On Sanskrit commentaries dealing with mathematics (VIIth-XIIth century)

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Abstract

This article will explore the shifting perceptions that past scholars had on Sanskrit commentaries dealing with mathematics. We will concentrate on texts composed between the VIIth to the XIIth century. We will try to specify how they have been collected in libraries and then how they were studied by historians of mathematics. We will end this exploration by evoking some of the new perspectives for the history of positional arithmetics in India opened by the study of such texts, when focussing on the mathematical and linguistical link they have with the text they comment.

Cet article examinera les regards changeants qu'ont porté des savants du passé, sur les commentaires en langue Sanskrite rédigés entre le VIIème et le XIIème siècle après J. -C, portant sur les mathématiques. Nous nous demanderons comment ces texts ont été collectés dans des bibliothèques puis comment ils ont été étudiés par des historiens des mathématiques. Nous achèverons cette étude sur les perspectives nouvelles que l'étude du lien mathématique et linguistique qui lie le commentaire à ce qu'il commente ouvre pour l'histoire de l'arithmétique positionnelle en Inde.

*The adjustment of this draft to a regular format, is being worked on.

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List of Abbreviations

Ab Āryabhaṭa I 's *Āryabhaṭīya*

APG Anonymous and undated commentary on the *Pāṭīgaṇita* of Śrīdhara

BAB Bhāskara I 's *Āryabhaṭīyabhāṣya*

BG *Bijagaṇita* of Bhāskara II

BM *Bhakshālī* manuscript

BSS *Brahmasphuṭasiddhānta* of Brahmagupta

CESS *Census of the Exact Sciences in Sanskrit*

GT *Gaṇitatilaka* of Śrīpati

GSS *Gaṇitasārasaṃgraha* of Mahāvīra

L *Līlāvatī* of Bhāskara II

MS *Mahāsiddhānta* of Āryabhaṭa II

PBSS Pṛthudakśvamin's commentary on the *Brahmasphuṭasiddhānta* of Brahmagupta

PG *Pāṭīgaṇita* of Śrīdhara

SAB Someśvara's commentary on the *Āryabhaṭīya* of Āryabhaṭa

SYAB Sūryadeva Yajvan's commentary on the *Āryabhaṭīya* of Āryabhaṭa

T *Triśatika* of Śrīdhara

Introduction

In the wake of a renewed interest for the contextualization of indological studies[1], it is striking that publications on Indian mathematics are often devoid of historical contextualisation. This is partly due to a historiographical trend of technical and patriotic history of mathematics, but also to the little information we indeed have on the context in which mathematics was practiced in India in the past. To overcome this problem some historians of science have turned to periods (XVIth-XIXth century) and places where institutions, libraries and a dirth of texts help us contextualize the mathematical and astronomical ideas produced in these places[2].

I would like to argue that a focus on the kind of texts produced by astronomers and mathematicians of the Indian subcontinent and the history of how they were transmitted to us can also help us contextualize the knowledge they disclose. What was the use of these texts? What does this tell us of the mathematical practices they testify of? Taking in account the diverse textual forms that were produced, trying to understand the function they had, their self-proclaimed aim as well as what they show us of their own conceptions of mathematical practices and ideas can also be fruitful to the history of math-

[1] Proemenintly in the project headed by Sheldon Pollock, the "Sanskrit Knowledge Systems at the Eve of Colonialism" (SKEC). See http://www.columbia.edu/itc/mealac/pollock/sks, [34] and the outcome for literature [35].

[2] Among the publications on history of science, produced within Pollock's SKEC, see the works of Christopher Minkowski and Dominik Wujastyk, listed at http://www.columbia.edu/itc/mealac/pollock/sks/papers/index.html

ematics in India[3]. And these questions can be addressed to texts of earlier periods for which very little background information is known.

Such a focus on text implies additionally to clarify what is the status of the manuscripts that we can find in libraries today and that serve as basis for editions and studies. Springs here many questions[4]: Who then copied the texts we now have at our disposal? Who had them copied? How where these texts collected into the library, through which we now have access to them? What was the aim of these different actors? How have these texts been used and read? What does this tell us of the changing conception of them as texts? etc. A set of questions that C. Minkowski and D. Raina address more directly in their contributions to this volume.

This article will attempt to draw some answers, when turning to the specific case of commentaries on mathematics written in Sanskrit between the VIIth and the XIIth century A. D.

The first part of this paper will sketch out what we know of the commentaries on mathematics of this period, reflecting on how they were handed down to us. We will then look at the texts that served as a basis for writing the history of mathematics in India from the XIXth century onwards, giving a special attention to the story of the rediscovery and edition of the works of Āryabhaṭa (ca. 499 [Billard 512][5]), Brahmagupta (ca. 628) and Śrīdhara (ca. 950). We will then give an example of how their study, taking in account

[3]This approach is inspired by K. Chemla, who has extensively published on the question. Her latest synthesis is [11].

[4]A similar set of questions addressed to another kind of texts, colonial archives, can be found in [50].

[5]See [5].

these relations, can shed a new light on the conceptions and practices of the decimal place-value notation while revealing different functions of treatises and commentaries.

1 Commentaries on mathematics from the VIIth to the XIIth century

Indologists are overwhelmed by the number of manuscripts that are at their disposition today to make new editions. The case of astronomy and mathematics is quite exceptional in this respect, since a census has been undertaken, which enables us to measure the relative amount of manuscripts and published editions for the specific field of *jyotiṣa*. Indeed, what we know today of the existing manuscripts on the astronomical and mathematical tradition/style in Sanskrit is mainly contained in David Pingree's *Census of the Exact Sciences in Sanskrit* (CESS)[6]. A close look[7] at this census shows that mathematical commentaries in Sanskrit are over-studied when looked at in the farther landscape of texts on *jyotiṣa*.

[6][31]. This is noted by [28] concerning the early modern period:

> "Because of the meticulous and comprehensive survey of the history of Jyotiṣ texts being done by David Pingree, we are in a position to make an assessment of the history of Jyotiṣ in the early modern context in a way that cannot yet be imagined for the other *śāstras*".

But this remark, on the early modern period, can probably be extended to earlier periods as well.

[7]I have undertaken a preliminary manual count of the census, some of the conclusions and more details on the Census are given in Appendix 1.

Before starting this article, I actually thought the opposite: that commentaries were neglected in the historiography, and that they were on the contrary an important part of the past tradition. I have to recognize that this initial evaluation was wrong both ways[8]. Part of this article then is an effort to try to pinpoint why I had developed such a wrong idea about Indian mathematical commentaries in Sanskrit.

1.1 A limited number of known texts on mathematics

What are the texts we will focus on in this article?

All the presently identified texts on mathematics of the Vth to VIIth century[9], for which we have manuscripts are enumerated in Table 3.3 in Appendix 2 at the end of the article.

The VIIth to the XIIth century is the beginning of an expanding mathematical and astronomical tradition, which will permeate not only the Indian subcontinent but extend in the East to China and in the West to the Arabic peninsula[10]. During this period, the question of the autonomy of mathemat-

[8]I indeed, mentioned this in the introduction to my book, [24]. This was noted and criticized quite rightly by S. R. Sharma. [43, p. 144].

[9]Some mathematical commentaries of this period are lost to us for now at least, such as Prabhākara's commentary on the *Āryabhaṭīya* (ca.VIth century) (CESS 4 227 a), and Balabadhadra's (fl. VIIIth century) commentary to the *Brahmasphuṭasiddhānta* (CESS 4 255 a), but we have not taken them into account here.

[10]We have chosen as an upper boundary the time before the works of Bhāskara II (ca.1114-1183) started to have an impact (which has enabled us to include Sūryadeva Yājvan's commentary, which dates from after Bhāskara II, but is ignorant of it) and after the vedic period, which had its own specific mathematical tradition or style. The period we are considering thus ranges thus from 499 AD to 1200 precisely.

ics (*gaṇita*) with astronomy is problematic.

Jyotiṣa (lit. "the ⟨sky's⟩ luminaries"), which we conveniently translate as "astronomy" is a field which includes in fact horoscopy and mathematics together with observational and computational astronomy[11]. Thus mathematics seems to have been to a certain extent a sub-discipline of astronomy (*jyotiṣa*). Indeed, *gaṇita* is sometimes understood as meaning "computational astronomy". In the period we are focussing on, we will see that a number of preserved texts on mathematics belong to astronomical treatises. However, some Sanskrit authors of astronomical texts insisted that mathematics also existed outside of astronomy[12]. And there existed also independent texts on mathematics, from the vedic time onwards. In this article, we will collect these texts of different kinds, focussing on the fact that they concentrate on a same subject matter, which is given a specific name, *gaṇita*.

Table 3.3 gives a list of 15 texts. It includes in fact two sets of texts. Primary texts : treatises and all the texts that stand alone; and secondary texts, those that need another to exist : commentaries. The whole of the texts on mathematics known for this period are also summarized graphically in Figure 1, stressing primary and secondary texts.

The texts so far identified as being mathematical commentaries written during our chosen period[13] and for which we have existing manuscripts are thus, in a chronological order:

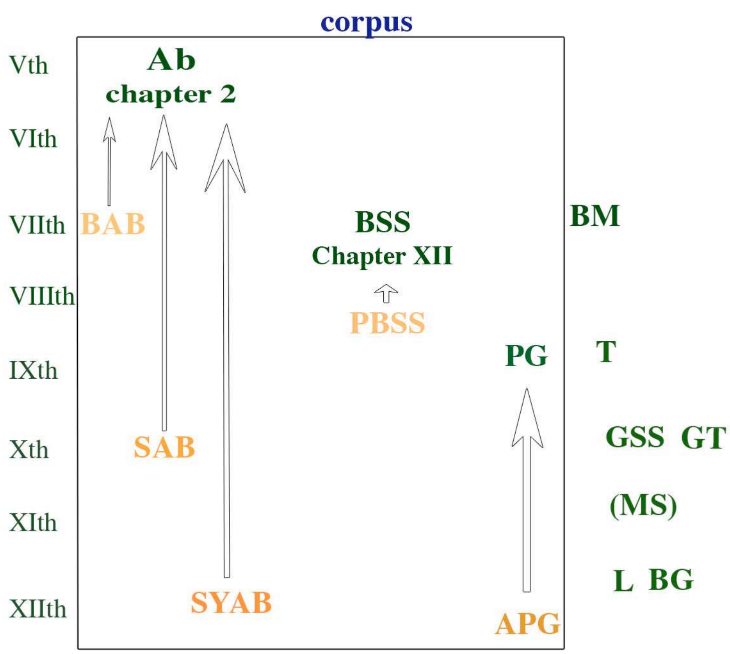
- Bhāskara's commentary on the second chapter of the *Āryabhaṭīya* (629

[11]See [32, Introduction and Table of contents].

