Opportunist politicians and the evolution ofelectoral competition

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Abstract

We study a unidimensional model of spatial competition between two parties with two types of politicians. The office oriented politicians, referred to as "opportunist" politicians, care only about the spoils of the office. The policy oriented politicians, referred to as "militant" politicians have ideological preferences on the policy space. In this framework, we compare a winner-take-all system, where all the spoils go to the winner, to a proportional system, where the spoils of office are split among the two parties proportionally to their share of the vote. We study the existence of short term political equilibria and then, within an evolutionary setup, the dynamics and stability of policies and of party membership decisions.

1 Introduction

In this paper we use a classical model of electoral competition to study the evolution of party membership. The citizens choose the government in the elections and thereby the policy to be implemented. The parties compete by offering policies and make credible commitments to implement these policies in case they are elected. The preferences of voters and the political competition together determine the collective outcome and the membership decisions of opportunist politicians are driven by the prospects of being elected. A main feature of political parties is that they are composed of factions who differ in their political motivations: one faction is an office seeker while the other faction cares about the ideological platform of the party. The former will be referred to as the opportunist faction and the latter will be referred to as the militant faction, following Roemer (1999). The previous formal analysis about political competition suppose that there are opportunist politicians in each party competing in the elections, but we need to answer why the opportunist politicians, who care only about the prospects of winning the elections and holding an office, choose to be in one party rather than the other or whether there should be opportunist politicians in each party. This paper attempts to answer these questions by treating the membership decision of a politician to a party as a strategic choice and by introducing an evolutionary setup. While the membership decisions of opportunist politicians is analysed within an evolutionary setup, the political outcome is identically to previous formal analysis given by the equilibrium of the simultaneous move electoral competition. This context allows us to view the political competition as a dynamic process and provides an explanation to the dynamic aspect of the political competition.

1.1 Related literature

The political interpretation of spatial models of competition dates back to the famous discussion of duopolists by Hotelling (1929). Hotelling formulated the tendency of competitors to be exactly alike under the principle of minimum differentiation and suggested that this principle can be applied to a wide range of social phenomena including the political competition, referring to the ideological similarities between the Republican and the Democratic platforms in the elections of 1928. This intuition was later formulated as "the median voter theorem" by Black (1948) in the case voters were characterised by single peaked preferences.

Downs (1957) extended the spatial model of competition to representative democracy where two candidates competed by offering policies from a unidimensional policy space. Under the Downsian approach the political parties are considered to be organised for the purpose of winning the elections, therefore the policy makers are supposed to shape their policy proposals in order to please the majority. In this sense the competitors are identical in all respects and the common equilibrium policy proposal is found at the preferred policy of the median citizen.

Along with this oversimplified view of the politicians as vote maximisers, there has been a long tradition from Michels (1915) to Lipset (1959) in which parties are ideological and they have policy preferences. This idea has later been formalised by Wittman (1983), Calvert (1985) and Hansson and Stuart (1984) who characterised parties as institutions that represent contesting interest groups in the society. Thus the competing candidates become differentiated as each represents an interest group in the society and seeks publicity for a different ideological platform. These politicians are supposed to have preferences over the policy space and to propose policies accordingly with the essential feature that parties and their ideologies are exogenously given.

These two approaches have been combined by Roemer (1999) who conceptualised parties as consisting of an opportunist faction and a militant faction[1]. Opportunists are those who wish to maximise the probability of victory, and militants are those who want to maximise the utility of the citizen the party represents. The opportunists belong to Downs's (1957) conception of politics and the militants to Wittman's (1983) conception of politics. In this context, a party-unanimity Nash equilibrium (PUNE) is defined. This consists of policies which are in equilibrium in the following sense: in neither party can the (internal) factions agree on a deviation from the proposed policy, given the other party's policy proposal.

1.2 Outline of the model

This paper adopts the previous approach but adds a dynamic dimension. Roemer (1999) studies a static model where there is a fixed number of opportunist politicians in each party. Here, we will suppose that this number is fixed in the short term but can evolve in the long run. The opportunist politicians are supposed to review their membership decisions in the pre-election period according to evolutionary dynamics. Moreover a technical difference from the previous analysis is that Roemer (1999) adopted a decision rule where each faction in the party has veto power over the policy proposals. Here, the internal decision mechanism is simply taken to be a weighted average of proposals of different factions[2].

More precisely, we consider the following electoral cycle:

[1] In Roemer (1999), there is a third faction, the so-called reformists, who maximise the expected utility of party constituents. We do not include the reformist faction in our model.

[2] In Roemer (2001), it has been also shown that when each party works out a method of inner-party bargaining, the policy proposal that they reach as a consequence of inner-party bargaining is a PUNE since at that proposal, no parties' factions would agree to deviate to another policy.

