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Taxation and The Crowding-Out Effect of Corporate Social Responsibility

J. Ballet, D. Bazin, A. Lioui and D. Touahri

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Abstract

We address in this paper the issue of the existence or not of a crowding-out effect of Corporate Social Responsibility by government intervention through a lump sum tax. For this purpose, we build a model of impur altruism for firms. We show that in general it will happen to be that public policy crowds out corporate (private) contribution but the crowding-out will not be complete. Two interesting findings are that \( i \) the intensity of the crowding-out depends upon the relative performance of the government in producing the public good and \( ii \) that public policy has an impact on wages in the economy since it is the opportunity cost for firms that spend time on Corporate Social Responsibility.

*Keywords*: Corporate Social Responsibility, Crowding-out effect, Taxation.

*JEL Classification*: H21, H32, H41.
1 Introduction

It is now well understood that Public Policy may crowd-out private contribution to public good. Frey and Jegen (2001) and Nyborg and Rege (2003) provide extensive literature review on this issue\(^1\). Most of the models built to address such an issue focus mainly on individual agents and their intrinsic motivation, even when a certain production sector is explicitly taken into account\(^2\). The lack of analysis of firms' behaviour is all the more problematic because they are active players in the area of voluntary contributions to a public good (be it the environment, health...). This activity by firms is of such importance that the OECD and the European Union found it necessary to publish their directives as to their understanding of Corporate Social Responsibility. It has been defined\(^3\) by the EU, for example, as the fact that companies integrate social and environmental concerns in their business, on a voluntary basis.

Our objective in this paper is to investigate whether public policy may crowd-out the voluntary contribution of firms to a public good. To the best of our knowledge, little if any attention has been devoted to this issue.

Public policy is introduced in this paper through a green tax which is considered here as an extrinsic motivation or stimulus introduced in the aim of achieving something good (by the State). We present our analysis in the context of corporate environmentalism, that is voluntary effort to improve the environment, but it should be clear from the analysis that the basic story has broader applicability. Since we focus on the contribution to improve the environment, this corresponds to the definition of corporate social responsibility mentioned above. Consequently, we will alternately use the terms contribution to a public good and corporate social responsibility (CSR).

The motivations encouraging firms to act voluntarily, \(i.e.\) to contribute voluntarily to a public good has already been the subject of various analyses. Indeed Brekke and Nyborg (2005) suggest that firms engage in voluntary activity linked to Corporate Social Responsibility with the aim of screening. That is the firm attracts good workers who prefer to compromise their wages to a small extent as long as they are assured that the firm they belong to respects the environment (Coca-Cola for example indicates on its web site the costs it incurs in terms of the price of carrying out measures to preserve and improve the Environment). Others suggest that by doing so firms preempt the State in the sense that they

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\(^1\)Recent contributions include Andreoni and Payne (2003) who found that government contributions may crowd-out fund raising activity which may in turn explain the crowding-out of private contributions and Eckel et al. (2005) showed that the intensity of the crowding-out depends upon the intensity of fiscal illusion.

\(^2\)See for example Brekke et al. (2003)

\(^3\)The document has been published by the EU under the title "Communication on Corporate Social Responsibility." It is available for download at http://europa.eu.int/comm/employment_social/soc-dial/csr/.
prefer to invest in the improvement of the environment before the State begins to regulate; intervention by the State being liable to be seen as harmful (Maxwell et al. (2000)). Other approaches suggest that firms hope to attract consumers who attach importance to the environment in their purchasing decisions (Arora and Gangopadhyay (1995), Moon et al. (2002) and Bjorner et al. (2004)). Finally, Goyal (2005) suggests that a firm that has the intention of proceeding with a FDI (Foreign Direct Investment) will have a valid reason to engage in Corporate Social Responsibility in order to convey its good intentions to the host country. Kim and Van Dam (2003) identify at least two dimensions along which CSR adds value, namely economic performance and reputation value.

In our article, we consider that the firms invest in Corporate Social Responsibility with the aim of improving in fine its economic returns. The firm therefore does not have an altruistic motivation in itself, even if it adopts an altruistic behaviour. It is on the contrary, faithful to its objective of generating profits. Consequently, it awaits a return on investment. This hypothesis falls in line with the literature on impure altruism (Andreoni 1989).

