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Physical quantum states and the meaning of probability*

Michel Paty

Ëquipe REHSEIS (UMR 7596), CNRS & Université Paris 7-Denis Diderot,
37 rue Jacob, F-75006 Paris, France.
E-mail : paty@paris7.jussieu.fr

Abstract.

We investigate epistemologically the meaning of probability as implied in quantum physics in connection with a proposed *direct interpretation* of the state function and of the related quantum theoretical quantities in terms of *physical systems* having *physical properties*, through an extension of meaning of the notion of physical quantity to complex mathematical expressions not reductible to simple numerical values. We show how the changes occurred in the implication of probabilities in quantum physics, from a penetration tool (probability as statistics and frequencies of occurencies of events) to a theoretical concept (probability given by a physically aimed at «amplitude of probability») actually make this view a somewhat «natural» one.

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1. Introduction

It is intended, in what follows, to perform an epistemological analysis of probability statements in quantum physics, in the perspective of an *extension of meaning* of the *notions of physical state* and of *physical quantity*, restricted up to now to numerically valued forms, to more complex mathematical expressions of the type that are used in the quantum theoretical formalism, i.e., state functions as coherent superposition of basis (eigen)state vectors of a Hilbert space and, for quantities or magnitudes, matrix or linear hermitian operators acting on the state functions.

In the usual interpretation of the «quantum formalism», these forms are considered as purely mathematical, their physical meaning being considered as given from *interpretation rules* relative to *measurements* performed on «quantum systems» with apparatuses obeying the laws of classical physics. The proposed extension for the concept of physical state and quantity would allow to claim that, contrary to the received interpretation of quantum mechanics, the *state function represents directy the physical system* in the considered state (instead of being viewed as a «catalogue of our knowledge of the system»), and that *the theoretical quantum quantities* (the «observables», as they are currently called) *represent physical magnitudes*, the dynamical variables, that *are properties of the system*. Such a view would considerably simplify the «interpretation problems» of quantum physics, for one could henceforth speak, for this domain, not only of phenomena related with our observation of them but, in the same way as in the other areas of physics, of physical systems having properties, i.e. *objects*, standing independently of our knowledge of them and that are fully described by quantum theory.

We have argumented elsewhere about various aspects of this unorthodox point of view, concerning the physical as well as the philosophical aspects of interpretation, the first one including the question of the physical meaning of theoretical (mathematically expressed) quantities, and also the problem of the quantum to classical relationships[1]. Such a *direct physical interpretation of the quantum variables* seems actually to correspond to the implicit conception of the quantum physicists at work, not only at the theoretical level, but also when considering the physical implications of it as manifested through experiments. This implicit conception, in our view, can rightly be made explicit, being justified with sound arguments, considering the

[1] Paty [1999, 2000a, b , forthcoming, b].

quantum phenomena in their variety, from the «simplest» ones (such as quantum interferences) up to the most elaborated ones (taken in subatomic physics and quantum theory of gauge fields). It might therefore appear nowadays as the most natural interpretation of the quantum formalism.

We shall concentrate here on the consistency of this «direct interpretation» conception with our understanding of probabilities in quantum physics. As a matter of fact, a questioning of the peculiar character of probabilities at work in quantum physics (through the concept of «probability amplitude») leads, as we shall see, to clarifications and distinctions concerning probability and statistics, theoretical (quantum) quantities and (classical) measured ones ; such clarifications call to the forefront a fundamental property attached originally to the notion of *physical quantity*, that of being *relational*. These clarifications are fully consistent with the proposed direct interpretation for the state function and the quantum theoretical variables (as operators) in terms of a *theoretical description* (or representation) *of physical states and of their physical properties*.

We shall sketch briefly how the concept of probability allowed to conceptually penetrate the world of *atomic* (and henceforth *quantum*) *physics*, by concomitantly undergoing changes of meaning and afterward of role. From a pure mathematical function used in auxiliary reasoning, it became «physically interpreted» as frequency for occurrence of events, and turned in this way into a fecund heuristic tool that helped revealing specific features of quantum phenomena and systems. It happened further, in the course of the evolution of ideas in quantum physics, to structure progressively the physicists' comprehension of the peculiarity of the new quantum domain, and at the same time it was merged into the problems and ambiguities of the «interpretation» of quantum mechanics, being tightly connected with the «measurement problem», as well as with the quantum to classical relationship.

It is this problem of interpretation that we have in mind, and we shall try, from our analysis, to clarify the exact *meaning of probability statements* in quantum physics. Meaning, or meanings ?… For, we shall establish conceptual distinctions between probability in the theory and probability from results of experiment : two different meanings of probability as implied in quantum physics, respectively a *quantum* and a *classical* one, a *relational* and a *statistical* one, that the «probability interpretation» taken together with the «measurement rule» had merged into one another, which had led to some confusion.

