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An experience with *Mathematica* in the field of economics of the environment: DIAM.

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Abstract

Climate change is one of the biggest challenges facing humankind for the next century. We expose how we build a Model about the Dynamics of Inertia and Adaptability in energy systems (DIAM) to study the question "Shall we wait another decade before taking costly measures to curb greenhouse gases emissions". Using primarily *Mathematica*, we had to use also the *GAMS* language, not only to be understood by other researchers in our field, but also to gain access to a powerful non linear constrained optimisation solver. It seems urgent to bridge the gap between these kind of solvers and *Mathematica* because when one builds models of complex systems, intertemporal optimisation is often preferred to recursive simulation.

1 Introduction to the issue of climate change

The Intergovernmental Panel on Climate Change[1] introduces the issue of climate change as follows : "During the past few decades, two important factors regarding the relationship between humans and the Earth's climate have become apparent. First, human activities, including the burning of fossil fuels, land use change and agriculture, are increasing the atmospheric concentration of greenhouse gases (which tend to warm the atmosphere) and, in some regions, aerosols (microscopic airborne particles, which tend to cool the atmosphere). These changes in greenhouse gases and aerosols, taken together, are projected to change regional and global climate and climate-related parameters such as temperature, precipitation, soil moisture and sea level. Second, some human communities have become more vulnerable to hazards such as storms, floods and droughts as a result of increasing population density in sensitive areas such as river basins and coastal plains. Potentially serious changes have been identified, including an increase in some regions in the incidence of extreme high temperature events, floods and droughts, with resultant consequences for fires, pest outbreaks, and ecosystem composition, structure and functioning, including primary productivity."

To control climate change, the ultimate objective of the UN Framework Convention on Climate Change, signed in 1992 in Rio, is expressed in Article 2: "... stabilisation of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system. Such a level should be achieved within a timeframe sufficient to allow ecosystems to adapt naturally to climate change, to ensure that food production is not threatened and to enable economic development to proceed in a sustainable manner."

Stabilisation of greenhouse gas concentrations implies switching from the present growing emissions trend to a decreasing emissions path. That raises many complex issues that can't be adressed all at the same time. In what follows, we neglected the multiplicity of countries and of greenhouse gases, to focus on the question of the timing of abatement: "How fast should we reform the energy systems to use less fossil fuels ?"

There are reasons to think that it would be cheaper to wait a decade or two before implementing energy-saving policies comparable to the ones many countries led in the late 70's. First, technical progress implies that in the future, reforming the energy system will be easier. Second, discounting makes the present value of any cost incurred less if it is deferred. Third and finally, the existing installed physical capital stock implies that any reform should start very slowly: there is inertia in energy systems. But because the climate system is non linear, we expect surprises regarding the magnitude of climate damage. So the choice involves balancing the economic risks of rapid abatement now (that premature capital stock retirement will later be proven unnecessary), against the corresponding risks of delay (that more rapid reduction will then be required, necessitating premature retirement of future capital).

The balance of the risks described above is above all an empirical question, so it makes sense to try to represent the discussion with a numerical model. The following section describes our experience with *Mathematica* and such a model we called DIAM (acronym standing for a Model on the Dynamics of Inertia and Adaptability in energy systems). The purpose of DIAM lies not in its predictive power. We build it to serve as a guide for thought: we wanted to highlight the role of inertia and adaptability in energy systems. This illustrates the difference between optimisation models and simulation models, a point we will address in the third section of this paper.

2 A model of optimal global pollution control

First, rather that a technical explanation for economists of the environment, will try to give an intuition on what DIAM does. Then we will expose our personal journey through the successive versions of the model, a path that led us to complement *Mathematica* with another programming language: *GAMS*.

DIAM, fully defined , computes the emissions reduction path $(\boldsymbol{x}(\boldsymbol{t}))_{1990\boldsymbol{t}2140}$, that minimises the discounted sum of the total cost associated to the greenhouse issue. It has five parameters, an optional parameter and an endogenous variable. The optional parameter **refEmis(*t*)** represents the CO_2 emissions in a scenario without intervention to reduce greenhouse gases emissions: they are projected to grow at a rate of 1% a year from the 1990 figure of 7.4 Gigatons. The endogenous variable is ***x***, its interpretation is that CO_2 emissions at date ***t*** are given by (1 - ***x*(*t*)**) **refEmis(*t*)**. Parameters have different economic interpretations related to discounting (**discount**), technical progress (**techProg**), the cost of reducing CO_2 emissions (**a** and **b**), and the damage of climate change (**c**).

