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Are risk averse agents more optimistic? A Bayesian estimation approach

Selima Ben Mansour*, Elyès Jouini‡, Jean-Michel Marin§, Clotilde Napp* and Christian Robert†

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Abstract

Our aim is to analyze the link between optimism and risk aversion in a subjective expected utility setting and to estimate the average level of optimism when weighted by risk tolerance. This quantity is of particular importance since it characterizes the consensus belief in risk-taking situations with heterogeneous beliefs. Its estimation leads to a nontrivial statistical problem. We start from a large lottery survey (1,536 individuals). We assume that individuals have true unobservable characteristics and that their answers in the survey are noisy realizations of these characteristics. We adopt a Bayesian approach for the statistical analysis of this problem and use an hybrid MCMC approximation method to numerically estimate the distributions of the unobservable characteristics. We obtain that individuals are on average pessimistic and that pessimism and risk tolerance are positively correlated. As a consequence, we conclude that the consensus belief is biased towards pessimism.

Keywords: Bayesian estimation, MCMC scheme, importance sampling, pessimism, risk tolerance, risk aversion, consensus belief.

1 Introduction

Optimism is usually conceived as a personal trait leading to overestimate the likelihood of good outcomes and to underestimate the likelihood of bad outcomes. Previous work related to this issue includes psychological surveys as well as empirical studies on professionals' economic forecasts (see, e.g., Fried and Givoly, 1982, O'Brien, 1988, Francis and Philbrick, 1993, Kang et al., 1994, Dreman and Berry, 1995, Giordani and Söderlind, 2005).

Risk aversion in decision making intuitively corresponds to the fact that when facing choices with comparable returns, agents tend to choose the less risky alternative. This feature has been largely studied in particular in relation with other personal characteristics like gender, age, income, education (see, among others, Binswanger, 1980, 1981, Palsson, 1996, Barsky et al., 1997, Eisenhauer and Halek, 2001, Eisenhauer and Ventura, 2003, Felton et al., 2003, Guiso and Paiella, 2001).

In the setting of a Subjective Expected Utility (SEU) model, risk aversion is measured by the curvature of the utility function and optimism is characterized by the individual subjective probability. Risk aversion and individual subjective probability completely determine the individual decision-making behavior, at least locally. The SEU model is particularly well adapted to the aggregation problem and recent papers show how individual risk aversions and subjective probabilities are aggregated at the collective level in such a setting (Calvet et al. 2002, Gollier, 2007, Jouini and Napp, 2006, 2007, Li, 2006).

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From an empirical point of view, these two notions are hard to disentangle. For instance, in the context of lotteries, which is often used to measure the risk aversion level, the reservation price of a risk averse (resp. risk tolerant) and unbiased individual is empirically indistinguishable from the reservation price of a risk neutral and pessimistic (resp. optimistic) individual; there is here an identification problem that confounds data interpretation.

In this paper, we start from a lottery survey, in which data on both risk aversion (reservation price) and subjective beliefs (subjective probabilities) is available in the same context. In the setting of a SEU model, this survey enables us to disentangle both notions and to infer about the individual levels of optimism and of risk aversion. Our aim is then to determine and to understand the nature of the link between optimism and risk aversion. More precisely, we want to estimate the average level of optimism weighted by the risk tolerance[1] as well as the unweighted average.

The estimation of these quantities is important for both following reasons. First, since the difference between the weighted average and the unweighted average of the individual level of optimism is precisely given by the covariance between risk tolerance and optimism, the estimation of these averages will give us the nature of the correlation between risk tolerance and optimism. The determination of this correlation should shed some light on the determinants of the level of optimism and/or risk aversion at the individual level. Second, the estimation of the weighted average of the individual beliefs is of particular importance at the collective level. Indeed, in the setting of collective risk-taking decisions with heterogeneous beliefs (and SEU), this average subjective belief represents the consensus of the group (see Gollier, 2007). Analogously, in CCAPM[2]-like equilibrium models with heterogeneous beliefs (and SEU), the weighted average of the individual beliefs[3] represents the belief of the representative agent (see Rubinstein, 1976, Detemple and Murthy, 1994, Calvet et al., 2002, Abel, 1989, Zapatero, 1998, Jouini and Napp, 2006, 2007, Li, 2006). This consensus belief plays a crucial role on the Pareto optimal risk sharing rules and on the equilibrium properties. In particular, a pessimistic consensus belief might help in explaining the risk premium and risk free rate puzzles[4].

We analyze answers to a large survey (1536 individuals). The main problem we encounter in this survey is related to the fact that some observations correspond to risk neutral or even risk seeking behaviour. Note that the presence of risk seeking or risk neutral individuals in the panel is not surprising. Similar results have been obtained in the literature (Guiso and Paiella, 2001, Eisenhauer and Ventura, 2003, Hartog et al., 2002). However, these observations are not compatible with the standard assumptions of decision theory and of CCAPM-like equilibrium models nor with the existence of an equilibrium with positive risk-premia since at such an equilibrium, agents are, at least locally, risk averse. Since our aim is to estimate the risk tolerance weighted average of the individual levels of optimism at the equilibrium, these observations require a specific analysis. It is not possible to simply reject these observations, since they are associated with high levels of risk tolerance and their rejection would have a strong impact on the quantities to be estimated. We can not keep them without a specific treatment since, for these individuals, the risk tolerance level is infinite or negative, and with such infinite or negative weights, an average weighted by the risk tolerance becomes meaningless. We propose then to interpret both risk neutrality and risk seeking as noise and we adopt the following approach; we assume that individuals have true unobservable characteristics that respect the assumptions of decision theory, and we interpret the answers in the survey as noisy realizations of those characteristics. This is not an original treatment of the data and it is in the spirit of the Quantal Response Equilibrium model of McKelvey and Palfrey (1992) where it is assumed, in a game theoretic framework, that with some probability, and for reasons that are completely

[1] Risk tolerance is defined as the inverse of risk aversion.

[2] Consumption-based Capital Asset Pricing Model.

[3] When agents are assumed to have log utilities, this corresponds to the wealth weighted average belief.

[4] As shown by Mehra and Prescott (1985) and Weil (1989), standard models of financial markets equilibrium predict risk-premia (resp. risk-free rates) that are too low (resp. too high) to fit observed data.

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orthogonal to the model, a player might choose actions that are not in accordance with the predictions of the model: agents do not select systematically their best-responses, but they play these strategies with a higher frequency. This approach has been shown to be powerful to organize behavior in numerous experimental settings.

