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School Choice: Income, Peer effect and the formation of Inequalities.

Saïd HANCHANE^a; Tarek MOSTAFA^{a*1}

Abstract: In this paper, we analyze the equilibrium on the market for schooling where both public and private schools coexist and where individuals are differentiated by income and ability. We introduce a non linear in means model of peer effect by shedding the light on the fact that school quality is not solely dependent on mean ability but also on the dispersion of abilities. We study the distribution of students across sectors while examining the conditions for the existence of a majority voting equilibrium in the context of non single peaked preferences. Finally, we examine the presence of a hierarchy of school qualities. In the paper we shed the light on equity problems related to the access to educational quality while analyzing the functioning of the educational system.

JEL Classification: I20, I21, H52.

Keywords: Education market, majority voting equilibrium, peer group effect, pricing discrimination, educational opportunity.

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Reducing inequalities both in the access to education and in the outcomes of education has become a major driving force of educational reforms in most developed countries. Unequal access to education, unequal outcomes, disparities of earnings, disparities of educational qualities between communities, and the quality of public education constituted an important subject of debate in the recent French presidential elections. Recent and ongoing international surveys by the OECD – PISA and IALS – showed that the outcomes of education in major western countries tend to have large levels of inequalities. For instance in PISA2000, Belgium, Germany, the USA, The UK and France turned out to be the most unequal states in term of the dispersion of students' achievements over reading, numeracy and basic science tests¹. The ongoing debate over these issues provides the motivation for our article as well as to the growing literature on school choice, college choice and urban economics.

The aim of our paper is to study the formation of inequalities in the access to education and in educational outcomes through an analysis of stratification on the market for schooling when both private and public education alternatives coexist. We analyze stratification by contrasting individual behavior in term of voting over tax rates and private schools profit maximizing behavior. We shed the light on the fact that school quality and students' achievements depend not only on mean ability in the school but also on the distribution of students thus on the dispersion of ability². One reads for instance in a mimeo of Caroline Hoxby and Gretchen Weingarth (Hoxby, Weingarth, 2005 pp. 2 and 30).

“The linear-in-means model assumes that each student has the same effect on each other student (a homogeneous treatment effect). It also assumes that a single student whose achievement raises a class's mean achievement by two points has precisely the same effect as several students whose combined achievement raises the class's mean by two points (that is, all effects operate through one moment: the mean of peers)... Moreover, most applications of peer effects—school desegregation, school choice, college choice, urban economics—need to have non-linear peer effects to generate results that are interesting and that mimic the facts.” pp.30 “Our finding support for the Boutique and Focus models suggests that schools, colleges, and workplaces should be wary of creating peer groups in which some people are isolated. However, they should also avoid creating critical mass around a certain type of person. Some focus is good”.

¹ Chapter 5 in “Education, Equality and Social Cohesion” by Green, Preston and Janmatt provides interesting results on outcomes' inequalities in five groups of countries: East Asia, the Nordic countries, the Germanic region, the Mediterranean countries and the predominantly English speaking countries.

² In our model, we account for nonlinearity in mean ability by taking the variance of abilities into consideration.

The philosophy behind the introduction of peer ability dispersion in the quality function can be understood in two ways³. The first is related to the impact of the dispersion on students' achievements. Schools with low ability dispersions tend to be more appreciated by students. When teaching, a teacher normally targets the mean ability students; students with ability higher than the mean understand more easily than those with mean ability and conversely for students with ability lower than the mean. In a highly dispersed class we have two groups, very high ability and very low ability students; when aiming at the mean ability holders, the high dispersion will negatively affect the transmission of knowledge since the teacher will no longer be able to cater to the needs of each group⁴. The second is understood through the signaling effect of education when information about an individual's ability is incomplete⁵; belonging to a school with very high dispersion of ability reduces the quality of the signal that this individual generates, since outsiders with limited information on ability will no longer be able to identify his type. Thus, we can consider that school quality is decreasing in the variance of abilities.

Our model constitutes an integrated analysis of school choice, stratification, and the provision of education by public and private sectors. It coincides and supports the empirical findings of Hoxby and Weingarth; in fact, since mean ability and the variance of abilities will be working in opposite directions, schools tend to have relatively homogenous student composition, in other terms students will not be isolated and a critical mass will not be built around a particular type. This was found by Hoxby and Weingarth to be an efficient form of mixing.

The analysis of public versus private school choice started with Yoram Barzel (1973); in a critical study of an article of Robin Barlow (1970) he introduced what is known as the "Ends against the middle phenomenon" for which we have a coalition of the rich and the poor against the middle class. The study of Barzel constituted an introduction to a more

³ M Dertouzos et al (1989; pp. 84-85) underlined the need to take the dispersion in test scores into account. R. Benabou (1996) explored the effect of human capital dispersion on economic growth.

⁴ This is consistent with the Boutique and the Focus models. See Hoxby and Weingarth 2005, pp 6.

⁵ A school known to be highly selective can be imagined to have a high mean ability and a low ability variance, since selection leads to the homogenization of the student population.

formalized theoretical approach of school choice, starting with Joseph Stiglitz in (1974), Clotfelter (1976) and Sonstelie (1979).

Stiglitz analyzed the demand for schooling under different institutional arrangements. In his study education is a complex good; it is consumption and an investment good as well as a screening instrument. He notes that preferences are not single peaked when public and private alternatives coexist. All these characteristics may prevent the existence of a majority voting equilibrium. At this level, two categories of articles are relevant to our analysis.

The first concerns the article written by Fernandez and Rogerson 1996 on the provision of public goods in a multicomunity model, the one written by Epple and Romano 1996, and the one written by Glomm and Ravikumar. The latter constructed a majority voting equilibrium determined by the individual with median income in the context of non single peaked preferences; they provided the necessary conditions for the existence of this equilibrium. In their article individuals are differentiated only by income, and quality is only dependent on public expenditure per pupil. The objective of these articles was the study of the provision of public services.

The second concerns the series of articles written by Epple et al (1998, and 2006), two major characteristics are the linearity in mean ability and the single crossing assumption. In 1998, Epple and Romano considered an economy where individuals are differentiated by income and ability; the private sector is active and private schools maximize their profit, this strategic behavior combined to a single crossing assumption leads to the existence of a hierarchy of school qualities. In their article quality is only dependent on peer group effects. In 2006 the authors added an income diversity measure in order to estimate a model for higher education. The objective of these articles was the analysis of stratification between schools. Some related literature may include Calabrese et al (2006), Epple and Platt (1998), Epple, Filimon and Romer (1984-1993).

In this paper, we carry on the work made by Glomm, Ravikumar (1998), Epple and Romano (1998), by combining the study of stratification and the provision of public services when private alternatives exist. Our model differs in a number of ways, in comparison to the study of Glomm and Ravikumar; we consider an economy where individuals are differentiated by both income and ability. We present the necessary conditions for the existence of a majority voting equilibrium in the context of non single peaked preferences. We analyze the distribution of students across sectors and schools, while presenting two equity related problems. First, private schools will operate to the detriment of public schools by siphoning high ability high income students. Second, inequalities of educational opportunity exist when a fraction of the population does not have an equal access to educational achievement, given its allocation of income and ability.

In our model, similarly to Epple and Romano, the private sector is active. Private schools maximize profits under a quality constraint. Unlike them, we consider a school quality that is dependent on peer effect and the dispersion of peer ability in a school and we do not impose a single crossing assumption. Individual utility will be maximized through a utility taking assumption. Peer group effect and the dispersion of peer ability are built into our model; these factors are the corner stone of the discriminating pricing strategy. The inclusion of peer group effect and the dispersion effect is consistent with the literature on this subject; Summers and Wolfe (1977), Henderson (1978), Sorensen and Hallinan (1986), Kulik (1992), Benabou (1996) and Hoxby and Weingarth (2005) have provided evidence that such an effect exists.

Our equilibrium will be characterized by the existence of an indifference point between the two sectors determined by a couple of income and ability. The distribution of students across sectors and schools, given the discriminating pricing strategy will happen to the detriment of public schools. Individuals with high ability and income levels will be in the private sector; individuals with high ability and low income levels will be in the private sector and will receive financial aides; individuals with high income and low ability levels will be in the private sector and will pay tuition premia; individuals with low

ability and income levels will be in the public sector. Individual behavior and preferences will determine the existence conditions of a majority voting equilibrium over tax rates.

Section I presents the model, equilibrium conditions follow in section II, section III examines the distribution of students across sectors, the conditions for a majority voting equilibrium and the existence of a hierarchy of school qualities, section IV concludes. An appendix contains mathematical details.

