Appendix on some elements of Indian Astronomy

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Chapter 1

Appendix: Some elements of Indian Astronomy

1.1 Generalities

The sky is considered as a sphere (*gola*) whose radius is 3438 minutes (*kalās*)[1], with the Earth at its center. Stars are fixed on the sphere, which is thus called *bhagola*, "sphere of the asterisms/stars". We will call it here the Celestial Sphere. Tradition states that the Earth does not move, and that the Celestial Sphere turns daily around the line going from the North pole (P) to the South pole (P') called the Celestial axis. Āryabhaṭa however considered that the Earth rotated from West to East, and therefore that the movement of the Celestial Sphere was only apparent. Because of the violent reactions such a statement provoked, later commentators changed the verse in order for it to mean exactly the contrary[2]. The planets, among which the Sun and the Moon, revolve in the space between the Earth and the Celestial Sphere. The Celestial Equator (*viṣuvat*) is defined as the great circle (i.e. a circle belonging to the sphere and having the earth for center) perpendicular to the Celestial axis.

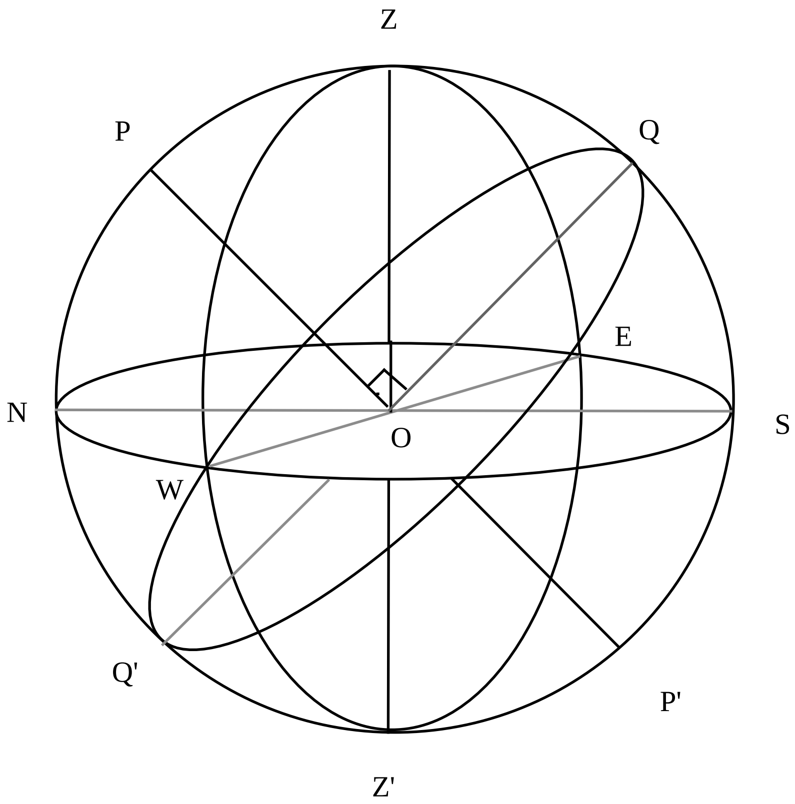
Let us imagine an observer (O) on Earth. Since the Earth and thus the point where the observer stands is very small compared to the radius of the Celestial Sphere, both are collected together. Apart from the Celestial axis and Equator, in the following representations, all the other planes and lines will be defined according to this observer.

The imaginary vertical line which through the observer's feet extends itself

[1]The reason why a circular measuring unit is used here remains mysterious to me.

[2]See for instance [Sharma-Shukla 1976; Intro, p.xxix; p. 8; p. 119-120], [Yano 1980], and [Bhattacharya 1991].

Figure 1.1: The Celestial Sphere
$NESW$ is the Horizon for the observer in O;
$ZPZ'P'$ is the Celestial Meridian;
$ZEZ'W$ is the prime vertical;
$WQEQ'$ is the Celestial Equator.



to two points on the surface of the sphere defines respectively the Zenith (Z, *nata*), which is the point above, and the Nadir (Z'), which is the point below. This is illustrated in Figure 1.1.

