Logics for Dialogue

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CLIPS-IMAG et Université Pierre Mendès-France, Grenoble
Alain.Lecomte@upmf-grenoble.fr

Abstract

This paper is essentially a survey of some logical approaches to dialogue. We start with Dialogical Logic, which was initiated by Lorenzen and has mainly been explored as a new foundation for logics. It continues with Hintikka's Game Theoretical Semantics, which has been more developed in contact with Natural Language. For instance, we show how to deal with generalized quantifiers by using games, after ideas taken from Ahti Pietarinen. The two perspectives, if different in their objectives, could be mixed for applicative purposes like the treatment of argumentative dialogues: this requires that they be recast in a neutral form, which consists in Dialogue Games in Extensive form. Nevertheless, to stay at one level of elementary language games is not sufficient: in every day life, games are combined. At this point, it seems that the Game-Theoretic interpretation of Linear Logic provides us with the appropriate tool for combining elementary games of various kinds.

1 Introduction

Logical concepts have shown to be helpful in language clarification for more than one century with the works done by Frege, Russell and their numerous followers. A little less attention has been given to dialogue during the same period. Yet dialogue is fundamental as soon as we want to understand pronouns (or *shifters* in Benveniste's terms) and other indexicals, but also, as we shall see further, other linguistic phenomena, like generalized quantifiers. Moreover, it has been shown by many researchers since Lorenzen [Lorenzen, 1960] and Hintikka [Hintikka, 1968] that logical connectives and natural language particles could be explained by referring to a dialogical interpretation of language, which is also known as Game-Theoretical Semantics in Hintikka's case. It is an open question whether those Game-Theoretic approaches are able to provide models of true dialogues, as they occur in the every day life.
Solving such a question has many consequences and applications, among which are applications to Human-Machine Dialogue and to dialogue between humans via a machine.
From a theoretical viewpoint, if it appears that, like some contemporary researchers point out ([Hurford, 2002], [Bickerton, 1990]), predicate logic is appropriate for describing thought as pre-existing to language, it also seems that interaction structures have played a major role for humans before the emergence of language and that language received much of its features from these structures. It is therefore no surprise to find very early formulations of logic (sometimes assimilated with dialectic) which make use of dialogical concepts. We may mention here of course Aristotle, in *Topica VIII*, for whom dialectical situations involve two participants: *Answerer*, who must defend his or her thesis or *positum*, and *Questioner*, who tries to make *Answerer* to change his *positum*. In the modern times, Leibniz invented the famous 'epsilon-delta' definition of continuity and thought of it as a simple dialogue (give me one 'epsilon', I shall give you one 'delta').
In this paper, we consider different conceptions of games with regards to logic and language. A first conception, dating from Lorenzen [Lorenzen, 1960, Lorenz, 1961] amounts to use games as a foundation for logic: we call it a *procedural approach* because it is essentially based on rules: rules for particles and global rules. A second one, coined by Hintikka

[Hintikka, 1968] amounts to use games as an alternative to the classical models used for evaluating the truth of a formula, we call it a *model-based approach*. There is also an other viewpoint, called *geometry of interaction* by Girard [Girard, 1995] which uses games as altogether semantics and rules for defining a particular logic: Linear Logic.
Our main thesis is that in order to treat real dialogues (which include language and non-language games) we must altogether mix the two first approaches (the procedural one and the model-based one) and combine the elementary games by means of Linear Logic used as a "glue" logic.

2 Games as a foundation

2.1 Argumentation forms

Lorenzen's goal in the late fifties was to give a justification to intuitionistic logic. His purpose consisted in arguing that intuitionistic logic was the right one since it is the logic we obtain when we adopt the "natural" dialogical meaning of the connectives. This goal was not really achieved, because it was shown later on that other logics were as well obtainable simply by changing the so-called "global rules", but we may consider Lorenzen an ancestor of Girard's approach to logic that we shall look at soon.
Let us simply recall here that we need a denumerable set of variables x, y, z, …, the constant connectives $\wedge$, $\vee$, $\Rightarrow$, $\neg$ and the quantifiers $\forall$ and $\exists$. Let **P** and **O** two constant symbols (respectively representing *Proponent* and *Opponent*). For every expression *e*, we have two distinct expressions: **P**:*e* and **O**:*e* which are said to be respectively signed by **P** and **O**. X and Y are two variables which take their values in {**P**, **O**}. We assume that in every situation, X $\neq$ Y.
An *argumentation form* is a schematic presentation of how an assertion made by X can be attacked by Y and how, if possible, it can be defended by X. In order to have an argumentation form for each formula, it is enough to define argumentation forms for connectives and quantifiers.
We thus have for the connectives:

$\wedge$	assertion	X:$w_1 \wedge w_2$	
	attack	Y: $\wedge_i$	(i = 1,2)
	defence	X:w_i	

$\vee$	assertion	X:$w_1 \vee w_2$	
	attack	Y: $\vee$	
	defence	X:w_i	(i = 1,2)

$\Rightarrow$	assertion	X:$w_1 \Rightarrow w_2$
	attack	Y: w_1
	defence	X:w_2

$\neg$	assertion	X: $\neg$w
	attack	Y: w
	defence	no defence!

and for the quantifiers:

$\forall$	assertion	X: $\forall$xw	
	attack	Y: t	(Y chooses t)
	defence	X:w(t)	

$\exists$	assertion	X: $\exists$xw	
	attack	Y: $\exists$	
	defence	X:w(t)	(X chooses t)

Technically, a *dialogue* is a sequence δ of signed expressions, alternatively supported by **P** and **O** and satisfying at each step some argumentation form. This needs some more precision. Let also η be a function on natural numbers. Let A and D the two letters for *Attack* and *Defence*.

Definition 1 ([Felscher, 1986]) A *dialogue* is a couple (δ, η) such that:

- δ is a sequence of signed expressions,
- η is a function on the ranks of the signed expressions in δ such that $\eta(n)$ is a couple [m, Z] where $m \leq n$ and Z = A or D, which satisfies the following three conditions (D00), (D01), (D02).

