the second period, he transfers resources to the resident \( (B_2 = -T) \).

We wan now deﬁne the players’ strategies knowing that the resident seeks to maximize \( Z \), the migrant seeks to maximize \( \Sigma \).

The resident’s strategy \((S^r)\) can be represented by his choice of working hours at each period, given his wage (which is private information): \( S^r(i) = \{(h_1, h_2)|w^i, \text{with } i \in \{H, L\}\} \).

The migrant decides how much he is going to remit \((T)\) and how long he is going to work during the two periods \((\tau_1 \text{ and } \tau_2)\), given his income and his expectations about the resident’s wage. At the beginning of the game, the migrant’s beliefs are given. The migrant’s hours supply during the first period \((\tau_1)\) is independent from the resident’s behavior. At the beginning of the second period, the migrant chooses his remittances \((T)\) and working hours for the second period \((\tau_2)\) after having observed the resident’s working hours (during the first period). Any forecasting error implies an (ex-post) utility loss for the migrant. Thus, his strategy \((S^m)\) is made up of his rational guess about the resident’s wage; at \( t = 1 \), his expectations build on his prior beliefs, and at \( t = 2 \), his expectations take into account the resident’s working hours during the first period, \( h_1 \). In a compact form, we can write: \( S^m = (E[w^1|I_1], E[w^2|I_2]) \) where \( E[-] \) is the expectation operator and \( I_t \) is the information set at time \( t \), with \( t \in \{1, 2\} \). In our simple problem, \( E[w^t|I_1] \) is predetermined.

A Bayesian equilibrium of the game is a situation in which the resident’s strategy \( S^r \) maximizes his utility given the migrant’s beliefs, and the migrant’s beliefs \( S^m \) are correct given the optimal strategy of the resident.

The decision tree presented in Figure 2 summarizes the former elements. At the outset of the game, Nature picks the state of the world. We will study two classes of equilibria, depending on whether poor residents resort to strategic signaling by undercutting their first-period working hours to \( h_1^* \), below the lowest working time that would prevail with perfect information \((h_L^1)\). If
all poor residents decide to signal themselves ($\mu = 0$), the imperfect information branches of the game disappear and the game only has the upper and lower branches. If poor residents cannot resort to strategic signaling, the upper branch vanishes ($\mu = 1$) and the game features the three lower branches. The dotted line that connects the migrant’s dots is then representative of the imperfect information set-up: if the resident supplies $h^L_1$ hours, the migrant cannot determine his type exactly.

Figure 2: Decision Tree.

In the next section we analyze the equilibrium of the game when the resident subject to a bad economic situation cannot signal his type, i.e. cannot resort to strategic signaling. The case where the migrant can resort to strategic signaling will be analyzed in Section 4. It will then be shown that in some cases, even if the resident can signal his type by drastically reducing his working hours, he will not choose to do so because this strategy is dominated. In this case, the equilibrium
developed in the previous section will prevail.

3 Equilibrium without strategic signaling ($\mu = 1$)

3.1 The resident’s choice of working hours during the last period

This sequential game is solved by backward induction. At the beginning of the second period ($t = 2$), the resident has already received the remittances $T$. Thus, he can decide his optimal working hours $h^*_2$, given his wage $w^i$, without any strategic consideration.

To determine his optimal working hours during the second period, the resident maximizes his second-period utility given his budget constraint $c_2 = w^i h^*_2 + T$:

$$\max_{h_2} \{U(c_2(h_2), h_2) = (w^i h^*_2 + T)(1 - h_2)\}.$$  \hspace{1cm} (5)

The first order condition is: $dU(\cdot)/dh_2 = 0$. Thus, his optimal working hours are:

$$h^*_2 = 0.5 \left(1 - T/w^i\right), \quad \text{with } i \in \{H, L\}.$$ \hspace{1cm} (6)

The resident’s second-period labor supply increases with his wage and decreases with the amount remitted.

Finally, replacing the expression of his labor supply in the utility function, we can write the resident’s indirect second-period utility as a function of his wage and remittances: $U^*_2 = u_2(T, w^i) = \max\{U(c_2(h_2), h_2)\}$, with the explicit form:

$$u_2(T, w^i) = \frac{0.25}{w^i} \left(T + w^i\right)^2 \quad \text{with } i \in \{H, L\}.$$ \hspace{1cm} (7)

3.2 The migrant’s choice of remitted amount and working hours during the last period

At the beginning of the second period ($t = 2$), the first-period migrant’s utility ($W_1$) has already been realized. Therefore his decision problem of maximizing $\Sigma = W_1 + W_2$ is truncated: his choices will have an impact only on his second-period utility. Hence, he is concerned only about maximizing $W_2$. Given that he does not know the resident’s wage, he decides on the amount of remittances according to his wage estimate, which depends on the information available at the beginning of the second period ($I_2$). The expected wage is denoted by $E[w^i|I_2]$. 
The migrant must take into account the fact that once the resident gets his remittances, he is going to decide his second-period working hours such as to maximize his utility. Hence the migrant’s optimal choice takes the form of a standard Stackelberg decision problem (where the migrant is the "leader" and the resident is the "follower"). Let $\hat{U}_2$ denote the migrant’s estimate of the resident’s utility maximum, given his expectations about the resident’s wage (Eq. 7). The migrant’s decision problem can be stated as:

$$\max_{T, \tau_2} \left\{ W_2 = [V(x_2, \tau_2)]^{(1-\beta)} \left( \hat{U}_2 \right)^{\beta} \right\}$$

with (1): $x_2 = s\tau_2 - T$  
and (2): $\hat{U}_2 = u_2(T, E[w^i|I_2]) = \frac{0.25}{E[w^i|I_2]} (T + E[w^i|I_2])^2$

where the constraint (1) is the second-period budget constraint and (2) is the indirect utility of the resident as estimated by the migrant.

To solve the problem, we introduce the constraints into the objective and write the first order conditions, $dW_2/d\tau_2 = 0$ and $dW_2/dT = 0$. After some elementary calculations, we obtain the optimal amount remitted:

$$T^*(s, E[w^i|I_2]) = \beta s - (1 - \beta) E[w^i|I_2].$$

It turns out that remittances decrease with the resident’s wage (as anticipated by the migrant) and increase with the migrant’s wage and degree of altruism.

As previously mentioned, in this paper we will focus only on interior solutions to various optimization problems. Thus, we assume that remittances are strictly positive. This requires some additional constraints on the parameters values. Since $T^*$ decreases with $E[w^i|I_2]$, the most general condition for interior solutions is:

$$T^*(s, w^H) = \beta s - (1 - \beta) w^H > 0 \Leftrightarrow s > \hat{s} \equiv \frac{1 - \beta}{\beta} w^H \text{ or } \beta > \hat{\beta} \equiv \frac{w^H}{s + w^H}. \quad (10)$$

Within our framework, for a given level of altruism, the existence of remittances implies a minimum wage for the migrant ($s > \hat{s}$). Or, for a given migrant’s wage, the existence of remittances implies a minimum degree of altruism ($\beta > \hat{\beta}$).
We can also determine the migrant’s labor supply during the second period:

\[ \tau_s^* = 0.5 \left[ (1 + \beta) - (1 - \beta) \frac{E[w^s|I_2]}{s} \right], \tag{11} \]

which increases with the migrant’s wage \( s \), and decreases with the expected value of the resident’s wage \( E[w^s|I_2] \).