I- The model

In our model, we consider an economy populated by a continuum of agents; the number of types is largely superior to that of schools, we do not make a difference between households and students, each household has one student and decisions are made by the household. Students have identical preferences over consumption; they are differentiated by their income and ability levels, an individual i (with $i = 1, 2, 3, 4, \dots, P$) has an income y_i and an ability b_i , i is also used to denote a type of individuals. Income and ability are distributed in the population according to $f(b, y)$ which is positive, and continuous on its support $S \equiv (0, b_{\max}) \times (0, y_{\max})$.

Individual utility is assumed to be a function of consumption goods other than education and school quality. It is denoted $U(c, q)$, U is continuous, twice differentiable, and increasing in both arguments. Education is considered to be a superior good $\lim_{c \rightarrow \infty} U(c, q) = 0$, in other terms individuals always choose the highest school quality they can afford given their disposable income.

School quality is determined by school expenditure per pupil E , by peer group effect θ_j , and by the dispersion of peer ability in a school σ_j^2 . Quality is increasing in the first two arguments and decreasing in the last. j is an index denoting a particular school, (with $j = 1, 2, 3, \dots, j$). Educational achievement is given by $a = a(b, q)$; a is continuous and

increasing function of both arguments. Thus, the access to a higher quality school is translated through higher achievements. In term of inequalities, unequal access to education generates unequal outcomes.

Education production cost depends only on the number of students enrolled in a particular school, it is denoted $C(k) = V(k) + F$ with $V' > 0$ and $V'' < 0$ or $V'' > 0$ ⁶. k is the number of students in this school; F is a constant reflecting a fixed cost when no student are enrolled. The existence of economies of scale in the production of educational services is realistic; it will prevent the existence of an infinite number of private schools. In our model we suppose that the number of student types is largely superior to the number of schools.

Each private school will retain a fraction of the student population that has chosen the private sector, by applying tuition and admission policy permitting the maximization of each individual's utility through a utility taking assumption. The pricing strategies will be characterized by a meritocratic system.

In the private sector, students with ability higher than the mean receive a tuition reduction, conversely, students with ability lower than the mean pay a tuition premia. Similarly, students with ability close to the mean or (an ability pushing the variance downward receives tuition reductions, conversely, students with ability far from the mean (an ability pushing the mean upward) pay tuition premia. This discriminating pricing strategy is justified in the context of internalizing the externality generated by the position of a student's ability in the distribution of abilities in the school.

It is important to note that there will be no supplementary educational investments in students with low ability levels, as a consequence school expenditure per student will be the same for all students in a particular school.

⁶ The existence or nonexistence of economies of scale in the production of education is discussed in the optimization section.

Individuals in the private sector can always choose public sector schools, conversely individuals obliged to enroll in the public sector given their allocation of income and ability do not have the opportunity to enroll in private schools.

The public Sector:

Public schools are income tax financed; all individuals pay taxes dedicated to financing public education even if they do not use public schools. Public sector resources are determined by tY ; and the expenditure per student is determined by $\frac{tY}{N}$. Public sector schools do not receive tuition, thus $p_{ju} = 0$.

With, t : The tax rate determined through majority voting.

Y : Total income.

N : The number of students in the public sector. $N \in [0, P]$.

P : The number of students in the economy.

u is an index denoting a particular public school.

The quality of education in the public sector for a school j is written as follows:

$$q_{ju} = \left[\frac{tY}{N}, \theta_{ju}, \sigma_{ju}^2 \right]$$

$\frac{tY}{N}$ is homogenous for all schools and students. However peer group effect and dispersion effect may be different or homogenous depending on whether public schools have strategical selection procedures⁷.

We denote $\alpha_{ju}(b, y)$ as the conditional distribution, it represents the number of students of type (b, y) enrolled in a particular public school.

$k_{ju} = \iint_s \alpha_{ju}(b, y) f(b, y) db dy$ is the global distribution; with $\sum k_{ju} = N$.

⁷ The analysis of public schools' strategical behavior will be seen in the last section.

Individual utility in the public sector is of the following form:

$$U_{iu} = U[y_i(1-t), q_{ju}]$$

Indirect utility can be written as:

$$U_{iu} = W_{iu} = W(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i)$$

In the public sector, utility maximization is trivial. A student will allocate his disposable income (after the deduction of taxes dedicated to public school finance) to private consumption.

The private sector:

In the private sector, schools are tuition financed. Parents pay a positive tuition equal to p_{ijr} . Private school resources can be determined by $\sum p_{ijr}$. These resources are not entirely distributed over students; per student expenditure is determined by dividing the production cost of education by the number of students in the school. The difference between resources and expenditure represent private school's profit. This positive profit will encourage new schools to enter the market until it vanishes (in a competitive market).

Expenditure per student is given by:
$$\frac{C(k_{jr})}{k_{jr}} = \frac{V(k_{jr}) + F}{k_{jr}}$$

r is an index denoting a particular private school.

We denote $\alpha_{jr}(b, y)$ as the conditional distribution, it represents the number of students of type (b, y) enrolled in a particular private school.

$k_{jr} = \iint_s \alpha_{jr}(b, y) f(b, y) db dy$ is the global distribution; with $\sum k_{jr} = P - N$.

The quality of education in the private sector for a school j is written as follows:

$$q_{jr} = q \left[\frac{C(k_{jr})}{k_{jr}}, \theta_{jr}, \sigma_{jr}^2 \right]$$

It should be noted that private school quality is positively correlated with individual income; when income increases, the private quality that can be purchased is much higher. When an individual compares quality in both sectors; his ability level has no influence because it is the same in all schools.

Private sector quality is heterogeneous between schools in all arguments. Private school expenditure is homogenous for students attending the same school, and heterogeneous between schools. Peer group effect and dispersion effect are heterogeneous between schools.

Individual utility in the private sector is of the following form:

$$U_{ir} = U[y_i(1-t) - p_{ijr}, q_{jr}]$$

Individuals are going to choose an optimal combination between consumption goods and school quality in order to maximize their utility.

$$\text{Max } U_{ir} = U[y_i(1-t) - p_{ijr}, q_{jr}]$$

Subject to $p_{ijr} = y_i(1-t) - c_i$ with c as consumption.

Indirect utility can be written as: $U_{ir}^* = W_{ir} = W(t, y_i, \theta_{jr}, \sigma_{jr}^2, b_i)$, it represents the only solution to this optimization.

Private schools behave strategically on the market; they maximize profits as utility takers by conditioning admission and tuition according to ability levels. The utility obtained in a school should be at least equal to that obtained elsewhere. This behavior is similar to that of private clubs with non anonymous crowding, see Scotchmer and wooders (1987). It should be noted that individual ability is perfectly observable by private schools.

Private schools have to maximize the following function:

$$\text{Max } \pi_{jr} = \iint_s [p_{ijr}(b, y) \alpha_{jr}(b, y) f(b, y) db dy] - V(k_j) - F \quad (1)$$

Subject to:

$$\alpha_{jr}(b, y) \in [0, P - N] \quad \forall (b, y) \quad (1a)$$

$$U_r = U[y_i(1-t) - p_{ijr}(b, y), q_{jr}] \geq \text{Max}U[y_i(1-t) - p_{ijr}(b, y), q_{jr}] \quad \forall(b, y) \quad (1b)$$

$$k_{jr} = \iint_s \alpha_{jr}(b, y) f(b, y) dbdy \quad (1c)$$

$$\theta_{jr} = \frac{1}{k_{jr}} \iint_s b \alpha_{jr}(b, y) f(b, y) dbdy \quad (1d)$$

$$\sigma_{jr}^2 = \frac{1}{k_{jr}} \iint_s (b - \theta_{jr})^2 \alpha_{jr}(b, y) f(b, y) dbdy \quad (1e)$$

Constraint (1c) determines the number of students enrolled in a particular private school. Constraint (1d) determines peer group effect given by mean ability. Constraint (1e) determines dispersion effect given by the variance of peer ability. Constraint (1b) represents the utility taking assumption.