D00 $\delta(n)$ is **P**-signed if n is even and **O**-signed if n is odd; $\delta(n)$ is a compound formula,

D01 If $\eta(n)$ = [m, A], then $\delta(m)$ is a compound formula and $\delta(n)$ is an attack on $\delta(m)$ which corresponds to a regular argumentation form,

D02 If $\eta(p)$ = [n, D], then $\eta(n)$ = [m, A], and $\delta(p)$ is an answer to the attack $\delta(n)$ which corresponds to a regular argumentation form.

2.2 Global rules

Global rules are used according to the logic we aim at expressing. Let us envisage the following rules (some of which are not compatible with others):

D10 **P** may assert an atomic formula only if **O** asserted it previously,

D11 If at a position p – 1, several attacks are still not answered, it is only to the last not already defended attack that it can be answered at p,

D12 An attack can have at most one answer,

D12' **P** may repeat an attack or a defence if and only if **O** has introduced a new atomic formula (which can now be used by **P**),

D13 A **P**-signed formula can be attacked at most once,

D13' In any move, each player may attack a complex formula asserted by his partner or he may defend himself against any attack (including those which have already been defended)

Playing this kind of games obviously needs a *winning rule*:

D2 X wins if and only if it is Y's turn but Y cannot move

Definition 2: A formula is said to be valid in a dialogical system (defined by the previous definition + some of the global rules) if and only if **P** has a winning strategy for it, that is, **P** can win whatever moves are done by **O**.

Theorem 1: *Classical theses coincide with formulae valid for the system which includes D10, D12', D13', D2*

Theorem 2: *Intuitionistic theses coincide with formulae valid for the system which includes D10, D11, D12, D13, D2*

2.3. Examples

It is fundamental to notice here that in order to defend a formula (either in classical or in intuitionistic logic) **P** has not only to use more or less automatically the argumentation forms, he has also to choose some strategy which compels **O** to give him or her the most information as possible. This is particularly clear with regards to the global rule D10 which allows him or her to assert an atomic formula, provided that his or her partner already asserted it. In this case he or she has therefore to oblige **O** to *concede* some fact. This is clearly showed in the following cases. (The two first examples come from [Felscher, 1986], the third one from [Rahman and Rückert, 2001].)

Example 1

0.	**P**: $(a \wedge b) \Rightarrow (a \wedge b)$				
1.	**O**: $(a \wedge b)$	[0, A]			
2.	**P**: $\wedge_1$	[1, A]			
3.	**O** : a	[2, D]			
4.	**P**: $\wedge_2$	[1, A]			
5.	**O**: b	[4, D]			
6.	**P**: $(a \wedge b)$	[1, D]			
7.	**O**: $\wedge_1$	[6, A]	7'.	**O**: $\wedge_1$	[6, A]
8.	**P**: a	[7, D]	8'.	**P**: b	[7' D]

Example 2

0.	**P**: $((a \Rightarrow a) \Rightarrow b) \Rightarrow b$			
1.	**O**: $(a \Rightarrow a) \Rightarrow b$	[0, A]		
2.	**P**: $(a \Rightarrow a)$	[1, A]		
3.	**O**: b	[2, D]	3. **O** : a	[2, A]
4.	**P** : b	[1, D]	4. **P** : a	[3, D]
5.	**O** : a	[2, A]	5. **O** : b	[2, D]
6.	**P** : a	[5, D]	6. **P** : b	[1, D]

Example 3

0.	**P**: $\forall x(P(x) \vee \neg P(x))$	
1.	**O**: τ	[0, A]
2.	**P**: $P(\tau) \vee \neg P(\tau)$	[1, D]
3.	**O**: $\vee$	[2, A]
4.	**P**: $\neg P(\tau)$	[3, D]
5.	**O**: $P(\tau)$	[4, A]
6.	**P**: $P(\tau)$	[3, D]

Notice that at step 4, **P** cannot choose $P(\tau)$ because it is an atomic formula, not already asserted by **O**. He therefore chooses $\neg P(\tau)$, which obliges **O** to assert $P(\tau)$ and then **P** can choose it, thus providing a complete argument for his or her thesis. Notice also that this is only possible in classical logic since in the intuitionistic one, **P** would only be allowed to answer last **O**'s attack, which is the attack performed at step 5.

2.4. Non classical logics

Among non classical logics, we include *free* logics and *paraconsistent* ones. These logics are treated in the dialogical framework by changing global rules (or adding new ones). A good illustration for that is provided by S. Rahman's treatment of free logics ([Rahman, 2000]). Let us suppose that in a dialogue similar to Example 3 above, *only the opponent is allowed to introduce a constant* when playing a move associated with a quantifier, then it becomes possible to have a logic where it is impossible to show that, say, *if there is no vampire,* *Nosferatu is not a vampire*, thus opening room to fiction: even if we know that vampires do not exist, we can accept the idea that Nosferatu is a vampire. Let us formulate the classical dialogue:

Example 4:

0.	**P**: $(\forall x \neg vampire(x)) \Rightarrow \neg vampire(n)$	
1.	**O**: $\forall x \neg vampire(x)$	[0, A]
2.	**P**: $\neg vampire(n)$	[1, D]
3.	**O**: $vampire(n)$	[2, A]
4.	**P**: n	[1, A]
5.	**O**: $\neg vampire(n)$	[4, D]
6.	**P**: $vampire(n)$	[5, A]

At step 4, **P** attacks **O**'s move at 1, thus obliging **O** to assert that Nosferatu is not a vampire. This statement is a compound one and **P** may still attack it by asserting that Nosferatu is a vampire, an atomic formula that he or she can use because it was already asserted by **O** at step 3. After this move, there is no possible answer by **O** and therefore **P** wins, but we can see that if we introduce the new global rule concerning *the use of constants*, move 4 becomes illegal, and **P** looses. Further, he or she has no winning strategy: this entails that **P**'s proposition is not a thesis in such a logic.