The great circle perpendicular to ZOZ' is called the Horizon. The plane it encloses is the plane of the observer. It intersects the Celestial Equator in two points called the East (E) and West (W).

The great circle which passes through the Zenith, nadir and the poles is called the Celestial Meridian for this observer. It intersects the Horizon at the North (N) and South (S).

The great circle perpendicular to the Celestial Meridian, passing through the Zenith and Nadir, and the the East (E) and the west (W) is called the prime vertical (*samamaṇḍala*).

1.2 Coordinates

The latitude of the observer, O, usually noted ϕ, is the angular distance between the Equator and the Zenith (the arc ZQ as illustrated in Figure 1.2.)

The distance of the pole to the Horizon (the arc PN) is called the altitude of the pole. Because the angles ZOQ and PON are equal, the altitude of the pole and the latitude of the observer are equal. The co-latitude is $90° - \phi$ (as the arc QS).

Let us now consider the orbit of the Sun.

The path of the Sun in the sky relatively to the stars, and to a fixed earth, when noted during a year, at a given time, in a given place, every day, draws an ellipse. This ellipse is in fact a mirror of the motion of the Earth around the Sun. The plane defined by this ellipse intersects the Celestial Sphere in a great circle called the Ecliptic (*apamaṇḍala*). The Ecliptic intersects the Celestial Equator in two points γ and Ω. The angle of the Sun with the Equator is constantly changing. In γ and Ω it is zero. The points where it is the greatest is called the obliquity of the Ecliptic (*paramāraprama*, lit. "greatest declination"). This is illustrated in Figure 1.3.

Today this angle, which is also that of the Ecliptic with the Equator, is roughly considered to be $23°7'$. γ is the point of the Equator through which the Sun is considered to move from the southern hemisphere to the northern hemisphere. It is called the vernal equinox. Ω is the point on the Equator through which the Sun is considered to move from the northern hemisphere to the southern hemisphere. It is called the autumnal equinox. The two points where the sun is at its greatest angular distance from the Celestial Equator are

Figure 1.2: Coordinates
ϕ is the latitude of the observer in O;
$90 - \phi$ is the co-latitude.

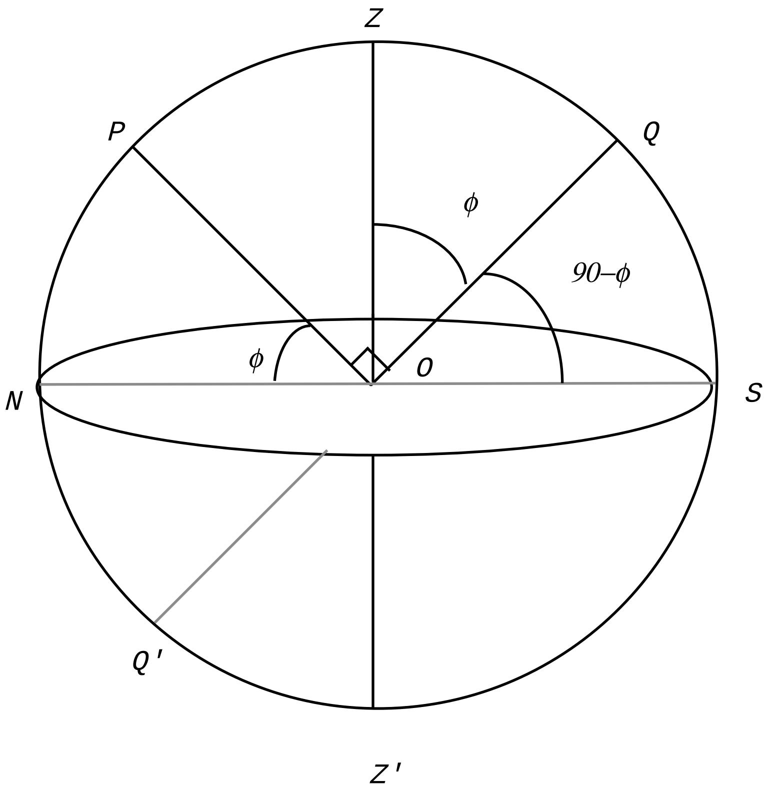


Figure 1.3: Apparent motion of the Sun in a year
γ is the vernal equinox;
Ω is the autumnal equinox;