Besides, there are several ways of modeling voluntary contribution of firms to the quality of the environment. Indeed, this can not only be a flat monetary contribution of firms since this social responsibility is certainly time consuming. It is not moreover illogical to think that the time spent on this act of donating is probably more costly than the donation itself. An anonymous donation is not beneficial to a firm and consequently the folklore around the donation is at least as important as the donation itself. Indeed, if I am satisfied in sending a cheque, people will never know that I have contributed. It is only if I say it aloud that they will know it. A recent study of Kaltenegger (2004) on a region in Austria is quite instructive on this point of view. Indeed the main barrier to Corporate Social Responsibility which was given by interviews was personal costs, that is, the time that employees have to spend on Corporate Social Responsibility.

In this paper we have consequently decided to model the cost of voluntary contribution rather in terms of hours of work. That is to say that on a unit of labour recruited, the firm has to decide what it will dedicate to production and what it will dedicate to its effort of Corporate Social Responsibility. Thus, the natural cost of effort will be nothing more than wages, whereas for individuals this cost is in general incorporated in the model as an abstract cost (Brekke et al. (2003)). Since wages in the economy are generally an endogenous variable (determined at equilibrium), we have an interesting interaction between the cost of effort (expressed in wages) and the effort itself (expressed in hours of work). We can then show for the first time in literature how public policy linked to a

\[\text{If the cost of effort is determined in an endogeneous manner, then the total cost of effort is not necessarily an increasing function of effort. Indeed, an increase in effort can reduce wages when the latter is endogeneous and therefore the total cost of effort does not necessarily increase.}\]
public good can have an impact on crucial macroeconomic variables such as wages and production. The consideration of a monetary contribution in addition to the temporal contribution will have effects that could slightly hide the basic results as in the case of individuals (see Andreoni et al. (1996) and Duncan (1999)). For this reason we concentrate on the effort expressed in terms of hours of work. The combination of the two is nonetheless a natural extension of what we propose in this paper. Another advantage of this approach is that the problem of free rider does not really arise in that if the firm does not contribute, it has nothing to communicate. Consequently, what counts is mainly the contribution of the firm itself and accessorily that of the others.

The clear cut impact that we show of public policy with regard to firms is that there will also be a crowding-out effect as long as the State is capable of producing a public good with the tax, but in this hypothesis, the quality of the environment (in this case the public good) is always improved. The crowding out effect is generally partial and its intensity depends on the State’s capacity to produce quality relative to firms themselves. Consequently, the impact of public policy is linked to its relative efficiency (with regard to firms) in the production of the public good. In other words, it is not so much the intervention of the State quantitatively that counts but the efficiency of its action. This implies that if the firm is happy to communicate its actions but is in reality not efficient in its contribution to the public good, State intervention will prove relatively efficient. Moreover, the cost of effort (wages) is endogeneised and we give the conditions under which the State can have a positive impact on wages in the economy.

In the following section we will present the model, then the main results and finally, the last section will conclude the analysis. We have also included the proofs of the Propositions in the appendix.

2 The Model

We consider a simplified setting in order to illustrate the issue at hand. We consider an economy in pure and perfect competition. In its production process firm uses labour and capital. The standard model of production for a firm states that production is such that:

\[ Y = F(K, L) \]  

(1)

where \( Y \) is the output, \( K \) the capital needed to produce this output and \( L \) the labour required. We assume that this function has the usual properties, namely that it is increasing and concave in each of its arguments, and the cross derivative is positive (complementarity of two factors of production).
As stated in the introduction of our article, we assume that the firm has an impure altruistic motivation to participate voluntarily in the improvement of the quality of the environment. Indeed, it is a question of positive interaction between the profit of the company and the conservation of the quality of the environment. For example, individuals who show ethical behaviour with regard to the environment (respect of nature, belief in solidarity economics) will adopt an identity approach and will develop common points towards firms defending and representing their life ethic. Thus, persons will tend to buy more products manufactured in a production process that respects the environment. In other words, the firm has an interest to contribute to a public good. We can then write this production function as follows:

\[ Y = A(E; Q) F(K, L - E) \]  

where \( Q \) represents the global level of a public good to which the firm participates and \( E \) represents the effort produced by the firm to participate in this good. A key point in this approach is that the firm gains in participating in a public good since \( A(E; Q) \) is supposed to be greater than or equal to 1, and expresses an increasing and concave function for the public good to which the firm participates. \( A(E; Q) \) symbolises a technological factor related to the quality of the environment and the responsibility.