II- Equilibrium conditions

Three types of conditions are necessary for the existence of equilibrium:

A- The conditions for private sector equilibrium are:

- Individual utility maximization:

$$U^*(b, y) = \text{Max}U[y_i(1-t) - p_{ijr}, q_{jr}]$$

- Private schools' profits maximization:

$$[\theta_{jr}, \sigma_{jr}^2, k_{jr}, p_{ijr}(b, y), \alpha_{jr}(b, y)] \quad \text{Satisfies equation (1).}$$

- New entries are expected so long as private schools expect to make a profit; the private sector equilibrium will be defined by.

$$\pi_{jr} = 0 \quad j=1, 2, 3, 4, \dots, n. \quad \text{new entries are no longer profitable.}$$

B- The conditions for public sector equilibrium are:

- Public schools are not tuition financed:

$$p_{iju}(b, y) = 0$$

- $\alpha_{ju}(b, y) \in [0, N] \quad \forall(b, y)$

- The number of students in a public school is given by:

$$k_{ju} = \iint_s \alpha_{ju}(b, y) f(b, y) db dy$$

- Public school's mean ability is given by :

$$\theta_{ju} = \frac{1}{k_{ju}} \iint_s b \alpha_{ju}(b, y) f(b, y) db dy$$

- Public school peer ability variance is given by:

$$\sigma_{ju}^2 = \frac{1}{k_{ju}} \iint_s (b - \theta_{ju})^2 \alpha_{ju}(b, y) f(b, y) db dy$$

C- All students in the age for schooling are enrolled in one of the two sectors:

$$\sum k_{ju} + \sum k_{jr} = N + (P - N) = P \quad \forall (b, y)$$

This condition represents market clearance; public schools are preferred to no schooling.

III- Theoretical results:

A- Private schools' profit maximization results.

The optimal function combining tuition and individual utility is given by:

$$U_{ir} [y_i (1 - t) - p_{ijr}^*, q_{jr}] = U^*(b, y) = W_{ir}(t, y_i, \theta_{jr}, b_i) \quad \forall (b, y) \quad (2)$$

The optimal level of tuition is:

$$\alpha_{ju}(b, y) \in [0, N] \quad (3)$$

$$\alpha_{jr}(b, y) \in [0, P - N]$$

$$p_{ijr}^* = V'(k_{jr}) + \mu_{jr}(\theta_{jr} - b) + \mu'_{jr} [(b - \theta_{jr})^2 - \sigma_{jr}^2] \quad \forall (b, y)$$

With μ_{jr} and μ'_{jr} are the Lagrangian multipliers⁸.

⁸ Result (3) are obtained by forming a Lagrangian function to take account of constraints (1c) and (1d) and (1e), then by optimizing over α_{jr} . Mathematical details are to be found in the appendix.

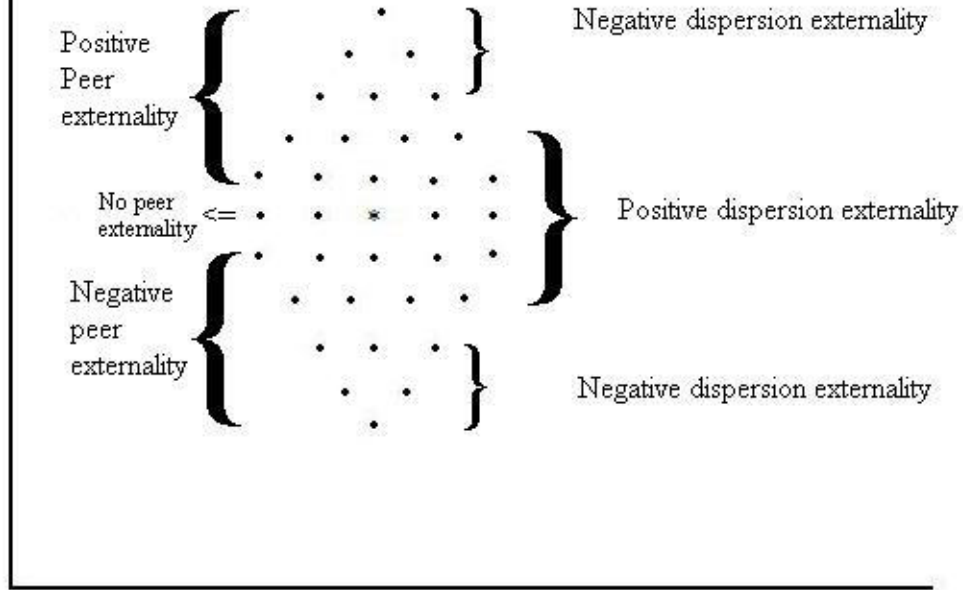
This price is what an individual of type (b, y) should be able to pay in order to be admitted in school j . This price is formed of two parts; fraction $V'(k_{jr})$ represents a homogenous price for all students resulting from the production cost of education, fraction $\mu_{jr}(\theta_{jr} - b_i)$ represents the price resulting from the difference between own ability and mean ability, and fraction $\mu'_{jr}[(b - \theta_{jr})^2 - \sigma_{jr}^2]$ represents the price resulting from the positioning of an individual's ability around the mean. Students with ability levels lower than school mean ability $b_i < \theta_{jr}$ will have to pay a tuition premia equal to $\mu_{jr}(\theta_{jr} - b_i) > 0$. Students with ability levels higher than school mean ability $b_i > \theta_{jr}$ will receive tuition discount equal to $\mu_{jr}(\theta_{jr} - b_i) < 0$. Conversely, students with $(b - \theta_{jr})^2 > \sigma_{jr}^2$ will pay a tuition premia equal to $\mu'_{jr}[(b - \theta_{jr})^2 - \sigma_{jr}^2] > 0$; students with $(b - \theta_{jr})^2 < \sigma_{jr}^2$ will receive a tuition reduction equal to $\mu'_{jr}[(b - \theta_{jr})^2 - \sigma_{jr}^2] < 0$. It should be said that the distance of one's ability from the mean determines the importance of $(b - \theta_{jr})^2$ relatively to σ_{jr}^2 ; students with ability equal to the mean receive maximum reduction equal to $\mu'_{jr}[\sigma_{jr}^2]$ and as we go away from the mean this price reduction decreases until it becomes a positive price⁹. An example of the distribution of externalities in a school is given in figure 2.

⁹ Note that $(b - \theta_{jr})^2$ is always positive and has the effect of increasing price.

b

Figure 2

- * Mean ability and income in a school.
- An individual.



y

An interesting result is related to the difference between peer group effect and the dispersion effect; peer effect has these two properties

$$\lim_{b \rightarrow b_{\max}} \mu_{jr}(\theta_{jr} - b) = \mu_{jr}(\theta_{jr} - b_{\max}) \text{ and } \lim_{b \rightarrow 0} \mu_{jr}(\theta_{jr} - b) = \mu_{jr}\theta;$$

while dispersion effect has these two properties $\lim_{b \rightarrow b_{\max}} \mu'_{jr}[(b - \theta_{jr})^2 - \sigma_{jr}^2] = \mu'_{jr}[(b_{\max} - \theta_{jr})^2 - \sigma_{jr}^2]$ and

$$\lim_{b \rightarrow \theta} \mu'_{jr}[(b - \theta_{jr})^2 - \sigma_{jr}^2] = -\mu'_{jr}\sigma_{jr}^2.$$

In other words, for a student the benefit of having ability largely higher than the mean is high in term of peer effect and the cost is also high in term of dispersion effect; similarly, the cost of peer effect attains its maximum $\mu_{jr}\theta$ at $b = 0$ and the benefit of dispersion effect attains its maximum

$$-\mu'_{jr}\sigma_{jr}^2 \text{ at } b = \theta_{jr}.$$

In our pricing function, schools admit all students who are able to pay a price $p \geq p_{ijr}^*$, however no student would accept to pay a price superior to the Pareto optimal one p_{ijr}^* unless there is no school that cater to his needs. The existence of economies of scale in the production of education $V''(k_j) < 0$ limits the number of schools and thus some students will be obliged to find a second best solution. Otherwise, $V''(k_j) > 0$ the market produces a large number of schools catering to each type. For the rest of the paper we consider $V''(k_j) > 0$ to be true¹⁰.

It is of a great importance to note that the price of private education can be negative. For some students we may have $\mu_{jr}(\theta_{jr} - b) < 0$,

$\mu'_{jr}[(b - \theta_{jr})^2 - \sigma_{jr}^2] < 0$ and $p_{ijr}^* = V'(k_{jr}) + \mu_{jr}(\theta_{jr} - b) + \mu'_{jr}[(b - \theta_{jr})^2 - \sigma_{jr}^2] < 0$. This negative price represents a financial aid to students permitting an important amelioration of school quality. This pricing strategy represents the corner stone of our meritocratic system.

Our results converge towards the empirical findings of Hoxby and Weingarth 2005; schools looking for a quality peer effect tend to have a high mean ability and a low variance. However since mean ability and the variance of abilities operate in opposite directions, schools have to make a compromise between the two effects, this leads to a relative homogenization of the students' population without creating a critical mass around one type. These results are consistent with the Boutique and the focus models and diverge considerably from the linear in means models¹¹.

¹⁰ Lawrence Kenny 1982 provided some evidence that economies of scale do exist in the production of education. A counterargument was provided by Ferris and West 2004; it relies on the existence of external costs that increase with size and can be related to social problems present in large schools.

¹¹ Note that in the work of Epple and Romano 1982, subsidization between low ability high income students and high ability low income students is pushed to the limits. In our model schools can no longer admit students with abilities so far from the mean since the variance has a negative effect on pricing and quality.