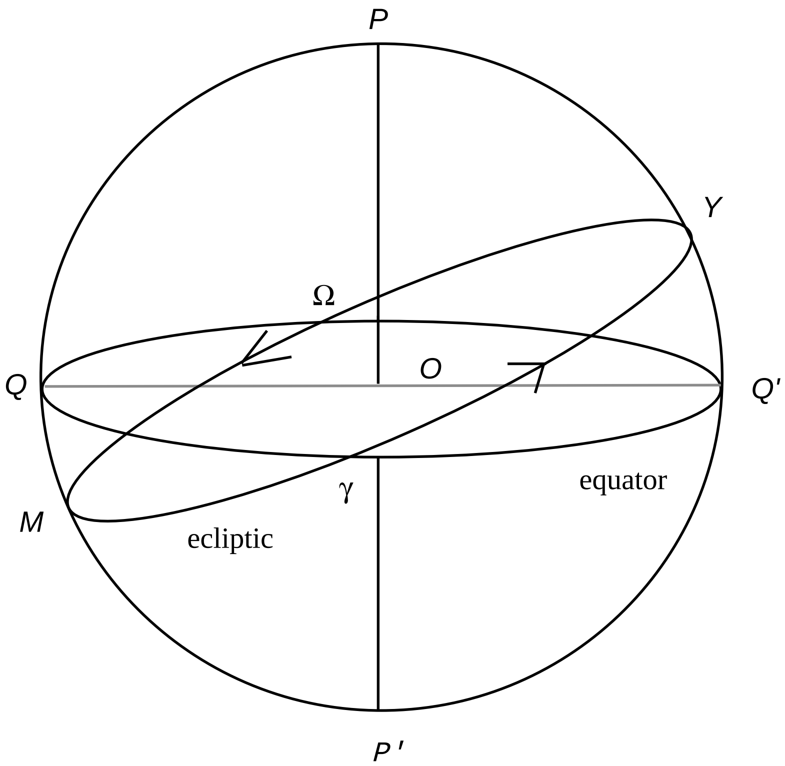
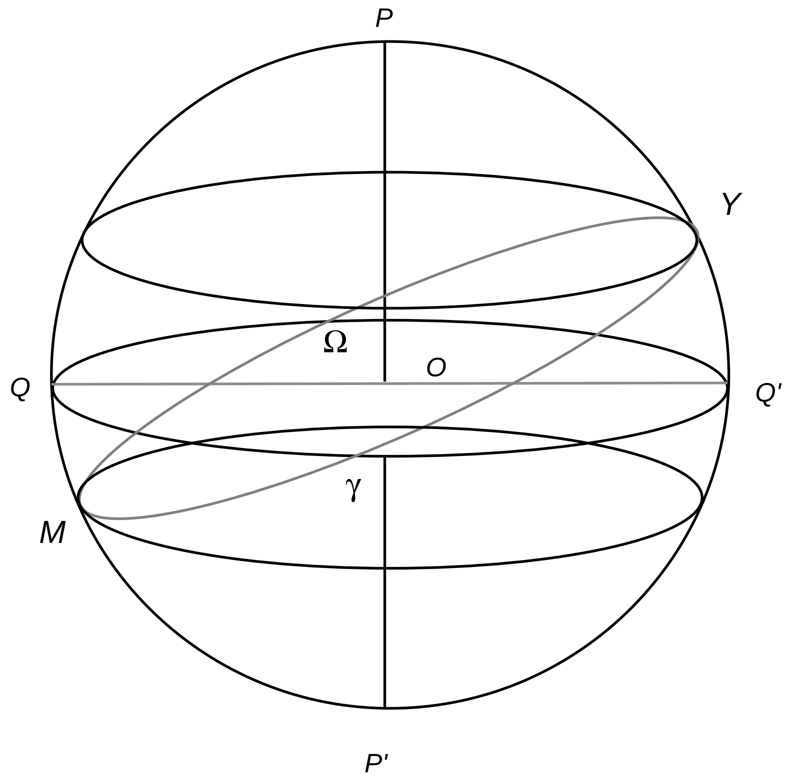


Figure 1.4: Daily and yearly apparent motions of the Sun



called the Summer (Y) and Winter (M) solstice.

