# Capital accumulation, welfare and the emergence of pension fund activism

Pascal Belan, Philippe Michel, Bertrand Wigniolle

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**Abstract**

This paper presents an overlapping generations model with altruistic consumers, in which pension funds, by holding a significant share of capital assets, produce non competitive behavior. We study the consequences of such behavior on capital accumulation and welfare in the long run when subsidies are associated with contributions to pension funds. If bequests are operative and the subsidy rate is not too high, the capital stock increases with the introduction of pension funds, and this increases long run utility. If bequests are not operative without pension funds, the rise in long-run welfare is no longer guaranteed, even if the subsidy rate is low.

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[‡]GREQAM, Université de la Méditerranée and EUREQua, Université de Paris I. The current version of this paper was completed after Philippe Michel's sudden death. We want to express our deep sorrow for the loss of a close friend and an excellent economist, from whom we had learned much over the years.

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# 1 Introduction

This paper explores the consequences of imperfect competition on capital accumulation. We consider an equilibrium concept using the Cournot-Walras equilibrium, according to which some agents, having a significant size compared to the whole economy, take into account the influence of their choice on the equilibrium. A Walrasian equilibrium is formed, which depends on the quantities chosen by the strategic agents who play between themselves a game of the Cournot-Nash type.

From a theoretical point of view, our work can be seen as a first attempt to develop this concept within a dynamic framework. Indeed, the literature in dynamic macroeconomics has mainly focused on the study of monopolistic competition (following Dixit and Stiglitz, 1977). In overlapping generation models, few studies have explored other concepts of imperfect competition. For instance, Laitner (1982) studies the consequences on long-run capital accumulation of the existence of oligopolies on commodity markets. D'Aspremont, Dos Santos-Ferreira and Gerard-Varet (1991, 1995) consider an overlapping generations model without capital, where firms act as Cournot oligopolists in the good market.

From another perspective, we argue that this equilibrium concept turns out to be fruitful for analyzing some features of contemporary economies, in particular, the impact of pension funds. Indeed, the growing size of pension funds has led to the emergence of economic agents who hold a significant share of the capital assets of the whole economy[1], and who may therefore influence equilibrium prices and quantities. In practice, the concentration of capital gives pension funds the power to influence the managers of the firms in order to increase the return on their investment.[2] At the macroeconomic level, this may modify the distribution of income between workers and pensioners and may have consequences for capital accumulation and welfare in the long-run.

Until now, the literature on the transition from pay-as-you-go to funded pension systems has eluded pension funds activism[3]. The conventional wisdom about private funded systems is that they are neutral with respect to capital accumulation and welfare. Indeed, from a theoretical point of view, assuming perfect capital markets, a fully-funded system has no impact on aggregate savings. On the contrary, we argue that the activism of pension funds induces a capital market imperfection at the macroeconomic level, when these funds acquire sufficient market power.

[1] In the US, pension funds assets were equivalent to 24.3 % of GDP in 1981 and grew to 73.9 % in 1999. In the same period, pension funds assets in the United Kingdom were growing from 22.4 % to 87.8 % of GDP.

[2] The literature on corporate governance gives some theoretical justifications for the activism of pension funds on the management of firms. As soon as a shareholder holds a sufficiently large fraction of capital, his gain from activism can exceed the monitoring cost (see, for instance, Shleifer and Vishny (1986), Holmström and Tirole (1993), Huddart (1993) and Burkart, Gromb and Panunzi (1997)).

[3] See Feldstein (1998) or Belan and Pestieau (1999) for a survey. Breyer (1989) analyzes the transition issue in the standard Diamond (1965) model with inelastic labor supply. Homburg (1990), Breyer and Straub (1993) and Brunner (1994, 1996) have considered the case of elastic labor supply. Furthermore, Belan, Michel and Pestieau (1998) and Gyárfás and Marquardt (2001) study the transition in an endogenous growth model with technological externality.

In this respect, our paper can also be viewed as a contribution to a growing literature about the relationship between financial structure and economic growth. The noncompetitive behavior of pension funds introduces a market imperfection in financial intermediation. In particular, our contribution shares some concerns with recent papers that focus on banking market structure and its relationship with capital accumulation (*e.g.* Cetorelli, 1997, Cetorelli and Peretto, 2000 and Guzman, 2000).

In order to capture the consequences of pension funds activism on capital accumulation and welfare, we consider an overlapping generation model where people live for two periods and are altruistic to their offspring (Barro, 1974). Consumers allocate their savings between pension funds and personal savings. With these contributions, pension funds hold a significant part of the productive sector. They are able to intervene in the management of the firms that they hold, and consequently, in the equilibrium resulting from the competitive behavior of the other agents. The strategic variable of pension funds is the demand for labor by firms that they control. They take into account the effect of their labor demand on the equilibrium prices and display Nash behavior. This equilibrium concept follows the line of the Cournot-Walras equilibrium defined by Gabszewicz and Vial (1972) and studied by Codognato and Gabszewicz (1993) and Gabszewicz and Michel (1997).

In this framework, Belan, Michel and Wigniolle (2002) consider agents without bequest motive (as in Diamond, 1965) and show that the introduction of pension funds modifies the income distribution between labor and capital, by reducing wages and increasing savings returns. Since savings are based on wages, the distribution of income that favors capital income will diminish savings and capital stock in the long run. The present paper questions whether this mechanism is relevant when savings are based on labor earnings and past savings returns. We consider dynasties of altruistic agents whose savings depend on current wages and bequests that they receive from their parents. In these cases, past capital returns, which should be higher with pension funds activism, will influence current and future savings and may compensate for the impact of the fall in wages.

In practice, fiscal incentives are usually associated with contributions to pension funds. For instance, in the US, employers can deduce contributions to pension funds from their profits. We model such a tax exemption as subsidies on savings invested in pension funds. These subsidies will increase the importance of the pension funds in the economy and allows us to parameterize their size. To some extent, such fiscal incentives will qualitatively have the same consequences as subsidies on savings in a model without imperfect competition. But in addition to these usual effects, they introduce some supplementary distortions related to the noncompetitive behavior of pension funds.

We study the impact of imperfect competition and subsidies on capital accumulation and welfare in the long run. In particular, when bequests are operative, we show that, despite the fall in wages, capital stock increases with the introduction of pension funds and is an increasing function of the subsidy rate. As a consequence, long-run utility increases for small values of the subsidy rate. But it decreases above a certain threshold, since the economy is in overaccumulation, too far from the

Golden Rule, and the distortions introduced by the savings subsidies and imperfect competition become too high. Nevertheless, when bequests are constrained in the economy without pension funds, there exist situations where long-run welfare with pension funds is lower than without pension funds, whatever the rate of subsidy.

The paper is organized as follows. Section 2 presents the model and notably introduces the game between firms managed by pension funds. In section 3, we analyze the effect of pension funds in the long run when bequests are positive. In section 4, we consider the situation where the constraint of non-negative bequests may be binding. Section 5 concludes. The most demanding proofs are given in the Appendix.

# 2 The model

## 2.1 Consumer behavior

We consider an overlapping generations model of agents living for two periods. The size of generation $t$ is $N_t$ and each agent has $(1+n)$ children. We assume that parents care about their children's welfare by weighting the children's utility in their own utility function (Barro (1974)). The utility of a generation born at time $t$, $V_t$, is given by

$$V_t = U(c_t, d_{t+1}) + \gamma V_{t+1}, \quad 0 < \gamma < 1$$

where $U$ satisfies the following assumption

**Assumption 1** *$U$ is twice continuously differentiable, increasing with respect to both consumptions, strictly concave and satisfies, for all positive $c$ and $d$: $U_c'(0,d) = +\infty$, $U_d'(c,0) = +\infty$. Moreover, for all positive $c$ and $d$,*

$$U_d' U_{cc}'' < U_c' U_{cd}'' \text{ and } U_c' U_{dd}'' < U_d' U_{cd}''$$

*which implies that both consumptions are normal goods.*

In their first period of life, individuals born in $t$ work and receive a wage $w_t$. In addition to wage income, they receive a bequest $x_t$ from their parents and pay a lump-sum tax $\tau_t^1$. They consume $c_t$ and save the remainder. We assume that individuals allocate their total savings between two types of investments: personal savings $s_t^0$ and contributions to pension funds $s_t^1$. Gross returns are respectively denoted by $R_{t+1}^0$ and $R_{t+1}^1$. Contributions to pension funds are subsidized at rate $\theta_t$ per unit saved. In their second period of life, people receive returns on savings, pay a lump-sum tax $\tau_{t+1}^2$ and allocate net resources between consumption $d_{t+1}$ and bequests $x_{t+1}$ to their $(1+n)$ children. Thus

$$x_t + w_t - \tau_t^1 + \theta_t s_t^1 = c_t + s_t^0 + s_t^1$$

$$R_{t+1}^0 s_t^0 + R_{t+1}^1 s_t^1 - \tau_{t+1}^2 = d_{t+1} + (1+n) x_{t+1}$$

Bequests must be non-negative : $x_{t+1} \geq 0$.

With perfect capital market, the arbitrage between both types of savings implies the equality of net returns, i.e.

$$\frac{R^1_{t+1}}{1-\theta_t} = R^0_{t+1} \tag{1}$$

The net investment expenditure of an agent of generation $t$ is

$$\sigma_t = s^0_t + (1-\theta_t)s^1_t \tag{2}$$

The maximum of total utility is given by the following recursive relation:

$$V^*_t(x_t) = \max_{c_t,\sigma_t,d_{t+1},x_{t+1}} \left\{U(c_t, d_{t+1}) + \gamma V^*_{t+1}(x_{t+1})\right\}$$

subject to

$$x_t + w_t - \tau^1_t = c_t + \sigma_t \tag{3}$$

$$R^0_{t+1}\sigma_t - \tau^2_{t+1} = d_{t+1} + (1+n)x_{t+1} \tag{4}$$

$$x_{t+1} \geq 0 \tag{5}$$

For any positive $t$, $V^*_t(x_t)$ represents the maximum utility of a young agent born in $t$ when he inherits $x_t$. These are the value functions of the infinite horizon problems $\max \sum_{j=0}^{+\infty} \gamma^j U(c_{t+j}, d_{t+j+1})$ subject to (3), (4) and (5).

This maximization problem leads to the following first-order conditions

$$U^{'}_c(c_t, d_{t+1}) = R^0_{t+1}U^{'}_d(c_t, d_{t+1}) \tag{6}$$

$$-(1+n)U^{'}_d(c_t, d_{t+1}) + \gamma U^{'}_c(c_{t+1}, d_{t+2}) \leq 0 \tag{7}$$

The second condition holds with equality if $x_{t+1} > 0$. Equation (6) is the standard condition for individual life-cycle allocation. Condition (7) is a condition for optimal allocation between parent and children. If $x_{t+1} > 0$, it states that the marginal utility loss from reduction of a parent's consumption will equal the marginal utility gain of an increase in the bequest.

