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The effects of technological innovations on employment: a new explanation

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2005.13
The Effects of Technological Innovations On Employment: A New Explanation *

Chahnez BOUDAYA†

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*I’m grateful to A. Dib, J-O. Hairault, R. Perotti, L. Phaneuf and N. Rebei. I also thank participants at the T2M conference (Lyon, 2005) and ADRES meeting (Paris, 2005) for stimulating suggestions and comments. The usual disclaimer applies.

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Résumé

Ce papier examine l’effet des chocs technologiques sur l’emploi principalement. Notre objectif est de reproduire la baisse initiale des heures travaillées suivant un choc technologique favorable, largement soutenue depuis les travaux de Gali (1999), indépendamment de la nature de la politique monétaire. Le modèle simulé sur des données américaines d’après guerre, révèle que l’introduction d’un input intermédiaire dans la structure de production permet d’obtenir l’effet escompté même si la politique monétaire est fortement accommodante. En effet, la prise en compte d’une telle structure de production produit trois mécanismes de propagation dans le modèle (i) elle augmente le degré de rigidités de prix; (ii) la demande de travail est décroissante dans la part de l’input intermédiaire dans la fonction de production; (iii) au fur et à mesure que la part de l’input intermédiaire augmente pour atteindre des valeurs plausibles pour les données d’après guerre, l’input intermédiaire et les heures travaillées deviennent des substituts.


Mots clés: Chocs technologiques, prix rigides, emploi, inputs intermédiaires, règle de Taylor, politique monétaire exogène.

Abstract

This paper investigates principally the effects of a technological innovation on hours worked in a sticky price model. Our challenge is to reproduce the short-run decline in employment supported by a large range of recent works, inspired by Gali (1999), regardless of any monetary policy consideration. The model simulations concern the postwar U.S. economy under two different monetary policy: an exogenous rule targeting money supply and a simple Taylor rule. The most interesting result is that the introduction of an input-output production structure counterbalances the full-accommodation of a technological innovation when monetary policy is governed by a Taylor rule, by (i) providing the model with more price rigidities; (ii) inducing a substitution effect between intermediate goods and labor input for plausible values of intermediate inputs share.

JEL Classification: E23, E24, E31, E32, E52.

Keywords: Technology shocks, sticky prices, labor, intermediate inputs, Taylor rule, exogenous monetary policy.
1 Introduction

Competing business cycle models are typically evaluated on the basis of their ability to reproduce the comovements of macroeconomic variables observed in the data. Recently, a large broad of studies has drawn the attention to the correlation between labor input and technology shocks. For instance, Gali (1999), Basu, Fernald and Kimball (1998) (hereafter BFK), and Kiley (1998) have documented for the U.S. and other G7 economies a provocative evidence\(^1\) that employment \textit{falls} following technology improvements, at least in the short-run. This controversial result has stimulated subsequent empirical contributions to qualify Gali’s claim. It also motivated a debate on what theories can account for this empirical evidence.

Empirical studies on the effects of technology improvements on labor fall into three categories. The first category is based on a structural vector autoregressive approach (VAR) initially developed by Olivier Blanchard and Danny Quah (1989) with long-run restrictions to identify technology shocks. For instance, in the same spirit, Gali (1999) assumes that the technology shock is the only shock that affects labor productivity in the long run. He estimated in a first step a bivariate VAR with changes in labor productivity and labor input using U.S. quarterly data covering the period 1948:1-1994:4. He finds that a favorable technology shock causes labor to decrease. Particularly, a 1\% positive shock on the technological change results in an initial drop of labor by 0.4\%\(^2\). Moreover, a five-variable VAR specification using data on money, interest rates and inflation in addition to labor input and labor productivity shows that a favorable “... technology shock leads to an immediate increase in productivity that is not matched by a proportional change in output...implying a transitory -though persistent-decline in hours.”\(^3\). This contractionary effect is robust for all G7 countries but Japan. Thus, that result is at odds with the predictions of standard business cycle models and contrasts sharply with the mechanisms underlying fluctuations emphasized in the RBC literature. Besides, the prediction of a negative comovement

\(^1\)In fact, this contractionary effect is contrasting with the standard business cycle models predictions that technological progress not only expands the production frontier but also creates jobs.

\(^2\)see Gali 1999, figures 2 and 3, pp261-262.

\(^3\)Gali (1999), pp261.
between technology and employment is supported by other structural VAR models concerning the effects of identified technology shocks. Francis and Ramey (2001) confirmed Gali’s results by enlarging the number of identifying long-run restrictions also used as overidentifying tests. For instance, analogous for the labor productivity, they assume that only technology shocks can have a permanent effect on the real wages whereas they should have only a temporary effect on hours. In a first step, using quarterly data from 1947:1 to 2000:4, they compare the technology shocks effects across different identification schemes in a bivariate VAR with labor productivity and input. In their first model, only technology shocks can have permanent effects on labor productivity which corresponds to Gali’s identification scheme. In the second model, only technology shocks can have permanent effects on real wages and in the final one, the technology shocks cannot have permanent effects on hours. Despite the differences in long-run identification schemes, they produce reactions of hours and productivity (or wages) that were similar across the systems; in all three schemes, a positive technology shock appears to lead to a decline in hours for at least one year. The initial response hinged between -0.3 and -0.4 depending on the scheme considered. In a second step, they estimate a five-variable Vector-Error Correction model (VECM) containing the logs of productivity, labor input, private output, real product wage, investment and consumption. Consistent with the bivariate results, a positive technology shock raises productivity and real wages permanently and lowers hours in the short-run by 0.25%. Also, following Gali (1999), Kiley (1998) identifies technology shocks by imposing the restriction that only fluctuations in technology have long run effects on labor productivity in separate sectoral VARs involving employment and labor growth in the sector. He estimates a VAR for quarterly data from 1968:2-1995:4 for aggregate manufacturing and each of the 17 two-digit U.S. manufacturing industry. He shows that “...technology-induced fluctuations ....yield negative comovement between employment and output, and labor productivity and employment, within sectors.”

A second category of empirical evidence is based on an accounting approach well-exemplified by the works of Susanto Basu, John Fernald and Miles Kimball (1998, 2004). BFK use a sophisticated growth accounting methodology allowing for increasing returns, imperfect competition, variable

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4Kiley (1998), pp21
factor utilization and sectoral compositional effects in order to construct a time series for aggregate technological change in the postwar U.S. economy. Following Hall (1990), they assume cost minimization and relate output growth to inputs growth rate. The first-order conditions provide the weights on growth of each input. The "purified" technology change is a weighted sum of industry technology change; the weights are given by the firm's share of aggregate nominal value added after converting gross-output technology shocks to a value-added basis. Finally, they estimate bivariate VARs with some function of the log of hours worked and their "purified" technology measure covering the period 1951 to 1996. They identify "true" long-run technology shocks as the estimated VAR shock that affects the long-run level of their purified technology series. They estimate VARs for both log-levels and log-differences of hours worked. Both specifications show strong evidence that technology improvements reduce hours worked (the initial drop is by 1% in level and 0.75% in log-differences). Also, Basu (1998) investigates the effects of a technology shock by estimating a VAR with changes in BFK's new measure of technology and other macroeconomic variables, e.g. output, a measure of capital-labor input, total hours worked and a measure of capital and labor utilization. Basu (1998) findings confirm the contractionary effect of a technology improvement on hours worked. Relative to the previous empirical approach, we should notice that BFK's and Basu's accounting approach does not rely on any long-run restrictions since the series for technology are measured.