Let us express a similar dialogue in ordinary language:

I :	if there is no vampire, of course Nosferatu is not a vampire!	
You:	OK, let us admit that there is no vampire	Attack
I:	then Nosferatu is not one!	Defence
You:	I pretend it is	Attack
I:	take Nosferatu	Attack
You:	I concede that Nosferatu is not vampire	Defence
I:	but you told me it is one!	Attack
You :	but I did not concede it was existing!	

This opens the field to *a conception of Fiction* which involves two partners who have an asymmetric relation, that we could call *Reader* and *Narrator*. In a fiction, *Narrator* is the only master. She may choose any character she wishes with any kind of properties even if both (*Reader and Narrator*) agree on the point that these properties are not satisfied in reality. We may notice that even contradictory properties could be used in the case of a paraconsistent logic (a case we do not develop here, see Rahman, 2000).

3 Model-based games

Hintikka's Game Rules slightly differ from Lorenzen's, but these small differences entail important consequences. The main difference lies in the rule for asserting atomic statements. Because Hintikka's games are based on a model M of the language L, the only requirement on asserting atomic formulae is that these formulae be *true in M*. Moreover, no repetition of "attack" or "defence" is allowed: at each stage, the game-formula simplifies and the run cannot be expected to go back to an earlier stage ([Hintikka and Kulas, 1985], [Hintikka and Sandu, 1997]).

Example 5
Let us suppose we have a structure <D, R> where D = {1, 2, 3, 4} and R is a binary relation on D : R = {(1,2), (1, 4), (2,3), (3, 1), (4, 2), (4, 3)}, and let us suppose we have to evaluate the formula: $\forall x \forall y(R(x,y) \vee \exists z(R(x,z) \wedge R(z,y)))$,
A possible game is the following one:

0. **P**: $\forall x \forall y(R(x,y) \vee \exists z(R(x,z) \wedge R(z,y)))$	
1. **O**: 2	[0, A]
2. **P**: $\forall y(R(2,y) \vee \exists z(R(2,z) \wedge R(z,y)))$	[1, D]
3. **O**: 4	[2, A]
4. **P**: $R(2,4) \vee \exists z(R(2,z) \wedge R(z,4))$	[3, D]
5. **O**: $\vee$	[4, A]
6. **P**: $\exists z(R(2,z) \wedge R(z,4))$	[5, D]
7. **O**: $\exists$	[6, A]
8. **P**: $R(2,3) \wedge R(3,4)$	[7, D]
9. **O**: $\wedge 2$	[8, A]
10. **P** looses	

P simply looses because the only defence he could have would be to assert R(3,4), which is false in M. In fact, it is easy to check that P has no winning strategy: the formula is false in M.

Theorem 3 ([Hintikka and Sandu, 1997]) *For any first-order sentence S and model M, Tarski-type truth and GTS-truth coincide, ie: M* $|=_{Tarski}$ *S if and only if M* $|=_{GTS}$ *S, where M* $|=_{GTS}$ *S means that Proponent has a winning strategy for S with regards to M.*

4 IF-logic

4.1. Informational independence

One way of dealing with incomplete information amounts to use Hintikka's notion of informational independence. It is well known that in a formula like $\forall x \exists y A(x,y)$, *y* functionally depends on *x*. Skolemization of this formula yields $\exists f \forall x A(x, f(x))$. In formulae with several universal quantifiers, like $\forall x \forall y \exists z \exists u A(x,y,z,u)$, the two existential quantifiers depend on *x* and *y*. Skolemization would give: $\exists f \exists g \forall x \forall y A(x,y,f(x,y),g(x,y))$.
Henkin was the first to point out this kind of limitation, intrinsic to first-order logic and provided a solution using what he called *branching quantifiers*. In a notation like:

$$\begin{pmatrix} \forall x & \exists z \\ \forall y & \exists u \end{pmatrix} A(x,y,z,u)$$

$\exists z$ only depends on *x* and $\exists u$ only depends on *y*, so that the skolemized form is:
$\exists f \exists g \forall x \forall y A(x,y,f(x),g(y))$
In Hintikka's notation, this is written: $(\forall x)(\forall y)(\exists z / \forall y)(\exists u / \forall x) \exists u A(x,y,z,u)$
Informational independence receives a convenient foundation in Game-Theoretical Semantics. It suffices to say that when defending a formula $(\exists z / \forall y)A$ against a $\exists$-attack, the player *does not know* the choice for *y* made by his partner when he chooses a value for *z*. Hintikka says in this case that the move prompted by ∃z is *informationally independent* on the move prompted by ∀y. The logic obtained by supplementing first-order formulae with those formulae in which informational independence is allowed is called *IF-logic* (for independence friendly logic).
Hintikka proved the following:

Theorem 4: *the IF first-order logic is a non recursively axiomatizable but compact extension of first-order logic*.

4.2. Informational independence and incomplete information

It happens that if we play dialogical games with informational independence, that amounts in fact in playing strategic games with incomplete information. Such a property may be seen on the following example.

Example 6

Let us suppose we have a structure <D, R> where D = {s, t} and R is a binary relation on D : R = {(s, t), (t, s)}, and let us suppose we have to evaluate the formula: $\forall x \exists y R(x, y)$, there is of course a winning strategy for the proponent **P**: to choose the object non selected by **O**. Let us suppose now we have to evaluate $\forall x(\exists y / \forall x) R(x, y)$: there is no longer any winning strategy (neither for P nor for O). The situation is the same as in *a Nash game without equilibrium*.