2. Probability as a tool to explore quantum phenomena

and the classical physical meaning

Probabilities play in quantum physics a much more fundamental and deeper role than in classical physics. Actually, probabilities entered physics in a quite classical way, that is to say, in the ancillary position of a mere pragmatical means, and such was still their status when they happened to be taken as a powerful and indispensable tool in the first physical investigations of the microphysical (atomic) world[2]. But afterwards, when this atomic world revealed itself qualified in a decidedly non-classical way as a world of *quantum phenomena* and supposedly of *quantum entities* responsible for these phenomena, probability began showing more than indispensable : it entered the very foundations of the description of this world. Let us recall the gross features of this status transformation of probability when its use was moved from classical to quantum physics, which one could summarize as : *from distributions to structuration*. Such a change would correspond to an underlying modification in our understanding of the basic notions used in physics, such as that of quantity : we shall retain, as a great epistemological lesson of these developments, the move of our notion of *physical quantities*, *from measurement to relationship*. Further on we shall make these statements more explicit .

For the atomic domain, which escaped the possibility of direct observation, statistical mechanics happened to be, after chemical analysis, the most natural tool to explore it, in a conceptual continuation of statistical mechanics along Ludwig Boltzmann's path, as Max Planck did with the study of radiation emitted and absorbed by atomic matter in a «black-body» (heated closed cavity at thermal equilibrium). The indirect exploration thus opened would reveal for the new invisible area, notwithstanding the methodological continuity, characteristics escaping any reduction to the usual conceptual schemes of «classical» physics. As a matter of fact, Planck had to perform empirically some modifications to the usual mode of counting complexions *à la* Boltzmann for elements of radiation energy, which led him to discover the discontinuity of energy exchanges between atomic matter and radiation[3].

The first moment of the introduction of probability in what would become quantum physics, was that of the mathematical treatment by Planck of

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[2] On statistical mechanics, see Boltzmann [1896-1898], Gibbs [1903]..

[3] In his one hundred years ago celebrated publication, Planck [1900], although he would try for many years to restore some fundamental continuity. Planck's energy relation is $\Delta E = nh\nu$ (ΔE : radiation energy absorbed or emitted, ν : radiation frequency, h : Planck's constant of action, n : integer number).

the energy distribution of radiation in analogy with statistical mechanics : Planck adapted to his problem the method of statistical physics developed by Boltzmann for atomic gases : he transferred a procedure adequate to a discontinuous sample (an atomic gas distributed in a volume of phase space) to the case of a continuous one (that of radiation energy distribution). Clearly, the probability considered in both cases had not the same meaning from the physical point of view. With Boltzmann's statistical physics of gases, counting particles and states leads to a combinatorics of numbers of particles for given volume elements of phase space, giving probabilities from integer values that had somehow a physical significance. Planck was well aware that the probability distributions he attained at for radiation in a similar way had not the same physical meaning, and considered the calculated probabilities in his problem as purely mathematical intermediaries in the reasoning. His theoretical treatment of radiation, although fruitful with regard to the result obtained, looked artificial and had no justification if not only a pragmatical one. The procedure used to obtain it was rather obscure[4].

Some time later, Einstein radicalized the idea of the discontinuity of energy, making of it and of Planck's quantum of action a general property of radiation[5] as well as of atomic matter[6] themselves, and not only of their mutual exchanges, arriving soon at the conclusion that this property was irreductible to classical physics and implied fundamental modifications in both the electromagnetic theory and the mechanics of bodies at atomic level. To get at this conviction (as soon as 1906), he made use of a physical reasoning he had set up (already in 1903) about the physical meaning that was to be given to the probability implied in the theoretical calculations. The probability involved in the combinatory of complexions of energy cells was purely mathematical, like counting balls in urns, which would look rather improper if it were used physically for radiation, considered as continuous according to the electromagnetic theory (as already emphasized above).

For physical phenomena and quantities, Einstein thought necessary to *reinterpret physically* the mathematical probability : he gave to that one present in Boltzmann's entropy formula (which he would use to call «Boltzmann's principle»[7]), the meaning of a frequency in time (for a system to

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4 See Jammer [1966], Kuhn [1978], Klein [1962], Kastler [1983].

5 First in his 1905 paper «On a heuristic point of view …» (Einstein [1905]), and in a 1906 one where fluctuation calculations around statistical mean values led him to diagnose in radiation a combination of wave interference and corpuscular contributions, Einstein [1906].

