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Negotiating remedies: revealing the merger efficiency gains

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Negotiating Remedies:
Revealing the Merger Efficiency Gains∗

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Abstract

This paper aims to contribute to the normative economic analysis of merger control by taking into account the possible efficiency gains for the design of structural merger remedies. We show that a larger asset transfer should be requested from a less efficient merged firm than from a more efficient one, which conforms with the recommendations of competition policy practitioners. However, since cost savings are private information of merging firms, the Competition Authority will require them to reveal their efficiency gains, so as to tailor the optimal remedy. We propose a revelation mechanism combining the use of divestitures with the regulation of their sale price. We discuss the opportunity of such a merger policy tool, and argue that in practice it may be used to signal the efficiency gains of notified mergers.

Keywords: merger control, structural merger remedies, asymmetric information

JEL: D82, L41

Résumé

Cet article propose une analyse normative du contrôle des fusions qui prend en compte les gains d’efficacité générés par la concentration pour la détermination des remèdes structurels. On montre que la cession d’actifs requise sera supérieure dans le cas d’une fusion générant moins de synergies. En asymétrie d’information, l’autorité de la concurrence doit faire révéler les gains d’efficacité pour pouvoir exiger des transferts d’actifs proportionnels aux dommages concurrentiels. Le mécanisme de révélation proposé est basé sur la réglementation du prix de vente des actifs transférés. L’utilité de cet outil de révélation justifie une réflexion approfondie des autorités de la concurrence sur le processus de vente d’actifs lors de l’application des remèdes structurels.

Mots-clé: contrôle des fusions, remèdes structurels, information asymétrique
1. Introduction

Faced with a potentially anticompetitive merger, the Competition Authority (CA) has basically the choice among three alternatives: rejecting it, accepting it if the efficiency gains are overwhelming, or accepting it provided that corrective remedies are adopted. A recent study of the European Commission DG Comp counted no less than 190 concentrations cleared with commitments from a total of 2469 mergers since the introduction of the European Merger Regulation in 1990. It might look like a small number, but a closer look at the Commission’s statistics reveals that remedies mostly apply to important mergers. Moreover, about 80% of those commitments address horizontal concerns. Divestitures, i.e. structural remedies, are typically employed for horizontal mergers, and account for more than 60% of all remedies. Structural remedies are thought to be easier to apply than behavioral ones, since they change the allocation of property rights within the industry, and therefore need no monitoring once implemented. Yet, their application and effects have often been subject to questioning, because "the fashioning of merger remedies is not materially governed by case law[...]" and therefore "is subject to standards that are not well-defined or consistent" (see Blumenthal (2001)).

To begin with, this paper aims to contribute to the economic analysis of structural merger remedies by taking into account the possible efficiency gains for the design of divestitures. Our model formalizes the intuition of a link between the amount of efficiency gains that the merging partners can achieve and the amount of assets they will have to divest for the merger to be accepted. Such an idea is in line with the current consensus among competition policy practitioners, according to which the required divestiture should neither exceed the net competitive harm caused by the merger, nor prove insufficient to correct it (that is, both overfixing and underfixing should be avoided). Consequently, the required divestiture should obey the proportionality principle, meaning a larger transfer

2 Suggestive examples for such mergers, cleared with commitments under the new European Merger Regulation 139/2004 are Pernod Ricard - Allied Domecq, Sanofi - Aventis, Alcan - Pechiney.
being requested from a less efficient merged firm than from a more efficient one. The important consequence is that in order to tailor the optimal remedy, the CA needs to learn the efficiency gains generated by the merger, which are private information for the merger partners.

Thus, the second objective of the paper is to shed light on the design of optimal structural divestitures when merging firms are better informed than the CA with respect to the synergy level of the merger. The U. S. Merger Guidelines acknowledge that "mergers have the potential to generate significant efficiencies", but warn at the same time that "efficiencies are difficult to verify and quantify, in part because much of the information relating to efficiencies is uniquely in the possession of the merging firms".

Competition authorities do try to extract this private information. A possible "approach in screening mergers would be to implement a revelation mechanism through the institution of merger license fees to be paid to the government", as Röller et al. (2000) suggested. An alternative powerful way to extract private information on efficiency gains is quoted by Brodley (1996), who reminds that unfounded efficiency claims may be deterred by bonding procedures. He provides the suggestive American example of the Pennsylvania versus Providence Health Sys., Inc. case, where the consent decree negotiated between the Pennsylvania Attorney General and the undertakings provided that if argued efficiencies do not translate into net cost savings directly passed on to consumers five years later, the merging parties engaged to pay to the Treasury the shortfall from the claimed efficiency gains. This solution is nevertheless subject to the specific critique against behavioral remedies, concerning the ex-post monitoring cost.

We do not formalize here the implications of such a "put-up-or-shut-up" clause. In turn, we model a competition authority requiring the merging firms to reveal efficiency gains so as to tailor the optimal structural remedy. We propose a revelation mechanism combining the use of divestitures with the regulation of their sale prices. We supply as a matter of fact a possible answer to the following question: "What role, if any, should a competition authority play in the pricing of assets to be divested?"

For the time being, Competition Authorities do not tamper with the sale price

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of the divested assets. However, in this paper we intend to check the relevance of a regulated sale price as a possible screening device, even if this item rather belongs to a regulator’s instruments. As a matter of fact, merger control is typically a mixed area of antitrust and regulation, where the Competition Authority is supposed to directly impact on the market structure by rejecting anticompetitive mergers (see Motta et al. (2002)). Besides, since divestitures alone may not suffice to screen merger proposals, it is natural to look for complementary screening devices that might prove effective. Note that as far as merger control is concerned, the scope of instruments employed has constantly increased. For instance, the very structural remedies themselves represent a rather recent instrument introduced in the 70s by the FTC\textsuperscript{7}. In addition, whenever the CA appoints a trustee to implement the divestitures it requires, merger control comes close to an indirect determination of the price of divested assets: according to the European Merger Regulation\textsuperscript{8}, the trustee may apply the fire-sale clause and organize the sale of the divested assets without a minimum price, subject only to the Commission’s approval. After all, though in a different context, the CA would rather interfere with the price setting process, although maybe not quite ready to do it: indeed, in the battle to force Microsoft to open up the market for media-playing software, the European Commission may not yet be likely to order a lower price for Windows without Media Player, even if it does ponder it\textsuperscript{9}. Concerning merger control, our screening mechanism incidentally lends itself to an equivalent and appealing signaling-like interpretation: notwithstanding that competition laws do not currently allow antitrust agencies to explicitly fix the price of divestitures, the latter might nevertheless reveal efficiency gains. More precisely, a more efficient merged entity will accept a lower price for any given level of divestiture than a less efficient one, and by doing so it will signal itself as such.