A third category of empirical evidence is provided by several studies exploiting disaggregated data. For instance, Shea (1998) examines the time series interactions between measures of technological change, such as patents and research and development, and measures of economic activity. He uses panel data on inputs, total factor productivity and technological indicators for 19 U.S. manufacturing industries covering 1959-1991 in a structural VAR. He finds that favorable R&D or patent shocks tend to increase inputs, especially labor, in the short run, but to decrease inputs in the long run. In addition, Marchetti and Nucci (2004) investigate the relationship between technology shocks and labor inputs for the Italian economy using highly detailed panel data of a representative sample of Italian manufacturing firms for the period 1984-1997. Following BFK (1998), they derive a measure of technology change from a theoretical model based on a dynamic cost min-

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5 see BFK(2004) pp.6, equation (1.4).
imization set-up that controls for imperfect competition, increasing returns and variable utilization of labor and capital. Finally, they estimate the model by GMM and conclude that a negative relationship between a technology improvement, labor and other inputs emerges from their data.

Thus, despite differing data, countries and methods, the bottom line is that the three different empirical approaches give similar results: technology improvements lead to a contractionary impact on labor input. These results are clearly inconsistent with standard parameterizations of frictionless RBC models.

The empirical evidence of a contractionary technology improvement has been supported by several theoretical models. While, Francis and Ramey (2001) propose a variant of the standard RBC model with inertial consumption and investment (coming from habit formation and investment adjustment costs), Basu and al. (1998) and Gali (1999) have interpreted their findings as an evidence in favor of sticky price models. Consider the simple case where the quantity theory governs the demand for money, so output is proportional to real balances. In the short run, if money supply is fixed and prices cannot adjust, then real balances and output are also fixed. If technology improves, firms need less labor to produce the same output, so they lay-off workers. Over time, with price adjustments, the underlying real-business-cycle dynamics take over and output rises. Marchetti and Nucci (2004) find that technology improvements reduce the input use only for the firm that had sticky prices for a year or more. Of course, within a sticky-price model, the pattern of correlation between input growth and technology shocks hinges crucially on the response of the monetary authorities to technology shocks, which depends, in turn, on the characterization assumed for the systematic part of monetary policy. In particular, in the context of a general equilibrium model with staggered price settings, Dotsey (1999) shows that when the central bank follows the optimal monetary policy or a Taylor (1993) rule or the rule estimated by Clarida and al. (2000), the effect of a favorable technology shock on employment is no longer negative since the monetary policy provides a full accommodation of the shock (which implies a jump in output and labor). The explanation of this finding is that, with a staggered price setting, a technology improvement decreases firms’ marginal costs and generates a reduction in the aggregate price level that is smaller than that obtained under perfect price flexibility. Consequently, aggregate demand increases but less than under price flexibility. A wedge between output and its natural level is created, therefore, the output gap decreases and
so does the inflation. When the monetary authority responds to deviations in output from its natural level, it would reduce the policy rate to provide full accommodation of the shock. This implies a positive correlation between the technology shocks and the labor input. Dotsey (1999a) however, shows that the initial drop in labor following a positive technology shock is possible with a modified Taylor rule where the output gap is replaced by the growth rate of output or under a constant money growth rule. Basu (1998) allows the monetary policy to follow a Taylor rule, setting the nominal interest rate in response to lagged inflation and the lagged output gap. He finds that inputs fall sharply initially. Monetary policy is insufficiently loose under a Taylor rule, in part, because the Federal Reserve bank reacts only with a lag. Gali (1999) and King and Wolman (1996) show that under a constant money growth rule, labor decreases in response to technological innovation as long as the response of the monetary authority falls short of full accommodation. In addition, Gali, Lopez-Salido and Valles (2003) show that only a monetary targeting rule can allow for the initial drop in labor input following a technology improvement.

The results summarized above imply that only economies where the monetary policy is well characterized by a money growth pegging or a simple rule failing to fully respond to the technology shocks would allow for the initial drop in labor input in response to a favorable technology shock.

These considerations motivate our empirical investigation of the relationship between technology shocks and labor input. Our model follows the same spirit of analysis by Gali (1999). In fact, our challenge is to reproduce the contractionary effect of technology innovation upon hours worked but regardless of the monetary policy concerns. In particular, we focus on a money supply rule (MS Model) versus a simple Taylor rule (TR Model). While, in the MS model, nominal price rigidities are sufficient to reproduce the expected contractionary effect on labor, this could not happen when we consider a simple Taylor rule. More endogenous propagation dynamics are needed. This motivates our major contribution to introduce an input-output production structure as suggested by Atta-Mensah and Dib (2003), Huang, Liu and Phaneuf (2004) and Basu (1995). However, while Huang and al. (2004) focus on a business cycle model driven solely by demand shocks to explain the evolution in real wages cyclicality during the 20th century, we take into account technology shocks and concentrate on labor-technology shocks relationship. For that, we develop a model with monopolistic competition between firms producing intermediate goods and perfect competition between
those producing final goods. Besides, we consider nominal price rigidities in the spirit of Gali (1999) but with capital accumulation. However, our model differs from Gali (1999) in several important aspects: First, in our model, we abandon the labor effort variable which seems not to modify the response of other variables to technological innovation. Besides, firms set their prices optimally in a randomly staggered fashion as suggested by Calvo (1983). More specifically, each firm resets its price in any given period only with the probability \((1 - \theta)\), independently of other firms and of the time elapsed since the last adjustment. Thus, a measure \((1 - \theta)\) of producers reset their prices each period, while a fraction \(\theta\) keep their prices unchanged. This assumption is more realistic than the one-period price rigidity suggested by Gali. The last difference concerns the shocks. While Gali considers only technology and monetary shocks, we study the results of an exogenous monetary policy (a money supply shock) versus an endogenous monetary policy which is determined by a simple Taylor rule. Under this rule, the nominal interest rate deviates from the level consistent with the economy’s equilibrium rate and the target inflation rate if the output gap is nonzero or if inflation deviates from the target. A positive output gap leads to a rise in the nominal interest rate as does a deviation of actual inflation above the target. In this case, a favorable technology shock is followed by a significant increase in output and labor as monetary policy is fully accommodating the shock. The intuition is that, with staggered price settings, a technology improvement decreases firms’s real marginal costs and generates a reduction in the aggregate price level that is smaller than that obtained under perfect price flexibility. Consequently, aggregate demand increases, but by less than under price flexibility. This creates a wedge between output and its natural level (achieved when prices fully adjust), and therefore, the output gap diminishes and so does the inflation. This results in a reduction in the monetary policy rate to provide full accommodation of the shock. Therefore, output rises and there is a positive relationship between labor input and technology. As noticed above, only a simple monetary rule which doesn’t accommodate technology shocks could reproduce the initial decline in labor input when combined with price rigidities. However, when we consider a Taylor rule, introducing an input-output production structure counterbalances the reaction of monetary authorities to a technology improvement through three effects: First, it provides the model with more price rigidities. In fact, the intermediate input is a part of the final good and could be either consumed or invested, or used as an
intermediate production input. So, the rigid intermediate input price corresponds to the aggregate price level. Second, the intermediate input price level becomes a more significant component of marginal cost which records not only the real wages and the capital rental rate but also the intermediate input price. This causes marginal cost to become more rigid. Therefore, a less variable marginal cost increases the rigidity in firms’ pricing decisions. As a consequence, with intermediate inputs, a technology improvement reduces marginal costs but less than with no intermediate goods. Hence, the induced increase in labor input is less important. Finally, as the intermediate input share gets greater, the conditional demand for intermediate input derived from cost-minimization problem becomes more important, whereas the demand for labor input decreases. There is a substitution effect between intermediate inputs and labor inputs through intermediate inputs share. Thus, the combined effect of a plausible value for the intermediate inputs share for the postwar U.S. economy and more price rigidities induced by the presence of intermediate goods should provide the expected short-run decline in hours following a favorable technology shock. Moreover, as the intermediate input share gets greater, the initial drop should be more important as the substitution effect becomes stronger.