### 3.3 The migrant’s expectations over the resident’s wage

Notice that when there is perfect information about his wage, the resident cannot aim at manipulating expectations, and must choose his first-period working hours with the only objective of maximizing his first-period utility \( (U_1 = U(c_1, h_1)) \), given his first-period budget constraint \( c_1 = A + w^s h_1 \).

In the perfect information case, the resolution of the resident’s optimization program implies that a resident who earns \( w^H \) (respectively \( w^L \)) supplies in the first period \( h^H_1 \) hours (respectively \( h^L_1 \)), with \( h^H_1 \equiv 0.5 \left( 1 - A/w^H \right) \) and \( h^L_1 \equiv 0.5 \left( 1 - A/w^L \right) \), and \( h^H_1 > h^L_1 \).

As mentioned earlier, we assume that \( A \) is such that the resident’s optimal hour supply during the first period is strictly positive. Thus, we impose a first restriction on parameters \( A \) and \( w^L \), that is \( A/w^L < 1 \). This is a reasonable assumption, the public aid is probably not very important in developing countries; here we assume that it is lower than the total income of a poor worker over the period (recall that maximum working time has been normalized to one).

We now turn back to our framework with imperfect information. At the beginning of the game, information available to the migrant about the resident’s economic situation is summarized by his prior beliefs: \( \Pr[w^H] = \Pr[w^L] = 0.5 \). In this first part of the analysis, we assume that the poor resident cannot undercut his first-period working hours below \( h^L \) in order to signal his type. Remember that the remitted amount decreases with the resident’s expected wage. Thus, when the resident’s economic situation is truly bad, he has no incentive to behave as if his situation were good (by choosing \( h^1 = h^H \)), because, not only would he incur a first-period utility loss, but he would also receive a smaller amount of remittances. On the other hand, if he gets the high wage, in case of asymmetric information, the resident may decide to work less as if his wage were low, in order to make the migrant believe that he is in a bad economic situation. In that case, the
migrant would remit a higher amount and the resident’s second-period utility might be higher.

Let us introduce now an important endogenous variable, the probability denoted by \( q \), that a resident earning \( w^H \) decides to implement the manipulating strategy (supplies \( h^L_1 \)). If \( q = 1 \) or \( q = 0 \), the rich resident plays pure strategies; if \( q \in [0, 1] \), the rich resident uses this probability for randomizing between the manipulating and the fair strategies. In equilibrium, the frequency of rich residents who have implemented the manipulating strategy will be equal to \( q \) as well.

Thus, under imperfect information and without strategic signaling, residents earning \( w^L \) will supply \( h^L_1 \) hours of work in the first period; among residents earning \( w^H \), a proportion \( (1 - q) \) will reveal their type by working \( h^H_1 \), and the remaining \( q \) will choose to mimic the \( w^L \) type and work \( h^L_1 \).

The migrant’s equilibrium beliefs can then be written as the contingent probability that a resident earning \( w^i \) (with \( i \in \{H, L\} \)) supplies \( h^L_1 \):

\[
\Theta = \begin{cases} 
\Pr[h^L_1 | w^L] = 1 \\
\Pr[h^L_1 | w^H] = q, \text{ with } q \in [0, 1]
\end{cases}
\] (12)

where \( \Pr[h^L_1 | w^L] = 1 - \Pr[h^L_1 | w^H] \) and \( \Pr[h^H_1 | w^H] = 1 - \Pr[h^L_1 | w^H] \).

The migrant is able to revise his ex ante probabilities \( \Pr[w^H] \) and \( \Pr[w^L] \) after observing the resident’s working hours at the first period. Given that the poor resident will never play \( h^H_1 \), if the migrant observes this strategy he can infer that the resident is rich. If the strategy played by the resident is \( h^L_1 \), the migrant must make a rational guess on whether the resident is rich or poor.

The expected wage will be an weighted average between \( w^L \) and \( w^H \), with weights \( \Pr[w^L|h^L_1] \) and \( \Pr[w^H|h^L_1] \). We can use Bayes rule to determine these probabilities, given \( q \) and \( \Pr[w^H] \). It turns out that:

**Proposition 1** In the imperfect information case, the migrant’s expectations over the resident’s wage are:

- **a)** if the resident works \( h^H_1 \), then:

  \[
  E[w^i | h^H_1] = w^H
  \] (13)

- **b)** if the resident works \( h^L_1 \) then:

  \[
  E[w^i | h^L_1] = \frac{q}{1 + q} w^H + \frac{1}{1 + q} w^L
  \] (14)
Proof. The formal proof can be found in Appendix A.1.

It can be easily checked that \( E[w^i | h^L_1] \) is an increasing function in the manipulating probability \( q \), and varies between \( w^L \) and \( 0.5(w^L + w^H) < w^H \). Thus, the optimal amount of remittances (Eq. 9) is bigger if the resident chooses \( h_1 = h^L_1 \) than if he chooses \( h_1 = h^H_1 \).

### 3.4 The resident’s choice of working hours during the first period

Given former developments, it turns out that when the resident is in a poor economic situation \((w^i = w^L)\), he will always choose to work the small amount of time \((h_1 = h^L_1)\): he does not want the migrant to believe that he is well paid because he would then get less remittances. On the other hand, if the resident is in a good economic situation \((w^i = w^H)\), he will manipulate migrant’s anticipations by choosing to work \( h^L_1 \) with probability \( q \), and will be honest by choosing to work \( h^H_1 \) with probability \( 1 - q \). Extreme cases \( q = 0 \) or \( q = 1 \) correspond to pure strategies.

In the following, we focus on the mixed strategy case \((q \in]0, 1[)\), which encompasses the two pure strategies as special situations.

The Nash mixed strategy \( q \in]0, 1[ \) is implemented if a "rich" resident \((w^i = w^H)\) is indifferent between playing \( h^H_1 \) or \( h^L_1 \):

\[
Z(h^H_1, w^H) = Z(h^L_1, w^H)
\] (15)

\[
\iff u_1(h^H_1, w^H) + u_2(T^*(w^H), w^H) = u_1(h^L_1, w^H) + u_2(T^*(E[w^i | h^L_1]), w^H) - \theta.
\] (16)

After making the necessary substitutions and some calculations presented in Appendix A.2., this equilibrium condition can be written as \( s = g(q, \beta, \theta, A, w^H, w^L) \), where \( g() \) is a function in \( q \) and parameters. Hence, \( q \) is implicitly defined as a function of the various parameters, including the migrant’s wage \( s \). For our analysis, the most important property is:

**Proposition 2** In the hybrid equilibrium, the manipulation probability \( q \) is an increasing function in the migrant’s wage \( s \).

**Proof.** The proof is provided in the Appendix A.2. ■
working \( h^H \), the others aim at manipulating information and work \( h^L \). In a first step, if \( s \) increases, both the amount of remittances sent to the honest and to the manipulating resident goes up. Yet, it can be shown that, ceteris paribus, an infinitesimal increase in \( s \) brings about a larger increase in the utility of the shirkers than of the honest residents. Hence, when \( s \) increases, more residents are tempted to manipulate information. But if \( q \) goes up, the amount of remittances provided to those who choose the strategy \( h^L \) declines to some extent, and the indifference condition is recovered.