B- Majority voting results.

Proofs of propositions and lemmas are to be found in the appendix.

Notation remark: If (b', y') is better than (\hat{b}, \hat{y}) . It is denoted $(b', y') > (\hat{b}, \hat{y})$. This means that $b' > \hat{b}$ and $y' > \hat{y}$, or $b' > \hat{b}$ and $y' = \hat{y}$, or $b' = \hat{b}$ and $y' > \hat{y}$, or b' is weakly inferior to \hat{b} while y' is largely superior to \hat{y} , or finally b' is largely superior to \hat{b} while y' is weakly inferior to \hat{y} . With (b', y') and (\hat{b}, \hat{y}) two couples of ability and income. For the last two cases the superiority of one of the couples depends on the structural form of the utility function.

Proposition 1: we suppose that $\lim_{c \rightarrow \infty} U(c, q) = 0 \quad \forall q > 0$.

Given $t \in [0, 1]$, $N \in (0, P)$ and $Y \in \mathfrak{R}^+$. There exists a unique couple (\hat{b}, \hat{y}) with $\hat{b} > 0$ and $\hat{y} > 0$ such that $W_{iu} = W(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i) \geq W_{ir} = W(t, y_i, \theta_{jr}, \sigma_{jr}^2, b_i)$ if and only if $y_i < \hat{y}$ and $b_i < \hat{b}$.

In this proposition, we ignore extreme cases. For $t=0$, we have $N=0$, nobody will choose the public sector. Conversely for $t=1$, we have $N=P$, everybody will choose the public sector. An important remark to take into account is that the positioning of (\hat{b}, \hat{y}) will depend on income and ability distribution and the quality of education in the public sector determined by t , Y and N .

Lemma 1: as in proposition 1 we suppose that $\lim_{c \rightarrow \infty} U(c, q) = 0 \quad \forall q > 0$.

Given $t \in [0, 1]$, $N \in (0, P)$ and $Y \in \mathfrak{R}^+$. (i) Individuals with $y_i < \hat{y}$ and $b_i < \hat{b}$ will be enrolled in the public sector. (ii) Individuals with $y_i > \hat{y}$ and $b_i > \hat{b}$ will be enrolled in the private sector. (iii) Individual with $y_i < \hat{y}$ and $b_i > \hat{b}$ will be enrolled in the private sector and will receive tuition discounts. (iiii) Individuals with $y_i > \hat{y}$ and $b_i < \hat{b}$ will be enrolled in the private sector and will pay tuition premia. (iii) and (iiii) depend on certain conditions.

(i) And (ii) represent the outcome of proposition 1. The proof of (iii) and (iiii) depends on the fact that the disposable income should cover the price of private education when it is positive; secondly on the fact that $\lim_{c \rightarrow \infty} U(c, q) = 0$. When consumption becomes largely superior to school quality (when most of individual income is dedicated to consumption), individual utility tends to decrease. In other words, Individuals accept a decrease in consumption if school quality increases; they will choose the highest level of quality given their disposable income.

In this lemma we show that given our discriminating private pricing function, compensation between ability and income is possible. Sufficiently high abilities allow a student to compensate his lack of income by having a tuition reduction or a scholarship. Conversely, sufficiently high levels of income allow a student to compensate his lack of ability by giving him a higher purchasing power in order to buy private school quality. We can see that compensation will allow private schools to operate to the detriment of public schools by siphoning high ability high income students. Two uncommon cases shall be mentioned: individuals with abilities weakly higher than \hat{b} and very low income levels, and individuals with incomes weakly higher than \hat{y} and very low ability levels will not be admitted in the private sector since b and y are not sufficient to compensate the lack on income and ability respectively.

Lemma 2: in this lemma we establish the properties of the couple (\hat{b}, \hat{y}) relatively to N , Y and t . (i) For $N \in (0, P)$, the couple (\hat{b}, \hat{y}) is decreasing in N . (ii) For $Y \in \mathfrak{R}^+$, the couple (\hat{b}, \hat{y}) is increasing in Y . (iii) For $t \in [0, 1]$, the couple (\hat{b}, \hat{y}) is increasing in t .

Proofs for (i) and (ii) are based on the fact that private school quality is not related to N and Y . These factors only influence school choice through public school quality. Proof for (iii) is based on the fact that t has the same influence on consumption in both sectors. t also has an effect on public school quality without affecting that of private schools.

When we say (\hat{b}, \hat{y}) increases, this means that both \hat{b} and \hat{y} increase, or one of them increases while the other remains the same, or one of them largely increases while the other weakly decreases. The reverse is true when we say (\hat{b}, \hat{y}) decreases.

Proposition 2: for $t \in [0,1]$ and $Y \in \mathfrak{R}^+$. There exists a unique N^* which determines the number of students in the public sector as a result of majority voting. The number of individuals with $y_i < \hat{y}$ and $b_i < \hat{b}$ must be exactly equal to N^* .

So far we have considered an exogenous tax rate; in the following sections we endogenize the tax rate through majority voting. For an individual i , the most preferred tax rate is denoted as:

$$t^*(b_i, y_i) = \text{Arg max} \{ \max[W_{iu}(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i), W_{ir}(t, y_i, \theta_{jr}, \sigma_{jr}^2, b_i)] \}$$

This tax rate will maximize individual utility in the sector offering the highest indirect utility.

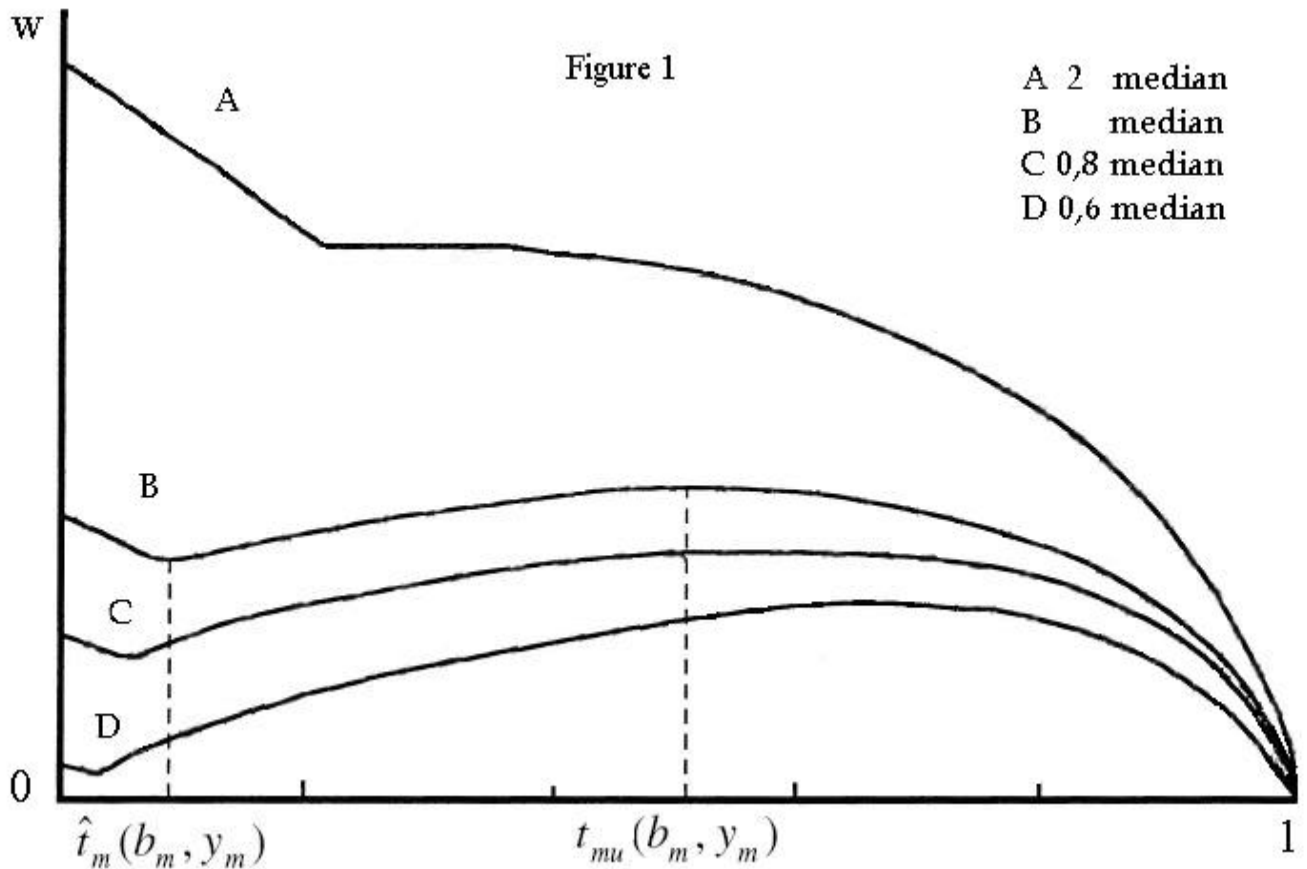
We denote $t_{iu}(b_i, y_i)$ the tax rate that maximizes utility in the public sector for an individual i with a couple (b_i, y_i) . $t_{iu}(b_i, y_i) = \arg \max W_{iu}(t, y_i, Y, N, \theta_{ju}, b_i)$.

