Combinatoric, mimetic and non mimetic aspects of creativity

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Abstract In the present paper, we try to demystify the notion of «creativity». Creativity is not mysterious : all the universe is creative (it is the spinozist «natura naturans»), and especially the living beings on Earth. Regarding men, who do not just create but think a lot about their creations, they have long understood that certain aspects of creativity are purely combinatorial : this is the case, in particular, in many technical creations and in a certain number of intellectual inventions. However, many of them - and the most beautiful, no doubt - escape, at least partially, this explanation. We try to report without pretending, nevertheless, always be able to lift the veil.

Key words. Creativity in nature, inventive stimulation, combinatorics, Lull, mathematical creativity, Ramanujan, Grothendieck.

1 Introduction

We usually define «creativity» - American neologism from the 1940s, but already used in 1868 by the French writer and art critic Antoine Mollière in his *Métaphysique de l'Art* - as the ability of an individual or a group to imagine, build and implement a new concept, a new object, or discover an original solution to a problem. Creativity appealing to the imagination, a psychological faculty actually quite mysterious, its presence in certain individuals (artists, scientists, strategists ...) often seems to be a miracle if not a kind of natural genius some of which would be provided, others not. We would like to demystify this view by showing how a number of adjuvants, including technical ones, have long been artificial supports for creative activity, certainly present in nature at all levels, but then carried by man to its highest power (from the first attemps of artificial creativity by Ramon Lull to the most sophisticated mathematical models of inventive stimulations by Moles or Kaufmann). Still, a number of creative approaches in Art - presumably the most spectacular ones, seem to escape this description. It is also the case in mathematics (see the surprising works of Ramanujan or Grothendieck). With some development, however, we think we can bring them back into the fold of a rational construction, even if, in these cases, combinations, variations and extrapolations are partially blind.

2 Is nature creative ?

Spinoza – the Dutch philosopher – was not a creationist, in the sense of God, for him, is only Nature. But then, taking up – in his *Ethics* (I, prop. XXIX, sch.)– the old concept of "natura naturans", he affirmed, in the 17th century, that nature was an immense productive machine, each attribute of which, *causa sui*, was expressed in a multiplicity of modes and affections inheriting its productive capacities. In the 19h century, the mathematician Hoëne-Wrónski gave an even more dynamic meaning to this statement. For Wrónski, the vocation of human reason is the fulfillment of a certain unique "law of creation", presiding over all the activities of the mind, so that "the different branches of our knowledge, which form respectively the various science and philosophy, all of which, as so many separate realities of the universe, must be identically constituted by this unique law of creation."([Wro 78], 20). But, as Philippe d'Arcy comments very well, the meaning of the Wonskian law of creation is not only that beings are created or formed according to a law which is their essence, but that "creation is for every thing a law, or a duty, that every being must create, that only are beings who can create, generate effects, consequences, and, in the case of man, acts by which he accomplishes and engenders himself (autogeny)" ([Arc 70], 5). No doubt, therefore, that creativity is inherent to Nature, even at the smaller levels of its evolution.

As Quantum Mechanics tells us today, any particle, which is also a wave, already explores the universe with a kind of freedom (see [Con 06], [Con 09]) and the entire universe has emerged from the quantum vacuum, as shown by the famous Wheeler-de Witt equation (see [Elb 92]) :

$$\bar{H}\Psi(a, \phi) = 0,$$

($\bar{H}$ being the hamiltonian, Ψ the wave function, a the scale factor, fulfilled by the scalar field ϕ.)

Among numerous other solutions, this equation admits the so-called semi-classical solutions of Hartle and Hawking :

$$\Psi_0(a, \phi) \sim -\exp(-S_E(a, \phi)),$$

where $S_E(a, \phi)$ is the euclidian action corresponding to the euclidian solutions of the Lagrange's equation for $a(\tau)$ and $\phi(\tau)$ with the initial conditions $a(0) = a$ and $\phi(0) = \phi$.