6 Einstein published his first paper on specific heats in 1907 (Einstein [1907]). See also Einstein [1912].

7 The Boltzmann's formula is : $S = kLogW$ (S : entropy, W : probability of the state, k :

come to the same physical state). Frequency in time is a physical quantity that can be empirically determined, with a mean value and fluctuations around the mean value[8]. Such fluctuations were not considered with pure combinatoric calculations although they effectively characterize the distributions of quantities relative to physical phenomena.

From this *physical reinterpretation* of a *mathematical quantity* implied by idealized processes of combinatorics, calculating fluctuations became Einstein's favoured exploration instrument for the atomic and quantum domain, which led him to his major contributions in this field from 1905 to 1925 : evidence for molecular motion, light quanta as shown in particular in the photoelectric effect, both in 1905[9] ; some dual wave-corpuscle aspect of radiation in energy distributions, from 1906 to 1909 ; extension of Planck's quantum of action to the atomic structure itself and specific heats, in 1907 up to 1911 ; first synthetic theory of quanta with evidence that light quanta carry momentum, in 1916-1917 ; statistical specific properties of radiation and similarities with monoatomic gases, i.e. Bose-Einstein statistics for indistinguishable quantum particles, in 1924-1925[10], related with the generalization of the wave-particle duality for any elementary material system as formulated by Louis de Broglie in 1923[11].

Calculating fluctuations in the statistical distributions of significant dynamical quantities was a powerful tool to getting a knowledge of characteristics of quantum phenomena. As a matter of fact, the properties of the quantum domain have been mostly expressed in a statistical-probabilistic way. For instance, the first indices for some kind of particle-wave dualism obtained as soon as in 1906 had the form of a juxtaposition, in an energy fluctuation formula, of a statistical mechanics type term and of an interference one.

Fluctuations, that were the mark of a physical meaning for probability distrinutions, revealed actually in Einstein's hands properties of the new radiation and atomic domain of phenomena (the quantum domain) that were decidedly not classical ones. Among such properties were the energy discontinuity of radiation and of atomic levels, dual wave-particle aspects for light, and quantum statistical behaviour (for bosons, revealed also for fermions with Dirac) referred to the indistiguishability of identical quantum systems. All

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Boltzmann's constant.. See Boltzmann [1896-1898].

8 Einstein [1903].

9 The light quantum is characterized by its energy (E)-frequency (ν) relation : $E = h\upsilon$.

10 Respectively, Einstein [1905a and b ; 1906, 1909 ; 1907, 1911; 1916-1917] ; 1924, 1925]. See Kuhn [1978], Darrigol [1988, 1991], Paty [forthcoming, a].

11 See his thesis : de Broglie [1924].

three appeared to be fundamental properties of the quantum systems, interconnected with each other. The latter, in particular, entailed powerful consequences that have all been verified afterwards (explanation of the periodical table of the chemical elements, constitution of degenerate stars, Bose-Einstein condensation of many identical atoms falling into the same «zero enegy point» ground state, …)[12].

It must be noted, incidentally, that probability as statistics entered also early in quantum physics from another, independent, way : the law of radioactive decays formulated in 1903 by Ernest Rutherford and Frederic Soddy, expressing a constant rate of desintegrations in time, which corresponds to the independence of successive events in radioactivity[13]. This law was extended to atomic transtions, from Bohr's and Sommerfeld's 1913-1916 atomic model to Einstein's first (semi-classical) quantum theory of 1916[14].

The consideration, in Einstein's 1916-1917 quantum papers, that the light energy quantum has a momentum[15], which entailed its full particle property, came out from the condition of equilibrium of a statistical ensemble of quanta of radiation and of atomic states emitting and absorbing this radiation with amplitudes of transitions ruled by statistical laws. The relative frequencies of the states could be obtained from thermodynamical considerations or from «Boltzmann's principle», and the transition probability between two states was expressed in the same way as that of radioactive decay[16]. Einstein obtained from it Bohr's quantum condition for transitions between atomic levels[17] and a derivation of Planck's radiation formula. In this first theory of quanta, yet a semi-classical one, the *amplitudes* of transitions between atomic levels were characteristics of the time probability law of the process, given by statistical

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12 See Paty [1999, forthcoming, b].

13 In Rutherford [1962-1965], vol. 1. See Amaldi [1979].

14 See Bohr 's 1913 aper in Bohr [1972], Einstein [1916-1917]. See Paty [1988 & tforthcoming, a].

15 $p = \frac{h}{\lambda}$ (p : impulsion ; λ : wave length). Einstein [1916-1917].

16 $dW = A_m^n dt$. The transition amplitude A_m^n has the same paper as a radioactive constant, characteristic of a given radioactive substance.