We denote $\hat{t}_i(b_i, y_i)$ the tax rate for which an individual is indifferent between the two sectors. In this case $W_{iu}(\hat{t}_i, y_i, Y, N, \theta_{ju}, b_i) = W_{ir}(\hat{t}_i, y_i, \theta_{jr}, b_i)$. It is clear that for each couple of ability and income, there exists a critical tax rate which determines the state of indifference. For low levels of t , the private sector is preferred; for high level of t , the public sector is preferred. While passing from an extreme to the other, there exists a tipping point of preferences determined by the tax rate $\hat{t}_i(b_i, y_i)$.

Joseph Stiglitz (1974) has presented the fact that, preferences over tax rates are not single peaked. For low tax rates, public school quality is low and individuals prefer private schools. A marginal increase of the tax rate will not increase public school quality that much, but will reduce consumption; private schools will still be preferred and utility will

be reduced. An important increase of the tax rate will induce an important increase of public school quality; at $\hat{t}_i(b_i, y_i)$ individuals will be indifferent between the two sectors; from this point on, an increase in the tax rate will reverse tendencies and the public sector becomes preferable, utility is increasing in t . When the tax rate reaches $t_{iu}(b_i, y_i)$ utility is maximal in the public sector; from this point on utility is decreasing in t . Thus we find that utilities may have two peaks, one in each sector. This is represented in figure 1.

For extreme cases where individuals have low ability and income levels; they can not choose private schools and thus they are obliged to enroll in public ones; they have single peaked preferences. Conversely, individuals with very high income and ability levels will choose private schools (See Lemma 1); they have single peaked preferences. Households with old or no children can be regarded as individuals with high ability and income levels; they have single peaked preferences and always vote for a zero tax rate, this can be seen on the preference curve A.



Assumption 1: students with higher abilities can get higher educational qualities. This assumption is intuitive, even if in the same school a higher ability may have contradicting effects, because mean ability is higher and dispersion is higher too. Since $V''(k_j) > 0$, the market produces a large number of schools catering to each type. High ability students will always be able to find a higher quality school willing to enroll them given their endowments in ability and income and the discriminating pricing function.

Assumption 2: students with higher income levels can get higher educational qualities. This assumption is simple to understand, higher income students have higher purchasing power. They can afford higher levels of education in the private sector.

For the same level of income; when b increases, the quality that can be obtained increases. In the public sector the quality that can be obtained increases with b until reaching its maximum in the school with the highest public quality¹². From this point on any increase in ability provokes a tipping point, students will opt out of public schools and will join the private sector. If for a sufficiently high ability, a student stays in the public sector; he will have his utility reduced. We can write:

$$\lim_{b \rightarrow +\infty} W_{iu}(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i) = 0 \text{ and } \lim_{b \rightarrow +\infty} W_{ir}(t, y_i, \theta_{ju}, \sigma_{ju}^2, b_i) = +\infty$$

For the same level of ability; when y increases, the quality that can be purchased increases. In the public sector, since education is free, an income increase permits only a higher consumption and thus a higher utility. Since $\lim_{c \rightarrow \infty} U(c, q) = 0$, when income becomes sufficiently large, students opt out of the public sector and join the private one. If for sufficiently high income, a student stays in the public sector; he will have a high consumption and a low school quality, and thus he will have his utility

$$\text{reduced. } \lim_{y \rightarrow +\infty} W_{iu}(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i) = 0 \text{ and } \lim_{b \rightarrow +\infty} W_{ir}(t, y_i, \theta_{ju}, \sigma_{ju}^2, b_i) = +\infty$$

¹² The fact that public schools may have different quality levels will be discussed in the last section.

By combining these previous explanations we can write:

$$\lim_{(b,y) \rightarrow +\infty} W_{iu}(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i) = 0$$

$$\lim_{(b,y) \rightarrow +\infty} W_{ir}(t, y_i, \theta_{ju}, \sigma_{ju}^2, b_i) = +\infty$$

When the couple (b,y) becomes sufficiently high, staying in the public sector becomes costly.

For two individuals with $(b_1, y_1) > (b_2, y_2)$ we can write:

If $(b_1, y_1) > (b_2, y_2) > (\hat{b}, \hat{y})$ both individuals are in the private sector; with:

$$W_{1u}(t, y_1, Y, N, \theta_{ju}, \sigma_{ju}^2, b_1) < W_{2u}(t, y_2, Y, N, \theta_{ju}, \sigma_{ju}^2, b_2)$$

$$W_{1r}(t, y_1, \theta_{jr}, \sigma_{jr}^2, b_1) > W_{2r}(t, y_2, \theta_{jr}, \sigma_{jr}^2, b_2)$$

Lemma 3: The indifference tax rate $\hat{t}_i(b_i, y_i)$ is increasing in the couple (b_i, y_i) over the interval $(\hat{b}, b_{\max}) \times (\hat{y}, y_{\max})$. This can be explained intuitively; higher income and ability individuals have higher indifference tax rate since they demand higher public educational quality.

In this part of our analyses we study the existence of a majority voting equilibrium determined by the individual with the mean couple of income and ability that we denote (b_m, y_m) . We establish the necessary conditions for this existence.

We denote $t_{mu}(b_m, y_m)$ the tax rate which maximizes the utility of this individual; $\hat{t}_m(b_m, y_m)$ the tax rate for which the individual is indifferent between public and private sectors. We denote N_m the number of students in the public sector under $t_{mu}(b_m, y_m)$.

The individual with the median couple of income and ability separates the population into two fractions of 50%. In order to demonstrate that a majority voting equilibrium exists and is determined by this individual; we have to demonstrate that there does not exist a tax rate that is preferred to $t_{mu}(b_m, y_m)$ by more than 50% of the population. It is clear

that for the individual with the median couple of income and ability; $t_{mu}(b_m, y_m)$ is preferred. Thus, it will be sufficient to demonstrate that the fraction of the population with $(b, y) > (b_m, y_m)$ or $(b, y) < (b_m, y_m)$ prefer the tax rate $t_{mu}(b_m, y_m)$ to prove the existence of a majority voting equilibrium determined by the median individual.

Two conditions are necessary for the existence of a majority voting equilibrium determined by the individual with the median couple of income and ability:

Condition 1: $t_{iu}(b_i, y_i)$ is decreasing in the couple (b_i, y_i) . For two individuals having $(b_2, y_2) > (b_1, y_1)$ we have $t_{1u}(b_1, y_1) > t_{2u}(b_2, y_2)$.

Condition 2: $W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m] < W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_m]$, this means that the individual with the median couple of income and ability prefer public education financed at a positive tax rate $t_{mu}(b_m, y_m)$ to private education when the public sector does not exist.

Given individual behavior shown in figure 1; we are going to study the existence of majority voting equilibrium over 3 intervals: $[0, \hat{t}_m(b_m, y_m)]$, $[\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)]$ et $[t_{mu}(b_m, y_m), 1]$.

Lemma 4: over the interval $[t_{mu}(b_m, y_m), 1]$, there does not exist a tax rate which is preferred to $t_{mu}(b_m, y_m)$ by more than 50% of the population.

Lemma 5: over the interval $[\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)]$, there does not exist a tax rate which is preferred to $t_{mu}(b_m, y_m)$ by more than 50% of the population.

An important property used in the proofs of lemma 4 and 5 is based on the fact that public sector utility is increasing over the interval $[\hat{t}_i(b_i, y_i), t_{iu}(b_i, y_i)]$ and decreasing over the interval $[t_{iu}(b_i, y_i), 1]$ in the tax rate.

Lemma 6:

(i) If $W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m] < W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N, \theta_{ju}, \sigma_{ju}^2, b_m]$

Then $W_{lr}[0, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] < W_{lu}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1]$ For $(b_1, y_1) < (b_m, y_m)$.

(ii) If $W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m] > W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N, \theta_{ju}, \sigma_{ju}^2, b_m]$

Then $W_{lr}[0, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] > W_{lu}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1]$ For $(b_1, y_1) > (b_m, y_m)$.

