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Abstract

We analyze the relationship between imperfect competition and capital accumulation in a dual economy, with traditional and modern sectors and two types of agents (workers and capitalists). Workers allocate their time endowment between the two sectors. Capitalists accumulate wealth in the modern sector. The economy is open to capital flows, but capitalists face borrowing constraints. Non-competitive behavior of capitalists results in a rent which is extracted from the workers and lowers employment in the modern sector. In the long-run, if capitalists are unconstrained, imperfect competition is beneficial for capital accumulation and growth, while it is detrimental in the converse case. Moreover, not-binding borrowing constraints lead to higher employment and wages. This can motivate the introduction of a subsidy on bequests which allows the economy to reach the unconstrained regime, and is welfare-enhancing for workers.

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1 Introduction

Does imperfect competition foster capital accumulation in a developing economy ? This paper is an attempt to answer this question. We explore the relationship between imperfect competition and capital accumulation, in a dual economy. Considering an economy with two sectors, a traditional agricultural sector and a modern capitalistic one, we assume a non-competitive behavior of capitalists in the modern sector. Capitalists take into account the effect of their labor demand on the equilibrium wage. This behavior results in an extra rent for them, but introduces a distortion in factor prices. We aim at evaluating its effect on capital accumulation.

Since Lewis (1954) contribution (developed through a formal model in Ranis and Fei (1961)), numerous articles have explored issues relating to the migration of workers from a traditional sector to a modern one. Development is viewed as an unbalanced growth process, in which labor shifts toward the manufacturing sector. Capital accumulation in this sector increases labor productivity and results in higher wages, attracting more and more workers. The article by Kongsamut, Rebelo and Xie (2001) can be viewed as a modern reformulation of the argument, which deals with three sectors of activity (agriculture, manufacturing and services), and shows how such models are consistent with historical stylized facts.

As it is apparent in Lewis' model, a direct consequence of the dual production structure is that the two sectors have conflicting interests. Capitalists in the modern sector can hire more workers at lower costs if the labor productivity in the traditional sector is low enough. The mechanism would be reinforced if the capitalists had a non-competitive behavior taking account of the effect of their decisions on the traditional sector. Worsening the situation in the traditional sector by such a behavior could favor a quicker development of the modern sector. Typically, by strategically reducing their labor demand, capitalists can maintain lower wages.

In order to formulate this idea, we consider a framework that retains the main ideas of Lewis (1954) and Ranis and Fei (1961) models. We use a model with successive generations of workers and capitalists. Workers are identical within a generation and consume their whole revenue at each period. Capitalists are altruistic and leave as a bequest a stock of wealth to their descendants. Thus, our model is close to Mankiw (2000)'s model with savers and non-savers agents.

We consider a dual economy with two production sectors, a traditional agricultural sector and a modern capitalist one. The traditional sector employs labor as the only input, when the modern one uses capital and labor.

Workers must allocate their time endowment between the two activities. The optimal allocation of workers time determines labor supply for the modern sector.

We assume that capitalists behave non-competitively. They maximize their profit taking into account the impact of their labor demand on the equilibrium wage. Under this assumption, they tend to demand less labor than in the competitive case, in order to decrease the equilibrium wage. The equilibrium concept is in the line of the Cournot-Walras equilibrium (cf. Gabszewicz and Vial (1972), Codognato and Gabszewicz (1993) and Gabszewicz and Michel (1997)) : some agents, called strategic agents, take into account the influence of their choice on the walrassian equilibrium and play between them a game of the Cournot-Nash type. In our framework, strategic agents are the capitalists, who consider the effect of their labor demand on the walrassian equilibrium wage.

The economy is small and open to capital flows and the international interest rate is exogenous and constant. Capital markets are imperfect in the sense that capitalists cannot borrow more than a given fraction of their wealth. Such kind of borrowing limit can result from an enforcement problem, as in Matsuyama (2000).

Whether borrowing constraints are binding or not will be crucial for the consequences of imperfect competition. If the borrowing constraint is not binding, capital return is determined by the international interest rate. Thus, the non-competitive behavior of capitalists cannot affect the marginal return of capital, and the economy works as a small open economy. On the contrary, if the borrowing constraint is binding, the total capital stock of each capitalist is proportional to his own wealth, and the economy works as a closed economy. The capital return now is endogenous and depends on the capital-labor ratio. Therefore, the non-competitive behavior of capitalists tends to decrease the marginal return of capital as it leads to a smaller labor demand. To sum up, in both cases, the non-competitive behavior of capitalists provides them an extra rent due to a lower wage. In the case of a non-binding constraint, the marginal return of capital is not affected. But in the case of a binding constraint, the marginal return of capital decreases.

We study the long run equilibrium of this economy, and we analyze how the non-competitive behavior of capitalists affects capital accumulation. Two long run growth rates can be define. One can be called the international growth rate, driven by the international rate of return of capital. The other one is the natural growth rate corresponding to the growth rate in the population of workers. If the international growth rate is greater than the natural one, the borrowing constraint is not binding in the long run. In this case, the rentability of wealth is exogenously given by the international rate of return.

Capitalists benefit from an extra rent induced by their non-competitive behavior and the marginal return of their wealth is not affected. Therefore, capitalists non-competitive behavior favors capital accumulation.

On the contrary, if the international growth rate is smaller than the natural one, the borrowing constraint is binding in the long run. Capitalists benefit from an extra rent provided by their non-competitive behavior but the marginal return of their wealth decreases. In this case, the economy works as a closed economy. Capitalists non-competitive behavior leads to smaller employment and capital stock in the modern sector. Thus, imperfect competition involves very different consequences on capital accumulation according to the borrowing constraint is binding or not. Capital accumulation is fostered in the unconstrained regime and is weaken in the constrained one.

Our work can be viewed as a contribution to the literature about inequality and economic growth, following Stiglitz (1969) and Bourguignon (1981). If the saving function is convex, inequality is growth-enhancing. In our model, savers are the capitalists and their non-competitive behavior that impoverishes workers may have a positive or negative effect on growth.