The fact that \( A \) is a function of \( E \) serves to bring out the fact that the voluntary contributions that we analyse are those that are beneficial only if they are carried out. The free rider does not gain anything in our setting. Moreover, the cost of its participation is explicit here, being the wages. That is, the firm recruits \( L \) units of work, but part of this time \( (E) \) is dedicated to the public good. We will name this phenomenon as effort.

This form of modeling belongs to the literature on the theory of public good and consequently is known to produce a crowding-out effect in the case of individuals.

By assuming that the production function has the usual properties and that we consider a representative firm, we can write output \textit{per capita} as:

\[ y = \alpha(q) f(k, 1 - e) \]  

where the function alpha \( \alpha \) is always strictly positive, increasing and concave of the quality of the environment \( q \), that is \( \alpha(q) > 0, \frac{\partial \alpha}{\partial q} > 0, \frac{\partial^2 \alpha}{\partial^2 q} < 0 \).

The firm must choose between the level of its effort and capital that maximises its profit:

\[ \pi = \alpha(q) f(k, 1 - e) - rk - w \times 1 - \tau \]  

where \( r \) is the return required per unit of capital, \( w \) unit wages and \( \tau \) the green tax imposed by the State on the firm. The cost of effort is incorporated by
the fact that the firm pays a wage for a unit whereas in the production function there is only $1 - e$ that comes in. The green tax is assumed to be a flat tax.

We will assume that the quality of the environment is perceived as:

$$q = P(e, \theta_e) + T(\tau, \theta_\tau)$$  \hspace{1cm} (5)

where $P$ the production function of the public good through effort and $T$ that of the production of a public good through tax. The functions $P$ and $T$ are assumed to be increasing and concave. $\theta_e$ and $\theta_\tau$ are technological factors linked to the environment. Moreover, we shall interpret the technological factor of effort here as a link between effort made visible through communication and concrete or real actions, which will allow us to clearly show that the quality of the environment is all the more improved by the action of the State when this link is weak, i.e. firms do a lot of communication and make little real voluntary contribution. In this case the crowding-out effect only exerts a little influence on the quality of the environment. The separability between production issuing from the effort of the firm and that issuing from the tax is only assumed for simplification purposes.

We assume in what follows that the representative firm is a price taker. Certain results can be obtained generally, but in order to simplify and clarify matters, we will assume that the production function is a Cobb-Douglas production function:

$$f(k, 1 - e) = \sigma k^\varepsilon (1 - e)^{1-\varepsilon}$$  \hspace{1cm} (6)

### 3 Taxation and Corporate Motivation

The question that we would like to answer is the following: Would state intervention have consequences on the behaviour of firms? The response is in the following proposition:

**Proposition 1:** The public policy always crowds out the firm's social responsibility.

As in the case for individuals, public intervention *via* a tax has secondary effects. The intensity of the crowding-out effect depends on an essential parametre indicating that intensity is always increasing. This parametre is the efficiency of the State (measured by the marginal productivity of the State). The reduction of responsibility is partial and its intensity depends on the relative efficiency of the State in the production of the public good. This is a new idea insofar as it points out that what is most important in a crowding-out situation is not merely
the presence of public policy, but also its performance. Our model corresponds in this respect to the empirical results of Brooks (2004) who shows that individuals also react to the government’s performance. It turns out that firms will have the same considerations.

A key assumption in order that the previous result be valid is that marginal productivity of the State is always positive, that is for each dollar of tax levied by the State, there is production of quality. If this is not the case, and it is definitely possible because of high transaction costs for example, then the crowding-out effect is not systematic and sometimes transforms itself into a crowding-in effect. The intuition being that firms have a certain objective of visibility of effort with regard to consumers that they want to attract. The taxes imposed by the State liberates them a little of their responsibility, but only in the measure that the State effectively produces quality. Otherwise, they will pay their tax but will continue to invest in order to maintain the desired level.

The direct implication of the crowding-out effect is to free the working time that firms can dedicate to production. In the appendix, we will also see that the capital used by the firm will also increase, which is not surprising since capital and labour are complementary. However, this phenomenon does not imply that the output will increase and the main reason is the following: The production function has two components, one purely technical \((\sigma k^e (1 - e)^{1-\epsilon})\) which will be increasing with the tax because of the crowding-out effect. The second component is linked to the behaviour of firms and what it gains from it in terms of brand image, and this is the multiplying factor \(\alpha (q)\). So as to have an increase in output, the term \((\alpha (q))\) should not be decreasing with tax. Whereby the interest of the following result:

**Proposition 2:** The public policy always increases environmental quality.

Before interpreting this result, it is important to note the immediate consequence: Output will increase with the introduction of a public tax. There are two contradictory effects that come into effect. Indeed, on the one hand the tax reduces the effort and the quality, but on the other hand, the State uses the money from the tax to produce environmental quality. By doing this, the quality of the environment will be increased. This cross effect is dominated by the impact of the State and consequently the intervention of the State will not lead to a reduction of the quality of the environment. In order to understand this mechanism, it must be noted that the impact of the tax on the overall quality of the environment *via* the production of quality of the firm is an indirect impact. Indeed, the

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5 The mechanism of crowding-out consists in a reduction of the voluntary contribution of the individual following State intervention. The crowding-in effect takes place when individuals increase their voluntary contribution following state intervention.
tax reduces the effort which consequently will reduce the private production of quality. However, the impact of the tax on the quality of an indirect impact via the production of the State. This direct effect has an impact greater than the indirect effect. The direct impact of the tax is all the more positive and dominant when firms do not have a real contribution with regard to environmental quality. In other words, this contribution is nevertheless weak enough compared to the efforts made in communication, i.e., firms communicate a lot but do not really contribute by their actions except marginally to the quality of the environment.

So far, we saw that State intervention crowds-out the responsibility of firms but at the same time improves the quality of the environment. This seems very encouraging in the case where the State is in a position to take into hand the provision of this public good, which is environmental quality. However, it will be interesting to study the impact of State intervention on total investment of firms. The latter can be written as followed:

\[ \text{inv} = \text{we} + \tau \]  

(7)

This means that the contribution of firms is double, with a contribution via effort and a contribution via the tax. We will maintain for the moment the assumption of exogenous wages. Taxation produces two simultaneous contradictory effects on total investment with regard to the improvement of the environment: a positive direct effect \( (\tau) \) and an indirect negative effect on effort due to the crowding-out. The final outcome is given in the following:

**Proposition 3:** Assume \( \frac{1}{w} \frac{\partial P}{\partial e} - \frac{\partial T}{\partial \tau} > 0 \). It follows that the total investment will increase with respect to the tax level. Otherwise, the impact of taxation on total investment cannot be determined.

Let us first interpret the condition given at the outset of the preceding Proposition. The term represents the difference between an increase in environmental quality due to an increase of a unit of effort, weighted by the cost of the effort on the one hand, and on the other hand, the increase of environmental quality due to the increase of one taxation unit \( (\frac{\partial T}{\partial \tau}) \). This term reflects the difference in relative efficiency between public policy and private effort of firms to improve the environment.

If the State is less efficient than firms in producing quality, then the crowding-out effect will be of less than one for one. That is $1 of tax levied by the state will lead to an effort of reduction of less than $1. If the State is more efficient, then the crowing-out effect will be of more than one for one. Therefore, what matters here is the way firms perceive not only State intervention but also the ability of the State to produce the public good compared to them. The condition given in the Proposition brings out the importance of the role of the technological factor of firms.
A more challenging perspective on the impact of taxation on total investment would be to endogeneize the cost of the effort. As explained in the Introduction, writing the cost of effort as an abstract cost would not allow us to investigate this extension of the previous analysis. We will begin by studying the impact of the tax on the cost of effort, therefore on wages, and then on total investment by firms.

**Proposition 4:** Assume that the following condition holds:

\[
\frac{\partial T}{\partial \tau} \frac{\partial \alpha}{\partial q} \frac{\tau}{\tau} \left( \frac{y - r_i k_i}{\tau} \right) \geq 1 - \epsilon
\]

Then taxes increase the wages in the economy. Otherwise, the impact of the tax on wages cannot be determined.

Let us first interpret the condition given at the outset of the Proposition. It turns out that on the right hand side we have the elasticity of the "technological" part of the output to the labour. On the left hand side, this term \(\frac{\partial T}{\partial \tau} \frac{\partial \alpha}{\partial q} \frac{\tau}{\tau} \left( \frac{y - r_i k_i}{\tau} \right)\) can be interpreted as the elasticity with regard to the tax of the term linked to responsibility in the production function; this elasticity being weighted by the ratio of wages plus the tax divided by the tax.

Let us first note that the ratio \(\frac{y - r_i k_i}{\tau}\) is always greater than 1 because it can be written as: \(\left( \frac{w + \tau}{\tau} \right)\). Therefore, in order for that tax to increase wages, the gain of one unit of extra tax in terms of the increase in output must be greater than the production gain for an increase in a unit of labour, which seems intuitive.

