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Klaus HAMBERGER[1], Michael HOUSEMAN[1], Isabelle DAILLANT[1],
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RÉSUMÉ – Les anneaux matrimoniaux : une approche formelle
Les anneaux matrimoniaux sont un type particulier de cycles qui se constituent dans les réseaux de parenté lorsque les conjoints sont liés entre eux par des liens de consanguinité et d'affinité. En adoptant une approche relevant de l'analyse des réseaux, l'article tente d'aborder la théorie structurale de la parenté sur une base générale permettant de dépasser l'examen de types singuliers d'anneaux (comme le « mariage de cousins croisés », l'« l'échange de sœurs », etc). Il offre une définition et une analyse formelle des anneaux matrimoniaux, une méthode d'énumération de toutes leurs classes d'isomorphisme au sein d'un horizon généalogique donné. Cette approche permet d'étudier les configurations d'anneaux dans des réseaux de parenté empiriques. L'article fournit aussi les moyens techniques pour mener ces analyses avec le programme informatique PAJEK. *Un dossier contenant les macros nécessaires peut être téléchargé sur le* web. *La méthode est illustrée à l'aide de réseaux de parenté provenant de quatre continents (Amérique du Sud, Afrique, Australie et Europe.*

MOTS-CLÉS – Anneaux matrimoniaux, Parenté, Analyse des réseaux sociaux, Théorie des graphes, Théorie de l'énumération, Anthropologie sociale

SUMMARY – *The paper deals with matrimonial rings, a particular kind of cycles in kinship networks which result when spouses are linked to each other by ties of consanguinity or affinity. By taking a network-analytic perspective, the paper endeavours to put this classical issue of structural kinship theory on a general basis, such as to allow conclusions which go beyond isolated discussions of particular ring types (like "cross-cousin marriage", "sister exchange", and so forth). The paper provides a definition and formal analysis of matrimonial rings, a method of enumerating all isomorphism classes of matrimonial rings within given genealogical bounds, a series of network-analytic tools – such as the census graph – to analyse ring structures in empirical kinship networks, and techniques to effectuate these analyses with the computer program* PAJEK. *A program package containing the required macros can be downloaded from the* WWWeb. *The working of the method is illustrated at the example of kinship networks from four different parts of the world (South-America, Africa, Australia and Europe).*

KEY WORDS – Matrimonial rings, Kinship, Social network analysis, Graph theory, Enumeration theory, Social anthropology

INTRODUCTION

One of the fundamental hypotheses of kinship theory is that marriages not only create ties of kinship and affinity, but also are determined by them – be it positively (by

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preferences) or negatively (by avoidance), directly (by following certain conventional marriage patterns) or indirectly (by copying or avoiding the marriage patterns followed by certain kin or affines).

Whatever the precise manner of determination, the results will be the same: within the network of kinship and marriage ties, the appearance of closed circuits of relatedness – "matrimonial rings" – other than those one would expect if marriages were at random within a certain horizon of endogamy [White, 1999].

The pattern of cycles in a given kinship network is thus the starting point of any analysis of how kinship and affinity actually determine matrimonial choices – in ways which are not necessarily reflected by the explicit matrimonial standards (if any) of the societies in question.

The most simple, traditional approach to this problem would be to start from the hypothesis of a rule to produce or not to produce a certain type of cycle (e.g., to marry one's MBD[4] or to avoid one's MZD) and then to count the frequency of marriages which form part of a cycle of that type. A slightly more sophisticated approach would be to count the relative frequencies of several different cycles (e.g., all first-cousin marriages) in order to establish a ranking among them. This is the way in which many anthropological studies have treated genealogical data.

The problem with such an approach is twofold.

To begin with, simply counting the occurrences of a single type of cycle, or even of several types chosen according to the theoretical expectations of the analyst, often proves misleading. It may turn out, for example, that a high frequency of MBD-marriages is in fact a consequence of a tendency for men to marry their FWBD. As Dumont [1953, 1975(a), 1957, 1975(b)] has argued for Dravidian-type systems, what may be taken to be a preference for marriage with a certain type of consanguine is actually a predilection for marriage with a certain type of affine. In order to avoid such mistakes, it is necessary to consider *all* possible matrimonial configurations within a given horizon of consanguinity and/or affinity[5]. The latter configurations, involving marriages between people linked through marriages – globally called "relinkings" (*renchaînements*), or "redoublings" (*redoublements*) when only two consanguineous groups are involved (cf. [Jolas *et al*., 1970; Héritier, 1981; White, 1997, Brudner and White, 1997; Harary and White, 2001]) – has, until recently, received little attention by anthropologists [Houseman and White, 1996; Richard, 1993; Segalen, 1985]. However, in our perspective, their inclusion is essential: not only do relinkings represent by far the most frequent matrimonial configurations in any marriage network (including random networks), but in those societies in which close kin marriage is prohibited or avoided, it is, one may suppose, the coordinate aggregation of such relinkings that underlie the patterning of cycles within the matrimonial universe.

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[4] Throughout this paper, we shall use the conventional anthropological notation for kinship relations by abbreviated English kin terms: single capital letters denote the simple relations F(ather), M(other), B(rother), Z (Sister), S(on), D(aughter), H(usband), W(ife), sequences denote their composites (FB = Father's Brother, etc.).

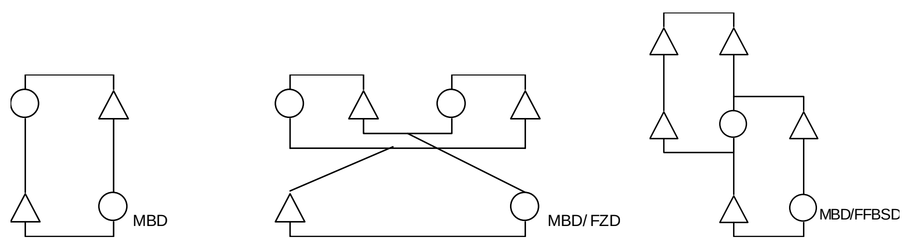
[5] "Consanguines" are defined here as persons having a known common ascendant (not all kinship systems distinguish between consanguines and affines in the same manner).

Now, such a comprehensive screening of the marriage network rests on two conditions: a complete and structured list of non-isomorphic matrimonial rings, and the technical means to count their occurrences in a given network so as to establish a “matrimonial census”.

Following a general introduction in Section 1 to the concept and theory of matrimonial rings, Section 2 presents the necessary operations to solve the first of these two tasks and provides the numbers of all possible matrimonial ring types involving up to 3 marriages between families within the bounds of first degree cousinhood.

The second task, that is, the matrimonial census itself, has been made feasible by the recent development of appropriate computer programs, in particular PAJEK which allows for repeated fragment searches in large networks. This procedure is described in Section 3**.**

However – and this brings us to the second problem of the traditional approach to kinship network analysis – even a complete census of elementary marriage types does not by itself give a sufficient representation of the matrimonial ring structure. The reason is that the significance of a marriage belonging to a certain type may depend crucially on the *other* types to which it also conforms. As is well known since Lévi-Strauss [1949], an isolated MBD-marriage (Figure 1(a)) is something completely different from a MBD which is at the same time a FZD-marriage, by virtue of a ZHZ marriage in the preceding generation (Figure 1(b)).



Figures 1(a), 1(b), 1(c)

Similarly, as Barry [1998, 2000] has argued in the context of the debate on "Arab marriage", an apparent preference for MBD-marriages along with a high rate of FBD-marriages may well be the result of a general preference for unions with close agnatic kin (FBD or FFBSD) combined with a rule that spouses not repeat their father's type of marriage (Figure 1(c)).

One way to investigate this mutual interdependence of matrimonial rings is the construction, from the same kinship network, of second-order networks in which types of marriages appear as nodes and their combinations as lines. While such networks – which we call “census graphs” – contain the same information as the simple matrimonial census, they complement it by informing on the frequencies of the combined marriage types, that is, the number of marriages which conform to two types at the same time. By applying different criteria of classification to the list of elementary marriage types, differing census graphs may be compared with one another to test alternative models of kinship structure and alternative logics that may be at play (cf. [Denham and White, 2005]).

The virtues of such network-analytic second-order tools are manifold and go far beyond the simple advantage of visualizing the interdependences of the various types of matrimonial rings involved. The manipulation and analysis of second-order networks with graph-theoretical methods allow for the study of these interdependences both within and between theoretical classes, this being an indispensable precondition for the development of hypotheses regarding the underlying matrimonial precepts or preferences which generate the pattern of cycles under consideration.

A method for constructing census graphs with PAJEK software as well as a number of techniques for manipulating and analysing them are also described in Section 3. The use of these techniques will be illustrated by applying them to several sample data sets. As our aim is above all to present new analytical tools, the latter presentations can of course be nothing more than a sketch to illustrate their potential. The development of more sophisticated network-analytic methods using these tools is a task for the near future.

1. THE CONCEPT OF MATRIMONIAL RINGS

1.1 FUNDAMENTALS

As mentioned at the start, understanding empirical marriage patterns amounts to analysing, within a given matrimonial universe, the frequency and interdependence of particular matrimonial configurations in which persons who are related to each other as spouses are *also* related to each other through either consanguinity (e.g., a man who marries his MBD) and/or affinity (e.g., a man who marries his BWZ). Because such configurations constitute cycles in kinship networks, it is appropriate to approach them with graph-theoretic tools. Box 1 gives the definitions of the graph-theoretical concepts we shall use in this article.

Box 1. Basic graph-theoretical concepts

A *simple digraph* is a pair $< N, L >$, where N is a set of *nodes*, and L is a set of *lines* each of which corresponds to a pair of nodes (the nodes being said *incident* with the line) and no pair is repeated twice. A line is called an *arc* if its corresponding node pair is ordered; otherwise it is called an *edge*.

A *network digraph* is a triple $< N, L, \sim >$ that combines a simple digraph with an equivalence relation $\sim$ on L that partitions L into a set of *line classes* (the line classes may be distinguished by distinct values).

Let us call a *mixed graph* a network digraph with two line classes consisting of arcs and edges.

A node v_1 is *edge-adjacent* to a node v_2 if there is an edge connecting v_1 and v_2. It is *arc-adjacent to* v_2 if there is an arc from v_1 to v_2, and *arc-adjacent from* v_2 if there is an arc from v_1 to v_2. The *degree* of a node is the number of incident lines. More particularly, its *edge-degree* is the number of incident edges, its *arc-degree* the number of incident arcs, its *indegree* the number of arcs to it and its *outdegree* the number of arcs from it. A node with zero (edge- or arc-) degree is called an (edge- or arc-) *isolate*.

A *path* of length n ($n \geq 1$) is a sequence of $n + 1$ nodes $v_0, v_1, \ldots, v_n$ joined by n lines such that v_i is adjacent to v_{i+1} for all $i = 1,\ldots, n - 1$, where all lines and all nodes are distinct.

A graph is *connected* if there is a path between any two nodes.

A *cycle* of length n ($n \geq 2$) is a path of that length plus a line between nodes v_n and v_0.

A path or cycle is *directed* if each line in it is an arc directed from v_i to v_{i+1}, and for a cycle from v_n to v_0.

A (mixed) graph is *acyclic* if it contains no cycles and *quasi-acyclic* if it contains no directed cycles.

A *tree* is a connected acyclic graph.

A set S is *maximal* (*minimal*) with respect to some property if no proper superset (subset) of S, containing more (fewer) elements than S, has the property but S does.

An *ancestral tree* or out-tree (tree from a root) is a tree in which the only lines are arcs along directed paths from a given node, called the *root,* to every other node in the tree. A *maximal node* is one, in an ancestral tree, with zero arc-indegree. Similarly, a *minimal node* is one with zero arc-outdegree. A *tip* is any minimal node, in an ancestral tree, possibly a root. *Sibling nodes,* in an ancestral tree, are arc-adjacent from the same node having a common "parent." A *linknode* in an ancestral tree is any node that is neither root nor tip. A *branch* of an ancestral tree is a path from a root to a tip. A root is *branching* if its degree is greater than 1. A branch is *singular* if its root has degree 1 (i.e., there is no other branch in the tree). A root is *singular* if there is no other node in the tree (and so it is also a tip and has zero degree).

A graph H is a *subgraph* of a graph G if $V(H) \subseteq V(G)$ and $L(H) \subseteq L(G)$. The *edge-part* of G is a maximal subgraph of (a mixed graph) G with an empty arc set. The *tip-part* of G is a maximal subgraph of an edge-part with only tips and edges. The *arc-part* of G is one with an empty edge set.

A *component* of G is a maximal connected subgraph of G. A component of the arc- (edge-) part of G is called an *arc- (edge-) component* of G. An *induced* subgraph $< S >$ of G is the maximal subgraph with node set S.

Two nodes are called *partner nodes* if they are arc-adjacent to a same node (i.e., having a common "child").

A set of cycles is *independent* and its cycles mutually independent if each cycle contains at least one line which is not contained in any other cycle of the set. The *cyclomatic number* $\gamma(G)$ of a graph G is the maximum number of mutually independent cycles in G. For a graph with m lines, n nodes, and c components, $\gamma(G) = m - n + c$. Any maximal set of mutually independent cycles is called a *cycle basis* of the graph. Two cycles that are subgraphs of a graph G and that have lines in common are said to *compose* a third cycle in G that is defined by keeping the nodes and lines in either subgraph and removing the common lines. Any maximal set of mutually independent cycles is called a *cycle basis* of the graph because all cycles in G can be composed from those in the basis.

A *configured* graph is a graph G together with a value set V and a mapping $g : N \rightarrow V$ (called a *configuration* of G) which assigns a value to each node of G.

Two configured graphs G_1 and G_2 are *isomorphic* ($G_1 \cong G_2$) if there exists a bijective mapping between their respective node, line and value sets which preserves incidence and configuration. The relation of isomorphy is an equivalence relation which partitions any set of graphs into a set of *isomorphism classes*.

Consider a network of individuals linked by relations of kinship and marriage. This kinship network may be represented as a graph whose nodes correspond to the individuals, whose edges correspond to marriage relations, whose arcs correspond to filiation relations leading from parent to child, and whose node values correspond to the male or female sexes of the individuals. Suppose that all marriages are same sex and no individual has more than one parent of a given sex, so that any two nodes adjacent to the same edge have different value, as do all nodes with arcs to a common node (marriage networks allowing for same-sex marriages and multiple same-sex parents would require broader definitions and a different formalization for analysis). The fact that no individual can be at once an ascendant and a descendant of the same individual excludes the possibility of directed cycles. We can thus provide a formal definition of a kinship graph (Box 2):

Box 2. Kinship Graphs

A *kinship graph* or *k-graph* is a configured quasi-acyclic mixed graph, with a binary value set, in which all partner nodes, as well as all edge-adjacent nodes, have different value. A *k*-graph is *canonical* if all partner nodes are edge-adjacent and *inversely canonical* if all edge adjacent nodes are partner nodes (i.e., nodes with children). A k-graph is *regular* if every non-maximal node (i.e., indegree greater than zero) has (arc) indegree 2 (i.e., one parent implies a second of opposite sex).

A *parental triad* is every triple of nodes (v_1, v_2, v_3) in a *k*-graph such that v_1 and v_2 are arc-adjacent to v_3 (in a canonical *k*-graph, this implies that they are edge-adjacent to one another). In standard triad census terms, the parental triads are the *021u*-triads in the arc-part of a canonical *k*-graph.

The *canonical closure* of a *k*-graph is the canonical graph which results from it by adding an edge between all partner nodes which are not yet edge-adjacent. The *canonical closure of a cycle* in a *k*-graph is the subgraph which results from it by adding an edge between all partner nodes which are not edge-adjacent and eliminating the node to which they are arc-adjacent (i.e., their "child").

A *matrimonial ring* is every cycle in a *k*-graph which is its own canonical closure and induced subgraph.