\[^6\] Hence, the aggregate demand constraint becomes \( Y_t = C_t + I_t + X_t \) where \( X_t \) is the intermediate input.

\[^7\] In fact, the cost function is:

\[
\frac{W}{P}L + r_kK + PX
\]

and marginal cost function \( MC = cons \tan tP^{\Psi}[r_k^{1-\alpha}W^{\alpha}]^{1-\Psi} = f\left(\frac{W}{P}, r_k, P\right)\)

where \( P \) is the aggregate price level, \( r_k \) is the real capital return, \( \frac{W}{P} \) are real wages, \( \Psi \) is the intermediate input share, \( \alpha \) is the labor share and \( cons \tan t \) depends on \( \alpha \) and \( \Psi \).

\[^8\] In fact, the optimality condition relative to the price level is

\[
P_t(i) : \frac{e}{e-1} \frac{E_t \sum_{j=0}^{\infty} (\theta \beta)^j \Lambda_{i+j} Y_{t+j}(i) k_{i+j}}{E_t \sum_{j=0}^{\infty} (\theta \beta)^j \Lambda_{i+j} Y_{t+j}(i) k_{i+j}}
\]

which relates the optimal price to the expected future price of the final good and to the expected future real marginal costs (which depends in turn on rigid the final good price among others).

\[^9\] The conditional demand for intermediate goods and labor inputs are:

\[
X_t = \frac{\Psi Y}{\mu}
\]

\[
L_t = \frac{(1-\Psi)\alpha Y}{\mu}
\]

where \( \mu \) is the markup.
When monetary policy is exogenous and governed by a money supply rule, nominal price rigidities à la Calvo are sufficient to generate the initial drop in labor following a favorable technology shock regardless of the form of price rigidities. This is an expected result. In fact, suppose that in addition to staggered price settings, the quantity theory governs the demand for money so that the output is proportional to the real balances. In the short run, if money supply remains unchanged and prices cannot adjust, then real balances and hence output are also fixed. Even though all firms will experience a decline in their marginal cost following a favorable technology shock, only a fraction of them will adjust their prices downwards in the short run. Accordingly, the aggregate price level will decline increasing the markup and aggregate demand will rise less than proportionally to the increase in productivity. This implies an increase in the wedge between the marginal product of labor and the real wage. Since the wedge will eventually return to its steady state level, there is a strong substitution effect that causes labor input to fall in the impact period. A more interesting result emerges from TR Model where a technology improvement causes hours worked to drop in the short-run even when monetary policy is fully accommodating the shock.

The paper is organized into three sections. Section II presents the benchmark model with monopolistically competitive firms, capital accumulation, nominal price rigidities à la Calvo and a money supply rule (henceforth MS Model). In section III, monetary policy becomes endogenous and determined by a simple Taylor rule (henceforth TR Model). Under this rule, we show that hours worked show a significant rise following a favorable technology shock when we set the intermediate input share to zero. We show that taking into account an input-output production structure allows the initial drop in employment in response to a positive technology shock for a plausible value of intermediate input share. Here, we examine different cases when intermediate input share is different from zero. In section IV, we give the preliminary conclusions.

2 MS Model: A Money Supply Rule

We assume an economy populated by a representative household, a representative finished goods-producing firm, a continuum of intermediate goods-producing firms, and a monetary authority. The finished goods-producing firm produces a finished good, that is sold on a perfectly competitive market,
while each intermediate goods-producing firm produces a distinct, perishable intermediate good, sold on a monopolistically competitive market.

2.1 Households

The representative household carries real balances and bonds $B_t$ into period $t$. At the beginning of the period, she receives lump-sum nominal transfer $T_t$ from monetary authority in addition to work revenues, capital returns and nominal profits $D_t$ as a dividend from each intermediate goods-producing firm $i$. Next, the household’s bonds mature, providing it with $B_t$ additional units of money. The household uses some of this money to purchase $B_{t+1}$ new bonds at the nominal cost $\frac{B_t}{R_{t-1}}$; hence, $R_{t-1}$ denotes the gross nominal interest rate between $t-1$ and $t$. Besides, the household maximizes her utility by the choice of consumption, the level of real balances to hold for the next period, the labor supply $N_t$, the stock of capital to lend and the bonds to hold. Total time available to the household in the period is normalized to equal one. The maximization program would be:

$$\max_{C_t, \frac{M_t}{P_t}, N_t, K_{t+1}, B_{t+1}} E_t \sum \beta^t \left\{ \frac{\gamma}{\gamma - 1} \log \left( C_t^{\frac{\gamma - 1}{\gamma}} + b \left( \frac{M_t}{P_t} \right)^{\frac{\gamma - 1}{\gamma}} \right) + \eta \log(1 - N_t) \right\}$$

subject to:

$$C_t + \frac{M_t}{P_t} + \frac{B_{t+1}}{P_t} + I_t = \frac{W_t}{P_t} N_t + \frac{R_{k,t}}{P_t} K_t + \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} + \frac{R_{t-1}}{P_t} B_t + \frac{D_t}{P_t}$$

where $\gamma$ and $\eta$ are positive structural parameters denoting the constant elasticity of substitution between consumption and real balances, and the weight on leisure in the utility function, respectively. $b$ is a positive parameter, $\beta \in (0, 1)$ is the discount factor.

$$P_t = (\int_0^{1} P_{it}^{1-\varepsilon} di)^{\frac{1}{1-\varepsilon}}$$

$\varepsilon > 1$ is the constant elasticity of substitution between different intermediate goods.

$$D_t = \int_0^{1} D_t(i) di$$
The motion law of capital is:

\[ K_{t+1} = (1 - \delta)K_t + I_t \]

The resources constraint is given by:

\[ Y_t = C_t + I_t \]

in the absence of government spending.