## 2.2 Firms

Firms live for one period. Their capital stock is fixed at the beginning of the period, with total depreciation. There exist two types of firms. Some are controlled by pension funds; their entire capital stock consists of contributions to pension funds in the preceding period. Other firms are independent of pension funds ; their capital stock consists of personal savings in the preceding period. Both types of firm have the same technology using capital and labor as inputs: $F(K, L)$. $F$ is linear homogeneous. Marginal products are positive and decreasing.

The firms independent of pension funds behave competitively. Thus, one can consider a representative firm for this sector (which we call respectively competitive firm and competitive sector in the following). At period $t$, its capital stock is $K_t^0 = N_{t-1}s_{t-1}^0$.

We assume pension funds are represented by $m$ firms, with capital stocks denoted by $K_t^i$, $i = 1, ..., m$, such that $\sum_{i=1}^m K_t^i = N_{t-1}s_{t-1}^1$. By holding a significant share of the total capital stock, these firms have a market power and behave noncompetitively. For simplicity, we assume that each pension funds holds the whole capital of a single firm. But, our model encompasses a wider range of occurrences. For instance, if several pension funds share the capital stock of a single firm, the objective of this firm would remain the same since pension funds would be unanimous for maximizing total profits. Moreover, our framework can also represent the case of one pension fund that holds the capital of several firms: since technology exhibits constant returns to scale, those firms can be aggregated.

Firms owned by pension funds act noncompetitively on the labor market. They do not take the wage rate as given. They maximize their profits taking into account the impact of their labor demand on the equilibrium wage. So doing, they exert a detrimental effect on the wage rate which increases capital return. Their behavior harms the workers for the benefit of capital owners. Note that in our two-period OLG model, the workers are the young and the capital owners are the old. Therefore, here, pension funds represent interests of the old.

## 2.3 Equilibrium between noncompetitive firms

The behavior of noncompetitive firm is described by a game that we call the *Firms' Cournot-Walras Game*:[4]

**Definition 1 (*Firms' Cournot-Walras Game, FCWG*)** *1. Let $K_t^0$ be the capital stock of the competitive firm and $K_t^i$, $i = 1, ..., m$, the capital stocks of the noncompetitive firms. Labor supply is $N_t$.*

2. *The strategy of player $i$ (the noncompetitive firm $i$) is its labor demand $L_t^i \geq 0$ $(i = 1, ..., m)$.*

3. *For any vector of strategies of the noncompetitive firms $(L_t^i)_{i=1,...,m}$, a competitive equilibrium exists if $\sum_{i=1}^m L_t^i < N_t$ [5] and it consists of a price $w_t$ and a quantity $L_t^0$, such that the competitive firm maximizes its profits $L_t^0 = \arg\max_L F(K_t^0, L) - w_t L$ and labor supply equals labor demand : $L_t^0 = N_t - \sum_{i=1}^m L_t^i$.*

[4] The definition of the Cournot-Walras equilibrium rests on the study of the general competitive equilibrium when the strategic decisions of the non-competitive players are fixed (Gabszewicz and Vial (1972)). In our model, given the capital stocks (resulting from past decisions) and the total labor supply (which is inelastic), it is possible to define such a static game *for firms*.

[5] If $\sum_{i=1}^m L_t^i \geq N_t$, a rationing scheme could be considered. To simplify, we assume that in that case, payoffs of the $m$ players are equal to 0. Of course, this case never does occur at equilibrium.

4. *Payoff of player $i$ is its profits :* $\Pi_t^i = F\left(K_t^i, L_t^i\right) - w_t L_t^i$ *if* $\sum_{i=1}^m L_t^i < N_t$. *Payoff is* $0$ *if* $\sum_{i=1}^m L_t^i \geq N_t$.[6]

Given this definition, it is now possible to consider the equilibrium of the game.

**Definition 2 (*Firms' Cournot-Walras Equilibrium, FCWE*)** *A Firms' Cournot-Walras Equilibrium is a Nash Equilibrium* $(L_t^i)_{i=1,\ldots,m}$ *of the FCWG. It is non-trivial if* $\sum_{i=1}^m L_t^i < N_t$.

Note that with the assumption of inelastic labor supply, the competitive behavior of the consumers does not interact with the firms' Cournot-Walras equilibrium.

Let us compare this economy with the standard framework where all firms behave competitively. The game under consideration involves an additional type of agent. Besides competitive firms, there exist noncompetitive firms with labor demand as a strategic variable. Given the labor demands of all strategic agents, there exists a Walrasian equilibrium that determines the value of their profits (payoffs). These agents internalize the effect of their labor demand on the Walrasian equilibrium in order to maximize their profits. In addition, following a Cournotian approach, they take the strategies of other players as given. This results in a Nash equilibrium. The following proposition characterizes the FCWE and states existence and uniqueness. Since capital stocks are given, one can consider labor capital ratios $l_t^i = L_t^i / K_t^i$ instead of labor demand $L_t^i$ as the strategic variable for firm $i$.

**Proposition 1** *Assume* $F_{LLL}''' \geq 0$.[7] *For given positive capital stocks* $K_t^0$ *and* $(K_t^i)_{i=1,\ldots,m}$, *there exists a unique non-trivial FCWE. This equilibrium is characterized by the labor-capital ratios* $l_t^0$, $l_t^1$, .... $l_t^m$ *which satisfy for* $i = 1, \ldots m$,

$$F_L'\left(1, l_t^i\right) - F_L'\left(1, l_t^0\right) + l_t^i \frac{K_t^i}{K_t^0} F_{LL}''\left(1, l_t^0\right) = 0, \tag{8}$$

*and the equilibrium condition on the labor market*

$$K_t^0 l_t^0 + \sum_{i=1}^m l_t^i K_t^i = N_t. \tag{9}$$

*Proof.* See Appendix 1. ■

As shown in Appendix 1, equation (8) represents the best-response function at equilibrium. Noncompetitive firms have a higher marginal productivity than competitive firms. The difference comes from their strategic behavior which internalizes

[6] Notice that the payoff is expressed in real terms. As for the Cournot-Walras equilibrium, the choice of the *numéraire* will not affect the equilibrium of the game.

[7] With a CES production function $F(K, L) = A\left[aK^\rho + bL^\rho\right]^{1/\rho}$, $A > 0$ $a > 0$, $b > 0$ and $\rho < 1$, $F_{LLL}'''$ has the same sign as $(1+\rho)b + a(2-\rho)K^\rho L^{-\rho}$. Thus the condition $F_{LLL}''' \geq 0$ is satisfied with $\rho \geq -1$, i.e. when the elasticity of substitution is not too low (larger or equal to 1/2). Note also that if $K_t^i$ is small enough, the assumption $F_{LLL}''' \geq 0$ is no longer necessary.

the effect of their labor demand on the equilibrium wage and, therefore, on their profits.

Since $F_{LL}^{"} < 0$, equation (8) implies

$$l_t^i < l_t^0, \text{ for } i = 1, ...m.$$

By demanding less labor by unit of capital, noncompetitive firms tend to push down the equilibrium wage, in order to raise their capital return.

Proposition (1) characterizes the FCWE for all initial allocations of capital stocks. Nevertheless, the arbitrage conditions resulting from the consumer program govern the allocation of capital stock between firms. Indeed, with perfect capital markets, arbitrage conditions (1) at equilibrium imply the equality of all net capital returns

$$R_t^i = R_t^1 = (1 - \theta_{t-1}) R_t^0.$$

At the FCWE, the equilibrium wage is:

$$w_t = F_L^{'}\left(1, l_t^0\right). \tag{10}$$

Thus, the return to capital is

$$R_t^i = \Pi_t^i / K_t^i = F(1, l_t^i) - l_t^i F_L^{'}(1, l_t^0) \tag{11}$$

for a noncompetitive firm $i$ and,

$$R_t^0 = F_K^{'}\left(1, l_t^0\right) \tag{12}$$

for the competitive firm. The following proposition features the FCWE which satisfies arbitrage conditions of the consumers.

**Proposition 2** *The equality of all returns to capital $R_t^i$ implies that noncompetitive firms have identical labor capital ratios $l_t^i = l_t^1$ and identical capital stocks $K_t^i = K_t^1$. Moreover, the equality of returns to capital between competitive and noncompetitive firms results in the following relation between $l_t^0$ and $l_t^1$:*

$$F\left(1, l_t^1\right) - l_t^1 F_L^{'}\left(1, l_t^0\right) = (1 - \theta_{t-1}) F_K^{'}\left(1, l_t^0\right) \tag{13}$$

*Proof.* See Appendix 2. ■

At equilibrium, the labor-capital ratios of the noncompetitive firms are equal and smaller than the total labor-capital ratio in the economy, $\bar{l}_t = N_t/(K_t^0 + mK_t^1)$. Reducing their labor demand, the noncompetitive firms induce a lower wage in the competitive sector. This increases the capital return in the noncompetitive sector and in the competitive sector. On the one hand, for the competitive sector, as $\bar{l}_t < l_t^0$, it follows $R_t^0 = F_K^{\prime}(1, l_t^0) > F_K^{\prime}(1, \bar{l}_t)$ where $F_K^{\prime}(1, \bar{l}_t)$ is the capital return at the competitive equilibrium. On the other hand, for the noncompetitive sector at equilibrium, by definition (cf. appendix 1),

$$l_t^1 = \arg\max_l \qquad \pi_t(l, l_t^1)$$
$$\text{s. t. } 0 \leq l \leq \frac{N_t}{K_t^1} - (m-1)l_t^1$$

where

$$\pi_t(l, l_t^1) \equiv F(1, l) - lF_L'\left(1, \frac{N_t - lK_t^1 - (m-1)l_t^1 K_t^1}{K_t^0}\right)$$

Since

$$0 \leq \bar{l}_t \leq \frac{N_t}{K_t^1} - (m-1)l_t^1$$

the competitive labor-capital ratio $\bar{l}_t$ is achievable for the noncompetitive firm. Thus, we have:

$$\pi_t(l_t^1, l_t^1) > \pi_t(\bar{l}_t, l_t^1)$$

and since $\pi_t(l, l_t^1)$ is decreasing with respect to $l_t^1$, we obtain:

$$R_t^1 = \pi_t(l_t^1, l_t^1) > \pi_t(\bar{l}_t, l_t^1) > \pi_t(\bar{l}_t, \bar{l}_t) = F_K'(1, \bar{l}_t)$$

At equilibrium, the capital return in the noncompetitive sector is greater than its level at the competitive equilibrium without pension funds.