Lemma 6 means that if the individual with the mean couple of income and ability prefers (does not prefer) the positive tax rate $t_{mu}(b_m, y_m)$ over zero; the fraction of the population having an inferior (a superior) couple of income and ability has the same behavior as him.

Proposition 3: if $W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m] < W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N, \theta_{ju}, \sigma_{ju}^2, b_m]$ is true, then the couple (t_m, N_m) is a majority voting equilibrium.

Conversely, if $W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m] > W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N, \theta_{ju}, \sigma_{ju}^2, b_m]$ is true. All individuals with a couple of income and ability higher than (b_m, y_m) prefer a zero tax rate because $W_{lr}[0, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] > W_{lu}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1]$. The majority voting equilibrium is determined by the couple (0,0).

If the first equilibrium condition is not satisfied, then a majority voting equilibrium may not exist. If the second condition is not satisfied then (0,0) is a majority voting equilibrium.

C- The existence of a hierarchy of school quality.

In their analyses, Epple and Romano (1998) have demonstrated the existence of a hierarchy of school quality in the private sector. They have argued that if two schools have the same level of quality, they can increase their profit by modifying the student composition of the school by exchanging the same number of different ability and income students. Quality will increase for one and decrease for the other. This strategy will allow a profit increase following a differentiation of quality. The single crossing

condition is central to the demonstration; it implies that higher ability higher income students value increased quality by more than lower ability lower income types.

Two major differences exist in our model relatively to Epple and Romano.

The first difference is related to the fact that school quality in our model is dependent on expenditure per pupil, mean ability and the dispersion of abilities. Expenditure per pupil is not an additional variable relatively to Epple and Romano since it is determined

by $\frac{C(k_{jr})}{k_{jr}} = \frac{V(k_{jr}) + F}{k_{jr}}$, exchanging the same number of students would not have an

effect on expenditure. However the inclusion of the dispersion of abilities has an important effect, it is no longer sufficient to exchange (b_1, y_1) types for (b_2, y_2) when $b_2 > b_1$ and $y_2 > y_1$. Admitting higher ability students has contradicting effects on quality, it increases quality through peer effect and it reduces quality through the dispersion of abilities. The final effect of this exchange depends on the structural form of educational quality. The second difference is that in our model we did not impose a single crossing assumption.

These two differences may result in the absence of a strict hierarchy of private school qualities. Different schools may have the same quality through different combinations

of $\frac{C(k_{jr})}{k_{jr}}$, θ_{jr} and σ_{jr}^2 . Two private schools with the same quality will charge the same price¹³.

School hierarchy in the public sector:

Following the equilibrium between the two sectors, the number of students enrolled in the public sector is given by N. Since public schools are free entry free of tuition schools, the income level of individuals will not have an effect on their distribution between

¹³ See Epple and Romano 1998, pp. 56.

schools¹⁴. We have previously seen that expenditure per student is the same for all public schools and depend on income tax levels, thus public school quality can only be differentiated through peer effect or dispersion effect.

In the public system, the absence of restrictions on entry encourages individuals to choose the school (or the number of schools) with maximum quality levels, denoted $\max q_{ju}$. If all N individuals have the same behavior, they will all end up in the school with $\max q_{ju}$, and then $\max q_{ju} = \bar{q}_u$ with \bar{q}_u the level of quality when the N individuals are enrolled in the single existing public school u¹⁵.

Following this kind of equilibrium, a suboptimal solution is reached. Individuals providing a non-internalized positive externality are disadvantaged and individuals providing non internalized negative externalities are advantaged. In this situation, a hierarchy of qualities will not exist in the public sector, and public school quality is given by \bar{q}_u .

In what follows, we identify two different solutions for the problem.

- a- Screening by examination:** this is a trivial solution, if public schools are allowed to screen individuals according to their ability; we will have a number of schools that is at least equal to the number of types b_i in the population. If a school defines an access score by s_{ju} that is measured by ability units, it will accept all students with $b_i \geq s_{ju}$. An individual looking for a high level of educational quality will choose a school with $s_{ju} \geq b_i$ in order to be enrolled with students with at least the same ability level. The combination of $s_{ju} \geq b_i$ and $b_i \geq s_{ju}$ leads to the optimal equilibrium condition $s_{ju} = b_i$. Thus a hierarchy of qualities will exist and it follows the distribution of ability types in the population. It should be noted that a

¹⁴ In this paper, residential choice is not built into the model. In a multicomunity model, income will have an effect on the distribution of students between public schools through residential choice.

¹⁵ This public school can operate on multiple geographical locations (multiple campuses), but it can be regarded as a large single public school since expenditure per pupil, peer effect and dispersion effects would be the same.

number of different schools can share a group of students of the same type; in this case, these schools will have the same quality and the hierarchy will not be strict.

b- Compensation schemes: another optimality improving solution to the problem can be obtained through a compensation scheme; in this context public schools have to maximize a resources function. Similarly to private schools' profit maximization, resources maximization yields the marginal cost function of admitting a student in a public school. It is of the following form:

$$mc(b_i, y_i, \theta_{ju}) = V'(k_{ju}) + \mu_{ju} (\theta_{ju} - b) + \mu'_{ju} [(b - \theta_{ju})^2 - \sigma_{ju}^2].$$

Since public schools are free of tuition, the resources collected through income taxes should cover the total cost of students in the public sector. In this case the funding

condition is $\sum E = \sum [V'(k_{ju}) + \mu_{ju} (\theta_{ju} - b) + \mu'_{ju} [(b - \theta_{ju})^2 - \sigma_{ju}^2]]$; expenditure

per pupil should be able to compensate positive externality providers through financial aids, and should counterbalance the negative impact of negative externality providers, since they would not pay (free public schools) in order to internalize their negative externality. This solution does not lead to the creation of a quality hierarchy; instead, we still have the same large public school with \bar{q}_u . It is not fully optimal, since negative externality providers do not pay to compensate their negative impact on quality. The optimal solution would be like the one in private schools, negative externality providers compensate directly positive externality providers, tax revenues should only cover the part related to the functioning of public schools $V'(k_{ju})$. This may seem unacceptable since optimality contradicts the free entrance assumption to public schools; public schools will become private to a certain degree. It should be noted that a combination of screening by examination and compensation schemes can lead to the existence of a non-strict hierarchy of school quality with each school regrouping at least two different types of individuals. It should be noted that the equilibrium point between public and private sectors will be modified since public schools behave strategically through selection or resources maximization; this

entails a study of competition on the market of schooling which is not the object of this paper.

IV- Conclusion

In this paper, we have constructed an equilibrium on the market for education where individuals are differentiated by income and ability and where educational quality is not linear in mean ability. We have studied the distribution of students across sectors while mentioning the fact that private schools will operate to the detriment of public sector schools by siphoning high income high ability students. We have presented the necessary conditions for the existence of a majority voting equilibrium determined by the individual with the median couple of income and ability. Finally, we have analyzed the presence of a hierarchy of quality in both sectors.

According to our results, stratification by income and ability will prevent a fraction of the student population from having access to a more performing education, given its allocation of income and ability. In future studies, we may want to revisit models of urban economics contrasting school choice with residential choice in order to get a more generalized view of the provision of educational services, stratification, and the formation of inequalities in the access to education.

Appendix

Optimization in a private school:

Private schools profits maximization:

$$MAX \pi_{jr} = \iint_s [p_{jr}(b, y) \alpha_{jr}(b, y) f(b, y) dbdy] - V(k_{jr}) - F$$

Subject to:

$$k_{jr} = \iint_s \alpha_{jr}(b, y) f(b, y) dbdy \quad (1c)$$

$$\theta_{jr} = \frac{1}{k_{jr}} \iint_s b \alpha_{jr}(b, y) f(b, y) dbdy \quad (1d)$$

$$\sigma_{jr}^2 = \frac{1}{k_{jr}} \iint_s (b - \theta)^2 \alpha_{jr}(b, y) f(b, y) dbdy \quad (1e)$$

The three constraints can be integrated in two:

$$\theta_{jr} \iint_s \alpha_{jr}(b, y) f(b, y) dbdy - \iint_s b \alpha_{jr}(b, y) f(b, y) dbdy = 0$$

$$\iint_s (b - \theta_{jr})^2 \alpha_{jr}(b, y) f(b, y) dbdy - \sigma_{jr}^2 \iint_s \alpha_{jr}(b, y) f(b, y) dbdy = 0$$

The Lagrangian function is of the following form:

$$\begin{aligned} \Phi = & \iint_s [p_{ijr}(b, y) \alpha_{jr}(b, y) f(b, y) dbdy] - V(k_{jr}) - F - \mu_{jr} \left[\theta_{jr} \iint_s \alpha_{jr}(b, y) f(b, y) dbdy - \iint_s b \alpha_{jr}(b, y) f(b, y) dbdy \right] \\ & - \mu'_{jr} \left[\iint_s (b - \theta_{jr})^2 \alpha_{jr}(b, y) f(b, y) dbdy - \sigma_{jr}^2 \iint_s \alpha_{jr}(b, y) f(b, y) dbdy \right] \end{aligned}$$

With μ_{jr} and μ'_{jr} are the Lagrangian multipliers.