This paper is also related to an emerging literature that focuses on the interaction between long-run capital accumulation and imperfect competition. Laitner (1982) is one of the first attempt to study the consequences on long-run capital accumulation of the existence of oligopolies on commodity market. More recently, Sorger (2002) assumes, in the Ramsey model, that households exercise market power on the capital market: they take into account the impact of their own investment on the capital return. Still, Becker (2003) uses the same framework and studies the existence of stationary equilibria in a two players game. In these two contributions, the strategic variable is the investment in capital. Belan, Michel and Wigniolle (2002, 2004) have introduced the Cournot-Walras equilibrium in a growth model to represent the long-run effect of pension funds.

Section 2 presents the model. Section 3 studies the short run equilibrium, for given levels of wealth for capitalists. It analyzes the impact of imperfect competition on labor demand in the modern sector and on the marginal return of wealth for capitalists. Section 4 considers the long run equilibrium and the impact of non-competitive behavior on capital accumulation. Section 5 concludes. Section 6 is a final technical appendix.

2 The model

Throughout the paper, we consider a small economy open to capital flows, with an international constant gross interest rate R. In each period, population is divided between workers and capitalists. The workers live for one period and consume their income (non-savers in Mankiw (2000)). Capitalists are altruistic dynasties *à la* Barro (1974) who consume, save and accumulate wealth.

2.1 The workers

In each period, a new generation of identical workers is born and lives for one period. The number of workers, N_t in period t, grows at rate n: $N_t = (1+n) N_{t-1}$, where the growth factor $(1+n)$ is less that R. Each worker possesses a time endowment of one unit. He spends l units of time at work in modern firms for a wage w_t, and the remainder in the traditional activity which provides him a revenue $\phi(1-l)$. Labor supply l_t of a worker living in period t maximizes

$$w_t l + \phi(1-l), \qquad l \in [0,1] \tag{1}$$

where w_t is the real wage. Alternatively (1) can be interpreted as a utility function of consumption and leisure which is linear with respect to consumption $c_t = w_t l_t$.

We make the following assumption on ϕ.

Assumption 1. The function ϕ is three times continuously differentiable on the interval $(0,1)$. It satisfies[1]

$$\forall z \in (0,1), \ \phi'(z) > 0,\ \phi''(z) < 0 \text{ and } \phi'''(z) \geq 0$$
$$\lim_{z \longrightarrow 0} \phi'(z) = +\infty$$

Individual labor supply l_t satisfies $\phi'(1-l_t) = w_t$, if $l_t > 0$. The total labor supply is $L_t = N_t l_t$. Let us define the inverse function of aggregate labor supply

$$W_t(L) \equiv \phi'\left(1 - \frac{L}{N_t}\right) \tag{2}$$

for $0 < L < N_t$. Under assumption 1, this function is increasing from $\phi'(1)$ to $+\infty$ and convex.[2] These properties have two consequences. First, if

[1] The assumption $\phi''' \geq 0$ implies that the inverse function of aggregate labor supply is convex.

[2]

$$\left.\begin{array}{l} W_t'(L) = -\frac{1}{N_t}\phi''\left(1-\frac{L}{N_t}\right) > 0 \\ W_t''(L) = \frac{1}{N_t^2}\phi'''\left(1-\frac{L}{N_t}\right) \geq 0 \end{array}\right\}$$

$w_t \leq \phi'(1)$, labor supply is zero and home production is the only resource of the economy. In order to hire some workers, firms must pay a wage higher than $\phi'(1) \equiv W_{\min}$, that we call minimum wage in the following. Second, whatever the actual wage is, people will spend part of their time endowment in traditional production.

2.2 The capitalists

There are I dynasties of capitalists, $I \geq 1$. In period t, capitalist i allocates his wealth X_{it} between capital stock in the familial firm or investment on capital markets. All the firms produce the same commodity with capital K_{it} and labor L_{it} according to a constant-return-to-scale technology $F(K_{it}, L_{it})$ which includes capital after depreciation. Marginal products are positive and decreasing.

The international gross interest rate R applies for borrowing and lending. Let $Z_{it} = K_{it} - X_{it}$, the amount borrowed by family i in period t. A negative Z_{it} means that a fraction of family wealth is invested on international capital markets. Capitalists face a borrowing limit in the sense that Z_{it} must be less than a fraction μ of the wealth of family i [3]

$$Z_{it} \leq \mu X_{it},\ \mu > 0. \tag{3}$$

Capitalists have a Nash behavior on labor market. They make their decisions in factor demands by taking into account the effect of their own labor demand on the equilibrium wage w_t. At equilibrium of the labor market, labor supply L_t is equal to the sum of labor demands $L_{it} + L_{-it}$, where $L_{-it} = \sum_{j \neq i} L_{jt}$ is labor demand of other capitalists. From equation (2), real wage paid by all firms is then $w_t = W_t(L_{it} + L_{-it})$. Thus, income of capitalist i in period t writes

$$\Pi_{it} = F(X_{it} + Z_{it}, L_{it}) - RZ_{it} - W_t(L_{it} + L_{-it}) L_{it} \tag{4}$$

and is allocated between consumption C_{it} and savings

$$\Pi_{it} = C_{it} + X_{it+1} \tag{5}$$

[3] This kind of borrowing limit can result, for instance, from the enforcement problem (Matsuyama (2000)). Specifically, the capitalist would refuse to honour its payment obligation RZ_{it} if it is greater than the cost of default, which is taken to be a fraction of family wealth λX_{it}. Thus, the lender would allow the capitalist to borrow only up to μX_{it}, with $\mu = \lambda / R$.

Capitalist i behaves like an infinite-lived agent and maximizes the discounted sum

$$\sum_{t=0}^{\infty} \gamma^t \frac{C_{it}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}},\ 0<\gamma<1,\ \sigma>0 \tag{6}$$

subject to equations (3), (4) and (5), for a given initial wealth X_{i0} and taking labor demands of other capitalists $(L_{-it})_{t\geq 0}$ as given. The discount factor γ stands for the degree of altruism.