The Ecliptic represents the yearly path of the Sun on the Celestial Sphere. Daily, however, the Sun is considered to have a motion parallel to that of the Equator, because of the rotation of the Celestial Sphere around the axis of the poles. In fact, if we would represent the daily motions of the Sun in a year, it would appear as a spiral made of roughly 365 spins parallel to the Equator. It would be a spiral because in 24 hours the Sun slightly moves along the Ecliptic. During the vernal and autumnal equinox the apparent motion of the Sun is on the Equator. The days are equal to the nights. The day of the winter solstice is the shortest of the year. The day of the summer solstice the day is the longest of the year. Whatever the day, at mid-day the Sun is on the Celestial Meridian. This is illustrated in Figure 1.4.

Let's take any day of the year, and consider the Sun at mid-day. As illustrated in Figure 1.5.

The straight line $SuSu'$ represents the orbit of the Sun. At mid-day the Sun is in Su. The angular distance between the Zenith and the Sun at Su (the

Figure 1.5: Daily motion of the Sun

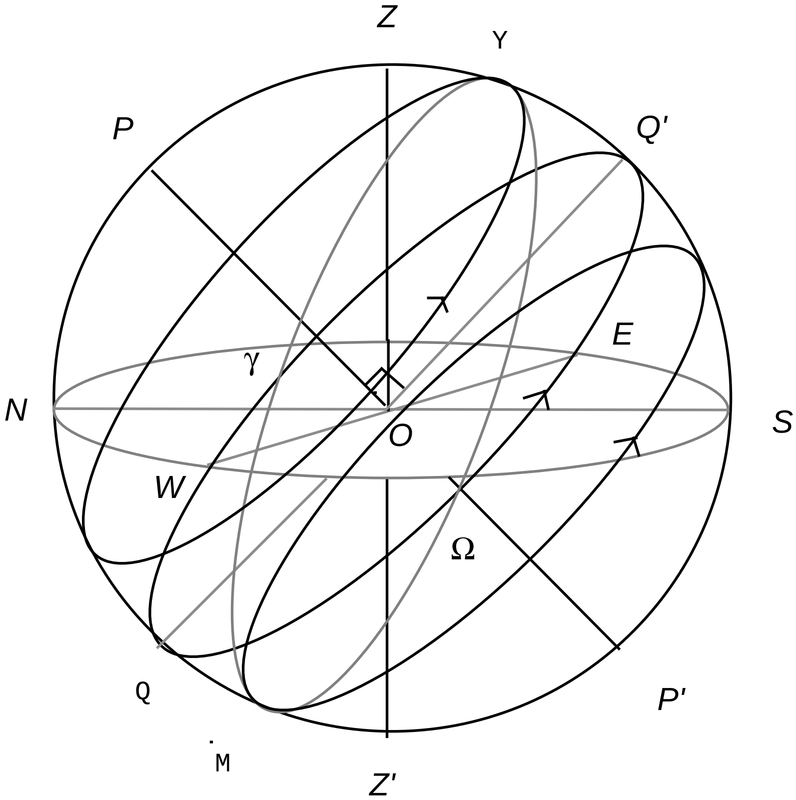
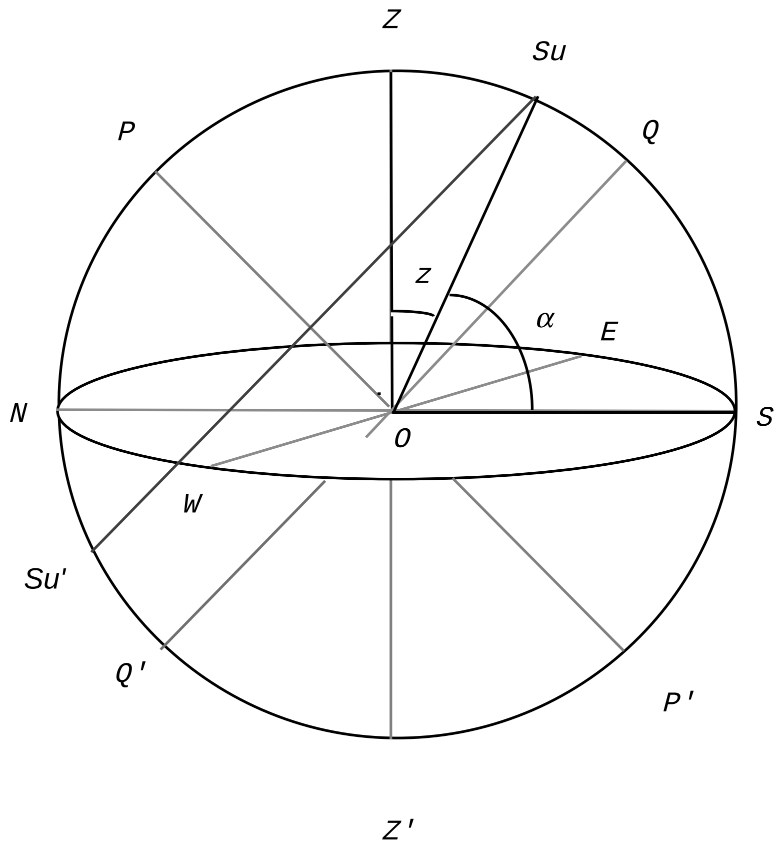