17 Bohr's condition is : $\left|E_m - E_n\right| = h\nu_{mn}$ (E_m and E_n : energies of atomic levels m and n ; ν_{mn} : frequency of the radiation emitted or absorbed between these levels).

distributions. (At this time they were given from experimental data. They would be calculable theoretically only with quantum field theory, shortly after the quantum mechanics formulation was obtained[18]).

The evidence for attributing radiation (defined by its frequency and wave length) both an energy and a momentum (property of a particle), seemed to show that atomic processes were defined at the level of individuals. But, as Einstein observed in his conclusion, by which he considered that a «proper quantum theory of radiation appear[s] almost unavoidable», such a full theoretical understanding was not yet achieved : «The weakness of the theory lies on the one hand in the fact that it does not get us any closer to making the connection with a wave theory ; on the other, that it leaves the duration and direction of the elementary process to «chance»»[19].

Refering probability to the law of chance was expressing its classical nature (with its «subjective», laplacean, interpretation[20]). As a matter of fact, the use of probability in quantum physics up to then had been able to shed light «from outside», so to speak (i.e., from the concepts for classical phenomena) on genuine characteristics of quantum phenomena, irreductible to classical ones. Among such characteristics were the energy discontinuity of radiation and of atomic levels, dual wave-particle aspects for light and matter, and quantum statistical behaviour refered to the indistiguishability of identical quantum systems. All three appeared to be fundamental properties of the quantum systems, interconnected with each other. To help going further into the quantum domain, probability would have to uncover a radical change in its function and meaning, as the problem at stake was to formulate a *proper quantum theory*, as Einstein said. This would not be unthinkable, since one would always have the possibility to afford probability, which as such is a purely mathematical concept, a different meaning for its use in physics[21]. The identification of a probability with a frequency was a choice justified in classical physics. Quantum physics may lead to give privilege to another kind of physical interpretation. Although the problem was not put in these terms at the time, we point out this alternative as a possible reading of quantum mechanics as a physical theory.

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18 See, for instance, Born and Heisenberg [1928].

19 Einstein [1916-1917], engl. transl.., Waerden, p. 76.

20 Laplace [1814].

21 On the mathematical theory of probability, see Kolmogorov [1933]. For discussions about the meaning attached to probability in physics, see, for instance, Popper [1957, 1982, 1990], Bunge [1985],.

3. Probability for individuals. Changes in the meaning

One of the major problems of the physics of quanta was set from that time onward : how to reconcile in a «proper quantum theory» a «probabilistic determination» of the properties of physical systems with the conception that these systems are made of individual entities ?

The Bohr, Kramer and Slater's episode might be viewed here as a symptom of this problem some time before quantum mechanics was formulated. Compton's experiment had confirmed by observation, in 1923, the momentum of quantum radiation (theoretically derived by Einstein), i.e. its corpuscular character, from a collision process such as the extraction of a atomic electrons by X-rays of a given wave-length and direction impinging on it (Compton effect)[22]. It was just a matter of writing the momentum-energy balance in the reaction for particles, and comparing with the values obtained from the detection of the emitted electron and of the scattered photon. However, Niels Bohr, Hendrik Kramer and John Slater put doubt on the conclusion, in an attempt to maintain continuous energy exchanges inside the atom that they wanted to conciliate with discontinuity in quantum phenomena[23]. They argued that momentum-energy conservation might not hold at the atomic level for individuals but might be only statistically verified. The experiment performed in 1925 by Geiger and Bothe, setting evidence for an individual correlation between the electron and the X-ray photon simultaneously emitted was an unambiguous proof of the particle property of radiation[24]. It was at the same time a proof of the «individual reality» of light quanta, as Einstein stated in his own exposition of the result[25].

Significantly enough, it was this individual physical reality of quantum systems that became thenafter Einstein's main concern about quantum mechanics[26]. How far was this theory able to describe individual physical systems ? The answer was not a priori obvious, neither for the opponents to the standard Copenhagen interpretation such as Einstein, nor for its proponents, such as Bohr or Born, and others. The fact that probabilities had an important paper did not forbid the possibility of getting at some description, even indirect,

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22 Compton [1923]. The experiment was done with low atomic number elements.

23 Bohr, Kramer and Slater [1924]. On this thory, the Compton scattering of the electron would be continuous proces in which all atomic electrons of the scattering body take part, only emitted electrons beiing individuals and being submtted to statistical laws.

24 Geiger and Bothe [1924, 1925].

25 Einstein [1926]. See also Einstein'ss correspondence of the time with Langevin, and others, mentioned in Paty [forthcoming, a].

26 For his further inquiries, and in partivular the EPR argument, see Paty [forthcoming, a].

of individual quantum processes and systems. The problem with quantum physics was that such individual systems were exhibiting unusual properties such as the wave-particle dualism and types of correlations showing up in scattering, in interferences, in «quantum statistical behaviour» and, finally, in non-local separability of subsystems, all of them appearing to have something to do with a non-local character.