The aim is then to estimate the distribution of the unobservable characteristics and to derive the value of the average level of optimism weighted by the risk tolerance. We follow a Bayesian approach for this purpose and use an MCMC approximation method for computational reasons. We emphasize that we do not pretend to compute a precise level of optimism of the consensus belief in a general setting but rather to determine whether there is a clear spread between the weighted and unweighted averages, which would indicate a behavioral correlation between optimism and risk aversion and a pessimistic bias at the collective level.

The paper is organized as follows. Section 2 presents the data as well as the measures of individual optimism and risk aversion. Section 3 presents the statistical estimation. Section 4 concludes.

2 The data

We first describe the survey, then we present the adopted risk aversion and optimism measures. The last subsection deals with data consistency results.

2.1 The survey instrument

We start from answers to a survey conducted face-to-face in the field by professional interviewers. The sample is made of adults, between 22 and 55, that are randomly chosen, and yielded 1,536 responses. The questionnaire consists of essentially five parts[5].

In the first part, the participant is asked about his reluctance to take part in a game of chance for religious grounds. Only those who do not have such interdict go on with the questionnaire.

In the second part, the participant is offered the opportunity of entering a head and tail game for ten rounds. More precisely, a coin is being flipped ten times; each time head occurs, the participant is supposed to get 10 Euros. The participant is then asked for his own estimation of the number of times head will occur. The participant is also asked to reveal the maximum amount he is willing to pay in order to take part in this game. The aim of this second part is to obtain, in the same setting, measures of the individual levels of optimism and of risk aversion.

The third part deals with questions similar to those in Barsky et al. (1997) which lead to the elicitation of the level of individual relative risk aversion. This part will enable us to verify that the data on absolute risk aversion is consistent with the data on relative risk aversion at the individual level. The fourth part deals with optimism questions as in Wenglert and Rosen (2000) while the fifth part deals with personal questions, such as gender, age, marital status, employment status, education and income.

These last three parts will make it possible to compare the results obtained on our sample with those usually obtained in the literature in terms of Risk aversion, optimism and their determinants.

2.2 Risk aversion and optimism measures

We derive individual measures of risk aversion and of optimism from the answers to the survey.

As in Ben Mansour et al. (2006), we derive a measure of the degree of optimism from the answers to the first question of the second part of the questionnaire "Imagine that a coin will be flipped 10 times. Each time, if head, you win 10€. What is your own estimation, according to your experience and your luck, of the number of times head will occur, i.e. how many times

[5]It is available at http://www.ceremade.dauphine.fr/jouini.

(out of ten) do you think you are going to win (and get 10€)?" This measure of optimism[6] is directly given by the number of times x_i that the individual i thinks he is going to win. An optimistic individual is characterized by a value of x_i above the objective value of 5 and the distance to 5 measures the intensity of his optimism.

This measure is obviously compatible with the definition of optimism as a transformation of the objective distribution into a more favorable one. Indeed, the subjective probability of individual i associated to the favourable event "head occurs" is approximated by $\pi_i = \frac{x_i}{10}$ and an optimistic individual is characterized by a π_i strictly higher than 0.5. We will use this measure of individual subjective probability when determining a measure of risk aversion in the framework of Subjective Expected Utility. Notice that the introduction of a ten times head-tail game is intended to provide the individuals with a simple framework, helping them to reveal their subjective probability associated with the event "head occurs".

As far as risk aversion is concerned, we recall that absolute risk aversion is defined by the quantity $A(c) = -\frac{u''(c)}{u'(c)}$, whereas relative risk aversion is defined by $R(c) = -c\frac{u''(c)}{u'(c)}$, where u denotes the agent's utility function. Adopting an approach similar to Guiso and Paiella (2001), we will use the answers to the question "What is the maximum amount that you are willing to pay to take part in the game?" to derive a measure of the individual level of absolute risk aversion. The answer of individual i, denoted by y_i, corresponds to his reservation price, which leaves him indifferent between his initial wealth and the expected gain from the game. This means that if we adopt the SEU model and if we let $\widetilde{X}$ denote the random payoff associated with the head-tail game, the answer y_i satisfies

$$\begin{aligned} u_i(w_i) &= E^i\left[u_i\left(w_i + \widetilde{X} - y_i\right)\right] \\ &= \sum_{k=0}^{10} C_{10}^k \pi_i^k (1-\pi_i)^{10-k} u_i(w_i + 10k - y_i), \end{aligned}$$

where u_i denotes the utility function of agent i, w_i his initial wealth and E^i the expectation operator under his subjective probability. In the case of exponential utility functions[7], a measure for (corrected) absolute risk aversion of individual i is then given by the nonzero solution a_i of

$$1 = \sum_{k=0}^{10} C_{10}^k \pi_i^k (1-\pi_i)^{10-k} \exp a_i (y_i - 10k).$$

It is easy to obtain through a simple factorization that this equation is equivalent to

$$1 = \pi_i \exp a_i \left(\frac{y_i}{10} - 10\right) + (1-\pi_i) \exp a_i \frac{y_i}{10}. \tag{1}$$

We denote by CA_i the nonzero solution, obtained by numerical approximation, of Equation (1). We refer to it as Corrected absolute risk Aversion because we take into account the subjective probability π_i in order to determine the individual level of risk aversion. For example, a pessimist who believes $\pi = 3$ and chooses $y = 10$ has an absolute risk aversion of $a = 0.35$, whereas an optimist who believes $\pi = 3$ and yet chooses $y = 10$ has an absolute risk aversion of $a = 1.2$. This approach is different from the classical approach in experimental or empirical literature on risk aversion in an expected utility framework, where the possibility of subjective beliefs is usually not taken into account. For instance, in Guiso and Paiella (2001), when the individual is told to have a chance of 1 over 10 of winning, one does not take into account, both in the interpretations of the results and in the inferred levels of risk aversion, the fact that some

[6] Ben Mansour et al. (2006) provide a rationale for this measure as well as an analysis of its main features.

[7] The choice of exponential utility functions amounts to assuming constant absolute risk-aversion. It permits numerical computations of the level of absolute risk-aversion without information about the individual level of wealth.

people believe that they are lucky and overweight their probability of winning while others do the opposite. The same remark holds for empirical approaches. For instance, Eisenhauer and Halek (2001) estimate the level of risk aversion using life insurance data. The underlying model assumes an age based estimated probability of mortality, i.e. assumes that all people in the same age class share the same objective estimation of their probability of mortality, whereas it is intuitive that the estimation of one's probability of mortality is subjective and highly dependent upon one's optimism.