Figure 1.6: The Sun on an equinoctial day



arc ZSu) is called the Zenith distance of the Sun (z). The angular distance between the Horizon and the Sun at Su is the altitude of the Sun (a).

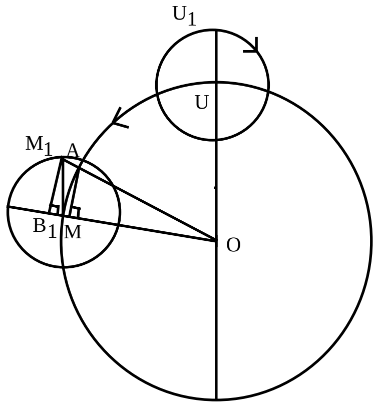
On an equinoctial day, the Sun is on the Celestial Equator. As illustrated in Figure 1.6. At mid-day the Sun is in Q. The Zenith distance of the Sun in Q is then the latitude (*akṣa*) of the observer. And its altitude becomes the colatitude (*avalambaka*) of the observer.

These concepts are used in Bhāskara's commentary, when studying the astronomical interpretation of the shadow cast by a gnomon, at mid-day (in BAB.2.14).

1.3 Movement of Planets

One aspect of the Hindu planetary theory bearing trace of a Hellenistic influence concerns the description of the apparent motion of planets. These are

Figure 1.7: Orbit of a planet



rendered through an epicycle theory: the problem then being the constant discrepancy between the mean motions and the true ones. We will expose very briefly here some elements of Bhāskara's epicyclic theory. For a more detailed analysis see the explanations given in Chapter IV of [Shukla 1960].

A planet G (*graha*) has a mean circular motion, along a great circle of the Celestial Sphere, the deferent, called in Bhāskara's commentary *vyāsārdha-maṇḍala* ("the circle ⟨of that⟩ semi-diameter"). Āryabhaṭa calls it *kakṣyāmaṇḍala* (Ab.3.18) "orbit's circle". Let O, the earth, be its center, and R, the radius of the celestial sphere, its radius. This is illustrated in figure 1.7.

However, at a specific time of a specific day, the tabulated position of G is considered to be on a second smaller circle, the epicycle (*pratimaṇḍala*), which revolves in a direction opposite to the revolution described by the deferent. Although the point on the epicycle, representing G at that time on that day is not yet the true position of G it is considered a first, better approximation of it.