1. Given the party membership profiles each party announces the party platform which is obtained by the aggregation of the policy proposals of the factions. The aggregation rule is the weighted average of the policy proposals of each faction. The influence of each faction is proportional to its weight in the party. The proposal of the militant faction is given exogenously. The proposal of the opportunist faction is obtained through the maximisation of their utilities given the policy proposal of the competing party. However, the opportunists are supposed to be subject to a certain constraint in their decision making processes. The opportunist faction of the party L (R) can not propose a platform greater (less) than the platform of the party R (L). This is taken as an assumption but could be rationalized in several ways. For instance, one can think that, in the process of choosing the party platform, a politician would be expelled from the party if he was to propose a platform "of the opposite side", precisely for party L a platform at the right of party R platform.

2. The political outcome is determined by the elections. We consider two political systems differing in the way that votes are translated into seats in an assembly: the *proportional* system and the *winner-take-all* system. These two systems differ by the rewards that accrue to vote shares. In the winner-take-all system, all the spoils of office go to the winner. In the proportional system, the spoils of office are split among the candidates proportionally to their share of the vote. The spoils of office represent the benefits for a party of being able to implement its policy, and the rents from power.

3. Between elections, the opportunist candidates review their membership decisions to the political parties for the following elections. The standard approach is to assume that the rational and optimising politicians can collectively locate the equilibrium of the model and then to compute the equilibria of the two stage game where the opportunists review their membership decisions at the first stage and the parties make their policy proposals at the second stage[3]. This approach will not provide insight about the evolution of the behaviour of opportunist candidates from any initial state. As we attempt to study the disequilibrium dynamics, we will suppose that initially there are opportunist politicians in each party. The proportion of candidates in each party will then evolve based upon the standard *replicator dynam-*

[3] The formal results are provided in Appendix.

ics. This rule essentially means that opportunists candidates tend to enroll preferably into the party that offers them better perspectives. The militants by definition are not supposed to review their membership decisions[4].

1.3 Results

We analyse the behaviour of the opportunist politicians under the proportional and the winner-take-all systems. There are two kinds of states of the opportunist population: the *pure states* where all the opportunists are in one party or the other and the *mixed states* where the opportunists are distributed in both parties. The pure states are rest points of evolutionary dynamics in both systems. We show that, in the winner-take-all system, one only one mixed state is a rest point of the dynamics. In the proportional system, depending on the distribution of voters and the distribution of politicians, there may be zero, one, or more mixed state rest points. In the winner-take-all system, only the pure states are stable regardless of the distribution of the voters. The mixed state is not stable. In the proportional system, the stability of the pure and mixed states depend on the distribution of voters and the distribution of politicians. Only in this case can there be stable mixed states.

The paper is organised as follows. In the next section the political economic environment is formulated. Section 3 describes the aggregation of policy proposals of different factions within each party and provides the short term equilibrium results. Section 4 introduces the evolutionary process used to analyse the behaviour of opportunist politicians and gives the long run stability results. Section 5 concludes with a discussion of the results.

2 The model

2.1 The voters

A society has to decide collectively on a policy t such as a redistributive income tax levied by the government in order to finance a public good that is equally valued by all citizens. The policy space is the unit interval. Each citizen evaluates the policies according to their utility and uses the only one

[4] Other behavioral models of dynamic political competiton use the techniques of agent-based modeling (Kollman, Miller and Page 1992; Kollman, Miller and Page 1998; De Marchi 1999; Kollman, Miller and Page 2003; Laver 2005).

vote he has for the policy he likes best. Each voter has well-defined single-peaked political preferences given by an ideological position. The voters are distributed according to their ideal policies on the unit interval by a cumulative distribution function F, so that $F(t) = \Pr(T \leq t)$ where T is a random variable describing the voters' ideal policies. The average ideal policy is

$$\overline{t} = \int_0^1 t dF(t) \tag{1}$$

and the median voter is given by

$$t^* = F^{-1}(\frac{1}{2}) \tag{2}$$

The utility for a voter of a given policy t is given by minus the square of his actual distance with the policy. Then, the utility of a voter whose ideal policy is t_i is given by the following equation:

$$u(t) = -(t - t_i)^2 \tag{3}$$

This utility definition implies that all voters will prefer the policy which is closer to their ideal policy.

2.2 The parties

There are two political parties: party L and party R. These parties compete by offering the policies t_L and t_R. The citizens vote for the parties according to these proposals. Thus they indirectly choose the policy. In other words, the voters' preferences and the parties' proposals together determine the policy to be implemented.

A majority (or Condorcet) winning tax policy is the policy t^c that is preferred by some majority of individuals to any other policy $t \in [0, 1]$. In this setting, we define by $\pi(t_L, t_R)$ the probability that the party L wins when party L propose t_L and party R propose t_R. Consequently, the probability that party R wins will be $1 - \pi(t_L, t_R)$. If the majority of the population prefers t_L to t_R then $\pi(t_L, t_R) = 1$. If the majority of the population prefers t_R to t_L then $\pi(t_L, t_R) = 0$. If the same number of people vote for t_L and t_R, we have $\pi(t_L, t_R) = \frac{1}{2}$; in this case each party is elected with probability $\frac{1}{2}$.