5 Dialogical semantics for natural language

5.1. Language as an infinite game

Following Wittgenstein's conception on language and meaning, we can assume that interactions we have in language can always be regarded as *games*: they are very ordinary games played in every day life. More than that, we can assume that *semantics is dialogical*, in the sense that *the meaning of a sentence is precisely a class of dialogues*. Of course, most of them are not necessarily won by *one* player, but they can be won by *both* (or all players in case of more than two). These assumptions depend on a more general one, according to which language makes us enter into an *infinite dialogue* where we fortunately can isolate *subdialogues* which can be seen as *subgames*. One purpose of dialogical semantics is to study these subgames and to try to represent their combination.
We will see further that linear logic provides us with good tools for expressing the combination part, but of course it is necessary to study the elementary (atomic) subgames: they are intrinsically attached to expressions in natural language. Hintikka's views on the semantics of natural language give us a natural way of solving this task. He notes that natural languages differ from formal ones mainly on some essential points:

- natural sentences don't have parentheses which could disambiguate them, like it is the case in formal languages,
- natural sentences don't have variables,
- the domain from which individuals are extracted is not fixed once and for all, it is always evolving.

5.2. Some rules

In consequence, game rules must have a different formulation, (particularly those associated with quantifiers) : their order of application is determined by specific *ordering principles* (it cannot simply follow from the ordering of connectives in the tree of subformulae, because such a tree cannot be built up), and the selection of individuals is always made in a *choice set* which changes when a play of the game progresses. This leads Hintikka to the formulation of rules like :
<R. some> If the game G(S; M) [where S is the sentence and M a model where it has to be evaluated] has reached an expression of the form:

Z – some X who Y – W

then the proponent may choose an individual from the appropriate domain, say b, and the game is continued as G(Z – b – W, b is an X and bY; M). The individual b is added to the choice set I_S.
In such a formulation, we say that the game G reaches an expression e if e is the currently parsed sentence in a text. That e is of the form "Z – some X who Y – W" means that it contains the words *some* and *who* in the order indicated in that pattern, assuming that X does not contain a similar pattern.

Example :
P : *I know somebody who came here*
O : (who?)
P : *I know Mr Nightingale*, (Mr Nightingale is a person), *Mr Nightingale came here*.

<R. every> If the game G(S; M) has reached an expression of the form:
Z – every X who Y – W
then the opponent may choose an individual from the appropriate domain, say b, and the game is continued as G(Z – b – W, b is an X and bY; M). The individual b is added to the choice set I_S.

<R. any> like <R. every>

Example :
P : *Everybody wants peace*.
O : *George W. Bush?*
… P fails!

<R. negation> If the game G(S; M) has reached an expression –P which is the negation of P, then the players exchange roles, i.e. the proponent will take the role of the opponent and vice versa. The game goes on as G(P;M).

5.3. Ordering principles

As said above, natural language cannot rely on a notion of specific scope for each particle. The priority scope is therefore handled by means of certain ordering principles. General principles interact with specific ones, we have for instance:

<O. Left-Right> for any semantical game G(S; M), the game rules are applied from left to right, in a same clause (general rule).

<O. comm.> a game rule must not be applied to an ingredient of a lower clause if it can be applied to an ingredient of a higher one (general rule).

<O. any> the rule <R. any> has priority over <R. negation>, <R. or> and <R. conditional> (specific rule).

Example :
We haven't got any bananas

0	P : *We haven't got any bananas*	
1	O : b?	Attack 0 <R. any>
2	P : *we haven't got b*, b is a banana	Defence 1
3	O : *you have got b*, b is a banana	Attack 2 <R. negation>

because O is asserting an atomic fact, it is either true or false, if it is true, P is wrong, if it is false, P is right.
We must notice on that example that the normal order would be: <R. negation> and then <R. any> because of <O. LeftRight>, but <O. LeftRight> is overruled by <O. any>.
If we had not this specific rule ordering, the game would have been:

0	P : *We haven't got any bananas*	
1	O : *you have got any bananas*	Attack 0 <R. negation>
2	P : b?	Attack 1 <R. any>
3	O : *you have got b*, b is a banana	Defence 2

which is actually a win for O if P has got any particular banana and is a win for P if she did not get a particular banana, which is obviously not the expected meaning of the sentence. (The winning strategies are obviously not the same in the two games).

5.4. On generalized quantifiers

It is known that generalized quantifiers (like *most A's*, *more A's than B's*, *as many A's as B's* and so on) are not definable in first order logic ([van Benthem and Doets, 1986]). A. Pietarinen [Pietarinen, 2001] demonstrates that they are in IF-logic. That shows that some verbal expressions can (must?) be interpreted dialogically. What is the reasoning?
Let us consider the generalized quantifier "at least as many B's as A's" and let us try to represent the expression "there are at least as many B's as A's". Mathematically speaking, such an expression means that there is an injection from A to B, or said differently that there is some mapping from A to B and that there is some inverse mapping from a subpart of B to A. This therefore can be expressed by the following second order formula:

$$(\exists f)(\exists g)(\forall x)(\forall y)((y = f(x)) \Leftrightarrow (x = g(y)) \wedge (A(x) \Rightarrow B(f(x))))$$

But this formula has an equivalent in IF-logic, which is:

$$(\forall x)(\exists y)(\forall z)(\exists u / x, y)((x = u \Leftrightarrow y = z) \wedge (A(x) \Rightarrow B(y)))$$

which means that the expression may be evaluated by means of a game where Opponent chooses at first an x in A, then Proponent chooses a y in B, moreover to ensure that it is not always the same y which is chosen, Opponent chooses a value in B, and then, ignoring what values were chosen previously for x and y, Proponent has to choose a value for u in A, in such a way that if $x \neq u$, then $y \neq z$ must hold.

6 Dialogue Games in Extensive Form

6.1. Extensive form of a game

In the general setting issued from early works by von Neumann, extensive-form games are defined by five-tuples $\mathbf{G}_A = \langle H, Z, P, N, (u_i)_{i \in N} \rangle$ where:

- A is set of possible actions,
- H is a set of finite sequences of actions $h = \langle a^i \rangle_{\{i=1\ldots n\}}$ from A, called *histories* such that:
    - the empty sequence <> is in H,
    - if $h \in H$ then any initial segment of h is in H too

- Z is the set of maximal histories,
- $P:H - Z \rightarrow N$ is the player function which assigns to every non terminal history a player in N whose turn is to move,
- each u_i where $i \in N$ is the payoff function, that is the function which gives for each maximal history the payoff for player i.