[12]See for instance [25], [33].

[13]Commentaries on the treatises enumerated here have sometimes been written after our period but are not listed here. We will come back to this bellow.

Figure 1: Texts on mathematics in Sanskrit, VIIth-XIIth century



A. D.; hereafter abbreviated for the treatise into Ab and for the commentary into BAB, implicitly referring to chapter 2 when quoted in this way),

- Pṛthudakśvamin's mathematical commentary on the XIIth chapter of the *Brahmasphuṭasiddhānta* of Brahmagupta (the treatise is of 628 A. D. and the commentary of ca 864 A. D. ; hereafter abbreviated into BSS for the treatise and PBSS for the commentary, implicitly referring to chapter XII when quoted in this way),

- Someśvara's commentary on the second chapter of the *Āryabhaṭīya* (ca. 1040, hereafter abbreviated as SAB, implicitly referring to chapter 2),

- Sūryadeva Yajvan's mathematical commentary on the *Āryabhaṭīya* (Sūryadeva Yajvan is thought to have been born in 1191 A. D., his commentary is hereafter abbreviated into SYAB, implicitly referring to chapter 2).

To this we can maybe add:

- the anonymous and undated commentary on the *Pāṭīgaṇita* of Śrīdhara (fl. 850-950 A.D , date unknown for the commentary[14]; hereafter abbreviated into PG for the treatise and APG for the commentary.).

These are the texts we are going to focus on, in the following discussion; more specifically on BAB, SYAB and APG.

[14] Shukla who edited the text, considers that the commentary bears features of texts of the time span we have chosen, he especially draws similarities with the Bhakshālī Manuscript and the BSS. [46, pp. xxviii-xxxiv].

Let us turn to what we know of the manuscripts and commentaries through which these texts are known.

The *Āryabhaṭīya* has been extensively commented upon.

Thus K. S. Shukla and K. V. Sarma count 19 commentators of the *Āryabhaṭīya*[15], 12 of which are in Sanskrit. Half of them are from after the XVth century.

For Brahmagupta's BSS, on the other hand, two commentators are known. And we only have seven manuscripts, for one of these author's - PBSS, two manuscripts only containing commentaries of Chapter XII. Four manuscripts additionally provide anonymous commentaries on this text, 4 out of the 34 remaining manuscripts of the BSS[16]. Finally, as far as I know, the PG is known through a unique manuscript, containing a similarly unique anonymous commentary on this text. Thus there is a great diversity in the modes and ways the texts seem to have come down to us.

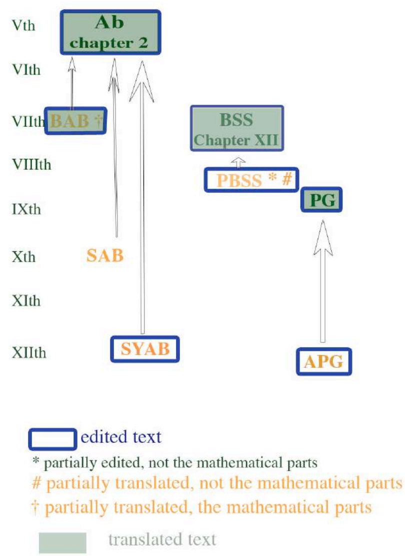
All the primary texts we have evoked here have been entirely edited and translated into English. The BSS is an exception, and has been but partially translated in English, in bits and pieces. Its special situation is probably due to the fact that no extant ancient commentaries in extant manuscripts are known for this text, part of which thus remains hard to understand. Concerning commentaries, two (PBSS and BAB) have been partially translated into English, only one translated for its mathematical part (the BAB).

Edited and translated texts are given in Figure 7, in Appendix 2.

[15] [48, p. xxv-lviii], we have included in this account, Prabhākara for which no extant commentary is known, allthought he is quoted by Bhāskara.

[16] CESS IV 255 b; V 239 b.

Figure 2: Edited and translated commentaries on mathematics, Vth-XIIth century



The case of BSS and PBSS set aside, strikingly, we can notice that if commentaries are edited, they have usually not been translated. The editorial situation is highlighted in Figure 2.

We can take this as a symptom of commentaries special treatment as texts. Is it because historians of science were not interested in them? Does this reflect a state of the manuscript collections? Is this due to the way ancient collectors of texts, and maybe the authors of the commentaries themselves, thought of the text they were composing?

Before looking at how these commentaries were transmitted to us, let us specify some general facts on these commentaries.

1.2 A first rough description of commentaries on mathematics

There is something tricky here in attempting to individualize mathematical commentaries among other texts on mathematics in Sanskrit, since the aim of this article is precisely to underline that much stills needs to be done in this respect.

Titles of texts, and expressions used in them, testifies that the Sanskrit scholarly tradition distinguished between treatises (*śāstra* , *tantra*) and commentaries (*vyakhyā*, *bhāṣya, ṭīkā*). Concerning the texts that have our attention, the Ab is referred to as a *tantra* by Bhāskara[17] and as a *śāstra* by Sūryadeva Yajvan. APG refers to the text it comments as a *śāstra*[18].

Similarly a dirth of names are used by authors to evoke the commentaries they are writing. Thus, Bhāscara calls his commentary a *vyākhyā*[19]. The tradition, as reflected in the title given to his commentary, evokes however a *bhāṣya*. Sūryadeva Yajvan sometimes refers to his own text as a *vyākhyā*, and, more commonly, as a *prakāśa*, "light"[20]. The APG calls itself a *ṭīkā*[21].

[17] To be more specific, it is the three last chapters of the *Āryabhaṭīya* which are referred to in this way, when concluding the commentaries to each of these chapters. [41, p. xxv], [47, p. 171; p. 239; p. 288].

[18] [46, p.1].

[19] For the reference to his own work as a written *vyākhyā*, see the *maṅgalācaraṇam* of BAB.2 : *vyākhyānaṃ gurupādalabdham adhunā kiñcin mayā likhyate*, [47, p. 43].

[20] Thus as the end of the introduction which begins his commentary he writes *evam upodghātaṃ pradarśya śastraṃ vyākhyāyate*. [41, p. 7] For references of the commentary as a *prakāśa* see the end of the chapter's commentary. [41, p. 32, p. 79 (note 11); p. 117, p. 185]. The end of SYAB.2 uses also again the verbal root *vyākh-*.

[21] [46, p.1].

We do not know then if these names are just synonyms or if they tell us something of the kind of commentary, authors and readers had in mind, when they were addressed to in this way. A historical perspective on these names is lacking as well.

A commentary, by definition, is a secondary text, a deuteronomic text, a text that needs another to exist[22]. Is this definition however sufficient to characterize it as a text?

Indeed, there seems to be a diversity in kind that runs through Sanskrit commentaries in general which also reflects itself in the mathematical tradition. Some commentaries respect the original order of the text, when others do not[23]. Thus BAB, SAB, SYAB and APG[24] respect the order of the text whereas PBSS does not[25]. SYAB develops a long introduction (*upodghata*) before the gloss of the text, this isn't the case in the other commentaries evoked here. So that if they have in common the fact that they comment a text, the way they do comment on it is variable. To study them we thus need tools to characterize these different ways of commenting, and of thinning the relation between a commentary and the treatise it comments.

Of course, all the commentaries of our corpus quote entirely the text they

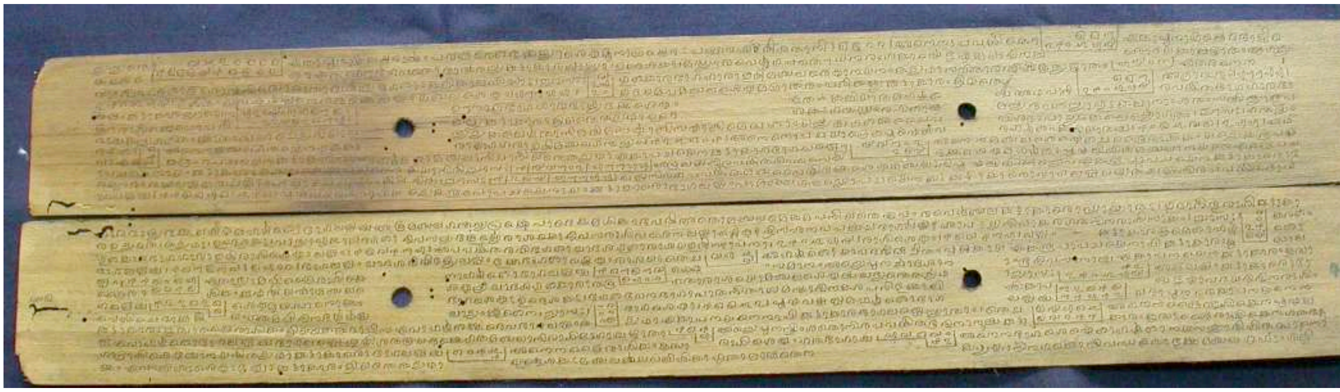
[22]On the question of "secondary texts" in the history of mathematics, see [29], [10], [4] and of course Chemla (and Brown ?) in this volume.

[23]See [8].

[24]One should recall however that PG as given in Shukla's edition is known in this unique recension only [46].

[25]According to CESS IV 221b the order is, for chapters noted in roman numbers: I 1-3; XXI 1-XXII 3; I 4-II 68; XV 1-9; III 1-XIV 55; XV 1à-XX 19; and XXII 4-XXIV 13. The question being then, what is the logic found by PBSS for this order.

Figure 3: Palm leaf manuscript of BAB in a copy of the Kerala University Oriental Manuscripts Library



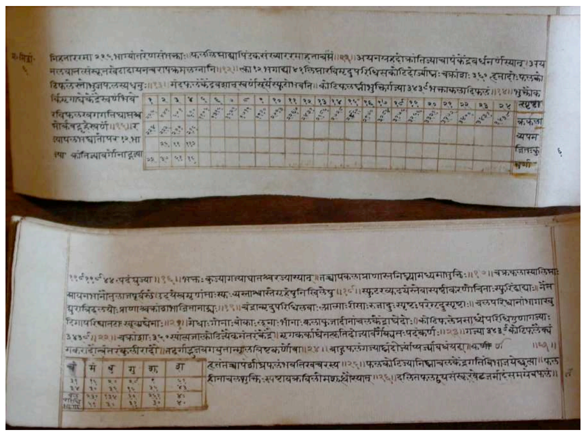
comment[26].

But it is hard, in fact, to find a stylistic criteria separating treatises from commentaries. As soon as one criteria is brought up, an example can be given which blurs it immediately. For instance, manuscripts, didn't always graphically distinguish the treatise from the commentary. Thus, as illustrated in Figure 3, the treatises seems to be an undifferentiated part of the commentary.