Let U_1 be the apogee (*ucca*) of G. Bhāskara defines in BAB.3.4ab [Sharma-Shukla 1976; p.179, line22-23], the *ucca* as follows:

> *yatra grahāḥ sūkṣmā lakṣayante (Shukla's readings)/labhyante (Mss. reading) karṇasya mahattvāt sa ākāśapradeśa uccasaṃjñitaḥ*

That we can understand as follows:

> A spot in the sky where a planet is perceived to be small because of the greatness of the hypotenuse (*karṇa*) is called *ucca* (high).

The apogee is the apparent remotest point of G along its orbit. And U its mean position along its orbit. UU_1 serves as reference both for the radius of the epicycle at any time, and for the exact place on the epicycle where the tabulated position of G on the epicycle should be.

Let M be the mean position of G on its circular orbit on a given day at a given time. The arc UM represents the mean arcual distance of G to its apogee at that given time, and is called the *bhujā*. Let M_1 be an approximation of the true position of G when its mean position is in M. M_1 is such that $MM_1 = UU_1$. This defines the epicycle. In his commentary to Ab.2.26-27.ab. Bhāskara does not consider the epicycle itself, but the circle having for radius OM_1: *tatkālotpannakarṇaviṣkambhārdhamaṇḍala* (the circle which has for semi-diameter the hypotenuse produced at that time).

Let A be the point of OM_1 that intersects with the mean orbit of G. Let B be a point of (MO) such that AB is perpendicular to (MO). Let B_1 be a point of (MO) such that M_1B_1 is perpendicular to (MO). Both AB and M_1B_1 are called the *bhujāphala* (the correction of the *bhujā*). OA is the radius of the orbit (*vyāsārdha*) and OM_1 is called the hypotenuse (*karṇa*).

Bhāskara states in BAB.2.26-27.ab. that:

$$\frac{AB}{OA} = \frac{B_1M_1}{OM_1}.$$

And thus that AB is inversely proportionate to OM_1.

This section and the following, gives several supplementary remarks on the astronomical aspects of BAB.2.32-33.

1.4 Time cycles

Traditional hinduism considers time as cyclical: there are four ages, called *yugas*, at the end of which the universe is destroyed and reborn again. The four *yugas*, in which the conditions of life increasingly deteriorates are in due order: the *kṛtayuga*, the *tretāyuga*, the *dvāparayuga*, and the *kaliyuga* in which we presently live.

Ab.1.3-4 gives the numbers of revolutions of the sun, moon, earth etc. in a *yuga*, and the date of the beginning of the current *yuga*. Ab.3.5 defines solar years (*saṃvatsara*), lunar months and civil and sidereal days. A solar year is defined by the time taken by the Sun, apparently, to make a full rotation around the earth. The number of solar revolutions, which gives the number of years, in a *yuga* is stated to be 4 320 000.

Traditional astronomy also distinguishes between civil days (*bhūdivasa/dina*, lit. terrestrial days) and celestial ones (*nakṣatradivasa*). A celestial day, corresponds to one apparent rotation of the celestial sphere from East to West.

A civil day, corresponds to the daily apparent rotation of the sun around the earth: since the sun every day slides slightly on the ecliptic there is a discrepancy between celestial and civil days.

The civil days are defined in Ab.3.5: "The conjunctions of the Sun and the Earth are (civil) days"[3]. The computation of the number of conjunctions in a *yuga* is defined in Ab.3.3ab: "The difference between the revolution-numbers of any two planets is the number of conjunctions of those planets in a *yuga*." [4] The "revolution-number" (*bhagaṇa*) of a planet is the number of revolutions of a planet in a *yuga*: these are constant and given in Ab.3-4. The number of terrestrial revolutions in a *yuga* is given by Āryabhaṭa in Ab.1.3: 1582237500. So that the number of civil days in a *yuga* (A_y[5]) is equal to the number of revolutions of the Sun in a *yuga* minus the number of revolutions of the earth in a *yuga*: $1582237500 - 4320000 = 1577917500$. Therefore $A_y = 1577917500$.