2.3 Intertemporal equilibrium

Given the initial wealth of capitalists, X_{i0} $(i=1,\ldots,I)$, an intertemporal equilibrium consists of sequences of

- wages $(w_t)_{t\geq 0}$,
- decisions of capitalists $(L_{it}, Z_{it}, C_{it}, , X_{it+1})_{t\geq 0}$, $i=1,\ldots,I$,
- decisions of workers $(l_t)_{t\geq 0}$,
- aggregate labor $(L_t)_{t\geq 0}$,

which, in addition to all the individual optimality conditions, satisfy the equilibrium conditions

$$L_t = N_t l_t = \sum_{i=1}^{I} L_{it}, \text{ and } w_t = W_t(L_t).$$

2.4 The competitive economy without restriction on borrowing

We first study the benchmark case of an economy where capitalists have a competitive behavior and face no borrowing constraint. It will allow us to measure the impact of imperfect competition and imperfect capital market and to introduce some basic assumptions that will be maintained throughout the paper.

In the competitive economy without borrowing restriction, the marginal productivities of capital and labor are respectively equal to the gross interest rate R and the wage w_t in period t

$$F'_K(K_t, L_t) = R \text{ and } F'_L(K_t, L_t) = w_t$$

The competitive capital-labor ratio $K_t/L_t = k^c$ is the solution of $F'_K(k^c, 1) = R$. The equilibrium of the labor market implies

$$\phi'\left(1 - \frac{L_t}{N_t}\right) = w_t = F'_L(k^c, 1) \equiv W^c$$

Let $l^c = L_t/N_t$ be the competitive labor supply of each worker, solution of $\phi'(1 - l^c) = F'_L(k^c, 1)$. It is positive if and only if W^c is larger than the minimum wage $W_{\min} = \phi'(1)$.

Assumption 2. $W^c = F'_L(k^c, 1) > W_{\min} = \phi'(1)$.

The dynamics of wealth of the competitive capitalist i are then

$$X_{it+1} = RX_{it} - C_{it}$$

and consumptions satisfy the Euler equation

$$\frac{C_{it+1}}{C_{it}} = (\gamma R)^{\sigma}.$$

As a consequence of Lemma A1 in the appendix, the intertemporal budget constraint of capitalist i is satisfied with equality ($\lim_{t \longrightarrow +\infty} R^{-t} X_{it} = 0$) if and only if the growth factor of consumption is smaller than the gross interest rate R. This property is rather intuitive. Indeed, if $(\gamma R)^{\sigma} \geq R$, capitalist consumption and wealth would grow at a higher rate than his income, which is unfeasible. The assumption $(\gamma R)^{\sigma} < R$ is equivalent to

Assumption 3. $\gamma < \bar{\gamma} \equiv R^{\frac{1}{\sigma} - 1}$.

Assumptions 2 and 3 made in the benchmark case of a competitive economy will be maintained for analyzing imperfect competition. Under these assumptions, the properties of the competitive equilibrium can be further analyzed. It is straightforward to show that wealth evolves according to:

$$X_{it} = (\gamma R)^{\sigma t} X_{i0} \tag{7}$$

and consumption is such that:

$$C_{it} = (\gamma R)^{\sigma t} X_{i0} \left(1 - \gamma^{\sigma} R^{\sigma - 1}\right)$$

3 Short run equilibrium

The problem of a capitalist is broken up into a static problem and a dynamic problem. The former consists in characterizing labor demand and capital demand in period t of a capitalist, given his current wealth and labor demands of the other firms. The resulting optimal income function of period t is studied. Then, we analyze the labor market equilibrium and we state conditions for the borrowing constraints to be binding or not in each period.

3.1 Capital and labor demands of capitalists

In period t, given wealth X_{it} and labor demand of other firms L_{-it}, capitalist i maximizes his income Π_{it} subject to the constraint $Z_{it} \leq \mu X_{it}$. We first treat the case where the borrowing constraint is not binding and state the condition ensuring that capitalist i will be unconstrained. Then, we will turn to the binding case.

The borrowing constraint is not binding. The problem of capitalist i is equivalent to choose capital stock $K_{it} = X_{it} + Z_{it}$ and labor L_{it}, which maximize

$$F\left(K_i, L_i\right) - R\left(K_i - X_{it}\right) - W_t\left(L_i + L_{-it}\right) L_i$$

with respect to K_i and L_i. The solution is independent of X_{it}. The capital-labor ratio has the same value as in the competitive economy

$$\frac{K_{it}}{L_{it}} = k^c. \tag{8}$$

Moreover, optimal labor demand L_{it} satisfies

$$W_t\left(L_{it} + L_{-it}\right) + W_t'\left(L_{it} + L_{-it}\right) L_{it} = W^c. \tag{9}$$

Since the LHS in (9) increases from $W_t\left(L_{-it}\right)$ to $+\infty$ when L_{it} increases from 0 to $N_t - L_{-it}$, there is a unique positive solution $L_{it} = \psi_t\left(L_{-it}\right)$ of (9) if and only if $W_t\left(L_{-it}\right) < W^c$.

Note that the borrowing constraint is not binding if and only if $Z_{it} = K_{it} - X_{it} \leq \mu X_{it}$ at the unconstrained optimum. Since $K_{it} = k^c \psi_t\left(L_{-it}\right)$, this inequality rewrites

$$X_{it} \geq \frac{k^c \psi_t\left(L_{-it}\right)}{1+\mu}. \tag{10}$$

The borrowing constraint is binding. From the above analysis, we deduce that, if $X_{it} < k^c \psi_t\left(L_{-it}\right) / \left(1+\mu\right)$, capitalist i cannot reach his unconstrained optimal income. In this case, labor demand L_{it} maximizes

$$F\left(\left(1+\mu\right) X_{it}, L_i\right) - R\mu X_{it} - W_t\left(L_i + L_{-it}\right) L_i$$

with respect to L_i. Since $F_L'(1,0) > F_L'\left(1, \frac{1}{k^c}\right) = W^c$, the assumption $W^c > W_t(L_{-it})$ implies that there exists a unique positive solution in L_{it} of

$$W_t(L_{it} + L_{-it}) + W_t'(L_{it} + L_{-it}) L_{it} = F_L'((1+\mu) X_{it}, L_{it}) \qquad (11)$$

In both cases, the effect of non-competitive behavior appears in equations (9) and (11) with the term $W_t'(L_{it} + L_{-it}) L_{it}$. It renders the fact that capitalists take into account the impact of their labor demands on wages.