Notice that the comparison of equilibrium capital returns leads to

$$R_t^0 = \frac{R_t^1}{1 - \theta_{t-1}} > R_t^1 > F_K'(1, \bar{l}_t)$$

Noncompetitive firms exert a positive externality on competitive firms. The former push down the equilibrium wage by reducing their labor-capital ratio, which is detrimental for their capital return. The latter benefit from this fall in the wage rate and take advantage of a higher labor-capital ratio. A positive subsidy rate allows for the equality of net returns.

**Remark 1** *The number of pension funds $m$ is exogenously given. An interesting extension would be to make this parameter endogenous. For instance, one may introduce a fixed cost for each pension fund that can be interpreted as the cost for activism: managers monitoring, lobbying, etc... Under this assumption, the gross capital return $R^1$ increases with the capital stock held by the pension fund. Therefore, the number of pension funds would be the maximum value of $m$ compatible with the equality between the gross capital return $R^1$ and the competitive return $R^0$.*

## 2.4 Intertemporal equilibrium

Pension funds are created in period 0. In period 0, the capital stock is given and all firms are competitive. Production is $F(K_0, N_0)$, and prices are

$$w_0 = F_L'(K_0, N_0) \quad \text{and} \quad R_0^0 = F_K'(K_0, N_0) \tag{14}$$

There are $N_{-1}$ old agents in $t = 0$, who hold an equal part $s_{-1} = K_0/N_{-1}$ of the initial capital stock. They allocate their income $R_0 s_{-1} - \tau_0^2$ between consumption and bequests

$$R_0^0 s_{-1} - \tau_0^2 = d_0 + (1+n)x_0, \quad x_0 \geq 0 \tag{15}$$

satisfying

$$-(1+n)U_d^{'}(c_{-1}, d_0) + \gamma U_c^{'}(c_0, d_1) \leq 0, \quad = 0 \text{ if } x_0 > 0 \tag{16}$$

where $c_{-1}$ is given.

At equilibrium, taxes are equal to subsidies: $N_t \tau_t^1 + N_{t-1}\tau_t^2 = N_t \theta_t s_t^1$, or

$$\tau_t^1 + \frac{1}{1+n}\tau_t^2 = \theta_t s_t^1 \tag{17}$$

and capital stocks result from savings of the preceding period. Its allocation with equality of $K_{t+1}^i$ satisfies

$$K_{t+1}^0 = N_t s_t^0 \ , \quad K_{t+1}^i = \frac{1}{m} N_t s_t^1 \tag{18}$$

**Definition 3** *Given $K_0$, $c_{-1}$ and a sequence of subsidy rates $(\theta_t)_{t\geq 0}$ and taxes $(\tau_t^1, \tau_t^2)_{t\geq 0}$, an intertemporal equilibrium is defined by wages and capital returns, individual choices, capital and labor allocation, which constitute at each period a non trivial FCWE with equal net returns to capital, and which satisfy all the equilibrium conditions (1)-(7) and (8)-(18).*

**Steady state**

Let us denote the capital stocks per young $k_t^0 = \frac{K_t^0}{N_t}$ and $k_t^i = \frac{K_t^i}{N_t} = k_t^1$. We assume that the subsidy rate $\theta$ and the tax levels taxes $\tau^1$ and $\tau^2$ are constant. A steady-state equilibrium consists of

(i) wages and capital returns

$$w = F_L^{'}(1, l^0), \quad R^0 = F_K^{'}(1, l^0) \tag{19}$$

$$R^1 = F(1, l^1) - F_L^{'}(1, l^0)l^1 = (1-\theta)R^0 \tag{20}$$

(ii) individual choices, such that

$$x + w - \tau^1 = c + \sigma, \quad R^0\sigma - \tau^2 = d + (1+n)x \text{ with } x \geq 0 \tag{21}$$

$$U_c^{'}(c, d) = R^0 U_d^{'}(c, d), \tag{22}$$

$$-(1+n)U_d^{'}(c, d) + \gamma U_c^{'}(c, d) \leq 0, \quad = 0 \text{ if } x > 0 \tag{23}$$

(iii) the long run FCWE equations

$$F_L^{'}(1, l^1) - F_L^{'}(1, l^0) + l^1 \frac{k^1}{k^0} F_{LL}^{''}(1, l^0) = 0 \tag{24}$$

$$l^0 k^0 + m l^1 k^1 = 1 \tag{25}$$

(iv) other equilibrium conditions (respectively the capital market equilibrium and the government budget constraint)

$$\sigma = (1+n)[k^0 + (1-\theta)mk^1] \tag{26}$$

$$\tau^1 + \frac{1}{1+n}\tau^2 = \theta mk^1 \tag{27}$$

**Remark 2** *If $\theta = 0$ (no subsidy), we obtain the standard competitive equilibrium. As we have mentioned above, competitive firms offer a higher capital return than noncompetitive firms since they benefit from the fall in the wage rate without having to reduce their labor-capital ratio. A positive subsidy rate allows the equality of net returns and then makes possible the noncompetitive behavior of the pension funds. If the subsidy rate is zero, pension funds are constrained to behave competitively, otherwise they would not collect any savings. But if they behave competitively, they have no impact on the equilibrium. Associating positive subsidy rate $\theta > 0$ with non-competitive behavior allows to go further the conventional wisdom of the neutrality of pension funds.*

Two types of long-run equilibria may exist: equilibrium with positive bequests ($x > 0$) and equilibrium with constrained bequests ($x = 0$). In these two types of equilibria, pension funds may involve very different consequences on long-run capital accumulation and welfare. If bequests are operative, both wages and capital revenues are sources of savings. If bequests are non operative, the only source of savings is wages. In the former case, since pension funds tend to increase capital revenues and to decrease wages, one would expect a positive effect of pension funds on capital accumulation. In the latter case, however, pension funds could diminish capital stock. We successively analyze both cases in the two following sections.

**Remark 3** *Existence of the equilibrium goes beyond this study and is deferred for further work. Several studies have analysed the existence issue in the standard Barro's model without pension funds ($\theta = 0$): see Thibault (2000) and Michel, Thibault and Vidal (2006). The introduction of pension funds adds a new difficulty: the game between non-competitive firms affects factor prices through the allocation of labor and capital between the two sectors.*

# 3 Operative bequests in the long run

The assumption of positive bequests implies that the arbitrage condition (23) is verified with equality. This determines return on capital of competitive firms (equations (19), (22) and (23))

$$R^0 = F_K^{'}(1, l^0) = \frac{1+n}{\gamma} \tag{28}$$

Thus, the labor-capital ratio of the competitive firms is the modified golden-rule level: $l^0 = l^*$, defined by $F_K^{'}(1, l^*) = \frac{1+n}{\gamma}$.

## 3.1 Effect of pension funds on capital accumulation

We analyze the effect of pension funds using the subsidy rate $\theta$. In fact, this parameter governs the size of pension funds and so, the fraction of total capital stock held by the noncompetitive firms. When $\theta = 0$, this fraction falls to zero and the economy reaches the standard competitive equilibrium without pension funds. Moreover, $\theta$ increases total capital accumulation by subsidizing savings. Thus, we proceed by analyzing the effect of $\theta$ on the capital accumulation and on the allocation of the capital stock between the two sectors.

Since the labor-capital ratio in the competitive sector corresponds to the modified golden rule, the arbitrage condition (20) determines the labor-capital ratio in the noncompetitive firms. Then, the long-run equilibrium conditions (24) and (25) determine the capital stocks. We study the effect of $\theta$ on capital accumulation, more precisely on both $k^0$ and $k^1$ in the long run FCWE.

The labor-capital ratio of the noncompetitive firms $l^1$ is the solution of

$$(1-\theta)\frac{1+n}{\gamma} = F(1,l^1) - F_L'(1,l^*)l^1 \equiv R(l^1) \tag{29}$$

When $l^1$ increases from 0 to $l^*$, $R(l^1)$ increases from $R(0) = F(1,0)$ to $R(l^*) = F_K'(1,l^*) = \frac{1+n}{\gamma}$. Thus, (29) defines a decreasing function $l^1(\theta)$, and $l^1(\theta)$ tends to 0 when $\theta$ tends to $\bar{\theta} = 1 - \frac{\gamma}{1+n} F(1,0)$. The condition on the subsidy rate for the existence of an equilibrium (with positive bequests) is $0 \leq \theta < \bar{\theta}$. We have $\bar{\theta} = 1$ if and only if $F(1,0) = 0$.

**Proposition 3** *When $\theta$ increases from 0 to $\bar{\theta}$, the capital per young agent in the competitive sector $k^0(\theta)$ decreases, the capital per young agent in each noncompetitive firm $k^1(\theta)$ increases, and total capital per young agent $k(\theta) = k^0(\theta) + mk^1(\theta)$ increases; $k^1(\theta)$ and $k(\theta)$ increase without limit when $\theta$ tends to $\bar{\theta}$. Given $\theta$, $0 < \theta < \bar{\theta}$, increasing the number $m$ of noncompetitive firms implies a decrease in $k^0$ and $k^1$ and an increase in $mk^1$ and $k$.*

*Proof.* See Appendix 3. ■

When the subsidy rate increases, the capital stock per young agent in the competitive sector decreases. The relative weight of the competitive sector diminishes. Since, at the steady-state, the labor-capital ratio in this sector is independent from the subsidy rate, the fraction of labor in the competitive sector decreases. Thus, necessarily, the fraction of labor in the noncompetitive sector increases. On the other hand, we have seen that the labor-capital ratio in the noncompetitive sector decreases and tends to zero when $\theta$ tends to the upper bound $\bar{\theta}$. Thus, the capital stock per young agent in the noncompetitive sector increases and grows without limit when $\theta$ tends to $\bar{\theta}$. The fraction of the capital stock invested in the competitive firms decreases with respect to $\theta$ and becomes negligible at the limit.

## 3.2 Effect of pension funds on welfare

An important question is for which values of the subsidy rate the long-run intertemporal utility with pension funds is larger than without pension funds. Recalling that the fully competitive case occurs when $\theta = 0$ (no subsidy), one only needs to study the effect of $\theta$ on welfare.