By setting \( q = 0 \) and respectively \( q = 1 \), we get the inferior and superior wage thresholds that separate the three types of equilibria:

\[
q = 0 \Rightarrow s_0 \equiv \frac{1}{2\beta} \left( \frac{(w^H - w^L)}{(1 - \beta)} \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] + (1 - \beta)w^L - (1 + \beta)w^H \right) \tag{17}
\]

\[
q = 1 \Rightarrow s_1 \equiv \frac{1}{4\beta} \left( \frac{4(w^H - w^L)}{(1 - \beta)} \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] + (1 - \beta)w^L - (1 + 3\beta)w^H \right) \tag{18}
\]

with \( s_1 > s_0 \).

As previously shown, the assumption that the remitted amount is strictly positive implies that, for a given level of altruism, the migrant’s wage rate is above a certain threshold (Eq. 10). Thus, in order to have the full range of equilibria, including the separating one, we impose the condition \( s_0 > \frac{1-\beta}{\beta} w^H > 0 \).\(^{11}\)

This implies:

\[
s_0 > 0 \iff \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} > (1 - \beta^2) + 2\beta(1 - \beta) \frac{w^L}{(w^H - w^L)}. \tag{19}
\]

We assume that this condition holds for \( \theta = 0 \), so it will hold for any \( \theta > 0 \).\(^{12}\)

When the migrant’s wage is low (lower than \( s_0 \)), then remittances are low too. Thus, the cost of working only a small number of hours for a rich resident is higher that the benefit linked to higher remittances. No resident in a good economic situation will find it beneficial to manipulate information.

---

\(^{11}\) If \( s_0 < \frac{1-\beta}{\beta} w^H \), the separating equilibrium cannot occur. If \( s_1 < \frac{1-\beta}{\beta} w^H \), the only possible equilibrium is the pooling one.

\(^{12}\) This condition, together with the former restriction \( (A/w^L) < 1 \), implies that a non empty set for \( A/w^L \) consistent with our framework exists if \( \beta > \beta_1 \equiv \frac{2k}{1+2k} \), with \( k \equiv \frac{w^L}{w^H - w^L} \). Note that the wider the gap between the two possible wages of the resident, the smaller \( k \) is and the easier it is for this condition to be fulfilled.
However, when the migrant’s wage is high (higher than \(s_1\)), then remittances are high too. Thus, the cost of working only a small number of hours for a rich resident is lower than the benefit linked to higher remittances. All residents in a good economic situation find it beneficial to manipulate information.

Finally, when the migrant’s wage is neither too low nor too high, some residents (\(q\)) in a good economic situation find it beneficial to manipulate information and some (1 – \(q\)) do not.

**Proposition 3** Depending on \(s\), one of the three types of equilibria can occur:

- When \(s < s_0\), the equilibrium is separating (\(q = 0\)): each type of resident implements a specific action, either \(h_L^1\) or \(h_H^1\), and this action signals his type without ambiguity.
- When \(s \in [s_0, s_1]\), the equilibrium is hybrid (\(q \in ]0, 1[\)): while the action \(h_H^1\) signals the migrant’s type, the action \(h_L^1\) does not; the rich resident plays a mixed Nash strategy.
- When \(s > s_1\), the equilibrium is of the pooling type (\(q = 1\)): all residents, whatever their wage, choose the same action \(h_L^1\), migrants can infer no information from observing resident’s first-period working time \(h_1^\text{L}\).

**Proof.** Residents in a good economic situation all choose to work \(h_H^1\) if they do not find it beneficial to pretend they are in a bad economic situation. Formally, \(Z(h_H^1, w^{H}) > Z(h_L^1, w^{H}) \iff s < s_0\).

Thus, if \(s < s_0\), then the equilibrium is separating. Residents in a good economic situation all choose to work \(h_L^1\) if they do find it beneficial to pretend they are in a bad economic situation.

Formally: \(Z(h_H^1, w^{H}) < Z(h_L^1, w^{H}) \iff s > s_1\). Thus, if \(s > s_1\), then the equilibrium is of the pooling type. Residents in a good economic situation are indifferent between working \(h_L^1\) and \(h_H^1\) if \(Z(h_H^1, w^{H}) = Z(h_L^1, w^{H}) \iff s_0 < s < s_1\). Thus, if \(s \in [s_0, s_1]\), then the equilibrium is hybrid.

Figure 3 displays the regionning of equilibria and the evolution of \(q\) with respect to \(s\).

### 3.5 The equilibrium relationship between remittances and the migrant’s wage

According to Eq. (9), optimal remittances depend on the migrant’s wage and on his evaluation of the resident’s wage. But the probabilities that enable him to determine the resident’s expected wage depend on his own wage, since the latter has a bearing on the resident’s behavior. More precisely, a raise in the migrant’s wage \(s\) generates two opposite effects: on the one hand, there is a wealth effect such that the migrant, richer, wishes to increase his remittances; on the other hand,
the rise in the amount remitted causes an increase in the probability of manipulation and thus in the resident’s wage as expected by the migrant, who is then prompted to reduce his remittances. In this model, the positive wealth effect overrides the negative moral hazard effect. Indeed, it can be shown that:

**Proposition 4** Remittances are an increasing function of the migrant’s wage.

**Proof.** Appendix A.3 shows that \( \frac{dT^e}{ds} = \beta - (1 - \beta) \frac{dE[w^1|h_1^L]}{dq} \frac{dq}{ds} > 0 \).

Finally, note that the resident’s working hours during the second period are a decreasing function of remittances (Eq. 9). Thus, the effect of a raise in the migrant’s wage on the resident’s hours supply is negative.

### 3.6 The migrant’s choice of working hours during the first period

In order to conclude the analysis of individual strategies, we can analyze the migrant’s choice of working hours during the first period. His decision problem is:

\[
\max_{\tau_1} \left\{ \Sigma = [V(x_1, \tau_1)]^{(1-\beta)} \left[ \dot{U}_1 \right]^\beta + [V(x_2, \tau_2)]^{(1-\beta)} \left[ \dot{U}_2 \right]^\beta \right\}
\]

with \( \forall t, \quad x_t = s \tau_t + B_t \), and \( B_1 = 0, B_2 = -T \)

\[
\text{with } \dot{U}_1 = u_1(A, E[w^1|I_1]) = \frac{0.25}{E[w^1|I_1]} (A + E[w^1|I_1])^2
\]

\[
\text{with } \dot{U}_2 = u_2(T, E[w^1|I_2]) = \frac{0.25}{E[w^1|I_2]} (T + E[w^1|I_2])^2
\]

and \( E[w^1|I_1] = 0.5(w^H + w^L) \).
Given the assumption that the migrant gets no exogenous income \((B_1 = 0)\), the migrant’s working hours at time \(t = 1\) do not depend on his expectations about the resident’s wage as determined at the beginning of the game, \(E[w^i|I_1]\).\(^{13}\) It is easy to check that the optimal solution is \(\tau_1^* = 0.5\).

### 3.7 A welfare comparison

This subsection aims at providing a comparison in terms of welfare between the perfect and imperfect information case. We focus on the case of the poor resident in the hybrid equilibrium. Indeed, if he incurs a welfare loss, he may try to implement a strategy of signalization (to be analyzed in the next section).

In the case of perfect information, the resident subject to the good economic situation cannot manipulate information because the migrant knows his wage. Therefore, like in the separating equilibrium, each type of resident has a specific first-period strategy: if \(w^i = w^L\) then \(h_1 = h_1^L\) and if \(w^i = w^H\) then \(h_1 = h_1^H\). In this case, the poor resident’s utility would be:

\[
Z^P(h_1^L, w^L) = u_1(h_1^L, w^L) + u_2(T^*(w^L), w^L)
\]

where superscript \(P\) stands here for perfect information.