Since the preferences of the voters are single-peaked, the majority of the votes will depend on the preferences of the median voter. Thus, the probability that the party L wins when party L proposes t_L and party R proposes t_R is given by the following equation:

$$\pi(t_L, t_R) = \begin{cases} 1 & \text{if} \quad \frac{t_L+t_R}{2} > t^* \\ \frac{1}{2} & \text{if} \quad \frac{t_L+t_R}{2} = t^* \\ 0 & \text{if} \quad \frac{t_L+t_R}{2} < t^* \end{cases} \tag{4}$$

Each party consists of two competing factions, the 'militants' and the 'opportunists'. The opportunists are only willing to maximise the probability of their party's victory. On the other hand, the militants always propose the party's ideal point. The factions have divergent interests, we make the assumption that the policy proposals of each party is the weighted average of the proposals of the factions. The electoral platforms are therefore determined according to the party aggregation rule –average proposal– and the strategic behaviour of the opportunist faction.

3 The policy proposals with given party membership

Political parties can choose among the same set of feasible policies. The decision within the parties for a policy proposal is the weighted average of the proposals of the factions. The weight of a faction is the proportion of its members in the party. The proportion of militants in party i is denoted by α_i where $i = L, R$. We denote by t_i^j the policy proposals of the militants and the opportunists in party i where j will stand for O, M. We suppose that the policy of the party i is as follows:

$$t_i = \alpha_i t_i^M + (1 - \alpha_i) t_i^O \tag{5}$$

The militant members propose always the party ideology. For simplicity, we suppose that the ideologies of the party L and the party R are 0 and 1 respectively.

When we define the utility of the opportunist politicians we have to consider the political systems since the opportunist politicians are seeking for the benefits from implementing their policies and the rents from power. We consider two political systems: the proportional system and the winner-take-all system. In the former, the rewards that accrue to votes are split proportionally to the share of votes of each party and the utility of an opportunist candidate is the share of the votes per candidate. In the latter the winning party has all the benefits and the utility of an opportunist candidate is the probability of winning divided by the number of candidates. This captures the ideas that the benefits from the elections are equally shared by all the politicians, or that the politicians have an equal opportunity to get the benefits.

The party L opportunists maximise $\pi(t_L, t_R)$ or $F(\frac{t_L+t_R}{2})$ per candidate depending on the political system, subject to the constraint $t_L^O \leq t_R$. Consequently, they will propose the highest possible value, namely

$$t_L^O = t_R. \tag{6}$$

Likewise, the party R opportunists maximise $1 - \pi(t_L, t_R)$ or $1 - F(\frac{t_L+t_R}{2})$ per candidate depending on the political system subject to the constraint $t_R^O \geq t_L$. They will propose the smallest possible value, namely

$$t_R^O = t_L. \tag{7}$$

Solving for $t_L^O = t_R$ and $t_R^O = t_L$ in equations (5) will result in a Nash equilibrium of the game which is a pair of policies (t_L^N, t_R^N) such that:

$$t_L^N = \frac{(1-\alpha_L)\alpha_R}{1-(1-\alpha_L)(1-\alpha_R)} \tag{8}$$

$$t_R^N = \frac{\alpha_R}{1-(1-\alpha_L)(1-\alpha_R)} \tag{9}$$

Given our hypothesis on the voters' preferences, in order to analyse the probability of victory, we just have to calculate the midpoint of the pair of policies $t^N = \frac{t_L^N+t_R^N}{2}$:

$$t^N = \frac{\alpha_R(2-\alpha_L)}{2(1-(1-\alpha_L)(1-\alpha_R))} \tag{10}$$

Notice that $0 \leq t_L^N \leq 1$, $0 \leq t_R^N \leq 1$ and $0 \leq t^N \leq 1$.

3.1 Party composition

By definition, the militants do not update their membership decisions since they are supposed to derive satisfaction only by being a member of their party and struggling for the ideology of the party. On the other hand, the opportunists are supposed to sometimes update their membership decisions between the elections. Consequently, the set of politicians has three subsets: the militants of party L, the militants of party R and the opportunists. We denote by l the number of militants in party L and by r the number of militants in party R. The number of the opportunists in the total politician population is n. The proportion of opportunists in party L is denoted by s. Thus, the proportion of opportunists in party R is denoted by $1-s$. The number of opportunists in party L is ns. The number of opportunists in party R is $n(1-s)$. Without loss of generality, we normalise the number of politicians to unity ($l+r+n=1$). We can express the previous values and results in terms of the new parameters as follows.