At the long-run steady state equilibrium, intertemporal utility

$$\sum_{t=0}^{+\infty} \gamma^t U\left(c_t, d_{t+1}\right)$$

is equal to $(1-\gamma)^{-1} U(c,d)$. So, we analyze the effect of pension funds of lifetime utility $U(c,d)$. Consumptions $c$ and $d$ can be determined by the arbitrage equation (22) and the resource constraint

$$c + \frac{d}{1+n} = k^0 F(1, l^*) + mk^1 F(1, l^1) - (1+n)(k^0 + mk^1) \tag{30}$$

With operative bequests, the long-run gross interest rate $R_0 = (1+n)/\gamma$ does not depend on the subsidy rate. Thus, the marginal rate of substitution between both consumptions $\frac{u_c'(c(\theta),d(\theta))}{u_d'(c(\theta),d(\theta))}$ is constant with $\theta$. It follows that the subsidy rate $\theta$ only affects consumptions $c$ and $d$ through the total production per young agent net of investment (RHS of equation (30))

$$z(\theta) = k^0 F(1, l^*) + mk^1 F(1, l^1) - (1+n)(k^0 + mk^1)$$

where $k^0$, $k^1$and $l^1$are functions of $\theta$.

The following proposition states conditions under which pension funds have a positive impact on long-run welfare. When bequests are operative ($x > 0$), the equilibrium does not depend on the allocation of taxes between young and old ($\tau^1$ and $\tau^2$). This results from the neutrality of net transfer between young and old. Without loss of generality, we focus on the case $\tau^2 = 0$.

**Proposition 4** *Assume $\tau^2 = 0$ and that bequests are operative in the economy without subsidy. Then, there exists a threshold value of the subsidy rate $\tilde{\theta}$ such that, for all $\theta \in \left(0, \tilde{\theta}\right)$, bequests are operative and total consumption per young agent is larger than its value in the economy without subsidy. As a consequence, the intertemporal utility of the agents at steady state is also larger than in the economy without subsidy.*

*Proof.* Let us define the net product function (see figure 1)

$$\phi(l) \equiv F(\frac{1}{l}, 1) - \frac{1+n}{l}. \tag{31}$$

which reaches its maximum at $\hat{l}$ (Golden-rule). The total production per young agent net of investment can be rewritten as

$$z(\theta) = \frac{L_t^0}{N_t}\phi\left(l^*\right) + \frac{mL_t^1}{N_t}\phi\left(l^1\right) \tag{32}$$

where $L_t^0 + mL_t^1 = N_t$. Thus, $z(\theta)$ appears as an average of $\phi(l^*)$ and $\phi(l^1)$, weighted by the shares of the total labor force $N_t$ employed in each sector. Moreover, without pension funds (i.e. $\theta = 0$), production per young agent reaches the modified golden-rule level $z(0) = \phi(l^*)$.

Let us define $\tilde{l}$, such that

$$\phi\left(\tilde{l}\right) = \phi\left(l^*\right). \tag{33}$$

Since $l^1(\theta)$ decreases from $l^*$ to 0 when $\theta$ goes from 0 to $\bar{\theta}$, then there exists a unique value $\tilde{\theta}$ of the subsidy rate such that $l^1\left(\tilde{\theta}\right) = \tilde{l}$ and we have[8]

$$\tilde{\theta} = (1-\gamma)\left(1 - \frac{\tilde{l}}{l^*}\right) \tag{34}$$

Thus, from equation (32), $z(\theta)$ is higher than $z(0)$ for all values of $\theta$ such that $\tilde{l} < l^1(\theta) < l^*$.

For any $\theta$ in $(0, \tilde{\theta})$, as long as bequests are positive, total consumption $z(\theta)$ is larger than its value for $\theta = 0$. Thus, at least one consumption will also increase. Moreover, differentiating $U_c^{'}(c,d) = \frac{1+n}{\gamma}U_d^{'}(c,d)$ with respect to $z$, one obtains

$$\frac{dc}{dz}\left[-U_{cc}'' + \frac{U_c'}{U_d'}U_{cd}''\right] = \frac{dd}{dz}\left[-\frac{U_c'}{U_d'}U_{dd}'' + U_{cd}''\right]$$

where Assumption 1 implies that both terms into brackets are positive. Thus, $c$ and $d$ are increasing with respect to $z$. Consequently, for any $\theta$ in $(0, \tilde{\theta})$, the life-cycle utility $U(c(\theta), d(\theta))$ and the altruistic utility $U(c(\theta), d(\theta))/(1-\gamma)$ are larger than their levels for $\theta = 0$.

If $\tau^2 = 0$, bequest writes

$$x(\theta) = c(\theta) + \sigma(\theta) + \tau^1(\theta) - w^* = c(\theta) + (1+n)k(\theta) - F_L^{'}(1, l^*).$$

From Proposition 3, $k(\theta)$ is increasing, and for $0 < \theta < \tilde{\theta}$, $c(\theta) > c(0)$. This implies $x(\theta) > x(0)$ and thus $x(\theta) > 0$ if $x(0) > 0$.■

[8] From (29) for $\theta = 0$ and $\theta = \tilde{\theta}$, one deduces that

$$\frac{1+n}{\gamma}\frac{1}{l^*} = F(\frac{1}{l^*}, 1) - F_L^{'}(1, l^*)$$

and

$$\left(1-\tilde{\theta}\right)\frac{1+n}{\gamma}\frac{1}{\tilde{l}} = F(\frac{1}{\tilde{l}}, 1) - F_L^{'}(1, l^*)$$

Taking the difference between the two equations, the characterization of $\tilde{\theta}$ is obtained from equation (33).

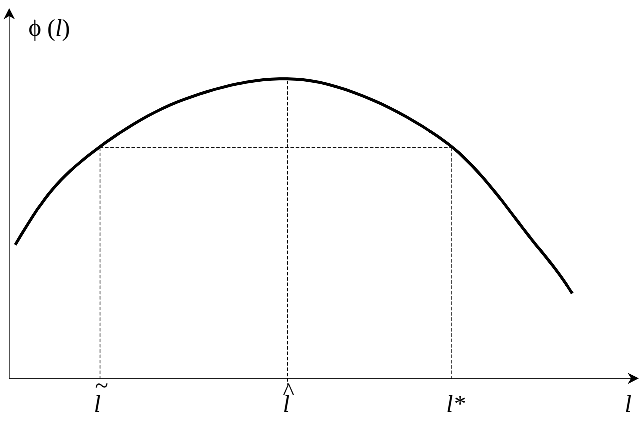


Figure 1: the function $\phi(l)$

The introduction of pension funds increases capital stock. In the economy without pension funds, a rise in capital stock leads to an increase in disposable product for consumption as long as the economy is in underaccumulation. However, pension funds introduce imperfect competition, which has a detrimental effect on the allocation of productive factors. Notably, capital marginal product in noncompetitive firms is equal to $(1-\theta)$ times capital marginal product in the competitive sector ($R^1 = (1-\theta)R^0$). Our results show that the disposable product for consumption continues to increase with $\theta$ above $\hat{\theta}$ such that $l^1(\hat{\theta}) = \hat{l}$. Thus, it increases with capital stock above the golden rule. But when $\theta$ (and capital stock) is larger, the share of the product used for investment becomes too high, and then consumptions decrease. At $\theta = \tilde{\theta}$, consumptions fall back to their level in the economy without pension funds. Thus utility is higher with pension funds when $0 < \theta < \tilde{\theta}$, and is maximum for some value $\theta^*$ such that $\hat{\theta} < \theta^* < \tilde{\theta}$.

**Remark 4** *For $\theta > \tilde{\theta}$, total consumption per young agent and welfare are lower in the economy with pension funds than in the economy without, as long as bequests are operative. When preferences are homothetic, bequests are operative for all $\theta$, $0 < \theta < \bar{\theta}$ (see Appendix 4).*

We represent the steady state utility level as a function of $\theta$ in the Cobb-Douglas case ($U(c,d) = \ln c + (1/3)\ln d$, $F(K,L) = K^{1/3}L^{2/3}$, $\gamma = 1/2$, $m = 1$, $n = 0$). In that case, $\bar{\theta} = 1$ and bequests are operative for all $\theta$. Figure 2 represents the life-cycle utility $U(\theta)$. In order to illustrate the distortions resulting from the noncompetitive allocation of productive factors, we also represent the utility $\hat{U}(\theta)$ which could be obtained with total capital stock $k(\theta)$, perfect competition and unchanged life-cycle arbitrage condition. In our model, the subvention rate has two effects: first, a rise in $\theta$ has the usual positive effect on capital accumulation; second, it raises the distortive effect of pension funds since their market power increases. Figure 2 illustrates the strength of this second effect. The larger $\theta$, the larger the difference $\hat{U}(\theta) - U(\theta)$ is.

[Insert Figure 2]

We have shown that pension funds could have positive effects on the steady-state intertemporal utility. Nevertheless, we are able to give some insights about the intertemporal utility of all generations from period 0. Let us assume that the subsidy rate allows a welfare improvement in the long-run. For the first generation, the competitive equilibrium (which is obtained without pension funds) is identical to the optimum of a social planner who would maximize the same discounted sum of utilities. Since the inception of pension funds introduces distortions through imperfect competition and the subsidy rate, the intertemporal utility of the first generation necessarily decreases. But pension funds increases capital accumulation in the long-run. Therefore, intertemporal utility increases from some future generation.

### 3.3 Effect of the number of noncompetitive firms

As seen in Proposition 3, the number $m$ of noncompetitive firms decreases the capital stock per young agent in the competitive sector but increases the total capital stock per young agent. We study the effect of $m$ on welfare in the following proposition.

**Proposition 5** *Assume $\tau^2 = 0$ and bequests are operative in the economy without pension funds. Then, for all $\theta$, $0 < \theta < \tilde{\theta}$, the utility of the agents at steady-state is increasing in $m$. For $\theta > \tilde{\theta}$, this utility is decreasing in $m$, as long as bequests are operative. The operative bequests property holds for all $\theta < \bar{\theta}$, when preferences are homothetic.*

*Proof.* The equilibrium value of noncompetitive labor-capital ratio $l^1$ is determined by the arbitrage condition (29) and does not depend on $m$. From Proposition 4,

$$z(\theta) - z(0) = mk^1 l^1 \left[\phi\left(l^1\right) - \phi\left(l^*\right)\right]$$

where $\phi(l^1) - \phi(l^*) > 0$ iff $\theta < \tilde{\theta}$. Thus, for $\theta < \tilde{\theta}$ ($> \tilde{\theta}$), the effect of $m$ on $z(\theta)$ is positive (negative) since $mk^1$ is increasing with $m$ (Proposition 3).

When preferences are homothetic, the property of increasing desired bequests (see Appendix 4) implies that if bequests are positive for $\theta = 0$, they are positive for all $\theta$, $0 < \theta < \bar{\theta}$.■

The addition of steady state resources for consumption, $z(\theta) - z(0)$ is proportional to $mk^1$, with a proportional factor independent of $m$ and positive (negative) if $\theta < \tilde{\theta}$ ($\theta > \tilde{\theta}$). As $mk^1$ increases with $m$, $z(\theta)$ increases if $\theta < \tilde{\theta}$ and decreases if $\theta > \tilde{\theta}$. Figure 3 represents $z(\theta) - z(0)$ for different values of $m$ and the same value of the other parameters as figure 2.