The results curves represent ***x*(*t*)** for the next half of a century. The larger is **x**, the tighter is the control on greenhouse gases emissions. The thick continuous line curve represents the reference case. For that case, the optimal path reduces CO_2 emissions by about 15% in 2020. Let us see the sensitivity of the results to discounting, technical progress and climate damage. The dashed line with long dashes represents the 5% discount rate case. High discounting decrease the importance of the future and therefore of climate damage. It is therefore natural to find a curve below the central case. It is also sensible to find that when climate damages c are doubled, optimal reduction increases significantly (thin continuous line). With a larger rate of technical progress, the costs of reducing emissions decreases over time, and therefore the optimal reduction progressively increases (dashed line, short dashes). For more details, the discussion about inertia and adaptability, and how the values in were defined, see Grubb & al.[2] and Ha-Duong & al.[3].

First, we solved the model analytically using variational calculus. We appreciated much the adaptability of *Mathematica*: In the beginning, it let us define optimisation problem with many simple commands. In the end, using the appropriate package led us to the very concise formulation of . This way, *Mathematica* helped us to understand the books about optimisation theory and about differential equations. Regarding *Mathematica* as an integrated modelling environment, we found it easier to use than spreadsheets when it came to draw plots that compared the results of several model runs.

Second, we solved numerically. Although the model did satisfy us in representing the discussion about inertia, adaptability and technical progress, when we submitted for publication the referees argued that we had to justify our choice of an analytically "exact" solution over a numerical solution. Surprised at first, we soon recognised that numeric optimisation has real advantages. Being much less restricted in the shape of functions in the model allows to relax many unnecessary hypothesis. We used *Mathematica* with the goal to integrate everything, from the numeric values to the graphics, in one file. Unfortunately, we couldn't achieve that goal. We missed most in *Mathematica 2.2* a word processor that could handle mathematical writing correctly. Writing the paper and the code separately implies time consuming and tedious verifications for version errors.

Third, uncertainty was introduced. By the time we completed the numeric version of DIAM 1.0, Professor Alan Manne, from the Stanford Optimisation Laboratory, was leading a study on climate change and optimal strategies under uncertainty. Future decisions will adapt to the new findings of climate science. To represent this in models, one have to consider sequential decision making. Mathematically, that leads to the theory of stochastic dynamic programming. Pr. Manne he was able to quickly rewrite our model in that framework. He used the General Algebraic Modelling System, described by Brooke & al.[4], known as the *GAMS* computer language. That high level language is specialised, efficient and allows to specify optimisation programs in a form easily readable by most human economists.

Fourth, we rewrote completely DIAM with *Mathematica*, as a cost-constraint analysis with uncertainty. But, to a certain extend, *GAMS* works more like magic than like mathematics. There are no garantee that the optimum found is global, nor that it is unique, nor that it will be found. Being not specialists in the solver we used, we had no idea on how the computer found its solution. A close search on the internet on optimisation and *Mathematica* yield us a few resources on optimisation, noticeably Varian[5] and Culioli[6] , but there seems to be no ready-made package for numeric stochastic dynamic programming. Rewriting the model forced us to a more rigorous mathematical analysis of the model. It also enabled some minor improvements like variable width time steps and convolutions, delicate to code in *GAMS*.

Fifth, we made cost-benefit analysis with uncertainty and non linearity. In the preceding version, we assumed that in year 2020 a CO_2 concentration ceiling will be enforced. The uncertainty was that we don't know at what level the ceiling will be set. But the idea of a concentration ceiling is questionable. It is unrealistic and economically irrational to impose a constraint to be respected at all cost. For example, we could realise that the safe CO_2 concentration is 450 parts per million in volume (ppmv), as the preindustrial level was about 280 ppmv. Present CO_2 concentration is 360 ppmv. It will increase more as many countries industrialise. So if we decide to stabilise CO_2 concentration at 450 ppmv, it is possible to imagine a temporary overshoot over that level.

To represent a damage from climate change suddenly increasing when the CO_2 concentration goes above 450 ppmv, it is necessary to use a function that is very non linear. Changing the *GAMS* code to replace a hard constraint by a non linear penalty function was easy and did not increased run time much. On the other hand, our *Mathematica* code was already rather slow, in spite of the fact that the Euler equations solved were linear. We judged that to include the non linearity in the *Mathematica* version would not be worth the effort.

The five steps summarised above took us three years, making the PhD. work of one of the authors. We are still discovering interesting new aspects of *Mathematica*, but we would use it more often if it was better at word processing and at numeric optimisation. In the next section, we would like to expose why, in our minds, intertemporal optimisation is a key direction for the future of computer modelling environments.

3 Broadening the discussion: optimisation and simulation

Figure : The god Janus, looking at the past and the future at the same time.

We examined the question of the timing of action against global warming by building a model drawing both from economics and climatology. The building of models linking human activities to the evolution of the terrestrial environment, a research field called 'Integrated assessment', seems to be caught between two traditions: the tradition of natural sciences, especially physics, established for more than three centuries and the traditions of social sciences which are only around 50 years old.