A decisive aspect in these circumstances was qualified when it was realized, in 1925-1926, that the quantum statistical behaviour of radiation (that of symmetric indistinguishable entities, bosons, with respect to their mutual exchange) was the deep reason that justified Planck's unusual counting of radiation energy distributions by substituting combinations to permutations, obtaining as a consequence discontinuous energy exchanges. Planck had not been aware of it ; he did it actually with a purely pragmatic purpose, that of recovering the observed spectrum of frequencies for radiation in the black body. This heterodox way of counting had been noticed and analyzed by Ladislaw Natanson and by Paul and Tania Ehrenfest already in 1911-1912[27], but it had to wait twelve more years to be fully acknowledged. This deep origin (which remained hidden underground for a long time) makes us aware that the quantum statistical or probabilistic dependence was, so to speak, co-natural to the quantum of action which is the mark of the discontinuity of energy and, more generally, of the quantum specificity.

And then goes on the quantum physics story (and history). Based on all the quantum properties known by then, whose deep root, or «essential feature», can be identified as *quantum statistics* or indistinguishability of quantum systems, quantum mechanics was built, with probability still being of help (although not reinterpreted), as a powerful tool, in the following sense : it led to theoretical prediction of phenomena that were confirmed by experiments, in statistical distributions. The theory, quantum mechanics, took the form of a mathematical-theoretical machinery aimed at the description (by then thought as an indirect one) of the states of a physical system, based on mathematically expressed quantities such as the *state function* (vector of a Hilbert space, denoted ψ) ruled by the (generally considered as «formal») *superposition principle* (by which, ψ actually appears as a *phase coherent* vector superposition of basis eigenstates), and the theoretical quantities, or quantum variables, usually called «observables», were represented by linear hermitian non-commuting operators (a «formal» property again)[28].

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27 Natanson [1911], P. and T. Ehrenfest. [1911]. See Kastler [1983], Darrigol [1988, 1991].

28 See, in particular, Dirac [1930], Neumann [1932]. For an historical recollection, see Jammer [1966, 1974].

At this stage, the theoretical structure was considered to be a mathematical one, whose (mathematical) entities had the function of *bearing the relationships* that were characteristic of the physical quantum phenomena and systems. But they were not considered by physicists, at least when trying to elucidate their relations to physical contents, as *physical quantities*, for the reasons many times discussed in the quantum debate about interpretation[29]. Physical contents were thought to come only from experiments, performed with macroscopic devices, whose results were stated in terms of classical quantities statistically distributed. These classical quantities showed being submitted to restrictions (or to «conditions of use») such as Heisenberg inequalies, which transcribe their being related to quantum systems.

4. Physical meanings : from measurement to relationship

By having been able to reveal, from a classical approach, unclassical features, probability underwent a change in its function and possibly in its nature, with respect to physics and to physical theory. And physical theory is generally made of physical quantities. Even if the nature of such quantities for quantum physics has remained for a long time unclear, the evolution just mentioned might be evaluated with regard to something of physical quantities. Let us state that this evolution has gone along not so much from classical measurements to quantum measurements, as it is usually presented, but *from a thought of measurement* (whatever the meaning of «measurement» was thought to be) to a thought about *relationships*. And, to introduce already what I have in mind and shall emphasize afterwards, this is precisely why probability got more importance in quantum than in classical physics, for wheras relationship is *internal* to the descriptive theoretical and conceptual scheme, measurement is only *external* to it.

Relationing (relationing quantities one to the others) corresponds to the essential *function* of the *quantities* that express physical concepts. In classical physics, the relationships of quantities have taken the form of causality in the «newtonian» sense of the differential dynamical law. The causality law entailed the requirement of precision in theoretical as well as in experimental determination, and this is how probability entered classical physics, by two ways. By the way of *measurement* of physical quantities, with the theory of errors, which was the means to counterbalance the *lack of experimental precision*, on the one hand. And, on the other hand, by the way of *observation* of quantities that were assigned to be the average of other ones, in order to counterbalance *uncomplete knowledge* as in statistical mechanics. With these

29 See, for instance Bohr [1957], Einstein and Born [1969].

two statistical-probability «repairs», the ideal of causality was, so to speak, recovered (in the vein of Laplace's philosophy of probability[30]).

With quantum mechanics, the causality scheme concerned the «formal» quantities related with the physical system, but not the directly «observable» ones, thought by then to be the only physical ones, which were considered only statistically. But the causal quantities, by being considered as «formal», were cleaned out of any physical content, because their (formal) properties appeared contrary to what was usually understood as physical properties, which ought to be expressed (so one thought) through numerically valued variables and functions. If the quantum theoretical quantities were only *mathematical*, the *physical content* of the theory was thought as being given through the *interpretation rules* relating the theoretical formalism and the observational data.