In such games, *strategies* can be defined as functions f_i which determine for any history where player i is to move a proper action in A.
Sandu and Pietarinen ([Sandu and Pietarinen, 2001]) propose to recast dialogical or semantic games inside extensive-form games. For that, the set of players is restricted to two players: **V**(erifier) and **F**(alsifier) (our previously called **P**roponent and **O**pponent) and the definition of an extensive-form game is enriched with a sixth component L which is a *labelling function* $L:H \rightarrow Sub(\Phi)$ where Φ is the game formula and $Sub(\Phi)$ is its set of subformulae. L has to verify the following properties:

- $L(<>) = \Phi$,
- for every maximal history h, L(h) is an atomic formula or its negation,
- if $L(h) = \neg\Phi$ and $P(h)=V$, then $h\Phi \in H$, $L(h\Phi) = \Phi$, $P(h\Phi)=F$,
- if $L(h) = \neg\Phi$ and $P(h)=F$, then $h\Phi \in H$, $L(h\Phi) = \Phi$, $P(h\Phi)=V$,
- if $L(h) = \Psi \vee \Phi$ or $L(h) = \Psi \wedge \Phi$, then $h.Left \in H$, $h.Right \in H$, $L(h.Left) = \Psi$,
- $L(h.Right) = \Phi$,
- if $L(h) = \Psi \vee \Phi$, then $P(h)=V$,
- if $L(h) = \Psi \wedge \Phi$, then $P(h)=F$,
- if $L(h) = \exists x\Phi$ or $L(h) = \forall x\Phi$, then $ha \in H$ for every $a \in D$ (D : domain of the model),
- if $L(h) = \exists x\Phi$ then $P(h)=V$,
- if $L(h) = \forall x\Phi$ then $P(h)=F$,
- for every maximal history $h \in Z$:
    - if $L(h) = Pt_1...t_m$ and $Pt_1...t_m$ true in the model, then $u_V(h) = 1$ and $u_F(h) = -1$,
    - if $L(h) = Pt_1...t_m$ and $Pt_1...t_m$ false in the model, then $u_V(h) = -1$ and $u_F(h) = 1$,

A *winning strategy* for $X \in \{V, F\}$ is a set of strategies f_X which leads X to $u_X(h) = 1$ no matter how the player Y decides to act.

6.2. Argumentative dialogues

Such definitions are interesting in that they allow more variability in the definition of games. One of the most obvious points where it is possible to make a change is the *payoff function*. In particular, we can expect now an account of *non strictly competitive* games, that argumentative dialogues often exemplify.
Let us take for instance a small dialogue like the following:

A. - *I would like to go to Paris this summer*
B. - *It will be very hot...*
A. - *Yes, I know, but it is our best opportunity to go there, during my mission to Europe*
B. - *I should better go to the Alps, we never saw the Mount Blanc*
A. - *You know I don't like climbing*
B. - *But there are many other things to do in mountains, like walking*
A. - *Well, I see, I cannot convince you... After all, why not to spend one week in Paris and one week in Chamonix?*
B. - *All right.*

We can analyse this dialogue like this:

0. A expresses a *goal* G (*to go to Paris this summer*),
1. B expresses a *disagreement* on G and she gives a *reason* (*Paris too hot in summer*),
2. A *agrees* with this last statement but A thinks he has a *better argument*,
3. B suggests *another goal* G' in substitution to G,
4. A *objects* to that goal,
5. B *objects* to the objection,
6. A *partially agrees*, he *retracts* and *suggests a compromise*,
7. B also *retracts* and *agrees with this compromise*

According to [Krabbe, 2001], we can schematize this kind of dialogue by a "profile":

0. A: p
1. B: **not** p (**because** q)
2. A: **concession**(q), q'
3. B: q **therefore** p'
4. A: **not** p' (**because** r)
5. B: **concession**(r), r'
6. A: **concession**(r'), C
7. B: **agreement**(C)

where we make appear some of the fundamental status any statement of a dialogue can have: *assertion, concession, reason* (preceded by *because*), *agreement, disagreement* (preceded by *not*). E. Krabbe suggests that it is possible to deal with argumentative dialogues by using these categories together with specific rules, and notably specific retraction rules which occur here when for instance each participant abandons his or her initial statement (which amounts to a wish). We don't enter into details of this approach here but simply wish to emphasize the fact that to deal with such a dialogue *not only needs a payoff function defined for terminations of histories*, but *a payoff function which attributes points to either participant at intermediary steps*. For instance, concessions may be more or less costly, but on another hand, agreements are strongly desirable goals and therefore give high benefits to both participants. Moreover, such games may have several kinds of issues. They can have issues where one of the participants is the winner (and the other one the looser), issues where both are winners (like in this example) and also, unfortunately, issues were both are losers (imagine the conflict leads to separate choices for A and B, who yet would have preferred to share their holidays...). In such games, *the extensive definition + the proper definition of the payoff functions* would be the appropriate means for describing the dialogue. Let us notice here that if Dialogical Logic has mainly been explored for providing logics with new (and better?) foundations, Game Theoretical Semantics has been more developed in contact with natural language interests. We think that they can complement each other. GTS is based on the use of models: we saw that the tarskian notion of truth and the GTS's notion coincide. This explains why GTS so well suits to all applications where facts are at disposal (for instance for questioning a database here represented by hotels registrations), but in many other cases, facts are either missing or difficult to access: *that does not prevent people to have debates*. In those precise cases, people must have strategies in order to enforce their partners *to concede facts* and this is more in the spirit of Lorenzen's dialogical logic. In order to clarify these relations between two conceptions, some neutral form of logical games is required: this role is played by extensive definitions of games which provide the most general frame for developing the study of dialogues and argumentation. Further works will develop these ideas.