In some cases, such as in Figure 4, the commented part was colored, but this wasn't a systematical rule.

[26]However, they all quote also other texts, although not completely. Thus SYAB quotes often PG, and APG, BSS. SYAB often paraphrases BAB. So that these commentaries are in fact composite texts made of parts of previously composed texts, that are sometimes rewritten (paraphrase), intertwined with their own original compositions. We will come back in our conclusion to the versified examples commentaries share with each others as well, and that can be seen as a form of quotation as well.

Figure 4: Paper manuscript with colored highlights of MS in a copy of the Mumbai University Library



So that even thought titles suggest a difference, the material culture doesn't always follow the linguistic difference. A strong stylistic criteria could help philologers then to determine, while editing manuscripts, what belongs to one and the other. The opposition of treatises and commentaries has often been seen as the written reflection of the opposition in an oral tradition of what was to be known by heart (the treatise) and the explanations that were given subsequently orally (the commentary)[27]. This is rooted in what commentators tell us. Thus Bhāskara insists on an opposition between Āryabhaṭa who states orally *āh-* the commented treatise, and Bhāskara's own writing, *likh-*. But in certain texts the word *likh-* is used as a synonym for *vac-*, as in the BM[28]. In this case, there appears to be no difference between a written text and an oral one.

Even the opposition between versified and prose text does not always hold

27 See for instance [47, Introduction].

28 [18, p. 85].

its way. Indeed, the treatises we are concerned with here are all versified. Thus chapter 2 of the AB, chapter XII of the BSS and the PG are composed in *ārya* verses. The commentaries all include prose section, which can present a great diversity : they can set up dialogs for instance, or grammatical analysis. But all include versified parts. Thus BAB and SYAB contain pages of versified tables? All commentaries contain versified examples[29]. Commentaries concerned with mathematics also contain non discursive parts, such as numerical tables and drawings, to which we will come back briefly in the last part.

So that the question of the link and characteristics of commentaries and treatises appears as a complex and difficult one. The plea of this article is that they may be more studied in the future.

1.3 Reading and collecting commentaries on mathematical subjects

Who collected manuscripts? Who had them made? Who copied them? As C. Minkowski and D. Raina's articles in this volume underline, we have, to this date, only sparse information enabling partial answers, varying from library to library, region to region, collection to collection. While restricting ourselves to our chosen corpus, information is especially sparse.

The first element, is that in certain cases there exists a great number of ancient hand made copies of a given text. We usually do not know the

[29]In his edition of PG and APG, Shukla seems to hesitate: are the examples part of the treatise, or part of the commentary?. Editions of L, all consider examples as part of the treatise. It is hard then to decide how both are separated

dates of these manuscripts nor their stories, but it is reasonable to believe that they are mostly results of the copying frenzy of the late modern period described by C. Minkowski in his article of this volume.

Thus the CESS counts for Ab [30] 149 manuscripts of which 47 are with commentaries (more or less 1/3rd). These manuscripts can be found in 28 different libraries. They are mostly of unknown origin and are either paper or palm leaf.

Comparatively, extent manuscripts of commentaries are less important. Bhāskara's commentary has been transmitted to us through 6 manuscripts, 5 of which are in the same library in Kerala. They are all incomplete[31]. Similarly Sūryadeva's commentary has been transmitted to us, through 8 south indian recensions and copies[32], while SAB is known through one copy only[33]. The BSS is known through 34 manuscripts[34] and PBSS is known in 7 manuscripts, two of which are fairly recent copy of two others, none of which are extant[35]. Only two contain the commentary on chapter XII which is explicitly devoted to *gaṇita*[36]. Finally the edited PG is known through one manuscript the one containing APG[37].

[30]CESS 1 51a-52b; 2 15b; 3 16a; 4 27b;

[31][47], CESS IV 297b.

[32][41, p. xvii to xxv].

[33][44, p. 202], CESS I-II 51a.

[34]CESS IV 254 b-255b, CESS V 239 b- 240 a.

[35]CESS IV 221 a, CESS V 224 a .

[36][19, p. S7].

[37][46]. [44, p. 204] Note a second incomplete manuscript of the PG *ṭīkā* in the Descriptive catalogue of the Oriental Mss in the Mackenzie Collection, compiled by H. H . Wilson in Madras in 1882, which would be in 54 folios. I do not know if this manuscript is still traceable to date at the GOML, where the collection formed the base of the original

This is summarized in Table 1.

Table 1: Number of manuscripts for commentaries and treatises on mathematics, Vth-XIIth century

Text	Treatise	Commentary	Number of remaining manuscripts
Ab	x		47
BAB		x	6
SAB		x	1
BSS	x		24
PBSS		x	7
PG	x		1
APG		x	1

We can thus note that in these collections and for the texts of our corpus, commentaries were much less numerous then treatises, and not always complete. Is this due to the hazards of preservation? Does it reflect a trend in the conception of commentaries when the texts were copied or collected sometime during the XVIIIth or XIXth century ? Did the authors of commentaries themselves think of their commentaries as fragmentary texts not to be spread? Does this explain the apparent "neglect" of commentaries in the historiography? To answer these questions with precision, we need to delve a bit more into the complexity of each text, and how it has been transmitted to us.

Let us turn then to how specific manuscripts can tell us something on who had them made, and maybe how they were read and copied.

library. CESS VI might contain more information.

1.4 An example: the manuscripts of the editions of SYAB

K. V. Sarma has given a description of the codexes in which the SYAB is found in Kerala[38]. It shows us a first set of manuscripts were part of a collection made by scholarly astronomers.

Indeed, the codex in which Manuscript A is contained, Codex C. 224-A , was made for the royal family of Eḍapaḷḷī of Kerala, in 1753. It included, the *Bhaṭaprakāśikā* of Sūryadeva Yajvan (SYAB) and also the *Laghubhāskarīya* and *Mahābhāskarīya* of Bhāskara I (VIIth century), the *Tantrasaṇgraha* of Nīlakaṇṭha Somayājin (fl. 1450), the *Sūryasiddhānta* (Xth century), the *Goladīpikā* of Parameśvara (XIVth) and the *Siddhāntaśekhara* of Śrīpati (fl. 1050). All these texts have one subject then: astronomy. Similarly manuscript D, also originating from a codex (C. 22475-A) belonging to the same royal family of Kerala, contains both SYAB and the first chapter of Ab, alone.

Manuscript E of the same edition belongs to a codex (C.2121 C and D) which originally came from the "library of a family of astronomical scholars, the Maṅgalappaḷḷī Illam, at Āranmuḷa, in southern Kerala"[39]. The codex contains with SYAb, the 3 last chapters of the Ab, T and also the *Mahābhāskarīya* of Bhāskara I, a *Mahābhāskarīyavyakhyā* (an anonymous commentary on the *Mahābhāskarīya*), the *Laghumānasakaraṇa* of Muñjāla (fl. 932) with an anonymous commentary, a commentary on chapter II and parts of chapter III of the *Sūryasiddhānta*, the whole of the *Sūryasiddhānta*,

[38] [41, Introduction, xvii-xx].
[39] [41, Introduction, p.xix]

and an anonymous prose *Rāmāyaṇa*. Setting aside the prose version of the famous Indian Epic, we are once again in presence of a codex that seems to concentrate essentially on astronomical and mathematical lore together. We can note here that in some cases only part of a commentary is copied (for the *Sūryasiddhānta*), when all of the treatise itself is extently copied, separately. Ab, similarly, is known to have been transmitted in two separate parts, the first chapter being copied separately (as in the codex for Manuscript A) from the 3 other parts (as in the codex of Manuscript E).

Commentaries then weren't copied together with other commentaries, but a codex could contain a treatise alone, and a commentary of a couple of chapters by a given author. Unlike the Chinese tradition, texts of important treatises could be copied and collected without any commentaries. Different commentaries of a same text were usually copied separately, one did not record in the same manuscript different commentaries of a same verse, as for the Chinese *Nine Chapters*.

Another codex, shows us a text copied for another kind of context: that of a brahmin cast of Kerala, the *nampūtiri*, whose priests follow vedic rituals (*śrauta*). The codex which contains SYAB, C. 2320-A, contains also a text describing the horse sacrifice and detailed accounts of expenses for a ceremony carried out in 1535. The codex seems a copy of a manuscript dating from 1536. Indeed, Sūryadeva understood Āryabhaṭa's text in relation to *śrauta*, being himself a performer of such rituals[40]. This small codex is of the kind

[40]SYAB states this clearly in its general introduction to the Ab ([41, xxv-xxvi, 2-4]). This is an originality of SYAB, Āryabhaṭa's text alone has so little to do with it, that some modern historiographers have even imagined him to be a materialist.

of *śrauta* collection C. Minkowski describes for the Toro family, although the copy of this *siddhāntic* text, may seem surprising. Was the use of *siddhāntic* astronomy by ritualist families a usual phenomena?

On the contrary, Manuscript B of Sarma's edition was given by a scholarly family coming from the *vaṭṭapaḷḷi maṭham* (e.g. a religious complex) near the southernmost tip of the indian sub-continent, *kanyākumārī* in Tamil Nadu. With the *Bhaṭaprakāśikā* (SYAB) the codex contains an *Āṣṭādhyāyīsūtrānukramaṇi*, eg an alphabetical index of Pāṇini's *sūtras*, in other words a text of grammatical lore[41].

In this case, SYAb seems to have been copied as part of a more general endeavor to collect general Sanskrit scholarship.

If the provenance of the manuscripts underline an already well known posterity of *Āryabhaṭa* in Kerala, they show that SYAB seems to have been used in three different, although probably not separate, social contexts: that of scholarly astronomers, that of priests performing vedic rituals, the more general scholarly atmosphere of south indians monasteries.

We have however but little information on how these texts were integrated into the Kerala University Oriental Manuscript Library were they are now to be found. The library was originally created by the government of Travencore (one of the autonomous states within the British Raj) in 1908, aiming at the preservation of their own heritage[42]. More on the history of this library needs

[41] [41, xviii].

[42] The "index of manuscripts" of the library notes that "In 1940 it possessed 3000 manuscripts, 142 publications in sanskrit, 63 in Malayalam. Travancore University (which became the University of Kerala) organized after its establishment (1938) a manuscript preservation and collection department. Both were amalgamated (sic) in 1940. In 1958

to be investigated, but the structure of the codexes in which SYAB is to be found recalls the kind of collection work C. Minkowski described.

On the contrary, existing manuscripts of the PBSS tell us more about their recent collecting, then how the ancient tradition has transmitted them to us existing manuscripts. Thus, the two manuscripts which explicitly contain chapter XII of the PBSS, are those that served respectively for the translation Colebrooke made of this text and the edition that S. Dvivedin made in Benares[43].