Our model builds on a simple framework, which nevertheless allows a consistent formal treatment of both efficiency gains and structural remedies. We consider a Cournot competition game with homogenous good, constant marginal costs and capacity constraints in a three-firm framework. Following a two-firm exogenous merger, the merged entity may enjoy cost savings. The latter stem from the efficiency gains brought about by the merger. The CA requires in turn asset transfers to the outsider, so as to fulfill its objective. Divestitures alter the distribution of capital assets between firms and thereby the capacity constraints

\textsuperscript{7}See the Hart-Scott-Rodino Act. 
\textsuperscript{8}See the Commission notice on remedies acceptable under merger regulation, OJEC 2.3.2001. 
and merger profitability for firms in the industry. We look for the optimal transfer to implement according to the objective of the CA: maximizing total welfare under the constraint that the Consumers’ Surplus does not fall. ((In our framework with capacity constraints, the same transfer will keep the market price constant. We show that the first best divestiture will be proportional to the level of cost savings, meaning that less efficient mergers will need to divest more. But as the cost savings are private information for merging partners (also called insiders), the latter are likely to cheat when declaring the amount of efficiency gains generated by the merger, so as to avoid higher asset transfers. Thus the divestiture alone will not be effective as a screening device.

We argue here that a non linear tariff for the divested assets can be employed as a screening device. More specifically, we consider a two-type model, where the insiders are either highly efficient (low-cost) or poorly efficient (high-cost). We show that the first best divestiture contract that the CA proposes to the merged entity consists of a menu of two tariffs. The first tariff consists of a lower divestiture for a lower average sale price than the second one. The efficient merged entity values its capacity more than the inefficient one does. As a result, the low-cost insiders are more reluctant to divest a large quantity of assets than the high-cost insiders, even though the corresponding monetary transfer is high. By the same token, since assets have lower value for the inefficient merged firm, the latter will find it profitable to divest a large quantity of assets for a higher average price.

In other words, the CA clears the merger provided the merged entity pays a "licence to merge", although not to the CA, we should stress, since the monetary transfer actually takes place between the insiders and the buyer of divested assets. The optimal licence schedule proposed by the CA takes two forms. The firms pay either by divesting a large quantity of assets, or by divesting a low quantity of assets at a depreciated price. Hence the licence is either a monetary payment whenever few assets are divested at a low price, or an asset payment whenever a large quantity of assets is divested. Facing this two possible types of licences to merge, the more efficient merged entity is induced to choose the least distorting form of payment, the monetary one. Basically, we provide an answer to Rey’s (2000) informal suggestion that "The firms could be asked to 'pay' [...] for any negative external effect of the merger, so as to ensure that only socially desirable mergers are proposed". The effectiveness of the sale price as a screening device (which is what basically our model shows) reveals that it can be successfully be employed by merging firms to signal their efficiency gains. Notwithstanding the reluctance of competition authorities to openly interfere with the pricing of the
divested assets, close attention should be paid to the asset prices as they result from the divestiture process, since they may very well signal the merger synergies.

As early as 1993, Yao and Dahdouh clearly stated that "In merger review, the selective provision of information creates problems for government antitrust officials because much relevant information is held privately by merging parties" (Yao and Dahdouh (1993), p.24). The asymmetric information problem for competition policy has been already tackled by Faulli-Oller and Corchon (1999), who study the implementation of socially optimal mergers with standard tools in dominant strategy implementation, but without allowing for merger remedies.

To our knowledge, few papers deal with structural merger remedies. Our model takes on Rey (2000) and Gonzalez (2003), who both account for the possibility of divestitures, besides their corrective role, to be used for screening. As compared with Gonzalez (2003), whose incentive mechanism relies on the choice of the market where the divestiture will apply if accepted, we restrict the use of divestiture to the same market on which the competitive harm is witnessed, and propose a second revelation instrument, the sale price. Medvedev (2004) proposed the first formalization of the intuition that the amount of asset transfer necessary to remedy the competitive harm depends on the amount of efficiency gains. We show in turn that this result can be obtained within a quite general framework with constant marginal costs, without assuming a particular relationship (substitutability in his case) between the capital-based cost-savings and the synergies following the merger.

This paper also belongs to a strand of literature analyzing the effects of capital transfers between firms. For instance, Farrell and Shapiro (1990,a) raise precisely the question of how capital transfers between Cournot oligopolists affect total industry profit or welfare. Compte et al. (2002) and Vasconcelos (2005,a) also look into the effects of asset transfers, but from a collusive industry point of view, whereas Vasconcelos (2005,b) models the CA’s incentives to apply divestitures within an overfixing yet symmetric information merger control framework. We completely overlook the possibility for collusion, but focus only on the unilateral effects of the concentration.

We present first the market equilibria both before and after merger, taking simultaneously into account efficiency gains as well as asset transfers. We then go on to present the game between industry firms and the CA. For the design of optimal remedies, we begin by the symmetric information benchmark, then deal with the asymmetric information framework. Finally, we comment our results and conclude on their relevance. Technical proofs are grouped in the Appendix.
2. The model

We present first the pre-merger equilibrium as a benchmark and then the post-merger framework.

2.1. Pre-merger market equilibrium

We consider as starting point a homogenous good, three-firm perfectly symmetric industry. Demand is linear: \( P(Q) = 1 - Q \), where \( Q \) is total output. Firms maximize individual profit. We place ourselves in a situation where firms face capacity constraints as in Dixit (1980). Explicitly, we assume that a two-stage capacity-quantity game took place before merger, where firms acquire first capacity at a positive unit cost \( c_k \) and then compete à la Cournot with a constant production marginal cost equal to \( c \). The Subgame Perfect Equilibrium of such a game is given by capacity and output equal to \( \frac{1-c-c_k}{4} \). We take this to be the operational capacity of a firm before merger and denote it by \( k \). Consequently, pre-merger equilibrium yields \( Q = 3k \) and \( P(3k) = 1 - 3k \). We denote \( \Pi \) the pre-merger individual profit, which writes \( \Pi = k(1 - 3k - c) \). We assume that the unit cost for the acquisition of additional capacity is prohibitive in the short-run.

2.2. Post-merger market framework

Since we only deal with exogenous market concentration, and that initially the three firms are identical, merger is assumed to take place between any two of them. We shall index the merged entity, i.e. the insiders, by \( M \), and the outsider by \( o \) respectively.

Following the merger, \( M \)'s capacity constraint changes, since now it holds the double of the pre-merger capacity. As far as the outsider is concerned, its capacity is unchanged, as well as its marginal cost. The insiders in turn may benefit from merger-specific cost savings\(^{10}\). More precisely, the marginal cost of the merged firm, denoted \( c_M \), satisfies the following: \( c_M = c - \alpha \), where \( \alpha \in [0, c] \) measures the amount of cost savings. Our framework is general enough to lend itself to different interpretations of this parameter. For instance, \( \alpha \) stands for the synergies that arise from the merger, i.e. substantial efficiency gains that would not have been obtained without it. More generally, \( \alpha \) measures the positive effect of an essential complementarity between the merger partners that allows them to lower

\(^{10}\)Our treatment of cost savings involves the marginal cost, so as to consistently follow the current CAs’ treatment of allowable merger efficiency gains.
their common marginal cost\textsuperscript{11}. Following Röller et al. (2000), these synergies can typically be due to complementarity between technological or administrative capabilities of firms. For example, firms may own complementary patents, which if employed together further improve the production process.