7 Geometry of interaction

7.1. Linear Logic

The study of Linear Logic was not intended to provide descriptions for dialogical situations when it was invented by J-Y. Girard ([Girard, 1987]) at the end of the eighties: it simply came from the study of denotational semantics for programming langages, but it revealed soon to have a nice Game-Semantics, as showed primarily by Andreas Blass ([Blass, 1992]), and then by many other researchers like Abramsky and Jagadeesan ([Abramsky and Jagadeesan, 1992]), F. Lamarche ([Lamarche, 1992]) and others. J-Y. Girard himself recently explored this subject in depth, by providing a new game-based theory known as *Ludics* (but a viewpoint which will not be taken in this paper).

7.2. A brief history

Girard's Linear Logic is now well known. Let us simply recall that it comes from problems of theoretical computer science. Deep insights into denotational semantics showed that if we want to have a more fine-grained theory of functions, suitable to computer science problems, we have to take into account not only a general notion of function like it was already expressed in topological terms by Dana Scott, but a notion of function which is sensitive to the number of times it accesses its input. Aiming at this, Girard found particular spaces and particular functions from and to these spaces which were intended to become the mathematical objects needed for this semantics, they were called *coherent spaces* and *linear functions*. Intuitively, a linear function is such that when computed from an input, its argument is accessed only once. If we want to speak of an argument which could be accessed an arbitrary number of times, we make use of another type of data, written !A instead of A. If the type of a *linear* function is denoted by A—o B, the type of an ordinary function (which could access to its input an arbitrary number of times) is therefore !A—o B, in such a way that, remembering of the well known Curry-Howard isomorphism, according to which the space $A \rightarrow B$ corresponds to the logical formula $A \Rightarrow B$, !A—o B is equivalent to $A \Rightarrow B$. This is the corner stone of Linear Logic, based on a decomposition of the implication into more primitive operators, ! (usually called *bang* and treated as a kind of modality, that Girard prefers to call an *exponential* by reference to its mathematical properties), and --o now known as the linear implication (or *lollypop*!).
Starting from this possible logical reading of this denotational semantics, Girard expressed it in a *sequent calculus*, which is very similar to the one Gentzen provided for classical logic, but with an important difference. Because Linear Logic is sensitive to the amount of resources, structural rules, usually called *Weakening* and *Contraction* are removed. The reason for that is clear. When translated into functional terms, the Weakening rule would say that we should be allowed to give more resources to the computation than required (and therefore "useless resources"), and the Contraction rule would say that we should be allowed to consider two accesses to the same input as being the same one, thus violating the limitation to the number of accesses.
Such a restriction has a very strong consequence. Now, two rules which can be very simply shown equivalent in classical logic (by means of Weakening and Contraction) like the following ones :

$$\frac{\Gamma \rightarrow A, \Gamma' \quad \Delta \rightarrow B, \Delta'}{\Gamma, \Delta \rightarrow A \wedge B, \Gamma', \Delta'} \qquad\qquad \frac{\Gamma \rightarrow A, \Delta \quad \Gamma \rightarrow B, \Delta}{\Gamma \rightarrow A \wedge B, \Delta}$$

are no longer equivalent, thus giving room for two different connectives, which behave according to the two following rules: ([⊗, R] and [&, R])

$$\frac{\Gamma\to A, \Gamma' \quad \Delta\to B, \Delta'}{\Gamma,\Delta\to A\otimes B,\Gamma',\Delta'} \qquad \frac{\Gamma\to A,\Delta \quad \Gamma\to B,\Delta}{\Gamma \to A\& B,\Delta}$$

By symmetry, we get four distinct connectives, and their rules for left and right introduction : (respectively [℘, L], [⊕, L], [⊗, L], [&, L], [℘, R], [⊕, R])

$$\frac{\Gamma,A\to\Gamma' \quad \Delta,B \to\Delta'}{\Gamma,\Delta,A\wp B \to\Gamma',\Delta'} \qquad \frac{\Gamma,A\to\Delta \quad \Gamma,B \to\Delta}{\Gamma, A\oplus B \to \Delta}$$

$$\frac{\Gamma, A, B \to \Gamma'}{\Gamma, A\otimes B \to \Gamma'} \qquad \frac{\Gamma,A \text{ [resp. B]}\to\Gamma'}{\Gamma, A\& B \to \Gamma'}$$

$$\frac{\Gamma\to A, B, \Gamma'}{\Gamma\to A\wp B, \Gamma'} \qquad \frac{\Gamma \to A\text{[resp. B]}, \Gamma}{\Gamma\to A\oplus B, \Gamma'}$$

This formulation also needs to express two identity rules: the axiom and the cut rule.
Axiom : $A \to A$

Cut-rule:

$$\frac{\Gamma\to A, \Gamma' \quad \Delta, A\to \Delta'}{\Gamma,\Delta\to \Gamma',\Delta'}$$

Girard showed that the latter is eliminable, and thus, that Gentzen's Hauptsatz applies to Linear Logic.
In all these formulations, block Greek letters indicate finite lists of formulas. The only structural rule is the *Permutation* rule, which allows us not to take care of the order of formulas. Intuitively, the linear implication should have as its rules:

$$\frac{\Theta\to A, \Gamma' \quad \Gamma,B,\Delta\to \Delta'}{\Gamma, \Theta, A\multimap B,\Delta\to \Gamma',\Delta'} \qquad \frac{\Gamma,A \to B, \Delta}{\Gamma \to A\multimap B,\Delta}$$

In order to have the same property as in classical logic concerning the implication, i.e. the fact that it can be expressed in terms of disjunction ($A\Rightarrow B = \neg A\vee B$), we have to introduce a special negation, called *linear negation*, denoted by $^\perp$ which has the following rules:

$$\frac{\Gamma \to A, \Delta}{\Gamma,A^\perp \to \Delta} \qquad \frac{\Gamma,A \to \Delta}{\Gamma \to A^\perp,\Delta}$$

by means of which it is possible to show the analogues of the De Morgan laws:

$$(A\otimes B)^\perp = A^\perp \wp B^\perp \quad (A\wp B)^\perp = A^\perp\otimes B^\perp \quad (A\& B)^\perp = A^\perp\oplus B^\perp \quad (A\oplus B)^\perp = A^\perp\& B^\perp$$