[Insert Figure 3]

As in Proposition 3, for a given value of $\theta$, an increase in the number of noncompetitive firms $m$ raises the share of capital stock held by the pension funds, leaving their labor-capital ratio $l^1$ unchanged. If the subsidy rate is sufficiently low ($0 < \theta < \tilde{\theta}$), the beneficial effect of pension funds is strengthened. Conversely, for a high subsidy rate ($\theta > \tilde{\theta}$), the detrimental effect of pension funds is worsened.

## 3.4 Robustness

We evaluate the robustness of our results in different contexts. First, we investigate the influence of distortive tax. Secondly, we study the impact of a change in the fertility rate in order to take the phenomenon of ageing into account.

### 3.4.1 Distortive taxes

Until now, we have assumed lump-sum taxation. One may wonder if pension funds would have similar consequences when tax instruments are distortive. In our model, labor supply is inelastic. Thus, tax $\tau^1$ paid by the young may be viewed as a tax on labor earnings without loss of generality. However, tax on the old should be distortive as it hits savings revenue. Let us assume that the lump-sum tax $\tau^2$ is replaced by a linear tax on savings revenue at rate $\eta$.

Under this assumption, equation (28) becomes

$$R^0 = F_K^{'}(1, l^0) = \frac{1+n}{(1-\eta)\,\gamma},$$

leading to a value $l^0(\eta)$ of the labor-capital ratio in the competitive sector which is greater than $l^*$. Tax on savings has the usual effect of increasing the labor-capital ratio. Nevertheless, the arbitrage condition (29) and the resource constraint (30) are not modified. Moreover, the marginal rate of substitution between both consumptions $\frac{u_c^{'}(c(\theta),d(\theta))}{u_d^{'}(c(\theta),d(\theta))} = (1-\eta)\,R^0 = \frac{1+n}{\gamma}$ remains unchanged.

The rise in the labor-capital ratio $l^0(\eta)$ enlarges the interval of values of labor-capital ratios of the noncompetitive sector $l^1$, leading to a higher level of long-run utility. This interval is denoted by $\left(\tilde{l}(\eta), l^0(\eta)\right)$. Figure 4 shows how this interval is widened when the distortive tax on savings is introduced. With equation (34), it appears that the threshold on the subsidy rate $\tilde{\theta}(\eta)$ that guarantees an increase of the long-run utility is higher.

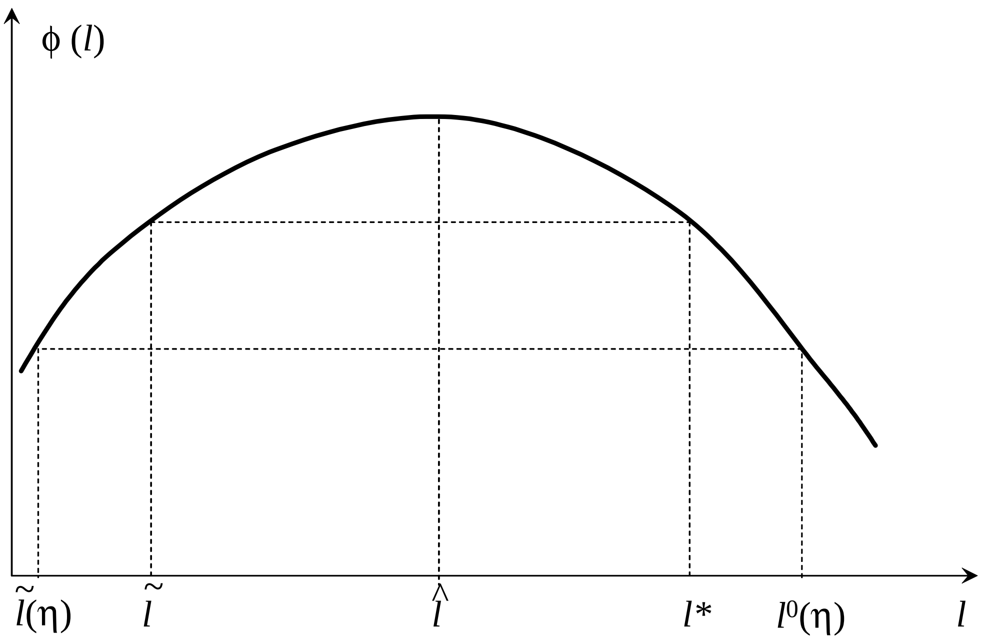


Figure 4: impact of distortive taxes

Linear taxes on savings lead to an underaccumulation of capital relative to the economy without distortive taxes. Since subsidies on pension funds tend to increase capital accumulation, taxes on savings increase the potential benefits of pension funds.

### 3.4.2 Ageing parameter

The fertility rate is the parameter of our model that allows us to consider change in the demographic structure. Whatever the fertility rate $n$ is, there exists an interval $\left(0, \tilde{\theta}(n)\right)$ of values of the subsidy rate such that the introduction of pension funds increases steady-state life-cycle utility. Let us study variations of $\tilde{\theta}(n)$ with respect to $n$. An increase in $n$ has two contradictory effects. (1) First, it results in a rise in the gross interest rate $R^0 = \frac{1+n}{\gamma}$ that leads to a larger labor-capital ratio $l^*(n)$. For a given function $\phi$, this implies a smaller value of $\tilde{l}(n)$. From equation (34), this first effect tends to increase the threshold $\tilde{\theta}$. (2) Secondly, the increase in $n$ moves the function $\phi$ downwards. This corresponds to the standard dilution effect: for a given labor capital ratio, an increase in the population growth rate leaves lower resources for consumption. This second effect increases the ratio $\tilde{l}(n)/l^*(n)$ and, from equation (34), reduces $\tilde{\theta}$. Consequently, the resulting effect is indeterminate.

From equation (33), we obtain

$$F(1, l^*) - (1+n) = F(\frac{l^*}{\tilde{l}}, l^*) - (1+n)\,\frac{l^*}{\tilde{l}} \tag{35}$$

where $F_K'(1, l^*) = \frac{1+n}{\gamma}$. From equation (34), the threshold $\tilde{\theta}$ is increasing with $\left(l^*/\tilde{l}\right)$. Furthermore, equation (35) determines $\left(l^*/\tilde{l}\right)$ as a function of the fertility rate $n$. By differentiation, one obtains

$$\frac{d\left(l^*/\tilde{l}\right)}{dn} = \frac{\left[F_L'(1, \tilde{l}) - F_L'(1, l^*)\right]\frac{dl^*}{dn} - \left(\frac{l^*}{\tilde{l}} - 1\right)}{(1+n) - F_K'(1, \tilde{l})}.$$

Recalling that $l^* > \hat{l} > \tilde{l}$, we deduce that the denominator is positive and that $F_L'(1, \tilde{l}) > F_L'(1, l^*)$. The first term in the numerator corresponds to the positive effect (1) on the threshold $\tilde{\theta}$. The second term corresponds to the negative dilution effect (2). The respective size of both effects depends on the shape of the production function.

For instance, for a Cobb-Douglas production function $F(K, L) = K^\alpha L^{1-\alpha}$, $0 < \alpha < 1$, the numerator can be rewritten as

$$\begin{aligned} & \frac{l^*}{1+n}\left(\left[F_L'(1, \tilde{l}) - F_L'(1, l^*)\right]\frac{1+n}{l^*}\frac{dl^*}{dn} - \left(\frac{1+n}{\tilde{l}} - \frac{1+n}{l^*}\right)\right) \\ = \; & \frac{l^*}{1+n}\left(\left[(l^*)^{-\alpha} - \frac{1+n}{l^*}\right] - \left[\left(\tilde{l}\right)^{-\alpha} - \frac{1+n}{\tilde{l}}\right]\right) = 0 \end{aligned}$$

where the last equality results from $\phi\left(\tilde{l}\right) = \phi\left(l^*\right)$. In this case, the fertility rate has no effect on the value of the threshold $\tilde{\theta}$.

# 4 Constrained bequests and change of regime

Until now, we have assumed that bequests were operative, so that savings were based on labor and capital incomes. In this section, we analyze the regime where the non-negativity constraint on bequests is binding. In this case, the intertemporal equilibrium with altruistic agents coincides with the equilibrium with non-altruistic agents: with no bequest, each agent simply consumes his life-cycle income and savings are only supported by current labor income.

As in the preceding section, we shall assume that the lump-sum taxes that finance the subsidies of pension funds are paid by young agents (i.e. $\tau^2 = 0$). This is the most favorable case for bequests to be operative. Moreover, suppose that, with $\tau^2 = 0$, bequests are zero in the economy with no pension funds ($\theta = 0$). By continuity, there exists an interval of values of the subsidy rate $(0, \underline{\theta})$ such that bequests are zero. The corresponding steady-state with zero bequest has been studied in detail by Belan, Michel and Wigniolle (2002, 2003) assuming a Cobb-Douglas production function. The main results are the following.

There exist three effects of pension funds on capital accumulation. On the one hand, there are two negative effects on the net wage income of young agents : taxation which finances the subsidies; a decrease in wages resulting from the behavior of the noncompetitive firms. On the other hand, there may be a positive effect of the increased savings returns, that occurs only if the substitution effect dominates the income effect (i.e. savings are an increasing function of capital return). When the substitution effect does not dominate the income effect, all effects are negative and capital accumulation decreases with $\theta$. This is in sharp contrast with the case of operative bequests.

When the economy is in underaccumulation without pension funds, steady-state welfare is always lower when pension funds are introduced (even if steady-state capital stock is higher). Only in overaccumulation, welfare is higher for small values of the subsidy rate $\theta$.

With altruistic agents when bequests are constrained, the equilibrium corresponds to the one obtained in an economy with egoistic agents. Thus, in the case of constrained bequests, the discussion above allows to understand the impact of pension funds on welfare. But, as stated in the following proposition, under the assumption of homothetic preferences, there exists a threshold $\underline{\theta}$ on the subsidy rate such that, if $\theta > \underline{\theta}$, bequests become operative. Therefore, if $\theta < \underline{\theta}$, the framework reduces to the one studied in Belan, Michel and Wigniolle (2002, 2003); if $\theta > \underline{\theta}$, results derived in section 3 of the current paper apply.

Let us define the desired bequest $x^d(\theta)$ as the solution of the steady-state equilibrium conditions with no restriction on the sign of $x$, i.e. condition (23) is verified with equality. It is supposed to be negative for $\theta = 0$. All the results (except the sign of $x$) of the preceding section apply to this "desired" steady-state.