In the post-merger framework, firms play a standard Cournot game - we rule out in our model any possibility for post-merger collusion. Firms obey the following capacity constraints\textsuperscript{12}: $q^M \leq 2k$ and $q^o \leq k$. Taking into account the cost savings for the merged entity, their Best Reply functions write as follows: $BR^M(q^o) = \min\left(\frac{1+\alpha-c}{2} - q^o, 2k\right)$ and $BR^o(q^M) = \min\left(\frac{1-c-q^M}{2}, k\right)$. It is straightforward to show that post-merger equilibrium price is less or equal to $P(3k)$ only if $\alpha \geq 5k - 1 + c$. In other words, a merger increases market price whenever its cost savings fail to exceed this minimum threshold.

To sum up, mergers have an ambiguous impact on the economy. Indeed, if the efficiency gains are sufficiently high, the merged entity is induced to use all its capacity and thus the price is unchanged with respect to its pre-merger level. In turn, with lower efficiency gains, the insiders hold slack capacity and the price is higher than its pre-merger level. In such a case, divestitures might be used by the CA to modify the post-merger market equilibrium so as to prevent any drop in Consumers' Surplus. These divestitures depend on the efficiency gains, and their design is studied in the next section.

3. Remedies as a screening device

3.1. Objective of merger control and terms of divestiture

As far as the CA’s objective goes, we wish to closely follow the current trend in merger control in as much as we assume total welfare maximization under the constraint of no drop in Consumers’ Surplus. Competition Authorities actually rather accept or reject mergers on the basis of the likelihood of a price increase. To put it short, a merger is approved if consumers will not be hurt. However, due to our modeling choice of capacity constraints, keeping the price constant is the best that can be done, so maximizing Consumers’ Surplus is equivalent to

\textsuperscript{11}For an explicit modeling of cost savings through the use of complementary assets by merger partners see Bensaid et al. (1994)

\textsuperscript{12}We consider here that in the short run firms cannot increase their production capacity. This result remains true as long as the unit cost of capacity acquisition $c_k$ is high enough, which is precisely our assumption here.
requiring the above constraint to bind\(^{13}\).

We use a two-type model, with the synergy parameter \(\alpha \in \{\underline{\alpha}, \overline{\alpha}\}\), where \(\underline{\alpha}\) stands for high efficiency gains, and \(\overline{\alpha}\) for low efficiency gains. Explicitly, \(\underline{\alpha} = c_\text{LM} - \overline{c}_\text{LM}\) and \(\overline{\alpha} = c_\text{HM} - \overline{c}_\text{HM}\), where \(c_\text{LM}\) and \(c_\text{HM}\) are the marginal costs of the low-cost merged firm and of the high-cost one respectively. Objective probabilities are \(\rho\) and \(1 - \rho\) respectively.

In our setting, divestitures will consist in transfers of assets to the outsider, whom we consider here to be the only possible buyer. We make this hypothesis for the sake of simplicity, but the framework lends itself well to the introduction of a new entrant on the market\(^{14}\). Moreover, by not allowing entry on the market, we want to remind that besides capacity, a certain know-how and experience of the market are necessary to guarantee actual competition on behalf of the buyer of divested assets.

We determine first the optimal divestiture when information on the merger type is symmetric, and then go on to study the role of asymmetric information for the design of the remedy.

### 3.2. Optimal divestitures with symmetric information

Given the screening stand we take in our model, the game we consider between the firms and the CA is the following:

In the first stage, the merging firms learn their efficiency gains level \(\alpha\) and submit a merger proposal to the CA. When information is symmetric the parameter \(\alpha\) is also observed by the outsider and by the CA. (Later on we detail the information structure of the game with asymmetric information).

In the second stage, the CA evaluates the consequences of the merger taking into account its own merger control objective. It proposes a divestiture contract accordingly, if such a contract exists; if not, it rejects the merger.

In the third stage, the insiders accept or reject the divestiture. If they accept, assets will be transferred to the outsider on a take-it-or-leave-it basis.

In the fourth stage, the outsider decides whether to take over or not the divested assets.

\(^{13}\)See also theoretical contributions supporting the choice of a pure Consumers' Surplus standard instead of the Total Welfare one, such as Besanko and Spulber (1993), Neven and Röller (2001), and Lyons (2002).

\(^{14}\)This would nevertheless go beyond the primary purpose of our model, since it would require the specification of the entrants' marginal cost and a detailed case discussion to determine then the optimal asset buyer from the CA’s point of view.
In the fifth stage, conditional on the divestiture contract being accepted, the Cournot market equilibrium is determined taking into account the amount of asset transfer required by the CA. If any of the parties rejects the contract, the merger project falls through.

This last assumption is quite in line with the current unfolding of a divestiture negotiation process. Indeed, whenever a divestiture injunction is being settled upon, it has previously gained approval of all involved parties: the divesting firms, the buyer, and the CA. The failure of such a three-party negotiation ends either in the appointment of a trustee to carry out the divestiture, or in the merger itself falling through.

At the last stage of the game, firms play a standard Cournot game. Note that before the divestiture requested by the CA, firms’ capacities amount to $2k$ for $M$ and $k$ for $o$. Once the transfer of $\Delta$ is made from $M$ to $o$, firms obey the following capacity constraints: $q^M \leq 2k - \Delta$ and $q^o \leq k + \Delta$. Taking into account both the cost savings and the required divestiture, the Best Reply functions of the Cournot game write as follows: $BR^M(q^o) = \min \left( \frac{1+\alpha-c-q^o}{2}, 2k - \Delta \right)$ and $BR^o(q^M) = \min \left( \frac{1-c-q^M}{2}, k + \Delta \right)$.

Profits are denoted $\Pi^M(\Delta; \alpha)$ and $\Pi^o(\Delta; \alpha)$ respectively, and depend on the cost savings parameter and the amount of divestiture.

Actually, for a given divestiture $0 \leq \Delta \leq (1-c) - 4k$, i.e. not too high so as to have the outsider operate at full capacity, profits write

$$\Pi^M(\Delta; \alpha) = \begin{cases} 
\left( \frac{1+\alpha-c-k-\Delta}{2} \right)^2, & \text{if } \alpha \leq 5k - \Delta - (1-c) \\
(1 - 3k - c + \alpha)(2k - \Delta), & \text{if } \alpha > 5k - \Delta - (1-c)
\end{cases}$$

and

$$\Pi^o(\Delta; \alpha) = \begin{cases} 
\left( \frac{1-c-k-\Delta}{2} \right)(k + \Delta), & \text{if } \alpha \leq 5k - \Delta - (1-c) \\
(1 - 3k - c)(k + \Delta), & \text{if } \alpha > 5k - \Delta - (1-c)
\end{cases}$$

Clearly, when the efficiency gains are high enough (i.e. $\alpha > 5k - \Delta - (1-c)$), the merged firm is also led to employ all of its capacity.