Although the stories still need to be written, emerges from what can be found today in these codexes, the idea that commentaries could have not been considered as extant texts by those who last copied them.

Did the authors themselves consider commentaries as whole texts on their own? And were treatises also considered such? A close look at the texts themselves may help us yield some answers. Another question that comes out of this enumeration, is how much of this attitude of copiors was subsequently reflected by the orientalists who looked at them. As slowly, we try to understand the influence that existing pandit's tradition articulated itself with "western" philology, we can also try to understand what affected the XIXthe century point of view on commentaries in the mathematical field.

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there was 28 000 Codices in Sanskrit; 5 000 in malayalam."

[43] See [31, op.cit], [19, op.cit] and [15].

2 Rediscovering Ab, BSS, PG and their commentaries in the historiography of Indian mathematics

This section is a first attempt to highlight textual aspects of the historiography of mathematics in India, when considering the way historians of mathematics looked at the Ab, the BSS and the PG and their VIIth-XIIth century commentaries. It is difficult to find a unifying trend in the way different historians dealt with commentaries, the key being, maybe, whether they were mathematicians or philologers. The more the historian was a mathematician, the less there seems to have been a sensitivity to text, the more he was a philologer, the more the commentary was given an important place. However, this does not mean that commentaries were then always seen as secondary texts by mathematicians and important texts by philologers, as the trend has been to pay more and more attention to the mathematical contents of commentaries as the XXth century came to a close. We cannot afford here to look closely at the shifting attitudes of all different actors, but we will try to draw out some of the characteristics of Colebrooke, Datta & Sing and K. S. Shukla's attitude towards commentaries.

As early as the XVIIth century a certain number of people in Europe knew of the existence of Sanskrit astronomical treatises, by the testimonies of travelers, academic envoys and especially jesuit missionaries, as D. Raina's article in this volume explains[44]. It took time for the Europeans curious of them, to get hold of the texts and be able to study them. By the early XIXth

[44]See also [37], [36].

century, the first translations concerning mathematics were made, aiming at a European audience. Thus in 1812 Stratchey translated in London for the first time the BG, in 1816 Taylor translated the BG in Mumbai, folowed by Colebrooke's translation of L and BG together with the mathematical chapters of Brahmagupta's BSS which was published in London in 1817[45].

2.1 Colebrooke and commentaries

Colebrooke's publication proved to be a landmark. He was the first serious and already well established indologist interested in mathematical texts from the Indian subcontinent. He was at the time a former director of the Asiatic Society of Bengal, a recognized specialist of Hindu Law, rites and Indian languages. His translations of these mathematical texts was preceded by a general "dissertation", followed by "notes and illustrations", which attempted to replace these texts in a general history of mathematics which would provide for India a name it had until then lacked[46]. His introduction takes a strong position on the antiquity of the indian tradition in mathematics, especially in algebra, in an ongoing controversy which it will heighten[47]. The quality of his translations, generally made in close collaboration with pandits, has made it an enduring reference.

Colebrooke included portions of commentaries in the footnotes of his translations. Thus Gaṅgadhāra (fl. 1420), Sūryadasa (1541), Gaṇeṣa's (fl.1520/1554) and also Ramakṛṣṇa's (? date unknown maybe 1687[48]) commentaries are par-

[45] [6].

[46] *Op.cit.*; p. xvi.

[47] [22, 111-112].

[48] CESS V 453 a.

tially included in the translation of L[49]; Kṛṣṇa's (ca.1615), Rangunāṭha's (?) and Ramakṛṣṇa's commentaries are used in the translation of BG. Finally, PBSS (noted as "CA", an abbreviation of *Caturdeva*, a part of his name) is used (and quoted) to translate the BSS. Furthermore, although the original text is not known to him or to his English readers, Colebrooke's introduction refers and discusses the work of Āryabhaṭa[50]. He was indeed discussed to by Brahmagupta and by diverse commentators. Colebrooke also, interestingly, refers to a work of Śrīdhara, which is not however our PG[51]. Thus the names of Āryabhaṭa, Brahmagupta and Śrīdhara were known to Colebrooke's readers.

Colebrooke's aim was to describe the mathematical tradition of India. This led him to consider only the mathematical part of Brahmagupta's treatise, the BSS. And consequently only a portion of PBSS. By doing so, was he prolonging an attested tradition of copying (and thus showing a separate interest) of mathematical chapters of astronomical treatise that existed in the Sanskrit tradition? This selection of BSS's chapters in a printed edition, may also mark the beginning of a long enduring historiographical trend noted in our introduction, the over-representation of mathematical chapters and texts over astronomical and astrological ones in the treatment of ancient *jyotiṣa* texts.

If we turn to Colebrooke's introduction, commentators first come as proofs of the authenticity and antiquity of the texts he is concerned with. Thus he

[49] *Op.cit.* Note A p. xxv and p.xxvii

[50] *Op.cit.* Introduction, sections G to I pp. xxxvii-xiv.

[51] *Op.cit.* p.v he writes that he has a copy of "Srìdhara's compendium of arithmetic", which is probably the *Triśatika*.

writes[52]:

> "The genuineness of the text is established with no less certainty [than its date] by numerous commentaries in Sanskrit, besides a Persian version of it. Those commentaries comprise a perpetual gloss, in which every passage of the original is noticed and interpreted : and every word of it is repeated and explained, a comparison of them authenticates the text where they agree; and would serve, where they did not, to detect any alterations of it that might have taken place, or variations, if any had crept in, subsequent to the composition of the earliest of them. A careful collation of several commentaries, and of three copies of the original work, has been made, and it will be seen in the notes to the translation how unimportant are the discrepancies. "

Commentaries are thus useful and necessary when one wants to edit a text, they are philological tools. However the way they are integrated in Colebrooke's translations, pin points to the fact that commentaries were far from being just that: they were used to understand the treatises, and were stimulating mathematically with their examples and proofs. A commentary however was not treated as a text in itself. It was given in bits and pieces. Selected.

Let us look graphically at the translation, as seen in Figure 5. The commentary appears typographically as a secondary text, written in a smaller font. It is fragmentary, given in different footnotes. However, it literally spills over and eats the space which is meant for the treatise. One can thus

[52] [6, iii]

graphically feel how important the commentary was in the understanding of the text for Colebrooke.

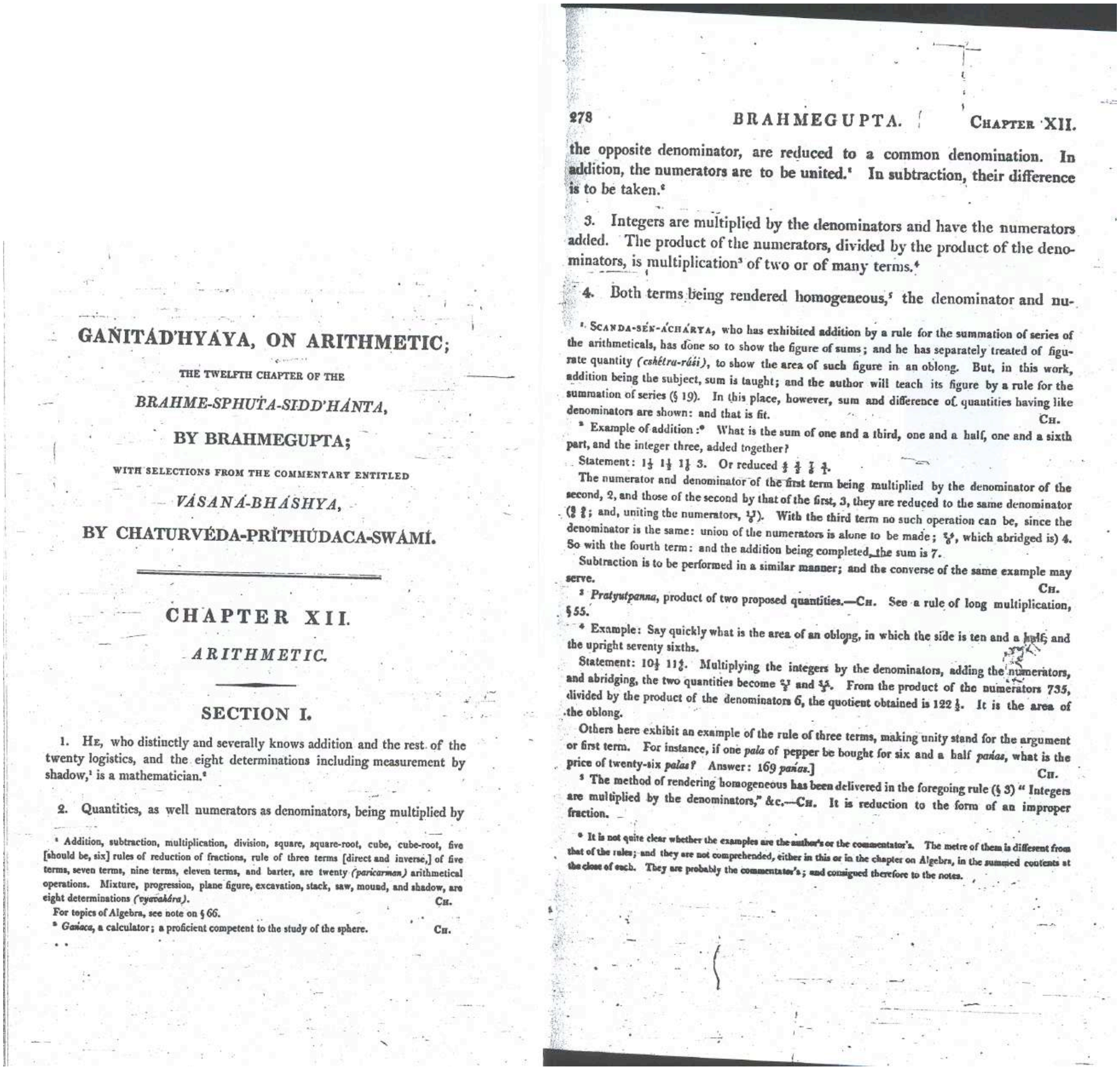
Recalling our reflections on the fact that the codex sometimes include only portions of commentaries, we can wonder how much of this attitude of regarding commentaries in bits and pieces, reflects the way the pandits that trained Colebrooke, and maybe helped him in collating and understanding the text, worked with these texts themselves. This is precisely where a history of reading and conceptions of commentaries as texts is still to be written, as much as Colebrooke's (and other orientalists) mode of working with pandits[53].