3.2 Optimal income function of a capitalist

Let us define $\Pi_{it}^*(X_{it}, L_{-it})$ as the maximum profit of capitalist i, that we call *optimal income function* of capitalist i. We have stressed that the characterization of the optimal factor demands of a capitalist depends on his level of wealth, but the differentiability of the optimal income function with respect to wealth is needed for obtaining the usual Euler equation. We prove this property in the following proposition.

Proposition 1. *Assume that L_{-it} satisfies $W_t(L_{-it}) < W^c$. Then, the optimal income function $\Pi_{it}^*(X_{it}, L_{-it})$ of capitalist i is continuously differentiable with respect to X_{it}.*

(i) *If the borrowing constraint is not binding ($X_{it} \geq k^c \psi_t(L_{-it}) / (1+\mu)$), then $(\partial \Pi_{it}^* / \partial X_{it})(X_{it}, L_{-it}) = R$.*

(ii) *If it is binding ($X_{it} < k^c \psi_t(L_{-it}) / (1+\mu)$), then $(\partial \Pi_{it}^* / \partial X_{it})(X_{it}, L_{-it}) = (1+\mu) F_K'((1+\mu) X_{it}, L_{it}) - R\mu > R$.*

Proof. The derivatives in (i) and (ii) are respectively obtained for $X_{it} < k^c \psi_t(L_{-it}) / (1+\mu)$ and $X_{it} > k^c \psi_t(L_{-it}) / (1+\mu)$, by the envelope theorem.

For $X_{it} = k^c \psi_t(L_{-it}) / (1+\mu)$, we have $F_K'((1+\mu) X_{it}, L_{it}) = R$, which implies that, at this point, left-hand derivative and right-hand derivative are equal. Thus, Π_{it}^* is continuously differentiable with respect to X_{it}.

Finally, if the borrowing constraint is binding, we have $(1+\mu) X_{it}/L_{it} < k^c$ which implies $(1+\mu) F_K'((1+\mu) X_{it}, L_{it}) - R\mu > R$. This concludes the proof. ■

This proposition shows that, when the borrowing constraint is not binding, the marginal return of wealth of some capitalist remains the same as in the benchmark competitive case, i.e. equal to R. On the contrary, when the borrowing constraint is binding, the return is endogenous. It depends on the non-competitive levels of the labor demands L_{it}, and thus, will be affected by imperfect competition.

The differentiability of the optimal income function in period $t+1$ allows to write the marginal optimality condition for the program of capitalist i (defined by equations (3) to (6)). The Euler equation of capitalist i is

$$\frac{C_{it+1}}{C_{it}} = \left(\gamma \frac{\partial \Pi^*_{it+1}}{\partial X_{it+1}}\right)^{\sigma}. \tag{12}$$

3.3 Equilibrium of the labor market

In order to analyze the equilibrium of the labor market in period t, we introduce the following deflated variables

$$x_{it} = \frac{X_{it}}{N_t},\ l_{it} = \frac{L_{it}}{N_t},\ \text{and } l_t = \frac{L_t}{N_t} = \sum_{i=1}^{I} l_{it}.$$

Using equations (2), the marginal cost of labor of the firm held by capitalist i, $W_t(L_t) + W_t'(L_t) L_{it}$, can be written as a function of aggregate labor per worker l_t and labor demand per worker l_{it}

$$m(l_t, l_{it}) \equiv \phi'(1-l_t) - \phi''(1-l_t) l_{it} \tag{13}$$

where m increases with respect to l_t and l_{it}.

The borrowing constraint is not binding for capitalist i. Then, using equation (9), the marginal cost of labor is

$$m(l_t, l_{it}) = W^c \tag{14}$$

which implies

$$l_{it} = g(l_t) \equiv \frac{W^c - \phi'(1-l_t)}{-\phi''(1-l_t)}.$$

Under Assumptions 1 and 2, the function g is positive and decreasing with l_t.

The borrowing constraint is binding for capitalist i. By equation (11), labor demand l_{it} satisfies

$$m(l_t, l_{it}) - F_L'((1+\mu) x_{it}, l_{it}) = 0. \tag{15}$$

We study this equation in l_{it}. The LHS is increasing with l_{it}, and goes to $+\infty$ (since $m(l_t, +\infty) = +\infty$ and $F'_L((1+\mu)x_{it}, +\infty) = F'_L(0,1)$ is finite). When l_{it} goes to zero, we have

$$m(l_t, 0) - F'_L((1+\mu)x_{it}, 0) < W^c - F'_L(1,0),$$

and the LHS is negative since $W^c < F'_L(1,0)$. Thus, there is a unique solution to equation (15)

$$l_{it} = h(x_{it}, l_t)$$

where h is increasing with respect to x_{it} and decreasing with respect to l_t.

Remark. It is possible to make a comparison with the case where capitalist i has a competitive behavior and a binding borrowing constraint. In this case, labor demand l_{it} satisfies:

$$\phi'(1-l_t) - F'_L((1+\mu)x_{it}, l_{it}) = 0. \tag{16}$$

This equation has a unique solution that can be written:

$$l_{it} = H(x_{it}, l_t)$$

where H is increasing with respect to x_{it} and decreasing with respect to l_t. Moreover, for given values of x_{it} and l_t, imperfect competition reduces labor demand : $h(x_{it}, l_t) < H(x_{it}, l_t)$.