2.3 Consistency of the data

In this section we verify the consistency of our data from different points of view.

First, the answers of our sample seem consistent with previous surveys. As far as relative risk aversion is concerned, we find that women are more risk averse (as in Eisenhauer and Halek, 2001, Barsky et al., 1997, Cohn et al., 1975, Palsson, 1996, Jianakoplos and Bernasek, 1998, Hariharan et al., 2000) and that relative risk aversion increases with age (as in McInish, 1982, Morin and Suarez, 1983, Riley and Chow, 1992, Palsson, 1996, Schooley and Worden, 1996, Wang and Hanna, 1997, Barsky et al., 1997).

The answers about optimism in the fourth part are also consistent with previous surveys. We obtain on our sample the same results as Wenglert and Rosen (2000) about personal optimism. Indeed, Wenglert and Rosen (2000) obtain an average level of "personal optimism" equal to 0.596 and we obtain a value of 0.536 (using the same methodology).

The consistency with previous surveys is also confirmed by the data on reservation prices. Let us denote by OA_i the nonzero solution of Equation (1) for $\pi_i = \frac{1}{2}$, i.e. the nonzero solution a_i of

$$1 = \frac{1}{2}\exp a_i\left(\frac{y_i}{10} - 10\right) + \frac{1}{2}\exp a_i\frac{y_i}{10}.$$

In other words, OA corresponds to the absolute risk aversion under the objective probability as it is usually computed in the literature. We obtain that OA is higher for women, increases with age and decreases with income as in Guiso and Paiella (2001), Donkers et al. (2001), Hartog et al (2002), Powell and Ansic (1997), Eisenhauer and Ventura (2003).

A possible drawback of our approach is that we use essentially two simple questions only to elicit individual risk attitudes. In fact, the survey contains various questions that involve risk aversion or optimism. In order to elicit one of these characteristics, the classical procedure would be a regression based on all the questions related to that characteristic. However, our aim is to determine both characteristics in the same context in order to disentangle the two notions and to obtain a measure of risk aversion that is corrected by the level of optimism.

For this reason, we have chosen to determine a measure of the individual levels of optimism and risk aversion from the answers to the head and tail questions only and then to verify their consistency with the answers to other questions in the survey.

We verify that the answers about the reservation price are compatible with those about relative risk aversion. By definition, relative risk aversion is equal to the product of absolute risk aversion by individual wealth. We consider individual income as a proxy for individual wealth. Since the level of relative risk aversion derived from our survey is a qualitative variable (individuals are sorted into seven groups), we establish the consistency between relative and absolute risk aversion measures through a comonotonicity test between relative risk aversion and the product of absolute risk aversion by the mean of the income band (see Table 1).

Finally, as far as the data on the subjective probabilities are concerned, three questions in the questionnaire on personal pessimism have attracted our attention since intuitively, they should have a link with our notion of pessimism : (Q1) You have no chance to win at a lottery game, (Q2) You have no chance to be selected for a television game and (Q3) You will win one day at the Promosport. As expected, we have obtained a decreasing relationship between (Q1)

(resp. Q2) and our measure of optimism and an increasing relationship between (Q3) and our measure of optimism.

3 Statistical estimation

In this section we adopt a Bayesian approach (see, e.g., Robert, 2001) for the estimation of the average of the subjective probabilities weighted by the risk tolerance. We also estimate the unweighted average of the subjective probabilities.

The estimation of these quantities is interesting at the individual level, and can help to understand the nature of the link between risk aversion and optimism. Indeed, the correlation between risk aversion and optimism is easily deduced from the difference between these two quantities.

Moreover, the estimation of the weighted and unweighted averages of the individual beliefs is of particular importance at the collective level. Indeed, in the setting of collective risk-taking decisions with heterogeneous beliefs (and SEU), this average subjective belief represents the consensus of the group. Analogously, in CCAPM-like equilibrium models with heterogeneous beliefs (and SEU), the weighted average of the individual beliefs represents the belief of the representative agent. The determination of the consensus belief, or more precisely of the extent to which it differs from the unweighted average of the subjective beliefs, is then particularly interesting.

We do not pretend here to be able to compute precise and reliable individual and collective levels of risk aversion or of optimism. We are interested in observing whether there is a clear spread between the weighted and unweighted averages, which would indicate a behavioral correlation between optimism and risk aversion and a pessimistic bias at the collective level. In particular, it is well-known that willingness-to-pay questions are biased but this bias should not induce by itself a correlation between optimism and risk aversion.

3.1 Descriptive statistics

The results on the measure of optimism introduced in Section 2 have been studied in Ben Mansour et al. (2006). We recall here the main features of this analysis. Figure 1 provides the distribution of our measure of optimism among individuals and shows that there is a great heterogeneity in the level of optimism among agents. For the considered sample of 1,536 individuals, the mean value for the number of times the individual thinks he is going to win is equal to 3.93. It is significant and quite striking to observe that when asked about the number of times he is going to win at a head and tail game for ten draws, an average individual does not answer 5 times as he should if thinking under the objective probability, but answers slightly less than 4 times. This result is in favor of the existence of a behavioral bias towards pessimism in individual beliefs.

Regarding risk aversion, the average announced price is 11.96€ which, as expected, is lower than the average subjectively expected payoff. However some agents announced a reservation price higher than their subjectively expected gain hence have a negative corrected risk aversion. This can be shown if we focus on the probabilities and prices that we denoted respectively by x_i and y_i and if we represent them on a diagram. Figure 2 contains a jitter[8] representation of this diagram. There are 55 points such that $y/100 = x/10$ and 40 points such that $y/100 > x/10$.

Based on Figure 2, it appears that there is a negative link between non corrected risk aversion and optimism, leading us to conclude that risk aversion (in the usual sense) decreases with optimism. Indeed, the average announced price for a given level of optimism clearly increases with the level of optimism. This correlation is confirmed as statistically significant by

[8] The jitter representation is used to improve the rendering on samples on grids by introducing a slight perturbation that exhibits the number of points at each location on the grid.

Pearson, Kendall and Spearman tests. This is clearly understandable since the more optimistic an individual, the lower his non-corrected risk aversion. In other words, the non-corrected measure of risk aversion contents a great dose of pessimism. This negative correlation does not hold for *corrected* risk aversion. On the contrary, we verified through Spearman's and Kendall's tests that the average of the corrected risk aversion for a given level of optimism significantly increases with the level of optimism (see Table 2). This is in favor of a positive correlation between risk aversion (in our sense) and optimism.