This gives a very strong probability for a creation – if not from nothing – at less from the quantum vacuum, because the process described above can be interpreted as a kind of "tunnel effect" from the vacuum, whose field is susceptible to fluctuations. The analogue of a "potential barrier" can, therefore, be crossed by a particle, and so, the inflationary process is on the way and everything flows from it : light, heavy elements (atoms, molecules), increasingly complex structures (stars, galaxies, clusters), even the planets and the Earth where, under the influence of a strong cosmic radiation, will be able to develop life.

Spontaneously, by self-ordering and self-organisation, or, maybe, cybernetic adaptation – a cornerstone of biological and technological evolution, as well as of artificial intelligence and cognition, chemical evolution and biological evolution may take place and the temporal survivability of information can be considered as a factor of general evolutionary fitness for all evolutionary adaptations. Because all experimentation spaces are finite these ones may become exhausted due to convergence towards optimal configurations, explaining perhaps, finally the observed decay of technological innovation and economic growth with time (see [Kra 17]).

However, this fantastic creativity has manifested itself on different levels as well, especially in the world of life, among plants and animals.

The former must be very inventive to survive. As one knows, the concentration of heavy metals in the soil can vary considerably. Some are trace elements that are necessary for the plant's metabolism ; others are toxic in the smallest concentrations. Scientists at the University of Hohenheim investigated the mechanisms that plants use to adapt to specific soil situations. For examples, plants suffer when they are not supplied with the necessary quantities of iron. The lack of iron results in yellow leaves and reduced growth. However, while people can compensate for the lack of iron by taking iron tablets, plants have to find other ways to counteract iron deficiency. So they have come up with two basic mechanisms to solve this problem :

The most common strategy of plants involves the roots releasing protons into the soil. As von Wirén shows ([Wir 95], this leads to the acidification of the soil around the roots resulting in a better iron solubility. The iron is further modified by enzymes before it is transported by a specific transport protein into the cell's interior where it acts as an important cofactor of many metabolic processes. When plants register the lack of iron, specific, finely tuned signal transduction pathways are induced resulting in the increased formation of transporter protein.

However, this iron transporter is not very targeted and it also transports heavy metals into the plant – like cadmium, toxic in very low concentrations. So, grasses – a relative recent plant family in evolutionary terms – have developed a new strategy : they are able to secrete what are known as phytosiderophores, neutral complexes with iron, which are then taken up by the roots through a specific transport system. Phytosiderophores not only bind iron, but are also involved in the uptake of other trace elements such as zinc, manganese and copper – a phenomenon that is often observed in plants suffering from iron deficiency. The phytosiderophores are also able to form a complex with cadmium, but this one is not transported into the plant. The toxic heavy metal therefore enters the roots through a different path. Nevertheless, the presence of cadmium leads to the increased release of phytosiderophores. Von Wirén believes that cadmium interferes with the determination of the intracellular iron content. This leads to the plants experiencing a much stronger iron deficiency than is actually present. The plants try to compensate for this deficiency through the generation of phytosiderophores.

At another level, for example, interrelationship between insects and plants, phenomena like mimicry (resemblance of one species (model) to another (mimic) living together in the same area, or homochromy (change of color to resemble the environment) are very common (see [Jol 98], 161).

Mimicry is especially remarkable in Insects. One usually distinguishes two kinds of mimicry :

– Müllerian mimicry is a natural phenomenon in which two or more unprofitable (often, distasteful) species, that may or may not be closely related and share one or more common predators, have come to mimic each other's honest warning signals, to their mutual benefit, since predators can learn to avoid all of them with fewer experiences. It is named after the German naturalist Fritz Müller, who first proposed the concept in 1878 (see [Mul 78]).

– The Müllerian strategy is usually contrasted with Batesian mimicry (see [Bat 61]), in which one harmless species adopts the appearance of an unprofitable species to gain the advantage of predators' avoidance ; Batesian mimicry is thus in a sense parasitic on the model's defences, whereas Müllerian is to mutual benefit.