7.3. Game theoretical aspects

Andreas Blass ([Blass, 1992]) gave an appealing interpretation of Linear Logic, based on the *client-server* metaphor. It is this interpretation that can help us for our purpose. He says:

"Until now, we have regarded types as essentially synonymous with sets. Computationally, a type could be regarded as a server from which a client can get, in one access, an element of that type; the client need not do anything more than show up". This kind of type would be said *simple*... simply because the client has nothing to do! But we could also build *complex* types. In fact, we can understand A&B as *a choice*, the right rule given above can be read: the resources you have can provide you with A as well as with B, therefore they provide you a *choice* between the two kinds of data. We may therefore have the complex type A&B which is a server from which the client can get either one A or one B: he must only make a choice. Continuing in this vein, because $\oplus$ has a symmetric law with regards to &, it seems natural to understand it as a dual choice : the client has nothing to do because the server makes the choice. *De Morgan laws interpret as a simple exchange of roles*: the server becomes the client and reciprocally!

Let us think, say, of a client who phones to a booking office in order to make reservations in a hotel: she asks for a room. The man or woman in the office gives her some choice : bath or shower? TV or not TV? The client makes her choice (the type is **bath** & **shower** or **TV** & **not-TV**}). The client may also choose the price. After that, the office *makes his own choice* concerning, say, the location, according to the rooms which are still free in the hotel. Having made her choice, the man or the woman in the office can still give the floor to the client and she may say whether she will come by car or not, so telling whether she needs a garage or not.

Taking again from Blass, we shall say that "we regard a data type as consisting not only of a set of possible data elements but also as the access protocol that is to be run before a data element is provided".

Formally, a *protocol* is a pair consisting of:

- a non empty set H of finite sequences, the possible histories of the protocol up to any point in its execution, such that every initial segment of a sequence in H is also in H and such that no infinite sequence has all its finite initial segments in H, and
- a function N from H to {c, s, t} such that a node h with N(h) = t cannot be a proper initial segment of any other node in H

(N(h) = c means that the next move is by the client, N(h) = s that the next move is by the server and N(h) = t that the protocol is terminated).

7.4. The dialogical meaning of the multiplicatives

It remains to interpret the other connectives in this light. It is suggested that $\otimes$, being cumulative, expresses several tasks to be performed necessarily before reaching the terminating point. Because Linear Logic is commutative, it seems that the order between these tasks does not matter: in fact, the dialogue will reach the same point if the man or woman in the office asks first whether the client comes by car or whether she wants a shower or a bathroom... But it can be otherwise in other cases, simply because some choices can depend on others, for instance it is unlikely for the server to propose a garage before knowing whether the client comes by car. A reasonable amount of non-commutativity seems to be required in this case (cf. [Retoré, 1997]).

Blass ([Blass, 1994], [Blass, 2002]) proposes a more elaborate interpretation of the multiplicatives. "A play of $A\otimes B$ consists of interleaved runs of the two constituents A and B." Whenever it is the proponent's turn (= server's turn) to move, he or she must move in the same component in which Opponent (= client) last moved, while Opponent is free to switch components. Let us suppose A and B be two games, like two different chess games, they are played "in parallel". At each move, Opponent chooses the game where he wants to play, and Proponent must reply in the same game. The complex game expressed by $A\otimes B$ is terminated when both games A and B are. If we take the dual formula $A^{\perp} \wp B^{\perp}$, we simply interchange the two roles: Proponent can switch and Opponent has to reply in the game Proponent

imposes to her. This situation is now familiar in the literature on Game-semantics, from the famous example of the game against Karpov and Kasparov: I have a winning strategy (called the *copy-cat* strategy) against Karpov *or* Kasparov. I simultaneously play against both (who cannot communicate) and I copy each move made by one of them on the game played against the other. So, if one game is A, the other (obtained by the role exchange) is $A^{\perp}$, and because Proponent (here *I*) can switch, the whole game is expressed by $A \wp A^{\perp}$, which is a provable formula in LL (simply coming from the axiom $A \rightarrow A$, which gives $\rightarrow A^{\perp}, A$ and therefore $\rightarrow A^{\perp} \wp A$).
In any case, $A \otimes B$ means a complex game with the impossibility for the Proponent to make use of information obtained from one component in playing in the other, while $A \wp B$ means such a game where she can.
This of course reflects in the behaviour of the linear implication. As we know, $A \multimap B$ is equal to $A^{\perp} \wp B$. Therefore players simultaneously play A and B, but in A, players' roles are reversed, and of course information coming from A can be used in playing B. Blass comments: "I could win B if I were shown how to win A", and further: "In effect, the players are playing B but the proponent may, whenever he wishes, temporarily suspend the play of B to consult an expert oracle about how to play A. The consultation consists of the proponent acting as the opponent in a play of A while the oracle, acting as proponent in that play, shows him how to win A". The proponent for A is also the opponent for B, thus really entering into dialogue.
We can take as a funny example of such a situation the well known following joke: two persons sit before two unequal pieces of meat, one big and one small. One of them, B takes the biggest one, they have the following dialogue:

A - you are not fair, you took the biggest one
B - what should have you done if you had to choose?
A - I should have taken the smallest one, of course!
B - So you are happy, because you have it!

In this piece of dialogue, B acts exactly like we suggest in the previous paragraph, i.e. by making A playing a reverse game and by deducing its move from the one made by A. In fact, there are four games according to the player who plays first and whether he or she plays with politeness rules or not. B plays the game where she begins and where she does not use politeness rules, but she asks A to play the game where A begins and uses them.