Considering quantum theory in this manner would actually be to make of it a phantom relating two disconnected orders of things : the formal and mathematical one and the empirical data given in experiments. Strictly speaking, physical theory would reduce to the quantum interpretation rules that connect, in a purely conventional way, the mathematical or formal and the empirical. Up to that circumstance, physical theory had used to be considered as a theoretical structure of concepts having physical content. One may ask whether separating in two distinct moments and functions *probability statements* and *measurement* would not help to recover some scheme of this kind for the physical theory of the quantum domain.

5. Probability interpretation and the measurement rule. How to escape the Procuste's bed ?

Consider these two rules of the quantum formalism : the *probabilistic or statistical interpretation* of the state function or state vector, and the *measurement or reduction rule* of the state function to one of its components in the observation process. These two rules have been tightly connected in the formulation of quantum mechanics as an axiomatic theory[31], due to the fact that measurements yield statistical distributions for the various states of the system and for the corresponding values of the compatible variables[32] characterizing these states. In effect, the measurement interpretation

30 Laplace [1814].

31 In particular, von Neumann [1932], Dirac [1930].

32 i.e. whose operators mutually commute.

rule had been formulated in such a way as to seal quantum mechanics as a closed system, as if it were a *principle* for quantum physics, when it is merely a pragmatic statement, a recipe for use, imposed by the necessity in which we are to get informations on quantum physical systems through classical observation and measurement devices. This Procuste's bed condition can in no case be invoked as defining quantum systems, since these systems cannot be reduced to classical properties. It is obviouly a human (macroscopic) observer's limitation with respect to the entities, whatever they are, of the quantum domain of physical reality.

We might actually consider the two statements separately, and the first one (the probabilistic interpretation) had indeed been formulated independently from the second, and previously to it by Max Born[33]. Some distance must be taken from the historical circumstances of the edification of quantum mechanics, when the new pieces of the quantum puzzle seemed to organize themselves in a so incredibly consistent and powerful way as to inspire its inventors the conviction that the theory was already (in 1927) complete and definitive[34]. Now that we have no doubt that quantum physics is a sound piece of knowledge about a large part of world phenomena, we may allow ourselves to loosen the elements of the logical construction and think afresh the meanings of these statements. Actually, the two rules are by no means tied together in essence. Let us first consider separately the meaning of the first one, up to the point when we shall need the second.

The probabilistic interpretation of the state function is obviously one of the main foundational statements of quantum mechanics as a physical theory : it defines the correspondence between a chosen mathematical quantity and a physical concept. Or, in other words, it defines a physical quantity with the help of a mathematical expression, and this is, indeed, how physical theory usually proceeds. In this respect, we may consider that such a quantum physical construction of concept does not differ from what one has been used to do in classical physics (including relativity theory), a process that is responsible for the «success of mathematics in physics», which is the counterpart of the mathematization of this science, entangled with its conceptual edification.

If we take this path, we meet the question of the *physical meaning* or *content* of a *state probability* in quantum physics. Actually, probability is only one of the steps of the interpretation, so to speak the last (or the synthetical) one and, significantly enough, physicists have been led to identify, as conceptually previous to it, an *amplitude*, which they called *amplitude of*

33 Born [1926].

34 Born and Heisenberg [1928].

probability. Although such an expression sounds somewhat non realistic (a probability could hardly be some kind of physical object propagating through space), it expressed at the same time a *necessity* and an *impossibility*. The impossibility was clearly realized as soon as Schrödinger's equation had been established[35]. The «wave equation» adequate to the description of a quantum system (for instance, an hydrogen atom) is about something else than a physical wave, whose propagation in space would not hold the quantum properties of the considered system, which would be spread away[36]. Nevertheless, if there is no wave for the wave equation, there is *something* having properties *mathematically* (or *formally*) *analogous to the amplitude of a wave*, and this is the *state function* itself. Entered mathematically in the equation for the physical system obtained through an hamiltonian formalism, the state function must be given back a physical content from the successful application of the mathematical formalism to the physical properties of the system.