However, if the divestiture exceeds the threshold above mentioned, regardless of the level of merger synergies, the outsider will never operate to full capacity. Thus, for $\Delta > (1-c) - 4k$,

$$\Pi^M(\Delta; \alpha) = \left( \frac{1+2\alpha-c-2k+\Delta}{2} \right)(2k - \Delta)$$

and

$$\Pi^o(\Delta; \alpha) = \left( \frac{1-c-2k+\Delta}{2} \right)^2$$
At stage four, as far as the outsider is concerned, the decision to accept to take over the divested assets depends on his willingness to pay for them, equal to $\Pi_o(\Delta; \alpha) - \Pi$. We denote by $P$ the price of divestitures proposed by the merged entity and observed by the CA. The outsider accepts the divested assets iff $\Pi_o(\Delta; \alpha) - P \geq \Pi$.

At the stage before (stage three), the insiders make a take-it-or-leave-it offer to the outsider. Thus if the assets are transferred to the outsider, the insiders set a price equal to the outsider maximum willingness to pay: $P = \Pi_o(\Delta; \alpha) - \Pi$. Moreover, the merged entity agrees to divest iff $\Pi^M(\Delta; \alpha) + P \geq 2\Pi$.

At the second stage, since the CA observes the type of the merger submitted for approval, it makes its decision based on the following programme:

$$
\max_{\Delta \geq 0} W(\Delta; \alpha)
\begin{cases}
CS(\Delta; \alpha) = CS_0 \\
\Pi^M(\Delta; \alpha) + P \geq 2\Pi \\
\Pi^o(\Delta; \alpha) - \Pi = P
\end{cases}
$$

where $CS_0$ stands for the Consumers' Surplus level before merger. Since the Consumers' Surplus constraint is binding, we will denote by $\Delta^{FB}(\alpha)$ the solution of this programme, which is the lowest positive asset transfer that ensures production at full capacity on behalf of both firms

15 We remind that the market price increases whenever firms hold slack capacity. Thus, successful remedies in our context are necessarily those that make firms produce up to their full capacity. However, given that total capacity in the industry is fixed, firms can never produce more than they did before merger. Consequently, in our framework, maximizing consumers’ surplus after merger and keeping it constant are equivalent. Finally, the positivity constraint of this programme simply states that whenever there is no asset transfer able to keep constant the price, the merger is rejected.

We characterize the First Best asset transfers as follows:

**Lemma 1.** (i) For $k \in \left(\frac{2(1-c)}{9}, \frac{2-c}{9}\right)$, there exists a threshold $\tilde{\alpha} = 9k - 2(1 - c)$, $\tilde{\alpha} \in (0, c)$, such that:

- for any $\alpha \geq \tilde{\alpha}$, the merger is accepted with divestiture $\Delta^{FB}(\alpha)$, where $\Delta^{FB}(\alpha) = \max(0, 5k - \alpha - (1 - c))$. Also, $\Delta^{FB}(\tilde{\alpha}) < \Delta^{FB}(\pi)$.

15 According to the competition policy theory and practice, merger remedies are to be the slightest modification possible that is able to restore market competition.
for any $\alpha < \hat{\alpha}$, the merger is rejected.

(ii) For $k \leq \frac{2(1-c)}{9}$, $\hat{\alpha} \leq 0$, therefore all mergers are accepted with $\Delta^{FB}(\alpha)$

(iii) For $k \geq \frac{2-c}{9}, \hat{\alpha} \geq c$ and therefore all mergers are rejected.

Whenever the divestitures $\Delta^{FB}(\alpha)$ and $\Delta^{FB}(\bar{\alpha})$ exist, the assets are transferred to the outsider at prices equal to $P^{FB} = (P(3k) - c) \cdot \Delta^{FB}(\alpha)$ and $P^{FB} = (P(3k) - c) \cdot \Delta^{FB}(\bar{\alpha})$ respectively.

See proof in the Appendix.

The threshold $\hat{\alpha}$ corresponds to the shut-down limit. In the case where this threshold is zero, for all mergers there exists a positive transfer such that the CA’s objective is fulfilled. In the last case (case (iii)) the shutdown range covers the whole interval, so there are no transfers that can make both firms operate to full capacity. Henceforth we shall only consider $k \in \left(\frac{2(1-c)}{9}, \frac{2-c}{9}\right)$, so as to deal with an interesting case$^{16}$.

This lemma shows that our mechanism replicates the outcome of CAs’ behavior to the extent that merger control decisions obey threshold criteria. Here, whenever the notified merger does not generate enough cost savings, the CA rejects it.

Moreover, if firms anticipate the CA’s decision making process, only efficiently enough mergers will be proposed, and therefore all submitted mergers shall be accepted. This is a self selection effect. In turn, if $\alpha < \hat{\alpha}$, no merger is submitted. Henceforth we shall consider only the case where $\alpha > \hat{\alpha}$.

Finally, the required transfer is higher for the less efficient merger. The reason is quite simple: in this model with fixed total capacity, for the price to be kept constant, firms need to produce to their full capacity. But the more efficiency gains it generates, the more capacity will employ the merged entity. Therefore, more efficient insiders hold less slack capacity, so the price increase will be lower for a more efficient merger, and the corresponding divestiture as well. This conforms with the proportionality principle advocated by competition policy practitioners, and justifies the fact that the average sale price of divested assets is actually constant, since $\frac{P^{FB}}{\Delta^{FB}(\bar{\alpha})} = \frac{P^{FB}}{\Delta^{FB}(\alpha)}$. Nevertheless, such a proportionality principle is subject to an implementation problem if efficiency gains captured here by the parameter $\alpha$ are not observed by the CA.

$^{16}$If the industry capacity is large enough, a duopoly is never induced to produce at full capacity even if the cost of $M$ is zero, whereas if the capacity is low enough, a duopoly always produces at full capacity, even without efficiency gains.
3.3. Optimal divestitures with asymmetric information: a regulated sale price mechanism

The game with asymmetric information is basically the same as before, taking into account the changes due to the inobservability of the merger type:

In the first stage, the merging firms learn their efficiency level and submit a merger proposal to the CA. The parameter \( \alpha \) is now private information of the merging firms, but the latter may or may not report it truthfully.

In the second stage, the CA evaluates the consequences of the merger taking into account its own merger control objective. It proposes a divestiture contract accordingly, if such a contract exists; if not, it rejects the merger. The divestiture contract will contain the amount of assets to be divested \( \Delta \) and the corresponding sale price \( P \).

In the third stage, the insiders accept or reject the divestiture. If they accept, assets will be transferred to the outsider at the price \( P \) determined by the CA.

In the fourth stage, the outsider, having observed the insiders’ choice to accept or not the divestiture contract, decides whether to take over or not the divested assets.

In the fifth stage, conditional on the divestiture contract being accepted, the Cournot market equilibrium is determined taking into account the amount of asset transfer required by the CA. The merger is abandoned whenever one of the parties rejects the contract.