This uncertainty regarding the impact on wages is equally transmitted with the impact of the tax on total investment. Indeed, the latter depends among others, on wages which is the cost of voluntary investment. Consequently, we cannot say without ambiguity if, with regard to total investment, there will be a crowding-in or out effect.

### 4 Concluding Remarks

The previous analysis can of course be completed with an analysis of the impact of technological progress on all these variables.

The study by Kaltenegger (2004) is very instructive for another reason: it brings out the link between moral motivation of individuals and that of firms. Indeed, it showed that beyond the brand image of the firm and its credibility, the main motor of Corporate Social Responsibility remains personal concern. Consequently, the next natural step of our research project is to study the question of the interaction of these two types of motivation (that of the firm and that of individuals) and of course the impact of public policy on them.
5 Appendix

Proof of Proposition 1: The firm objective will be:

$$\max_{e,k} \pi = \alpha (q) \sigma k^{\epsilon} (1 - e)^{1-\epsilon} - rk - w - \tau$$  \hfill (8)

The first order conditions write:

$$-(1 - \epsilon) \alpha + \frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q} (1 - e) = 0$$  \hfill (9)

$$\epsilon \alpha \sigma k^{\epsilon-1} (1 - e)^{1-\epsilon} - r = 0$$  \hfill (10)

Writing the total differential of (8), and using the first order conditions yields:

$$0 = \left[ -(2 - \epsilon) \frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q} + \left( \frac{\partial P}{\partial e} \right)^2 \frac{\partial^2 \alpha}{\partial q^2} (1 - e) + \frac{\partial^2 P}{\partial e \partial q} \frac{\partial \alpha}{\partial q} (1 - e) \right] de$$

$$+ \left[ -(1 - \epsilon) \frac{\partial T}{\partial \tau} \frac{\partial \alpha}{\partial q} + \frac{\partial P}{\partial e} \frac{\partial T}{\partial \tau} \frac{\partial^2 \alpha}{\partial q^2} (1 - e) \right] d\tau$$

and therefore, at the optimum,

$$\frac{de}{d\tau} = \frac{- (1 - \epsilon) \frac{\partial T}{\partial \tau} \frac{\partial \alpha}{\partial q} + \frac{\partial P}{\partial e} \frac{\partial T}{\partial \tau} \frac{\partial^2 \alpha}{\partial q^2} (1 - e)}{(2 - \epsilon) \frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q} - \left( \frac{\partial P}{\partial e} \right)^2 \frac{\partial^2 \alpha}{\partial q^2} (1 - e) - \frac{\partial^2 P}{\partial e \partial q} \frac{\partial \alpha}{\partial q} (1 - e)} \frac{\partial T}{\partial \tau}$$  \hfill (11)

Noting that the numerator is always negative and the denominator positive under the condition given in the Proposition yields the desired result.

Proof of Proposition 2: We have:

$$\frac{dq}{d\tau} = \frac{\partial P}{\partial e} + \frac{\partial T}{\partial \tau} \frac{\partial \alpha}{\partial q}$$  \hfill (12)

$$= \frac{\frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q}}{(2 - \epsilon) \frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q} - \left( \frac{\partial P}{\partial e} \right)^2 \frac{\partial^2 \alpha}{\partial q^2} (1 - e) - \frac{\partial^2 P}{\partial e \partial q} \frac{\partial \alpha}{\partial q} (1 - e) \frac{\partial T}{\partial \tau}} > 0$$

which yields the desired result.

Proof that output is increasing with the Tax level Using the first order condition (8) yields:

$$\frac{dk}{d\tau} = \frac{1}{\alpha (1 - \epsilon) \frac{\partial T}{\partial \tau} \frac{\partial \alpha}{\partial q}} k_i > 0$$  \hfill (13)
and therefore:

\[
\frac{dy}{d\tau} = \frac{dq}{d\tau} \frac{\partial \alpha}{\partial q} \sigma k^\epsilon (1 - e)^{1-\epsilon} + \frac{dk}{d\tau} \alpha e \epsilon k^{\epsilon-1} (1 - e)^{1-\epsilon} - \frac{de}{d\tau} \alpha (1 - e) \sigma k^\epsilon (1 - e)^{-\epsilon}
\] (14)

Since from (10) we have:

\[
\frac{dq}{d\tau} = \frac{\partial P}{\partial e} \frac{\partial e}{\partial \tau} + \frac{\partial T}{\partial \tau}
\] (15)

It turns out that:

\[
\frac{dy}{d\tau} = \frac{\partial T}{\partial \tau} \sigma k^\epsilon (1 - e)^{1-\epsilon} + \frac{\partial P}{\partial e} \frac{\partial e}{\partial \tau} \frac{\partial^2 \alpha}{\partial q^2} \sigma k^\epsilon (1 - e)^{1-\epsilon} - \frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q} - \frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q} \sigma k^\epsilon (1 - e)^{1-\epsilon} > 0
\]

which yields the desired result.