Examples of matrimonial rings (containing trees with 0, 1 or 2 branches) are given in Figure 2:

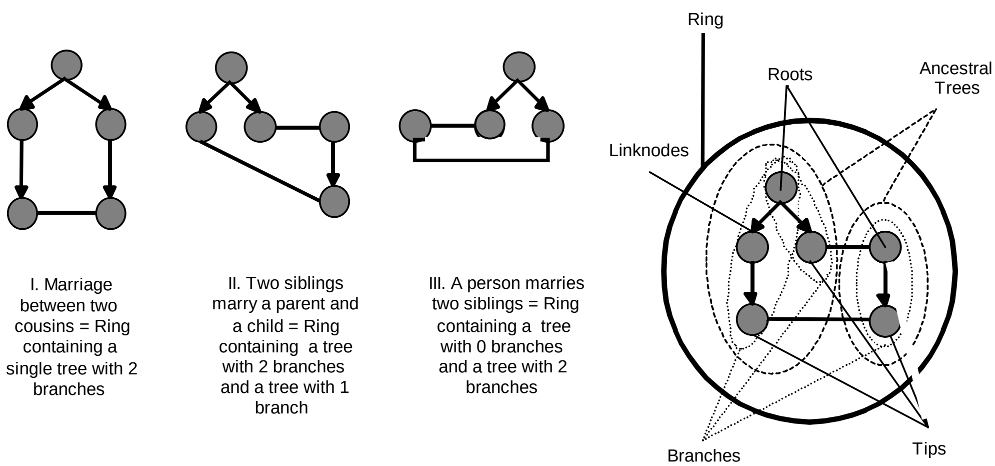


Figure 2

1.2 FUNDAMENTAL PROPERTIES OF MATRIMONIAL RINGS

From these definitions follow several important properties of k-graphs and matrimonial rings:

1. *No node in a k-graph can have indegree greater than 2.* If they did, then of three parents and two sexes, two parents must be of the same sex, which is disallowed. As a corollary, no two parental triads can have an arc in common, which means that *all parental triads are mutually independent*. For a k-graph with m_a arcs, and n nodes, of which n* are maximal nodes (ancestors), this means that the set of parental triads constitutes a cycle basis for itself, and for the subgraph S of parental triads, $\gamma(S) = n - n^*$. The cyclomatic number of G_a, the arc-part of a k-graph, is $\gamma(G_a) = m_a - n + c$, but in the arc-part of a regular k-graph G every node that is not a root has maximum indegree of 2, so subtracting the ancestors, it follows[6] that the cyclomatic number

[6] This relationship has been shown by White, Batagelj and Mrvar (1997) and Mrvar and Batagelj (2004) to define the index of relinking in the context of p-graphs, which show the same indegree-property. A *p-graph* (cf. White and Jorion 1992, 1996, White 1997, Brudner and White 1997, White and Schweizer 1998, Harary and White 2001) is a multigraph with two arc classes which can be derived from a canonical k-graph by defining a p-node for each couple of edge-adjacent k-nodes as well as for each k-edge-isolate, and by defining an arc between two p-nodes whenever there is at least one arc between the corresponding k-nodes, where two p-arcs are of the same class if the corresponding k-nodes point to nodes of the same

$\gamma(G_a) = 2(n - n^*) - n + c = n - 2n^* + c$ and $\gamma(S) = \gamma(G_a) + n^* - c$, so that the cyclomatic number of a regular k-graph is independent of the number of edges.

2. *For every edge contained in precisely two different rings of a k-graph there exists another matrimonial ring which does not contain it*, namely, the canonical closure of their composition.[7] Figure 1(b) is an example. This can be proven as a theorem: The given marriage edge must be connected by two non-identical paths, so that when all of the edges they have in common are removed, what remains of these paths must still connect to form a cycle; further, this cycle is its own reduced subgraph because if not, there would be three distinct rings containing the original marriage edge. As a corollary, *For every edge contained in any two different rings of a k-graph there exists another matrimonial ring which does not contain it*, namely, the canonical closure of their composition.
3. *The canonical closure of every cycle other than a parental triad in a k-graph* (i.e., including edges between partner nodes but taking away the child nodes in parental triads) *is itself a cycle.* This procedure cannot destroy the cycle property, except if the edge of the triad already forms part of the cycle, which is only the case for parental triads. Further, *in an inversely canonical k-graph* (i.e., where all partner nodes have children but at most one parent is identified per child)*, every matrimonial ring will contain only arcs.* The number of independent matrimonial rings in an inversely canonical k-graph is thus equal to the cyclomatic number of its arc-part $\gamma(G_a)$.
4. *The arc-components of a matrimonial ring are ancestral trees with at most two branches, connected by edges between their tips*. The proof of this theorem [White 2005] follows from the fact that a *k*-graph contains no directed cycles. Every cycle consisting purely of arcs therefore must contain at least one pair of arcs incident to the same node, which are removed by canonical closure and replaced by an edge. Removal of all edges of a matrimonial ring thus necessarily decomposes it into a number of trees which, since there are no more partner nodes, are all ancestral trees. Since no node in a cycle can have a degree greater than 2, the maximum number of branches of each component tree of a matrimonial ring is 2.

The last theorem permits us to derive some useful equations for the numerical characterization of matrimonial rings. We shall present them together with a notation convention and several additional definitions which we shall use in the remainder of this paper (Box 3).

Box 3. Matrimonial rings

Let there be a matrimonial ring composed of *n* nodes in *k* trees with *t* tips in *b* branches (in all trees; $t > b$ because some roots are minimal) of maximal length *d*. We also say that the ring has *length n*, *width k* and *depth d* (width, depth and length thus denote, respectively, one plus the maximum affinal, consanguineal and total distances between any two nodes in the ring). Then there are:

$n - k - b$ linknodes
$k - t + b$ branching roots (roots with degree 2)
$2(t - k) - b$ singular branches (roots with degree 1)
$2k - t$ singular roots (roots with degree 0)

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value. Since all arcs to the same k-node are transformed into a single arc to the corresponding p-node, and no p-node can correspond to more than 2 k-nodes, the maximal indegree of a p-node is 2.

[7] In a canonical regular graph,every consanguineous marriage is also a relinking marriage but in a trivial way (e.g., FBD = MHBD) that does not constitute a ring.

We say that a matrimonial ring is given in *neutral* form if the values of its nodes are undetermined. It is given in *semi-neutral* form if only the values of its tips are determined (i.e., only the sex of married individuals is considered). It is given in *reduced* form if the values of its branching roots are undetermined (i.e., the sex of apical ancestors is not considered unless their marriage forms part of the ring[8]. The number of valued nodes in a reduced-form ring is therefore $n^* = n - k + t - b$.

Let $i = 1,\ldots, k$ be the index of the i[th] tree of the ring (counted from left to right), and let $s = \{-1, 0, 1\}$ be the index of the left branch, the root, and the right branch of any tree. Let $n_i^s \in \{1, \ldots, d\}$ be the number of nodes in the s-part of the i[th] tree, which means that n_i^s is equal to the length of the s-branch for $s \neq 0$ and equal to 1 for $s = 0$. Then every ring in neutral form with δ a maximum d for all rings in the graph be uniquely characterized by its *branch configuration number* $z_b(\delta)$:

$$z_b(\delta) = \sum_i \sum_{s\neq 0} n_i^s \cdot \delta^{q-1} \leq \delta^{2k} \text{ (where } q = 2i + (s-1)/2)$$

We shall also define the *skewedness degree* of a ring, which is a measure for the generational distance between spouses:

$$dg_s = \left|\sum_i \sum_s s \cdot n_i^s\right|$$

Let $j = 1, \ldots, n_i^s$ be the index of the j^{th} node in the s-part of the i^{th} tree (counted from left to right). Let $x_{ij}^s \in \{1,0\}$ be the value of that node (the node being *male* if it has value 1 and *female* if it has value 0). Then every ring with given branch configuration number can be uniquely characterized by its *value configuration number* z_v:

$$z_v = x_{ij}^s \cdot 2^{q-1} \leq 2^n \text{ (where } q = \sum_{u<i} \sum_w n_u^w + \sum_{w<s} n_i^w + j \text{)}.$$

For a given δ, every matrimonial ring can thus be unambiguously identified by two numbers (z_b z_v). Conversely, however, not every possible number pair will correspond to a matrimonial ring.

Due to the condition of opposite values for edge-adjacent nodes, there are at least $t - k$ male (female) nodes in every ring. We define the *agnatic (uterine) degree* of a ring, which is the number of intervening male (female) nodes, as a percentage of their maximum possible number:

$$dg_a = \left(\sum_i \sum_s \sum_j x_{ij}^s - 1\right) \Big/ \left(n + k - t - 1\right), \quad dg_u = \left(n - \sum_i \sum_s \sum_j x_{ij}^s - 1\right) \Big/ \left(n + k - t - 1\right) \quad \text{for}$$

$n \geq t - k + 1$, while $dg_a = dg_a = 1$ for $n = t - k + 1$.

If a ring is in reduced form, n_i^0 has to be set equal to zero when calculating z_v, n has to be replaced by:

$n - k + t - b$ in the numerator of the formula for d_u, and the denominator in the formulae for d_a and d_u has to be replaced by $n - b - 1$.

The representation of a matrimonial ring in *HF-notation* (cf. [Barry 1996][9]) consists in a string of capital letters $X_{ij}^s \in \{H,F\}$, dots, and parentheses, such that $X_{ij}^s = H$ ("homme") or F ("femme") for x_{ij}^s = 1 or 0, all X_{ij}^0 are inserted in parentheses, and all X_{i0}^s are preceded by a dot (which can be omitted for $i = 1$)[10]. Rings in reduced HF- form are represented by eliminating all capital letters in parentheses not followed or preceded by a dot, and replacing them by a hyphen, which could represent a sibling relation.

A set of matrimonial rings has *bounds* (κ, δ) if $k \leq \kappa$ and $d \leq \delta$ for all rings contained in it (where κ and δ are, respectively, the maximum width and the maximum depth of any ring in it).

[8] This reduction is convenient wherever kinship runs through both apical ancestors, or if we do not wish to differentiate between full and half siblings.

[9] This notation is also used by the genealogical computer program GENOS 2.0 (© 1997) which counts, for every edge in a kinship network, all matrimonial rings (including isomorphs) containing it.

[10] Capital letters thus represent the male and female-valued nodes of the ring, those in parentheses represent roots, dots represent marriage edges, dots and parentheses demarcate the branches of the ring, and direct juxtaposition of letters represents a filiation arc pointing to the node represented by the letter which is closer to the limiting dot and farther from the limiting parenthesis of the branch. For instance, HF-HF denotes a marriage with MBD in conventional notation, H.(F)F a marriage with WD, etc. The notation can be studied in detail at the ring list in appendix 1, which gives all rings both in analytical HF-notation and in the conventional anthropological notation by abbreviated English kin terms.

A *matrimonial universe* with bounds (κ, δ) – or briefly a (κ, δ)-matrimonial universe – is a set of isomorphism classes of a matrimonial ring set with these bounds. We shall also call these isomorphism classes *matrimonial ring types*. Their number μ is the *extension* of the matrimonial universe. As a standard representation of each ring type we chose the ring in it which has lowest branch and value configuration numbers (z_b z_v).

We shall briefly speak of *the* (κ, δ)-matrimonial universe when referring to the *maximal* matrimonial universe (i.e., the set of all logically possible ring types) within these bounds. If we want to restrict the universe to ring types of width k, we shall speak of a [k, δ]-universe, using brackets instead of parentheses.

We shall call a matrimonial universe *semi-reduced* if all rings are in reduced form. We shall call it *reduced* if all rings are in reduced form and no branch of any ring contains δ valued nodes (i.e., the maximum length of singular branches is $\delta - 1$)[11].

The next section will deal with the problem of determining the maximum number of non-isomorphic matrimonial rings within given genealogical bounds. The third section will deal with the technique of identifying and counting these rings in particular kinship networks (establishing a matrimonial census), and the problem of analysing their interrelationship in the composition of the global ring structure of these networks.

2. MATRIMONIAL RING ENUMERATION

2.1 CONCEPTUAL PREREQUISITES

This section deals with the problem of constructing a list of all isomorphism classes of matrimonial rings within given bounds (κ, δ). To illustrate the method, we will solve the problem for $\kappa = 3$ and $\delta = 2$ (which includes all matrimonial rings containing up to 3 marriages between groups of consanguines within the bounds of first degree cousinhood). All matrimonial rings will be treated in reduced form only.

The enumeration of isomorphism classes of matrimonial rings – as well as of any other kind of graphs – rests on a consideration of their symmetry properties, as given by their automorphism groups (Box 4).

Box 4. Automorphism groups and cycle indices

An *automorphism* of a configured graph G is a mapping of G on itself which preserves incidence and configuration. The set of all automorphisms of G forms a group, the *automorphism group* $A(G)$.

A *cycle generated* by an automorphism α acting on a set X is any sequence of elements of X such that $x_{i+1} = \alpha(x_i)$ for all $i = 1,\ldots, r$ (where r is the *length* of the cycle).

The *cycle index* of a structure with n elements and automorphism group A is the polynomial $Z_{A,n} = |A|^{-1} \sum_{\alpha \in A} \prod_{r=1}^{n} s_r^{j_r(\alpha)}$, where $j_r(\alpha)$ is the number of cycles of length r generated by the automorphism α.

The *configuration enumerator* of a graph G with respect to a set V of m values is a polynomial such that the coefficient of the summand having exponents h_i ($i = 0, \ldots, m-1$) gives the number of the non-isomorphic configurations which assign the value i to h_i elements of the structure (if $m = 2$, the index i can be dropped, for any element not having value 1 will necessarily have value 2 and vice versa).

[11] This means that ancestors at generational level δ are considered only as the common ancestors of distinct married individuals, but not in their own right. They could thus be entirely omitted if we introduced a separate "siblingship" relation.

Polya's enumeration theorem says that the configuration enumerator of a graph G with automorphism group A and value set V is obtained by substituting $s_r = 1 + x_1^r + \ldots + x_{m-1}^r$ in its cycle index $Z_{A,n}$ (for details see, e.g., [Harary and Palmer, 1973]).

2.2 THE SYMMETRY PROPERTIES OF MATRIMONIAL RINGS

A matrimonial ring with n nodes in k trees may be subject to two kinds of automorphisms:

- k possible *rotations* ρ_q defined by $\rho_q(i) = i + q \bmod k$, $\rho_q(j) = j$, $\rho_q(s) = s$ for all x_{ij}^s ($\rho_k = \rho_0 = \iota$ is the identity automorphism)
- k possible *reflections* σ_q defined by $\sigma_q(i) = k - q - i + 1 \bmod k$, $\rho_q(j) = j$, $\rho_q(s) = -s$ for all x_{ij}^s

We can thus characterize any automorphism group of a matrimonial ring by k and a pair of numbers (a, b), called its *symmetry index*, where $a = \Sigma a_q \cdot 2^q$ ($q = 0, \ldots, k-1$) is the *rotation configuration number* and $b = \Sigma b_q \cdot 2^q$ is the *reflection configuration number*, with $a_q = 1$ if the rotation ρ_q belongs to the group and 0 otherwise (and similarly for b_q and reflection σ_q). The minimal (identity) group then has symmetry index (1, 0), whereas the maximal group has symmetry index ($2^k - 1$, $2^k - 1$). For reasons of simplicity, we may also drop the symmetry index of a maximal group, denoting it simply by A_k.

Here are the possible automorphism groups for $k \leq 3$:

$A_{110} = \{\iota\}$, $A_{1(11)} = \{\iota, \sigma_0\}$

$A_{210} = \{\iota\}$, $A_{211} = \{\iota, \sigma_0\}$, $A_{212} = \{\iota, \sigma_1\}$, $A_{230} = \{\iota, \rho_1\}$, $A_{2(33)} = \{\iota, \rho_1, \sigma_0, \sigma_1\}$

$A_{310} = \{\iota\}$, $A_{311} = \{\iota, \sigma_0\}$, $A_{312} = \{\iota, \sigma_1\}$, $A_{314} = \{\iota, \sigma_2\}$, $A_{370} = \{\iota, \rho_1, \rho_2\}$, $A_{3(77)} = \{\iota, \rho_1, \rho_2, \sigma_0, \sigma_1, \sigma_2\}$

Each of the k possible rotations ρ_q generates n/r cycles of length r, where $r = \text{lcm}(k, q)/q$. The number of rotations which generate cycles of a given length r ($r \mid k$) is given by the Euler function $\varphi(r)$[12]. If k is odd, each of the k reflections generates $(n-1)/2$ cycles of length 2 and 1 cycle of length 1. If k is even, $k/2$ reflections generate $n/2$ cycles of length 2, while the remaining $k/2$ reflections generate $(n-2)/2$ cycles of length 2 and 2 cycles of length 1.

For $k \leq 3$, we thus have the following cycle indices[13]:

[12] Since $r = k$ if q and k are coprime, the number of possible rotations ρ_q which generate cycles of length r ($r \mid k$) is equal to the number of integers $q \leq r$ which are coprime with r. This number is given by the Euler function $\varphi(r)$: $\varphi(1) = 1$, $\varphi(2) = 1$, $\varphi(3) = 2$, $\varphi(4) = 2$, $\varphi(5) = 4$, $\varphi(6) = 2$, etc.

[13] In general, the cycle index of the maximal automorphism group A_k is:

$$Z_k^n = \frac{1}{2k}\sum_{r|k}\varphi(r)s_r^{n/r} + \frac{1}{2}s_1 s_2^{(n-1)/2} \text{ for } k \text{ odd}$$

$$Z_k^n = \frac{1}{2k}\sum_{r|k}\varphi(r)s_r^{n/r} + \frac{1}{4}\left(s_2^{n/2} + s_1^2 s_2^{(n-2)/2}\right) \text{ for } k \text{ even.}$$

$$Z_{110}^{n} = Z_{210}^{n} = Z_{310}^{n} = s_1^n, \qquad Z_{111}^{n} = \frac{1}{2}\left[s_1^n + s_1 s_2^{(n-1)/2}\right], \qquad Z_{211}^{n} = Z_{230}^{n} = \frac{1}{2}\left[s_1^n + s_2^{n/2}\right],$$

$$Z_{212}^{n} = \frac{1}{2}\left[s_1^n + s_1^2 s_2^{(n-2)/2}\right],\ Z_{233}^{n} = \frac{1}{4}\left[s_1^n + 2s_2^{n/2} + s_1^2 s_2^{(n-2)/2}\right],\ Z_{311}^{n} = Z_{312}^{n} = Z_{314}^{n} = \frac{1}{2}\left[s_1^n + s_1 s_2^{(n-1)/2}\right],$$

$$Z_{370}^{n} = \frac{1}{3}\left[s_1^n + 2s_3^{n/3}\right],\ Z_{377}^{n} = \frac{1}{6}\left[s_1^n + 3s_1 s_2^{(n-1)/2} + 2s_3^{n/3}\right]$$

If the ring is given in *reduced form*, all cycles passing through branching roots have to be eliminated. We shall denote $Z_{kabc}^{n|(k-t+b)}$ the cycle index of a ring in reduced form with $k - t + b$ branching roots (recall that t is the number of tips and b the number of branches)[14].