At higher levels of evolution, some animals also surprise by their inventiveness : tool making, courtship displays, ingenious hunting techniques are often cited as evidence of animal creativity (see [Kau 15]). The construction of habitats (bird nests, burrows or complex galleries of wild rabbits), protection systems (beaver dams) are marvels of engineering. The design of certain traps – for example the webs of Epeire spiders, whose silk threads, soaked in a 5-compound solution, giving them adhesive properties and viscoelasticity (see [Gos 86]) – testifies to an extraordinary inventability, refined over the thousands years of evolution.

Better, some animals seem to look like human artists : in 2005, at an auction in London, three works a little special were presented. These were paintings made in the 1950s by Congo, a chimpanzee artist. Encouraged by his master, the ethologist Desmond Morris, Congo had developed a taste for brushes and colors and made a total of more than 400 works. And apparently this performance was noticed, since its last works went to several thousand euros. But Congo is not unique. There are other examples of animals that have developed a "passion" for the visual arts. Among them, the horse Cholla (who paints on an easel, holding the brush in his mouth) or the Jack Russel Terrier Tillie, a dog that spreads colors on a canvas with his paws and nibbles here and there more or less randomly. However, we must not dream about those pseudo-human imitations : since in these cases there is usually no deliberate intention in these productions, it is difficult to speak of creativity. As for natural ingenuity, which is instinctual and plays an essentially adaptive role, it can hardly be called creativity, even if, in this case, the word would be less usurped (see [Kau 15]).

Indeed, the other phenomena mentioned above (chemical survival strategies, homochromy or mimicry, hunting strategies and so on) seem to prove that there exists a kind of intelligence in nature. This seems to be of a different order than cybernetics can explain with feedback systems and feedback mechanisms. The difference is precisely that the latter - for example, "thermostat" type assemblies - can not invent anything new. On the contrary, as Jeremy Narby writes, "living forms are endowed with a capacity for creative knowledge, while thermostats tend not to do anything new"(see [Nar 05], 163).

3 A brief history of inventive stimulation

As we know, some ancient societies have often used very clever artificial procedural supports to solve non-trivial problems. This is the case in China where, next to technological achievements often ahead of their time (powder, compass, lock lift) one finds also many beautiful forms of theoretical inventiveness. Thus, in the domain of mathematics, besides the very pragmatic recipes of the Treaty of the *Nine Chapters* (see [Guo 17]), there exist intellectual jewels (such as the famous Chinese remainder theorem). But whereas the Greek thought, makes little use of this kind of procedures (if one excepts the Euclid's algorithm to determine the greatest common divider (PGCD) of two integers without knowing their factorization), it is with the Arabic algebra – especially Al-Kwarizmi (about 780-850), the man who gives his name to the notion of "algorithm" – that these tools have developed and then spread to other domains like the sciences of history (see, for instance, Ibn Kaldhûn).

Another stream of ideas coming from the "Arts of Memory"([Yat 66]) have influenced artificial creativity. The ancients had often noticed the importance of the dispositions ordered in the acquisition of a "good memory". They had thus developed a "mnemonic", or "art of the memory", which consisted in associating with the things of which one wanted to remember certain particular "places". This inscription of time in space has not only given birth to the "mnemonic". It is also the idea that a datum can only be found by placing it in space at a certain "address", which is already a kind of pre-computing. The culture of memory thus gives rise to an "artificial" memory alongside "natural" memory. Soon, at "places" (or loci) intended to fix memories in space, will be added "images" (or imagines), which represent them with economy. The treatises on memory will then be replaced by a real "theater of memory" (like that of Giulio Camillo in the 16th century or Robert Fludd in the 17th century).