3.2 The Bayesian model

We now want to estimate the average level of optimism weighted by the risk tolerance. The problem is that there is a non negligible number of individuals for which $y/100 = x/10$ (i.e. zero corrected risk aversion and infinite corrected risk tolerance) or even $y/100 > x/10$ (i.e. negative corrected risk aversion and risk tolerance). The presence of such individuals raises some specific problems since with such negative or infinite weights, the weighted sum is not a weighted average any longer. Furthermore, we can not simply eliminate these observations; indeed, the individuals for which $y/100$ is near $x/10$ are those for which the corrected risk tolerance level is high, hence they are heavily weighted in the considered average. Finally, such observations are incompatible with the classical assumptions of decision theory and CCAPM-like equilibrium models.

For all these reasons, a specific treatment of the data needs to be undertaken. One should bear in mind the fact that any slight modification of an observation such that $y/100$ is near $x/10$ has a strong impact on the risk tolerance level and then on the weight in the considered average.

Note that the presence of risk seeking or risk neutral individuals in the panel is not surprising. Similar results have been obtained in the literature (Guiso and Paiella, 2001, Eisenhauer and Ventura, 2003, Hartog et al., 2002). However, our aim is to provide a proxy for the risk aversion at the equilibrium where at least locally, the agents are necessarily risk averse. For this reason, we propose to interpret both risk neutrality and risk seeking as resulting from noise.

More precisely, we adopt the following approach. We assume that individuals have true unobservable characteristics (π_i, p_i), where π_i corresponds to the subjective probability of success at each draw and p_i corresponds to the price that the individual is willing to pay for the ten draw game normalized to one (i.e. divided by 100). The answer (x_i, y_i) is then interpreted as a noisy realization of (π_i, p_i).

More precisely, we assume that $(\mathbf{x}, \mathbf{y}) = ((x_1, y_1), \ldots, (x_n, y_n))$ is a set of independent realizations such that

$$(x_i, y_i) \,|(\pi_i, p_i) \sim B(10, \pi_i) \otimes B(100, p_i),$$

where n is equal to 1536.

We now follow a Bayesian strategy to infer about the parameter (π_i, p_i), first defining a prior distribution on this parameter as

$$f_{(\pi,p)}(\pi_i, p_i) = \frac{\pi_i^{a-1}(1-\pi_i)^{b-1} p_i^{c-1}(1-p_i)^{d-1}(\pi_i - p_i)^e}{g(a,b,c,d,e)} \mathbb{I}_{0<p_i<\pi_i<1} \tag{2}$$

where a, b, c, d, e are positive constants and

$$\begin{aligned} g(a,b,c,d,e) = {} & B(a+c+e, b)B(c, e+1) \\ & + \sum_{i=1}^{\infty} \frac{(-d+1) \times \ldots \times (-d+i)}{i!} B(a+c+e+i, b)B(c+i, e+1). \end{aligned}$$

is the normalizing constant of the probability density (2).

We have then

$$f((x_1,y_1,\pi_1,p_1),\dots,(x_n,y_n,\pi_n,p_n)|a,b,c,d,e) =$$

$$\prod_{i=1}^{n}\Big[C_{10}^{x_i}\pi_i^{x_i}(1-\pi_i)^{10-x_i}\mathbb{I}_{\{0,\dots,10\}}(x_i)C_{100}^{y_i}p_i^{y_i}(1-p_i)^{100-y_i}\mathbb{I}_{\{0,\dots,100\}}(y_i)$$

$$\frac{\pi_i^{a-1}(1-\pi_i)^{b-1}p_i^{c-1}(1-p_i)^{d-1}(\pi_i-p_i)^e}{g(a,b,c,d,e)}\mathbb{I}_{]0,1[}(\pi_i)\mathbb{I}_{]0,\pi_i[}(p_i)\Big]$$

which is a joint distribution on the data $(\mathbf{x},\mathbf{y})$ and on the parameter. The Bayesian estimation is based on the so-called posterior distribution of the parameter, which simply is the conditional distribution of this parameter given the data, $f_{(\pi,p)}(\pi_1,p_1,\dots,\pi_n,p_n|\mathbf{x},\mathbf{y},a,\cdots,e)$.

The sample $(\boldsymbol{\pi},\mathbf{p}) = ((\pi_1,p_1),\dots,(\pi_n,p_n))$ is not observed and, in addition, we first assume that the so-called hyperparameters a,b,c,d,e are also unknown and thus need to be estimated from the data $(\mathbf{x},\mathbf{y})$ (using similar Bayesian principles).

Once (a,b,c,d,e) is estimated, we want to estimate the average subjective probability weighted by the risk tolerance as well as the unweighted average subjective probability. For a given pair (π,p), the associated level of risk aversion $z(\pi,p)$ is defined as the solution in z of

$$\pi\exp(z(p-1)) + (1-\pi)\exp(zp) - 1 = 0$$

which is the analog of Equation (1). Hence, based on the data $(\mathbf{x},\mathbf{y})$, we want to estimate the ratio $\mathbb{E}[\pi/z(\pi,p)]/\mathbb{E}[1/z(\pi,p)]$, which is given by

$$\mathcal{I} = \frac{\displaystyle\int_0^1\left(\int_0^u\left[\frac{u}{z(u,v)}\right]\frac{u^{a-1}(1-u)^{b-1}v^{c-1}(1-v)^{d-1}(u-v)^e}{g(a,b,c,d,e)}dv\right)du}{\displaystyle\int_0^1\left(\int_0^u\left[\frac{1}{z(u,v)}\right]\frac{u^{a-1}(1-u)^{b-1}v^{c-1}(1-v)^{d-1}(u-v)^e}{g(a,b,c,d,e)}dv\right)du} = \frac{\mathcal{I}_3}{\mathcal{I}_4}.$$

The natural Bayes estimate of this ratio is then the posterior expectation, $\mathbb{E}[\mathcal{I}_3/\mathcal{I}_4|\mathbf{x},\mathbf{y}]$ (see Robert, 2001, Chapter 2).