2.2 An Indian scholarship with commentaries: Datta & Singh and K. S. Shukla

During the XIXth century, the history of mathematics in India slowly opened officially to Indian scholars, who elaborate a scholarship that is as much directed towards an inner audience, as an answer and a discussion with European interlocutors. Their presence is first seen in articles discussing authorships of texts, before being felt in a series of editions and translations of texts on mathematics. Thus, the names of Datta, Sengupta and Dvivedin appear in an initial period of confusion were the Vth century Āryabhaṭa was confused with his XIth century namesake, and the VIIth century Bhāskara I was confused with his XIIth century namesake. By the end of the XIXth century this movement of edition, translation and analysis of texts on mathematics

[53] The latter has been studied partially in [22], [1], [3] and others .

Figure 5: The BSS and PBSS in Colebrooke's translation



GANITÁD'HYÁYA, ON ARITHMETIC;

THE TWELFTH CHAPTER OF THE

BRAHME-SPHUTA-SIDD'HÁNTA,

BY BRAHMEGUPTA;

WITH SELECTIONS FROM THE COMMENTARY ENTITLED

VÁSANÁ-BHÁSHYA,

BY CHATURVÉDA-PRĬT'HÚDACA-SWÁMÍ.

CHAPTER XII.

ARITHMETIC.

SECTION I.

1. HE, who distinctly and severally knows addition and the rest of the twenty logistics, and the eight determinations including measurement by shadow,[1] is a mathematician.[2]

2. Quantities, as well numerators as denominators, being multiplied by

[1] Addition, subtraction, multiplication, division, square, square-root, cube, cube-root, five [should be, six] rules of reduction of fractions, rule of three terms [direct and inverse,] of five terms, seven terms, nine terms, eleven terms, and barter, are twenty *(paricarman)* arithmetical operations. Mixture, progression, plane figure, excavation, stack, saw, mound, and shadow, are eight determinations *(vyavahára)*. CH.

For topics of Algebra, see note on § 66.

[2] *Ganaca,* a calculator; a proficient competent to the study of the sphere. CH.

the opposite denominator, are reduced to a common denomination. In addition, the numerators are to be united.[1] In subtraction, their difference is to be taken.[2]

3. Integers are multiplied by the denominators and have the numerators added. The product of the numerators, divided by the product of the denominators, is multiplication[3] of two or of many terms.[4]

4. Both terms being rendered homogeneous,[5] the denominator and nu-

[1] SCANDA-SÉN-ÁCHÁRYA, who has exhibited addition by a rule for the summation of series of the arithmeticals, has done so to show the figure of sums; and he has separately treated of figurate quantity *(cshétra-ráśi)*, to show the area of such figure in an oblong. But, in this work, addition being the subject, sum is taught; and the author will teach its figure by a rule for the summation of series (§ 19). In this place, however, sum and difference of quantities having like denominators are shown: and that is fit. CH.

[2] Example of addition:* What is the sum of one and a third, one and a half, one and a sixth part, and the integer three, added together?

Statement: $1\frac{1}{3}$ $1\frac{1}{2}$ $1\frac{1}{6}$ 3. Or reduced $\frac{4}{3}$ $\frac{3}{2}$ $\frac{7}{6}$ $\frac{3}{1}$.

The numerator and denominator of the first term being multiplied by the denominator of the second, 2, and those of the second by that of the first, 3, they are reduced to the same denominator ($\frac{8}{6}$ $\frac{9}{6}$; and, uniting the numerators, $\frac{17}{6}$). With the third term no such operation can be, since the denominator is the same: union of the numerators is alone to be made; $\frac{24}{6}$, which abridged is) 4. So with the fourth term: and the addition being completed, the sum is 7.

Subtraction is to be performed in a similar manner; and the converse of the same example may serve. CH.

[3] *Pratyutpanna,* product of two proposed quantities.—CH. See a rule of long multiplication, § 55.

[4] Example: Say quickly what is the area of an oblong, in which the side is ten and a half, and the upright seventy sixths.

Statement: $10\frac{1}{2}$ $11\frac{2}{3}$. Multiplying the integers by the denominators, adding the numerators, and abridging, the two quantities become $\frac{21}{2}$ and $\frac{35}{3}$. From the product of the numerators 735, divided by the product of the denominators 6, the quotient obtained is $122\frac{1}{2}$. It is the area of the oblong.

Others here exhibit an example of the rule of three terms, making unity stand for the argument or first term. For instance, if one *pala* of pepper be bought for six and a half *paṅas,* what is the price of twenty-six *palas?* Answer: 169 *paṅas.*] CH.

[5] The method of rendering homogeneous has been delivered in the foregoing rule (§ 3) "Integers are multiplied by the denominators," &c.—CH. It is reduction to the form of an improper fraction.

* It is not quite clear whether the examples are the author's or the commentator's. The metre of them is different from that of the rules; and they are not comprehended, either in this or in the chapter on Algebra, in the summed contents at the close of each. They are probably the commentator's; and consigned therefore to the notes.

in Sanskrit comes to a peak. Kern in 1874[54] edits for the first time [55] the Ab. His edition is printed with Parameśvara (XIVth century) 's commentary. His introduction evokes SYAB, which is quoted and referred to by Parameśvara; but not the BAB. In 1879, chapter 2 of the Ab is translated and analysed into French by Leon Rodet[56]. In 1896 Dikshit gives an edition of the BSS with his own commentary[57]. These editions are then followed by a number of translations in English and the first studies in this language. Thus in 1907 and 1908 G. R. Kaye publishes his controversial *Notes on Indian Mathematics*, part two of which is devoted to Āryabhaṭa[58]. These are followed by a set of translations. Sengupta publishes a first English translation of the Ab[59], followed by Clark in 1930[60]. B. Datta and S. N. Singh publish in 1937 the enduring classic "Hindu Mathematics"[61]. Their main aim is to provide a general description of all the different ways Hindu mathematicians practiced elementary and sometimes higher mathematics, each author adding a stone to this description. But they also want to answer G. R. Kaye's claims on the Arabic or European origin of indian mathematics in general and Ab's mathematics in particular. To do so, part of their effort consists in comparing the history of mathematics in Europe with what has been discovered about the history of mathematics in the Indian subcontinent. Datta & Singh

[54] [27].

[55] The tradition of copying manuscripts can of course also be seen as an editorial tradition of classical India, but we are referring here to printed books.

[56] [40]

[57] [14].

[58] [21].

[59] [45].

[60] [12].

[61] [13].

write then, to a certain extent, a history of mathematics in India with its great authors and treatises. As trained mathematicians, their focus is essentially on the mathematical contents of the texts[62]. B. Datta and A. N. Singh consider then mathematical commentaries essentially as mathematical texts. Thus Bhāskara I, whose text was known but was not published, is referred to several times in this text as an astronomer sometimes dealing with mathematics[63]. His commentary, referred to as such, is sometimes quoted as explaining or interpretating Ab's verses[64], sometimes even in footnotes like in Colebrooke's text[65], and often with other commentators of Ab such as Nīlakaṇṭha, none of which were either at the time edited texts. Essentially, however, BAB is referred to for its mathematical contents, outside of its relation with the text of the Ab[66], and mingled sometimes with the contents of Bhāskara's other astronomical texts[67].

With the independence and the creation of institutions for the history of science, a new wave of editions is set forth, of which K. S. Shukla is an important actor. In 1959 he publishes, at the university of Lucknow, an edition of PG with APG, together with an English translation of APG. He

[62]But they also embody a dying tradition of mastering Sanskrit texts on *jyotiṣa*. After all, B. Datta was at the end of his life addressed as *pandit*. This mastering is attested by the enduring quality of their translations. The study of how the blend of these two traditions can be felt in their manual is open for further research.

[63][13, Volume I, 125].

[64][13, Volume I, 66-67; 196; 211. Volume II, 93-95].

[65][13, Volume I, 170]. SYAB also such as in *op. cit.*[Volume II, 91, footnote 4].

[66][13, Volume I, 80, 82, 87, 130, 204, 239; Volume II, 87, 238].

[67]Thus the entire part devoted to the *kuṭṭaka* in Volume 2, quotes all the different texts of Bhāskara I, sometimes to present his own algorithms, sometimes as explaining Ab's algorithm.

then turns to Bhāskara's work, first editing and translating his treatises that can be seen as elaborations of Āryabhaṭa's astronomy, the *Laghubhāskarīya* and the *Mahābhāskarīya*. Finally in 1976 are conjointly published an edition and translation of the AB, and editions of BAB and SYAB within the Indian National Science Academy.

By the 80's a new generation of mathematicians will take interest in commentaries. On the one hand, publications on the Mādhava school of mathematics will call the attention of historians of mathematics on scholiasts of Āryabhaṭa; on the other hand the Japanese students of D. Pingree, T. Hayashi in particular, will start publishing articles on the mathematical contents of different commentaries on Ab, BG and L. To this we can add the recent publications of F. Patte's PhD and my own, to testify of the growing interest for mathematical commentaries in Sanskrit as the XXth century came to an end[68].

Shukla's editions uses commentaries in three separate ways: commentaries are first of all used, as in Colebrooke's case, as a philological assessment of the original text one wants to edit. They are also used to explain the text. Thus the APG is not translated but sometimes referred to in the comments that Shukla has written with the translation. It is not always

[68] How much were these attitudes towards commentaries linked to the more general developments in the field of indology? Indeed Indology developed a special focus on the study of treatises and the contents of important commentaries, somewhat overlooking to reflect on the commentary as a specific kind of text. The last five, ten years has seen however a renewed interest in this kind of texts, as testifies by the conference "forms and uses of the commentary in the Indian world", which was held in Pondicherry in february 2005, see http://www.ifpindia.org/Forms-and-Uses-of-the-Commentary-in-the-Indian-World.html.

clear then how much Shukla consider's commentators as interpretating or explicitating the contents of the treatise. In his co-edition and translation of the Ab, commentators are summoned to add a mathematical depth to the algorithms, and sometimes quoted for their conflicting interpretations. Finally in some instances, especially concerning BAB, commentaries appear as a mathematical text in its own right[69]. This trend that sees commentaries not only as philological aids, but also as independent texts on mathematics has been growing as testifies the works of the scholarship in history of mathematics in India in the last twenty years, evoked above. This has no doubt to do with a general trend in the history of mathematics at large, as such approaches seem to have been characteristic of the Chinese corpus as well.

Commentaries then were used, read and analyzed but seldom translated by modern historians of mathematics. This made quite sense when they where thought of, as often they were, as philological tools to edit and understand the texts they commented, but remains quite surprising when they are studied as texts on mathematics in their own right. This is but one symptom of the fact that the question of their mathematical relation to the text they comment, other than that of interpretating them (in a right or wrong way), was left unresolved.

[69]Before publishing his edition of BAB, Shukla published a number of analysis in the Indian Journal of History of Science, pinpointing the mathematical importance of the text.