**First Order Conditions:**

\[ C_t : \frac{C_t^{-\frac{1}{\gamma}}}{C_t^{\frac{\gamma - 1}{\gamma}} + b\left(\bar{M}_t\right)^{\frac{\gamma - 1}{\gamma}}} = \Lambda_t \]  \hspace{1cm} (1)

\[ M_t : \frac{b\left(\bar{M}_t\right)^{\frac{1}{\gamma}}}{C_t^{\frac{\gamma - 1}{\gamma}} + b\left(\bar{M}_t\right)^{\frac{\gamma - 1}{\gamma}}} = \Lambda_t - \beta E_t(\Lambda_{t+1}\frac{1}{\pi_{t+1}}) \]  \hspace{1cm} (2)

\[ N_t : \frac{\eta}{1 - N_t} = \bar{W}_t\Lambda_t \]  \hspace{1cm} (3)

\[ K_{t+1} : \Lambda_t = \beta E_t(\Lambda_{t+1}(r_{k,t+1} + 1 - \delta)) \]  \hspace{1cm} (4)

\[ B_{t+1} : \Lambda_t = \beta R_tE_t(\Lambda_{t+1}\frac{1}{\pi_{t+1}}) \]  \hspace{1cm} (5)

where \(\bar{M}_t = \frac{M_t}{P_t}\) denotes real balances at time \(t\), \(\pi_t = \frac{P_t}{P_{t-1}}\) is the gross inflation rate, \(\bar{W}_t = \frac{W_t}{P_t}\) are real wages and \(r_{k,t+1} = \frac{R_{k+1,t+1}}{P_{t+1}}\) corresponds to real capital return.

Equations (1) and (3) equate the marginal rate of substitution between consumption and labor to the real wage rate. Equation (2) states that the marginal utility of real balances is equal to the difference between the marginal utility of consumption in period \(t\) and the discounted expected marginal utility of consumption in \(t+1\). Equation (4) indicates the optimal intertemporal wealth allocation. Equation (5) displays that the net nominal interest rate between \(t\) and \(t+1\), \(1 - \frac{1}{R_t}\), is equal to \(1 - \beta E_t(\Lambda_{t+1}\frac{1}{\pi_{t+1}\Lambda_t})\).
2.2 Firms

2.2.1 The representative finished goods-producing firm

It uses $Y_t(i)$ units of each intermediate good $(i)$ during each period $t$ to produce $Y_t$ units of the finished good according to the constant returns to scale technology described by:

$$Y_t = \left[ \int_0^1 Y_t(i) \frac{\varepsilon - 1}{\varepsilon} di \right]^{\frac{\varepsilon}{\varepsilon - 1}}$$

where $\varepsilon > 1$ is the constant elasticity of substitution between intermediate goods. The representative firm maximizes its profits by choosing $Y_t(i)$.

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to:

$$Y_t = \left[ \int_0^1 Y_t(i) \frac{\varepsilon - 1}{\varepsilon} di \right]^{\frac{\varepsilon}{\varepsilon - 1}}$$

The optimality condition gives the demand level for intermediate goods:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$$  \hspace{1cm} (6)

Competition in the market for the finished good drives the representative firm’s profits down to zero in equilibrium.

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di = 0$$

Along with the demand level for intermediate goods, this zero profit condition determines $P_t$ as:

$$P_t = \left( \int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$  \hspace{1cm} (7)
2.2.2 The intermediate goods-producing firm

$N_t(i)$ units of labor and $K_t(i)$ units of capital are demanded to the representative household during period $t$ in order to produce $Y_t(i)$ units of intermediate good $i$ according to the following constant returns to scale technology described by:

$$Y_t(i) = (Z_tN_t(i))^{\alpha}K_t(i)^{1-\alpha}$$

(8)

where $Z_t$ is the labor-augmenting technology shock, $\alpha \in (0, 1)$ denotes the share of labor.

The firm maximizes the expected discounted flow of its real profits.

$$Max E_t \sum \beta^t \Lambda_t \{ \frac{D_t(i)}{P_t} \}$$

subject to (6) and:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$$

The instantaneous profit function is given as follows:

$$D_t(i) = P_t(i)Y_t(i) - P_t \widetilde{W}_t N_t(i) - P_t r_{kt} K_t(i)$$

Assume that $\xi_t$ is the multiplier associated to the constraint.

**First Order Conditions**

$$K_t(i) : (1 - \alpha) \frac{Y_t(i)}{K_t(i)} = \frac{\Lambda_t}{\xi_t} r_{kt}$$

$$N_t(i) : \alpha \frac{Y_t(i)}{N_t(i)} = \frac{\Lambda_t}{\xi_t} \widetilde{W}_t$$

Let $\mu_t = \frac{\Lambda_t}{\xi_t}$ be the markup. The FOC’s become:

$$\mu_t = \frac{(1 - \alpha) Y_t(i)}{K_t(i)} r_{kt}$$

(9)

and

$$\mu_t = \frac{\alpha Y_t(i)}{N_t(i)} \frac{1}{\widetilde{W}_t}$$

(10)
Nominal price rigidities We assume that prices $P^*_t(i)$ are determined by a Calvo contract with a probability $\theta$ that the firm $i$ keeps its price unchanged at the period $t$. In that case, the aggregate price level is given by:

$$P_t = [\theta P_{t-1}^{1-\varepsilon} + (1 - \theta) P^*_t]^{1/\varepsilon}$$

where $P_t$ is the logarithm of aggregate price level and $P^*_t$ is the logarithm of the price fixed by the firms adjusting their prices in $t$. The optimization problem of the firm adjusting its price is:

$$\max_{P_t^*} \sum_{j=0}^{\infty} (\beta \lambda)^j E_t \{ Y_{t+j}(i) \}$$

subject to (6) and the following demand function:

$$Y_{t+j}(i) = \frac{P_t(i)}{P_{t+j}}^{-\varepsilon} Y_{t+j}$$

$P_t(i)$ determines $P_t^*$ at the optimum. The first order condition with respect to $P_t(i)$ is:

$$P_t(i) = \frac{E_t \sum_{j=0}^{\infty} (\beta \lambda)^j \Lambda_{t+j} Y_{t+j}(i) \xi_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta \lambda)^j \Lambda_{t+j} Y_{t+j}(i) / P_{t+j}}$$

The previous equation relates the optimal price to the expected future price of the final good and to the expected future real marginal costs.

For any variable $x_t$, we define $\tilde{x}_t = \log(\frac{x_t}{x})$ as the deviation of $x_t$ from its steady-state value.

Equation (12), together with (11) allow us to derive the following log-linearized New Phillips Curve:

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} - \frac{(1 - \theta)(1 - \theta \beta)}{\theta} \tilde{\mu}_t$$

2.3 Technology and money supply Shocks

There are two shocks in our model: a technological and a money supply shocks. The first one is common to all intermediate goods and it follows a random walk process:

$$Z_t = Z_{t-1} \exp(\varepsilon_{z,t})$$
where the zero-mean, serially uncorrelated innovation $\varepsilon_{z,t}$ is normally distributed with standard deviation $\sigma_z$.

Money supply is introduced by the following relation:

$$M_t = g_t M_{t-1}$$

where $g_t$ denotes money supply growth rate.

Using the definition of $\widetilde{M}_t$, we obtain:

$$\frac{\widetilde{M}_{t-1}}{\widetilde{M}_t} = \frac{\pi_t}{g_t}$$  \hspace{1cm} (15)

The monetary growth rate shock is defined by the following stationary AR(1) process:

$$\log g_t = \rho_g \log g_{t-1} + (1 - \rho_g) \log q + \varepsilon_{g,t}$$  \hspace{1cm} (16)

where $\rho_g \in (0, 1)$ and the zero-mean, serially uncorrelated innovation $\varepsilon_{g,t}$ is normally distributed with standard deviation $\sigma_g$.