Note that the CA and the outsider have a common prior on the merger’s types \( (\rho \text{ and } 1 - \rho) \). At stage four, the outsider observes the menu of contracts proposed by the CA as well as the contract chosen by the merged entity and thus revises its prior beliefs.

Following the revelation principle we restrict to truthful direct revelation mechanisms, and look for a separating perfect Bayesian equilibrium of the game.

When information is asymmetric and the only instrument employed is the asset transfer, the CA is no longer able to make the insiders reveal truthfully their efficiency level, i.e. their type, since they always choose the lowest level of divestitures that is proposed. Indeed, should the CA propose the former levels of divestiture \( (\Delta^{FB}(\alpha) \text{ and } \Delta^{FB}(\bar{\alpha})) \), associated with the previous First Best prices, i.e. equal to the outsider’s willingness to pay, we can show that the high-cost merged firm has incentives to choose the low asset transfer destined to the low-cost merged entity. Explicitly, the following holds:
\[ \Pi^M(\Delta^{FB}(\pi); \pi) + \Pi^o(\Delta^{FB}(\pi); \pi) - \Pi = \Pi^M(\Delta^{FB}(\alpha); \pi) + \Pi^o(\Delta^{FB}(\alpha); \alpha) - \Pi = P^{FB} < \Pi^M(\Delta^{FB}(\pi); \pi) + \Pi^o(\Delta^{FB}(\pi); \pi) - \Pi = P^{FB} \]

See proof in the Appendix.

In other words, given the first best levels of divestitures, mimicking is more profitable than truth-telling for the less efficient type. Indeed, when optimal divestitures are sold for a price equal to the outsider’s willingness to pay (\(P^{FB}\) or \(P^{FB}\)), the less efficient merged entity prefers the lower asset transfer, which allows it to hold spare capacity and thus increase its profit by means of a price raise. As a result, in order to induce truthful revelation, a second instrument is needed.

We model here a CA regulating the monetary transfer between the insiders and the outsiders that accompanies the asset takeover. In case this instrument should trigger the critique that the CA behaves as a sheer market regulator, we claim that it is worth analyzing it for three main reasons. Firstly, it allows us to test its theoretical relevance for the merger control. Secondly, because the frontier between pure regulation and merger control has already been blurred by the very use of structural divestitures, which are basically meant to modify the market structure (see Motta et al. (2004)). After all, "introducing the possibility of remedies ... puts the merger control office in a position close to that of an industry specific regulator" - Rey (2003, p.130). Thirdly, regulating asset prices already concerns other competition-related fields than pure sector regulation, such as intellectual property rights, where the price of the licence would be set by the Patent Office\(^{17}\).

The incentive contract fixed by the CA will thus contain a given sale price \(P\) for an amount of divested assets \(\Delta\). Two Incentive Constraints (IC) are added to the programme of the CA to induce revelation of information. Hence, the programme of the CA writes:

\(^{17}\)Such a "buy out" mechanism is formally analyzed in Llobet et al. (2001).
\[ \max_{\{(\Delta; P), (\overline{\Delta}; \overline{P})\}} \rho W(\Delta; \alpha) + (1 - \rho) W(\overline{\Delta}; \overline{\pi}) \quad (\text{AS}) \]

\[
\begin{align*}
&\text{s.t.} \\
&\quad CS(\Delta; \alpha) = CS(\overline{\Delta}; \overline{\pi}) = CS_0 \\
&\quad \Pi^M(\Delta; \alpha) + P \geq \Pi^M(\overline{\Delta}; \overline{\alpha}) + \overline{P} \\
&\quad \Pi^M(\overline{\Delta}; \overline{\pi}) + \overline{P} \geq \Pi^M(\Delta; \overline{\alpha}) + P \\
&\quad \Pi^M(\Delta; \alpha) + P \geq 2\Pi \\
&\quad \Pi^M(\overline{\Delta}; \overline{\alpha}) + \overline{P} \geq 2\Pi \\
&\quad \Pi^o(\Delta; \alpha) - P \geq \Pi \\
&\quad \Pi^o(\overline{\Delta}; \overline{\alpha}) - \overline{P} \geq \Pi \\
\end{align*}
\]

where the contracts \((\Delta; P)\) and \((\overline{\Delta}; \overline{P})\) are destined for types \(\alpha\) and \(\overline{\pi}\) respectively.

Note that unlike a standard screening programme, there is no direct transfer between the agent and the principal. Instead, the CA fixes the monetary transfer between the agent and a third party, the outsider, and uses it as an incentive device. The lump-sum transfer between firms does not affect total industry profit, and is different from the suggestion of Röller et al. (2000) to introduce an explicit licence to merge, to the extent that in our model the monetary transfer does not benefit directly the CA. In this way we actually avoid the direct implication of the CA in the merger process as an explicit regulator, although the revelation mechanism is indeed based on the taxation principle.

Note equally that since the contract we look for induces information revelation, i.e. separation of types, in equilibrium the priors of the outsider necessarily coincide with its revised beliefs. As in the symmetric information configuration, CA employs a supplementary choice variable to solve the screening problem, namely the shut-down policy (i.e. refuse the merger of the less efficient type), hence the positivity constraints on the asset transfers. Finally, the equality constraints on Consumers’ Surplus as well as the participation constraints correspond actually to a three-party negotiation. Indeed, the additional instrument we propose to screen mergers, namely the sale price of the divested assets, is merely a lump-transfer between the outsider and the insiders, hence it stands for a particular distribution of total industry profit. To achieve screening, the contracts proposed by the CA need to ensure the industry firms’ participation. Thus, the above programme is actually designed to make the industry parties involved agree on the incentive-compatible sharing of their total profit.
Before presenting the optimal contract we give the following lemma:

**Lemma 2.** For any $\Delta < \overline{\Delta}$, $\Pi^M(\Delta; \overline{\alpha}) - \Pi^M(\overline{\Delta}; \alpha) > \Pi^M(\overline{\Delta}; \overline{\alpha}) - \Pi^M(\Delta; \alpha)$. 

See proof in the Appendix.

This inequality stands for a standard single crossing condition. It states that the efficient merged firm benefits more from a low asset divestiture than the inefficient firm. In other words, the efficient merged firm attaches more value to capacity than the inefficient one. The intuition goes as follows: following merger and the ensuing increase in capacity, the efficient insiders are always able to produce the same quantity as the inefficient ones, and can thus guarantee themselves the same revenue. Yet, thanks to the synergy cost reduction, their profit increase is actually higher, hence their willingness to receive w.r.t. the monetary payment of a given divestiture is lower. As in any standard Principal Agent model, this lemma makes room for screening. We can therefore derive the optimal contract summarized in the following proposition:

**Proposition 1.** Denote $(\Delta, \overline{\Delta}, P)$, $(\overline{\Delta}, \overline{\overline{\Delta}}, \overline{P})$ the divestiture contracts proposed with asymmetric information when they exist. The solution of the programme (AS) is such that:

(i) No shut-down
When $\overline{\alpha} > \hat{\alpha}$, then $\Delta = \Delta^{FB}(\alpha)$ and $\overline{\Delta} = \Delta^{FB}(\overline{\alpha})$. Prices are $P = \Pi^o(\Delta; \overline{\alpha}) - \Pi^M(\Delta; \alpha)$ and $\overline{P} = \overline{P} - \Pi^M(\overline{\Delta}; \overline{\alpha}) + \Pi^M(\overline{\Delta}; \alpha)$. Moreover, there exists a threshold $\hat{\alpha}$ such that for $\alpha > \hat{\alpha}$, $\overline{P} < 0$.