For $t - b = k$ (zero branching roots), we clearly have $Z_{kab}^{n|0} = Z_{kab}^{n}$.

For $t = b$ (k branching roots), we get[15]

$$Z_{k10}^{n|k} = s_1^{n-k}, \qquad Z_{111}^{n|1} = \frac{1}{2}\left[s_1^{n-1} + s_2^{(n-1)/2}\right], \qquad Z_{211}^{n|2} = Z_{230}^{n|2} = Z_{212}^{n|2} = \frac{1}{2}\left[s_1^{n-2} + s_2^{(n-2)/2}\right],$$

$$Z_{311}^{n|3} = Z_{312}^{n|3} = Z_{314}^{n|3} = \frac{1}{2}\left[s_1^{n-3} + s_2^{(n-3)/2}\right], \qquad Z_{370}^{n|3} = \frac{1}{3}\left[s_1^{n-3} + 2s_3^{(n-3)/3}\right],$$

$$Z_{377}^{n|3} = \frac{1}{6}\left[s_1^{n-3} + 3s_2^{(n-3)/2} + 2s_3^{(n-3)/3}\right]$$

For $0 < t - b < k$, no automorphism (other than the identity) is possible for $k = 2$, while in the case of $k = 3$ there remains the possibility of a reflection with the branching root in the axis. We thus have the possible cycle indices $Z_{k10}^{n|t-b} = s_1^{n-t+b}$ and $Z_{311}^{n|b-t+3} = Z_{312}^{n|b-t+3} = Z_{314}^{n|b-t+3} = \frac{1}{2}\left[s_1^{n-b+t-3} + s_2^{(n-b+t-3)/2}\right]$.

2.3 THE ENUMERATION PROCEDURE

Consider a matrimonial ring in reduced semi-neutral form with k trees and symmetry index ab, and let $h = n - k - b$ (recall that n is the number of nodes and b the number of branches) be the number of linknodes (whose values are still to be determined). The cycle index of this ring is $Z_{kab}^{n-t|k-t+b} = Z_{kab}^{n|k+b}$. According to Polya's theorem, substitution of $s_r = 1 + x^r$ in $Z_{kab}^{n|k+b}$ then yields the polynomial which counts all isomorphism classes of its complete value configuration. Multiplication of this polynomial by the number N_{kab}^{h} of isomorphism classes of semi-neutral rings with parameters (k, h, ab) then counts

[14] Their particular configuration can be neglected for $k \leq 3$ since in this case all configurations of the same number of branching roots are isomorphic. To see this, note that each possible branching root configuration corresponds to a value configuration of a ring consisting entirely of singular trees. Enumerating these configurations by substituting $s_r = 1 + x^r$ in Z_k^k yields, for each k, a polynomial where all coefficients are equal to 1. The case is equivalent to that of the enumeration of tip-configurations discussed below.

[15] For general value of k, we have $Z_{kabc}^{n|k} = \frac{1}{2k}\sum_{r|k}\varphi(r)\cdot s_r^{(n-k)/r} + \frac{1}{2}s_s^{(n-k)/2}$.

the isomorphism classes of fully configured (reduced-form) matrimonial rings. Summation over all *ab*, *h* and *k* then yields the desired result.

To determine N^h_{kab}, we start with counting the non-isomorphic configurations of tips in a *k*-tree ring. Since each tree can contain either 1 or 2 tips (the first case indicates a polygamous marriage), the problem is equivalent to that of enumerating the value configurations of a ring consisting of *k* singular trees. This is solved by substituting $s_r = 1 + x^r$ in Z^k_k. For $k \leq 3$, the resulting polynomials have only coefficients 1, so the number of isomorphism classes of tip-configurations is 1 + *k*! Summing over all $k \leq 3$, we thus have 2 + 3 + 4 = 9 isomorphism classes. Representing each of them by the element with the lowest *tip configuration number* $z_t = \Sigma z_k \cdot 2^k$, where $z_k = 0$ if the k^{th} root is singular and 1 otherwise, we get the following list (ordered by *k* and z_t:

1.0. X [0], 1.1. XX [2], 2.0. X.X [2], 2.1. XX.X [2], 2.3. XX.XX [4], 3.0.X.X.X [0],
3.1. XX.X.X [2], 3.5. XX.XX.X [4], 3.7 XX.XX.XX [8]

The numbers in brackets count the possible value configurations (including isomorphic copies) of each of these structures. To determine them, we take account of the fact that edge-adjacent tips must have different value. As there are *t* tips and *k* edges, there are thus 2^{t-k} possible value configurations for $t > k$. For $t = k$, there are 2 configurations if *k* is even and 0 if *k* is odd. For $k \leq 3$, we thus have 24 possible value configurations. Denoting each by its value configuration number z_v (as defined in the preceding section for reduced-form rings), we get the following list (ordered by *k*, z_t and z_v):

1.1.1. HF, 1.1.2. FH, 2.0.1. H.F, 2.0.2. F.H, 2.1.3. HH.F, 2.1.4. FF.H, 2.3.3. HH.FF, 2.3.5. HF.HF,
2.3.10. FH.FH, 2.3.12. FF.HH, 3.1.5. HF.H.F, 3.1.10 FH.F.H, 3.5.11. HH.FH.F, 3.5.13. HF.HH.F,
3.5.18. FH.FF.H, 3.5.20. FF.HF.H, 3.7.11. HH.FH.FF, 3.7.13. HF.HH.FF, 3.7.19. HH.FF.HF,
3.7.21. HF.HF.HF, 3.7.42. FH.FH.FH, 3.7.44. FF.HH.FH, 3.7.50. FH.FF.HH, 3.7.52. FF.HF.HH

Having thus enumerated the configurations of tips and their possible values, let us now turn to the configurations of branches. As each of the *k* trees contains at most 2 branches, the problem of determining the number of non-isomorphic configurations of *b* branches is equivalent to that of determining the configurations of *b* same-sex nodes in the perfectly symmetrical (reduced form) matrimonial ring with $n^s_i = 1$ for all *i* and *s*. We thus substitute $s_r = 1 + x^r$ into the cycle index $Z^{3k|k}_k$ (which is that for a maximal automorphism group acting on $n = 3k$ nodes) and get the desired numbers as coefficients of summands x^b in the resulting polynomial:

k / *b*	0	1	2	3	4	5	6	Σ
1	1	1	1					3
2	1	1	3	1	1			7
3	1	1	4	4	4	1	1	16
Σ	1	3	8	5	5	1	1	**26**

Table 1. Enumeration of matrimonial ring tip-configurations within bounds (2,3)

Representing each of these 26 isomorphism classes by the element with the lowest branch configuration number $z_b(1)$ (as defined in the preceding section), we get the following list (ordered by *k* and z_b):

1.0. X, 1.1. X(X), 1.3. X-X, 2.0. X.X, 2.1. X(X).X, 2.3. X-X.X, 2.5. X(X).X(X), 2.6. (X)X.X(X), 2.7. X-X.X(X), 2.15. X-X.X-X, 3.0. X.X.X, 3.1. X(X).X.X, 3.3. X-X.X.X, 3.5. X(X).X(X).X, 3.6. (X)X.X(X).X, 3.7. X-X.X(X).X, 3.9. X(X).(X)X.X, 3.11. X-X.(X)X.X, 3.15. X-X.X-X.X, 3.21. X(X).X(X).X(X), 3.22. (X)X.X(X).X(X), 3.23. X-X.X(X).X(X), 3.27. X-X.(X)X.X(X), 3.30. (X)X.X-X.X(X), 3.31. X-X.X-X.X(X), 3.63. X-X.X-X.X-X.

Combination of each of these 26 non-isomorphic branch configurations with those of the 24 value configurations enumerated before which are based on the same tip-configuration and prove non-isomorphic *given* the branch configuration in question then yields the isomorphism classes of all (reduced-form) matrimonial rings consisting entirely of tips (or equivalently, of all the tip-parts of matrimonial rings within the chosen bounds). In our case of $k \leq 3$, the derivation of their automorphism groups from those of the constituent branch- and value-configurations is largely facilitated by the fact that no groups other than the maximal groups have proper subgroups, so that the intersection of any two different non-maximal groups is the identity group. Since no configured matrimonial ring can have a maximal automorphism group (the condition of opposite values for edge-adjacent nodes makes at least one reflection impossible), none of the resulting automorphism group has proper subgroups. Table 2 lists the symmetry indices for all non-isomorphic tip-parts of matrimonial rings for $k \leq 3$. There are 80 such configurations (3 for $k = 1$, 16 for $k = 2$ and 61 for $k = 3$):

			t	2	2	2	2	3	3	4	4	4	4	4	4	5	5	5	5	6	6	6	6	6	6	6	6
			k. z_b. z_v	1. 1. 1	1. 1. 2	2. 0. 1	2. 0. 2	2. 1. 3	2. 1. 4	2. 3. 3	2. 3. 5	2. 3. 10	2. 3. 12	3. 1. 5	3. 1. 10	3. 5. 11	3. 5. 13	3. 5. 18	3. 5. 20	3. 7. 11	3. 7. 13	3. 7. 19	3. 7. 21	3. 7. 42	3. 7. 44	3. 7. 50	3. 7. 52
t	b	k.z_b	*ab*	10	10	12	12	12	12	12	30	30	12	10	10	10	10	10	10	10	10	10	70	70	10	10	10
1	**0**	1.0	11																								
2	**1**	1.1	10	**10**	**10**																						
2	**2**	1.3	11	**10**																							
2	**0**	2.0	33			**12**																					
3	**1**	2.1	10					**10**	**10**																		
3	**2**	2.3	12					**12**	**12**																		
4	**2**	2.5	30							**10**	**30**	**30**															
4	**2**	2.6	11							**10**	**10**																
4	**3**	2.7	10							**10**	**10**	**10**	**10**														
4	**4**	2.15	33							**12**	**30**																
3	**0**	3.0	77																								
4	**1**	3.1	10											**10**	**10**												
4	**2**	3.3	14											**10**													
5	**2**	3.5	10													**10**	**10**	**10**	**10**								
5	**2**	3.6	12													**10**		**10**									
5	**3**	3.7	10													**10**	**10**	**10**	**10**								
5	**2**	3.9	12													**10**		**10**									
5	**3**	3.11	10													**10**	**10**	**10**	**10**								
5	**4**	3.15	12													**10**		**10**									
6	**3**	3.21	70																	**10**	**10**		**70**	**70**			
6	**3**	3.22	10																	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**
6	**4**	3.23	10																	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**
6	**4**	3.27	14																	**10**	**10**	**10**	**10**				
6	**4**	3.30	11																	**10**	**10**	**10**	**10**				
6	**5**	3.31	10																	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**
6	**6**	3.63	77																	**10**	**10**	**10**	**70**				

Table 2. Isomorphism classes of the tip-parts of matrimonial rings within bounds (3,2)

Having thus determined all non-isomorphic configurations of the *married* individuals in a matrimonial ring, let us now look at the *linking* individuals, to begin with their number. Given a maximum depth of δ, each of the b branches may contain up to δ - 1 linknodes. If we let b_l be the number of branches with length $l + 1$ ($l = 0, \ldots, \delta$), the total number of linknodes is $h = \sum_{l=0}^{\delta} h_l \cdot l$ (which reduces to $h = h_1$ for $\delta = 2$). For each of the 79 configurations, the number of isomorphism classes of matrimonial rings containing h_l branches of length $l + 1$ (in semi-neutral form, i.e., without yet determining the values of the linknodes) is given by the coefficient of the summand $x_1^{h_1} x_2^{h_2} \ldots x_{\nu-1}^{h_{\nu-1}}$ in the polynomial obtained by substituting $s_r = 1 + x_1^r + \ldots + x_{\nu-1}^r$ in the cycle index $Z_{kab}^{(b+k)|k}$. Multiplication of this polynomial with the number N_{kab}^{b} of non-isomorphic tip-parts of rings with parameters k, b and ab (which is obtained by counting the appropriate cells of the Table 2) then counts the isomorphism classes of all matrimonial rings (in semi-neutral form) with parameters k, b and (h_l) whose tip-part has symmetry index ab. The resulting numbers still have to be decomposed according to the symmetry index of the complete (semi-neutral) ring. Now, since for $k \leq 3$ no automorphism group of any configured tip-part has proper subgroups (cf. above), the symmetry index of the complete ring must either remain unchanged or reduce to the minimal symmetry index 10. It suffices therefore to inspect the branch-configurations of rings whose configured tip-parts have a non-minimal symmetry index, and count those which destroy the symmetry[16]. Summation over all l and all b then yields the numbers N_{kab}^{h} of all isomorphism classes of (reduced semi-neutral) rings with h linknodes in k trees and symmetry index ab.

In the case of a *reduced* matrimonial universe (in which no matrimonial rings contain singular branches of length δ), calculation has to be modified. For $k = 1$, this can easily be done by reducing δ to $\delta - 1$ for $b = 1$. For $\delta = 2$ (where the restriction excludes any linknodes in singular branches), an alternative method consists in replacing b by $2(k - t + b)$ (the number of branches from branching roots).

Table 3 contains the coefficients for the (3, 2)-universe. The number of symmetric structures (if they exist) is given in parenthesis, the numbers for the reduced universe (if they diverge) are given in brackets.

k	*b*	*ab*	N_{kab}^{b}	$Z_{kab}^{(b+k)\|k}$	*h* = 0	*1*	*2*	*3*	*4*	*5*	*6*	Σ
1	*1*	*10*	2	s_1	2	2 [0]						**4 [2]**
1	*2*	*10*	1	s_1^2	1	2	1					**4**
			N_{110}^{h}		**3**	**4 [2]**	**1**					**8 [6]**
2	*0*	*12*	1	1	1							**1**
2	*1*	*10*	2	s_1	2	2 [0]						**4 [2]**
2	*2*	*10*	3	s_1^2	3	6 [0]	3 [0]					**12 [3]**
2	*2*	*12*	2	$(s_1^2 + 1)/2$	2 (2)	2	2 (2)					**6**
2	*2*	*30*	2	$(s_1^2 + 1)/2$	2 (2)	2 [0]	2 (2) [0]					**6 [2]**

[16] For $\delta = 2$, these are (with automorphism group index *kab* and total linknode number *h* in brackets]: (1)1.(1)1 [30,0], (1)2.(1)2 [30,2], 1.1-1 [12,0], 1.2-2 [12,1], 1-1.1-1 [30,0],1-2.1-2 [30,2], 2-1.2-1 [30,2], 2-2.2-2 [30,4], 1-1.1-1 [12,0], 1-1.2-2 [12,2], 2-2.1-1 [12,2], 2-2.2-2 [12,2], (1)1.(1)1.(1)1 [70,0], (1)2.(1)2.(1)2 [70,3], 1-1.1-1.1-1 [70,0], 1-2.1-2.1-2 [70,3],2-1.2-1.2-1 [70,3], 2-2.2-2.2-2 [70,6].