In this context, the incessant meditation, of cabalistic inspiration, on the Names or fundamental attributes of God, who knew in Spain in the 13th century a particular development, will be also at the source of the current of thought of a great posterity that we want now to mention. From this meditation, indeed, arises the idea that, just as the combination of the letters of the Hebrew alphabet made it possible to enumerate all the Names of God, so the combination of all its fundamental properties would make it possible to know what He is. These speculations led the philosopher Ramon Lull to the origins of combinatorial science.

From this point of view, his *Ars Brevis*, which allows to generate combinations of

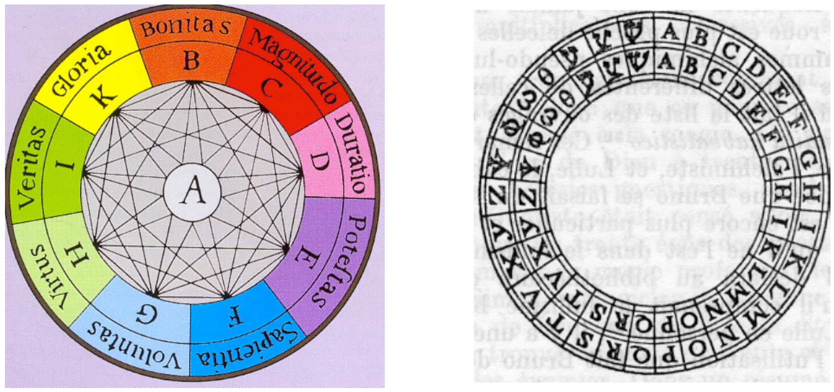

FIGURE 1 – R. Lull's *Art Brevis* and G. Bruno's *De Umbris*

concepts on the basis of existing ones, had to influence a little later G. Bruno, whose system called "Shadows of ideas" made it possible to calculate, thanks to concentric wheels, the action of the outside world (the planets of the zodiac) on the inner world (the spirit) through the combination of celestial configurations formed of various constellations. At the end of this evolution will be, in the 17th century, the *De Arte Combinatoria* of Leibniz.

But we can say more : the *Ars Magna* of Lull contained "tables of reflection" testifying to a method similar to the morphological method of the modern theoreticians of inventive stimulation that we will soon meet. See the following example of Fig. 2.

Using this table, by randomly combining B_1 and D_1, H_2 and G_2, one could formulate a question such as : "Will greatness and power one day bring more equality ?" As Arnold Kaufmann writes, this morphology, which seems a little naive, "is in reality very rich in important and realistic questions" ([Kau 79], 28).

It is in fact the starting point of an associationist and combinatorial conception of creativity, which amounts to reducing it and, in a way, to denying it. Under the influence of empiricism (Locke) but also of mathematical combinatorics (Leibniz), it finds its most famous representatives in the 18th century. For example, Denis Diderot felt that imagination is merely "the memory of forms and contents," and "creates nothing" but only combines, magnifies or diminishes. It was precisely in 18th-century France, indeed, that the idea of man's creativity met with resistance. Charles Batteux wrote that "The human mind cannot create, strictly speaking ; all its products bear

	1	2	3
	Absolute principles	Relative principes	Questions to be asked
A	Goodness	Difference	Does it exist ?
B	Greatness	Concordance	What is it ?
C	Duration	Opposite	From where ? - From who ?
D	Power	Beginning	Why ?
E	Wisdom	Middle	How much ?- how many ?
F	Appetite	End	Which ?
G	Virtue	Majority	When ?
H	Truth	Equality	Where ?
I	Glory	Minority	How ?

TABLE 1 – Artificial creativity according to Ramon Lull's *Ars Magna*

the stigmata of their model ; even monsters invented by an imagination unhampered by laws can only be composed of parts taken from nature." Vauvenargues (1715-47), and Étienne Bonnot de Condillac (1715-80) spoke to a similar effect.

Their resistance to the idea of human creativity had a triple source. The expression, "creation", was then reserved for creation *ex nihilo*, which was inaccessible to man. Second, creation is a mysterious act, and Enlightenment psychology did not admit of mysteries. Third, artists of the age were attached to their rules, and creativity seemed irreconcilable with rules. It is only gradually that one came to think rules ultimately are a human invention.