**Proposition 6** *Assume $\tau^2 = 0$, $x^d(0) < 0$ and homothetic preferences. The desired bequest $x^d(\theta)$ is an increasing function of $\theta$ and it becomes positive after some threshold $\underline{\theta} < \bar{\theta}$. As a consequence, in the long run, bequests are constrained if $0 \leq \theta < \underline{\theta}$ and bequests are positive if $\underline{\theta} < \theta < \bar{\theta}$.*

*Proof.* See Appendix 4. ■

As a consequence, if bequests are constrained without pension funds, the introduction of pension funds with some appropriate subsidy rate ($\theta > \underline{\theta}$) will make bequests operative.

When bequests are operative, the analysis of the preceding section applies. Assuming underaccumulation of capital at the competitive equilibrium without pension funds ($\theta = 0$), utility decreases on the interval $(0, \underline{\theta})$. Then there are two cases:

- either utility increases in the operative bequests regime, reaches a maximum and this maximum is larger than the utility at $\theta = 0$ (case 1),
- or, for all $\theta > 0$, the utility remains lower in the economy with pension funds than in the economy without (case 2).

In the first case, the introduction of pension funds with an appropriate value of the subsidy rate can improve long-run intertemporal utility of the agents. In the second case, the welfare effect of pension funds is always negative whatever the subsidy rate is.

[Insert Figures 5 and 6]

The parameters are the same as before, except $\gamma$. A lower $\gamma$ increases modified Golden-Rule labor-capital ratio $l^*$, and reduces the desired bequest. This bequest at $\theta = 0$ is zero for $\gamma = 1/2$. Thus for $\gamma > 1/2$, bequests are always operative. For $\gamma < 1/2$, the desired bequest is negative at $\theta = 0$, and there is a change of regime at $\underline{\theta}$. Near $\gamma = 1/2$ (Figure 5 with $\gamma = 2/5$), we are in the first case : utility becomes larger than its value in the economy without pension funds, but for low $\gamma$, it does not (Figure 6 with $\gamma = 1/5$).

# 5 Conclusion

This paper studies the long-run effect of pension funds in an economy where people are altruistic. We assume that pension funds behave noncompetitively and that contributions are subsidized.

The main results are the following. If bequests are operative, total capital stock with pension funds is higher than without pension funds and increases with the subsidy rate. So, despite the fall in wages that occurs with the noncompetitive behavior of pension funds, savings increase because of the rise in capital income. This leads to an increase in long-run welfare as long as the subsidy rate remains under some threshold. Above this threshold, capital stock becomes too high, putting the economy in overaccumulation, and the distortions created by imperfect competition are too great. Moreover, given a not too large subsidy rate, a rise in the number of pension funds increases long-run utility.

Finally, with homothetic preferences, we show that an increase in the subsidy rate can move the economy from constrained bequests to operative bequests. Then, if bequests are constrained in the economy without pension funds, the positive effect of pension funds on welfare is no longer guaranteed.

## Appendix 1. Characterization of the Firms' Cournot-Walras Equilibrium (FCWE)

We first characterize the best-response function of noncompetitive firms.

**Lemma 1** *Under assumption $F_{LLL}''' \geq 0$ and given labor capital ratios of other non-competitive firms $\left(l_t^j\right)_{j\neq i}$ which satisfies $\sum_{j\neq i} l_t^j K_t^j < N_t$, there exists a unique best response function $l_t^i$ for the firm $i$. It is the solution of*

$$F_L'\left(1, l_t^i\right) - F_L'\left(1, l_t^0\left(\cdot\right)\right) + l_t^i \frac{K_t^i}{K_t^0} F_{LL}''\left(1, l_t^0\left(\cdot\right)\right) = 0 \tag{36}$$

*where $l_t^0\left(\cdot\right) \equiv \frac{N_t - \sum_{j=1}^m l_t^j K_t^j}{K_t^0}$ is a function of all labor capital ratios $(l_t^i)_{i=1,...,m}$. It satisfies $l_t^i < l_t^0\left(\cdot\right)$.*

*Proof.* Since labor supply to the competitive firm is $N_t - \sum_{i=1}^m L_t^i$, wages are a function of labor demands $L_t^i$, $i = 1, ..., m$,

$$w_t = F_L'\left(K_t^0, N_t - \sum_{i=1}^m L_t^i\right) \equiv \omega_t\left(L_t^1, ..., L_t^m\right).$$

We write profits of firm $i$ as a function of the labor demands $\left(L_t^1, ..., L_t^m\right)$ :

$$\Pi_t^i = F\left(K_t^i, L_t^i\right) - \omega_t\left(L_t^1, ..., L_t^m\right) L_t^i.$$

Let us define

$$\pi_t^i \equiv \frac{\Pi_t^i}{K_t^i} = F\left(1, l_t^i\right) - l_t^i F_L'\left(1, \frac{N_t - l_t^i K_t^i - L_t^{-i}}{K_t^0}\right)$$

where $L_t^{-i} \equiv \sum_{j\neq i} L_t^j < N_t$. Profits maximization is equivalent to maximization of $\pi_t^i$ with respect to $l_t^i \in \left(0, (N_t - L_t^{-i})/K_t^i\right)$. First order condition is

$$\frac{\partial \pi_t^i}{\partial l_t^i} = F_L'\left(1, l_t^i\right) - F_L'\left(1, \frac{N_t - l_t^i K_t^i - L_t^{-i}}{K_t^0}\right) + l_t^i \frac{K_t^i}{K_t^0} F_{LL}''\left(1, \frac{N_t - l_t^i K_t^i - L_t^{-i}}{K_t^0}\right) = 0.$$

At $l_t^i$ goes to 0, we have $\frac{\partial \pi_t^i}{\partial l_t^i} > 0$, since $F_L'(1,0) > F_L'(1, (N_t - L_t^{-i})/K_t^0)$. Moreover, if $l_t^i \geq (N_t - L_t^{-i})/(K_t^0 + K_t^i)$, then $l_t^i \geq (N_t - l_t^i K_t^i - L_t^{-i})/K_t^0$, and we have $\frac{\partial \pi_t^i}{\partial l_t^i} < 0$. Thus, there exists $l_t^i \in \left]0, (N_t - L_t^{-i})/K_t^i\right[$ where $\pi_t^i$ reaches its maximum.

The second partial derivative of $\pi_t^i$ is

$$\frac{\partial^2 \pi_t^i}{\partial l_t^{i2}} = F_{LL}''\left(1, l_t^i\right) + 2\frac{K_t^i}{K_t^0} F_{LL}''\left(1, l_t^0\right) - l_t^i \left(\frac{K_t^i}{K_t^0}\right)^2 F_{LLL}'''\left(1, l_t^0\right)$$

where $l_t^0 \equiv \frac{N_t - \sum_{j=1}^m l_t^j K_t^j}{K_t^0}$. Since $F_{LLL}''' \geq 0$, $\pi_t^i$ is strictly concave and the first order conditions are sufficient.

The first-order condition (36) implies $F_L^{'}(1,l_t^i) > F_L^{'}(1,l_t^0)$. Thus $l_t^i < l_t^0$. ■

We now prove Proposition 1. Equation (36) writes as:

$$G_t\left(l_t^i, l_t^0(\cdot)\right) = 0$$

where

$$G_t\left(l^i, l^0\right) \equiv F_L^{'}\left(1, l^i\right) - F_L^{'}\left(1, l^0\right) + l^i \frac{K_t^i}{K_t^0} F_{LL}^{''}\left(1, l^0\right).$$

The function $G_t\left(l^i, l^0\right)$ has the following properties, for any $l^0 > 0$:

$$G_t\left(0, l^0\right) = F_L^{'}(1,0) - F_L^{'}\left(1, l^0\right) > 0$$

and

$$G_t\left(l^0, l^0\right) = l^0 \frac{K_t^i}{K_t^0} F_{LL}^{''}\left(1, l^0\right) < 0.$$

As

$$\frac{\partial G_t}{\partial l^i} = F_{LL}^{''}\left(1, l^i\right) + \frac{K_t^i}{K_t^0} F_{LL}^{"}\left(1, l^0\right) < 0$$

and

$$\frac{\partial G_t}{\partial l^0} = -F_{LL}^{''}\left(1, l^0\right) + l^i \frac{K_t^i}{K_t^0} F_{LLL}^{'''}\left(1, l^0\right) > 0$$

the equation $G_t\left(l^i, l^0\right) = 0$ defines a function $l^i = \xi_t^i\left(l^0\right)$, which is continuous and increasing on $\mathbb{R}_+^*$. The equation $G_t\left(l^i, l^0\right) = 0$ implies $F_L^{'}\left(1, l^i\right) > F_L^{'}\left(1, l^0\right)$, and thus $l^i = \xi_t^i\left(l^0\right) < l^0$. It follows that $\xi_t^i\left(l^0\right)$ tends to 0 when $l^0$ tends to 0.

Let us now consider the market equilibrium condition

$$G_t\left(l_t^i, l_t^0\right) = 0$$

which is equivalent to

$$l_t^i = \xi_t^i\left(l_t^0\right).$$

The equilibrium on the labor market is equivalent to:

$$K_t^0 l_t^0 + \sum_{i=1}^{m} \xi_t^i\left(l_t^0\right) K_t^i = N_t$$

The left-hand side goes from 0 to $+\infty$ as $l_t^0$ goes from 0 to $+\infty$. Thus, this equation defines a unique value for $l_t^0$ and $l_t^i = \xi_t^i\left(l_t^0\right)$, $i = 1, ...., m$. ■

## Appendix 2. Proof of Proposition 2.

The equality of gross returns on all investments

$$R_t^i = R_t^1, \ \forall i = 2, ..., m.$$

and

$$R_t^1 = (1-\theta_{t-1})R_t^0.$$

implies the equality $l_t^i = l_t^1$, $i = 2, ..., m$. Indeed, for the equilibrium value $l_t^0$, the derivative of $F(1,l^i) - l^i F_L^{'}(1,l_t^0)$ with respect to $l^i$ is $F_L^{'}(1,l^i) - F_L^{'}(1,l_t^0)$, and is positive for $l^i < l_t^0$. Thus the labor-capital ratio $l_t^i < l_t^0$ such that $R_t^i = (1-\theta_{t-1})R_t^0$ is unique and the equality of returns implies the equality of labor-capital ratio of all noncompetitive firms $l_t^i = l_t^1$, $i = 2, ..., m$. Given $l_t^0$, equation (8) $F_L^{'}(1,l_t^1) - F_L^{'}(1,l_t^0) + l_t^1 \frac{K_t^i}{K_t^0} F_{LL}^{"}(1,l_t^0) = 0$ implies $K_t^i = K_t^1$, $i = 2, ..., m$. We deduce immediately $L_t^i = L_t^1$, $i = 2, ..., m$.