In the case of imperfect information, we have shown that some rich residents may implement the manipulation strategy. In the hybrid equilibrium (and the pooling one as well), the poor resident’s utility is:

\[
Z^I(h_1^L, w^L) = u_1(h_1^L, w^L) + u_2(T^*(E[w^i|h_1^L]), w^L)
\]

where superscript \(I\) indicates imperfect information. The utility loss (in absolute value) of the poor resident due to the imperfection of information can be written:

\[
Z^P(h_1^L, w^L) - Z^I(h_1^L, w^L) = u_2(T^*(w^L), w^L) - u_2(T^*(E[w^i|h_1^L]), w^L).
\]

Since \(T^I = T^*(E[w^i|h_1^L]) < T^P = T^*(w^L)\), it is easy to see that in the hybrid equilibrium, the poor resident undergoes a utility loss:

\[
u_2(T^*(w^L), w^L) > u_2(T^*(E[w^i|h_1^L]), w^L) \iff Z^P(h_1^L, w^L) > Z^I(h_1^L, w^L).
\]

\(^{13}\) Since in this problem \(E[w^i|I_1]\) is a constant, this simplification does not modify the basic structure of the game.
Appendix A.4 presents the exact expression of \( Z_P(h_L^1, w^L) - Z_I(h_L^1, w^L) \) with respect to \( q \) and parameters.

Likewise, we can write the utility gain of the rich resident due to the imperfection of information. The intertemporal utility of a rich resident is the same in case of perfect information and in the hybrid equilibrium in case of imperfect information. Indeed, in the hybrid equilibrium, we know that: \( Z^I(h_L^1, w^H) = Z^I(h_H^1, w^H) \). Yet, \( Z^I(h_H^1, w^H) = Z^P(h_H^1, w^H) \). Note that in the pooling equilibrium, the intertemporal utility of a rich resident is higher than in the perfect information set-up since rich residents get a higher amount of remittances than in the case of perfect information \( (Z^I(h_L^1, w^H) > Z^I(h_H^1, w^H) = Z^P(h_H^1, w^H)) \).

Finally, in the hybrid equilibrium (and in the pooling one as well), imperfect information entails an ex-post welfare loss for the migrant, because he makes his decisions based on an inaccurate expected value of the resident’s wage; he would remit too much to a "rich" resident, and too little to a "poor" one.

Thus, the total welfare in the hybrid equilibrium is lower than in the case of perfect information (the poor resident and the migrant undergo a welfare loss, the rich resident is indifferent).

4 Equilibrium with strategic signaling (\( \mu = 0 \))

We have shown that when rich residents have an incentive to manipulate (for \( s > s_0 \)), if poor residents cannot reduce their first-period working hours \( h_1 \) below the perfect information lowest working time \( (h_L^1) \) in order to signal their type, they incur a welfare loss as compared to a situation with perfect information. In this section we relax the constraint on working hours, and allow poor residents to adjust working hours strategically. Indeed, according to the traditional argument (Vickers, 1986; Spence, 2002), the poor resident may try to signal his real situation (unfavorable) by undercutting working hours and accepting a degradation of his utility during the first period, provided that the reduction will not be implemented by a possible manipulator. Let us denote by \( \bar{h}_1 \) the working hours that allow signalization, with \( \bar{h}_1 < h_L^1 \).

In this paper we will study only the case where the poor resident strictly prefers to signal himself to not signalling; in equilibrium, all poor residents undercut working hours (\( \mu = 0 \)). If this
form of strategic signaling is effective, then the prevailing equilibrium is of the separating type.\footnote{Notice that this is not the only equilibrium with strategic signaling. The game also presents an equilibrium where a poor resident is indifferent between signaling or not by undercutting his working hours; in this hybrid equilibrium, a fraction $\mu \in [0, 1]$ of poor residents signal themselves.} 

The equilibrium with strategic signaling is defined here as a situation where residents earning $w^L$ all supply $\bar{h}_1$ hours (with $\bar{h}_1 < h^H_1$), and residents earning $w^H$ all supply $h^H_1$.

In this context, migrant’s equilibrium beliefs can be written:

$$\Theta = \begin{cases} 
\text{Pr}[\bar{h}_1|w^L] = 1 \\
\text{Pr}[h^H_1|w^H] = 1
\end{cases}$$  \hspace{1cm} (25)$$

The strategy of voluntary contraction of working hours is dominant if the two following conditions are fulfilled.

**Condition 1 or incentive constraint:** signalization has to be effective; in other words, it has to dissuade the manipulator (who is inevitably in a favorable situation, $w^H$) from choosing the same strategy as the poor resident. A manipulator does not find it beneficial to work $\bar{h}_1$ and, under the separating conditions, to be considered without ambiguity as a poor resident, if his gains are higher when he is honest (he then works $h^H_1$ and signals his type):

$$Z(\bar{h}_1, w^H) < Z(h^H_1, w^H)$$  \hspace{1cm} (26)$$

$$u_1(\bar{h}_1, w^H) + u_2(T^*(w^L), w^H) - \theta < u_1(h^H_1, w^H) + u_2(T^*(w^H), w^H).$$  \hspace{1cm} (27)$$

**Condition 2 or participation constraint:** signalization has to be profitable for the poor resident. If he undergoes the cost of reduced working hours during the first period, his intertemporal utility with signalization must nevertheless be higher than in the absence of signalization (and thus without cost during the first period):

$$Z(\bar{h}_1, w^L) > Z(h^L_1, w^L)$$  \hspace{1cm} (28)$$

$$u_1(\bar{h}_1, w^L) + u_2(T^*(w^L), w^L) > u_1(h^L_1, w^L) + u_2(T^*(E[w^L|h^L_1]), w^L).$$  \hspace{1cm} (29)$$

Appendix B.1 shows that Condition 1 is satisfied if:

$$\bar{h}_1 < h^H_1 - \sqrt{\bar{\gamma}_1},$$  \hspace{1cm} (30)$$
with:
\[
z_1 = \frac{(1 - \beta)(w^H - w^L)[2\beta(s + w^L) + (1 + \beta)(w^H - w^L)] - 4\theta w^H}{4(w^H)^2} > 0. \tag{31}
\]

Threshold \(z_1\) depends on \(s\), but not on \(q\), because in the separating equilibrium, \(q\) is null. Rational residents will choose the highest working hours that guarantee signalization:
\[
\bar{h}_1 \approx h_1^H - \sqrt{z_1}. \tag{32}
\]

In Appendix B.1, we prove that when \(s > s_0\), \(\bar{h}_1\) is always strictly inferior to \(h_1^L\). It implies that, in this game, signalization by reduction of working hours is always a possible strategy for a resident in an unfavorable economic situation.

As for Condition 2, it is satisfied if (see Appendix B.1):
\[
\bar{h}_1 > h_1^L - \sqrt{z_2}, \tag{33}
\]
with:
\[
z_2 \equiv q \frac{(w^H - w^L)^2}{4(w^L)^2} \left\{ \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] - \frac{1 - \beta^2}{1 + q} \right\} > 0. \tag{34}
\]
(Condition (19) enables us to make sure that \(z_2 > 0\) and \(dz_2/dq > 0\)).