Characterization of the labor market equilibrium. The condition for capitalist i to be constrained is $(1+\mu)x_{it}/l_{it} < k^c$, or equivalently

$$F'_L((1+\mu)x_{it}, l_{it}) < W^c = F'_L(k^c, 1).$$

From equation (14) and (15), we deduce that capitalist i is constrained if and only if

$$h(x_{it}, l_t) < g(l_t).$$

Therefore, labor demand of capitalist i can be written as a function λ of wealth x_{it} and aggregate labor l_t,

$$l_{it} = \lambda(x_{it}, l_t) \equiv \min\{h(x_{it}, l_t), g(l_t)\}$$

where λ is non-decreasing with respect to x_{it} and decreasing with respect to l_t. An equilibrium of the labor market l_t is a solution of

$$l_t = \sum_{i=1}^{I} \lambda(x_{it}, l_t).$$

Proposition 2. *Under Assumptions 1 and 2, and given wealths of capitalists $(X_{it})_{i=1,...,I}$, there is a unique positive equilibrium l_t^* of the labor market in period t. Labor per worker l_t^* is lower than its competitive level l^c (without borrowing constraint).*

Proof. The LHS of the equilibrium condition $l_t - \sum_{i=1}^{I} \lambda(x_{it}, l_t) = 0$ is a continuous increasing function of l_t. Its limit when l_t goes to 0 is negative, since both $g(0)$ and $h(x_{it}, 0)$ are positive. Indeed,

$$g(0) = \frac{W^c - \phi'(1)}{-\phi''(1)} > 0$$

and $h(x_{it}, 0)$ is such that

$$\phi'(1) - \phi''(1) h(x_{it}, 0) - F_L'((1+\mu) x_{it}, h(x_{it}, 0)) = 0.$$

Moreover, since $g(l^c) = 0$, we have $\lambda(x_{it}, l^c) = 0$. Thus, when l_t goes to l^c, the limit of $l_t - \sum_{i=1}^{I} \lambda(x_{it}, l_t)$ is positive. By consequence, there is a unique l_t^* in the interval $(0, l^c)$ satisfying the equilibrium condition of the labor market. ■

At equilibrium of the labor market, labor demand of capitalist i is $l_{it}^* = \lambda(x_{it}, l_t^*)$. The borrowing constraint of capitalist i is binding if and only if $h(x_{it}, l_t^*) < g(l_t^*)$.

3.4 Comparison of competitive and non-competitive equilibria

What is the impact of capitalists non-competitive behavior on the labor demand, wages and profits ? To answer this question, we first compare the resulting equilibrium with the benchmark competitive case where there is no restriction on borrowing. Then, we analyze how imperfect competition modifies the equilibrium when the borrowing constraint is binding.

The borrowing constraint is not binding. If all capitalists have enough wealth to be unconstrained, total labor is $L_t^* = N_t l^*(I)$, where $l^*(I)$ is the solution of $l - I g(l) = 0$. In this case, all capitalists employ the same quantity of labor given by:

$$N_t \frac{l^*(I)}{I}$$

From the properties of g, we deduce that, if the number of capitalists I augments, total labor increases and tends to its competitive level $N_t l^c$ when

I goes to infinity. The difference $l^c - l^*(I)$ is a consequence of the Nash behavior of capitalists on labor market, and reflects the effect of imperfect competition.

The optimal income function $\Pi^*_{it}(X_{it}, L_{-it})$ of capitalist i can be written at equilibrium:

$$RX_{it} + N_t \frac{l^*(I)}{I}\left[-\phi''(1 - l^*(I))\frac{l^*(I)}{I}\right]$$

The term RX_{it} is the income of the capitalist in the case of a competitive behavior. The supplementary term, which is positive, corresponds to the extra rent of the capitalist, due to its non-competitive behavior. Thus, when the borrowing constraint is not binding, the marginal return of capital is unchanged by the non-competitive behavior, and the level of profit is increased by the extra rent due to the non-competitive behavior.

The borrowing constraint is binding for at least one capitalist i. Then, his labor demand is $l^*_{it} = h(x_{it}, l^*_t) < g(l^*_t)$ where $l^*_t = \sum_{j=1}^{I} l^*_{jt} < Ig(l^*_t)$. Thus, if some capitalist is constrained, equilibrium labor l^*_t is smaller than $l^*(I)$. The difference $l^*(I) - l^*_t$ is a consequence of an additional effect that results from the borrowing constraint.

To obtain more precise results about the role of imperfect competition, we must focus on the special case of a symmetrical equilibrium with a binding borrowing constraint for all agents.

The symmetrical equilibrium with a binding borrowing constraint. We assume that x_{it} and l^*_{it} are equal in all firms and we note the common value of the capital stock x_t. In this case, labor demand in each firm is equal to l^*_t/I, with l^*_t solution to the equation:

$$l^*_t/I = h(x_t, l^*_t)$$

Let us compare the equilibrium labor demand l^*_t with the equilibrium labor demand obtained when capitalists have a competitive behavior and a binding borrowing constraint. The latter denoting by l^{*c}_t is solution of:

$$l^{*c}_t/I = H(x_t, l^{*c}_t)$$

It is straightforward to prove that $l^*_t < l^{*c}_t$. Indeed, we proceed by contradiction, assuming $l^*_t > l^{*c}_t$. Since H is a decreasing function of l and, using the property that, for all (x, l), $h(x, l) < H(x, l)$, we deduce

$$\frac{l^*_t}{I} = h(x_t, l^*_t) < H(x_t, l^*_t) < H(x_t, l^{*c}_t) = \frac{l^{*c}_t}{I}$$

which contradicts the initial statement. Thus, for a given capital stock, we conclude that the non-competitive behavior of the capitalists results in a smaller value for equilibrium employment. Indeed, capitalists demand less labor in order to decrease the level of wages.

The optimal income function $\Pi_{it}^{*}(X_{it}, L_{-it})$ of capitalist i can be written at equilibrium:

$$X_{it}\left[(1+\mu) F_K'\left((1+\mu) x_t, l_t^*/I\right) - R\mu\right] + N_t l_t^*/I\left[-\phi''\left(1-l_t^*\right) l_t^*/I\right]$$

The return on capital $\left[(1+\mu) F_K'\left((1+\mu) x_t, l_t^*/I\right) - R\mu\right]$ appears in the first term, while the second term consists of the extra rent due to the non-competitive behavior. If capitalists behave competitively, their optimal profit becomes

$$X_{it}\left[(1+\mu) F_K'\left((1+\mu) x_t, l_t^{*c}/I\right) - R\mu\right]$$

and the rent vanishes.