### 2.4 The stationary system

Solving the model requires working with stationary transformations of the variables containing unit roots. We use therefore the following transformations:

$$m_t = \frac{\widetilde{M}_t}{Z_t}, \quad w_t = \frac{\widetilde{W}_t}{Z_t}, \quad k_t = \frac{K_t}{Z_{t-1}}, \quad \Lambda_t = \frac{\lambda_t}{Z_t}$$

All other variables are transformed according to the formula: $x_t = \frac{X_t}{Z_t}$.

We consider the stationary technology variable:

$$z_t = \frac{Z_t}{Z_{t-1}} \implies z_t = \exp(\varepsilon_{z,t})$$

In a symmetric equilibrium, $P_t(i) = P_t$, $N_t(i) = N_t$, $K_t(i) = K_t$, $Y_t(i) = Y_t$ and $D_t(i) = D_t$ for all $i=1,2,...$ and all $t$.

The complete system of equations in stationary variables that characterize the model’s equilibrium, steady states and log-linearized equations system are reported in appendix A.
This system is composed of 14 equations and 14 variables. The log-linearized version of the model can be written under its state-space form:

\[
\begin{align*}
\tilde{s}_{t+1} &= \Phi_1 \tilde{s}_t + \Phi_2 \tilde{\varepsilon}_{t+1} \\
\tilde{d}_t &= \Phi_3 \tilde{s}_t
\end{align*}
\]

where \( \tilde{s}_t = (\tilde{k}_t, \tilde{m}_{t-1}, \tilde{g}_t)' \) is a vector of state variables that includes predetermined variables and exogenous shocks, \( \tilde{d}_t = (\tilde{\lambda}_t, \tilde{\mu}_t, \tilde{\pi}_t, \tilde{w}_t, \tilde{N}_t, \tilde{i}_t)' \) is a vector of control variables.

### 2.5 Calibration and results

Following Dib and Phaneuf (2001), we assume that the share of labor \( \alpha \) in the production function is equal to 0.64, the depreciation rate of capital \( \delta \) is set equal to 0.025, the subjective discount factor \( \beta \) is equal to 0.992. The parameter \( \eta \) that measures the weight on leisure in the utility function is determined in such a way that the representative household spends roughly one third of her time working at the steady state (which gives a value of 1.42 as in Dib and Phaneuf (2001)). The parameter \( b \) determining the steady-state ratio of real balances to consumption is set equal to 0.014, implying that the steady-state consumption velocity of money in the model matches the average consumption velocity of M2 in the U.S. data. The parameter \( \gamma \) is assigned the value of 0.2141 as estimated by Dib and Phaneuf (2001) for the postwar U.S. economy. The parameter that measures monopoly power in the market for intermediate goods, \( \varepsilon \), is set equal to 6, which implies a steady-state markup of 1.2. Following Ireland (1997), the autocorrelation coefficient of monetary shock \( \rho_g \) is set equal to 0.68. The probability, \( \theta \), that prices are kept unchanged at \( t \) is assumed equal to 0.75 which corresponds to a one-year contract duration.

Figure 1 shows the impact of 1% increase in the innovations of technology shocks when prices are flexible. The response of hours worked is positive and highly persistent as prices can adjust immediately to accommodate the shock.

In figure 2, we plot the impulse responses when prices are sticky. In this case, a favorable technology shock leads to a persistent increase in consumption level. Labor decreases for about six quarters (trough near to 0.6) then it increases gradually to its long-run level. The technology improvement is also followed by a permanent increase in output level. We should note that,
in this case of exogenous money economy, where prices are sticky but the monetary authority fails to respond to the shocks, a part of the increase in output caused by a positive technology shock is delayed; nominal prices cannot fall fast enough to generate the appropriate increase in demand. Real wages are initially countercyclical then they increase and become strongly procyclical. However this result is not supported by empirical evidence, it would be possible theoretically for sticky price models when monetary policy does not fully accommodate the shock. In fact, following a favorable technology shock, nominal price rigidities imply sluggishness in output response as the monetary policy is exogenous and does not respond systematically to the shock. Thus, output adjustment cannot catch up with technology improvement, leading to a fall in the labor demand at any given real wage, so that the labor demand curve would shift to the left, which causes equilibrium real wage to fall. When the monetary policy is exogenous, the initial response of hours worked is negative which confirms Gali (1999) conclusions even though we’ve modified the price rigidities structure.

Figures 1 and 2 highlight the importance of price rigidities to reproduce the initial drop in labor. In fact, suppose that prices are determined by a Calvo contract and that quantity theory governs the demand for money so output is proportional to real balances. In the short run, if money supply remains unchanged and prices cannot adjust, then real balances and hence output are also fixed. Even though all firms will experience a decline in their marginal cost following a favorable technology shock (as shown in figure 1), only a fraction of them will adjust their prices downwards in the short run. Accordingly, the aggregate price level will decline increasing the markup and aggregate demand will rise less than proportionally to the increase in productivity. This implies an increase in the wedge between the marginal product of labor and the real wage. Since the wedge will eventually return to its steady state level, there is a strong substitution effect that causes labor input to fall in the impact period. Gali (1999) made it clear that hours response depends on price stickiness as long as monetary policy falls short of full accommodation whereas Dotsey (1999) focuses on the importance of the systematic part of monetary policy.

So, what happens when monetary policy becomes endogenous and governed by a Taylor rule? Could we reproduce the initial drop in hours worked following a favorable technology shock? The response is therefore "no". How would we reproduce the contractionnary effect?
3 Endogenous monetary policy: A simple Taylor rule

In this section, we consider the interest rate channel as a propagation mechanism of the shocks. More specifically, we modify the monetary policy and introduce a simple Taylor rule (1993). In this article, he showed that the behavior of the federal funds interest rate in the United States from the mid-1980s through 1992 could be fairly well matched by a simple rule of the form:

\[ \widehat{R}_t = \alpha_y \widehat{y}_t + \alpha_{\pi} \widehat{\pi}_t + \varepsilon_{Rt} \]  

where \( \pi_t = \frac{P_t}{P_{t-1}} \) denotes the gross rate of inflation, \( R_t \) is the nominal interest rate and \( y_t \) is the output gap between actual transformed output \( (Y_t) \) and its steady state value. As defined below, for any variable \( x_t \), we define \( \widehat{x}_t = \log(\frac{x_t}{x_{t-1}}) \) as the deviation of \( x_t \) from its steady-state value.

Under this rule, the nominal interest rate deviates from the level consistent with the economy’s equilibrium rate and the target inflation rate if the output gap is nonzero or if inflation deviates from target. A positive output gap leads to a rise in the nominal interest rate as does a deviation of actual inflation above target.

3.1 Firms

In this section, we model the use of intermediate goods in an input-output production structure, so all firms use intermediate inputs in production. As prices are rigid for all firms-including those producing intermediate goods-, intermediate goods should have rigid prices. Intermediate goods, however, act as a multiplier for price stickiness: a little price rigidity at the level of an individual firm leads to a large degree of economy-wide price inflexibility.

The maximization problem for the representative firm producing final goods remains the same as in the previous section.