We can state now that:

**Proposition 5** There is a signalization strategy by reduction of first-period working hours which is effective and profitable if and only if:
\[
\sqrt{z_1} - \sqrt{z_2} < h_1^H - h_1^L, \quad \text{with } h_1^H - h_1^L = \frac{A}{w^L} \frac{(w^H - w^L)}{2w^H} > 0. \tag{35}
\]

**Proof.** We have shown that the smallest hours supply that guarantees signalization is \(\bar{h}_1 = h_1^H - \sqrt{z_1}\) (condition 32) and that it is worth signaling if \(\bar{h}_1 > h_1^L - \sqrt{z_2}\) (condition 33). Both conditions are simultaneously fulfilled if \(h_1^H - \sqrt{z_1} > h_1^L - \sqrt{z_2} \iff \sqrt{z_1} - \sqrt{z_2} < h_1^H - h_1^L\), with \(h_1^H - h_1^L = \frac{A}{w^L} \frac{(w^H - w^L)}{2w^H} > 0\). \(\square\)

Of course, whether condition (35) is met or not depends on the parameters of the problem. Since \(z_1\) and \(z_2\) have rather complex mathematical expressions, it is impossible to put forward a simple principle of existence of the equilibrium with strategic signaling. However, some intuition can be brought for the polar cases where \(s\) is close to \(s_0\) and \(s\) is close to \(s_1\) (with \(s \in [s_0, s_1]\), specific to the hybrid equilibrium).
• When $s$ is close to $s_0$, $q = 0$ and thus $z_2 = 0$. The previous condition becomes: $h_t^L < h_t^H - \sqrt{z_1} = \bar{h}_1$ which is impossible because it was shown that $\bar{h}_1 < h_t^L$. Thus, signalization by modulating working hours is not profitable when foreign wages are close to $s_0$. This result seems quite logical: when $s$ is close to $s_0$, almost nobody is cheating hence signalization is unnecessary.

• When $s$ is close to $s_1$, Appendix B.2 shows that condition (35) can be fulfilled for a broad range of parameters if $w^H$ is large enough. Indeed, if $s$ is close to $s_1$, $q$ is close to one and poor residents get too low an amount of remittances. They thus have a strong incentive to signal themselves. On the other hand, if $w^H$ is large, rich residents have little incentive to implement the same strategy. Should they do so, they lose too much: the opportunity cost $(h_t^L - \bar{h}_1)w^H$ is too big.

If strategic signaling is a dominated strategy for the poor residents, the equilibrium of the game is the one analyzed in the previous section.

5 Conclusion

In general, empirical studies highlight the positive effect of migrants’ remittances on poverty reduction in developing countries. Some economists noticed that remittances could nevertheless bring about adverse effects on recipients’ work effort. In this paper we aim at analyzing on the one hand the relationship between remitters’ wage and the amount of remittances, and on the other hand, the relationship between the amount of remittances and residents’ labor supply.

The model is cast as a two-period game between an altruistic migrant and a resident who receives remittances, under the assumption of imperfect information concerning the resident’s economic situation. Optimal hour supply of both the remitter and the recipient is the outcome of a traditional arbitrage between consumption and leisure, given the various transfers. It was shown that in the Hybrid Bayesian Equilibrium, a resident in a good economic situation can try to manipulate the migrant’s expectations by adopting the same behavior as a resident subject to a bad economic situation. The imperfection of information is thus detrimental to poor residents,
because, not being able to signal their type, they receive a reduced amount of remittances. It is also prejudicial to the altruistic migrant who remits less (more) than he would like to a poor (rich) resident. Therefore manipulation leads to a fall in the labor supply of the receiving country that may harm economic growth in the long run, in particular if time saved by shirkers is not used in a productive way (for instance, for investment in human capital). In some cases, poor residents can implement an expensive signaling strategy, which consists in drastically reducing their hour supply. This strategy is likely to reinforce the income precarity of residents right when they meet the worst economic outlook.

The model is based on several assumptions, and some of them are simplifying. In particular, we did not take into account the possibility for the resident to save resources during the first period which he could consume during the second period. The problem that integrates the intertemporal choice of consumption would require an even more complex formalization. Besides, it could be interesting to study the virtues of alternative contracting mechanisms between the migrant and the resident. For instance, if the migrant could commit to the amount of remittances at the beginning of the first period, this would dissuade the rich resident from cheating. Yet this contract might be dominated, since it would imply less insurance for the poor resident.

Simplifications used in this paper are the price to pay to get a straightforward analysis of the influence of imperfect information on the amount remitted on the one hand, and on labor supply on the other hand. Compared to existing theoretical models, this model submits an explanation of remittances linked not only to the resident’s wage but also to the migrant’s wage. This relationship between the migrant’s wage and the amount remitted is complex, because the traditional wealth effect can be partly offset by the reinforcement of the incentive to cheat for recipients.

If it is difficult to draw strong conclusions in terms of economic policy from a model which remains very stylized, results call for a cautious assessment of the macroeconomic impact of private intrafamily remittances. In the light of our analysis, any reform able to reduce the asymmetry of information between migrants and recipients should contribute to improve the situation of the poorest residents. There is no miracle solution able to achieve this result, but the reduction in telecommunications or in travelling costs should go in the right direction. It seems logical to
assume that migrants can better observe the residents’ situations when they make frequent trips back to their origin country. It should be noticed that for illegal immigrants it is almost impossible to travel back and forth, thus the degree of asymmetry is the largest. Shifting the structure of the immigration flow, from illegal to official, may also be taken into consideration.

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References


A Appendix

A.1 The resident’s expected wage

Proof of Proposition 1.

a) If the resident chooses to work $h^H_1$, given that $\Pr[h^H_1|w^L] = 0$, Bayesian calculation of conditional probabilities are:

$$\Pr[w^H|h^H_1] = \frac{\Pr[h^H_1|w^H] \Pr[w^H]}{\Pr[h^H_1|w^H] \Pr[w^H] + \Pr[h^H_1|w^L] \Pr[w^L]} = 1$$  \hspace{1cm} (A.36)

$$\Pr[w^L|h^H_1] = 0.$$  \hspace{1cm} (A.37)

The expected value of the resident’s wage is simply: $E[w^i|h^H_1] = w^H$.

b) If the resident chooses to work $h^L_1$, Bayesian calculation of probabilities yields:

$$\Pr[w^H|h^L_1] = \frac{\Pr[h^L_1|w^H] \Pr[w^H]}{\Pr[h^L_1|w^H] \Pr[w^H] + \Pr[h^L_1|w^L] \Pr[w^L]} = \frac{q}{1 + q}$$  \hspace{1cm} (A.38)

$$\Pr[w^L|h^L_1] = 1 - \Pr[w^H|h^L_1] = \frac{1}{1 + q}.$$  \hspace{1cm} (A.39)

The information set $I_2$ used by the migrant when $t = 2$ to revise probabilities includes as the single salient piece of information the resident’s working hours during the first period: $I_2 = \{h_1\}$.

The expected value of the resident’s wage conditional on $I_2$, $E[w^i|I_2]$, can then be written:

$$E[w^i|h^L_1] = \frac{q}{1 + q} w^H + \frac{1}{1 + q} w^L$$  \hspace{1cm} (40)

with $E[w^i|h^L_1] \in [w^L, 0.5(w^L + w^H)]$.

The expected value of the resident’s wage increases with the probability of adopting the strategy of manipulating expectations:

$$\frac{dE[w^i|h^L_1]}{dq} = \frac{w^H - w^L}{(1 + q)^2} > 0,$$  \hspace{1cm} (41)

to reach its highest value for $q = 1$ (when everybody works $h^L_1$, the migrant cannot revise prior probabilities, therefore $\Pr[w^H|h^L_1] = \Pr[w^L|h^L_1] = 0.5$).