The physical content of the state function and of the related dynamical quantities expressed mathematically lies in the relationships they ascribe to the corresponding physical quantities that were considered as such from the start (those being measurable), i.e. the basis state functions as the solutions of the state equation (eigenfunctions), and the eigenvalues of the «observables» as operators. In particular, the overall («mathematical») state function works, in this respect, in a fashion similar to that of an *amplitude* of wave in wave theory : in particular, as a coherent linear superposition of basis states functions, it entails interferences between these states, that are indeed observed. The mathematical overall state function (ψ) can therefore be itself interpreted as a physical *amplitude*, in the sense of giving rise to interference phenomena whose intensity is given by the squared modulus of it. It thus is the *amplitude of something*, but *of what*, if not of a wave ? But although suggestive, the wave analogy is restrictive, and the state function of a quantum system is actually much richer of physical content than that of a mere wave amplitude, as it holds all the specific (non classical) properties of quantum systems, such as, for instance, interference of a single quantum system with itself, non local separability of correlated sub-states, Bose-Einstein condensation for identical bosons, etc.)[37].

We have obtained, at this stage, an important conceptual result about the physical meaning of the «mathematical» quantum variables or dynamical quantities : they express *relations* that are characteristic of quantum

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35 Schrödinger [1926].

36 I refer to the debate that took place shortly after Schrödinger's derivtion of his equation, in 1926 (Schrödinger [1926]. And see, in particular, Jammer [1974], Paty [1993b].

37 Paty [forthcoming, b].

phenomena and are revealed in observation and measurement. These ascertained relations hold on *quantities prepared for measurement* with classical devices and which we can consider as the classical projections of «quantum quantities» characteristic of the quantum systems. Such *relations* are actually *deduced from statistical results* obtained for these classical quantities which, taken together, carry in a way or another the specific quantum character of the intitial system before its measurement. Conceptually and theoretically one has to reconstitute this quantum characteristics that shows in the relationship between the classical quantities projected from the quantum ones, and that is given to us from their statistical distributions.

6. Theoretical and empirical probabilities

Clearly, what precedes suggests the need, at this stage, of conceptual distinctions between two different effective uses of probability in quantum physics : on one side, a *theoretical* definition of *probability* in the quantum description of physical systems, even of *individual* systems ; on the other side, an *empirical* acception, where it refers to the statistical results of experiments. In the first sense, probability properly speaking, expressed as a mathematical function, is afforded a *theoretical physical meaning*, enrooted in the specific, physically elaborated, concept of «probability amplitude» (whose denomination, historically determined, remains somewhat ambiguous) ; it is *quantum theoretical and relational*. In the second sense, it is given a purely statistical meaning in the same way as when it is used in classical physics, in statistical mechanics for instance : it has no more a theoretical (and quantum) function, but a practical one, that of expressing results of experiments and of measurements, performed on quantum systems, in terms of classical quantities[38].

«Probability amplitude» is a bizarre expression for a concept or a quantity in physics. This oddness may have been the signature of its impossibility to be physically thought in a direct way. Made on the mold of «wave amplitude», which it coud not be, it does not either correspond semantically to something analogous : «probability» being the squared modulus

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38 This distinction between probability and statistics in the quantum context has been emphasized by Mario Bunge (in part.icular, Bunge [1985], see Paty [1990]). The propension conception of Karl Popper (Popper [1957, 1982, 1990]) would, in some respect, be alike our «theoretical» one, but it remains vague, because it does not make a clear distinction with the measured one. Among the analyses and possible interpretations of probability in quantum physics, l would like to mention also (non exhaustively) : Reichenbach [1944, 1978], Suppes [1961, 1963, 1970], Schushurin [1977], Mugur-Scächter [1977]. About probability in the strict Copenhagen sense, see, for instance Rosenfeld [1974].

of amplitude, its analogous for classical waves would be «intensity». But «intensity amplitude» would be taulogical and is not, indeed, in use for waves. The exact analogous to «wave amplitude» woud be «quantum state amplitude», which is effectively used also by to-day quantum physicists. As for them, the «founding fathers» of quantum mechanics preferred to speak in this sense of «state function» or «state vector», which they thought as mathematical quantities, and not as physical ones. As an effect, they forged this queer expression, «probability amplitude», when they realized that the solution of a wave equation for a quantum system could not be the amplitude of a wave, as recalled before.

Although of an obscure meaning literally speaking, the expression «probability amplitude» used to designate the state function must be given the credit of referring it, even if reluctantly, to something physical (an amplitude) and at the same time to its correspondents on the side of usual (classical) quantities (probability related to statistics). Note that the word «probability» gets a significant position here, as it could not be substituted by «statistics» (for, what would be the meaning of an expression such as «statistics amplitude» ? it would be not only queer, but nonsense), notwithstanding the lack of precision already diagnosed among the founding fathers concerning probability and statistics. «Probability amplitude» is indeed a quaint and at the same time a significantly penetrant concept as, by juxtaposing two terms so much foreigner one to the other, it gives (quantum) probability a *physical content* and provides a mathematical (state) function with a *precise theoretical meaning* related through a straightforward correspondence with empirically determinable quantities. Such was the insight, may we think from our proposed point of view of a «direct interpretation», but it was by then inhibited by the compelling orthodoxy… Even scientific terminology is affected by historical contingency. Let us keep the queer expression, «probability amplitude», as culturally useful, reminding of the uncertain paths of the discovery, of how one has come to know what was unknown.