3 The relation of commentaries with their treatises: the example of the extraction of square roots

Scholarship on the mathematical contents of commentaries, has often focussed on the interesting mathematical ideas, like the proofs that they contained[70]. I would like to focus here on non-textual practices of mathematics that the texts hint to, and to the mathematical reflection that can be seen through one mode of relation of the commentary with the treatise: word substitutions. To do so we will focus on the rule to extract square roots, which is found in Ab and PG, but not in BSS.

3.1 Positional notation and extracting square roots

I have shown elsewhere, that if the history of mathematics in the Indian subcontinent, has long insisted on showing that the decimal place value notation came from India, it reflected little on the concept different authors had of this system, and in particular how they thought and used the idea of position. And indeed, for BAB, SYAB and APG the decimal place value is a conventional notation for which places where digits are noted to make a number are an ordered set on a horizontal line[71]. The important idea stressed with

[70] [49], [30]. Strangely enough little reflection has been published on the comments contained in commentaries on the field of *gaṇita* itself, which however could help us understand why chapters on *gaṇita* containing algorithms with little astronomical applications, where included in treatises on astronomy. (See [32], [25], [33]).

[71] [26].

a great continuity by one author after another, is that a "place" (*sthāna*) does not exist alone, but only in relation to another, in an ordered relation. This is how a "place" then becomes a "position", although there is no new Sanskrit word that expresses this conceptual change. Consequently, is this positional notation thought of as a positional *system*? A close look at the way commentators treat the extraction of square root procedures enables us to approach more closely their conception of position, and sometimes tell us something of what the authors of the treatise wanted to emphasis as well.

Now, the square root extraction procedures during this period use the decimal place-value notation as a basis. The algorithms given by our authors are not "useful" procedures to extract square roots, such as the interpolations usually described in astronomical (parts of) treatises. They all suppose that one extracts the root of a perfect square[72]. The idea is to recognize in a number written with the decimal place-value notation the hidden development of a square expansion. To say it in other words, this would mean to recognize in a number written as $a_{2n} \times 10^{2n} + a_{2n-1} \times 10^{2n-1} + \ldots + a_1 \times 10^2 + a_0 \times 10 + c^2$ the development of a square of the $(b_n \times 10^n + \ldots + b_i \times 10^i + \ldots + c)^2$ $(i < n)$ kind.[73] Crucial then to this algorithm is to distinguish between powers of ten that are also squares (the even powers of ten), and those that are not.

[72]We can wonder why then such procedures were given. This may have to do with the fact that this procedure enables the inversion of a squaring procedure, in a mathematical tradition where the correction of an algorithm is sometimes verified by inverting it, and finding the initial input. Solved examples, all revert the squarings that where illustrated in the squaring procedure.

[73]For general explanations on the different methods see [13, Volume I, 170-171] and [2, 78-79].

Let us note here that Ab's algorithm provides directly the square root. PG's algorithm gives first a procedure to extract a double square root, and then says that it should be halved. Although they are grounded on the same idea, these two procedures differ in their intermediary steps then. But we will not expose this here, concentrating on what the texts tells us of the places and positions with which the algorithm is carried out.

3.2 Changing names in the procedure to extract square roots

This is how Ab gives the rule to extract square roots[74]:

> **Ab.2.4. One should divide, constantly, the non-square ⟨place⟩ by twice the square root|**
> **When the square has been subtracted from the square ⟨place⟩, the quotient is the root in a different place||**

Without a commentary the algorithm itself is hard to understand. Part of the difficulty rises from the fact that Āryabhaṭa uses a pun which makes us confuse which of the digits are squares (*varga*) and which of the digits are noted on "square positions"[75]. This pun however hightlights exactly

[74]

> *bhāgaṃ hared avargān nityaṃ dviguṇena vargamūlena*|
> *vargād varge śuddhe labdhaṃ sthānāntare mūlam*||

See [47, 36-37] for an explanation of the algorithm.

[75]Another difficult aspect of the verse, is the fact that it states the algorithm, starting from its middle, emphazing its iterative aspect. This we will come back to in a forthcoming article [23].

the mathematical idea behind the root extraction: “square places” of the decimal place value notation are the places where we get hints to which digits have been squared in order to produce the number we are dealing with. Ab considers the decimal place-value notation as an ordered line of places, for increasing powers of ten. To this reading, he adds a new grid to qualify the places, distinguishing the powers of ten which are squares from those that are not squares. Ab's readings of positions are concerned simultaneously with the mathematical dimension of the decimal place value notation and with the mathematical idea on which the algorithm rests. BAB followed by SYAB together help us understand Ab's verse by giving new names to these places. They both start from the decimal place value notation, but consider it outside of its mathematical signification. They count the different places were digits are noted, starting on the right, from the lowest power of ten, and continuing on the left. All the even numbers of this enumeration indicate “even places”, and odd numbers, “odd places”. Since, 10^0 starts this enumeration, what Ab calls “square places”, BAB and SYAB call “uneven place”, and what Ab calls “non-square places”, BAB and SYAB call “even place”:

10^5	10^4	10^3	10^2	10^1	10^0
non-square place (*avarga*)	square place (*varga*)	non-square place	square place	non-square place	square place
6	5	4	3	2	1
even place (*sama*)	odd place (*viṣama*)	even place	odd place	even place	odd place

Commentators then add here their own grid to the ordered list of places that define the decimal place value notation, a grid that considers the notation outside of its mathematical content, as a tabular form with a numbered list of items on a line. It is to this formal way of looking at the decimal place value notation, that they apply a mathematical assessment: with odd and even numbers. This mathematical assessment is not directly related to the algorithm, until the commentators in fact relate both this grid to the one used by Ab. It is thus their commentary that will link this reading to the one offered in Ab's verse. All this is done simply by substituting one word to another, in general word glosses. Thus, BAB states:

> In this computation (*gaṇita*), the square (*varga*) is the odd (*viṣama*) place.

And SYAB[76]:

> In the places where numbers are set-down, the odd places are square places. The even places are non-square places.

The name substitution they make is synthesized in Table 3.2.

The fact that these places are used and qualified in different classifications, some of them underlining their values, others their positions in an ordered line, others again pinpointing their mathematical qualities (as squares) points to the fact that all the authors considered here, do indeed use the decimal place value as a system of positions which can be qualified in as many different ways that an algorithms requires. The notation itself is given a

[76] [41, p. 36, line 15].

Table 2: Names of places in the algorithm to extract square roots

Texts	Even powers of ten	Uneven powers of ten
Ab	*varga*	*avarga*
BAB	*viṣama*	*sama*
PG	idem	nihil
APG	idem	*sama*
SYAB	idem	idem

specific grid by the commentators which relate it to the mathematically significant grid of the author of the treatise. Furthermore, it is because they give new names to these the places, that we can unravel the confusion the pun provokes in the verse.

To sum it up, the commentarial act of substituting a name given in the treatise by another has a first function then, that of explaining the literal meaning of the verse. It also points to the fact that the pun had a mathematical meaning, and what the meaning was. Then, it tells us something on how the *sūtra* was composed: using a mathematical pun so that one remembers the mathematical idea behind the algorithm. Finally, it creates a new grid, a new system of positions and links it to the one it has explicited for the treatise. If Ab gives the core mathematical ideas of the algorithm, BAB and SYAB hightlight them by explaining how they are associated with the notations of numbers and the algorithm on a working surface. However they do not detail the process as it is really worked out on a working surface, as Ab, they just give the general idea that rules the working out of the procedure.

Ab's confusing pun was not taken up in later literature. PG uses a vocabulary that is the same as BAB[77]:

> **PG.24.Having removed the square from the odd term, one should divide the remainder by twice the root that has trickled down to the place|**
> **⟨And⟩ dispose the remainder on a line (*paṅkti*)||**
> **PG.25. Having subtracted the square of that, one should divide the previous result|**
> **That has been doubled. Thus again and again, ⟨finally⟩ one should halve twice the square.||**

APG comments the verse regarding questions of place, as follows [78]:

> One should subtract a possible square (*saṃbhavinaṃ vargaṃ*), from the *viṣama* ⟨place⟩ of the square quantity, ⟨in other words⟩

[77] See [18 for the Sanskrit, 9-10 of the part in English for an explanation of the procedure as described in APG][46]

> *viṣamāt padas tyaktyvā vargaṃ sthānacyutena mūlena|*
> *dviguṇena bhajec cheṣaṃ labdhaṃ viniveśayet paṅktau ||*
> *tadvargaṃ saṃśodhya dviguṇaṃ kurvīt purvaval labdham|*
> *utsārya tato vibhajec śeṣaṃ dviguṇīkṛtaṃ dalayet ||*

[78] [46, 18, line 10-12]

> *vargarāśer viṣamāt padād ojākhyād ekatṛtīyapañcamasaptamāder ekaśatāyutaprayutādisthānebhyo 'nyatamasthānād antyāt padāt sambhavinaṃ vargaṃ tyajet*

from what is called odd (*oja*), that is from the first, third, fifth, or seventh etc. ⟨place⟩, from the places for one, one hundred, ten thousand, or one million, etc., from the *pada*, that is from the last among all other places.

And considering the square root of 188624, it adds[79]:

In due order from the first place which consists of four, making the names: "odd (*viṣama*), even (*sama*), odd (*viṣama*), even (*sama*)".

Setting down:

sa	vi	sa	vi	sa	vi
1	8	6	6	2	4

In this case, the odd terms which are the places one, a hundred, ten thousands, consist of four, six and eight. Therefore the last odd term is the ten thousand place which consists of eight.

[79][46, 18, line 19-22]

ānulomyena ekasthānāc catuṣkāt prabhṛti viṣamaṃ samaṃ viṣamaṃ samam iti saṃjñākaraṇam /

sa	sa	sa	vi	sa	vi
1	8	6	6	2	4

atra catuḥṣaḍaṣṭakāni ekaśatāyutasthānāni viṣamapadāni tebhyo 'yutasthānastham aṣṭakam antyaṃ viṣamapadaṃ

The distinction between the place, its value within a power of ten, the place it has in the row of numbers noted on the line, and the digit which is noted in this place, is underlined by the fact that the commentator uses a different syntactic expression for each . The values in power of tens that a place stands for are noted within a *tatpuruṣa* compound ending with *sthāna* and incorporating an inner enumerative *dvandva*, which gives the concerned powers of ten in increasing order (*ekaśatāyutasthānāni*, "the places for one, a hundred, ten thousands"). The place it has in the row of numbers is referred to in several ways. APG numbers the places, starting with one for the lowest power of ten and increasing successively. These are enumerated in a *dvandva* giving the ordinals of the concerned places (*ekatṛtīyapañcamasaptamāder*, "for the first[80], the third, the fifth, the seventh, etc.). APG, following PG, reproduces then the *viṣama* (odd) / *sama* (even) terminology we found in BAB and SYAB. But PG and APG insist that the places are used within an ordered set, or *series* of numbers for which there is a first and a last term: PG uses the word *pada*, which is used for the terms of a series, and APG glosses *pada* with the expression *anyatamasthānāt antyāt*, "the last among all the other places". Finally the digits noted are understood as tools to be worked with, withing these positions. This is how we can maybe understand the fact that the names of numbers are given ending with the suffixe -*ka* (*catuḥṣaḍaṣṭakāni* , "consists of four, six and eight").