The first tradition describes physical phenomena that can be qualified as « pushed by the past ». These are treated in the framework of differential equations or finite difference equations. They are accessible through models where the flow of the calculation follows the « natural time »: Given the state of the system at date 0, state at date 1 is computed first, then date 2 is examined..., recursively up to date T. In the second tradition, the phenomena are « driven by the future ». In economics and human affairs, there is often co-ordination (like general equilibrium), the existence of anticipations or a goal. Such phenomena can be best studied in the context of intertemporal optimisation where an optimal path is computed globally, taking into account initial and final conditions, as well as limited resources conditions. From a computational point of view, the solution is sought using the vector of states **V** = (S_1, S_2, .. , S_T). The number of generic equations can be small, but multiplied by the number of time periods, probabilistic outcomes, agents, production technologies and economic goods, they can generate a large, non linear, optimisation problem.

The question of the magnitude of global warming illustrates a typical physical phenomena "pushed by the past". The question of building international co-operation to reduce polluting emissions is "driven by the future", because anticipations play the main role in it. Models about economic responses to the issue of global warming needs to integrate human activities (demography, growth, the energy sector, agriculture, forestry ...) with the evolution of the terrestrial environment (the climate, the carbon cycle...). By essence, this is at the confluence of the two traditions defined above, as illustrates. Yet today, two types of models are build in our field:

policy analysis models, recursively calculated and usually quite detailed, as the IMAGE model by Rotmans[7].

policy optimisation models, intertemporally calculated and usually rather compact, as DIAM; DICE (Nordhaus[8]); or MERGE (Manne & Richels[9]).

Policy analysis simulation models raise problems when it comes to analysis with scarce resources (like fossil fuel resources) or environmental constraints (like a ceiling on CO_2 concentration). They tend to lead to « overshoot and collapse », and to have many ad hoc parameters. For these reasons, a lot of analysts prefer intertemporal models. This implies a modelling environment that is adequate to solve large optimisation programs.

The *GAMS* language is often chosen, as it is a user friendly interface to powerful solvers. There are many different solvers, each adapted to different kind of optimisation problem : linear programming, non linear programming with *MINOS*, integer programming with *ZOOM*, and so on. But this choice is done at the expense of the commodity of a higher symbolic expression of the model, as possible in *Mathematica*.

For the time being the gap between large optimisation program solvers and the symbolic mathematics is a big obstacle to the penetration of *Mathematica* in the fields of general equilibrium economics and integrated assessment of the environmental issues. If *Mathematica* wizards and solvers specialists got together, we have no doubt that they could write a package to give access to the power of, for example, *MINOS*, from within *Mathematica*.

Bridging the gap would be a great help in our everyday research. It would also facilitate a better study of the mathematical structure of models (in the spirit of analysis, control theory, theory of variations etc...). More generally, such an evolution would allow to bring closer policy analysis simulation models and policy optimisation intertemporal models. Ideally in fact, notwithstanding calculation limits of computers, the two types of models might converge as the equations are mainly the same, the main difference being more in the way the solution is sought.

4 Conclusion : the usefulness of powerful optimisation tools

In this paper, we exposed our personal experience with DIAM, an integrated assessment model about climate change. Using primarily *Mathematica*, we had to use also the *GAMS* language, not only to be understood by other researchers in our field, but also to gain easy access to powerful, non linear constrained optimisation solvers. That point shows that we still need a link, a bridge, between *Mathematica* and these solvers. This need arises because we model complex systems, where the future, not only the past, matters. Can *Mathematica* be the universal modelling environment we all want without including facilities for intertemporal optimisation ?

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Figure : DIAM without surprises and nonlinearities, definition

```
Needs["Calculus`VariationalMethods`"]
diam[discount_, techProg_, a_, b_, c_,
   refEmis_:(7.4 E^(0.01 #)&) ] :=
DSolve[
 {ExpandAll[
  VariationalD[ E^(-discount t) *
   (E^(-techProg t) refEmis[t] / refEmis[0] *
     (a x[t]^2 + b x'[t]^2) -
    c refEmis[t] x[t]
   ), x[t], t
  ] * E^((discount+techProg) t) / refEmis[t]
 ]      == 0,
 x [0]   == 0,
 x'[150] == 0
 }, x[t], t
] [[1,1,2]] // Simplify
```

Figure : Central parameters set (a) and sensitivity analysis (b-e)

```
a[t_] = diam[0.03, 0.01, 1.31, 1570, 0.124];
b[t_] = diam[0.05, 0.01, 1.31, 1570, 0.124];
c[t_] = diam[0.03, 0.02, 1.31, 1570, 0.124];
d[t_] = diam[0.03, 0.01, 1.87,  561, 0.124];
e[t_] = diam[0.03, 0.01, 1.31, 1570, 0.248];
```

Figure : Plot of the results (SetOptions for plot below are not shown.)

```
Plot[{a[t],b[t],c[t],d[t],e[t]}, {t, 0, 50}];
```