The proportion of militants in party L as a function of the share of the opportunists in party L is:

$$\alpha_L(s) = \frac{l}{l+ns} \tag{11}$$

The proportion of militants in party R as a function of the share of the opportunists in party L is:

$$\alpha_R(s) = \frac{r}{r+n(1-s)} \tag{12}$$

Consequently, the average of the equilibrium pair of policies is given by the following equation:

$$t^N(s) = \frac{r(2s(1-l-r)+l)}{2(l(1-l)(1-s)+sr(1-r))} \tag{13}$$

Remark. Notice that $t^N(0) = \frac{r}{2(r+n)} \leq \frac{1}{2}$ and $\frac{1}{2} \leq t^N(1) = \frac{2n+l}{2(l+n)} \leq 1$. The first and second derivatives of $t^N(s)$ with respect to s are

$$\frac{\partial t^N(s)}{\partial s} = \frac{nrl(r+2n+l)}{2\left(lr+n(l(1-s)+sr)\right)^2} \geq 0$$

and $\frac{\partial^2 t^N(s)}{\partial s^2} = \frac{n^2 r(l-r)l(r+2n+l)}{(lr+n(l(1-s)+sr))^3} \geq 0$ if $l \geq r$ and $\frac{\partial^2 t^N(s)}{\partial s^2} = \frac{n^2 r(l-r)l(r+2n+l)}{(lr+n(l(1-s)+sr))^3} \leq 0$ if $l \leq r$ so that we have the graph (1) for $t^N(s)$ when $l \geq r$ and $s \in [0,1]$. Notice also that $t^N(s)$ has its minimal value $\frac{r}{2(r+n)}$ when $s=0$ i.e. when all the opportunists are in party R and $t^N(s)$ has its maximal value $\frac{2n+l}{2(l+n)}$ when $s=1$ i.e. when all the opportunists are in party L.

4 Evolution

4.1 The replicator dynamics

The rationalistic approach to game theory assumes that players are perfectly rational, the game is played once and the game and the equilibrium are common knowledge. On the other hand, the evolutionary approach assumes that

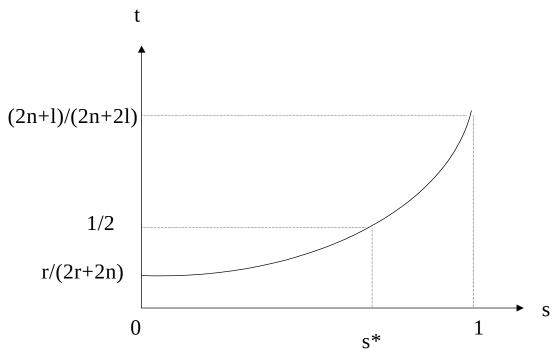

Figure 1: The average of the equilibrium pair of policies

boundedly rational players who are randomly drawn from large populations and who have little or no information about the game, play the game repeatedly. Thus, the evolutionary approach allows us to analyse a game theoretic situation when we relax the perfect information and unbounded rationality assumptions. The main difference between these approaches is that the rationalistic approach analyses the individual behaviour while the evolutionary approach analyses the population distribution of behaviours (strategies).

Classically, in Biology, the analysis of population dynamics includes two processes: *the selection process* favoring better performing strategies and *the mutation process* introducing varieties.

Non-biological interpretations of evolutionary game theory have been proposed. Borgers and Sarin (1997) and Laslier, Topol and Walliser (2001) show that models of individual learning by reinforcement can be approximated by the replicator dynamics. Bjonerstedt and Weibull (1996) show that the replicator dynamics may be derived from a number of learning by imitation models, where revising individuals imitate other individuals. Schlag (1998) shows that the behavioural rule which outperforms the other improving rules is the one where agents imitate the action of an observed individual and when each individual imitates the aggregate population behaviour is approximated by replicator dynamics.

In this article, we analyse the distribution of behaviours of opportunist politicians. They can belong to party L or party R. The utility of being a candidate of the party L is $u_L(t_L(s), t_R(s))$. The utility of being a candidate of the party R is given by $u_R(t_L(s), t_R(s))$. The average utility of the opportunist candidates is $\bar{u}(t_L(s), t_R(s))$.

$$\bar{u}(t_L(s), t_R(s)) = s u_L(t_L(s), t_R(s)) + (1-s) u_R(t_L(s), t_R(s)) \tag{14}$$

The selection process determines how population shares corresponding to different pure strategies evolve over time. This process is based on the survival of the fittest. In other words, the share of the population playing relatively better performing strategies increases. The selection dynamics governing change are in continuous time and are regular selection dynamics[5]. Taylor and Jonker (1978) defined a particular monotonic selection dynamics called the replicator dynamics. In our model, the replicator dynamics differential equation writes:

$$\dot{s} = (u_L(t_L(s), t_R(s)) - \bar{u}(t_L(s), t_R(s)))s \tag{15}$$

It is clear that better performing strategies have a higher growth rate, but this does not necessarily imply that the average payoff grows. The reason is that even if a player is replaced by a player playing a better performing strategy, this new distribution of players may reduce the payoffs of some other players.