(ii) Shut-down of less efficient type
When $\overline{\alpha} < \hat{\alpha}$, type $\overline{\alpha}$ merger is rejected by offering a single contract: $\Delta = \Delta^{FB}(\alpha)$ and $\overline{\Delta} = 2\Pi - \Pi^M(\Delta; \alpha)$.

See proof in the Appendix.

The optimal contract has two main characteristics. First, there is no distortion of asset divestitures and the merger clearance decision is unchanged as compared with the case of symmetric information. Second, the price of the low divestiture is distorted downwards, whereas the price of the high divestiture is not distorted at all (meaning it still equals the outsider’s willingness to pay), so that the average divestiture price increases now with the level of divestiture. To sum up, we obtain no distortion at all in terms of asset transfers, and distortion ‘at the top’ for the monetary transfers.

The intuition behind the design of these contracts proceeds in two steps.
First of all, we have previously emphasized that the CA must distort asset prices and possibly divestiture levels so as to obtain separation of types. Otherwise, both firms are induced to choose the low level of divestiture. Note however that since the asset price is a lump-sum transfer between firms without impact on the CA’s objective, the CA can distort prices at no cost, so as to induce the inefficient firm to choose the high divestiture.

Second, it remains to show that price distortion alone is sufficient to lead both firms to choose the optimal levels of divestiture disclosed in Lemma 1. For that purpose the CA must lower the price of the low level of divestitures so as to deter the inefficient firm from choosing such a contract. According to Lemma 2, a given divestiture is more distorting for the efficient merged firm than for the inefficient one, and thus we have seen that the willingness to receive for the First Best $\Delta$ is lower for the efficient entity. Hence we can find a price for this low level of asset divestiture that induces the inefficient entity to give it up, and this price will consequently need to be lower than the outsider’s willingness to pay. To sum up, it is enough to keep the price of the high First Best divestiture equal to the outsider’s willingness to pay, and in turn to set for the low First Best divestiture a price inferior to the outsider’s willingness to pay.

Moreover, in one particular configuration, the asset price can be negative, so that the CA would require the efficient firm to subsidize the outsider. Specifically, if the efficient firm has very substantial cost savings, the optimal level of divestiture required to the efficient firm is so low, that in order to prevent the inefficient merged firm from choosing it, the corresponding distortion on the sale price will make it negative.

Finally, the CA might have to reject the less efficient merger due to the absence of optimal divestiture, i.e. when $\alpha < \bar{\alpha}$. But, with asymmetric information and not distorted prices, both types of insiders will submit their merger. Again, to extract information, the CA will distort downwards the price for the low divestiture, so as to incite the submission of the highly efficient merger only. Point (ii) of our Proposition actually gives the value of $P$ that violates the participation constraint of $\pi$, while still ensuring that of $\alpha$. For this price, only the efficient merger will be submitted.

A low price of divested assets is often interpreted as a signal of failure of the divestiture process. The European Commission’s Merger Remedies Study, October 2005 (p.103), argues that "remedies were less effective in at least three divestiture cases where the purchaser had acquired the divested business for free,
or at a negative price\(^{18}\). We claim however that a more efficient merger can signal itself as such by accepting a low average asset price combined with low quantity of assets divested, whereas an inefficient merged firm reveals itself as such by divesting a large quantity of assets for a high average price. In the present model, the inefficient entity owns more slack capacity and values assets less than the efficient one. This is the reason why the inefficient insiders accept to divest a larger quantity of assets, while the efficient ones prefer a low price, provided that the quantity of divested assets remains low. Hence, one way to interpret our result is that a trustee appointed by the CA might tell an efficient merger proposal from an inefficient one using such a non linear tariff for asset divestitures.

Note that this mechanism bears no risk of inefficient or distorting lobbying activities on behalf of parties involved. Indeed, our programme is designed to make all three parties agree on an incentive-compatible distribution of industry profit. Thus, the more efficient merged entity will transfer less assets at a depreciated price, and the profit sharing is favourable to the outsider, who pays less than his maximum willingness to pay. In other words, our mechanism has the initial informational conflict between the CA and the insiders solved as soon as the diverging interests of the insiders and the outsider are reconciled. Thus, by making use of a three-party incentive-compatible negotiation, the CA can make sure that no useless lobbying activities occur.

Within the debate on the frontier between competition policy and regulation, our proposition is meant to draw attention to the best instruments that should be used to address the anticompetitive effects of merger. We argue that while price distortion is considered as highly interventionist and prohibited in an orthodox view of competition policy, in a merger control context the most distorting tool is more likely to be the asset transfer, rather than a lump-sum monetary transfer between industry firms. Indeed, whereas the monetary transfer does not affect market behavior of firms, the level of assets divested has a direct impact on firms’ production decisions. As a result, in order to induce firms to reveal efficiency gains, the use of monetary transfer appears less interventionist than the transfer of physical assets.

Our proposition suggests that firms be asked to pay a kind of licence to merge. Indeed, the divestiture implies a cost imposed to the merged entity which will give up some of its assets. The way it will pay to be allowed to merge depends on its level of efficiency. Basically, the inefficient merged firm pays by giving up assets,

\(^{18}\) See also Farrell (2003), or the Antitrust Division Policy Guide to Merger Remedies, by the U.S. Department of Justice, October 2004.
whereas a very efficient one pays by means of a monetary transfer to the outsider.

4. Conclusion

This paper contributes to the economic analysis of merger remedies. We propose a revelation mechanism allowing the design of optimal merger divestitures when information is asymmetric between firms and the CA with respect to the synergy generated by the merger. Our revelation mechanism replicates the typical behavior of a CA, namely decision making based on thresholds of announced efficiencies. In our framework, shut-down of the least efficient type is possible. Basically, mergers will only be accepted if they generate enough synergies, and this is what the merger control practically aims at.

Our results show that the sale price of divested assets is a powerful screening device. Despite the information asymmetry, types are perfectly screened, and only divestiture sale prices get distorted. Complete distortion, i.e. also affecting divestiture levels, would be possible in our framework if several modifications were performed, such as introducing a no longer prohibitive cost of capacity acquisition, or imposing a positivity constraint on asset sale prices.

We acknowledge of course the modelling of a CA actively modifying the market structure, but then any structural merger remedy is precisely meant to do this. Taking into account the reluctance of competition authorities to actually employ this instrument for the screening of merger projects, we can nevertheless insist on the ability of the price of divested assets to signal on behalf of merging partners the efficiency potential of their merger.