7.5. Problems of a complete semantics for linear logic

Connectives of linear logic have thus been shown to be naturally interpreted by game semantics, but if compared with dialogues, linear logic quickly shows its limitations. A classical theorem of Zermelo asserts that if a game of perfect information, between two players, in which every play is a win for exactly one player, is such that it always ends up in a finite number of moves, then the game is determined in the sense that one of the players has a winning strategy ([Blass, 2002}). This has an important consequence for linear logic: if we identify valid formulae with those for which Proponent has a winning strategy and invalid ones with those for which Opponent has a winning strategy, Zermelo's theorem implies that if we limit ourselves to this particular class of games, all the formulae which express them are either valid or invalid, and it can be shown then that the distinctions between $\wp$ and $\oplus$ on one hand and between $\otimes$ and & on the other collapse, thus leading us back to classical logic. In order to save the completeness of Game semantics for Multiplicative Linear Logic, Abramsky and Jagadeesan ([Abramsky and Jagadeesan, 1992]) are therefore led to admit *infinite games*

and to restrict admissible strategies (to those which are "uniform" and "history-free"). But there could be as well another issue.
Zermelo's theorem says that games of *perfect* information, between two players, in which every play is a win for exactly one player are in some sense trivial, but these are not actually plays we enter into in real life, simply because we play games with *imperfect* information or games in which every play is not necessarily a win for one of two players, but perhaps a win for both or... a win for none!
We shall therefore suggest further that games can be composed by linear logical operators, say in *protocols*, but that at some stage of the decomposition, we have particular (imperfect information) games which have no necessary winning strategy for one player or the other and which are not necessarily of complete information.

7.6. Dialogical acts and combinations of games

If elementary sentences can be understood as simple dialogical games using specific rules together with ordering principles, as soon as they enter into real *dialogical acts*, they are parts of processes which are analogous to *protocols*. By a dialogical act, we mean any use of a verbal expression in a context given by two or more participants. The clearest examples we have of such acts are given by requests. We may have very simple requests, like:

(1) (in a library) A: *I would like a novel by Zola*

or more complex ones like:

(2) *I would like a particular room in your hotel, such that somebody that I know slept in it… I would like the same room.*

We make the assumption that such acts are made by combining elementary atomic games by means of the linear connectives. In a simple request like (1), the interaction may be represented by a game between a client and a server such that after the client asserts (1), the server embeds the verbal material in a linear formula which expresses the fact that “if you give me the title of the novel by Zola that you want to get, then I will (try to) provide you with it”. Such an exchange can be represented by a linear implication A –o B where A is the game consisting for the server in asking to the client what novel she has in mind, and B is the game played by the server trying to find it in the library.
A request like (2) is more complex because it contains more elementary pieces to combine. We would like to associate with it a class of dialogues exemplified by the following one:

0. **P**: *give me your request*
1. **O**: *I would like the same room as the one where slept one of my friends*
2. **P**: *give me the name of your friend*
3. **O**: *Mr. Nightingale*
4. **P**: *OK, I shall try to give you the room where Mr. Nightingale slept*
5. **O**: *?*
6. **P**: *he slept in room 203*
7. **O**: *can you provide me that room?*
8. **P**: *yes, I can/ no, I can’t*

This dialogue shows that before the server playing the consequent (the game in which the server answers the request), two elementary games are necessarily played: one where the client has to give the name of somebody she knows who stayed in that hotel some times ago, and another one where the server has to find the right number of the room, depending on the name of the person. These two games are connected by a multiplicative connective (because there is no choice, the two pieces of information are needed), and they are dual to each other because in the first one, the client has to give an information, and in the second one, it is server's turn to give a correct information. Expressing the exact combination is not easy because of the quantifiers. We shall use the same trick as in Hintikka & Sandu, 1997, concerning the difference between *priority scope* and *binding scope*. Let us recall that the priority scope (indicated by brackets) indicates the relative priority of different logically active ingredients of the sentence and that the binding scope (indicated by parentheses) indicates for each quantifier the segment where pronouns have that particular quantifier as their heads. Following these distinctions, we shall write the formula (3) as representing the combination of elementary games which enter into (2):

(3) $\exists_1 x \exists_2 y \Big([[(known(x)] \otimes [slept-in(x,y)^{\perp})_1]] \multimap can\text{-}provide(y) \Big)_2$

This formula implies that the first game inside brackets is played first, with an existential (A asserts that she knows somebody who slept in that hotel), then the second one is played, also with an existential, but played by B (this is expressed by the linear negation). The values of x and y are kept all over the segment delimitated by parentheses having same index as their corresponding binder.

Of course, in such a formula, the order of quantifiers is relevant (even if they are of same type). If we had:

(4) $\exists_2 y \exists_1 x \Big([[(known(x)] \otimes [slept-in(x,y)^{\perp})_1]] \multimap can\text{-}provide(y) \Big)_2$

the first game to be played would be the second one, represented by [slept-in(x, y)$^{\perp}$]. Winning strategies would be different. This complex game would suppose that the client actually does not *a priori* know the person who slept there before, she only knows that there has been someone, and the server tries to solve the problem by enumerating a list of persons who slept recently in that hotel. Therefore (4) would be associated with a non specific reading of the same request. We can see that this has something to do with the ambiguities of the indefinite *a(n)*, when inserted into a request (cf. *Peter wishes to marry a Swiss girl…*).

8 Conclusion

This paper was mainly a survey of existing approaches which mix logic and dialogue. We think that they complement each other. Concerning their application to language, we saw that GTS leads to interesting views, expressed in the IF-logic, according to which a proper semantics for elementary sentences in ordinary language is dialogical (cf. the examples of *any* and of the generalized quantifiers like "as many B's as A's"). But *speech acts*, which are in fact *dialogical* ones, require several such elementary expressions be put together: this seems to require another kind of logic, a *glue logic*. We showed that this role could be played by *linear logic*. Finally, we sketched an approach of *argumentative dialogues*, where Lorenzen's concepts seem to be helpful, because in many debates, points are scored not by asserting some atomic facts which happen to be true in some model, but by forcing the partner to concede facts. This kind of debate is in fact closer to Lorenzen's procedural approach than to Hintikka's model based one, but those two approaches can be mixed in the neutral form of Extensive Games.

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