4.2 The evolutionary stability

The replicator dynamics describes how the population shares of candidates playing different strategies change over time. The next step will be to determine the rest points of the replicator dynamics and analyse their stability under the assumption that the parties continuously play their instantaneous

[5] The evolution of the composition of the population is given by a system of continuous-time differential equations: $\dot{s} = \xi(s)$. The function ξ is said to yield a monotonic selection dynamic if the following conditions are satisfied:

i. ξ is Lipschitz continuous

ii. $s = 0 \Rightarrow \xi(s) \geqslant 0$ and $s = 1 \Rightarrow \xi(s) \leqslant 0$

iii. $\lim\limits_{s \to 0} \frac{\xi(s)}{s}$ exists and is finite.

iv. $u_L(t_L(s), t_R(s)) > (=)\ u_R(t_L(s), t_R(s)) \Rightarrow \frac{\xi(s)}{s} > (=)\ 0$

These conditions ensure that s remains in $[0, 1]$, its growth rates are defined and continuous at all points $s \in [0, 1]$ and the growth of the share of the opportunists is proportional to its relative payoff.

equilibrium strategies. The utilities of the opportunist candidates are defined following the political system considered. There are two political systems differing in the way that votes are translated into seats in an assembly: the proportional system and the winner-take-all system.

4.2.1 The proportional system

In the proportional system, the rents from power are split among the candidates proportionally to their share of the vote. Each party gets seats in the parliament equal to its vote share. The utility of an opportunist candidate is the share of the votes per candidate of his party.

Formally, the utility of an opportunist candidate of the party L is:

$$u_L(t_L(s), t_R(s)) = \frac{1}{l + ns} F\left(\frac{t_L(s) + t_R(s)}{2}\right) \tag{16}$$

and the utility of an opportunist candidate of the party R is:

$$u_R(t_L(s), t_R(s)) = \frac{1}{r + n(1-s)} \left(1 - F\left(\frac{t_L(s) + t_R(s)}{2}\right)\right) \tag{17}$$

As we study the rest points of the replicator dynamics when the parties play their equilibrium strategies, the replicator dynamics will be given by the following equation:

$$\dot{s} = \left(\frac{F(t^N(s))}{l + ns} - \frac{1 - F(t^N(s))}{r + n(1-s)}\right) s\,(1-s) \tag{18}$$

The replicator dynamics has the trivial rest points at $s = 0$ (the state where all opportunists are in party R) and $s = 1$ (the state where all opportunists are in party L). The other rest points of the replicator dynamics are given by the following equation:

$$F(t^N(s^*)) = l + ns^* \tag{19}$$

which is the case where the number of votes of the party L is equal to the share of left party candidates in total candidate population.

Proposition 1 *In the proportional system the state where all opportunists are in party L ($s = 1$) is stable if $F(t^N(1)) > l + n$. The state where all opportunists are in party R ($s = 0$) is stable if $F(t^N(0)) < l$.*

The proof is provided in the Appendix.

Example 2 *The distribution of voters is given by $F(x) = x$. This is the case where the population is distributed uniformly along the unit line.*

The nontrivial rest points of the replicator dynamics are given by the following condition:

$$F(t^N(s)) = \frac{2nsr + lr}{2\,(lr + n(l(1-s) + sr))} = l + ns \tag{20}$$

When the number of militants of party L and R are equal ($l = r$), this equation has the solution $s = \frac{1}{2}$ with no additional assumptions about the weight of the opportunists in the candidate population. The right hand side and the left hand side of the equation (20) have been depicted by the graph (2) for the case $n = 0.3$. The right hand side is depicted by the dashed line and the left hand side is depicted by the solid line. We have $F(t^N(0)) < l$. In this case, since all the opportunist candidates are in party R and the utility of being a candidate of party L is less than the utility of being a candidate of party R, $s = 0$ is stable. Notice that $F(t^N(1)) > l + n$. As all the opportunists are in party L and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, $s = 1$ is stable. From the previous results, we conclude that $s = \frac{1}{2}$ is not stable.

When the number of militants of the party L and R are different ($l \neq r$), the equation (20) has at most two solutions. We have also the constraint $s \in [0, 1]$ and the condition $r + l < 1$ to be satisfied. $F(t^N(0)) < l$ if and only if $r < 2l\,(1 - l)$. In this case, since all the opportunist candidates are in party R and the utility of being a candidate of party L is less than the utility of being a candidate of party R, $s = 0$ is stable. We have $F(t^N(1)) > l + n$ under $l < 2r(1-r)$. As all the opportunists are in party L and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, $s = 1$ is stable. Notice that if we satisfy both conditions there will be only one solution to the equation (20) since the distribution function is always increasing in the interval $[0, 1]$ and this solution will not be stable. The case where $s = 0$ is the only stable stable i.e. $F(t^N(0)) < l$ and $F(t^N(1)) < l + n$ (or $r < 2l\,(1 - l)$ and $l > 2r(1 - r)$) there will be no solution to the equation (20). The only stable outcome is the case where all the opportunists are in party R. The case where only $s = 1$ is stable i.e. $F(t^N(0)) > l$ and $F(t^N(1)) > l + n$ (or $r > 2l\,(1 - l)$ and $l < 2r(1 - r)$) there will be no solution to the equation (20). The only stable outcome is the case where all the opportunists are in party L.