2	*3*	*10*	4	s_1^3	4	12 [8]	12 [4]	4 [0]				**32 [16]**
2	*4*	*12*	1	$\left(s_1^4+s_2^2\right)/2$	1 (1)	2	4 (2)	2	1 (1)			**10**
2	*4*	*30*	1	$\left(s_1^4+s_2^2\right)/2$	1 (1)	2	4 (2)	2	1 (1)			**10**
			N_{210}^h		**9**	**28 [14]**	**19 [8]**	**8 [4]**	**0**			**64 [35]**
			N_{212}^h		**4**		**4**		**1**			**9**
			N_{230}^h		**3**		**4 [2]**		**1**			**8 [6]**
3	*1*	*10*	2	s_1	2	2 [0]						**4 [2]**
3	*2*	*10*	9	s_1^2	9	18 [2]	9 [1]					**36 [12]**
3	*3*	*10*	18	s_1^3	18	54 [16]	54 [8]	18 [0]				**144 [42]**
3	*3*	*70*	2	$\left(s_1^3+2s_3\right)/3$	2 (2)	2 [0]	2 [0]	2 (2) [0]				**8 [2]**
3	*4*	*10*	18	s_1^4	18	72 [40]	108 [28]	72 [8]	18 [2]			**288 [96]**
3	*5*	*10*	8	s_1^5	8	40 [32]	80 [48]	80 [32]	40 [8]	8 [0]		**256 [128]**
3	*6*	*10*	3	s_1^6	3	18	45	60	45	18	3	**192**
3	*6*	*70*	1	$\left(s_1^6+2s_3^2\right)/3$	1 (1)	2	5	8 (2)	5	2	1 (1)	**24**
			N_{310}^h		**58**	**208 [110]**	**303 [135]**	**236 [106]**	**108 [60]**	**28 [20]**	**3**	**944[492]**
			N_{370}^h		**3**			**4 [2]**			**1**	**8 [6]**

Table 3. Enumeration of matrimonial ring types in semi-neutral form within bounds (3,2)

The final step consists in substituting $s_r = 1 + x^r$ into the weighted cycle index $N_{kab}^h \cdot Z_{kab}^{n|k+b}$ to obtain the numbers of all isomorphism classes of matrimonial rings with *h* linknodes in *k* trees and symmetry index *ab*. The coefficient of the summand x^g counts the isomorphism classes of matrimonial rings containing *g* same-sex linknodes. Summing over all *g*, *h* and *ab* then gives the total number of isomorphism classes of matrimonial rings with given *k* and *δ*. They are given in Table 4:

k	*h*	*ab*	N_{kab}^h	$Z_{kab}^{n\|k+b}$	*g = 0*	*1*	*2*	*3*	*4*	*5*	*6*	Σ
1	*0*	*10*	3	1	3							**3**
1	*1*	*10*	4 [2]	s_1	4 [2]	4 [2]						**8 [4]**
1	*2*	*10*	1	s_1^2	1	2	1					**4**
					8 [6]	**6 [4]**	**1**					**15 [11]**
2	*0*	*10*	10	1	9							**9**
2	*0*	*12*	3	1	4							**4**
2	*0*	*30*	3	1	3							**3**
2	*1*	*10*	28 [14]	s_1	28 [14]	28 [14]						**56 [28]**
2	*2*	*10*	19 [8]	s_1^2	19 [8]	38 [16]	19 [8]					**76 [32]**

2	*2*	*12*	4 [2]	$s_1^2 + s_2/2$	4	4	4					**12**
2	*2*	*30*	4	$s_1^2 + s_2/2$	4 [2]	4 [2]	4 [2]					**12 [6]**
2	*3*	*10*	8 [4]	s_1^3	8 [4]	24 [12]	24 [12]	8 [4]				**64 [32]**
2	*4*	*12*	1	$\left(s_1^4 + s_2^2\right)/2$	1	2	4	2	1			**10**
2	*4*	*30*	1	$\left(s_1^4 + s_2^2\right)/2$	1	2	4	2	1			**10**
					81 [50]	**102 [52]**	**59 [34]**	**12 [8]**	**2**			**256 [146]**
3	*0*	*10*	58	1	58							**58**
3	*0*	*70*	3	1	3							**3**
3	*1*	*10*	208 [110]	s_1	208 [110]	208 [110]						**416 [220]**
3	*2*	*10*	203 [135]	s_1^2	303 [135]	606 [270]	303 [125]					**1212 [540]**
3	*3*	*10*	236 [106]	s_1^3	236 [106]	708 [318]	708 [318]	236 [106]				**1888 [848]**
3	*3*	*70*	4 [2]	$\left(s_1^3 + 2s_3\right)/3$	4 [2]	4 [2]	4 [2]	4 [2]				**16 [8]**
3	*4*	*10*	108 [60]	s_1^4	108 [60]	432 [240]	648 [360]	432 [240]	108 [60]			**1728 [960]**
3	*5*	*10*	28 [20]	s_1^5	28 [20]	140 [100]	280 [200]	280 [200]	140 [100]	28 [20]		**896 [640]**
3	*6*	*10*	3	s_1^6	3	18	45	60	45	18	3	**192**
3	*6*	*70*	1	$\left(s_1^6 + 2s_3^2\right)/3$	1	2	5	8	5	2	1	**24**
					952 [498]	**2118 [1060]**	**1993 [1065]**	**1020 [616]**	**298 [210]**	**48 [40]**	**4**	**6433 [3493]**

Table 4. Enumeration of matrimonial ring types within bounds (2,3)

Within the bounds of up to three affinally linked consanguineous “families”, each delimited by first degree cousinhood, and without differentiating between full-sibling and half-sibling ties, there are thus 15 consanguineous marriage structures (k = 1), 256 classes relinking marriage structures with two families involved (k = 2), and 6433 relinking marriage structures with three families involved (k = 3). If apical ancestors of generational level δ are only considered as representations of a sibling tie, these numbers reduce to 11, 146 and 3493, respectively.

Appendix 1 gives an analytical overview of the (reduced) (2, 2)-universe, containing 11 consanguineous marriages and 146 relinking marriages. Each matrimonial ring type is represented by the ring with the lowest branch and value configuration numbers[17]. We have listed the rings both in conventional and in *HF*-notation, together with the indices of their most important structural characteristics (configuration numbers, symmetry indices, skewedness, agnatic and uterine degrees, length, width, depth, and the numbers of tips, linknodes, singular and non-singular branches and singular and branching roots).

[17] We have assigned number zero to the ring *H.F*, since it is an open question among anthropologists whether a remarriage with a former wife constitutes a proper relinking marriage or not.

3. THE MATRIMONIAL CENSUS

Having established a set of matrimonial ring types, we now turn to the question of analysing their distribution and mutual interconnection in empirical kinship networks. To this purpose, we first introduce some additional conceptual tools (Box 5):

Box 5. Matrimonial Census

Let K be a kinship graph with edge set M and a matrimonial universe U of extension μ. We define the *elementary marriage type* $M_h \subseteq M$ ($h = 1,\ldots, \mu$) as the set of all edges in K which form part of a matrimonial ring of type h[18]. Let $T^*_U(K)$ be the set of all elementary marriage types in K with respect to U.

Let M_0 be the set of all marriage edges which do not enter in a matrimonial ring of any type from $\mathbf{U}$. A *marriage type* in general is every subset of M which can be represented as the union, the intersection or the complement in $M \backslash M_0$ of elementary marriage types. The set T_U of all marriage types constitutes a topology (the *matrimonial topology*) on $M \backslash M_0$.

Let $m_h = |M_h|$ be the number of all elementary marriages of type h in K. The vector (m_h) is called the *matrimonial census* of K with respect to U (or briefly the (κ, δ)-matrimonial census of K if U is the maximal (κ, δ)-universe).

The *census graph* of K with respect to U is a 2-mode graph consisting of M and T^* as node sets, and an arc from a node in T^* (representing an elementary marriage type) to a node in M (representing a marriage edge) if the edge belongs to that type.

The *1-mode reduction* of this census graph (or briefly the corresponding second order or *1-mode-census graph*) is a valued graph with node set T^*, where any two nodes are linked by an edge if they are partner nodes in the corresponding 2-mode census graph (in other words if the corresponding marriage types have a non-empty intersection), and the edge value corresponds to the number of nodes from M which are adjacent from both (i.e., to the extension of the combined marriage type resulting from the intersection of the two elementary types).

3.1 ESTABLISHING A MATRIMONIAL CENSUS BY MEANS OF pajek

The first step in the analysis of a matrimonial ring structure consists in establishing the matrimonial census of the kinship network under consideration. This entails (1) identifying all distinct matrimonial rings in the network (i.e., all rings that do not contain the same nodes), (2) assigning them to their appropriate isomorphism class and (3) counting the number of marriages which form part of a ring belonging to each class, i.e., in establishing the matrimonial census of the kinship network in question.

A powerful computer tool for doing that is the program PAJEK, developed by A. Mrvar and V. Batagelj (University of Ljubljana) for the explorative analysis of large networks [Batagelj and Mrvar, 2003; de Nooy *et al.*, forthcoming])[19]. PAJEK contains a function which makes it possible to scan a given graph G_1 for all induced subgraphs[20]

[18] Note that this definition does not differentiate marriages according to their *position* in the matrimonial ring (for instance, marriage with WD and WM, while quite different from Ego's point of view, will be considered of the same "marriage type", as they are part of a ring of the same type). A refined classification of marriage types would have to take into account not only the *cycle* passing through the marriage edge, but also of the *path* (other than that edge) which connects its two incident nodes.

[19] We have been using version 1.01f.

[20] There is also the option in PAJEK to restrict the scan to those subgraphs of G_1 which are identical to the induced subgraphs generated by their node sets. In our case, this excludes all matrimonial rings which contain still smaller rings (i.e., whose nodes are linked to each other by lines which form not part of the ring). This option proves especially useful in the search for "pure" relinking marriages which are not at the same time consanguineous marriages (while, from Theorem 2 in Section 1.2, every consanguineous marriage is also a relinking marriage if the graph is regular and canonical in its neighbourhood).

(“fragments”) which are isomorphic to a given second graph G_2, and to repeat this fragment search for an arbitrary number of times, where G_1 or G_2 may be fixed. As a by-product of each fragment search, PAJEK extracts a subgraph G_{12} from G_1 by eliminating all lines which are not contained in a fragment isomorphic to G_2. If the fragment search is repeated *n* times, PAJEK creates a vector which contains the *n* numbers of found fragments. These vectors can be saved as VEC-files (or TXT-files) and read in EXCEL or similar programs.

Applying this tool to establish the matrimonial census for a matrimonial universe of extension μ is straightforward. Having previously defined a set of μ abstract networks G_h ($h = 3,\ldots, \mu + 2$) each of which consists of a single matrimonial ring of a different class, and an additional network G_2 consisting of a single marriage line (these $\mu + 1$ networks being saved and loaded as a single PAJEK project file), repeated scanning of a kinship network G_1 for fragments isomorphic to each of the μ matrimonial rings generates the vector of ring frequencies in G_1, and subsequent scanning of each of the extracted subgraphs G_{1h} for fragments isomorphic to G_2 generates the vector of elementary marriage type frequencies, i.e., the matrimonial census.

Because PAJEK fragment searches check line values but not node values, all information on nodes has to be incorporated into the lines. This is done by transforming the original *k*-graph (which is a configured mixed graph) into a non-configured multigraph with 5 classes of arcs, such that each edge of the k-graph corresponds to an arc of class 1 (pointing from female to male node), and each arc of the *k*-graph connecting a node-pair with values (x_1, x_2) corresponds to an arc of class $2(x_1 + 1) + x_2$ ($x_1, x_2 \in \{0, 1\}$)[21]. We shall call this graph a *k_5-graph*. Transformation of a *k*-graph into a k_5-graph is accomplished by transformer programs like GEN2PAJEK, developed especially for the present purpose by Jürgen Pfeffer (FAS.research, Vienna)[22].

So as to be able to search for matrimonial rings in reduced form, it is further necessary to add a 6th class consisting of sibling edges for all sibling nodes in the original *k*-graph. As PAJEK contains a function for the addition of sibling edges, this can be easily done from the k_5-graph by means of a short series of commands available as a macro M1[23].

The (reduced) matrimonial universes with bounds (1, 4)[24] and [2, 2] – containing 239 consanguineous marriage types and 146 relinking marriage types – have been

[21] This means that arcs belong to classes 2, 3, 4 and 5 according as they point from mother to daughter, from mother to son, from father to daughter and from father to son. This coding makes it easy to shrink the network to the corresponding network of matri- or patri-“lineages” (the weak components of the graph which results from it by retaining only lines with values 2 and 3 or 4 and 5, respectively) linked by arcs that point from wife-giving to wife-taking lineages, as it is accomplished by the macros M5ab of the program package “pajek matrimonial census”.

[22] GEN2PAJEK (to be downloaded at *http://eclectic.ss.uci.edu/download/MarriageNetTools.htm*) reads an EXCEL .XLS file whose columns are, for each individual: ID number, name, sex (H/F or H/M), father's ID number, mother's ID number and spouse's ID number (each of the individual's spouses are placed in a separate row). It generates a PAJEK .NET (network) file in which the original k-graph has been redefined as a k_5-graph.

[23] All of the macros mentioned can be downloaded from the web at : *http://eclectic.ss.uci.edu/download/MarriageNetTools.htm*; the macros M2 and M3A used to generate census graphs are provided in Appendix 2.

[24] We shall not present the corresponding coefficient tables and ring lists for the (1,4)-universe here (it contains the 239 types of consanguineous marriage structures within the bounds of third cousinhood).

defined as PAJEK-project files and can be downloaded from the web at *http://eclectic.ss.uci.edu/download/MarriageNetTools.htm*. All census graphs which we shall present as examples in the remainder of this paper have been generated with reference to these two universes.

Figures 3(a) and 3(b) show two examples of a matrimonial census generated by PAJEK: Figure 3(a) shows the (1,4)-census for the West African Jafun Fulani (503 marriages, 180 of them in 385 consanguineous rings belonging to 83 types), (cf. [Barry, 1996, 1998, 2000]); and for the Amerindian Chimane (747 marriages, 252 of them in 844 consanguineous rings belonging to 51 types, (cf. [Daillant, 2003]), together with the census for a random kinship network[25]. Figure 3(b) shows the (2,2)-census for the families of the European city of Ragusa from the XIIth to the XVIth century (2002 marriages, 490 of them in 587 2-family-relinking rings belonging to 91 types; a PAJEK sample genealogy, cf. [Mahnken 1960]) and for the Australian Aboriginal Nyungar (338 marriages, 115 of them in 151 2-family-relinking rings belonging to 51 types, (cf. [Tilbrook, 1983]), both of whom avoid close kin marriage. Here also, the census for a random kinship network has been added for comparison[26]. The data, presented graphically by means of EXCEL and expressed as percentages of the total number of marriages in each sample corpus, derives from the .VEC (vector) files generated as a by-product of PAJEK fragment searches. It is clear that while, in general, certain ring types (e.g., marriages between first cousins) occur more frequently than others (e.g., marriages between siblings), there are nonetheless significant differences in the distribution of frequencies from one network to the next.

They can be easily generated without consideration of symmetry properties, since each of the three possible valued tip-configurations for $k = 1$ is perfectly asymmetric. The complete table of the 239 consanguineous ring types (analogous to the one provided in appendix 1 for 146 relinking ring types) is available from the authors.

[25] The random network has been generated by GENEORND 0.3. It contains 10.000 individuals, of which 2 % belong to the first generation. Men and women are uniformly distributed, annual death and divorce probability is 2 %, annual marriage probability (including polygamous marriages) is 10 %, annual reproduction probability 30 %. Women have children from 15 to 55 years, men from 20 to 60 years, the life span is 60 years for both sexes. The network contains 3501 marriages.

[26] The second random network has been generated under the same assumptions as the first, with the only difference that 10 % of all individuals belong to the first generation. It contains 3209 marriages.

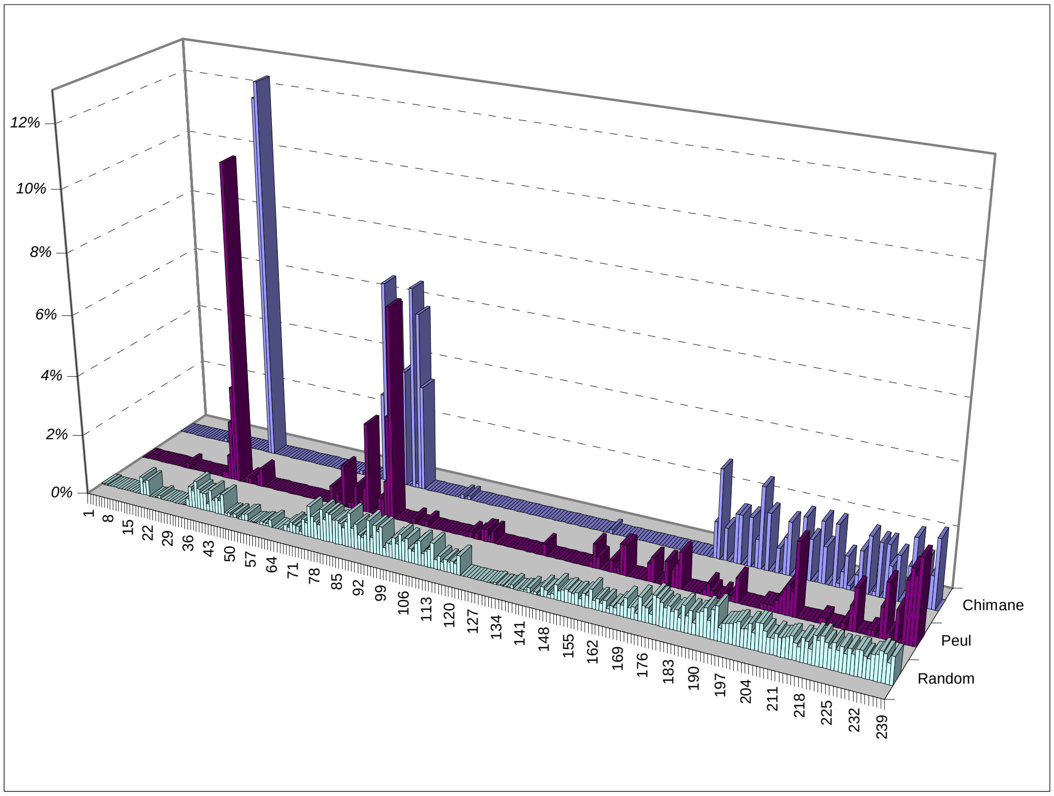


Figure 3(a)