## Appendix 3. Effect of pension funds on capital accumulation.

We define the labor ratio $\rho = \frac{L_t^1}{L_t^0} = \frac{l^1 k^1}{l^0 k^0} = \frac{F_L^{'}(1,l^1) - F_L^{'}(1,l^*)}{-F_{LL}^{"}(1,l^*)l^*}$ from (24).

When $\theta$ increases from 0 to $\bar{\theta}$, $l^1(\theta)$ decreases from $l^*$ to 0; thus $\rho(\theta)$ increases from 0 to

$$\bar{\rho} = \frac{F_L^{'}(1,0) - F_L^{'}(1,l^*)}{-F_{LL}^{"}(1,l^*)l^*}.$$

$\bar{\rho}$ is equal to $+\infty$ if and only if $F_L^{'}(1,0) = +\infty$.

With (25), $1 + m\rho = \frac{1}{l^* k^0}$. Thus, $k^0(\theta)$ decreases from $1/l^*$ to $1/\left[l^*(1+m\bar{\rho})\right]$ when $\theta$ increases from 0 to $\bar{\theta}$.

Thus, $ml^1k^1 = 1 - l^*k^0$ increases from 0 to $\frac{m\bar{\rho}}{1+m\bar{\rho}}$, and $k^1(\theta)$ increases from 0 to $+\infty$. Finally, total capital per young agent $k = k^0 + mk^1 = \frac{1}{l^*}\left(1 + mk^1(l^* - l^1)\right)$ increases from $1/l^*$ to $+\infty$.

Let us study the effect of $m$. For given $\theta$, $k^0 = \frac{1}{l^*(1+m\rho)}$ decreases with $m$ (since $\rho$ does not depend on $m$). Moreover $l^1k^1 = \frac{1}{m}\left(1 - l^*k^0\right) = \frac{\rho}{1+m\rho}$ decreases with $m$. Since $l^1$ and $\rho$ do not depend on $m$, $k^1$ decreases with $m$, but $mk^1$ increases with $m$. Thus, the total capital stock $k = \frac{1}{l^*}\left(1 + mk^1(l^* - l^1)\right)$ increases with $m$.

## Appendix 4. Study of the desired bequest function.

We study the steady-state equilibrium satisfying equations (19)-(27), with equation (23) satisfied with equality and assuming $\tau^2 = 0$.

The function $x^d(\theta)$ is defined by

$$x^d(\theta) = c(\theta) + (1+n)k(\theta) - F_L^{'}(1,l^*).$$

With the assumption of homothetic preferences, the condition $U_c^{'}(c,d) = \frac{1+n}{\gamma}U_d^{'}(c,d)$ implies that $c(\theta)$ is in proportion to $d(\theta)$. Thus, we have

$$c(\theta) = \lambda z(\theta), \quad \text{with} \;\; z(\theta) = c(\theta) + \frac{d(\theta)}{1+n} \;\; \text{and} \;\; 0 < \lambda < 1.$$

We deduce $c(\theta) + (1+n)k(\theta) = \lambda z(\theta) + (1+n)k(\theta)$ and

$$\frac{\partial x^d}{\partial \theta} = \lambda \frac{\partial z}{\partial \theta} + (1+n)\frac{\partial k}{\partial \theta}.$$

Since

$$z(\theta) = k^0 F(1, l^*) + F(k - k^0, 1 - k^0 l^*) - (1+n)k,$$

we obtain $\frac{\partial z}{\partial \theta} = A\frac{\partial k^0}{\partial \theta} + B\frac{\partial k}{\partial \theta}$, where $A(\theta) = F(1, l^*) - F_K^{'}(1, l^1) - l^* F_L^{'}(1, l^1)$, and $B(\theta) = F_K^{'}(1, l^1) - (1+n)$. The partial derivative of $A$ is

$$\frac{\partial A}{\partial \theta} = \frac{\partial l^1}{\partial \theta}\left[-F_{KL}^{"}(1, l^1) - l^* F_{LL}^{"}(1, l^1)\right] = \frac{\partial l^1}{\partial \theta}(l^1 - l^*)F_{LL}^{"}(1, l^1),$$

since $F_{KL}^{"}(1, l^1) = -l^1 F_{LL}^{"}(1, l^1)$. We have $\frac{\partial l^1}{\partial \theta} < 0$, $l^1 < l^*$ and $F_{LL}^{"} < 0$. This implies $\frac{\partial A}{\partial \theta} < 0$. Since $A(0) = F(1, l^*) - F_K^{'}(1, l^*) - l^* F_L^{'}(1, l^*) = 0$, we have $A(\theta) < 0$ for all $\theta > 0$ and $A\frac{\partial k^0}{\partial \theta} > 0$.

Thus, we obtain

$$\frac{\partial x^d}{\partial \theta} = \lambda A \frac{\partial k^0}{\partial \theta} + \left[\lambda F_K^{'}(1, l^1) + (1+n)(1-\lambda)\right]\frac{\partial k}{\partial \theta}.$$

Since $A\frac{\partial k^0}{\partial \theta}$ is non-negative and $\frac{\partial k}{\partial \theta}$ is positive (Proposition 3), we deduce that for all $\theta$, $0 < \theta < \bar{\theta}$, $\frac{\partial x^d}{\partial \theta} > 0$.

Let us now show that bequests are positive for $\theta$ sufficiently large. As long as bequests are zero, the income of young agents is $F_L^{'}(1, l^*) - \tau^1 = F_L^{'}(1, l^*) - \theta m k^1(\theta)$. For $\theta$ sufficiently high, we have $\theta m k^1(\theta) > F_L^{'}(1, l^*)$, since $\theta m k^1(\theta)$ tends to $+\infty$ when $\theta$ tends to $\bar{\theta}$. Thus, bequests are necessarily positive after some $\underline{\theta} < \bar{\theta}$.

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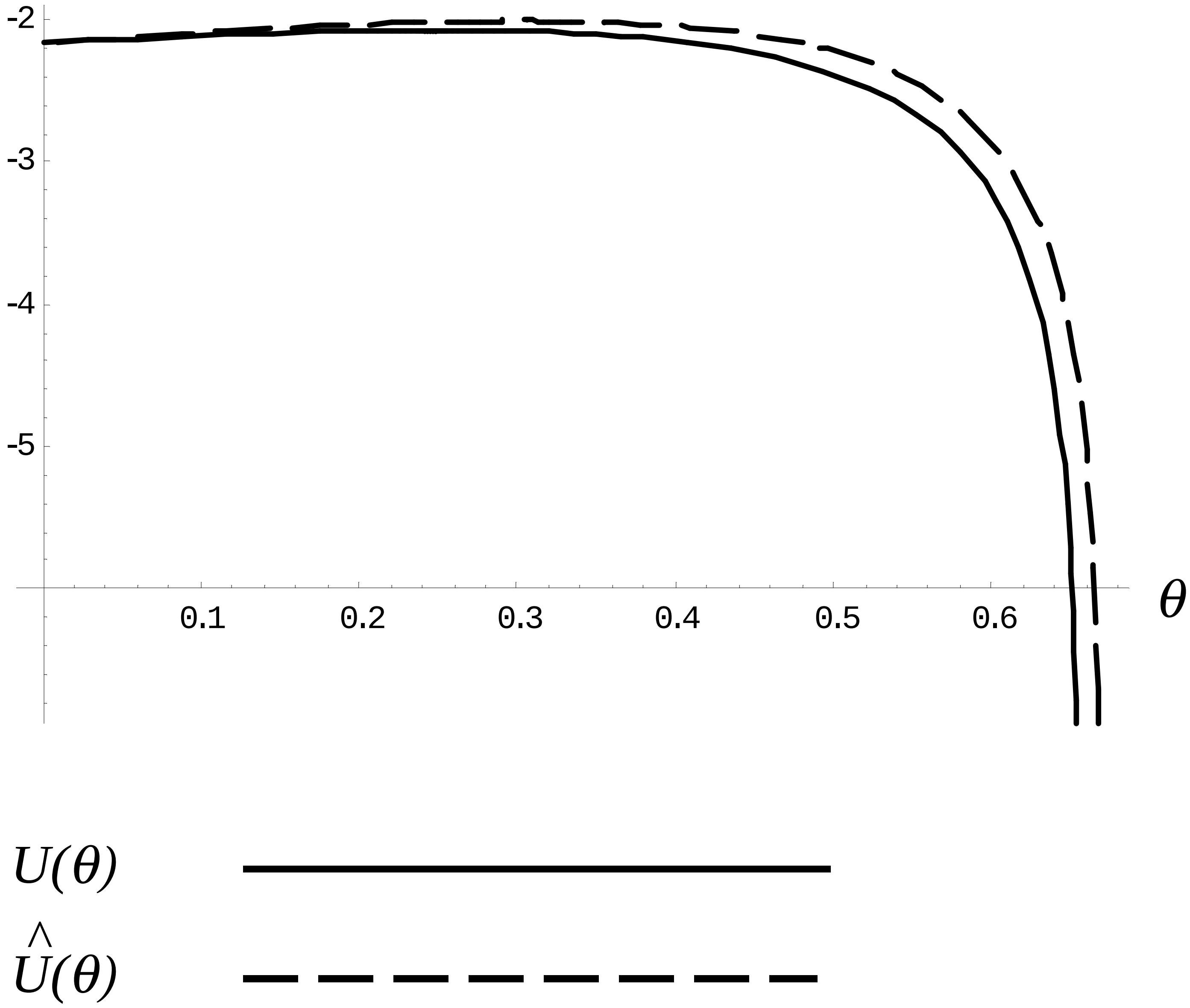