A.2 The manipulating probability $q$

Proof of Proposition 2.
The Nash mixed strategy \( q \in [0,1] \) is implemented if a "rich" resident \( (w^j = w^H) \) is indifferent between playing \( h^L \) or \( h^F \) (equation 15 in the main text):

\[
Z(h^L, w^H) = Z(h^F, w^H). \tag{42}
\]

In a first step, we estimate \( Z(h^L, w^H) \). Knowing that \( h^L = 0.5(1 - A/w^L) \), we can write the resident’s first-period utility as:

\[
u_1(h^L, w^H) = (w^H h^L + A)(1 - h^L) = 0.25(1 + A/w^L)[w^H(1 - A/w^L) + 2A]. \tag{A.43}
\]

Then, we know that \( E[w|h^L] = w^H q \frac{q}{1+q} + w^L \frac{1}{1+q} \). Thus, optimal remittances (Eq. 9) are:

\[
T^* = \beta s - (1 - \beta) \left[w^H \frac{q}{1+q} + w^L \frac{1}{1+q}\right]. \tag{44}
\]

so, the second-period indirect utility function can be written (Eq. 7):

\[
u_2(T^*(E[w|h^L]), w^H) = \frac{0.25}{w^H} \left\{ \beta s - (1 - \beta) \left[w^H \frac{q}{1+q} + w^L \frac{1}{1+q}\right] + w^H \right\}^2 = \frac{0.25}{w^H(1+q)^2} [\beta s(1+q) - (1 - \beta)w^L + (1 + \beta q)w^H]^2. \tag{A.45}
\]

In a second step, we calculate \( Z(h^H, w^H) \). We know that \( h^H = 0.5(1 - A/w^H) \), thus, \( U_1 = U_1(c_1(h^H), h^H) = u_1(h^H, w^H) \), with:

\[
u_1(h^H, w^H) = (w^H h^H + A)(1 - h^H) = \frac{0.25}{w^H} (A + w^H)^2. \tag{46}
\]

Knowing that \( E[w|h^H] = w^H \) and \( T^* = \beta s - (1 - \beta)w^H \), the second-period utility function (Eq. 7) becomes:

\[
u_2(T^*(w^H), w^H) = \frac{0.25}{w^H} (T^* + w^H)^2 = \frac{0.25\beta^2}{w^H} (s + w^H)^2. \tag{47}
\]

We can check that when \( s \) increase, for a given \( q \) the shirkers’ utility gain exceeds the utility gain of the honest residents:

\[
\frac{d}{ds} \left[Z(h^L, w^H) - Z(h^H, w^H)\right] = \beta (1 - \beta) \left(\frac{w^H - w^L}{w^H}\right) > 0. \tag{48}
\]

Taking into account the expressions of the one period utilities, the indifference condition (15)
becomes:

\[
\begin{align*}
  u_1(h^H_1, w^H) + u_2(T^*(w^H), w^H) &= u_1(h^L_1, w^H) + u_2(T^*(E[w^L|h^L_1]), w^H) - \theta \tag{A.49} \\
  &\Leftrightarrow (1 + q)^2 (w^H - w^L) \left(\frac{A}{w^L}\right)^2 = (1 - \beta) \left[ 2\beta s(1 + q) - (1 - \beta)w^L + (1 + 2\beta q + \beta)w^H \right] - (1 + q)^2 \frac{4\theta w^H}{w^H - w^L} \tag{A.50}\end{align*}
\]

This last equation can be written as a relationship between \( s \) and \( q \) of the form \( s = g(q, \beta, \theta, A, w^H, w^L) \):

\[
s = \frac{1}{2\beta(1 + q)} \left\{ \frac{(1 + q)^2 (w^H - w^L)}{1 - \beta} \left[ \left(\frac{A}{w^L}\right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] + (1 - \beta)w^L - (1 + 2\beta q + \beta)w^H \right\} > 0. \tag{51}
\]

Differentiating this expression, we get:

\[
\frac{dq}{ds} = \frac{2\beta(1 + q)^2(1 - \beta)}{(w^H - w^L) \left\{ (1 + q)^2 \left[ (A/w^L)^2 + 4\theta w^H / (w^H - w^L)^2 \right] + (1 - \beta)^2 \right\}} > 0. \tag{52}
\]

as has been stated in Proposition 2.

### A.3 Remittances and migrant’s wage

**Proof of Proposition 4.**

Let us study the formal relationship between \( T \) and \( s \). From the expression of optimal remittances, \( T^* = \beta s - (1 - \beta) E \left[ w^L|h^L_1 \right] \), we can write:

\[
\frac{dT^*}{ds} = \beta - (1 - \beta) \frac{dE \left[ w^L|h^L_1 \right]}{dq} \frac{dq}{ds}. \tag{53}
\]

We replace by the expressions (41) and (52) to get:

\[
\frac{dT^*}{ds} = \beta \left[ \left(\frac{A}{w^L}\right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] - \frac{(1 - \beta)^2}{(1 + q)^2}. \tag{54}
\]

The sign of \( \frac{dT^*}{ds} \) is the same as the sign of \( \left[ \left(\frac{A}{w^L}\right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] - \frac{(1 - \beta)^2}{(1 + q)^2} \). This term is positive. Indeed:

\[
\left[ \left(\frac{A}{w^L}\right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] - \frac{(1 - \beta)^2}{(1 + q)^2} \geq \left[ \left(\frac{A}{w^L}\right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] - (1 - \beta)^2 \forall q. \]

Yet, according to Condition (19), \( \left[ \left(\frac{A}{w^L}\right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] > (1 - \beta^2) + 2\beta (1 - \beta) k \). Thus, \[
\left[ \left(\frac{A}{w^L}\right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] - \frac{(1 - \beta)^2}{(1 + q)^2} \geq (1 - \beta^2) + 2\beta (1 - \beta) (1 + k) \geq 0 \forall q. \tag{A.55}\]

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A.4 Utility loss of a poor resident in case of imperfect information

When information is perfect, the poor resident’s utility is:

\[ Z^p(h^L_1, w^L) = u_1(h^L_1, w^L) + u_2(T^*(w^L), w^L). \]  (56)

When information is imperfect, in the hybrid equilibrium the poor resident’s utility is:

\[ Z^I(h^L_1, w^L) = u_1(h^L_1, w^L) + u_2(T^*(E[w^L|h^L_1]), w^L). \]  (57)

Knowing that:

\[ u_2(T^*(E[w^L|h^L_1]), w^L) = 0 \]

and that, according to Eq. (9):

\[ u_2(T^*(E[w^L|h^L_1]), w^L) = 0 \]

we can write the resident’s loss depending on \( q \):

\[ Z^p(h^L_1, w^L) - Z^I(h^L_1, w^L) = 0 \]

with \( \Phi = 2\beta + 1 - \frac{1}{1+q} \left[ (2\beta + q + \beta q)w^L - (q - q\beta)w^H \right] \).