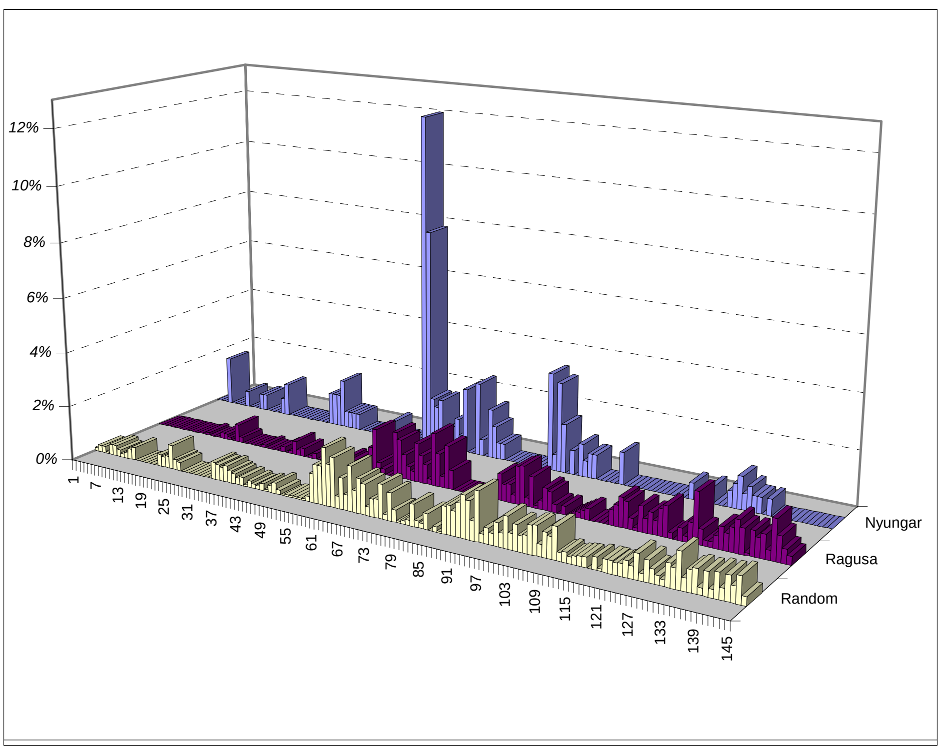


Figure 3(b)

3. 2 CENSUS GRAPHS

The comparison of such raw census data for empirical kinship networks, as well as for those of randomly generated kinship networks[27], may give a first idea of the possible determinants – sociological, statistical or purely mathematical – which may underlie their particular structural features. However, in order to grasp the logic of how a kinship structure is generated, it is not enough to merely consider the frequencies of elementary marriage types. The way various ring types are mutually related in the global structure must also be taken into account. A network is not simply the sum of its fragments, and a distribution of marriage type occurrences which differs significantly from a random distribution should not be taken as the direct reflection of a preference (or avoidance) ordering of the respective marriage types. In fact, many matrimonial rings may turn out as resulting from the combination of other matrimonial rings, such that their high or low frequency may just as well be interpreted as a preference for or an avoidance of this kind of combination (e.g., an avoidance of the repetition of the marriage type of a parent or sibling, rather than the avoidance of a certain marriage type as such).

To understand the composition of kinship structures, one must consider not only the elementary marriage types, but the entire matrimonial topology, that is, the set of *all* marriage types including those which are derived from the intersection, the union and the complements of elementary types: types of marriages which belong to different kinds of rings at the same time (e.g., "bilateral cross-cousin marriage" which is at once MBD and FZD), types of marriages which belong to at least one of several rings (e.g., "agnatic marriage" which may be as well FBD as FFBSD), and types of marriages which do not belong to certain rings (e.g., "distant cousin marriage" which is neither MZD, FZD, MBD nor FBD). An extended census for the complete list of (elementary and derived) marriage types gives a truer picture of the logical structure of the matrimonial network.

A first step towards an analysis of the complete matrimonial topology – rather than of the mere "surface" of simple ring occurrences – consists in examining the *census graph*: a 2-mode-affiliation network which links marriages to marriage types and thus gives a picture of the mutual interdependence and (in)compatibility of different marriage types.

The generation of census graphs from a given kinship network and a matrimonial ring list with PAJEK is facilitated by the fact that each PAJEK fragment search produces a hierarchy file which can be subsequently transformed into a 3-mode-network (actually an ancestral tree of uniform branch length 2, whose roots represent marriages, whose linknodes represent the found fragments for those marriages and whose tips represent their constituent nodes). Union of all these μ 3-mode networks (one for each fragment type), identification of nodes representing the same marriages, and removal of the third layer of tips (nodes for individuals) then results in the 2-mode matrimonial census graph. The entire procedure has been worked into a macro (M2) available on the WWWeb at *http://eclectic.ss.uci.edu/download/MarriageNetTools.htm* and given in appendix 2. The resulting census graph may then be reduced to 1-mode by means of another macro (M3A) also available from the WWWeb site.

[27] See [White, 1999]. V. Batagelj has recently created a generator of random kinship networks (GENEORND 0.3) which can also be read and analysed in PAJEK. The random networks of Figure 3 have been constructed with it.

Figures 4(a), (b), (c) and (d) show the 1-mode census graphs corresponding to the census data given in Figures 3(a) and (b). The size of the nodes corresponds to the number of marriages comprised in each ring type, the thickness of lines to that of their intersection, i.e., the number of marriages which belong to both types. The spatial layout of the nodes has been generated by the Fruchterman-Reingold algorithm implemented in PAJEK[28]. Larger versions of Figures 3-11 with readable labels may be found on the WWWeb at http://eclectic.ss.uci.edu/download/MSHfigs2005.htm.

Figure 4(a). Fulani

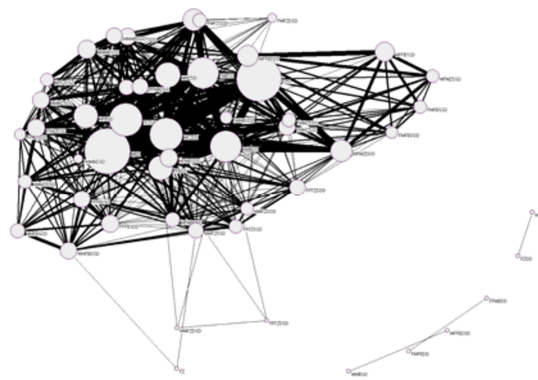


Figure 4(b). Chimane

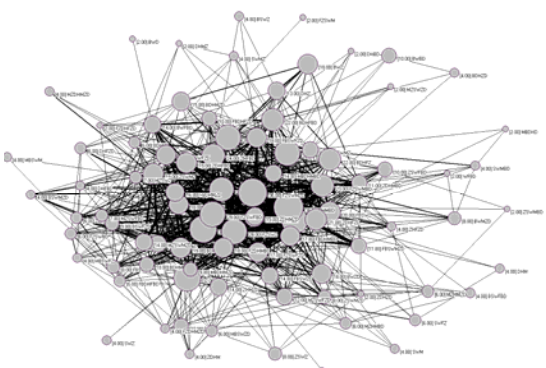


Figure 4(c). Ragusa

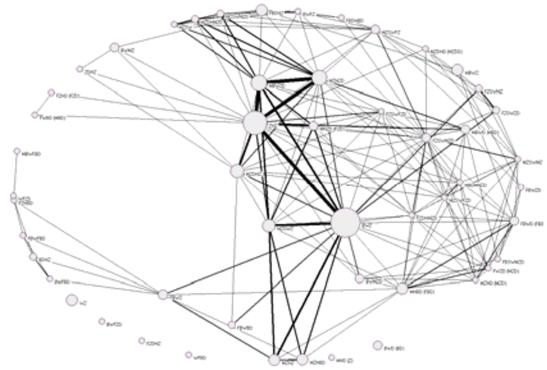


Figure 4(d). Nyungar

These census graphs can already give a first glimpse, however simplified, of the matrimonial topology, something which is impossible if one considers the matrimonial census alone (i.e., the sizes of the nodes only). Neglecting the relational structure of marriage types may indeed lead to erroneous conclusions. For instance, the presence of a large MBD-node in the Fulani-network – which might be presumed to be an exception to the overall pattern of agnatic marriage – acquires new significance when the thick lines connecting it to the FFBSD and other agnatic marriage ring nodes are taken into account. Similarly, the number of "incestuous" (not cross-cousin) marriages in the Chimane census, however small, would still be exaggerated if one didn't know that six

[28] The Fruchterman Reingold algorithm considers the network as a system of mass particles where the nodes represent mass points repelling each other while the lines simulate springs with attracting forces. It then seeks to find an equilibrium solution for this physical system such as to minimize its total energy.

of the nine occurrences of incestuous marriage types concern only three unions (each of which is incestuous twice over), while the three other types concern only two more marriages that are genealogically ambivalent as they also belong to at least two correct types each.

The importance of considering common intersections becomes especially clear when matrimonial rings are classified into broader, aggregate types, such as those comprising all relinking marriages between groups of same-sex-kin, or all consanguineous marriages implying a given generational distance between spouses, or all marriages between agnatic relatives, and so forth[29]. These broader classes represent *unions* of marriage types, which – except if these latter have no common intersections – will in general contain a smaller number of marriages than the sum of their constituent subclasses. Obviously, the aggregation of matrimonial census data cannot be effected by simple addition of frequencies, but requires that marriages belonging to more than one subtype of the aggregate type be consolidates. In other words, it is necessary to pass once again through the 2-mode census graph in order to shrink the network according to the chosen partition before counting the outdegrees (which are the net frequencies). This can be done in PAJEK by applying an alternative macro (M3B) for the reduction of the census graph to 1-mode and then inspecting the adjoined vector file which contains the consolidated census.

Consider, for example, the distribution of first, second and third cousin marriages among the Fulani and the Chimane. The first columns in Tables 5(a)and 5(b)are simple additions from the elementary matrimonial censuses represented in Figures 3(a) and (b) and 4(a) and (b). The second columns are obtained by reducing the census graph to a 1-mode graph containing the aggregate marriage types as nodes and deriving their true extensions directly from counting the marriages in the underlying census graph. As can be seen, the difference becomes substantial for third cousins which in most kinship networks are likely to be included simultaneously in more than one type. In the Chimane case, where 15 % of first cousin marriages are between bilateral cross cousins (note the line between the two largest nodes), this difference is already substantial for first cousins. By contrast, the network of the Fulani, who favour agnatic but avoid uterine kin marriage, shows no connection between first-cousin marriages whose number thus remains unchanged by consolidation.

[29] For the networks given above, this has be done by joining a suitably chosen partition of the matrimonial ring set, defined as a PAJEK CLU-file, before reducing the census graph to 1-mode form – the macro M3A automatically adjusts the partition to the 1-mode graph.

Chimane	Cumulate elementary census		True aggregate census	
1st cousins	182	*21.69 %*	158	*63.20 %*
2nd cousins	293	*34.92 %*	154	*61.60 %*
3rd cousins	364	*43.38 %*	127	*50.80 %*
All cousins up to 3rd	839		250	

Fulani	Cumulate elementary census		True aggregate census	
1st cousins	83	*27.39 %*	83	*55.70 %*
2nd cousins	92	*30.36 %*	74	*49.66 %*
3rd cousins	128	*42.24 %*	76	*51.01 %*
All cousins up to 3rd	303		149	

Tables 5(a) and 5(b). Census aggregation for the Chimane and Fulani networks

A second, related problem consists in the fact that the ranking of marriage types by frequencies may be radically changed if one eliminates from the network a marriage type which has large intersections with many other types. As a consequence, the *ordered* census which is generated by counting only those marriages of each type which have not yet been counted among the marriages of a preceding type may differ substantially from the raw census. To see this, consider again the example of cousin marriages among the Fulani and the Chimane (Tables 6(a) and 6(b)). The first column contains the absolute extension of each of the three marriage types (including marriages which may belong to more than one type), while the second counts only those second cousins marriages which are not also first cousin marriages, and only those third cousin marriages which are not second or first cousin marriages. In both cases, ties of third cousinhood between spouses exist in more than a half of all cousin marriages, but this proportion is reduced to about 1/8 when only "pure" third cousin marriages are considered[30].

Chimane	Raw census		Ordered census	
1st cousins	158	63.20 %	158	63.20 %
2nd cousins	154	61.60 %	58	23.20 %
3rd cousins	127	50.80 %	34	13.60 %
All cousins up to 3rd	250		250	

Fulani (Jafun)	Raw census		Ordered census	
1st cousins	83	*55.70 %*	83	*55.70 %*
2nd cousins	74	*49.66 %*	47	*31.54 %*
3rd cousins	76	*51.01 %*	19	*12.75 %*
All cousins up to 3rd	149		149	

Tables 6(a) and 6(b). Raw and ordered census for Chimane and Fulani cousin marriages

[30] When interpreting these numbers, it must however be kept in mind that third cousin marriages can only be found in a network from the 4th or 5th (documented) generation downwards, and are thus checked in a much smaller proportion of the network than first cousin marriages.

The progressive elimination of nodes may also be useful for problems other than frequency ranking. It may be used, for example, to examine the degrees of ring cohesion of the kinship network (White, this volume) by observing the rapidity with which census graph density decreases when nodes (or entire clusters of nodes according to a chosen partition) are successively eliminated by order of size. In PAJEK this is done by application of a macro (M4A to eliminate single nodes, M4B to eliminate clusters[31]) to the 2-mode census graph, and subsequent reduction to 1-mode. Figure 5 shows the difference of the raw and the ordered censuses for the Chimane marriages included in the different relinking types:

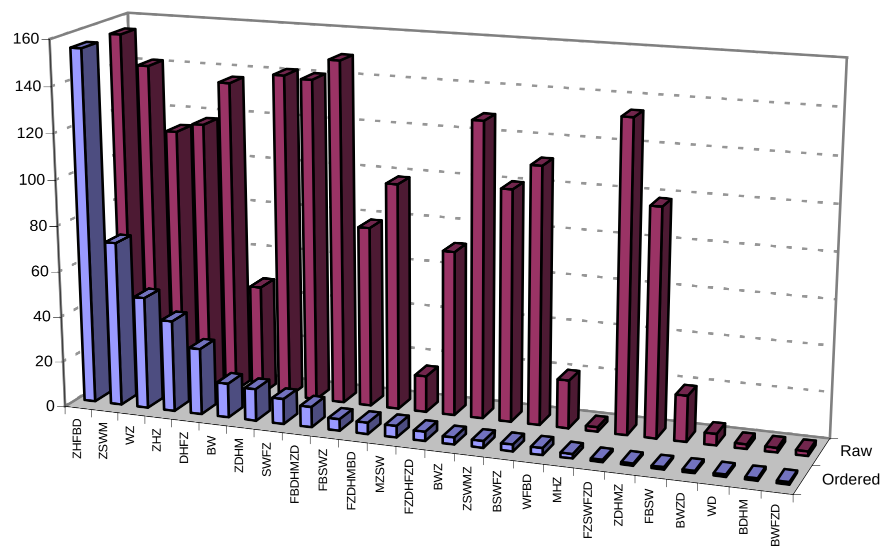
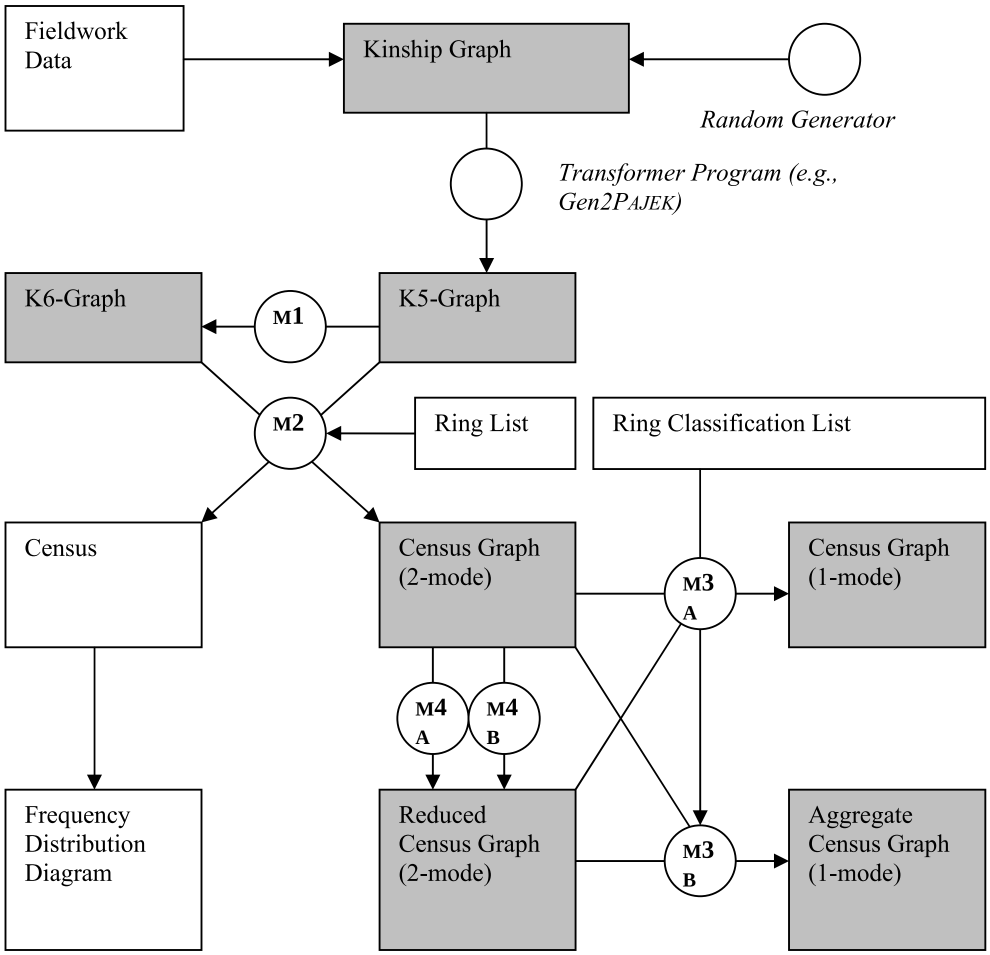


Figure 5. Chimane marriages implied in relinking rings
(raw and ordered censuses)

In the following section, we shall give some illustrations of various methods of census graph analysis. The aim of this is not to propose comprehensive analyses of particular marriage networks, but to suggest, by means of several brief examples, some of the possibilities offered by the conceptual and procedural tools just outlined. Figure 6 below summarizes the procedures of generation and manipulation of kinship and census graphs (squares indicate input and output data, shaded squares correspond to networks, circles to programs and macros).