References


Appendix

Post-merger Cournot equilibrium. Successful Remedies:
These are the transfers for which both firms produce up to their post-merger capacity: for \( M \), \( 2k - \Delta \), and for \( o \), \( k + \Delta \). Checking that the Best Reply function yield in equilibrium precisely the post-merger capacities allows us to compute the limits of the relevant range for the divestiture: for \( M \): \( BR^M(q^* = k+\Delta) = \frac{1+\alpha-c-(k+\Delta)}{2} \geq 2k-\Delta \leftrightarrow \Delta \geq 5k-\alpha-(1-c) = \Delta_1 \)
for $a$: $BR^a(q^M = 2k - \Delta) = \frac{1-c-(2k-\Delta)}{2} \geq k + \Delta \iff \Delta \leq (1-c) - 4k = \Delta_2$

It is straightforward to check that for $\Delta < \Delta_1$, the outsider produces up to its full capacity, but the merged firm holds slack capacity, whereas for $\Delta > \Delta_2$ the reverse is true. Note that $\Delta_1 > 0$ as long as $\alpha \leq 5k - (1-c)$, a necessary condition being $k \geq \frac{1-c}{5}$ so as to have positive cost savings.

**Proof of Lemma 1.** As seen before, successful remedies belong to $[\Delta_1, \Delta_2]$, with $\Delta_1 = 5k - \alpha - (1-c)$. The First Best level of divestiture is actually $\Delta_1$, since we take it to be the lowest positive asset transfer that satisfies the objective: $\Delta^{FB}(\alpha) = 5k - \alpha - (1-c)$.

The existence of $\Delta^{FB}(\alpha)$ is ensured as long as the interval $[\Delta_1, \Delta_2]$ exists. Define $\hat{\alpha}$ as the threshold value of cost savings for which $\Delta_1 = \Delta_2$: $\hat{\alpha} = 9k - 2(1-c)$. This threshold is positive provided that $k \geq \frac{2(1-c)}{9}$. Whenever $\alpha < \hat{\alpha}$, we have $\Delta_1 > \Delta_2$ therefore the CA rejects all mergers, since there is no transfer $\Delta$ for which firms produce both to full capacity. In turn, for $\alpha \geq \hat{\alpha}$, $\Delta^{FB}(\alpha) = 5k - \alpha - (1-c)$ and since $\alpha > \bar{\alpha}$ we obtain directly $\Delta^{FB}(\alpha) < \Delta^{FB}(\bar{\alpha})$.

Since assets are divested to the outsider at its willingness to pay, prices are given by: $\Pi^{FB} = \Pi^o(\Delta^{FB}(\alpha); \bar{\alpha}) - \Pi = (P(3k)-c)\Delta^{FB}(\alpha)$ and $\Pi^{FB} = \Pi^o(\Delta^{FB}(\bar{\alpha}); \bar{\alpha}) - \Pi = (P(3k)-c)\Delta^{FB}(\bar{\alpha})$.

**Proof - incentive to mimic.**

$$\Pi^M(\Delta^{FB}(\bar{\alpha}); \bar{\alpha}) + \Pi^o(\Delta^{FB}(\bar{\alpha}); \bar{\alpha}) - \Pi = \Pi^M(\Delta^{FB}(\alpha); \bar{\alpha}) + \Pi^o(\Delta^{FB}(\alpha); \alpha) - \Pi$$

$$\iff [P(3k) - c + \bar{\alpha}] \cdot (2k - \Delta^{FB}(\bar{\alpha})) + [P(3k) - c] \cdot (k + \Delta^{FB}(\bar{\alpha})) \leq [P(k + \Delta^{FB}(\alpha) + BR^M(k + \Delta^{FB}(\alpha))) - c + \bar{\alpha}] \cdot BR^M(k + \Delta^{FB}(\alpha))$$

$$- [P(3k) - c] \cdot (k + \Delta^{FB}(\alpha))$$

$$\iff [P(3k) - c] \cdot (\Delta^{FB}(\bar{\alpha}) - \Delta^{FB}(\alpha)) <$$

$$[P(k + \Delta^{FB}(\alpha) + BR^M(k + \Delta^{FB}(\alpha))) - c + \bar{\alpha}] \cdot BR^M(k + \Delta^{FB}(\alpha))$$

$$- [P(3k) - c + \bar{\alpha}] \cdot (2k - \Delta^{FB}(\bar{\alpha}))$$

$$\iff (1 - 3k - c) \cdot (\alpha - \bar{\alpha}) < \left(1 - 3k - c + \frac{\alpha + \bar{\alpha}}{2}\right)^2 - (1 - 3k - c + \bar{\alpha})^2$$

$$\iff \frac{1}{4} \left(\bar{\alpha} - \alpha\right) \cdot (\alpha + 3\bar{\alpha}) < 0$$ which is true since $0 \leq \bar{\alpha} < \alpha$. ■

**Proof of Lemma 2.** A necessary condition for the two incentives constraints to hold is $\Pi^M(\Delta; \alpha) - \Pi^M(\Delta; \alpha) > \Pi^M(\Delta; \bar{\alpha}) - \Pi^M(\Delta; \bar{\alpha})$.
The profit of the merged firm writes generally as follows:

\[ \Pi^M(\Delta; \alpha) = [P(k + \Delta + BR^M(k + \Delta) - c + \alpha] \cdot BR^M(k + \Delta) \]

where \( BR^M(k + \Delta) = \min \left( \frac{1 + \alpha - c - (k + \Delta)}{2}, 2k - \Delta \right) \)

We show next that \( \frac{\partial \Pi^M(\Delta; \alpha)}{\partial \Delta} < 0 \) and \( \frac{\partial \Pi^M(\Delta; \alpha)}{\partial \Delta \partial \alpha} < 0 \)

- Show \( \frac{\partial \Pi^M(\Delta; \alpha)}{\partial \Delta} < 0 \)

\[
\frac{\partial \Pi^M(\Delta; \alpha)}{\partial \Delta} = \frac{\partial BR^M}{\partial \Delta} \cdot [P(k + \Delta + BR^M(k + \Delta) - c + \alpha]
\]

\[ + BR^M(k + \Delta) \cdot \frac{\partial P(k + \Delta + BR^M(k + \Delta))}{\partial \Delta} \]

Note that \( \frac{\partial BR^M}{\partial \Delta} < 0 \) always, and that \( \frac{\partial P(k + \Delta + BR^M(k + \Delta))}{\partial \Delta} \leq 0 \).