By contrast with the non-binding case, imperfect competition changes the capital return. When the borrowing constraint is not binding, the capital return is equal to the international return R, whatever the behavior of capitalist is (competitive or non-competitive). When the borrowing constraint is binding, imperfect competition decreases equilibrium employment and consequently reduces capital return. Therefore, non-competitive behavior increases the income of capitalists by a rent extracted from workers, but it decreases the marginal productivity of capital at equilibrium. As we shall see, this second effect will always result in lower capital accumulation in the long-run.

4 Long run equilibrium

We prove the existence of two types of long-run balanced growth path. The first one is associated with the natural growth rate and arises when borrowing constraints are binding for all capitalists. When capitalists are unconstrained, consumption and wealth of every capitalist grow at a constant rate driven by the international gross interest rate. Finally, we study the possibility of a transition from the constrained regime toward the unconstrained regime.

4.1 Stationary growth path at rate n

Let assume that all variables grow at the same rate as N_t, i.e. ratios

$$\frac{C_{it}}{N_t} = \widehat{c}_i,\ \frac{X_{it}}{N_t} = \widehat{x}_i,\ \frac{L_{it}}{N_t} = \widehat{l}_i,\ \frac{\Pi_{it}^*}{N_t} = \widehat{\pi}_i,$$

are constant. Along such a path, labor per worker L_t/N_t is also constant and equal to $\widehat{l}$, solution of $l - \sum_{i=1}^{I} \lambda(\widehat{x}_i, l) = 0$. Consequently, labor demand of capitalist i is such that $\widehat{l}_i = \lambda\left(\widehat{x}_i, \widehat{l}\right)$.

From the Euler equation (12), the marginal revenue $\partial \Pi^*_{it+1}/\partial X_{it+1}$ must be constant and equal to $\frac{(1+n)^{1/\sigma}}{\gamma}$. Furthermore, we know from Proposition 1 that

$$\rho_i \equiv \frac{\partial \Pi^*_{it+1}}{\partial X_{it+1}} = \begin{cases} R, \text{ if } i \text{ is not constrained} \\ (1+\mu) F'_K\left((1+\mu)\widehat{x}_i, \widehat{l}_i\right) - R\mu > R, \text{ if } i \text{ is constrained} \end{cases} \tag{17}$$

Thus, we have

$$\rho_i = \rho \equiv \frac{(1+n)^{1/\sigma}}{\gamma} \geq R \tag{18}$$

and this inequality is strict if i is constrained.

Proposition 3. *If $\gamma < \frac{(1+n)^{1/\sigma}}{R} \equiv \tilde{\gamma}$, then there exists a unique stationary equilibrium growth path at the natural rate. Along this path, all dynasties of capitalists have equal wealth and their borrowing constraints are binding.*

Proof. From (17) and (18), along a stationary growth path, all capital-labor ratios $(1+\mu)\hat{x}_i/\hat{l}_i$ are equal to the solution $\hat{k}$ of

$$(1+\mu) F'_K(k,1) - R\mu = \rho > R. \tag{19}$$

Moreover, the marginal cost of labor $m(\hat{l}, \hat{l}_i)$ is equal to its marginal product $F'_L(\hat{k}, 1)$. This implies that all labor demands are equal to $\hat{l}/I$. This in turn leads to the equality of wealth $\hat{x}_i = \hat{k}\,\hat{l}/\left[(1+\mu)I\right]$.

Conversely, these variables satisfy all the equilibrium conditions when all capitalists are constrained. Indeed, from (19), we have $F'_K(\hat{k}, 1) > R = F'_K(k(R), 1)$ and we deduce that $(1+\mu)\hat{x}_i/\hat{l}_i = \hat{k} < k^c$, which implies that all the borrowing constraints are binding. ■

We are now in a position to discuss the effect of imperfect competition on labor and capital income in the long-run when all capitalists are constrained on their borrowing. Indeed, the long-run capital-labor ratio $\hat{k}$ does not depend on the competition environment. It would be the same in a competitive economy with binding borrowing constraints. In the short run, for a given level of wealth X_i, we have seen in Section 3.4 that non-competitive behavior leads capitalists to reduce their labor demand. The resulting marginal product of capital is smaller than in the competitive case. Now, in the long-run, capitalists wealths being endogenous, marginal product of capital remains unchanged whatever the competition environment is. Consequently, since labor demand is lower in the non-competitive case, wealth must also be lower. Imperfect competition leads to lower capital accumulation.

Increasing the number of families I allows to modify the intensity of competition, an infinite number of families corresponding to perfect competition. Note that the capital-labor ratio does not depend on I. Total labor per worker $\hat{l}$ satisfies $m(\hat{l}, \hat{l}\ /\ I) = F_L'(\hat{k}, 1)$ and is increasing with respect to the number of capitalists I. When I tends to $+\infty$, it converges to $\hat{l}_\infty$ solution of $m(\hat{l}_\infty, 0) = F_L'(\hat{k}, 1) < W^c$. Since $\hat{l}$ is increasing with I, total wealth per worker $\hat{x} = \sum_{i=1}^{I} \hat{x}_i = \hat{k}\ \hat{l}/\left(1+\mu\right)$ is also increasing with I. As a consequence, non-competitive behavior of capitalists results in lower capital accumulation, lower employment and lower income for workers.

Remark. If capitalists cannot borrow ($\mu = 0$) and if the population growth rate n is zero, the equilibrium capital-labor ratio corresponds to the Modified Golden-Rule level, solution of $\gamma F_K'(k, 1) = R$. When $n > 0$ and $\mu > 0$, two additional modifications affect the stationary capital-labor ratio. First, along a stationary growth path, a positive growth rate of worker population leads to a positive growth rate of the consumption of any capitalist. This augments long-run marginal return of wealth ρ. Second, with $\mu > 0$, capitalists can borrow and use higher capital stock than with $\mu = 0$. The capital-labor ratio $\hat{k}$ increases with respect to μ.