Example 3 *The distribution of voters is given by $F(x) = x(2 - x)$. This is a case where the population is distributed non-uniformly along the unit line.*

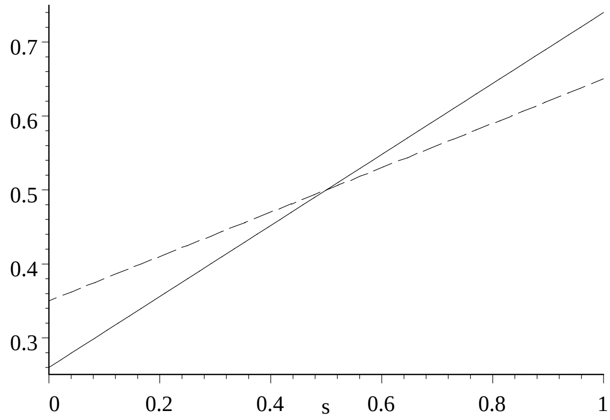

Figure 2: Uniform distribution of voters

Then the nontrivial rest points of the replicator dynamics are given by the following condition:

$$F(t^N) = \frac{2nsr + lr}{2\,(lr + n(l(1-s) + sr))}(2 - \frac{2nsr + lr}{2\,(lr + n(l(1-s) + sr))}) = l + ns \tag{21}$$

There are at most three solutions to this equation. When the number of militants of party L and R are equal ($l = r$), this equation has no solutions. We have $F(t^N(0)) > l$. In this case, since all the opportunist candidates are in party R and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, $s = 0$ is not stable. Notice that $F(t^N(1)) > l + n$. As all the opportunists are in party L and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, $s = 1$ is stable. In case there are equal numbers of militants in each party, only the situation where the opportunists are all in party L is stable. This result is due to the fact that the median is closer to the left party ideology. A symmetric result will apply when we have a nonuniform distribution with the median closer to the right party ideology. The right hand side and the left hand side of the equation (21) have been depicted by the graph (3) for the case $n = 0.3$. The right hand side is depicted by the dashed line and the left hand side is depicted by the solid line.

When the number of militants of the party L and R are different ($l \neq r$), $F(t^N(0)) < l$ if and only if $r(4(1-l)-r) < 4l(1-l)^2$ and in this case since all

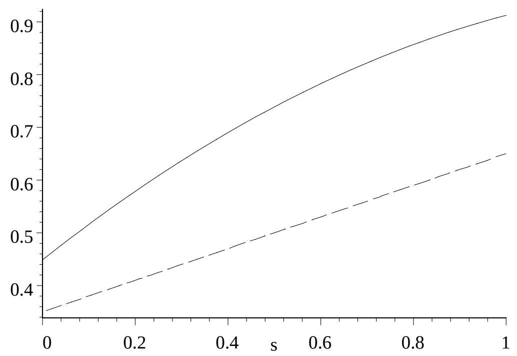

Figure 3: Non-uniform distribution of voters

the opportunist candidates are in party R and the utility of being a candidate of party L is less than the utility of being a candidate of party R, this point is stable. $F(t^N(1)) > l+n$ if and only if $4r(1-r)^2 > l^2$ the opportunists are in party L and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, this point is stable. The existence and the stability of the other rest points are determined accordingly.

The previous examples are provided to illustrate the relationship between the distribution of voters, the distribution of politicians and the stability of the *pure states*. In politics, the *mixed states* seem to be more common than the pure states. Next we provide an example where the *mixed state* as a rest point of the replicator dynamics is the only stable outcome.

Example 4 *The distribution of voters is given by a beta distribution* $F(x; v, w) = \frac{1}{B(v,w)} \int_0^x u^{v-1} (1-u)^{w-1}\, du$. *The beta function with parameters* v, w *is defined by the integral* $B(v, w) = \int_0^1 u^{v-1} (1-u)^{w-1}\, du$.

The nontrivial rest points of the replicator dynamics are given by the following condition:

$$F\left(t^N(s); v, w\right) = \frac{1}{B(v,w)} \int_0^A u^{v-1} (1-u)^{w-1}\, du = l + ns \tag{22}$$

with

$$A = \frac{2nsr + lr}{2\left(lr + n(l(1-s) + sr)\right)}$$

We will study the case where $v = w = \frac{1}{2}$. When the number of militants of party L and R are equal ($l = r$), this equation has the solution $s = \frac{1}{2}$ with no additional assumptions about the weight of the opportunists in the candidate population. We have $F(t^N(0)) > l$ when $r < 0.32$. In this case, since all the opportunist candidates are in party R and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, $s = 0$ is not stable. Notice that $F(t^N(1)) < l + n$ when $r < 0.32$. As all the opportunists are in party L and the utility of being a candidate of party L is less than the utility of being a candidate of party R, $s = 1$ is not stable. From the previous results, we conclude that $s = \frac{1}{2}$ is stable.

4.2.2 The winner-take-all system

In the winner-take-all system, all the rents from power go to the winner. As the winning party has all the benefits, the utility of an opportunist candidate is the probability of winning divided by the number of candidates having in mind that the benefits from the elections have to be equally shared by all the politicians or the politicians have and equal opportunity to get the benefits.