In the 19th century, art began to be seen as the true domain of creativity and it is only at the turn of the 20th century that there began to be discussion of creativity in the sciences (a thesis supported, in particular, by the polish logician Jan Lukasiewicz (1878-1956).

Then will come the theoricists. The start of the scientific study of creativity is sometimes taken as J. P. Guilford's 1950 address to the American Psychological Association, which helped popularize the subject.

As an extension of this current of thought, and as the fulfillment of Lull's ideas, which always put forward the existence of a (human or artificial) facilitator, we would find now, more than a hundred of existing creativity methods (maybe 172, according to Ngassa, identified in ([Nga 03]) and classified according to two categories by ([Sha 03]) :

1. The intuitive methods - which can be described as animation methods, which result depends on the involvement of the participants and their understanding of the problems proposed. The intervention of the facilitator with the participants allows to refocus thinking, explain the objectives of the method used and how it works. Six Thinking Hats and lateral thinking technics ([Bon 70]) can be cited as well as Brainstorming ([Osb 59], 9 windows ([Alt 84]), Design thinking ([Fas 93])... The principle is to solve the problem by plunging it into a larger or more precise context, in order to leave the "beaten track".
2. The more systematic methods - which can be designated as structuring methods, which result relies on the facilitator. For instance, one can mention the famous Discovering matrix ([Mol 54], [Mol 70]), the TRIZ methodology ([Alt 84], a theory of inventive problem solving, ASIT ([Hor 94], Advanced Systematic Inventive Thinking – coming from TRIZ, or the recent C-K Theory ([Hat 02], where C-K means "Concept-Knowledge", and which is based on set theory to generate conceptual and knowledge extensions...

One has also suggested combination between tools of these two categories, as in the work of Ambrosino and Legardeur (see [Amb 16]), which selects for instance the 9 windows (or 9 screens) tool proposed in TRIZ by Altshuller ([Alt 84]) and the "discovering matrix" described by Moles ([Mol 54], [Mol 70]). Those two tools were not initially designed to be used together but previous works on hybridization ([Leg 09]) highlighted that the use of combination of several methods, tools and techniques is a more flexible and agile approach to support complex creativity and innovation processes.

Let's say only a few words about the "discovering matrix" introduced by the French Abraham Moles ([Mol 54], [Mol 70]) and the so-called "morphological" method of the astronomer F. Zwicky ([Zwi 66]), two methods which mainly find practical applications in the field of social sciences and design.

The principle of the "discovering matrix" is quite simple : given a matrix M, crossing two sets of objects a_i and b_i), the boxes marked with a cross are those for which we found a relevant association, the others being left empty (we can also mark the significant boxes by the number 1 and the others by the number 0). For example, if a_3 denotes a truck, and b_3 a bottle, then the association (a_3, b_3) will be perfectly meaningful and may refer to a "truck-tank".

As we can see, the discovering matrix is based on the principle of bissociation : The bissociation is to associate or combine two distinct ideas (solutions or approaches) to give birth to a third unpublished element. This principle was well analyzed by

↱	b_1	b_2	b_3	b_4	b_5	b_6
a_1		x				
a_2	x			x		
a_3			x	x	x	x
a_4				x	x	
a_5						

TABLE 2 – Moles' discovering matrix

Arthur Koestler in a famous book published in 1964 ([Koe 64]), where he takes the example of Gutenberg, famous inventor of printing. In 1450, the monk wanted to reproduce the Bible. He is interested in the seals that are placed on the manuscripts but nothing comes. At the same time, he participates in harvesting and observes the operation of the grape press. He began to associate the use of the seal and the press and said that by applying a constant pressure on a seal, it would leave a mark on paper. The printing press was born!

But it should be noted here that the discovery matrix can go beyond bissociation. There are also the concepts of trissociation or multissociation, which relate here to the number of ideas that should be associated or combined.