The unusual prior distribution for (π,p) given in (2) is inspired from Beta distributions that are the conjugates of binomial distributions, the additional term $(\pi_i-p_i)^e$ being chosen for integrability reasons. Indeed, near the diagonal, we have that $z(\pi,p)\sim\pi-p$ and thus the nature of the integrals $\mathcal{I}_3$ and $\mathcal{I}_4$ depends on the position of e with respect to zero.

We will also estimate the unweighted average level of optimism $\mathcal{I}_1$ as well as the unweighted average level of risk aversion $\mathcal{I}_2$, respectively given by

$$\mathcal{I}_1 = \frac{\displaystyle\int_0^1\left(\int_0^u\frac{u^a(1-u)^{b-1}v^{c-1}(1-v)^{d-1}(u-v)^e}{g(a,b,c,d,e)}dv\right)du}{\displaystyle\int_0^1\left(\int_0^u\frac{u^{a-1}(1-u)^{b-1}v^{c-1}(1-v)^{d-1}(u-v)^e}{g(a,b,c,d,e)}dv\right)du}$$

$$\mathcal{I}_2 = \frac{\displaystyle\int_0^1\left(\int_0^u z(u,v)\frac{u^{a-1}(1-u)^{b-1}v^{c-1}(1-v)^{d-1}(u-v)^e}{g(a,b,c,d,e)}dv\right)du}{\displaystyle\int_0^1\left(\int_0^u\frac{u^{a-1}(1-u)^{b-1}v^{c-1}(1-v)^{d-1}(u-v)^e}{g(a,b,c,d,e)}dv\right)du}.$$

3.3 Parameter estimation and numerical results

To complete the definition of the prior distribution, we also introduce the following exponential prior on the hyperparameters:

$$a,b,c,d,e \overset{\text{iid}}{\sim} \mathcal{E}xp(1)\,.$$

Once the prior and the sampling distributions selected, we can formally draw inference about the true parameters (π_i, p_i) by computing their posterior distribution, that is, their distribution conditional on the observations, and using this distribution as a complete summary of the information available on the (π_i, p_i)'s (Robert, 2001). In practice, it is however quite complex to derive exactly the posterior distribution of a pair (π_i, p_i), given that this derivation implies the integration of all the other parameters conditional on the whole sample $(\mathbf{x}, \mathbf{y})$. We therefore need to resort to an approximate method, based on the simulation of Markov chains, namely the Markov Chain Monte Carlo (MCMC) method (Robert and Casella, 2004). Gibbs sampling is one of the most popular MCMC schemes. Its principle is quite straightforward: when faced with the simulation of a multivariate distribution $g(\theta_1, \ldots, \theta_p)$ $(p > 1)$, generate instead successively from the full conditionals $g_i(\theta_i|\theta_{-i})$ and let the corresponding Markov chain converge to the limiting distribution $\pi(\theta)$. In our case, the implementation of this principle translates into the generation of the (π_i, p_i)'s given (a, b, c, d, e) and $(\mathbf{x}, \mathbf{y})$, and of (a, b, c, d, e) given the (π_i, p_i)'s and $(\mathbf{x}, \mathbf{y})$. More precisely, the conditional independence relations imply that we simulate successively from $(1 \leq i \leq n)$

- $f_{(p,\pi)}(\pi_i, p_i|\ldots) \propto \pi_i^{a+x_i-1}(1-\pi_i)^{b+10-x_i-1} p_i^{c+y_i-1}(1-p_i)^{d+100-y_i-1}(\pi_i - p_i)^e \mathbb{I}_{0<\pi_i<p_i<1}$;
- $f_a(a|\ldots) \propto \frac{1}{g(a,b,c,d,e)^n} \exp\left(-(1 - \sum_{i=1}^n \log(\pi_i))a\right) \mathbb{I}_{]0,\infty[}(a)$;
- $f_b(b|\ldots) \propto \frac{1}{g(a,b,c,d,e)^n} \exp\left(-(1 - \sum_{i=1}^n \log(1-\pi_i))b\right) \mathbb{I}_{]0,\infty[}(b)$;
- $f_c(c|\ldots) \propto \frac{1}{g(a,b,c,d,e)^n} \exp\left(-(1 - \sum_{i=1}^n \log(p_i))c\right) \mathbb{I}_{]0,\infty[}(c)$;
- $f_d(d|\ldots) \propto \frac{1}{g(a,b,c,d,e)^n} \exp\left(-(1 - \sum_{i=1}^n \log(1-p_i))d\right) \mathbb{I}_{]0,\infty[}(d)$;
- $f_e(e|\ldots) \propto \frac{1}{g(a,b,c,d,e)^n} \exp\left(-(1 - \sum_{i=1}^n \log(\pi_i - p_i))e\right) \mathbb{I}_{]0,\infty[}(e)$.

Note that the dataset $(\mathbf{x}, \mathbf{y})$ only appears in the conditional distribution $f_{(p,\pi)}$. This is due to the hierarchical nature of the model that implies that $(a, \ldots, e)$ is independent of $(\mathbf{x}, \mathbf{y})$ given the (π_i, p_i)'s.

These distributions are moreover far from standard and their simulation requires another level of MCMC technology, namely the replacement of the exact simulation with a Markov step which guarantees convergence to the proper distribution. The corresponding algorithm is called *hybrid MCMC* and works as described in the Appendix.

We however abstain from reporting the full simulation experiment corresponding to this setting as it provides a non-identifiability conclusion on the parameter e. Indeed, the chain is always converging to a value of e very close to 0. This is particularly worrying because the parameter e has been introduced in order to ensure the finiteness of $\mathcal{I}_3$ and $\mathcal{I}_4$. A value of $e = 0$ would have been more natural for the distribution of (π, p) given the choice of binomial distributions for $(x_i, y_i)\,|(\pi_i, p_i)$. In order to examine this problem, we study instead the sensitivity of the model to changes in e. More precisely, we will fix different small values for e hoping that the ratio $\frac{\mathcal{I}_3}{\mathcal{I}_4}$ will remain stable when e goes to zero even if we know that $\mathcal{I}_3$ and $\mathcal{I}_4$ will diverge.

When setting the value of e to 0.1, 0.05, and 0.01, running an hybrid MCMC algorithm leads to a highly stable estimation of the other parameters. The results of this simulation for $e = 0.01$ are presented in Figures 3 and 4 while the Bayesian estimates of a, b, c, d are given in Table 3 for the different chosen values of e. The estimates are quite close and the mixing behavior of the MCMC chains seems to be fairly impervious to changes in e. We are therefore satisfied with this proxy to the full parameter estimation scheme and in the next we will retain a value of 0.01 for e.