3.1.1 The Representative Intermediate Goods-Producing Firm

Good \( Y_t(i) \) is produced using \( X_t(i) \) units of intermediate-good input (which is a quantity of the final output), \( K_t(i) \) units of capital and \( N_t(i) \) units of labor according to the constant-returns-to scale technology described by:
\[ Y_t(i) = X_t(i)^\Psi \left( (Z_t N_t(i))^\alpha K_t(i)^{1-\alpha} \right)^{1-\Psi} \]  \hfill (18)

where \( \Psi \in [0, 1] \) is the share of intermediate goods in the production function and \((1 - \alpha)\) is the weight of capital in value-added. There is a continuum of intermediate goods-producing firms indexed by \( i \). Since the intermediate goods substitute imperfectly for one another in the representative finished goods-producing firm’s technology, the representative intermediate goods-producing firms sees its output in a monopolistically competitive market. It maximizes its total market value:

\[
\max_{N_t(i), K_t(i), X_t(i)} E_t \sum_{i=1}^\infty \beta^t \Lambda_t \left\{ \frac{D_t(i)}{P_t} \right\}
\]

subject to (18) and

\[ Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \]

\[ D_t(i) = P_t(i) Y_t(i) - P_t w_t N_t(i) - P_t r_{kt} K_t(i) - P_t X_t(i) \]

where \( \frac{\beta^t \Lambda_t}{P_t} \) measures the marginal utility value to the representative household of an additional dollar in profits during \( t \).

**First Order Conditions:** The first order conditions with respect to \( X_t \), \( K_t \) and \( N_t \) are:

\[ X_t(i) : \frac{Y_t(i)}{X_t(i)} = \frac{\mu_t}{\Psi} \]  \hfill (19)

\[ K_t(i) : (1 - \alpha)(1 - \Psi) \frac{Y_t(i)}{K_t(i)} = \mu_t r_{kt} \]  \hfill (20)

\[ N_t(i) : (1 - \Psi) \alpha \frac{Y_t(i)}{N_t(i)} = \mu_t w_t \]  \hfill (21)

where \( \mu_t \) denotes the markup.

The conditional demand functions for intermediate input and for primary factors derived from cost-minimization are given by:

\[ X_t(i) = \Psi MC_t Y_t(i)/P \]
\[ N_t(i) = (1 - \Psi)\alpha MC_t Y_t(i)/W \]

\[ K_t(i) = (1 - \alpha)(1 - \Psi)MC_t Y_t(i)/r_k \]

where \( MC_t \) denotes the marginal cost function given by:

\[ MC_t = \Psi P^\Psi (W^\alpha r_k^{1-\alpha})^{1-\Psi} \]

There is a negative relationship between intermediate input share and labor demand function. In fact, as intermediate input share becomes more important, intermediate good is more productive while the marginal product of labor decreases. Thus, the conditional demand for intermediate good increases whereas labor input demand is lower. Second, labor demand is less important when we introduce intermediate input (\( \Psi \neq 0 \)) than the case where there is no intermediate good (\( \Psi = 0 \)). Therefore, it seems that taking into account an input-output structure would provides the model with the needed mechanism to reproduce the contractionary technological effect on labor input.

Besides, intermediate input acts as price rigidity multiplier: a little price rigidity at the level of an individual firm leads to a large degree of economy-wide price inflexibility. In fact, the intermediate input is a part of the final good and could be either consumed or invested, or used as an intermediate production input.\(^{10}\) So, the rigid intermediate input price corresponds to the aggregate price level. Second, the intermediate input price level becomes a more significant component of marginal cost which records not only the real wages and the capital rental rate but also the intermediate input price. This causes marginal cost to become more rigid. Therefore, a less variable marginal cost increases the rigidity in firms’ pricing decisions.

Moreover, the input-output structure makes intermediate inputs and labor input substitutes, following a technology improvement: a decrease in intermediate input price level generates a decrease in labor input demand. In fact, on one hand, the decrease in aggregate price level (which also corresponds to the intermediate input price) rising from a favorable technology shock increases the conditional demand for intermediate input. On the other hand, a (small) decrease in price level causes real wages to increase. Thus,

\[^{10}\text{As noted further, } Y_t = C_t + I_t + X_t\]
labor input demand drops as firms would replace the factor whose relative price becomes higher.

We consider the same New Phillips Curve:

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \frac{(1 - \theta)(1 - \theta \beta)}{\theta} \hat{\mu}_t \]  

(22)

The demand constraint is:

\[ Y_t = C_t + I_t + X_t \]  

(23)

The complete system of transformed equations with stationary variables, steady state values and log-linearized equations are reported in Appendix B. In this 14 equations and 14 variables system, control variables are noted as \( c_t, N_t, y_t, w_t, r_k, x_t, i_t \) and \( R_t \).

### 3.2 Calibration and results

Following Huang, Liu and Phaneuf (2003), we set \( \Psi \), the cost share of intermediate input, for the postwar U.S. economy equal to 0.7 which is not far from the value, suggested by the BEA (1997) for the manufacturing sector (0.68). The values of \( \alpha_x \) and \( \alpha_y \) are common in the literature and correspond to \( \alpha_x = 1.5 \) and \( \alpha_y = 0.5 \).

The results are reported in figures 3 to 7.

In the absence of intermediate goods (figure 3), it is well-known that a sticky price model tends to generate a significant increase in output and labor in response to a favorable technology shock since monetary policy is fully accommodating the shock. The intuition is that, with staggered price settings, a technology improvement decreases the firms’ real marginal costs and generates a reduction in the aggregate price level that is smaller than that obtained under perfect price flexibility. Consequently, aggregate demand increases, but by less than under price flexibility. This creates a wedge between output and its natural level (achieved when prices fully adjust), and therefore, the output gap diminishes and so does the inflation. This results in a reduction in the monetary policy rate to provide full accommodation of the shock. However, as shown in figure 8, it seems that staggered price setting à la Calvo is not sufficient to generate the expected drop in nominal interest rates and inflation. Output level and hours worked present a persistent increase following the shock (with a peak of 0.035 for hours worked). We’ll see
later that, for plausible values for intermediate input share, we obtain the expected dynamics for all considered variables.

For $\Psi=0.2$ (Figure 4), the reduction of aggregate price level generated by nominal price rigidities causes a sustained increase in unconditional intermediate input demand which causes a larger increase in output than in the previous case. Naturally, more output needs more labor. However, as prices become more rigid, hours cannot show the same sustained positive response as in the previous case with less price stickiness. In fact, labor jumps then presents a delayed drop (about 0.005%). Price rigidities is still not able to counterbalance the full accommodation of monetary policy to a favorable technology shock and generate the expected immediate decline in hours worked. However, the presence of intermediate inputs implies a smaller magnitude in the response of inflation and nominal interest rates as prices become more rigid. We should note that the presence of intermediate inputs creates a substitution effect with labor that would be intensified as intermediate input share gets greater.

For higher value of intermediate input share ($\Psi = 0.4$, figure 5), for the postwar U.S. economy, price stickiness caused by the intermediate input achieves to generate the immediate short-lived decline, however small, in labor following a positive technology shock. As the intermediate input share becomes greater, there are two effects that should cause hours worked to drop in response to a permanent technology shock: price rigidities which create an increase in the wedge between the marginal product of labor and the real wage. Since the wedge will eventually return to its steady state level, there is a strong substitution effect that causes labor input to fall in the impact period; and a substitution effect between intermediate inputs and hours worked being more important (as the intermediate input share grows, the unconditional demand level for intermediate input increases where the marginal product of labor decreases). An other remarkable effect is that, for a sufficient degree of price rigidities, we obtain the intuitive drop in inflation level and nominal interest rates which stimulates the economic activity.