However, in the hybrid equilibrium:

\[ s = \frac{1}{2\beta(1+q)} \left\{ (1+q)^2 \left( \frac{w^H - w^L}{1-\beta} \right) \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] + (1-\beta)w^L - (1+2\beta q + \beta)w^H \right\}. \]  (61)

Thus:

\[ \Phi = \frac{1}{1+q} \left[ \frac{(1+q)^2(w^H - w^L)}{1-\beta} \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] + (1-\beta)w^L - (1+2\beta q + \beta)w^H \right] + (1-\beta)w^L - (1+2\beta q + \beta)w^H + (2\beta + q + \beta q)w^L - (q - q\beta)w^H \]

\[ = (w^H - w^L) \left\{ \frac{1+q}{1-\beta} \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] - \frac{1 - \beta^2}{1+q} \right\}. \]  (A.62)
The difference between the two utilities is:

\[
Z^P(h^L_1, w^L) - Z^T(h^L_1, w^L) = q \left( \frac{w^H - w^L}{4w^L} \right)^2 \left\{ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right\} - \frac{1 - \beta^2}{1 + q}. \tag{63}
\]

The sign of \( Z^P(h^L_1, w^L) - Z^T(h^L_1, w^L) \) is the sign of the term:

\[
\Omega = \left[ \frac{A}{w^L} \right]^2 + \frac{4\theta w^H}{(w^H - w^L)^2} - \frac{1 - \beta^2}{1 + q}, \tag{64}
\]

which is an increasing function in \( q \). Its smallest value is obtained for \( q = 0 \), with \( \Omega_{q=0} = \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] - (1 - \beta^2) \). But, for \( s_0 > 0 \), which is the case under scrutiny, we have shown in Condition (19) that: \( \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] > (1 - \beta^2) + 2\beta(1 - \beta)k > (1 - \beta^2) \). Thus, \( \Omega_{q=0} > 0 \Rightarrow \Omega > 0 \ \forall q > 0 \). Thus \( Z^P(h^L_1, w^L) - Z^T(h^L_1, w^L) > 0 \ \forall q > 0 \).

\section*{B Appendix}

\subsection*{B.1 Signaling conditions}

\textit{Condition 1: Incentive constraint}

We study if signalization by the poor resident through reduction of his first-period labor supply is possible.

We calculate: \( u_1(h_1, w^H) = (w^H h_1 + A)(1 - h_1) \), \( u_1(h^L_1, w^H) = \frac{0.25}{w^H}(A + w^H)^2 \), \( u_1(h_1, w^L) = (w^L h_1 + A)(1 - h_1) \) and \( u_1(h^L_1, w^L) = \frac{0.25}{w^L}(A + w^L)^2 \).

Knowing that: \( u_2 = \frac{0.25}{w^H}(T^* + w^H)^2 \), \( T^* = \beta s - (1 - \beta)E[w^L|h^L_1] \), and \( E[w^L|h^L_1] = w^H \frac{q}{1 + q} + w^L \frac{1}{1 + q} \), we get:

\[
\begin{align*}
u_2(T^*(w^L), w^H) &= \frac{0.25}{w^H}(\beta s - (1 - \beta)w^L + w^H)^2 \\
u_2(T^*(w^H), w^H) &= \frac{0.25\beta^2}{w^H}(s + w^H)^2 \\
u_2(T^*(w^L), w^L) &= \frac{0.25}{w^L}(T^* + w^L)^2 = \frac{0.25}{w^L}(\beta s - (1 - \beta)w^L + w^L)^2 = \frac{0.25\beta^2}{w^L}(s + w^L)^2 \\
u_2(T^*(E[w^L]), w^L) &= \frac{0.25}{w^L} \left\{ \beta s - (1 - \beta) \left[ w^H \frac{q}{1 + q} + w^L \frac{1}{1 + q} \right] + w^L \right\}^2
\end{align*}
\]

We can then rewrite \textit{Condition 1}:

\[
\begin{align*}
u_2(T^*(w^L), w^H) - \nu_2(T^*(w^H), w^H) - \theta &< u_1(h^L_1, w^H) - u_1(h_1, w^H) \\
\frac{0.25}{w^H}(\beta s - (1 - \beta)w^L + w^H)^2 - \frac{0.25\beta^2}{w^H}(s + w^L)^2 - \theta &< \frac{0.25}{w^H}(A + w^H)^2 - (w^H h_1 + A)(1 - h_1) \\
(1 - \beta)(w^H - w^L)[2\beta s - (1 - \beta)w^L + (1 + \beta)w^H] - 4\theta w^H &< [(w^H - A) - 2w^H h_1]^2 \\
(1 - \beta)(w^H - w^L)[2\beta(s + w^L) + (1 + \beta)(w^H - w^L)] - 4\theta w^H &< 4(w^H)^2(h^L_1 - h_1)^2,
\end{align*}
\tag{B.65}
\]
where \( h^H_1 - \tilde{h}_1 > 0 \).

Let us denote:

\[
z_1 = \frac{(1 - \beta)(w^H - w^L)[2\beta(s + w^L) + (1 + \beta)(w^H - w^L)] - 4\theta w^H}{4(w^H)^2} > 0. \tag{66}
\]

Thus, separation is possible if there is a \( \tilde{h}_1 \in ]0, h^L_1[ \) such that:

\[
(h^H_1 - \tilde{h}_1)^2 > z_1 \iff \tilde{h}_1 < h^H_1 - \sqrt{z_1}. \tag{67}
\]

The resident chooses the highest working hours possible:

\[
\tilde{h}_1 \simeq h^H_1 - \sqrt{z_1}. \tag{68}
\]

We check that \( \tilde{h}_1 < h^L_1 \):

\[
\begin{align*}
(h^H_1 - \sqrt{z_1})^2 & < h^L_1 \\
(h^H_1 - h^L_1)^2 & < \frac{(1 - \beta)(w^H - w^L)[2\beta(s + w^L) + (1 + \beta)(w^H - w^L)] - 4\theta w^H}{4(w^H)^2} \\
(w^H - w^L)^2 \left( \frac{A}{w^L} \right)^2 & < (1 - \beta)(w^H - w^L) \left[ 2\beta(s + w^L) + (1 + \beta)(w^H - w^L) \right] - 4\theta w^H \iff \text{(B.60)}
\end{align*}
\]

In this inequality, the right term denoted \( Y(s) \) is a function increasing in \( s \).

We calculate \( Y(s_0) \), with \( s_0 \equiv \frac{1}{z} \left\{ \left( \frac{w^H - w^L}{1 - \beta} \right) \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} \right] + (1 - \beta)w^L - (1 + \beta)w^H \right\} :\)

\[
Y(s_0) = (w^H - w^L)^2 \left( \frac{A}{w^L} \right)^2. \tag{70}
\]

In the hybrid equilibrium, \( s > s_0 \). Thus:

\[
(w^H - w^L)^2 \left( \frac{A}{w^L} \right)^2 = Y(s_0) < Y(s), \forall s \iff \tilde{h}_1 < h^L_1, \forall s. \tag{71}
\]