Now, for a given sample $(\mathbf{x}, \mathbf{y})$, we want to estimate the quantity

$$\mathcal{I} = \frac{\displaystyle\int_0^1 \left(\int_0^u \left[\frac{u}{z(u,v)} \right] \frac{u^{a-1}(1-u)^{b-1} v^{c-1}(1-v)^{d-1}(u-v)^e}{g(a,b,c,d,e)} dv \right) du}{\displaystyle\int_0^1 \left(\int_0^u \left[\frac{1}{z(u,v)} \right] \frac{u^{a-1}(1-u)^{b-1} v^{c-1}(1-v)^{d-1}(u-v)^e}{g(a,b,c,d,e)} dv \right) du} = \frac{\mathcal{I}_3}{\mathcal{I}_4}.$$

Since this ratio depends on the unknown parameters (a, b, c, d), e being fixed, the natural estimator is the Bayes posterior expectation conditional on the sample $(\mathbf{x}, \mathbf{y})$, $\mathbb{E}[\mathcal{I}|(\mathbf{x}, \mathbf{y})]$. There is however a difficulty in that both integrals $\mathcal{I}_3$ and $\mathcal{I}_4$ are not available in closed form. We thus resort to separate evaluations of both integrals, with $\mathcal{I}_3$ and $\mathcal{I}_4$ estimated by importance sampling in order to ensure finite variance estimates (Robert and Casella, 2004). The importance distribution we use is a Dirichlet $\mathcal{D}(\psi, e-\epsilon, \lambda)$ with $\psi > 0$, $e - \epsilon > 0$ and $\lambda > 0$, and density

$$v_i^{\psi-1}(u_i - v_i)^{e-\epsilon-1}(1-u_i)^{\lambda-1}\Gamma(\psi + e - \epsilon + \lambda)/(\Gamma(\psi)\Gamma(e-\epsilon)\Gamma(\lambda)),$$

due to the similarity with the target distribution and the warranty of a finite variance. The importance sampling estimates of both integrals are therefore

$$\widetilde{\mathcal{I}_3} = \frac{1}{N}\sum_{i=1}^{N} \frac{u_i^{a_i}(1-u_i)^{b_i-1}v_i^{c_i-1}(1-v_i)^{d_i-1}(u_i-v_i)^e / g(a_i,b_i,c_i,d_i,e)}{z(u_i,v_i)v_i^{\psi-1}(u_i-v_i)^{e-\epsilon-1}(1-u_i)^{\lambda-1}\Gamma(\psi+e-\epsilon+\lambda)/(\Gamma(\psi)\Gamma(e-\epsilon)\Gamma(\lambda))},$$

and

$$\widetilde{\mathcal{I}_4} = \frac{1}{N}\sum_{i=1}^{N} \frac{u_i^{a_i-1}(1-u_i)^{b_i-1}v_i^{c_i-1}(1-v_i)^{d_i-1}(u_i-v_i)^e / g(a_i,b_i,c_i,d_i,e)}{z(u_i,v_i)v_i^{\psi-1}(u_i-v_i)^{e-\epsilon-1}(1-u_i)^{\lambda-1}\Gamma(\psi+e-\epsilon+\lambda)/(\Gamma(\psi)\Gamma(e-\epsilon)\Gamma(\lambda))},$$

where $((u_1, v_1), \ldots, (u_N, v_N))$ is an i.i.d. sample from $\mathcal{D}(\psi, e-\epsilon, \lambda)$ and the (a_i, b_i, c_i, d_i)'s constitute the MCMC sample from the posterior distribution of (a, b, c, d) given $(\mathbf{x}, \mathbf{y})$.[9]

The Bayes estimate of $\mathcal{I}$ is therefore approximated by the ratio of both importance sampling Bayes estimates, i.e. $\widetilde{\mathcal{I}_3}/\widetilde{\mathcal{I}_4}$.

The corresponding estimations of $\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4$ and $\mathcal{I}$ are given in Table 4, based on $200,000$ MCMC iterations. For $e = 0.1$ and $e = 0.05$, we used the last $190,000$ MCMC iterations in the summation and $\epsilon = 0.01$. For $e = 0.01$, we used only the last $150,000$ MCMC iterations in the summation and $\epsilon = 0.001$. The estimates of $\mathcal{I}_1, \mathcal{I}_2$ and $\mathcal{I}$ are very stable with respect to the variations of e even if $\mathcal{I}_3$ and $\mathcal{I}_4$ diverge when e goes to zero.

Recall that the objective probability is 0.5. As shown in Table 2, the average subjective probability which is given by $\mathcal{I}_1$ is around 0.37. The average subjective probability weighted by the risk tolerance, which is given by $\mathcal{I}_2$, is much lower than $\mathcal{I}_1$ and is around 0.3. As far as the link between optimism and risk aversion is concerned, these results indicate the presence of a positive correlation between optimism and risk aversion: more risk averse individuals are more optimistic.

4 Conclusion

In this paper, we have adopted a Bayesian estimation approach in order to analyze the link between risk aversion and optimism and to estimate the risk tolerance weighted average of subjective probabilities (which represents the consensus probability of the group) as well as the unweighted average. In the analyzed survey, while the objective probability is 0.5, the

[9] It may sound surprising to mix (u_i, v_i)'s simulated from the importance function with (a_i, b_i, c_i, d_i)'s simulated from the posterior distribution, but the average $\widetilde{\mathcal{I}_3}$ (resp. $\widetilde{\mathcal{I}_4}$) is converging to the double expectations $\mathbb{E}\{\mathbb{E}[\pi/z(\pi,p)]|(\mathbf{x},\mathbf{y})\}$ (resp. $\mathbb{E}\{\mathbb{E}[1/z(\pi,p)]|(\mathbf{x},\mathbf{y})\}$).

estimated (equally-weighted) average subjective probability reveals to be around 0.4 and the estimated average subjective probability weighted by the risk tolerance reveals to be around 0.3. It appears then, on our sample, that the group is pessimistic and more pessimistic than the average individual.

We do not pretend that 0.3 represents the collective subjective probability in a general setting, nor that $\frac{0.4-0.5}{0.5} = -0.2$ represents the average level of optimism in all risk-taking situations in a $[-1, 1]$ scale. We only claim that the difference between the unweighted and the risk tolerance weighted averages of subjective probabilities is substantial enough to conclude to a behavioral correlation between optimism and risk aversion. Analogously, the difference between the objective probability and the unweighted average of subjective probabilities is substantial enough to conclude to a behavioral bias toward pessimism at the individual level. As a consequence there should be a bias toward pessimism at the collective level. The impact on the equilibrium characteristics should then be towards an increase of the risk premium and a decrease of the risk free rate, which is interesting with regard to the risk premium and risk free rate puzzles.