[31] In applying M4A and M4B one has to enter the name (in conventional notation) of the representative ring of the type (or cluster) to be eliminated, preceded by a single cross (e.g., #MBD) in the case of a single node, or by a double cross (e.g., ##MBD) in the case of a cluster. A cluster is represented by the representative ring of the first ring type in the list which belongs to it; their names can also easily be read from the aggregate census graph created by macro M3B.



The program package "PAJEK MATRIMONIAL CENSUS" contains the transformer GEN2PAJEK, the ring lists RINGS 1.PAJ (239 consanguineous marriages) and RINGS 2.PAJ (145 relinking marriages) and the macros M1 – PREPARE BASIC NETWORK.MCR, M2 – CENSUS GRAPH 2-MODE (RING 1).MCR, M2 – CENSUS GRAPH 2-MODE (RINGS 2).MCR, M3A – CENSUS GRAPH 1-MODE.MCR, M3B – AGGREGATE CENSUS GRAPH 1-MODE.MCR, M4A – NODE ELIMINATION.MCR, and M4B – CLUSTER ELIMINATION.MCR.

Figure 6. Procedure of matrimonial census analysis

3.3 ILLUSTRATIVE APPLICATIONS

One way to explore the structural properties of ring census networks is to apply a number of rudimentary partitions. Consider, for example, a partition of the Ragusa and the Chimane census graphs for both consanguineous and relinking marriage types, which distinguishes both between these two broad classes (respectively, the left and right sides of Figures 7(a) and 7(b)), and between types entailing horizontal (dark grey or black), oblique (light grey) or alternate-generation (white) marriages; upper right dark grey nodes indicate rings in which both sets of couples are of the same generation (e.g., BWZ-marriages), whereas lower black nodes indicate rings in which they are not (e.g., FZHZD-marriages).

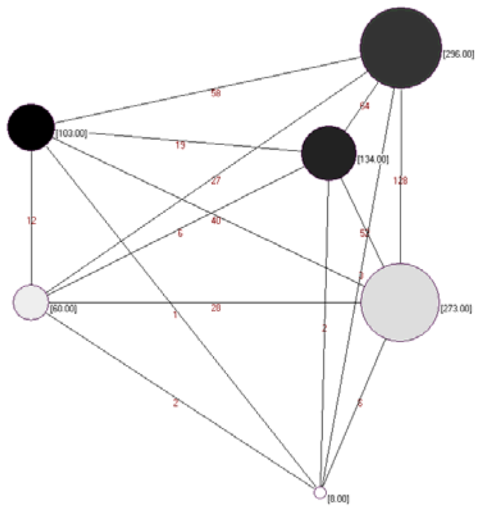


Figure 7(a). Ragusa

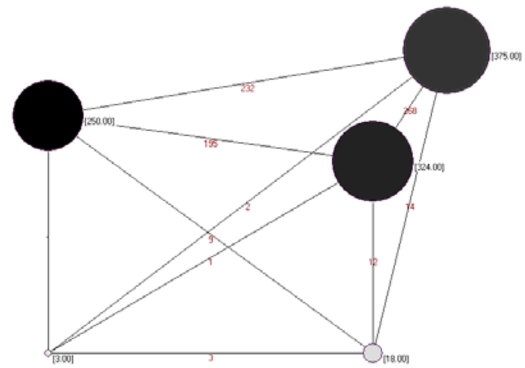


Figure 7(b). Chimane

To take another example, Figures 8(a) and 8(b) show the distribution of consanguineous and relinking matrimonial rings in the same networks according to criteria of "sidedness" (cf. [Houseman and White, 1996, 1998(a), 1998(b)]): consanguineous and relinking rings are represented by the left and right sides of the Figures respectively, the top row corresponding to unsided rings, the second row to virisided rings, the third to uxorisided rings and the bottom row to dual (both virisided and uxorisided) rings. Whereas the Ragusa network shows a homogenous distribution among these different classes (relinkings being proportionally greater in number), the Chimane network is shown to be overwhelmingly dual sided.

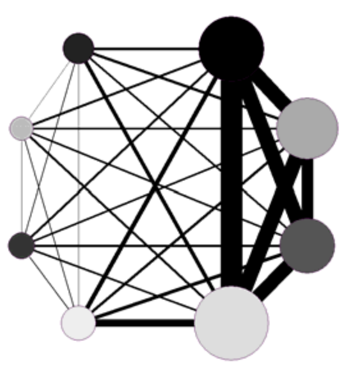


Figure 8(a). Ragusa

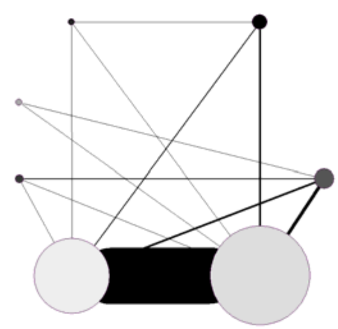


Figure 8(b). Chimane

Finally, Figures 9(a) and 9(b) represent a distribution of relinking matrimonial rings among the (Jafun) Fulani and among the Chimane according to the number of intervening nodes in the chain linking two spouses (which is equal to the ring length n diminished by 2). The ordering begins with the left-hand ("9 o'clock position") node corresponding to a minimum of 1 intervening node and moves clock-wise to a maximum of 8 intervening nodes (“8 o’clock position” node). One can see that whereas in the Fulani case, the majority of marriages take place between spouses separated by at least 4 intervening nodes, the Chimane network reveals a regular pattern in which rings entailing an even number of intervening nodes are clearly favoured.

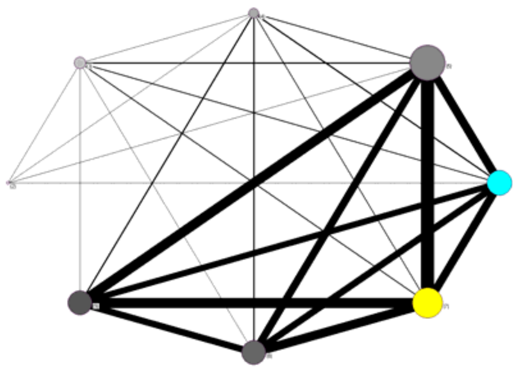


Figure 9(a). Fulani

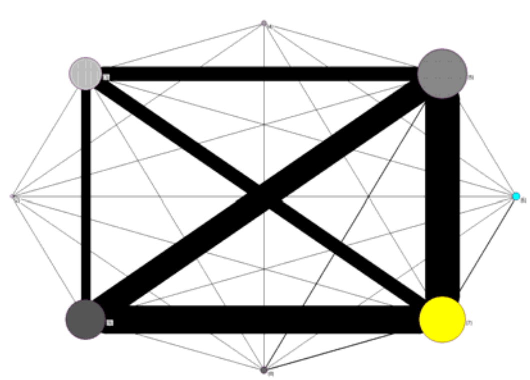


Figure 9(b). Chimane

Another, complementary approach consists in applying partitions that are motivated by particular hypotheses regarding the structural principles underlying marriage network patterns. Knowing for example that the Fulani explicitly express a preference for marriage with close agnatic kin, it would seem appropriate to apply a classification that classifies consanguineous marriage rings according to the number of uterine (female-valued) linknodes between spouses[32]. Figures 10(a) and 10(b) show this partition for the Fulani and the Chimane networks of consanguineous marriage rings; the ordering begins at the top with a minimum of 0 uterine linknodes and proceeds downwards to a maximum of 6. Figures 11(a) and 11(b) show graphs of the same partition but in which nodes represent the partitioned classes of matrimonial rings rather than the rings themselves (macro M3B). Here, the ordering begins with the left-hand (“9 o’clock position) node corresponding to a minimum of 0 uterine linknodes and proceeds clock-wise to a maximum of 6 (8 o’clock position node). In the Fulani case, both the absolute number of matrimonial rings and the degree of interconnection between them are clearly shown to be inversely proportional to of the number of uterine linknodes between spouses: 122 marriages entail no uterine linknodes, 114 entail 1 such linknode, 56 entail 2 linknodes, 22 entail 3 linknodes, 2 entail 4 linknodes, 1 entails 5 linknodes and no marriage entails 6 uterine linknodes. On the other hand, the Chimane network once again reveals a regular pattern in which marriages entailing an odd number of uterine linknodes are favoured.

[32] As we are treating rings in reduced form only, that number is equal to that of intervening female-valued nodes.

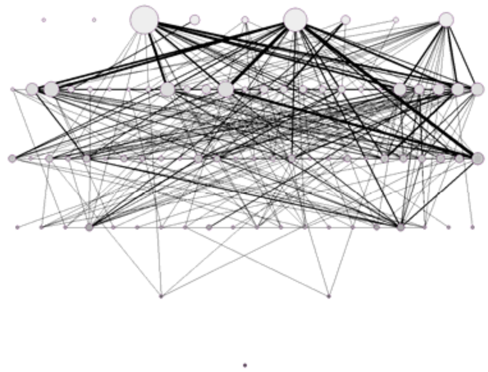


Figure 10(a). Fulani

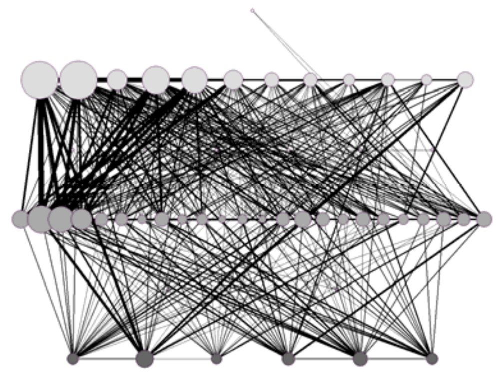


Figure 10(b). Chimane

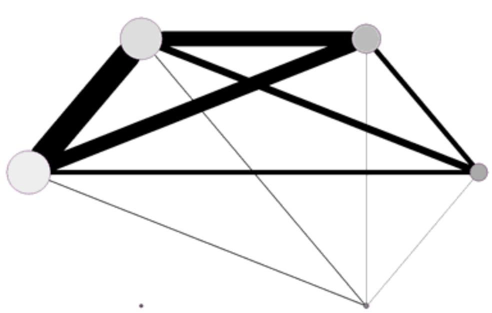


Figure 11(a). Fulani

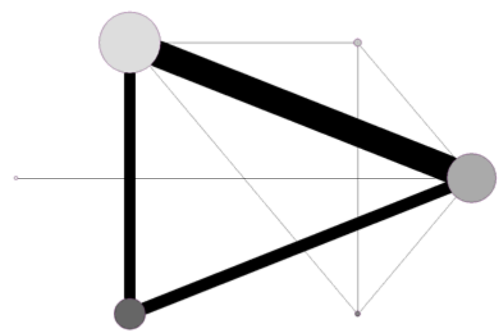


Figure 11(b). Chimane

As may be deduced from this and the preceding graphs of Chimane marriage rings, the marriage network of this society is organized according to bipartite "Dravidian" principles associated with bilateral cross-cousin marriage: a definite preference for unions between close kin belonging to same generation and linked by an odd number of cross-sex consanguineous ties (not counting the sex of Ego and Alter). Thus, applying such a "Dravidian crossness" partition to the Chimane network and then activating macro M3B results in two nodes, one of which contains 99 % of all marriage rings.

Such a clear-cut dominance of a certain class of marriage types is of course exceptional. In general, asserting the degree in which a network can be built up of matrimonial rings of a certain type requires the analytical manipulation of the census graph and cannot be directly "read" from it.

One method of doing this consists in deriving an "ordered census" for the various clusters, thus combining the procedures of network shrinking (macros 3AB) and successive node elimination (macros 4AB). As an illustration, Table 7 presents the results of this procedure for Nyungar relinking matrimonial rings (there are very few consanguineous marriages in this network). Starting from the 115 marriages in row 0, rows 1 – 11 show the results of successive elimination of the next most frequent marriage type (frequencies being calculated *after* the elimination of the preceding types). The criteria for elimination are founded on a partition, as shown in the columns, which distinguishes between different combinations of the various types of consanguineous "families" involved in these relinking rings: single individuals (in the case of polygamous unions), siblings, persons linked by uterine ties, persons linked by agnatic ties or persons linked by cognatic ties. More than one half (57 %) of all marriages in relinking rings involve unions with siblings or unions between pairs of siblings (rows 1 and 2 in Table 7), that is, persons who are at once uterine and agnatic relatives. 21 % (rows 3, 4 and 5) involve families whose members are linked by uterine ties without being linked by sibling-ties, 9 % involve one family whose members are linked by uterine ties and the other by agnatic ties without falling under any of the preceding headings (row 6), 10 % involve families whose members are linked by agnatic ties without being linked by uterine or sibling ties (rows 7, 8 and 9), and only 4 % involve any family whose members are linked by purely cognatic ties which cannot be characterized in any other manner. An implicit rule favouring the reiteration of the marriage of a "parallel" (agnatic or uterine) relative (95,7 %), with a strong uterine bias (77,4 %), could thus account for most of the relinking marriages among the Nyungar[33]. Note, however, that other elimination sequences should be tested in order to assess a comparative value to these hypothetical rule.

	A	B	C	D	E	F	G	H	I	J
	Classes eliminated	Marriages eliminated	%	Marriages remaining	Rings	Marriages/ Ring	Ring-Pairs	Connected Rings	Comp.	Density
0	**21**			**115**	**51**	**4.88 (6.47)** **3**	**188**	**7.37 (5.40)** **7**	**8**	**14.75**
1	Individual = siblings (1)	6	5.22	109	50	4.86 (6.53) 3	188	7.52 (5.34) 7	7	15.35
2	Siblings = siblings (2)	59 (65)	51.30 (56.5)	50	47	2.26 (1.25) 2	123	5.23 (4.00) 5	8	11.38
3	Individual = uterine relatives (1)	2 (67)	1.74 (58.3)	48	44	2.11 (1.25) 2	98	4.45 (3.77) 3.5	8	10.36
4	Siblings = uterine relatives (8)	18 (85)	15.65 (73.9)	30	33	1.91 (0.87) 2	81	4.91 (3.93) 4	7	15.34
5	Uterine relatives = uterine relatives (7)	4 (89)	3.48 (77.4)	26	22	1.95 (0.77) 2	24	2.18 (1.61) 2	8	10.39
6	Uterine relatives = agnatic relatives (11)	10 (99)	8.70 (86.1)	16	13	1.92 (0.73) 2	11	1.69 (1.59) 1	7	14.10
7	Individual = agnatic relatives (1)	2 (101)	1.74 (87.8)	14	12	1.92 (0.76) 2	11	1.83 (1.57) 1	6	16.67
8	Siblings = agnatic relatives (5)	7 (108)	6.09 (93.9)	7	6	1.33 (0.47) 1	1	0.33 (0.47) 0	5	6.67
9	Agnatic relatives = agnatic relatives (6)	2 (110)	1.74 (95.7)	5	3	1.67 (0.47) 2	0	0.00 (0.00) 0	3	0.00
10	Individual = cognatic relatives (1)	1 (111)	0.87 (96.5)	4	2	2.00 (0.00) 2	0	0.00 (0.00) 0	2	0.00
11	Siblings = cognatic relatives (2)	4 (115)	3.48 (100.0)	0	0	-	0	-	0	0.00

Table 7. Aggregate census analysis of Nyungar relinkings

[33] Column A indicates the successively eliminated clusters of marriage types (the initial number of types in each cluster is given in parentheses). Column B indicates the number of *remaining* marriages of the eliminated cluster after elimination of all preceding clusters, and, in parentheses, the number of all marriages thus far eliminated. Column C indicates these same two quantities as percentages of all marriages. The numbers of total marriages and marriage types remaining after elimination are indicated in columns D and E. Column F indicates the average and median number of marriages per type (the standard deviation is given in parentheses). Column G indicates the total number of marriage types which have a common intersection with each other (i.e., the number of lines in the reduced 1-mode census graph), whereas column H indicates the average and medium number of marriage types which have a common intersection with a given type. This quantity, together with the standard deviation (in parentheses), is a measure of the cohesion (and centralization) of the census graph (in 1-mode-form). Two alternative cohesion measures, the number of components (largest connected subgraphs) and the density (i.e., the number of lines – or binary non-empty type intersections – as a percentage of possible lines), are given in columns I and J.