Optimization is made through the partial derivation of the Lagrangian function over

$\alpha_{jr}(b, y)$, results are the following:

$$\frac{\partial \Phi}{\partial \alpha(b, y)} = p_{ijr}^* - V'(k_{jr}) - \mu_{jr}(\theta_{jr} - b) - \mu'_{jr} [(b - \theta_{jr})^2 - \sigma_{jr}^2] = 0$$

$$p_{ijr}^* = V'(k_{jr}) + \mu_{jr}(\theta_{jr} - b) + \mu'_{jr} [(b - \theta_{jr})^2 - \sigma_{jr}^2]$$

Proof of proposition 1:

The public sector is chosen when:

$$W_{iu} = W(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i) \geq W_{ir} = W(t, y_i, \theta_{jr}, \sigma_{jr}^2, b_i)$$

The couple (\hat{b}, \hat{y}) must satisfy the following equation:

$$W_{iu}(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i) = W_{ir}(t, y_i, \theta_{jr}, \sigma_{jr}^2, b_i)$$

It should be noted that private school quality is always superior to that of public schools.

This is consistent with the reality; if public schools have better qualities than private

schools, rational individuals will choose free public schools and the private sector will not exist. A private school with a quality lower than that of a public school will be eliminated from the market by the simple functioning of competition. Thus we can write:

$$q_{ju} < q_{jr}$$

In order to demonstrate the existence of (\hat{b}, \hat{y}) we analyze two extreme cases.

First case:

For very low income levels, individuals can not pay tuition for the private school with the lowest quality. For very low abilities, individuals can not benefit from a financial aide or a tuition discount; we have $p_{ijr}^* > 0$ and $(1-t)y_i - c_{ir}^* < p_{ijr}^*$ for this school; which means that the price for the lowest quality private school exceeds the disposable income.

These individuals can not pay the price in the private sector, and no private school accepts to admit them. Thus they are obliged to choose the public sector $W_{ir}^* = 0 < W_{iu}$.

Second Case:

For very high incomes, individuals can pay private school tuition. For very high abilities, individuals can obtain financial aide (scholarship) in a certain private school. We have $p_{ijr}^* > 0$ and small (or $p_{ijr}^* < 0$). As a consequence we have

$$(1-t)y_i = c_{iu}^* > c_{ir}^* = (1-t)y_i - p_{ijr}^* \text{ but the difference is small.}$$

$$\text{Or } ((1-t)y_i = c_{iu}^* \leq c_{ir}^* = (1-t)y_i - p_{ijr}^*)$$

$$c_{ir}^* = (1-t)y_i - p_{ijr}^* > 0 \text{ since income is high.}$$

Since $\lim_{c \rightarrow \infty} (c, q) = 0$. Individuals always prefer increased quality even if consumption in the private sector is weakly reduced. $W_{ir} > W_{iu}$. Individuals choose the private sector.

We have seen that for low levels of income and ability, the public sector is preferred; conversely for high levels of income and ability the private sector is preferred. The continuity of $f(b,y)$ implies that when ability and income levels increase (going from one extreme to the other) it is clear that there exists a point for which preferences will be reversed. This equilibrium is determined by a couple (\hat{b}, \hat{y}) for which individuals are indifferent between the two sectors.

Proof of Lemma 1:

(i) and (ii) are the outcome of proposition 1. (\hat{b}, \hat{y}) is the indifference point between the two sectors. For individuals with $y_i < \hat{y}$ and $b_i < \hat{b}$ the public sector is preferred. For individuals with $y_i > \hat{y}$ and $b_i > \hat{b}$ the private sector is preferred.

For (iii) and (iiii):

For sufficiently high ability $b_i > \hat{b}$ and low income $y_i < \hat{y}$. And for sufficiently high income $y_i > \hat{y}$ and low ability $b_i < \hat{b}$.

We have two cases: for the first $p_{ijr}^* > 0$ and for the second $p_{ijr}^* < 0$.

For the first case: $(1-t)y_i = c_{iu}^* > c_{ir}^* = (1-t)y_i - p_{ijr}^*$.

A necessary condition for admission in the private sector will be:

$$c_{ir}^* = (1-t)y_i - p_{ijr}^* > 0 \text{ in other terms } y_i \geq \frac{p_{ijr}^*}{(1-t)}.$$

$\lim_{c \rightarrow \infty} (c, q) = 0$ implies that this student prefers higher school quality over higher

consumption, thus he chooses the private sector. $W_{ir} > W_{iu}$.

For the second case: $(1-t)y_i = c_{iu}^* \leq c_{ir}^* = (1-t)y_i - p_{ijr}^*$

The student is admitted in the private sector without any condition. $W_{ir} > W_{iu}$ since school quality and consumption are higher in the private sector.

Proof of Lemma 2:

(i) For $N \in (0, P)$, the couple (\hat{b}, \hat{y}) is decreasing in N. An increase of N represents a congestion effect of public schools; thus public expenditure per pupil $\frac{tY}{N}$ decreases. As a consequence public school quality decreases. Students on the margin between sectors will opt out of the public sector to join the private one. In this case (\hat{b}, \hat{y}) decreases.

(ii) For $Y \in \mathfrak{R}^+$, the couple (\hat{b}, \hat{y}) is increasing in Y. An increase of Y implies a growth of public school quality following a growth of public expenditure per pupil $\frac{tY}{N}$. Students on the margin between sectors will opt out of the private sector to join the public one. In this case (\hat{b}, \hat{y}) increases.

(iii) For $t \in [0,1]$, the couple (\hat{b}, \hat{y}) is increasing in t. On one hand, an increase of t implies a decrease of consumption in both sectors, thus the effect of t on consumption as a factor of individual utility is neutral. On the other hand an increase of t induces a growth of public expenditure per pupil. As a consequence public school quality increases. Students on the margin between sectors will opt out of the private sector to join the public one. In this case (\hat{b}, \hat{y}) increases.

Proof of proposition 2:

In Lemma 2 we have presented the fact that public school quality is decreasing in N. Conversely, private school quality is not dependent on N¹⁶. Indifference is given by :

$$U_{iu} [(1-t)y_i, q_{ju}] = U_{ir}^* [(1-t)y_i - p_{ir}^*, q_{jr}^*]$$

The left hand side of the equation is decreasing in N; while the right hand side of the equation is independent of N. For N very low, public school quality is very high. For N very high, public school quality is very low.

¹⁶ Each school chooses its size according to its profit maximization problem.

While passing from one extreme to the other. The intersection between the decreasing and the steady function determines the unique equilibrium number of students N^* .

Proof of Lemma 3:

We consider two individuals, the first has a couple (b_1, y_1) and the second has a couple (b_2, y_2) . We consider that $(b_1, y_1) > (b_2, y_2) > (\hat{b}, \hat{y})$.

We suppose that $\hat{t}_1(b_1, y_1) < \hat{t}_2(b_2, y_2)$

For the individual with (b_1, y_1) , we have:

$$W_{1u}[\hat{t}_1(b_1, y_1), y_1, Y, N, \theta_{ju}, \sigma_{ju}^2, b_1] = W_{1r}[\hat{t}_1(b_1, y_1), y_1, \theta_{jr}, \sigma_{jr}^2, b_1] \quad (a)$$

For the individual with (b_2, y_2) , we have:

$$W_{2u}[\hat{t}_2(b_2, y_2), y_2, Y, N, \theta_{ju}, \sigma_{ju}^2, b_2] = W_{2r}[\hat{t}_2(b_2, y_2), y_2, \theta_{jr}, \sigma_{jr}^2, b_2]$$

According to figure 1 and the explanation of individual behavior, for each $t < \hat{t}_2(b_2, y_2)$, we have:

$$W_{2u}[t, y_2, Y, N, \theta_{ju}, \sigma_{ju}^2, b_2] < W_{2r}[t, y_2, \theta_{jr}, \sigma_{jr}^2, b_2]$$

As we have $\hat{t}_1(b_1, y_1) < \hat{t}_2(b_2, y_2)$, we can write:

$$W_{2u}[\hat{t}_1(b_1, y_1), y_2, Y, N, \theta_{ju}, \sigma_{ju}^2, b_2] < W_{2r}[\hat{t}_1(b_1, y_1), y_2, \theta_{jr}, \sigma_{jr}^2, b_2] \quad (b)$$

As we have $(b_1, y_1) > (b_2, y_2)$, we can write:

$$W_{2r}[\hat{t}_1(b_1, y_1), y_2, \theta_{jr}, \sigma_{jr}^2, b_2] < W_{1r}[\hat{t}_1(b_1, y_1), y_1, \theta_{jr}, \sigma_{jr}^2, b_1] \quad (c)$$

From (a), (b), and (c) we can write:

$$W_{2u}[\hat{t}_1(b_1, y_1), y_2, Y, N, \theta_{ju}, \sigma_{ju}^2, b_2] < W_{1u}[\hat{t}_1(b_1, y_1), y_1, Y, N, \theta_{ju}, \sigma_{ju}^2, b_1] \quad (d)$$

From assumption 1 and 2 we have for $(b_1, y_1) > (b_2, y_2)$ with $(b_1, y_1) > (b_2, y_2) > (\hat{b}, \hat{y})$:

$W_{2u}[\hat{t}_1(b_1, y_1), y_2, Y, N, \theta_{ju}, \sigma_{ju}^2, b_2] > W_{1u}[\hat{t}_1(b_1, y_1), y_1, Y, N, \theta_{ju}, \sigma_{ju}^2, b_1]$. This is a contradiction with (d).