To identify the origin of the elicited pessimism and correlation between optimism and risk aversion is a delicate question. The observed pessimism might result from an individual learning process, where individuals overestimate bad experiences (see Taylor, 1991) and the intensity of this effect might be related to the individual level of risk aversion. Besides, people seem to be disappointment averse in their choices (Bell, 1985). Disappointment avoidance may reflect a self deception mechanism designed to protect self esteem about decision making ability, i.e. a calculated avoidance of unpleasant future feelings (Joseph et al., 1996, Gollier and Muerman, 2006). The elicited pessimism could be interpreted as defensive pessimism, an anticipatory strategy that involves setting defensively low expectations prior to entering a situation so as to defend against loss of self esteem in the event of failure and here again the intensity of this effect might be related to the individual level of risk tolerance. This would require a deeper analysis of the process of belief formation.

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Appendix

Let us first define

$$f_G(y|x) = \frac{1}{4}\left(f_{G(\delta_1 x,\delta_1)}(y) + f_{G(\delta_2 x,\delta_2)}(y) + f_{G(\delta_3 x,\delta_3)}(y) + f_{G(\delta_4 x,\delta_4)}(y)\right).$$

The hybrid MCMC algorithm we used in our simulation is described by the following steps:

Hybrid MCMC algorithm

- Initialization: Simulate $\left(a^{(0)}, b^{(0)}, c^{(0)}, d^{(0)}\right)$ from $\otimes_{i=1}^{4} Ga(1,1)$
  For $i \in \{1,\ldots,n\}$, simulate $\left(\pi_i^{(0)}, p_i^{(0)}\right)$ from

$$Be\left(x_i + a^{(0)}, 10 - x_i + b^{(0)}\right) \otimes Be_{]0,\pi_i^{(0)}[}\left(y_i + c^{(0)}, 100 - y_i + d^{(0)}\right)$$

(truncated Beta distribution)

- For $t = 1,\ldots$:

1) For $i \in \{1,\ldots,n\}$, simulate $(\tilde{\pi}_i, \tilde{p}_i)$ from

$$Be\left(x_i + a^{(t-1)}, 10 - x_i + b^{(t-1)}\right) \otimes Be_{]0,\pi_i^{(t)}[}\left(y_i + c^{(t-1)}, 100 - y_i + d^{(t-1)}\right)$$

and take

$$\left(\pi_i^{(t)}, p_i^{(t)}\right) = \begin{cases} (\tilde{\pi}_i, \tilde{p}_i) & \text{with probability} \quad \rho_i^{(\pi,p)} \\ \left(\pi_i^{(t-1)}, p_i^{(t-1)}\right) & \text{with probability} \quad 1 - \rho_i^{(\pi,p)} \end{cases}$$

where

$$\rho_i^{(\pi,p)} = \min\left(\frac{(\tilde{\pi}_i - \tilde{p}_i)^{e^{(t-1)}} F_{Be(c^{(t-1)}+y_i, d^{(t-1)}+100-y_i, 0, 1)}(\tilde{\pi}_i)}{(\pi_i^{(t-1)} - p_i^{(t-1)})^{e^{(t-1)}} F_{Be(c^{(t-1)}+y_i, d^{(t-1)}+100-y_i, 0, 1)}(\pi_i^{(t-1)})}, 1\right)$$

2) Simulate $\tilde{a}$ from $f_G\left(\cdot|a^{(t-1)}\right)$ and take

$$a^{(t)} = \begin{cases} \tilde{a} & \text{with probability} \quad \rho^a = \min\left(\dfrac{f_a\left(\tilde{a}|\ldots\right)/f_G\left(\tilde{a}|a^{(t-1)}\right)}{f_a\left(a^{(t-1)}|\ldots\right)/f_G\left(a^{(t-1)}|\tilde{a}\right)}, 1\right) \\ a^{(t-1)} & \text{with probability} \quad 1 - \rho^a \end{cases}$$

3) Simulate $\tilde{b}$ from $f_G\left(\cdot|b^{(t-1)}\right)$ and take

$$b^{(t)} = \begin{cases} \tilde{b} & \text{with probability} \quad \rho^b = \min\left(\dfrac{f_b\left(\tilde{b}|\ldots\right)/f_G\left(\tilde{b}|b^{(t-1)}\right)}{f_b\left(b^{(t-1)}|\ldots\right)/f_G\left(b^{(t-1)}|\tilde{b}\right)}, 1\right) \\ b^{(t-1)} & \text{with probability} \quad 1 - \rho^b \end{cases}$$

4) Simulate $\tilde{c}$ from $f_G\left(\cdot|c^{(t-1)}\right)$ and take

$$c^{(t)} = \begin{cases} \tilde{c} & \text{with probability} \quad \rho^c = \min\left(\dfrac{f_c\left(\tilde{c}|\ldots\right)/f_G\left(\tilde{c}|c^{(t-1)}\right)}{f_c\left(c^{(t-1)}|\ldots\right)/f_G\left(c^{(t-1)}|\tilde{c}\right)}, 1\right) \\ c^{(t-1)} & \text{with probability} \quad 1 - \rho^c \end{cases}$$

5) Simulate $\tilde{d}$ from $f_G\left(\cdot|d^{(t-1)}\right)$ and take

$$d^{(t)} = \begin{cases} \tilde{d} & \text{with probability} \quad \rho^d = \min\left(\dfrac{f_d\left(\tilde{d}|\ldots\right)/f_G\left(\tilde{d}|d^{(t-1)}\right)}{f_d\left(d^{(t-1)}|\ldots\right)/f_G\left(d^{(t-1)}|\tilde{d}\right)}, 1\right) \\ d^{(t-1)} & \text{with probability} \quad 1-\rho^d \end{cases}$$

6) Simulate $\tilde{e}$ from $f_G\left(\cdot|e^{(t-1)}\right)$ and take

$$e^{(t)} = \begin{cases} \tilde{e} & \text{with probability} \quad \rho^e = \min\left(\dfrac{f_e\left(\tilde{e}|\ldots\right)/f_G\left(\tilde{e}|e^{(t-1)}\right)}{f_e\left(e^{(t-1)}|\ldots\right)/f_G\left(e^{(t-1)}|\tilde{e}\right)}, 1\right) \\ e^{(t-1)} & \text{with probability} \quad 1-\rho^e \end{cases}$$

After some experimenting, we found that the scales $\delta_1 = 1$, $\delta_2 = 10$, $\delta_3 = 100$ and $\delta_4 = 1000$ ensure a good behavior of the hybrid MCMC algorithm.