**Condition 2: Participation constraint**

We study if signalization by the poor resident through reduction of his first-period labor supply is profitable to him:

\[
Z(\tilde{h}_1, w^L) > Z(h^L_1, w^L) \\
u_1(\tilde{h}_1, w^L) + u_2(T^*(w^L), w^L) > u_1(h^L_1, w^L) + u_2(T^*(E[w^H|h^L_1]), w^L) \\
4w^L(w^L \tilde{h}_1 + A)(1 - \tilde{h}_1) + \beta^2(s + w^L)^2 > (A + w^L)^2 + \left[ \beta s - \frac{1 - \beta}{1 + q} (qw^H + w^L) + w^L \right]^2 \\
-4w^L w^L(\tilde{h}_1)^2 + 4w^L \tilde{h}_1 (w^L - A) + [4w^L A - (A + w^L)^2] > \left[ \beta s - \frac{1 - \beta}{1 + q} (qw^H + w^L) + w^L \right]^2 - \beta^2(s + B)^2.
\]
However, \((w_L - A) = 2h_L^L w_L\), thus the former inequality becomes:

\[
-4(w^L) \left[ (h_1^L)^2 - 2h_1^L + (h_1^L)^2 \right] > \left[ \beta s - \frac{1 - \beta}{1 + q} (qw^H + w_L) + w_L \right]^2 - \beta^2(s + w^L)^2
\]

\[
4(w^L)^2 (h_1^L - \bar{h}_1^L)^2 < (\beta s + \beta w^L)^2 - \left[ \beta s - \frac{1 - \beta}{1 + q} (qw^H + w_L) + w_L \right]^2
\]

\[
4(w^L)^2 (h_1^L - \bar{h}_1^L)^2 < \left\{ \frac{1 - \beta}{1 + q} (w^H - w_L) \right\} \left\{ 2\beta s - \frac{1}{1 + q} \left[ (1 - \beta)q w^H - (2\beta + q + \beta q)w_L \right] \right\}
\]

We know that in the hybrid equilibrium (eq. 61),

\[
s = \frac{1}{2\beta(1 + q)} \left\{ \frac{(1 + q)^2 (w^H - w_L)}{1 - \beta} \left[ \left( \frac{A}{w_L} \right)^2 + \frac{4\theta w^H}{(w^H - w_L)^2} \right] \right\}
\]

We can then rewrite Condition 2:

\[
4(w^L)^2 (h_1^L - \bar{h}_1^L)^2 < q \frac{1 - \beta}{(1 + q)^2 (w^H - w_L)} \left\{ \frac{(1 + q) \left( w^H - w_L \right)}{1 - \beta} \left[ \left( \frac{A}{w_L} \right)^2 + \frac{4\theta w^H}{(w^H - w_L)^2} \right] \right\}
\]

\[
4(w^L)^2 (h_1^L - \bar{h}_1^L)^2 < q (w^H - w_L)^2 \left\{ \left[ \left( \frac{A}{w_L} \right)^2 + \frac{4\theta w^H}{(w^H - w_L)^2} \right] - \frac{(1 + \beta)(1 - \beta)}{1 + q} \right\}
\]

Let us denote:

\[
z_2 = q \left( \frac{w^H - w_L}{4(w^L)^2} \right)^2 \left\{ \left[ \left( \frac{A}{w_L} \right)^2 + \frac{4\theta w^H}{(w^H - w_L)^2} \right] - \frac{1 - \beta^2}{1 + q} \right\}.
\]

Given that, according to condition (64), \(z_2 > 0\), and that \(h_1^L - \bar{h}_1^L > 0\), Condition 2 can be rewritten:

\[
(h_1^L - \bar{h}_1^L)^2 < z_2 \Leftrightarrow h_1^L - \sqrt{z_2} < \bar{h}_1^L.
\]

**B.2 Equilibrium with signalization: the case \(s\) close to \(s_1\)**

When \(s \to s_1\), \(q \to 1\). Replacing \(s\) by \(s_1\) (Eq. 18) in Eq. (31), threshold \(z_1\) becomes:

\[
[z_1]_{s=s_1} = \left( \frac{w^H - w_L}{4 w^H} \right)^2 \left\{ 2 \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w_L)^2} + 0.5(1 - \beta)^2 \right\}
\]

and threshold \(z_2\) becomes:

\[
[z_2]_{q=1} = \left( \frac{w^H - w_L}{4 w^L} \right)^2 \left\{ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w_L)^2} \right\} - 0.5(1 - \beta^2)
\]
Inequality (35) can be written:

\[
\left[ 2 \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} + 0.5(1 - \beta)^2 \right]^{1/2} - \frac{w^H}{w^L} \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} - 0.5(1 - \beta)^2 \right]^{1/2} < \frac{A}{w^L}.
\]  

(79)

Let us denote by \( \mathcal{C}(w^H) \) the left hand side term of this inequality. For \( w^H = 0 \), this term is positive. We can prove that above a certain threshold, i.e. for \( w^H \) high enough (as compared to \( w^L \)), \( \mathcal{C}(w^H) \) becomes negative and thus, condition (35) is fulfilled.

- Indeed, \( \mathcal{C}(w^H) < 0 \) is equivalent to:

\[
\left[ 2 \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} + 0.5(1 - \beta)^2 \right]^{1/2} < \frac{w^H}{w^L} \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} - 0.5(1 - \beta)^2 \right]^{1/2}
\]

\[2 \left( \frac{A}{w^L} \right)^2 + 0.5(1 - \beta)^2 < \frac{w^H}{w^L} \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} - 0.5(1 - \beta)^2 \right]^{1/2} - \frac{4\theta w^H}{(w^H - w^L)^2} \]

As \( \lim_{w^H \to +\infty} \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} - 0.5(1 - \beta)^2 \) and \( \lim_{w^H \to +\infty} \left( \frac{4\theta w^H}{(w^H - w^L)^2} \right) = 0 \), we can conclude that \( \lim_{w^H \to +\infty} \left( \frac{w^H}{w^L} \right)^2 \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} - 0.5(1 - \beta)^2 \right] - \frac{4\theta w^H}{(w^H - w^L)^2} \) = 

\(+/−\infty\), depending on the sign of \( \left( \frac{A}{w^L} \right)^2 - 0.5(1 - \beta)^2 \) when \( w^H \to +\infty \).

Yet, according to condition (19):

\[
\left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} > (1 - \beta^2) + 2\beta(1 - \beta) \frac{w^L}{w^H - w^L} \]

\[
\left( \frac{A}{w^L} \right)^2 - 0.5(1 - \beta^2) > 0.5(1 - \beta^2) + 2\beta(1 - \beta) \frac{w^L}{w^H - w^L} - \frac{4\theta w^H}{(w^H - w^L)^2} \]  

(B.81)

As \( \lim_{w^H \to +\infty} \left[ 0.5(1 - \beta^2) + 2\beta(1 - \beta) \frac{w^L}{w^H - w^L} - \frac{4\theta w^H}{(w^H - w^L)^2} \right] = 0.5(1 - \beta^2) > 0 \), we can conclude that \( \lim_{w^H \to +\infty} \left( \frac{w^H}{w^L} \right)^2 \left[ \left( \frac{A}{w^L} \right)^2 + \frac{4\theta w^H}{(w^H - w^L)^2} - 0.5(1 - \beta^2) \right] - \frac{4\theta w^H}{(w^H - w^L)^2} \) = +∞. Thus, above a certain threshold, i.e. for \( w^H \) high (compared to \( w^L \)), \( \mathcal{C}(w^H) \) is negative. Necessarily, condition (79) is fulfilled (since \( \frac{A}{w^L} > 0 \)). Therefore, signalization is possible and profitable for a resident in a difficult economic situation when \( w^H \) is relatively high.

Notice that in the case where the psychological cost linked to cheating is null (\( \theta = 0 \)), condition (79) becomes:

\[
\left[ 2 \left( \frac{A}{w^L} \right)^2 + 0.5(1 - \beta)^2 \right]^{1/2} - \frac{w^H}{w^L} \left[ \left( \frac{A}{w^L} \right)^2 - 0.5(1 - \beta)^2 \right]^{1/2} < \frac{A}{w^L}.
\]  

(82)
The left hand side term is obviously decreasing in $w^H$. It is then easy to see that above a certain threshold, i.e. for $w^H$ high (compared to $w^L$), condition (79) is fulfilled.