Thus here again a tightly knitted connection between commentary and treatise explains to us all the different ways in which a place becomes a position on an elaborate grid which rests not only upon the decimal place

[80] *Eka* however is used here and not *prathāma*.

value notation, and on the algorithm to be carried out, but also within the formal system created by the notation itself. However, if we compare Ab, BAB an SYAB, with PG and APG, with the latter the treatise itself hints to how the algorithm should be carried out within a working surface. APG then unravels the different steps precisely, but also, as the succession of glosses suggest, is the one to complete the mathematical background that the treatise just alludes to in relation to the decimal place value notation.

3.3 Extracting square roots along different lines

Indeed, if we turn again to PG's formulation of the algorithm, we can notice that it avoids puns on the words that would underline a mathematical idea behind the given algorithm: compared to Ab it doesn't seem so dense or confused. Let us note the elements hinting to the concrete carrying out of the algorithm on a working surface. A line (*paṅkti*) is evoked, and the movement of trickling down as a drop of water (*cyuta*) is used to characterize the apparition of a partial, double root, digit by digit.

This is further developed by APG, which multiplies the "setting down" of partial roots, and evokes repeatedly operations carried above (*uparita*), below (*adhas*) and the fact that the partial double root extracted slides like a snake (*sarpaṇa*, *sarpita*) to the next position[81]. The procedure then is described in the details of how practically it is to be carried out in a working surface. The different grids described by APG, have now both a meaning vertically and horizontally, and this is used to carry out the algorithm itself. Here then

[81] [46, p.18-19 of the Sanskrit, p. 9-10 of the English version to see how the numbers are disposed and change during the algorithm, according to APG]

the horizontal expansion of the decimal place value notation is extended into a table, with operations conducted within columns and others within lines, as for other elementary arithmetical operations. The intricate way the positionnal system works is quite precisely indicated by APG. However no reference is made to the idea we find in Ab's treatise evoking partial squares.

To sum it up, we have seen different authors use the places of the decimal notation in different manners: Ab underlining their mathematical powers, BAB and SYAB linking these mathematical meanings to the formal notation of numbers, and using mathematical properties of this formal notation to do so, PG describing only this formal notation and its mathematical properties, and APG linking this formal notation to their meaning in relation to the decimal place value notation. If we take then a chronological perspective it seems that over time the decimal place value notation, slowly took on a formal aspect, as a tabular form which could be used not referring systematically to what the positions meant in terms of values of powers of ten. It is difficult here to unravel here what belongs exactly to commentators and to the treatises: what we can read is what the commentaries saw in the treatise, and what they thought of it.

With this in mind, we see then the commentaries help us unveil how they perceived the different nature of Ab and PG as treatises : the emphasis is, on one side, on the idea behind the procedure (in Ab), on the other, on expressing all the different steps of the algorithm, including the fact that the double square is obtained on a separate line (for PG). And indeed, there are two different kind of texts involved here, Ab being a theoretical treatise while PG is explicitly devoted to practical mathematics (*vyavahāra*). The commentaries differ as well according to the type of texts they comment. If they all include illustrated examples, APG is the only one to follow precisely how the different intermediary operations are carried out, even evoking the possibility for a doubled number to become bigger than 10, during the steps of the algorithm[82]. While neither BAB nor SYAB insist on these intermediary steps, highlighting on the contrary the essential idea behind the algorithm. PG concentrates on whole numbers, whereas the emphasis in SYAB and BAB is on the fact that the square root of fractions is the fraction of the square roots. And indeed, SYAB has read PG and quotes it in this very commentary but not for the procedure to extract square roots on whole numbers, nor for the positioning of digits during the procedure. In all cases the role of the commentary seems to be to link different ways of understanding a same object. And the way it does it, is primarily through the technique of word substitution. Furthermore, eventhough the non textual practice of tabular computations seems to be the realm of the disposition and resolution parts of solved examples in commentaries, we have seen that PG alludes to it : treatises can also testify of these practices, although probably not in detail.

Conclusion

We have thus seen here two different kind of *śāstra*s: one whose emphasis is on mathematical ideas, an other on mathematical practices. This is not new for the history of mathematics in India, but this small analysis has enabled to approach a bit more closely what this means in terms of descriptive prac-

[82] *op. cit*[18, line 15-16]

tices and what commentaries should do according to the aim of the text. We have thus seen that it is not commentaries that embody "practical" knowledge, but the kind of treatise that determines the form of the commentary, whether it will concentrate on ideas or on how the algorithm is carried out on a working surface. However in all cases, the work of the commentaries is that of establishing relations, integrating what is hinted in the verse commented into a network of other systems. As underlined in the introduction, focussing on commentaries may help us understand what was the link between mathematical practices and the texts that practioners produced. Although this is but a beginning, we can hope that scrutiny of how commentaries and treatises relate will slowly enable us to specify what commentators thought of the roles of different kinds of texts.

The scholarly tradition in Sanskrit is often characterized by the specific kind of texts it produced: versified or aphoristic treatises (*śāstra*, *tantra*) and their commentaries (*ṭīkā, bhāṣya, vyākhyā*). But as the diversity of Sanskrit names suggest, these generic names cover many different sorts of texts[83]. In the case of astronomy and mathematics, theoretical treatises could have a technical name *siddhānta*, and there certainly existed a great range of commentaries[84]. One can also add for this technical field all sorts of other kinds of texts, such as handbooks for making calendars (*karaṇa*), almanachs (*pañcāṅga*), compendiums (*nibandha*), etc[85].

[83]See on this subject [39], [9], [7].

[84]See the Appendix 1, which lists the different names of commentaries as listed by D. Pingree in his CESS. On the status of Pingree's census, see below and the afore mentioned appendix.

[85][32].

With this diversity in mind, we can thus wonder if certain kinds of text were preferred to others in the past. Did such preferences vary over time, in different places? Does this tell us something of the conception the different actors had of what an astronomical or mathematical text was to be? Of how such disciplines were practiced? Did historians of science, discovering these texts, focus on one kind of text over another?

We have underlined, that the textual diversity of sanskrit commentaries on mathematics, seemed but a reflection of the even more diverse landscape of sanskrit commentaries in all sorts of technical fields. Can we speak of mathematical commentaries in sanskrit? E.g. is there a specific genre of commentaries pertaining to mathematics? A hypothesis to further explore is indeed the specificity of their solved versified examples, which includes through dispositions, the representations of a working surface on which diagrams or tables, such as those that can be seen in Figure 3 and Figure 2[86]. Indeed, the commentaries that constitute our corpus all seem to possess lists of solved examples which have the same standard structure[87]. This is

[86] A solved example consists of a versified problem, the disposition of the elements which will be used to solve the problem (numbers are disposed in a table, a diagram is drawn giving the values of the known segments), and a resolution. These three parts of a solved examples are often announced: *uddeśakaḥ, uddhāhṛtaḥ* (An example, an illustration:); *nyāsaḥ, sthāpanaḥ, sthāpitaḥ* (“setting down” “disposition”, “disposition”), and *karaṇaḥ* (”resolution”, “method”). The “disposition” part of solved example, is a unique feature. It figures a working surface on which numbers or diagrams are disposed, moved and erased. It opens a window onto written but non discursive computational practices, such as those we have seen above described in APG. It is one mathematical feature that does not appear in versified treatises.

[87] When Shukla wrote his introduction to his edition of PG, he seemed to have considered

especially clear in the edited commentaries we know of the *Āryabhaṭīya*, that is BAB, SAB, SYAB. There commentaries of the mathematical chapter of this treatise includes for almost all of the verses a list of solved examples. This is not the case in the three other chapters of the treatise dealing with astronomical constants, time reckoning and the movement of planets. This question is too vast to be delved upon here, but once again reflection on the relations between treatises and commentaries may help us specify to which texts the examples belonged, and how this may have changed over time.

We have thus seen that a late tradition may have considered commentaries as fragmentary explanations and not full texts. Looking at the texts themselves, one can wonder wether they were made to be read, verse-commentary by verse-commentary separately. In the case of the three commentaries on root extraction that we have looked at, the decimal place value notation and the rules to carry out elementary operations are used. If this is known, the commentary can be followed as if it was autonomous. This however may not be true for all algorithms. But we can imagine different verse commentaries by different authors being used simultaneously, leaving out how a full commentary would provide a more integrated vision of what the treatise was about. But how much of this imagined partial reading of commentaries was originally there when the authors wrote there commentaries? The examples we have taken here show us rather what looks like a late tradition considering commentaries as fragmentary explanations and not full texts. To which we

that the versified examples were part of PG, and that their resolution was a commentary. However he numbered them separately from the treatise. This is maybe once again an example of blurring of the frontiers between treatises and commentaries. Is it the case for PBSS? T. Hayashi has noted a similar structure in the BM [Hayashi 1995; p. 84-85].

can add this hypothetical evolution: This pandit tradition may have seeped into the way Colebrooke and other European orientalists considered these texts. This mingled with a European tradition of considering mathematical texts not for their textual characteristics but as just containing mathematical lore to give the kind of historiography, prevalent today when studying mathematical commentaries.

Appendix 1: Counting Authors and Texts in the CESS

Should I keep this part or leave it out all together?

D. Pingree started in 1955[88] a survey of manuscripts on *jyotiṣa* that was still left unfinished when he died fifty years later in november 2005. The CESS is in 5 volumes, volume 6 is still pending. Despite it's name, it lists texts in all sorts of languages of the Indian subcontinent and is not restricted to Sanskrit. If remaining manuscripts certainly exist in private collections, or are un- or mis- classified in libraries, we can still estimate that most of the manuscripts known in the world are included in his census, if we include the posthumous volume 6, which is at this date still to be published[89].

The near exhaustivity of the CESS, can thus enable us to reflect quantitavely on the manuscripts which have been collected and preserved for astronomical, astrological or mathematical texts. Since the census not only gives manuscript references but has also tracked published editions, it could help us evaluate how many of these texts couched in manuscripts have been edited and studied in the last two hundred years.

My own counting of the entries of the census was done manually as I had no electronic version of the text[90], so it is probably fought with errors. The

[88]See CESS I, preface.

[89]In the mean time, for the authors that haven't yet been treated in the published CESS, one can refer to [44] (which also includes text in Persian) and to individual library catalogs.