The theoretical probability (actually, its «amplitude», amplitude of something, whatever it be) is reconstituted from the set of values of measured quantities with their corresponding probabilities. This quantum state amplitude, or state function, is therefore identified as the true *source*, or to be more exact, as the representent (in the theoretical description) of the true *source of the ascertained physical relationships*. As such, this source is the very aim of quantum theory, and it is in right to be called physical : it is the (quantum) state of the system, beyond its (classical) projections. [For the sake of conceptual nuances in the meaning, I feel safer quoting it also in my own native language (in French) : Cette amplitude d'état quantique, ou fonction d'état, doit donc être identifiée comme étant la vraie *source*, ou plus exactement comme le

représentant (dans la description théorique) de la vraie *source des relations physiques* constatées, cette source qui est l'objet même auquel vise la théorie quantique, et qui peut à bon droit être appelée physique : l'état (quantique) du système, par-delà ses projections (classiques).]

The *relational disposition* of the state function (with its related quantum dynamical variables) given by its mathematical form, appears unambiguously as the theoretical counterpart of the physical properties that have been registered, and can therefore be endowed with a straightforward physical meaning : the *state function* may be taken as the very *theoretical expression of its physical content*. By «the very expression», I mean it as the full and most economical one. This means that the «mathematical quantities» (as physicists were used to think of them) of the quantum formalism should be considered as *physical* ones, but with two differences with respect to what is generally understood as *physical quantities* in the classical sense : they are expressed by mathematical quantities more complex than simple numerically valued ones, and they are only indirectly given (by rational reconstitution) through experiments of a classical type (with statistical distributions of classical quantities).

At this stage, a conceptual distinction needs to be emphasized between two different effective uses of probability in quantum physics : on one side, a *theoretical* definition of *probability* in the quantum description of physical systems, even of *individual* systems ; on the other side, an *empirical* acceptation, where it refers to the statistical results of experiments. In the first sense, probability properly speaking, expressed as a mathematical function and used in theoretical calculations, is afforded a *theoretical physical meaning*, enrooted in the specific, physically elaborated, concept of «probability amplitude» (despite the ambiguity of this denomination, historically determined) ; it is *quantum theoretical and relational*. In the second sense, probability is obtained from the statistical results of measurements. These measurements are performed *on quantum systems* but *in terms of classical quantities*. Probability obtained in that way have a purely statistical meaning, refering to the distributions of values of classical states and quantities as, for instance, in classical statistical mechanics ; with respect to quantum systems, it has no theoretical function, but only a practical one.

Through the measurement process of the quantum system, the theoretical, quantum, relational probability is put in correspondence with the empirical, classical, statistical one, the quantum proper theoretical description is confronted to the response of experiment. It is in this way that classical apparatuses of our macrocosm have been opening a window on the microcosm of the quantum world. The clear distinction which we have tried to establish

between a physical theoretical description and the corresponding empirical data, allows with full right to speak of a proper quantum world which can be *thought independently of measurement*, i.e. of our interacton with it. By «independently» it is meant that it stands on its own reality and soudness. As in other fields of knowledge, and particularly as in classical physics, it is known to us through the symbolic and conceptual, theoretical, representations that we make of it, and these representations are fed with the data of experiments.

The conviction that it is possible to conceive the reality of the quantum domain, by affording *a full direct physical meaning* to its theoretical representation in the way just exposed seems therefore to be rather firmly sustained. To get soundnes, two related conditions have been particularly operative : making a distinction between two different physical meanings of probabilities as used in quantum physics, one quantum theoretical (relational and mathematical) and one empirical (probability in a statistical sense), and disconnecting, from a fundamental point of view, the «probabilistic interpretation» rule of quantum theory from the «measurement» or «reduction» one.

Indeed, the *connexion* between these two *probabilities*, the *quantum theoretical and relational one* and the *classical empirical statistical one* might be viewed as the remaining most fundamental *interpretation problem* of quantum physics, intending this time not so much the physical as the philosophical interpretation, because it points directly at the modalities of knowledge. One might refer, up to some extent, such a distinction to the respectives roles of understanding and perception, rational elaboration and data acquisition. Let us content ourselves here by concluding with the simple remark that connecting is not identifying. It seems that, with the case just discussed, connecting opens intelligibility anew, whereas, on the contrary, identifying is limiting and shuts down to obscurity.

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