3.4 OUTLOOK: TOWARDS A THEORY OF MATRIMONIAL RINGS

The aim of the preceding section was to illustrate the application of census graphs to the exploratory analysis of kinship networks. Thus, we have largely concentrated on use of such graphs as an *instrument* for graph-theoretical analysis (e.g., for the consolidation of frequency distributions or the identification of dominant or subordinate ring types in the underlying "first order" graph) – rather than as an *object* of it. Put otherwise, the *lines* of the census graph have served us mainly to identify, shrink or rank the *nodes*. However, the structural features of census graphs may also be interesting in their own right. After all, both nodes and lines of a census graph *equally* represent sets of marriage edges contained in matrimonial rings, the only difference being that the sets represented by the lines are derived from the intersection of the sets represented by the nodes.

The very nature of kinship networks makes it impossible to understand the nodes in census graphs without considering the lines as well. To see this, recall (cf. Theorem 2 of Section 1.2. above and White, [2005]) that in kinship graphs, *every marriage which belongs to two matrimonial rings implies the presence of some other marriage belonging to another matrimonial ring* namely, to the canonical closure of the cycle composed of them. Consider the examples in Figure 1. A MBD-marriage which is at the same time an FZD-marriage implies a ZHZ-marriage for the spouses' parents (Figure 1(b)). Were it not for the unavoidable incompleteness of data (non-regularity) and the fact that some parents may remain unmarried (non-canonicity), the presence of a combined MBD-FZD-marriage in the absence of a ZHZ-marriage would be logically impossible. In the same manner, a combined MBD-FFBSD implies the presence of an FBD-marriage (Figure 1(c)), a combined MBD-FBD-marriage may co-occur with BZ-marriage, although this is not strictly implies (the FFBS need not be the WF; note also that the FFBSD marriage is not a ring in this case because this cycle is not its own induced subgraph). Lines between two nodes may thus implies a relationship with a third node in the network. Sizes of nodes in the census graph may be logically related to the thickness of lines between other nodes. This same reasoning can also be applied to the "third order" level, that is, to the analysis of *triads* insofar as they represent ternary intersections of marriage types, and so on. Not only is the thickness of lines related to the size of nodes elsewhere in the network, but so also is their density.

Results of matrimonial ring analysis have variable empirical and cautionary implications for theories of kinship. If all the lines in a census graph were like those between marriage types produced by the entailments in Figure 1(b), for example, one would be forewarned that if the marriage frequencies (number of individuals with both pairs of marriage types) on the MBD/FZD MBD/ZHZ, and ZHZ/FZD intersection lines were all 18 or greater then 36 of these intersection frequencies are nonindependent of 18 independent cycles in which any two of the three types entail the other. When there are many such structural entailments, a theorist of "immanent structure" of kinship rules might be tempted to argue for a logic immanent in the kinship structure, but the actual specification of these rules would be in doubt (are there two types that are primary, the other secondary? If so, which?). But if the lines in a census graph were like those between produced by the entailments in Figure 1(c), where the combination of FBD and MBD marriages in two successive generations entail in some cases FFBSD marriage and in other cases not, one is forced to consider a "logic of practice" analysis where the number of individuals for which this entails FFBSD marriage be made a focus of analysis. For this type of analysis, a two-mode analysis of the intersection frequencies of marriage types and individuals whose marriages conform to the types would be important. Further although when a MBD marriage is also a FFBSD the latter will not show up as a ring because the FFBSD marriage cycle is not its own induced subgraph, it cannot be assumed for the individual marrying that the marriage choice was not that of the FFBSD even though this does not show up as a ring but is masked by its embeddedness in a relationship that is also already constituted as MBD prior to the marriage.

The census graph represents a section – however small – of the topology which a matrimonial ring structure induces upon the set of marriages that compose the kinship network. Its analysis by proper graph-theoretic tools may thus be a first step towards the development of a true matrimonial ring theory. It is clear that it is by itself insufficient to grasp the total composition structure of matrimonial rings. The development of more sophisticated tools – such as lattice structures – will be necessary for that. The fertility of graph-theoretical concepts in analysing kinship patterns, and the recent progress of computer techniques which – in cases for the first time – permit their systematic and rapid application to real world data, however seems to encourage further efforts to attain this goal.

APPENDIX 1: MATRIMONIAL RING TYPES OF THE (REDUCED) UNIVERSE WITH BOUNDS (2, 2)

1. (Nr.)	Running identity number of the matrimonial ring type or isomorphism class
2. (Not. HF)	Representative ring in analytic notation
3. (Not. Conv.)	Representative ring in conventional notation (denoting the kin relationship of wife to husband – when the first node in the ring is female, the ring is read from right to left)
4. (v)	Number of variants (distinct rotations and reflections)
5. (k)	Number of trees (ring width)
6. (d)	Maximal branch length (ring depth)
7. (z_b)	Branch configuration number of the representative ring (the lowest of all rings of the type)
8. (z_v)	Value configuration number of the representative ring (the lowest of all rings of the type with branch configuration number z_b)
9. (z_b*)	Branch configuration number of the representative tip-part
10. (z_b*)	Value configuration number of the representative tip-part
11. (ab_1)	Symmetry index (tip-part)
12. (ab_2)	Symmetry index (semi-neutral form)
13. (ab_3)	Symmetry index (completely configured form)
14. n	Number of nodes (ring length)
15. n’	Number of valued nodes (reduced form)
16. b	Number of branches
17. b_2	Number of non-singular branches
18. t	Number of tips
19. k_0	Number of singular roots (roots with degree 0)
20. k_1 (= b_1)	Number of singular branches (roots with degree 1)
21. k_0	Number of branching roots (roots with degree 2)
22. h	Number of linknodes
23. m	Number of male nodes
24. dg_s	Skewedness degree
25. dg_a	Agnatic degree
26. dg_u	Uterne degree

22 consanguineous marriage rings in 11 classes

Nr	Not. HF	Not. Conv.	v	k	d	z_b	z_v	z_b*	z_v*	ab_1	ab_2	ab_3	n	n’	b	b_2	t	k_0	k_1	k_2	h	m	dg_s	dg_a	dg_u
1	H(F)	M	2	1	1	1	1	1	1	10	10	10	2	2	1	0	2	0	1	0	0	1	1	1,00	1,00
2	F(H)	D	2	1	1	1	2	1	2	10	10	10	2	2	1	0	2	0	1	0	0	1	1	1,00	1,00
3	H-F	Z	2	1	1	4	1	3	1	10	10	10	3	2	2	2	2	0	0	1	0	1	0	1,00	1,00
4	HF-F	MZ	2	1	2	5	1	3	1	10	10	10	4	3	2	2	2	0	0	1	1	1	1	0,00	1,00
5	HH-F	FZ	2	1	2	5	3	3	1	10	10	10	4	3	2	2	2	0	0	1	1	2	1	1,00	0,00
6	FF-H	ZD	2	1	2	5	4	3	2	10	10	10	4	3	2	2	2	0	0	1	1	1	1	0,00	1,00
7	FH-H	BD	2	1	2	5	6	3	2	10	10	10	4	3	2	2	2	0	0	1	1	2	1	1,00	0,00
8	HF-FF	MZD	2	1	2	8	1	3	1	10	10	10	5	4	2	2	2	0	0	1	2	1	0	0,00	1,00
9	HH-FF	FZD	2	1	2	8	3	3	1	10	10	10	5	4	2	2	2	0	0	1	2	2	0	0,50	0,50
10	HF-HF	MBD	2	1	2	8	5	3	1	10	10	10	5	4	2	2	2	0	0	1	2	2	0	0,50	0,50
11	HH-HF	FBD	2	1	2	8	7	3	1	10	10	10	5	4	2	2	2	0	0	1	2	3	0	1,00	0,00

532 relinking marriage rings in 146 classes

Nr	Not. HF	Not. Conv.	v	k	d	zb	zv	zb*	zv*	ab1	ab2	ab3	n	n'	b	b_2	t	k_0	k_1	k_2	h	m	s	a	u
0	H.F	W	2	2	0	0	1	0	1	12	12	12	2	2	0	0	2	2	0	0	0	1	0	0,00	0,00
1	H(H).F	FW	4	2	1	1	3	1	3	10	10	10	3	3	1	0	3	1	1	0	0	2	1	1,00	0,00
2	F(F).H	WD	4	2	1	1	4	1	4	10	10	10	3	3	1	0	3	1	1	0	0	1	1	0,00	1,00
3	H-H.F	BW	2	2	1	4	3	3	3	12	12	12	4	3	2	2	3	1	0	1	0	2	0	1,00	0,00
4	F-F.H	WZ	2	2	1	4	4	3	4	12	12	12	4	3	2	2	3	1	0	1	0	1	0	0,00	1,00
5	HF-H.F	MBW	4	2	2	5	5	3	3	12	10	10	5	4	2	2	3	1	0	1	1	2	1	0,50	0,50
6	HH-H.F	FBW	4	2	2	5	7	3	3	12	10	10	5	4	2	2	3	1	0	1	1	3	1	1,00	0,00
7	FF-F.H	WZD	4	2	2	5	8	3	4	12	10	10	5	4	2	2	3	1	0	1	1	1	1	0,00	1,00
8	FH-F.H	WBD	4	2	2	5	10	3	4	12	10	10	5	4	2	2	3	1	0	1	1	2	1	0,50	0,50
9	HF-FH.F	MZSW	2	2	2	8	9	3	3	12	12	10	6	5	2	2	3	1	0	1	2	2	0	0,33	0,67
10	HH-FH.F	FZSW	4	2	2	8	11	3	3	12	12	10	6	5	2	2	3	1	0	1	2	3	0	0,67	0,33
11	HH-HH.F	FBSW	2	2	2	8	15	3	3	12	12	12	6	5	2	2	3	1	0	1	2	4	0	1,00	0,00
12	FF-FF.H	WMZD	2	2	2	8	16	3	4	12	12	12	6	5	2	2	3	1	0	1	2	1	0	0,00	1,00
13	FH-FF.H	WMBD	4	2	2	8	18	3	4	12	12	10	6	5	2	2	3	1	0	1	2	2	0	0,33	0,67
14	FH-HF.H	WFBD	2	2	2	8	22	3	4	12	12	12	6	5	2	2	3	1	0	1	2	3	0	0,67	0,33
15	H(H).F(F)	FWM	4	2	1	10	3	5	3	10	10	10	4	4	2	0	4	0	2	0	0	2	2	1,00	1,00
16	H(F).H(F)	MHM	2	2	1	10	5	5	5	30	30	30	4	4	2	0	4	0	2	0	0	2	2	1,00	1,00
17	F(H).F(H)	DHD	2	2	1	10	10	5	10	30	30	30	4	4	2	0	4	0	2	0	0	2	2	1,00	1,00
18	(H)H.F(F)	SWM	4	2	1	12	3	6	3	10	10	10	4	4	2	0	4	0	2	0	0	2	0	1,00	1,00
19	(H)F.H(F)	DHM	4	2	1	12	5	6	5	10	10	10	4	4	2	0	4	0	2	0	0	2	0	1,00	1,00
20	H-H.F(F)	BWM	4	2	1	13	3	7	3	10	10	10	5	4	3	2	4	0	1	1	0	2	1	1,00	1,00
21	H-F.H(F)	ZHM	4	2	1	13	5	7	5	10	10	10	5	4	3	2	4	0	1	1	0	2	1	1,00	1,00
22	F-H.F(H)	DHZ	4	2	1	13	10	7	10	10	10	10	5	4	3	2	4	0	1	1	0	2	1	1,00	1,00
23	F-F.H(H)	SWZ	4	2	1	13	12	7	12	10	10	10	5	4	3	2	4	0	1	1	0	2	1	1,00	1,00
24	HF-H.F(F)	MBWM	4	2	2	14	5	7	3	10	10	10	6	5	3	2	4	0	1	1	1	2	2	0,50	1,00
25	HH-H.F(F)	FBWM	4	2	2	14	7	7	3	10	10	10	6	5	3	2	4	0	1	1	1	3	2	1,00	0,50
26	HF-F.H(F)	MZHM	4	2	2	14	9	7	5	10	10	10	6	5	3	2	4	0	1	1	1	2	2	0,50	1,00
27	HH-F.H(F)	FZHM	4	2	2	14	11	7	5	10	10	10	6	5	3	2	4	0	1	1	1	3	2	1,00	0,50
28	FF-H.F(H)	DHZD	4	2	2	14	20	7	10	10	10	10	6	5	3	2	4	0	1	1	1	2	2	0,50	1,00
29	FH-H.F(H)	DHBD	4	2	2	14	22	7	10	10	10	10	6	5	3	2	4	0	1	1	1	3	2	1,00	0,50
30	FF-F.H(H)	SWZD	4	2	2	14	24	7	12	10	10	10	6	5	3	2	4	0	1	1	1	2	2	0,50	1,00
31	FH-F.H(H)	SWBD	4	2	2	14	26	7	12	10	10	10	6	5	3	2	4	0	1	1	1	3	2	1,00	0,50
32	H-FH.F(F)	ZSWM	4	2	2	16	5	7	3	10	10	10	6	5	3	2	4	0	1	1	1	2	0	0,50	1,00
33	H-HH.F(F)	BSWM	4	2	2	16	7	7	3	10	10	10	6	5	3	2	4	0	1	1	1	3	0	1,00	0,50
34	H-FF.H(F)	ZDHM	4	2	2	16	9	7	5	10	10	10	6	5	3	2	4	0	1	1	1	2	0	0,50	1,00
35	H-HF.H(F)	BDHM	4	2	2	16	11	7	5	10	10	10	6	5	3	2	4	0	1	1	1	3	0	1,00	0,50
36	F-FH.F(H)	DHMZ	4	2	2	16	20	7	10	10	10	10	6	5	3	2	4	0	1	1	1	2	0	0,50	1,00
37	F-HH.F(H)	DHFZ	4	2	2	16	22	7	10	10	10	10	6	5	3	2	4	0	1	1	1	3	0	1,00	0,50
38	F-FF.H(H)	SWMZ	4	2	2	16	24	7	12	10	10	10	6	5	3	2	4	0	1	1	1	2	0	0,50	1,00
39	F-HF.H(H)	SWFZ	4	2	2	16	26	7	12	10	10	10	6	5	3	2	4	0	1	1	1	3	0	1,00	0,50
40	HF-FH.F(F)	MZSWM	4	2	2	17	9	7	3	10	10	10	7	6	3	2	4	0	1	1	2	2	1	0,33	1,00
41	HH-FH.F(F)	FZSWM	4	2	2	17	11	7	3	10	10	10	7	6	3	2	4	0	1	1	2	3	1	0,67	0,67
42	HF-HH.F(F)	MBSWM	4	2	2	17	13	7	3	10	10	10	7	6	3	2	4	0	1	1	2	3	1	0,67	0,67
43	HH-HH.F(F)	FBSWM	4	2	2	17	15	7	3	10	10	10	7	6	3	2	4	0	1	1	2	4	1	1,00	0,33
44	HF-FF.H(F)	MZDHM	4	2	2	17	17	7	5	10	10	10	7	6	3	2	4	0	1	1	2	2	1	0,33	1,00
45	HH-FF.H(F)	FZDHM	4	2	2	17	19	7	5	10	10	10	7	6	3	2	4	0	1	1	2	3	1	0,67	0,67
46	HF-HF.H(F)	MBDHM	4	2	2	17	21	7	5	10	10	10	7	6	3	2	4	0	1	1	2	3	1	0,67	0,67
47	HH-HF.H(F)	FBDHM	4	2	2	17	23	7	5	10	10	10	7	6	3	2	4	0	1	1	2	4	1	1,00	0,33
48	FF-FH.F(H)	DHMZD	4	2	2	17	40	7	10	10	10	10	7	6	3	2	4	0	1	1	2	2	1	0,33	1,00
49	FH-FH.F(H)	DHMBD	4	2	2	17	42	7	10	10	10	10	7	6	3	2	4	0	1	1	2	3	1	0,67	0,67
50	FF-HH.F(H)	DHFZD	4	2	2	17	44	7	10	10	10	10	7	6	3	2	4	0	1	1	2	3	1	0,67	0,67
51	FH-HH.F(H)	DHFBD	4	2	2	17	46	7	10	10	10	10	7	6	3	2	4	0	1	1	2	4	1	1,00	0,33
52	FF-FF.H(H)	SWMZD	4	2	2	17	48	7	12	10	10	10	7	6	3	2	4	0	1	1	2	2	1	0,33	1,00
53	FH-FF.H(H)	SWMBD	4	2	2	17	50	7	12	10	10	10	7	6	3	2	4	0	1	1	2	3	1	0,67	0,67
54	FF-HF.H(H)	SWFZD	4	2	2	17	52	7	12	10	10	10	7	6	3	2	4	0	1	1	2	3	1	0,67	0,67
55	FH-HF.H(H)	SWFBD	4	2	2	17	54	7	12	10	10	10	7	6	3	2	4	0	1	1	2	4	1	1,00	0,33
56	H-H.F-F	BWZ	2	2	1	40	3	15	3	12	12	12	6	4	4	4	4	0	0	2	0	2	0	1,00	1,00
57	H-F.H-F	ZHZ	2	2	1	40	5	15	5	30	30	30	6	4	4	4	4	0	0	2	0	2	0	1,00	1,00
58	HF-H.F-F	MBWZ	4	2	2	41	5	15	3	12	10	10	7	5	4	4	4	0	0	2	1	2	1	0,50	1,00
59	HH-H.F-F	FBWZ	4	2	2	41	7	15	3	12	10	10	7	5	4	4	4	0	0	2	1	3	1	1,00	0,50
60	HF-F.H-F	MZHZ	4	2	2	41	9	15	5	30	10	10	7	5	4	4	4	0	0	2	1	2	1	0,50	1,00
61	HH-F.H-F	FZHZ	4	2	2	41	11	15	5	30	10	10	7	5	4	4	4	0	0	2	1	3	1	1,00	0,50
62	FF-H.F-H	ZHZD	4	2	2	41	20	15	10	30	10	10	7	5	4	4	4	0	0	2	1	2	1	0,50	1,00
63	FH-H.F-H	ZHBD	4	2	2	41	22	15	10	30	10	10	7	5	4	4	4	0	0	2	1	3	1	1,00	0,50
64	FF-F.H-H	BWZD	4	2	2	41	24	15	12	12	10	10	7	5	4	4	4	0	0	2	1	2	1	0,50	1,00