[90]Note that the CESS has since then been partly digitalized (volumes 1, 2 and 4) on http://books.google.com/.

counting was spread on 5 volumes whose size increased with each volume; volume 5 has 756 pages. Although the counting is probably fought with errors, it still gives a general idea of the respective proportion of texts. I have counted texts that were manuscripts, and have thus excluded references to authors for which we do not have any remaining text, or texts for which we have XXth century published editions by XXth century authors and no manuscripts.

A first exploration then, has shown that texts on mathematics, are comparatively very few compared to the other censussed texts on *jyotiṣa*. Indeed, for the 3686 texts recorded in the first five volumes of the CESS only 102 (2,7 %) are clearly devoted at least partly to mathematics (*gaṇita*)[91]. I did not, when I was counting, list which texts were edited and translated. But knowing that some of these texts are in vernacular languages (oriya, tamil, notably), and that these texts have seldom been edited or even translated, I think it is safe to consider that an important part of the texts in sanskrit concerning *gaṇita* have been edited and translated compared to the texts solely concerned with astronomy or astrology, for which very few texts of the census have been published.

David Pingree cast a large net when undertaking his census, considering texts that may refer to some part of *jyotiṣa* in passing.

Under 2972 authors, and 3686 texts recorded devoted to *jyotiṣa*, 816 (22%) are commentaries, and 646 authors (21,7 %) are commentators. I have noted a very small number of compendiums, dictionaries and translations.

[91]This number will maybe increase as we slowly learn of the contents in all the listed texts of the Census.

The CESS notes a number of non-standard texts, such as the *Aparājitapṛcchā* of Bhuvanadeva (fl. XII-XIIth century) whose text on architecture is written in dialogue [92].

Concerning commentaries, many different technical names are recorded either in the titles or by D. Pingree, preeminently *ṭīkā*s (549- 67,2% of all commentaries), but also *vyākhyā*s (85-10,4%), *vivṛtti*s (50-6,1%) and *bhāṣya*s (34-4,1%) among *avacūrṇi*s, *vārtikka*s, *tippaṇaṇi*s, *vivāraṇa*'s are noted. The relative proportions of these different names for commentaries are illustrated in Figure 6

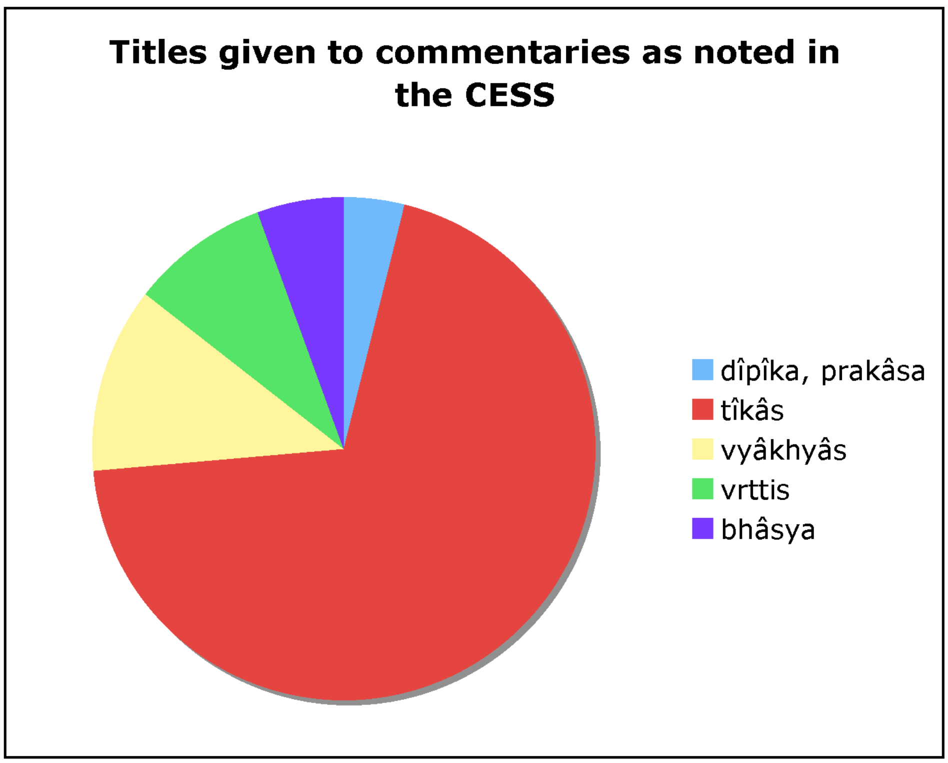
Of course this raises the question of how these names were given to texts[93]: do the authors themselves refer to these texts with these names? Are they given by later scholars? By those who copied the text? This set of questions can be extended to all the texts of the census in fact. I have counted 130 (3,5 %) texts bearing the name *karaṇa* or associated titles and 66 (1,7%) almanachs (*pac̃āṅgas*) and texts explaining how to make them. Only 205 (5,5%) texts bear *siddhānta* in their title, but this does not mean that a number of other theoretical texts are not listed. Thus we count 18 *śāstra*s and *tantra*s. Titles can express either that they bestow knowledge 141 (*saṅgraha, jñana*; 3, 8 %) or the idea of providing an essence of something 118 (*phala, sāra*; 3,2%); sometimes both (*sārasaṅgraha* being quite a common title compound). The metaphor of light (*dīpika*, *prakāśa*) is used also (289; 7,8%), for commentaries also (28).

I do not think that one should interpret too closely the figures here, but

[92]CESS V 264 a.

[93]We seldom encounter here the names of patrons, as in the cases examined by C. Minkowski.

Figure 6: Proportion of the different titles given to commentaries in the CESS



they should just be taken as giving us a taste of how diverse the texts on *jyotiṣa* in Sanskrit were.

More questions may be raised on the status of the Census, on what it represents of the existing manuscripts, etc. they are left open for further exploration.

Appendix 2: Texts on mathematics Vth-XIIth century

A list of all he known texts on mathematics, in Sanskrit, Vth-XIIth century is given in Table 3.3.

The editorial situation of these texts is summarized in Table 3.3.

The Ab, BSS, BM, GSS, PG, L and BG have been edited and translated into English[97]. All the other texts have been edited[98].

Concerning commentaries, two have been partially translated into English (PBSS and BAB), BAB is the only one which has been translated for its mathematical part[99]. SYAB and APG have been edited but not translated[100]. Part of PAB's astronomical commentary has been studied, edited and translated by Setsuro Ikeyama[101], and a portion of SAB was edited with BAB[102] . The (P)BSS is the only non-extently edited treatise containing a mathematical part for this period. Pṛthudakṣvamin's commentary on the XIIth chapter of the BSS, is found in a manuscript at the Indian Office and in what appears as a copy of this manuscript used by S. Dvivedi in Benares[103].

[97]See [48], [15], [18], [46], [38]. There has been many printed editions of L and BG two texts, which are noted in CESS 4 308 a and 311 b; we can note the translation given in [6].

[98]The T was edited in [17]. The GT was edited with a later commentary [20]. The *Mahāsiddhānta* was edited by Sudhākara Dvivedin in 1910 ([16]), and partially translated into English by S. R. Sarma, [42].

[99]See [47], [24].

[100][41], [46].

[101][19].

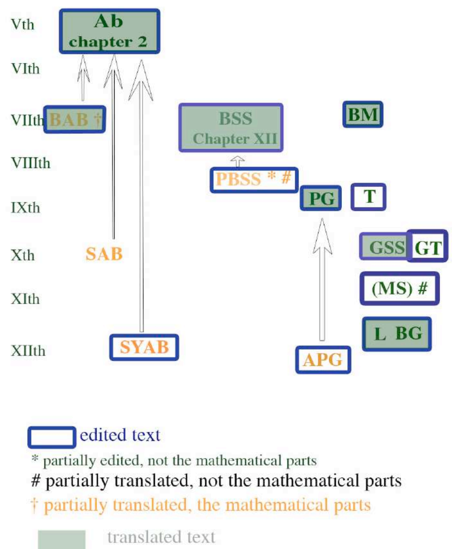
[102][47].

[103]CESS A. IV. 221 b, [19, S5-S7].

Table 3: A list of known texts on mathematics, in Sanskrit, Vth-XIIth century

Dates (A.D.)	Author	Title	Abbreviation
499	Āryabhaṭa	*Āryabhaṭīya* (Chapter 2)	Ab
628	Brahmagupta	*Brahmasputaṭasiddhānta* (Chapter XII)	BSS
629	Bhāskara	*Āryabhaṭīyabhāṣya* (Chapter 2)	BAB
VIIth-Xth century	unknown	*Bhakshālī* manuscript	BM
ca 864	Pṛthudakśvamin	*Vāsanābhāṣya* on the BSS (verses of Chapter XII)	PBSS
850-950	Śrīdhara	*Pāṭīgaṇita*	PG
idem	idem	*Triśatika*[94]	T
Xth century	Mahāvīra	*Gaṇitasārasaṃgraha*	GSS
c. 1039 A.D.	Śrīpati	*Gaṇitatilaka*	GT
ca. 1040	Someśvara	on the *Āryabhaṭīya* (chapter 2)	SAB
ca. 1150	Bhāskara II	*Līlāvatī*	L
idem	idem	*Bijagaṇita*	BG
[IXth-XIIth century	Āryabhaṭa II	*Mahāsiddhānta* (Chapters XV and XVIII)	MS][95]
ca. 1200	Sūryadeva Yajvan	on the *Āryabhaṭīya* (Chapter 2)	SYAB
[Uknown[96]	Unknown	commentary on the *Pāṭīgaṇita*	APG]

Figure 7: Edited and translated texts on mathematics, Vth-XIIth century



It has not been edited entirely, especially concerning the mathematical part, probably because the one recension is at times quite difficult to understand. The situation of these texts concerning editions and translations is given in Table **??**, and Figure 7.

Abbreviation	Treatise, Commentary, Others	Edited	Translated
Ab	Versified treatise	Yes	Yes
BSS	Versified treatise	Partly	Partly
BAB	Prose commentary	Yes	Partly
BM	Fragmentary text (versified rules and examples; prose resolutions)	Yes	Yes
PBSS	commentary	Partly	Partly
PG	Versified treatise	Yes	Yes
T	Treatise	Yes	No
GSS	Treatise	Yes	Yes
GT	Treatise	Yes	Partly (not the mathematical parts)
AB	Commentary	Yes	No
L	Treatise	Yes	Yes
BG	Treatise	Yes	Yes
[IMS	Treatise	Yes	Partly][104]
SYAB	Commentary	Yes	No
[APG	Commentary	Yes	No]

Table 4: A list of the editorial situation of texts on mathematics, in Sanskrit, Vth-XIIth century

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