**Proof of Proposition:** We have:

\[
\frac{\partial inv_i}{\partial \tau} = \frac{de}{d\tau} w + 1
\] (16)

which could be written as:

\[
\frac{\partial inv_i}{\partial \tau} = \frac{\frac{\partial P}{\partial e} \frac{\partial^2 \alpha}{\partial q^2} (1 - e) \frac{\partial T}{\partial \tau} - \frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q} - \frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q} \sigma k^\epsilon (1 - e)^{1-\epsilon} - \frac{\partial P}{\partial \tau} \frac{\partial \alpha}{\partial q} - \frac{\partial P}{\partial \tau} \frac{\partial \alpha}{\partial q} \sigma k^\epsilon (1 - e)^{1-\epsilon}}{(2 - \epsilon) \frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q} - \frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q} \sigma k^\epsilon (1 - e)^{1-\epsilon} - \frac{\partial P}{\partial \tau} \frac{\partial \alpha}{\partial q} - \frac{\partial P}{\partial \tau} \frac{\partial \alpha}{\partial q} \sigma k^\epsilon (1 - e)^{1-\epsilon}}
\]

and this yields the desired result.

**Proof of Proposition:** We have, at equilibrium,

\[
\pi = \alpha \sigma k^\epsilon (1 - e)^{1-\epsilon} - rk - w - \tau = 0
\] (17)

and therefore:

\[
w = \alpha \sigma k^\epsilon (1 - e)^{1-\epsilon} - rk - \tau
\] (18)
This yields:

\[ \frac{dw}{d\tau} = \frac{\partial y}{\partial \tau} - r \frac{\partial k}{\partial \tau} - 1 \]  

(19)

Substituting into (\#) yields:

\[ \frac{dw}{d\tau} = \frac{A}{(2 - \epsilon) \frac{\partial P}{\partial e} \frac{\partial \alpha}{\partial q} - \left( \frac{\partial P}{\partial e} \right)^2 \frac{\partial^2 \alpha}{\partial^2 q} (1 - e) - \frac{\partial^2 P}{\partial e \partial q} (1 - e)} \]  

(20)

where:

\[ A = -\frac{\partial P}{\partial e} \frac{\partial^2 \alpha}{\partial^2 q} \left( \frac{\partial \alpha}{\partial q} \right)^{-1} \left[ \frac{\partial T}{\partial \tau} \frac{\partial \alpha}{\partial \tau} \left[ \alpha \sigma k^\epsilon (1 - e)^{1 - \epsilon} - rk \right] - (1 - \epsilon) \alpha \right] \]  

(21)

\[ \frac{\partial \alpha}{\partial q} \frac{\partial^2 P}{\partial^2 e} (1 - e) \left[ \frac{\partial T}{\partial \tau} \frac{\partial \alpha}{\partial \tau} \left[ \alpha \sigma k^\epsilon (1 - e)^{1 - \epsilon} - rk \right] - \alpha (1 - \epsilon) \right] \]

\[ + \frac{\partial \alpha}{\partial q} \frac{\partial P}{\partial e} \frac{2 - \epsilon}{\alpha (1 - \epsilon)} \left[ \frac{\partial T}{\partial \tau} \frac{\partial \alpha}{\partial \tau} \left[ (2 + \epsilon) \alpha \sigma k^\epsilon (1 - e)^{1 - \epsilon} - rk \right] - \alpha (1 - \epsilon) \right] \]

This expression for A is positive when:

\[ \frac{\partial T}{\partial \tau} \frac{\partial \alpha}{\partial q} \left[ \alpha \sigma k^\epsilon (1 - e)^{1 - \epsilon} - rk \right] \geq (1 - \epsilon) \alpha \]  

(22)

or:

\[ \frac{\partial T}{\partial \tau} \frac{\partial \alpha}{\partial q} \frac{\tau}{\alpha} \left( \frac{y - rk}{\tau} \right) \geq 1 - \epsilon \]  

(23)

which yields the desired result.

6 References