In this case, our supposition that $\hat{t}_1(b_1, y_1) < \hat{t}_2(b_2, y_2)$ is wrong. This implies that the reverse is true $\hat{t}_1(b_1, y_1) > \hat{t}_2(b_2, y_2)$. As a consequence the indifference tax rate $\hat{t}_i(b_i, y_i)$ is increasing in the couple (b_i, y_i) over $(\hat{b}, b_{\max}) \times (\hat{y}, y_{\max})$.

The case where $(b_1, y_1) < (\hat{b}, \hat{y})$ or $(b_2, y_2) < (\hat{b}, \hat{y})$, does not have any influence, individuals are enrolled in the public sector and $W_r = 0$.

Proof of Lemma 4: For $t \in [t_{mu}(b_m, y_m), 1] \Rightarrow t > t_{mu}(b_m, y_m)$.

We consider an individual 1, he has $(b_1, y_1) > (b_m, y_m)$.

The first condition of the existence of a majority voting equilibrium determined by the individual with the median couple of income and ability implies:

$(b_1, y_1) > (b_m, y_m) \Rightarrow t_{mu}(b_m, y_m) > t_{lu}(b_1, y_1)$. We have two situations to analyze:

1- For an individual with (b_1, y_1) , we suppose that $\hat{t}_1(b_1, y_1) < t_{lu}(b_1, y_1)$

From figure 1, this individual will be in the public sector for $t \in [t_{mu}(b_m, y_m), 1]$. We can

write: $\hat{t}_1(b_1, y_1) < t_{lu}(b_1, y_1) < t_{mu}(b_m, y_m) < t \in [t_{mu}(b_m, y_m), 1]$

As public sector's utility is decreasing in t over the interval $[t_{lu}(b_1, y_1), 1]$ and thus

over $[t_{mu}(b_m, y_m), 1]$. The individual will prefer $t_{mu}(b_m, y_m)$ to any other tax rate

$t > [t_{mu}(b_m, y_m), 1]$ since $[t_{mu}(b_m, y_m), 1]$ is the lowest tax rate.

In this case, we have: $W_{lu}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1] > W_{lu}[t, y_1, Y, N, \theta_{ju}, \sigma_{ju}^2, b_1]$

2- For an individual with (b_1, y_1) we suppose that $\hat{t}_1(b_1, y_1) > t_{lu}(b_1, y_1)$.

From figure 1, this individual will be in the private sector.

As private sector's utility is decreasing in t . The individual will prefer the lowest tax rate over the interval, thus he chooses $t_{mu}(b_m, y_m)$.

In this case, we have: $W_{lr}[t_{mu}(b_m, y_m), y_1, \theta_{jr}, \sigma_{jr}^2, b_1] < W_{lr}[t, y_1, \theta_{jr}, \sigma_{jr}^2, b_1]$

Over the interval $[t_{mu}(b_m, y_m), 1]$, there does not exist a tax rate different from

$t_{mu}(b_m, y_m)$ which represents a majority voting equilibrium.

Proof of Lemma 5: For $t \in [\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)]$, we consider an individual, he has

$$(b_1, y_1) < (b_m, y_m).$$

$$t \in [\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)] \Rightarrow t < t_{mu}(b_m, y_m)$$

As public sector's utility is increasing in t over this interval. The individual will choose the highest tax rate over this interval, thus he chooses $t_{mu}(b_m, y_m)$.

$$\text{In this case, we have: } W_{lu}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1] > W_{lu}[t, y_1, Y, N, \theta_{ju}, \sigma_{ju}^2, b_1]$$

Over the interval $[\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)]$, there does not exist a tax rate different from $t_{mu}(b_m, y_m)$ which represents a majority voting equilibrium.

Proof of Lemma 6:

(i) For $(b_1, y_1) < (b_m, y_m)$ we can write:

$$W_{mu}[t_m(b_m, y_m), y_m, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_m] < W_{lu}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1]$$

With $(\hat{b}, \hat{y}) < (b_1, y_1) < (b_m, y_m)$. (See assumption 1 and 2).

If $W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m] < W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_m]$ is true.

And since $W_{lr}[0, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] < W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m]$ with $(b_1, y_1) < (b_m, y_m)$.

We can write:

$$W_{lr}[0, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] < W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m] < W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_m] \\ < W_{lu}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1]$$

Thus: $W_{lr}[0, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] < W_{lu}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1]$.

(ii) For $(b_1, y_1) > (b_m, y_m)$ we can write:

$$W_{mu}[t_m(b_m, y_m), y_m, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_m] > W_{lu}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1]$$

With $(\hat{b}, \hat{y}) < (b_m, y_m) < (b_1, y_1)$. (See assumption 1 and 2).

If $W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m] > W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_m]$ is true.

And since $W_{lr}[0, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] > W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m]$ as $(b_1, y_1) > (b_m, y_m)$.

We can write:

$$W_{1r}[0, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] > W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m] > W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_m] \\ > W_{lu}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1]$$

$$\text{Thus: } W_{1r}[0, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] > W_{lu}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1].$$

The case where $(\hat{b}, \hat{y}) > (b_1, y_1)$ and $(\hat{b}, \hat{y}) > (b_m, y_m)$, has no influence since individual 1 and individual m have no choice between the two sectors. They are obliged to enroll in public schools.

Proof of proposition 3:

In this proposition, we have to demonstrate that over the interval $[0, \hat{t}_m(b_m, y_m)]$, there does not exist a tax rate which is preferred to $t_{mu}(b_m, y_m)$ by more than 50% of the population. For an individual with $(b_1, y_1) < (b_m, y_m)$ we have two situations to analyze. The interval is divided into two $[0, \hat{t}_1(b_1, y_1)]$ and $[\hat{t}_1(b_1, y_1), \hat{t}_m(b_m, y_m)]$.

1- For $t \in [0, \hat{t}_1(b_1, y_1)] \Rightarrow t < t_{mu}(b_m, y_m)$.

$$W_{1r}[t, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] < W_{1r}[0, y_1, \theta_{jr}, \sigma_{jr}^2, b_1]. \text{ (As } W_{1r} \text{ is decreasing in } t).$$

$$\text{And since } W_{1r}[0, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] < W_{lu}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1].$$

(From the first part of Lemma 6 with:

$$W_{mr}[0, y_m, \theta_{jr}, \sigma_{jr}^2, b_m] < W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_m] \text{ true}).$$

$$\text{We can write: } W_{1r}[t, y_1, \theta_{jr}, \sigma_{jr}^2, b_1] < W_{lu}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1]$$

Thus the tax rate $t_{mu}(b_m, y_m)$ is preferred to any other tax rate over the interval $[0, \hat{t}_1(b_1, y_1)]$.

2- For $t \in [\hat{t}_1(b_1, y_1), \hat{t}_m(b_m, y_m)] \Rightarrow t < t_{mu}(b_m, y_m)$.

Over this interval, public sector's utility is increasing in t. The individual chooses the highest tax rate $t_{mu}(b_m, y_m)$.

$$W_{lu}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1] > W_{lu}[t, y_1, Y, N_m, \theta_{ju}, \sigma_{ju}^2, b_1]$$

In conclusion we find that over the three intervals $[0, \hat{t}_m(b_m, y_m)]$, $[\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)]$ and $[t_{mu}(b_m, y_m), 1]$. There does not exist any tax rate that is preferred to $t_{mu}(b_m, y_m)$ by more than 50% of the population. Thus the couple (t_m, N_m) is a majority voting equilibrium.

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