4.2 Dynamics with unconstrained capitalists

If no capitalist is constrained in period t, then all capital stocks K_{it}^* and labor demands L_{it}^* are identical among capitalists, and we have $L_{it}^* = L_t^*/I = N_t l^*$, where $l^* = l^*(I)$ is the solution of $Ig(l) - l = 0$. Optimal income of capitalist i is

$$\begin{aligned} \Pi_{it}^* &= F\left(K_{it}^*, L_{it}^*\right) - R\left(K_{it}^* - X_{it}\right) - W_t\left(L_t^*\right) L_{it}^* \\ &= RX_{it} + \left[W^c - W_t\left(L_t^*\right)\right] L_{it}^* \end{aligned}$$

where the second equality is obtained appealing to linear homogeneity of the production function F and to the equality $F_K'\left(K_i^*, L_i^*\right) = R$, which also implies $F_L'\left(K_i^*, L_i^*\right) = W^c$. Thus, we can rewrite the optimal income as

$$\Pi_{it}^* = RX_{it} + b^* N_t \tag{20}$$

where

$$b^* = [W^c - \phi'(1-l^*)]\, \frac{l^*}{I}.$$

is the rent per worker of each capitalist. Non competitive behavior allows the capitalist to levy the rent b^* on worker income. Differentiating the aggregate rent per worker Ib^* with respect to I leads to

$$[W^c - \phi'(1-l^*) + \phi''(1-l^*)\, l^*]\, \frac{dl^*}{dI} = \left[\phi''(1-l^*)\left(l^* - \frac{l^*}{I}\right)\right] \frac{dl^*}{dI}$$

which is negative since l^* is increasing with I. Therefore, the rent per worker for each capitalist b^* is also decreasing with I and tends to zero as I becomes infinite. As I represents the intensity of competition, stronger competition tends to diminish the rent.

From equations (12) and (17), we obtain

$$\frac{C^*_{it+1}}{C^*_{it}} = (\gamma R)^\sigma \equiv G.$$

From the budget constraint $\Pi^*_{it} = C^*_{it} + X^*_{it+1}$ and (20), we have

$$X^*_{it+1} = RX^*_{it} + b^* N_t - C^*_{it} \tag{21}$$

Dynamics of wealth and consumption of capitalist i are characterized by the two-dimensional system that consists of these two last equations, where initial wealth X_{i0} is given.

Proposition 4. *Let Assumptions 1, 2 and 3 be satisfied. If*

$$\gamma \geq \frac{(1+n)^{1/\sigma}}{R} \equiv \tilde{\gamma}$$

and

$$X_{i0} \geq \frac{k^c N_0}{1+\mu}, \text{ for } i = 1, \ldots, I,$$

then there exists a unique growth path where consumption and wealth of any capitalist asymptotically increase at factor G. Along this path, no capitalist is constrained. For the given initial wealth X_{i0} of family i, wealth X_{it} is given by

$$X_{it} = X_{i0} G^t + \frac{b^* N_0}{R-(1+n)} \left[G^t - (1+n)^t\right] \tag{22}$$

and consumption C_{it} grows at factor G.

Proof. If no capitalist is constrained, then consumption C_{it}^* of capitalist i grows at factor G and, from (21), dynamics of his wealth are

$$X_{it+1}^* = RX_{it}^* + b^* N_t - C_{i0} G^t.$$

Using Lemma A1 in the appendix, there exists a unique solution to this difference equation, given by (22). The assumption $\gamma \geq \tilde{\gamma}$ is equivalent to $G \geq 1+n$. Thus,

$$X_{it} \geq X_{i0} (1+n)^t \geq \frac{k^c N_0}{1+\mu} (1+n)^t \geq \frac{k^c \psi_t (L_{-it})}{1+\mu}.$$

If all capitalists are unconstrained in period 0 and $G \geq 1+n$, then they are still unconstrained in any future period. ■

Equation (22) can be compared to (7) obtained in the case of perfect competition when all capitalists are unconstrained : $X_{it} = X_{i0} G^t$. The extra rent due to imperfect competition enhances wealth accumulation. In contrast to the case with binding borrowing constraints, the imperfect competition provides extra rent without modifying the marginal productivity of capital.

Considering two families i and j with different initial wealths, $X_{i0} > X_{j0}$, the inequality among capitalists will also hold in the long-run since

$$\lim_{t \to +\infty} \frac{X_{it}}{X_{jt}} = \frac{X_{i0} + \frac{b^* N_0}{R-(1+n)}}{X_{j0} + \frac{b^* N_0}{R-(1+n)}},$$

which is larger than 1. But, it is also smaller than X_{i0}/X_{j0} that would be the limit of the ratio in the competitive case. Since the extra rent is the same for all capitalists, whatever their wealths, imperfect competition tends to reduce inequality among capitalists.

4.3 Transition

We have shown that, if $\gamma > \tilde{\gamma}$ and if capitalists are not constrained initially, they accumulate enough wealth in each period for remaining unconstrained in all future periods. In the long-run, the economy follows a growth path with an asymptotic growth factor equal to G. Now, even if $\gamma > \tilde{\gamma}$, capitalist could be constrained initially because of insufficient wealth. The following proposition proves that they will become unconstrained in the long-run.

Proposition 5. *If $\gamma > (1+n)^{1/\sigma} / R \equiv \tilde{\gamma}$, whatever the initial wealth of capitalists, all capitalists becomes unconstrained.*

Proof. In the constrained case, we have

$$C_{it} + X_{it+1} = \Pi_{it}^{*} = F\left(\left(1+\mu\right)X_{it}, L_{it}\right) - R\mu X_{it} - W_{t}\left(L_{t}\right)L_{it}$$

Since $X_{it+1} \geq 0$ and $0 \leq L_{it} \leq N_t$,

$$C_{it} \leq C_{it} + X_{it+1} \leq F\left(\left(1+\mu\right)X_{it}, L_{it}\right) \leq F\left(\left(1+\mu\right)X_{it}, N_{t}\right)$$

and we deduce that

$$\left(1+n\right)^{-t}C_{it} \leq F\left(\left(1+\mu\right)\left(1+n\right)^{-t}X_{it}, N_0\right).$$

Now, the condition $\gamma > \left(1+n\right)^{1/\sigma}/R$ is equivalent to $G > 1+n$ and implies

$$\lim_{t\to+\infty}\left(1+n\right)^{-t}X_{it} = +\infty.$$

A similar argument applies in the unconstrained case. Indeed, we have

$$C_{it} \leq C_{it} + X_{it+1} = \Pi_{it}^{*} < RX_{it} + \omega\left(R\right)L_{it}^{*} < RX_{it} + \omega\left(R\right)N_t$$

which implies

$$\left(1+n\right)^{-t}C_{it} < R\left(1+n\right)^{-t}X_{it} + \omega\left(R\right)N_0$$

and the result follows from the assumption $G > 1+n$.