It is good to know that many products and solutions have already been designed from this method. By way of illustration, we can cite the *snowmobile* that comes from the adaptation of a motorcycle to skis, *rollerblades* that come from sports shoes and roller skates or *kite surfing*, a combination of surf and a kite. The discovering matrix is a method that works and can be implemented in all types of creative processes, but especially commercial technology products.

Morphological analysis, coming from Zwicky, is well described in ([Kau 79], 24). It is presented in the following theoretical aspect :

Let $A, B, C, ..., H$ some sets defined as follows :

$$A = A_1, A_2, ..., A_m,$$

$$B = B_1, B_2, ..., B_n,$$

$$C = C_1, C_2, ..., C_p,$$

$$.............................$$

$$H = H_1, H_2, ..., H_v.$$

By taking any element in A, then any element in B, etc., we build what we call an "assembly" and, if it contains r elements, we call it a "r-assembly". We use the notation :

$$(A_2, B_1, C_5..., H_3),$$

to indicate that we meet in this assembly the components A_2, B_1, C_5... and H_3.

The set of all the assemblies is, mathematically, the "Cartesian product" of the starting sets. It is then easy to prove that if A contains m elements, B contains n elements, C contains p elements, ... H contains v elements, then there is :

$$N = m \times n \times p \times ... \times v$$

separate assemblies in the Cartesian product. Among them, some are sometimes impossible, others already known. But the method can also generate novelty, because of its combinatorial power (see [Kau 70] for some concrete examples of morphologies relative to an alarm device, aurban vehicle or the elderly behavior in the face of the medical profession).

We can also make use, as shown in [Kau 79] of all resources of graph theory, order theory or, eventually, fuzzy sets theory to give some structures to these morphologies.

And one could add to this panorama of combinatorial resources the reflections on problem-solving methods ([Pol 58], [Cas 78]) some of which are now supported by operational research ([Gon 78]) and artificial intelligence ([Lau 87]).

4 Creativity and unconscious scanning

All the previous methods may certainly stimulate inventivity. But it will probably be said that the kind of creativity we have been considering so far is not true creativity but a simple case of re-arrangement of already existing elements. Embedded in algorithms and capable of being assumed or relayed by mechanisms, its artificiality, precisely because it depends on a still limited technology, does not make use of this "freedom" which seems to be the preserve of the human species, and which, at the same time, does not meet the same constraints. It will be said, for example, that the most beautiful theories of modern science have not been produced by "combinatorics" and that, in this sense, the approach of the truly creative mathematician and physicist is more like that of the artist than that of a mechanic. Even if we need some knowledge of the existent to create, as Raymond Ruyer said, «we only imitate what we are almost capable of inventing» (see [Ruy 52], 138). In other words, in humans at least – but perhaps also in the whole nature if one considers the techniques of mimicry and camouflage (see also [Ruy 52], 27-34) – the capacity of invention comes first. In these cases of of actual creativity, it is not a question of applying rules to generate new from the old, but to overturn the existing rules to invent others.

This implies transgressions and breaks. Such has been the spirit of surrealism, the father of all the new movements of modern art. But impressionism, abstract art, cubism, action painting and all the great movements that shook painting or even music (serialism, jazz, etc.) follow from the same idea of substituting new rules for old, not for the pleasure of change, but because it is a matter of showing deeper phenomena beyond appearances and our surface sensitivity. The spraying of old forms always leads to stable new focuses, but often at the end of patient and difficult research.