65	FH-F.H-H	BWBD	4	2	2	41	26	15	12	12	10	10	7	5	4	4	4	0	0	2	1	3	1	1,00	0,50
66	HF-FH.F-F	MZSWZ	2	2	2	44	9	15	3	12	12	12	8	6	4	4	4	0	0	2	2	2	0	0,33	1,00
67	HH-FH.F-F	FZSWZ	4	2	2	44	11	15	3	12	12	10	8	6	4	4	4	0	0	2	2	3	0	0,67	0,67
68	HH-HH.F-F	FBSWZ	2	2	2	44	15	15	3	12	12	12	8	6	4	4	4	0	0	2	2	4	0	1,00	0,33
69	HF-FF.H-F	MZDHZ	4	2	2	44	17	15	5	30	10	10	8	6	4	4	4	0	0	2	2	2	0	0,33	1,00
70	HH-FF.H-F	FZDHZ	4	2	2	44	19	15	5	30	10	10	8	6	4	4	4	0	0	2	2	3	0	0,67	0,67
71	HF-HF.H-F	MBDHZ	4	2	2	44	21	15	5	30	10	10	8	6	4	4	4	0	0	2	2	3	0	0,67	0,67
72	HH-HF.H-F	FBDHZ	4	2	2	44	23	15	5	30	10	10	8	6	4	4	4	0	0	2	2	4	0	1,00	0,33
73	FF-FF.H-H	BWMZD	2	2	2	44	48	15	12	12	12	12	8	6	4	4	4	0	0	2	2	2	0	0,33	1,00
74	FH-FF.H-H	BWMBD	4	2	2	44	50	15	12	12	12	10	8	6	4	4	4	0	0	2	2	3	0	0,67	0,67
75	FH-HF.H-H	BWFBD	2	2	2	44	54	15	12	12	12	12	8	6	4	4	4	0	0	2	2	4	0	1,00	0,33
76	HF-H.FF-F	MBWMZ	4	2	2	50	5	15	3	12	10	10	8	6	4	4	4	0	0	2	2	2	2	0,33	1,00
77	HH-H.FF-F	FBWMZ	4	2	2	50	7	15	3	12	10	10	8	6	4	4	4	0	0	2	2	3	2	0,67	0,67
78	HF-F.HF-F	MZHMZ	2	2	2	50	9	15	5	30	30	30	8	6	4	4	4	0	0	2	2	2	2	0,33	1,00
79	HH-F.HF-F	FZHMZ	4	2	2	50	11	15	5	30	30	10	8	6	4	4	4	0	0	2	2	3	2	0,67	0,67
80	HF-H.FH-F	MBWFZ	4	2	2	50	21	15	3	12	10	10	8	6	4	4	4	0	0	2	2	3	2	0,67	0,67
81	HH-H.FH-F	FBWFZ	4	2	2	50	23	15	3	12	10	10	8	6	4	4	4	0	0	2	2	4	2	1,00	0,33
82	HH-F.HH-F	FZHFZ	2	2	2	50	27	15	5	30	30	30	8	6	4	4	4	0	0	2	2	4	2	1,00	0,33
83	FF-H.FF-H	ZDHZD	2	2	2	50	36	15	10	30	30	30	8	6	4	4	4	0	0	2	2	2	2	0,33	1,00
84	FH-H.FF-H	ZDHBD	4	2	2	50	38	15	10	30	30	10	8	6	4	4	4	0	0	2	2	3	2	0,67	0,67
85	FH-H.FH-H	BDHBD	2	2	2	50	54	15	10	30	30	30	8	6	4	4	4	0	0	2	2	4	2	1,00	0,33
86	H-FH.FF-F	ZSWMZ	4	2	2	52	5	15	3	12	10	10	8	6	4	4	4	0	0	2	2	2	0	0,33	1,00
87	H-HH.FF-F	BSWMZ	4	2	2	52	7	15	3	12	10	10	8	6	4	4	4	0	0	2	2	3	0	0,67	0,67
88	H-FF.HF-F	ZDHMZ	4	2	2	52	9	15	5	30	10	10	8	6	4	4	4	0	0	2	2	2	0	0,33	1,00
89	H-HF.HF-F	BDHMZ	4	2	2	52	11	15	5	30	10	10	8	6	4	4	4	0	0	2	2	3	0	0,67	0,67
90	H-FH.FH-F	ZSWFZ	4	2	2	52	21	15	3	12	10	10	8	6	4	4	4	0	0	2	2	3	0	0,67	0,67
91	H-HH.FH-F	BSWFZ	4	2	2	52	23	15	3	12	10	10	8	6	4	4	4	0	0	2	2	4	0	1,00	0,33
92	H-FF.HH-F	ZDHFZ	4	2	2	52	25	15	5	30	10	10	8	6	4	4	4	0	0	2	2	3	0	0,67	0,67
93	H-HF.HH-F	BDHFZ	4	2	2	52	27	15	5	30	10	10	8	6	4	4	4	0	0	2	2	4	0	1,00	0,33
94	HF-FH.FF-F	MZSWMZ	4	2	2	53	9	15	3	12	10	10	9	7	4	4	4	0	0	2	3	2	1	0,25	1,00
95	HH-FH.FF-F	FZSWMZ	4	2	2	53	11	15	3	12	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
96	HF-HH.FF-F	MBSWMZ	4	2	2	53	13	15	3	12	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
97	HH-HH.FF-F	FBSWMZ	4	2	2	53	15	15	3	12	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
98	HF-FF.HF-F	MZDHMZ	4	2	2	53	17	15	5	30	10	10	9	7	4	4	4	0	0	2	3	2	1	0,25	1,00
99	HH-FF.HF-F	FZDHMZ	4	2	2	53	19	15	5	30	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
100	HF-HF.HF-F	MBDHMZ	4	2	2	53	21	15	5	30	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
101	HH-HF.HF-F	FBDHMZ	4	2	2	53	23	15	5	30	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
102	HF-FH.FH-F	MZSWFZ	4	2	2	53	41	15	3	12	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
103	HH-FH.FH-F	FZSWFZ	4	2	2	53	43	15	3	12	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
104	HF-HH.FH-F	MBSWFZ	4	2	2	53	45	15	3	12	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
105	HH-HH.FH-F	FBSWFZ	4	2	2	53	47	15	3	12	10	10	9	7	4	4	4	0	0	2	3	5	1	1,00	0,25
106	HF-FF.HH-F	MZDHFZ	4	2	2	53	49	15	5	30	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
107	HH-FF.HH-F	FZDHFZ	4	2	2	53	51	15	5	30	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
108	HF-HF.HH-F	MBDHFZ	4	2	2	53	53	15	5	30	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
109	HH-HF.HH-F	FBDHFZ	4	2	2	53	55	15	5	30	10	10	9	7	4	4	4	0	0	2	3	5	1	1,00	0,25
111	FF-FH.FF-H	ZDHMZD	4	2	2	53	72	15	10	30	10	10	9	7	4	4	4	0	0	2	3	2	1	0,25	1,00
110	FH-FH.FF-H	ZDHMBD	4	2	2	53	74	15	10	30	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
112	FF-HH.FF-H	ZDHFZD	4	2	2	53	76	15	10	30	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
113	FH-HH.FF-H	ZDHFBD	4	2	2	53	78	15	10	30	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
114	FF-FF.HF-H	ZSWMZD	4	2	2	53	80	15	12	12	10	10	9	7	4	4	4	0	0	2	3	2	1	0,25	1,00
115	FH-FF.HF-H	ZSWMBD	4	2	2	53	82	15	12	12	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
116	FF-HF.HF-H	ZSWFZD	4	2	2	53	84	15	12	12	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
117	FH-HF.HF-H	ZSWFBD	4	2	2	53	86	15	12	12	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
118	FF-FH.FH-H	BDHMZD	4	2	2	53	104	15	10	30	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
119	FH-FH.FH-H	BDHMBD	4	2	2	53	106	15	10	30	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
120	FF-HH.FH-H	BDHFZD	4	2	2	53	108	15	10	30	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
121	FH-HH.FH-H	BDHFBD	4	2	2	53	110	15	10	30	10	10	9	7	4	4	4	0	0	2	3	5	1	1,00	0,25
122	FF-FF.HH-H	BSWMZD	4	2	2	53	112	15	12	12	10	10	9	7	4	4	4	0	0	2	3	3	1	0,50	0,75
123	FH-FF.HH-H	BSWMBD	4	2	2	53	114	15	12	12	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
124	FF-HF.HH-H	BSWFZD	4	2	2	53	116	15	12	12	10	10	9	7	4	4	4	0	0	2	3	4	1	0,75	0,50
125	FH-HF.HH-H	BSWFBD	4	2	2	53	118	15	12	12	10	10	9	7	4	4	4	0	0	2	3	5	1	1,00	0,25
126	HF-FH.FF-FF	MZSWMZD	2	2	2	80	9	15	3	12	12	12	10	8	4	4	4	0	0	2	4	2	0	0,20	1,00
127	HH-FH.FF-FF	FZSWMZD	4	2	2	80	11	15	3	12	12	10	10	8	4	4	4	0	0	2	4	3	0	0,40	0,80
128	HH-HH.FF-FF	FBSWMZD	2	2	2	80	15	15	3	12	12	12	10	8	4	4	4	0	0	2	4	4	0	0,60	0,60
129	HF-FF.HF-FF	MZDHMZD	2	2	2	80	17	15	5	30	30	30	10	8	4	4	4	0	0	2	4	2	0	0,20	1,00
130	HH-FF.HF-FF	FZDHMZD	4	2	2	80	19	15	5	30	30	10	10	8	4	4	4	0	0	2	4	3	0	0,40	0,80
131	HF-HF.HF-FF	MBDHMZD	4	2	2	80	21	15	5	30	30	10	10	8	4	4	4	0	0	2	4	3	0	0,40	0,80
132	HH-HF.HF-FF	FBDHMZD	4	2	2	80	23	15	5	30	30	10	10	8	4	4	4	0	0	2	4	4	0	0,60	0,60
133	HF-FH.FH-FF	MZSWFZD	4	2	2	80	41	15	3	12	12	10	10	8	4	4	4	0	0	2	4	3	0	0,40	0,80

134	HH-FH.FH-FF	FZSWFZD	4	2	2	80	43	15	3	12	12	10	10	8	4	4	4	0	0	2	4	4	0	0,60	0,60
135	HF-HH.FH-FF	MBSWFZD	4	2	2	80	45	15	3	12	12	10	10	8	4	4	4	0	0	2	4	4	0	0,60	0,60
136	HH-HH.FF-HF	FBSWMBD	4	2	2	80	47	15	3	12	12	10	10	8	4	4	4	0	0	2	4	5	0	0,80	0,40
137	HH-FF.HH-FF	FZDHFZD	2	2	2	80	51	15	5	30	30	30	10	8	4	4	4	0	0	2	4	4	0	0,60	0,60
138	HF-HF.HH-FF	MBDHFZD	4	2	2	80	53	15	5	30	30	10	10	8	4	4	4	0	0	2	4	4	0	0,60	0,60
139	HF-HF.HF-HF	MBDHMBD	2	2	2	80	85	15	5	30	30	30	10	8	4	4	4	0	0	2	4	4	0	0,60	0,60
140	HH-HF.HF-HF	FBDHMBD	4	2	2	80	87	15	5	30	30	10	10	8	4	4	4	0	0	2	4	5	0	0,80	0,40
141	HF-FH.FH-HF	MZSWFBD	2	2	2	80	105	15	3	12	12	12	10	8	4	4	4	0	0	2	4	4	0	0,60	0,60
142	HH-FH.FH-HF	FZSWFBD	4	2	2	80	107	15	3	12	12	10	10	8	4	4	4	0	0	2	4	5	0	0,80	0,40
143	HH-HH.FH-HF	FBSWFBD	2	2	2	80	111	15	3	12	12	12	10	8	4	4	4	0	0	2	4	6	0	1,00	0,20
144	HH-FF.HH-HF	FZDHFBD	4	2	2	80	115	15	5	30	30	10	10	8	4	4	4	0	0	2	4	5	0	0,80	0,40
145	HH-HF.HH-HF	FBDHFBD	2	2	2	80	119	15	5	30	30	30	10	8	4	4	4	0	0	2	4	6	0	1,00	0,20

APPENDIX 2. MACROS FOR GENERATING CENSUS GRAPHS IN pajek

M2 – Census Graph 2-mode 2 (145).mcr

NETBEGIN 1
CLUBEGIN 1
PERBEGIN 1
CLSBEGIN 1
HIEBEGIN 1
VECBEGIN 1

Msg Reading Networks
N 1 RDN ?
N 9999 RDPAJ ?
Msg Fragment Searches
OBJECTS1 145
C 1 FRAGSNL 4 1 TRUE FALSE TRUE TRUE TRUE FALSE 1
OBJECTS2 145
C 146 FRAGSNL 3 149 TRUE FALSE TRUE TRUE TRUE FALSE 1
Msg Making 3-Mode-Network
OBJECTS1 145
N 439 HIERNET 146 [1]
C 291 DEGC 2 [0]
N 584 EXT 2 291 [1,9999999,1]
OBJECTS1 145
OBJECTS2 145
N 585 ADDNET 439 584
Msg Reordering 3-Mode-Network
C 292 DEGC 729 [0]
C 293 BIN 292 [0,0]
P 1 MPER 293
P 2 MIRRORPERM 1
N 730 REOR 729 2
C 294 REORPART 292 2
N 731 EXT 730 294 [1,9999999,1]
C 295 DEGC 731 [0]
C 296 FUSEP 291 295
P 3 MPER 296
N 732 REOR 730 3
C 297 REORPART 296 3
Msg Labeling Rings
N 733 ADDNET 2 732
N 734 REMARC 733
C 298 COMP 734 [1] [1]
V 4 MVEC 298
C 299 BIN 298 [1,145]
V 5 MVEC 299
C 300 BIN 299 [0,0]
V 6 MVEC 300
N 735 REMARC 732
C 301 COMP 735 [1] [1]
C 302 FUSEP 291 301
V 7 MVEC 302
V 8 MULV 4 5
V 9 ADDV 7 8
C 303 MAKETRUNCPAR 9
N 736 SHR 733 303

Msg Identifying Individuals
C 304 DEGC 736 [0]
V 11 MVEC 304
C 305 DEGC 736 [1]
V 13 MVEC 305
V 14 MULV 11 13
C 306 MAKETRUNCPAR 14
C 307 BIN 306 [0,0]
V 15 MVEC 307
C 308 NAMEC 736
V 16 MVEC 308
V 17 MULV 15 16
C 309 MAKETRUNCPAR 17
N 737 SHR 736 309 [1 ,0,1]
Msg Identifying Marriages
N 738 ADDSIBL0 737
N 738 SIMPLS 738
N 739 REMLINL 738 2.0000
N 740 FUSE 737 739
N 741 EXT 740 310 [0,145,1]
C 311 EXTP 310 310 [0,145]
C 312 BIN 311 [0,0]
V 18 MVEC 312
C 313 COMP 741 [1] [1]
N 742 SHR 741 313 [1 ,0,1]
N 742 DLOOPS 742
V 19 SHRV 18 313 [0,3]
C 315 DEGC 742 [2]
V 21 MVEC 315
C 316 MAKETRUNCPAR 19

M3a – Census Graph 1-mode.mcr

NETBEGIN 2
CLUBEGIN 1
PERBEGIN 1
CLSBEGIN 1
HIEBEGIN 1
VECBEGIN 1
NETPARAM 1

Msg Create 1-Mode Network
C 1 DEGC 1 [1]
V 2 MVEC 1
N 2 ADDSIBL0 1
C 2 DEGC 1 [0]
N 3 EXT 2 2 [0,0,1]
V 4 EXTV 2 2 [0,0]
N 3 SIMPLS 3
N 4 EXT 2 1 [1,9999999,1]
V 5 EXTV 2 1 [1,9999999]
N 4 SIMPLS 4
C 1 DC
C 2 DC
N 2 DN

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