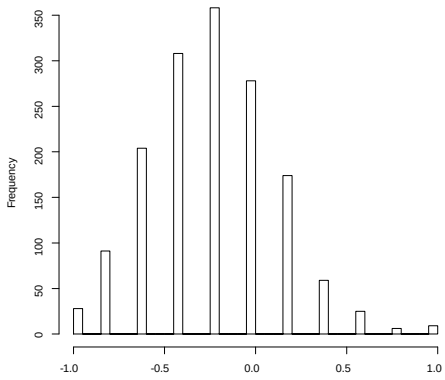


Figure 1: Distribution of pessimism. This figure represents the distribution of "pessimism" across our sample. A value of -1 (resp. 1) corresponds to people who think that they will never (resp. always) win at the head or tail game. The value of -0.1, which is endowed with the largest frequency of individuals, corresponds to people who think that they will win (or head will occur) 4 times out of ten.

c	$\mathrm{E}[OA_i * w_i \mid i \in c]$
1	515.8272
2	703.077
3	608.1076
4	951.7348
5	758.1568
6	1218.167
7	1206.866

Table 1: This table reports for each class c of relative risk aversion the average level over the individuals in that class of the product of absolute risk aversion (under the objective probability, OA_i) by the mean of the income band w_i. Kendall's Tau is equal to 0.71, Spearman's Rho is equal to 0.89 and Pearson's correlation coefficient is equal to 0.89. There is then a positive relationship between the elicited levels of absolute and relative risk aversion.

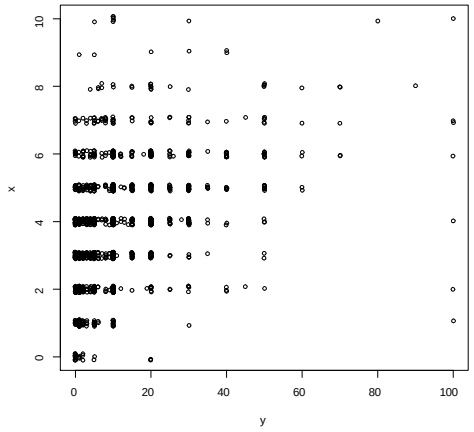


Figure 2: JITTER representation of the dataset. This figure reports on the horizontal axis the maximum price individuals are willing to pay in the game with ten draws and on the vertical axis the number of times they think that they will win.

x_i	$\mathrm{E}[CA_i \mid x_i]$
1	0.7732720
2	0.8224625
3	0.7881803
4	0.8831758
5	1.1054212
6	0.9278274
7	1.4897416
8	1.0668114
9	5.1140936

Table 2: Average level of corrected risk aversion for a given level of x (number of times the individual thinks he is going to win). Kendall's Tau is equal to 0.78 and Spearman's Rho is equal to 0.9 which indicates a positive link between optimism and corrected risk aversion.

e	$\hat{a}$	$\hat{b}$	$\hat{c}$	$\hat{d}$
0.1	4.6305	8.5007	0.7207	3.1777
0.05	4.5005	8.2155	0.7142	3.2730
0.01	4.7623	8.5973	0.7283	3.6653

Table 3: MCMC estimates of a, b, c, d (empirical means over the last $190,000$ points in the MCMC sample out of $200,000$ iterations).

e	I_1	I_2	I_3	I_4	I
0.1	0.3770	0.1161	1.2458	4.1281	0.3018
0.05	0.3755	0.1166	3.4904	11.7605	0.2968
0.01	0.3735	0.1153	22.1699	74.6999	0.2967

Table 4: MCMC estimates of $\mathcal{I}_1$,$\mathcal{I}_2$, $\mathcal{I}_3, \mathcal{I}_4$ and $\mathcal{I}$ based on $200,000$ iterations. It is immediate to see that $\mathcal{I}_3$ and $\mathcal{I}_4$ diverge when e goes to zero while their ratio $\mathcal{I}$ remains stable.

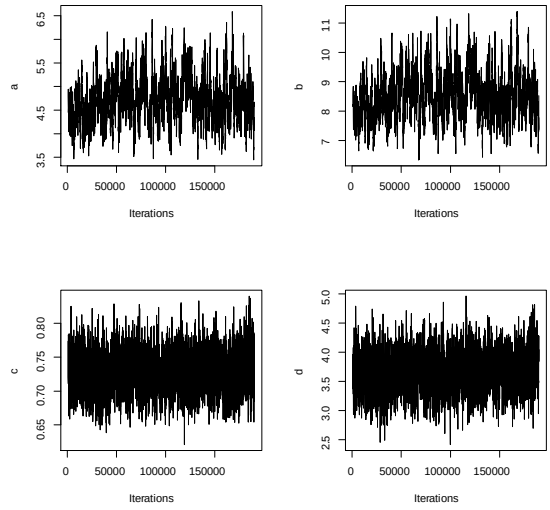


Figure 3: $e = 0.01$: evolution of the hybrid MCMC sample over $150,000$ iterations

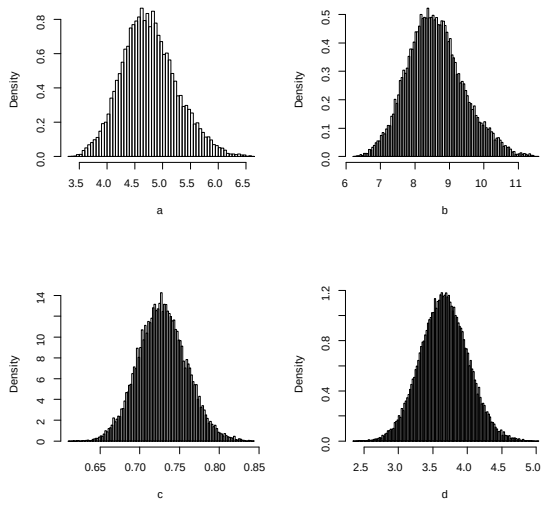


Figure 4: $e = 0.01$: histograms of the last $150,000$ MCMC iterations