This value is important when evaluating the number of days elapsed in the *Kaliyuga*, when the longitude of a given planet is known. This is one of the astronomical problems solved by a pulverizer computation, as described by Bhāskara in BAB.2.32-33.

1.5 Orbits and non integral residues of revolutions

The mean orbit (*kakṣyā*) of a planet, as we have seen above, is considered to be a circle (*kakṣyāvṛtta*). It represents tha apparent motion of a planet, around the earth, on the Celestial sphere. One movement of the planet along its orbit is called a revolution (*maṇḍala*). A revolution is divided into twelve equal signs (*rāśi*). A revolution is also divided into three hundred and sixty degrees (*bhāga*), so that there are thirty degrees per sign. A degree is divided into sixty minutes (*liptā*), a minute into sixty seconds (*vikalā*)[6]. This is summed up in Table 1.1.

At the beginning and at the end of a *yuga*, all planets are in conjunction. It is assumed, that along their respective orbits, all the planets cross the same distance in a *yuga*. This is stated in Ab. 3. 12. (op. cit. p. 100). The distance described by any planet in a *yuga* gives the "circumference of the sky"[7]. In

[3] [Sharma-Shukla 1976; p. 91].
[4] op. cit., p.86.
[5] This corresponds to the notations we have adopted in our supplement for BAB.2.32-33
[6] These subdivisions, of course, recover those that divide a circle in mathematics. See the Section of the Glossary on Time Units.
[7] op. cit., p. 14

Table 1.1: The different subdivisions of a Revolution

Sanskrit	English	Respective Amounts				
		Rev	Signs	Deg	Min	Seconds
maṇḍala	Revolution	1				
rāśi	Sign	12	1			
bhāga	Degree	360	30	1		
liptā	Minute	216000	300	60	1	
vikalā	Second	1296000	18000	3600	60	1

verse 6 of the *Gītikāpāda*, Āryabhaṭa gives the following rule (given here with the non-literal translation by K.S.Shukla and K.V. Sharma op. cit., p.13) to compute the length in *yojanas* of the orbit of any planet:

Ab.1.6.
khayugāṃśe grahajavo

The circumference of the sky divided by the revolutions of a planet in a *yuga* gives (the length of) the orbit on which the planet moves.

From this verse of the *Āryabhaṭīya* we also indirectly know that, the circumference of the sky in *yojanas* is: 12474720576000 *yojanas*. The orbit of the Moon, according to the value given in Ab.1.3 is

$$\frac{12474720576000}{57753336} = 216000 \textit{ yojanas}.$$

And the orbit of the Sun is

$$\frac{12474720576000}{4320000} = 2887666,8.$$

In the *Mahābhāskarīya*, the following verse gives a rule to find the mean longitude of a planet [8]:

Mbh.i.20
ambaroruparidhir vibhājito bhūdinair divasayojanāni taiḥ|
saṅguṇayya divasān athā haret kakṣyayā bhagaṇarāśayaḥ svayā||

Divide the (*yojanas* of the) circumference of the sky by the number of civil days (in a *yuga*): the result is the number of *yojanas* traversed (by a planet) per day. By those (*yojanas*) multiply the

[8] [Shukla 1960; Skt, p. 4; Eng, p.15]

> *ahargaṇa* and then divide (the product) by the length (in *yojanas*) of the own orbit of the planet. From that are obtained the revolutions, signs, etc. (of the mean longitude of the planet).

The *ahargaṇa*, is the number of days elapsed in the *Kaliyuga* at that time. If x is the *ahargaṇa*, since we know that the number of civil days in a yuga is 1577917500, then, for example, the mean longitude of the Sun (λ_S) is:

$$\lambda_S = \frac{12474720576000x}{1577917500 \times 2887666,8}.$$

We can recognize here the type of problem solved by pulverizer without remainder. Such problems are seen in Examples 24-26 of BAB.2.32-33. Note that there would be an obvious simplification here, that does not seem to be carried out in the resolution of these examples:

$$\lambda_S = \frac{12474720576000x}{1577917500} \times \frac{4320000}{12474720576000} = \frac{4320000x}{1577917500} = \frac{576x}{210389}$$