As Huang, Liu and Phaneuf (2000) notice, the intermediate input share values for the interwar period lie between 0.3 and 0.5. Here, we obtain the expected dynamics of the variables for both the interwar and postwar period.

For the calibrated value of intermediate input share for the postwar U.S. economy ($\Psi = 0.7$, figure 6), a positive technology shock is followed by a more important contractionary effect on hours worked. In fact, the increase in intermediate input share intensifies the two previous effects: the uncondi-
tional demand level of intermediate goods becomes more important and the substitution effect between labor and intermediate goods is magnified. We should notice that, when intermediate input share increases, output, consumption, investment, intermediate goods overshoot their steady state level then decrease gradually towards their long-run level. On the other side, the drop in inflation level (following a technology improvement) induced by price rigidities and combined with a decreasing output gap result in a fall of nominal interest rates.

But what happens when intermediate input share becomes greater and approaches one?

For an extreme value of the intermediate input share ($\Psi=0.9$, figure 7), all variable responses are stronger than before including the initial contractionary effect on labor. In fact, as the share of intermediate inputs grows larger, the rigid intermediate input price becomes a more significant component of the marginal cost. As a result, following a technology innovation, marginal cost drop magnitude is more important and so does the decrease in the price level. Hours worked decline by 0.5% in response to the shock. We get the same endogenous propagation mechanisms as before but with a greater magnitude.

4 Conclusion

In the present paper and following the works of Gali (1999), BFK (1998), etc., our challenge is to reproduce the contractionary effects of technological innovation on employment regardless of the monetary policy consideration. Developing a model with monopolistic competition between firms producing intermediate goods and perfect competition between those producing final goods and price rigidities à la Calvo, we can obtain the initial drop in hours worked following a favorable technology shock supported by many recent empirical works. The model is simulated under two different monetary policies. The most interesting result is that the contractionary effect of technological innovation persists even though monetary policy is endogenous and determined by a simple Taylor rule. In fact, this becomes feasible by considering an input-output production structure combined with staggered price-setting. In particular, this result is robust for a plausible range of intermediate input share for the postwar period even when monetary authorities fully accommodate the shock. Moreover, as it was expected, when intermediate input
share grows, the initial drop in labor following a favorable technology shock is more important. In fact, taking into account intermediate inputs as part of final output in addition to staggered price-setting introduces more price rigidities. Then, the aggregate price level becomes a significant part of marginal cost which becomes more rigid. This counterbalances the increase in labor that rises when monetary authorities fully accommodate technology improvement. Besides, there is a negative relationship between intermediate input share and labor input demand; more important is the intermediate input share, the weaker is the labor input demand. Finally, following a favorable technology shock, labor input and intermediate inputs are substitutes; the decrease in the intermediate input price level (corresponding to the aggregate price level) generated by the technology innovation induces a drop in labor input.

Our finding is contrasting with the claims of Dotsey (1999a) who concluded that only a rule targeting the money supply allows a decrease in hours worked following a favorable technology shock. In addition, this result casts skepticism on the conclusion of Dotsey (1999b) that a sticky price model predicts a drop in hours worked in response to a favorable technology shock only in the case of a modified Taylor rule where the output gap is replaced by output growth.

In this paper, we highlight the role of intermediate input as a mechanism of propagation of the contractionary effect of a technological innovation on labor. In a future work, we would enhance the role of technological innovation source to confirm the initial drop in hours worked in a two-sector model where we distinguish between intermediate goods sector and final good sector.
APPENDIX A

Equilibrium conditions with transformed equations

\[ \frac{1}{1 + b \left( \frac{m_t}{c_t} \right)^{\frac{\gamma - 1}{\gamma}}} = \lambda_t c_t \]  
\[ (24) \]

\[ \frac{b (m_t)^{\frac{\gamma - 1}{\gamma}}}{c_t^\gamma} + b (m_t)^{\frac{\gamma - 1}{\gamma}} = \lambda_t - \beta E_t \left( \frac{1}{\pi_{t+1} z_{t+1}} \right) \]  
\[ (25) \]

\[ \frac{\eta}{1 - \eta} = w_t \lambda_t \]  
\[ (26) \]

\[ \lambda_t = \beta E_t \left( \frac{\lambda_{t+1}}{z_{t+1}} (r_{k,t+1} + 1 - \delta) \right) \]  
\[ (27) \]

\[ y_t = N_t^{\alpha} h_t^{1-\alpha} z_t^{\alpha-1} \]  
\[ (28) \]

\[ \mu_t = \frac{\alpha w_t}{w_t} \]  
\[ (29) \]

\[ \mu_t = \frac{(1 - \alpha) w_t z_t}{r_{kt}} \]  
\[ (30) \]

\[ k_{t+1}z_t = (1 - \delta)k_t + z_t i_t \]  
\[ (31) \]

\[ y_t = c_t + i_t \]  
\[ (32) \]

\[ m_t = \frac{m_{t-1}}{\pi_t z_t} g_t \]  
\[ (33) \]

\[ z_t = Z \exp(\varepsilon_{zt}) \]  
\[ (34) \]

\[ \log g_t = \rho_g \log g_{t-1} + (1 - \rho_g) \log \bar{g} + \varepsilon_{g,t} \]  
\[ (35) \]
\[ \lambda_t = \beta R_tE_t \left( \frac{\lambda_{t+1}}{\pi_{t+1} z_{t+1}} \right) \]  

(36)

Dividing (25) by (24) and combining with (36), we obtain:

\[ b \left( \frac{m_t}{c_t} \right)^{-1} = 1 - \frac{1}{R_t} \]

**Steady state**

We assume that the household spends one third of her time working so that \( N = \frac{1}{3} \).

We have:

\[ r_k = \frac{z}{\beta} - 1 + \delta \]

\[ z = 1, g = 1, \pi = 1 \]

\[ \mu = \frac{\varepsilon}{\varepsilon - 1} \]

\[ R = \frac{z}{\beta} \]

\[ \frac{k}{y} = \frac{(1 - \alpha)z}{\mu r_k} \]

\[ \frac{i}{k} = 1 - \frac{(1 - \delta)}{z} \]

\[ \frac{c}{y} = 1 - \frac{i}{k} \frac{k}{y} \]

\[ \frac{m}{c} = \left( \frac{1}{b} \left( 1 - \frac{1}{R} \right) \right)^{-\gamma} \]

\[ \lambda c = \frac{1}{1 + b \left( \frac{m}{c} \right)^{\gamma - 1}} \]

\[ \mu w = \alpha \frac{y}{N} \]

\[ w \lambda = \frac{\eta}{(1 - N)} \]
The two previous equations give the value of $\eta$ when $N = \frac{1}{3}$

$$\eta = w\lambda(1 - N)$$

$$y = N \left( \frac{k}{y} \right)^{\frac{1-\alpha}{\alpha}}$$

$$c = \left( \frac{c}{y} \right) * y, k = \left( \frac{k}{y} \right) * y, i = \left( \frac{i}{k} \right) * k, \lambda = \frac{\lambda c}{c}, m = \left( \frac{m}{c} \right) * c$$