Figure 2

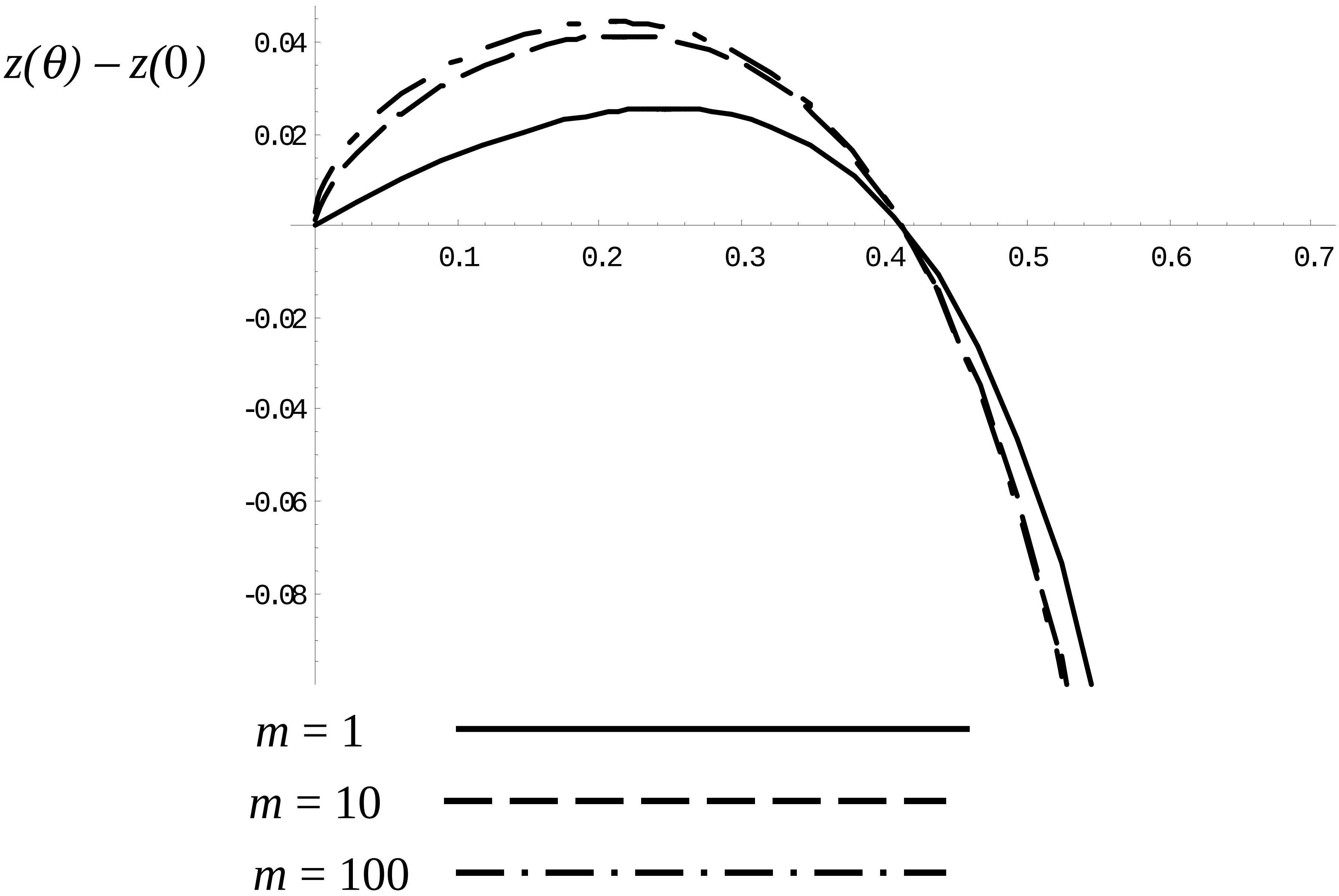
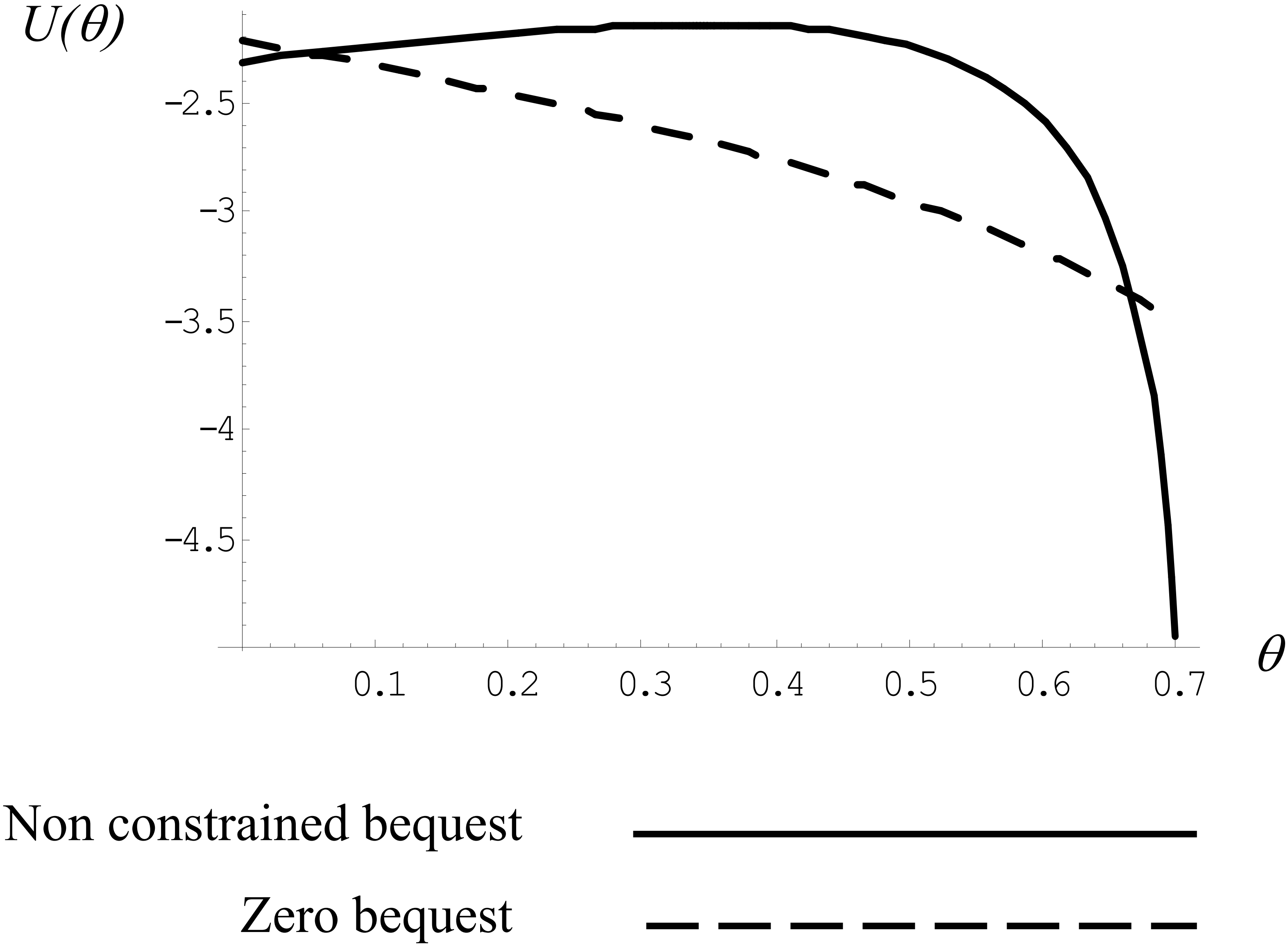



Figure 3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-3
-3.5
-4
-4.5
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0.3
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0.6
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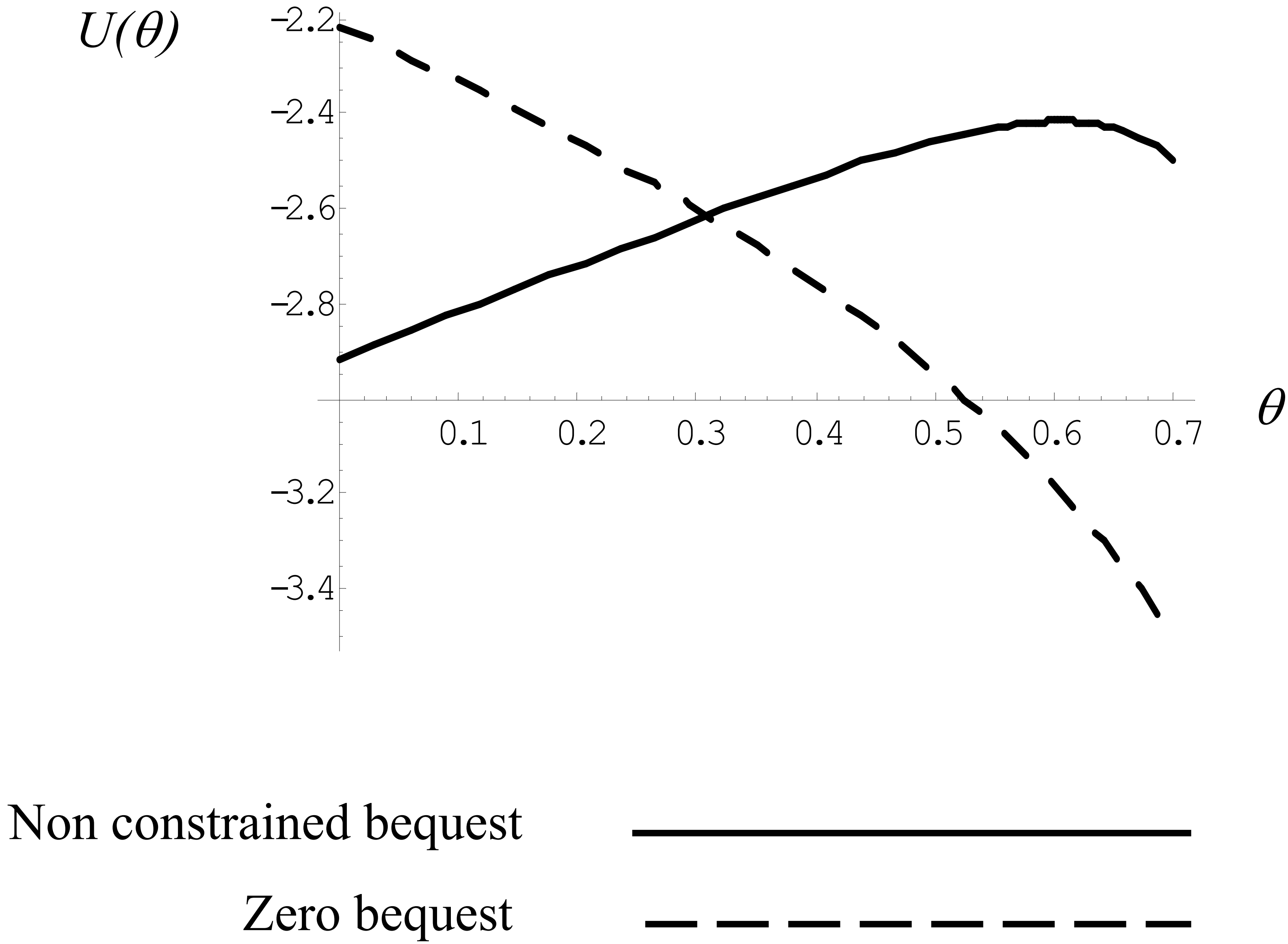
θ
Non constrained bequest
Zero bequest

Figure 5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-3.4
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0.2
0.3
0.4
0.5
0.6
0.7
θ
Non constrained bequest
Zero bequest

Figure 6