Actually, if \( BR^M(k + \Delta) = 2k - \Delta \) then \( P(k + \Delta + BR^M(k + \Delta)) = P(3k) \)

therefore \( \frac{\partial P(k + \Delta + BR^M(k + \Delta))}{\partial \Delta} = 0 \),

whereas if \( BR^M(k + \Delta) = \frac{1 + \alpha - c - (k + \Delta)}{2} \) then \( P(k + \Delta + BR^M(k + \Delta)) = \frac{1 + \alpha - c - k - \Delta}{2} \)

and thus \( \frac{\partial P(k + \Delta + BR^M(k + \Delta))}{\partial \Delta} < 0 \)

To sum up, \( \frac{\partial \Pi^M(\Delta; \alpha)}{\partial \Delta} < 0 \)

- Show \( \frac{\partial^2 \Pi^M(\Delta; \alpha)}{\partial \Delta \partial \alpha} < 0 \)

\[
\frac{\partial^2 \Pi^M(\Delta; \alpha)}{\partial \Delta \partial \alpha} = \frac{\partial}{\partial \alpha} \left( \frac{\partial \Pi^M(\Delta; \alpha)}{\partial \Delta} \right) = \frac{\partial^2 BR^M}{\partial \alpha \partial \Delta} \cdot [P(k + \Delta + BR^M(k + \Delta) - c + \alpha] +
\]

\[ + \frac{\partial BR^M}{\partial \Delta} \cdot \frac{\partial}{\partial \alpha} \left[ P(k + \Delta + BR^M(k + \Delta) - c + \alpha] \right]_{< 0} + \frac{\partial BR^M}{\partial \Delta} \cdot \frac{\partial P(k + \Delta + BR^M(k + \Delta))}{\partial \Delta} \left[ \frac{\partial}{\partial \alpha} \right]_{\geq 0} \]

\[ + BR^M(k + \Delta) \cdot \frac{\partial^2 P(k + \Delta + BR^M(k + \Delta))}{\partial \Delta \partial \alpha} \left[ \frac{\partial}{\partial \alpha} \right]_{\leq 0} \]

Conclusion: since \( \alpha > \overline{\alpha} \), the cross derivative \( \frac{\partial^2 \Pi^M(\Delta; \alpha)}{\partial \Delta \partial \alpha} < 0 \) yields equivalently

\( \frac{\partial}{\partial \alpha} \left( \frac{\partial \Pi^M(\Delta; \alpha)}{\partial \Delta} \right) < 0 \Leftrightarrow \frac{\partial \Pi^M(\Delta; \alpha)}{\partial \Delta} - \frac{\partial \Pi^M(\Delta, \alpha)}{\partial \Delta \partial \alpha} < 0 \)
\[ \frac{\partial}{\partial \Delta} (\Pi^M (\Delta; \omega) - \Pi^M (\Delta; \bar{\alpha})) < 0, \forall \Delta \]

\[ \Rightarrow \text{for any } \Delta < \bar{\Delta}, \Pi^M (\Delta; \bar{\alpha}) - \Pi^M (\bar{\Delta}; \bar{\alpha}) < \Pi^M (\Delta; \omega) - \Pi^M (\bar{\Delta}; \omega), \text{ q.e.d.} \]

**Proof of Proposition 1.**

- No shut-down

For \( \mathcal{P} = \Pi^o (\Delta; \bar{\alpha}) - \Pi \) and \( \mathcal{P}^o = \Pi^o (\Delta; \alpha) - \Pi \), both firms prefer the contract \((\Delta, \mathcal{P})\), since the following inequality holds for the inefficient type:

\[ \Pi^M (\Delta; \bar{\alpha}) + \Pi^o (\Delta; \alpha) > \Pi^M (\Delta; \alpha) + \Pi^o (\Delta; \alpha) \]

In turn, if \( \mathcal{P} = 2\Pi - \Pi^M (\Delta; \alpha) \), then \( \Pi^M (\Delta; \alpha) + \mathcal{P} < \Pi^M (\Delta; \alpha) + \mathcal{P} \)

so both firms prefer the contract \((\Delta, \mathcal{P})\).

By continuity of \( \mathcal{P} \), there exist \( \mathcal{P} \) and \( \mathcal{P}^o = \Pi^o (\Delta; \alpha) - \Pi \) with

\[ \Pi^o (\Delta; \alpha) - \Pi \geq \mathcal{P} > 2\Pi - \Pi^M (\Delta; \alpha) \]

such that \( \Pi^M (\Delta; \alpha) - \Pi^M (\Delta; \alpha) = \mathcal{P} - \mathcal{P} \)

This latter condition ensures that \( \mathcal{P} - \mathcal{P} > \Pi^M (\Delta; \bar{\alpha}) - \Pi^M (\Delta; \bar{\alpha}) \), i.e. separation of types, thanks to the single-crossing condition (see Lemma 2).

We show next there exists \( \bar{\alpha} \) such that for \( \alpha > \bar{\alpha}, \mathcal{P} < 0 \)

First of all, note that for \( \Delta = 0, \mathcal{P} < 0 \) necessarily, because:

\[ \mathcal{P} = \mathcal{P} - (\Pi^M (\Delta; \alpha) - \Pi^M (\Delta; \bar{\alpha})) \]

\[ = \Pi^o (\Delta; \bar{\alpha}) - \Pi - (\Pi^M (\Delta; \alpha) - \Pi^M (\Delta; \bar{\alpha})) \]

\[ = (P(3k) - c) \cdot (k + \bar{\Delta}) - (P(3k) - c) \cdot k \]

\[ - ((P(3k) - c + \alpha) \cdot (2k - \Delta) - (P(3k) - c + \alpha) \cdot (2k - \bar{\Delta})) \]

\[ = (P(3k) - c) \cdot \bar{\Delta} - (P(3k) - c + \alpha) \cdot (\Delta - \bar{\Delta}) \]

\[ \Rightarrow \text{for } \Delta = 0, \text{ this yields } (P(3k) - c) \cdot \bar{\Delta} - (P(3k) - c + \alpha) \cdot \bar{\Delta} < 0 \]

Moreover, \( \Delta = 0 \iff 5k - \bar{\alpha} - (1 - c) = 0 \iff \alpha = 5k - (1 - c) \)

To sum up, for \( \alpha = 5k - (1 - c) \) (which by the way is \( > \bar{\alpha} \)), \( \mathcal{P} < 0 \)

But, for \( \alpha = \bar{\alpha}, \mathcal{P} = \mathcal{P}^o \), thus \( \mathcal{P} > 0 \)

Therefore, by continuity and monotonicity\(^{19}\) of \( \mathcal{P} \), there exists a \( \tilde{\alpha} > \bar{\alpha} \) such that for \( \alpha > \tilde{\alpha}, \mathcal{P} < 0 \)

\(^{19}\)It is straightforward to show that \( P \) is decreasing with \( \alpha \)
Last but not least, we can show that a sufficient condition for this threshold \( \alpha \) to be \( < c \) is to have \( k < \frac{1}{c} \), which is compatible with our condition for positive cost savings, namely \( k \geq \frac{1-c}{\theta} \).

- Shut-down of less efficient merger
  
  When \( \pi < \alpha \), the optimal response from the CA is to reject the \( \pi \), simply because it is the only way to keep the price constant - see Lemma 1, which shows that when there is no transfer \( \Delta \) that can keep the price constant, the CA rejects the merger. To prevent therefore the submission of the \( \pi \) merger, it is enough to set \( P = 2\Pi - \Pi^M (\Delta; \pi) \), which violates the participation constraint of the \( \pi \) type.