The utility of being a candidate of the party L is $u_L(t_L(s), t_R(s))$.

$$u_L(t_L(s), t_R(s)) = \frac{1}{l + ns}\pi(t_L(s), t_R(s)) \tag{23}$$

The utility of being a candidate of the party R is given by $u_R(t_L(s), t_R(s))$.

$$u_R(t_L(s), t_R(s)) = \frac{1}{r + n(1-s)}(1 - \pi(t_L(s), t_R(s))) \tag{24}$$

As we study the rest points of the replicator dynamics when the parties play their equilibrium strategies, the replicator dynamics will be given by the following equation:

$$\dot{s} = (\frac{\pi(t_L^N(s), t_R^N(s))}{l + ns} - \frac{\pi(t_L^N(s), t_R^N(s))}{r + n(1-s)})s(1-s) \tag{25}$$

The replicator dynamics has the trivial rest points at $s = 0$ and $s = 1$. The other rest points of the replicator dynamics are given by the following equation:

$$\pi(t_L^N(s), t_R^N(s)) = l + ns \tag{26}$$

where the left hand side is the probability of victory and the right hand side is the weight of the politicians of the party L in the total population of politicians. When the parties play their equilibrium strategies the probability of victory becomes:

$$\pi(t_L^N(s), t_R^N(s)) = \begin{cases} 1 & if \quad t^N(s) > t^* \\ \frac{1}{2} & if \quad t^N(s) = t^* \\ 0 & if \quad t^N(s) < t^* \end{cases} \tag{27}$$

The graph of the probability of victory as we have defined is not continuous. From the definition of the probability of victory we can conclude that there may be at most one solution to the equation (26). There are three cases to analyse:

1. If $\frac{r}{2(1-l)} \leq t^* \leq \frac{1-r+l}{2(1-r)}$, the case where all the opportunists are in the party L $(s = 0)$ and the case where all the opportunists are in the party R $(s = 1)$ will both be stable. Let s^M be the solution to the equation (26) then s^M must satisfy the following conditions: $t^N(s^M) = t^*$ and $l + ns^M = \frac{1}{2}$. From the previous results, s^M will not be stable.

2. If $t^* < \frac{r}{2(1-l)}$, t^* is always less than $t^N(s)$ and $\pi(t_L^N(s), t_R^N(s)) = 1$. The case where all the opportunists are in the party L $(s = 0)$ is the only stable state.

3. If $t^* > \frac{1-r+l}{2(1-r)}$, t^* is always greater than $t^N(s)$ and $\pi(t_L^N(s), t_R^N(s)) = 0$. The case where all the opportunists are in the party R $(s = 1)$ is the only stable state.

5 Conclusion

This paper studies the evolution of party affiliation of opportunist politicians in different political systems. When all the benefits of office are shared proportionally to the share of the votes, a situation where the opportunist politicians are in both parties can be a stable outcome. When all the benefits go to the winner of the elections, there exist two possible outcomes: all the opportunists are in the party L or in the party R.

The model may be extended in two ways. The behaviour of policy oriented politicians is based on exogenously given ideologies. The model as such is not complete. The ideologies of the parties may be endogenised. Another extension of the model deals with the behavioural assumption about the opportunist candidates. The evolutionary dynamics adopted here is consistent with, for instance, the idea that opportunist politicians would decide about their membership based on random encounters with other opportunist politicians in the population. However we also suppose that they can perfectly calculate the optimal level of policy to maximise the probability of winning the elections. This assumption may be relaxed and an adjustment process may be coupled with the membership decision. Such extensions could be studied with examples and simulation techniques, but analytical results would become more complex.

A Appendix

A.1 The two-stage game for the electoral competition

In this section we analyse the model as a two stage game.We consider now that at the first stage the opportunists review their membership decisions and at the second stage the party platforms are determined. We can analyse the case where all the opportunists are in the party L or in the party R. Note that the following utilities apply at the second stage:

$$u_L(t_L(s), t_R(s)) = \frac{1}{l+ns} F(t^N(s))$$

$$u_R(t_L(s), t_R(s)) = \frac{1}{r+n(1-s)} (1 - F(t^N(s)))$$

where $t^N(s) = \frac{2nsr+lr}{2(lr+n(l(1-s)+sr))}$.

The opportunists will either choose the party L or the party R. As we would like to see whether the opportunists prefer the party L to party R, we have to analyse the difference of the utility of being in party L when they all choaose party L and the utility of being in party R when they all choose the party R. Let $\Delta = u_L(t_L(1), t_R(1)) - u_R(t_L(0), t_R(0))$. Then we have the following results.

$$\Delta = \frac{1}{l+n} F(t^N(1)) - \frac{1}{r+n} (1 - F(t^N(0)))$$

$$\Delta = \frac{1}{(1-r)} F\left(\frac{2n+l}{2(1-r)}\right) + \frac{1}{(r+n)} F\left(\frac{r}{2(r+n)}\right) - \frac{1}{(r+n)}$$

When we draw the previous function for different definitions of the distribution of voters we obtain the following graph (Fig.4) which describes the region where the utility of being in party L when they all choose party L is greater than the utility of being in party R when all choose the party R. The region is defined according to the values of the number of opportunist politicians and the militants in party R. In the region below the solid line in the graph (4), the opportunists will prefer party R when the citizens are uniformly distributed along the unit line. In the region between the dashed lines in the graph (4), the opportunists will prefer party L when the citizens are nonuniformly distributed along the unit line.