Thus, at some date, the condition $X_{it} \geq \frac{k(R)N_t}{1+\mu}$ is satisfied, for all i and all capitalists are unconstrained. ■

Proposition 3, 4 and 5 show that the long-run growth path depends crucially on the respective values of the natural growth factor $1+n$ and the internationally driven growth rate $G = \left(\gamma R\right)^{\sigma}$. When $1+n$ is higher (lower) than G, capitalists are constrained (unconstrained) in the long-run. Now, in the unconstrained case, workers benefit from higher wages and higher employment in the modern sector. These results suggest an interesting policy issue. In an economy with $1+n > G$, an appropriate economic policy can shift the economy towards the growth path where capitalists are unconstrained.

To this aim, the government may subsidy bequests and finance such payments with a lump-sum tax. The budget constraint of a capitalist writes

$$C_{it} + \left(1-\tau\right)X_{it+1} = \Pi_{it} - \theta_t$$

where τ is the share of the bequests financed by the government through a lump-sum tax θ_t. The corresponding Euler equation is

$$\frac{C_{it+1}}{C_{it}} = \left(\frac{\gamma}{1-\tau}\frac{\partial\Pi_{it+1}^{*}}{\partial X_{it+1}}\right)^{\sigma}.$$

The condition for capitalists to be unconstrained in the long-run rewrites

$$\left(\frac{\gamma R}{1-\tau}\right)^{\sigma} > 1+n.$$

Thus, the unconstrained regime will be reached if the subsidy rate τ is high enough.

This result shows that subsidizing capitalists bequests improve the welfare of workers. Nevertheless, this policy may increase inequality among capitalists since the subsidy is proportional whereas the lump-sum tax is uniform.

5 Conclusion

We have explored the relationship between imperfect competition and capital accumulation in a two-sector economy with a borrowing constraint. Non-competitive behavior of capitalists results in a rent which is extracted from the workers and lower employment in the modern sector.

In the long-run, the borrowing constraints remain binding if the internationally driven growth rate is lower than the natural one. In this case, imperfect competition decreases capital accumulation and is detrimental for growth. In the converse case, the borrowing constraint is not binding and imperfect competition is beneficial for capital accumulation and growth.

Finally, subsidy on bequests, that incites capitalists to accumulate wealth, allows the economy to take off and reach the unconstrained regime. Since employment and wages are higher in this case, such a policy is welfare-enhancing for workers.

6 Appendix : Equilibrium without borrowing constraint

If all capitalists are unconstrained, dynamics of consumptions and wealths are such that

$$X_{it+1} = RX_{it} + bN_t - C_{it}$$

$$C_{it+1}/C_{it} = (\gamma R)^{\sigma} \equiv G$$

where X_{i0} is given. We are looking for initial consumption level C_{i0} such that the intertemporal budget constraint is satisfied with equality

$$\lim_{t \longrightarrow +\infty} R^{-t} X_{it} = 0.$$

Lemma A1. *Assume* $R > N > 0$, $B > 0$ and $G > 0$. *For a given* X_0, *there exist* $C_0 > 0$ *such that the difference equation*

$$X_{t+1} = RX_t + BN^t - C_0 G^t \tag{23}$$

has a solution which satisfies $\lim_{t \longrightarrow +\infty} R^{-t} X_t = 0$ *if and only if* $G < R$. *This value of* C_0 *is unique and equal to*

$$C_0 = (R - G)\left(X_0 + \frac{B}{R - N}\right).$$

Proof. We distinguish three cases depending on the sign of $R - G$.

- $G > R$. The general solution of (23) is written as

$$X_t = \lambda_1 R^t + \lambda_2 N^t + \lambda_3 G^t \tag{24}$$

where λ_1, λ_2 and λ_3 have to be determined. The intertemporal budget constraint $\lim_{t \longrightarrow +\infty} R^{-t} X_t = 0$ implies $\lambda_1 = \lambda_3 = 0$. Thus $X_t = X_0 N^t$. Substituting in (23), one obtains

$$C_0 \left(\frac{G}{N}\right)^t = (R - N) X_0 + B$$

which contradicts $C_0 > 0$. The growth rate of consumption cannot be higher than the interest rate.

- $G = R$. The general solution of (23) is written

$$X_t = (\lambda_0 t + \lambda_1) R^t + \lambda_2 N^t$$

and the intertemporal budget constraint implies $\lambda_0 = \lambda_1 = 0$. Substituting $X_t = X_0 N^t$ in (23), one still obtains than $G = R$ is not consistent with $C_0 > 0$.

- $G < R$. The general solution of (23) is given by (24). The intertemporal budget constraint implies $\lambda_1 = 0$ and $X_t = \lambda_2 N^t + \lambda_3 G^t$. By substitution in (23), we obtain

$$\lambda_2 N^{t+1} + \lambda_3 G^{t+1} = R\left(\lambda_2 N^t + \lambda_3 G^t\right) + BN^t - C_0 G^t.$$

For $N \neq G$, we deduce that $\lambda_2 = -B/(R - N)$ and $\lambda_3 = C_0/(R - G)$. Since $X_0 = \lambda_2 + \lambda_3$, we have

$$C_0 = (R - G)\left[X_0 + \frac{B}{R - N}\right].$$

The same formula applies in the case $N = G$.■

Under the assumption $G < R$ (assumption 3 in the text), the unique solution of (23) writes

$$X_t = X_0 G^t + \frac{B}{R - N}\left(G^t - N^t\right)$$

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