Log-linearization
In this section, we log-linearize all the transformed first order conditions. The log-linearized system becomes:

$$-\left[1 + \frac{b}{\gamma} \left( \frac{m}{c} \right)^{\frac{\gamma-1}{\gamma}} \right] \tilde{c}_t = \left[ b(\frac{\gamma-1}{\gamma}) \left( \frac{m}{c} \right)^{\frac{\gamma-1}{\gamma}} \right] \tilde{m}_t + \left[ 1 + \frac{b}{\gamma} \left( \frac{m}{c} \right)^{\frac{\gamma-1}{\gamma}} \right] \tilde{\lambda}_t$$  \hspace{1cm} (37)

$$-\frac{b}{\gamma} \left( \frac{m}{c} \right)^{\frac{1}{\gamma}} (\tilde{m}_t - \tilde{c}_t) = \frac{\tilde{R}_t}{R}$$  \hspace{1cm} (38)

$$\frac{N}{1 - N} \tilde{N}_t = \tilde{\lambda}_t + \tilde{w}_t$$  \hspace{1cm} (39)

$$\tilde{\lambda}_t = \tilde{E}_t \tilde{\lambda}_{t+1} + \frac{r_k}{R} \tilde{E}_t \tilde{r}_{k,t+1} - d\tilde{z}_{z,t+1}$$  \hspace{1cm} (40)

$$\tilde{y}_t = (\alpha - 1) d\tilde{z}_{z,t} + \alpha \tilde{N}_t + (1 - \alpha) \tilde{k}_t$$  \hspace{1cm} (41)

$$\tilde{\mu}_t = -\tilde{w}_t + \tilde{y}_t - \tilde{N}_t$$  \hspace{1cm} (42)

$$\tilde{\mu}_t = \tilde{y}_t - \tilde{k}_t - d\tilde{z}_{z,t} - \tilde{r}_{k,t}$$  \hspace{1cm} (43)

$$z\tilde{k}_{t+1} + zd\tilde{z}_{z,t+1} = (1 - \delta) \tilde{k}_t + (z - 1 + \delta) \tilde{i}_t$$

$$\tilde{y}_t y = \tilde{c}_t c + \tilde{i}_t i$$  \hspace{1cm} (44)

$$\tilde{m}_t = \tilde{m}_{t-1} + \tilde{g}_t - \tilde{r}_t - d\tilde{z}_{z,t}$$  \hspace{1cm} (45)
\[ \tilde{z}_t = d \varepsilon_{z,t} \] (46)

\[ \hat{g}_t = \rho g_{t-1} + v_{g,t} \] (47)

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \frac{(1 - \theta)(1 - \theta \beta)}{\theta} \hat{\mu}_t \] (48)

\[ \hat{m}_t = \hat{m}_t \] (49)

\[ \hat{\lambda}_t = \hat{R}_t + E_t \hat{\lambda}_{t+1} - E_t \hat{\pi}_{t+1} - d \varepsilon_{z,t+1} \] (50)
APPENDIX B

Equilibrium conditions with transformed equations:

\[
\frac{-1}{c_t^{\gamma}} = \lambda_t
\]

\[
\frac{b(m_t)\frac{-1}{\pi}}{c_t^{\gamma} + b(m_t)^{\frac{\gamma - 1}{\pi}}} = \lambda_t - \beta E_t(\lambda_t+1 \frac{1}{\pi t+1 z t+1})
\]

\[
\frac{\eta}{1 - N_t} = w_t \lambda_t
\]

\[
\lambda_t = \beta E_t(\frac{\lambda t+1}{z t+1} (r_{k,t+1} + 1 - \delta))
\]

\[
\lambda_t = \beta R_t E_t(\frac{\lambda t+1}{\pi t+1 z t+1})
\]

\[
y_t = x_t^\Psi [N_t^\alpha k_t^{1-\alpha} z_t^{\alpha-1}]^{1-\Psi}
\]

(51)

\[
\mu_t = \frac{(1 - \Psi) a_\frac{\Psi}{N_t}}{w_t}
\]

\[
\mu_t = \frac{(1 - \Psi)(1 - a) \frac{\Psi}{k_t} z_t}{r_{k_t}}
\]

(52)

\[
\mu_t = \Psi \frac{y_t}{x_t}
\]

\[
k_{t+1} z_t = (1 - \delta) k_t + z_t i_t
\]

\[
y_t = c_t + i_t + x_t
\]

(53)

\[
z_t = Z \exp(\varepsilon_{z,t})
\]

\[
\pi_t = \beta E_t \pi_{t+1} - (1 - \theta)(1 - \theta \beta) \frac{1}{\theta} \mu_t
\]

30
\[ \hat{R}_t = \alpha_x \hat{x}_t + \alpha \hat{\pi}_t + \varepsilon_{Rt} \]

**Steady state**

\[ \mu = \frac{\varepsilon}{\varepsilon - 1} \]

\[ R = \frac{z}{\beta} \]

\[ r_k = \frac{z}{\beta} - 1 + \delta \]

\[ z = 1, g = 1, \pi = 1, N = 0.33 \]

\[ \frac{y}{x} = \frac{\mu}{\Psi} \]

\[ \frac{y}{k} = \frac{r_k \mu}{(1 - \alpha)(1 - \Psi)z} \]

Combining the two previous ratios gives:

\[ \frac{k}{x} = \frac{(1 - \alpha)(1 - \Psi)z}{r_k \Psi} \]

\[ y = \left( \frac{\Psi}{\mu} \right)^{\frac{\psi}{1 - \Psi}(1 - \alpha)} \left( \frac{k}{y} \right)^{\frac{\alpha}{1 - \alpha}} z N \]

\[ k = y * \text{inv}(\frac{y}{k}), x = y * \text{inv}(\frac{y}{x}), i = k * (1 - \left( \frac{1 - \delta}{z} \right)), \]

\[ c = y - i - x, m = c * (\frac{1}{b}(1 - \frac{1}{R}))^{-\gamma}, w = \frac{\alpha(1 - \Psi) y}{\mu} \frac{N}{N} \]

**Log-linearization**

The log-linearizes production function is given by:

\[ \tilde{y}_t = (1 - \Psi)(\alpha - 1)d\varepsilon_{z,t} + (1 - \Psi)\alpha \tilde{N}_t + (1 - \Psi)(1 - \alpha)\tilde{k}_t + \Psi \tilde{x}_t \]
The demand constraint and the foc with respect to intermediate input give

\[ \hat{y}_t y = \hat{c}_t c + \hat{i}_t i + \hat{x}_t x \]

\[ \hat{\mu}_t = \hat{y}_t - \hat{x}_t \]

respectively.
References


Figure 1: MS Model Technology Shock Effect with Flexible Prices
Figure 2: MS Model
Technology Shock Effect with Price Rigidities
Figure 3: TR Model with Price Rigidities
Technology Shock Effect for Intermediate Input Share=0
Figure 4: TR Model with Price Rigidities
Technology Shock Effect for Intermediate Input Share=0.2
Figure 5: TR Model with Price Rigidities
Technology Shock Effect for Intermediate Input Share=0.4
Figure 6: TR Model with Price Rigidities
Technology Shock Effect for Intermediate Input Share=0.7

Output

Consumption

Investment

Markup

Hours

Real Wages

Inflation and Prices

Real Balances

Nominal Interest Rate

$\pi_t$

$P_t$
Figure 7: TR Model with Price Rigidities
Technology Shock Effect for Intermediate Input Share=0.9