When the distribution of voters is uniform and there are equal number of militants in each party then $\Delta = u_L(t_L(1), t_R(1)) - u_R(t_L(0), t_R(0)) = 0$. In that case the opportunist politicians will be indifferent between two parties but as we have analysed in the chapter 3 the case where they are distributed equally is not a stable outcome.

A.2 Proof of Proposition 1

(**Proposition 1)** In the proportional system the state where all opportunists are in party L ($s = 1$) is stable if $F(t^N(1)) > l + n$. The state where all opportunists are in party R ($s = 0$) is stable if $F(t^N(0)) < l$.

To prove the proposition, we need to prove the following lemma.

Lemma *Given a population state s and a monotonic selection dynamic ξ, s is asymptotically stable if* $(2s-1)(u_L(t_L(s), t_R(s)) - u_R(t_L(s), t_R(s))) > 0$.

Proof. Let $s^* = 0$. If $(2s^*-1)(u_L(t_L(s^*), t_R(s^*)) - u_R(t_L(s^*), t_R(s^*))) > 0$, then $u_L(t_L(s^*), t_R(s^*)) > u_R(t_L(s^*), t_R(s^*))$. By continuity of payoffs in population share, there exists a neighborhood N of s^* such that, for all $s \in N - s^*$, $u_L(t_L(s), t_R(s)) < u_R(t_L(s), t_R(s))$. If the dynamics $\xi(s)$ are monotonic, then $\dot{s} < 0$. Let $L(s) = 1 - s$. $L(s)$ attains its maximum value of 1 when $s = s^*$, and is positive and increasing in $N - s^*$. This is a strict Liapunov function for s and s is asymptotically stable by Liapunov's Stability Theorem.

Let $s^* = 1$. If $(2s^* - 1)(u_L(t_L(s^*), t_R(s^*)) - u_R(t_L(s^*), t_R(s^*))) > 0$, then $u_L(t_L(s^*), t_R(s^*)) > u_R(t_L(s^*), t_R(s^*))$. By continuity of payoffs in population share, there exists a neighborhood N of s^* such that, for all $s \in N - s^*$, $u_L(t_L(s), t_R(s)) > u_R(t_L(s), t_R(s))$. If the dynamics $\xi(s)$ are monotonic, then $\dot{s} > 0$. Let $L(s) = s$. $L(s)$ attains its maximum value of 1 when $s = s^*$, and is positive and increasing in $N - s^*$. This is a strict Liapunov function for s and s is asymptotically stable by Liapunov's Stability Theorem. ■

The lemma simply says that $s = 1$ is stable if $u_L(t_L(s), t_R(s)) > u_R(t_L(s), t_R(s))$ and $s = 0$ is stable if $u_L(t_L(s), t_R(s)) < u_R(t_L(s), t_R(s))$. The proposition now may be proved.

Proof of the proposition. For $s = 1$ to be asymptotically stable we need

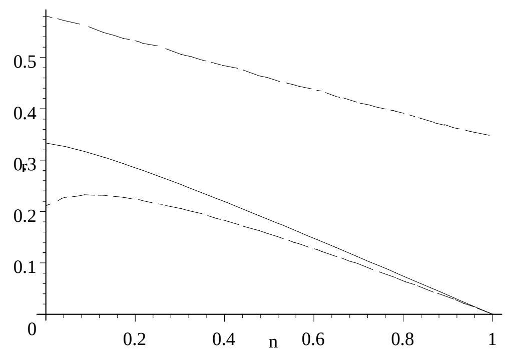

Figure 4: The values of Δ

the following condition:

$$u_L(t_L(1), t_R(1)) > u_R(t_L(1), t_R(1))$$

$u_L(t_L(1), t_R(1)) - u_R(t_L(1), t_R(1)) = \frac{1}{l+n} F(t^N(1)) - \frac{1}{r}(1 - F(t^N(1)))$
$u_L(t_L(1), t_R(1)) - u_R(t_L(1), t_R(1)) = \frac{F(t^N(1)) - l - n}{(l+n)r}$
$\frac{F(t^N(1)) - l - n}{(l+n)r} > 0 \Rightarrow F(t^N(1)) > l + n$
For $s = 0$ to be asymptotically stable we need the following condition:

$$u_L(t_L(0), t_R(0)) < u_R(t_L(0), t_R(0))$$

$$u_L(t_L(0), t_R(0)) - u_R(t_L(0), t_R(0)) = \frac{1}{l} F(t^N(0)) - \frac{1}{r+n}(1 - F(t^N(0)))$$

$$u_L(t_L(0), t_R(0)) - u_R(t_L(0), t_R(0)) = \frac{F(t^N(1)) - l}{(r+n)l}$$

$$\frac{F(t^N(0)) - l}{(r+n)l} > 0 \Rightarrow F(t^N(0)) > l.$$

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