As noted Anton Ehrenzweig ([Ehr 67]), in such domains, the approach turns out, at least initially, much more labyrinthine and hesitant. An artist does not always know a priori what he wants to paint or carve, a scientist does not always know exactly what he is looking for, or rather, what he will actually find. In fact, creation often encounters obstacles and sometimes dead ends, and the winning combinations must in fact be discovered through trials. Throughout this process, more or less unconscious conflicts can also manifest themselves, introducing breaks or dissociations that are far from the rational constructions of the preceding methods. This is why, according to Ehrenzweig, the research model corresponds to a complex graph, much fuzzier than the clear preceding combinatorial morphologies. It could be represented by the following image, which is perhaps closer to the more or less blind creative evolution of nature, in which Hegel asserted that the Concept was only conjecturing :

As Erhenzweig shows, in the research path, "any choice has the same crucial importance for further progression. The choice would be easy if we had at our disposal an aerial view of the entire network of nodal points and radiating paths that leave from there. This is never the case. If we could trace all the way to go, there would be no need for research. In fact, the creative thinker must make a decision on his way without having all the information he needs to choose."([Ehr 67], 71).

The important thing, then, in such a journey, is that it is not hindered by external considerations (prejudices, blockages), so that the creator always remains in touch with his creation, and especially, with his creative possibilities. Success comes when, throughout his trials and his quest, he is not cut off from what he can.

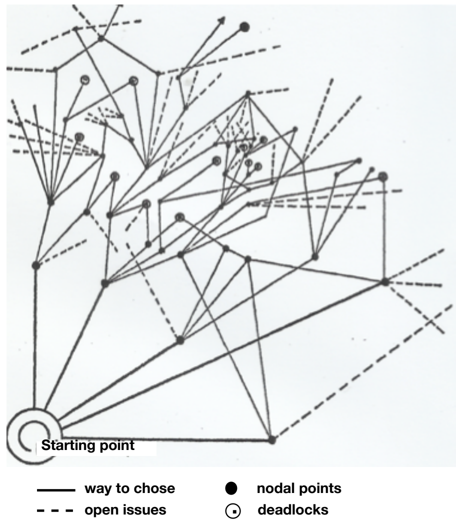

FIGURE 2 – The Labyrinth of a creative search

5 Mathematical creativity

Here we would like to give two examples of this unconscious scanning in mathematics, witnessing a remarkable creativity and leading to creations of a total novelty.

5.1 The Ramanujan case

The first is that of the Indian mathematician Ramanjuan :

In the appendix to the first of the two letters that Srinivasa Aiyangar Ramanujan (1887-1920), an almost self-taught Indian mathematician, sent in 1913 to Godfrey Harold Hardy (1877-1947), a professor at Trinity College in Cambridge and a specialist in number theory, figure among dozens of others, an extraordinary formula that takes the following form ([Ber 95], 25) :

$$\sqrt{\frac{5+\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{2}=\frac{e^{-\frac{2\pi}{5}}}{+1+\frac{e^{-2\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\frac{e^{-8\pi}}{1+\dots}}}}} \tag{1}$$

This formula, among a lot of others, was presented without demonstration, and Hardy confided, in 1937, his complete stupefaction, because he could not see where this kind of formulas could come from :

"The formulas (...) defeated me completely ; I have never seen anything before. A single look at them is enough to show that they could only be written down by a mathematician of the highest class. They must be true, if they were not true, no one would have had the imagination to invent them. "(See [Har 37], 144).

However, in front of a formula like (1), one cane make some observations :

1) The expression (1) is that of a continuous fraction establishing a link between different mathematical constants : π and e are immédiately visible but a trained mathematician will also recognize the presence of ϕ, the famous golden number of the Pythagoreans. Indeed,

$$\phi = \frac{\sqrt{5}+1}{2} \tag{2}$$

and a simple calculation also allows to pose :

$$\sqrt{\frac{5+\sqrt{5}}{2}} = \sqrt{\phi\sqrt{5}} = \sqrt{\phi+2} \tag{3}$$

(you just have to remember that $\phi^2 = \phi + 1$ and that $2\phi - 1 = \sqrt{5}$, so $2\phi^2 - \phi = \phi\sqrt{5} = 2(\phi+1) - \phi = 2\phi + 2 - \phi$, so $\phi + 2 = \phi\sqrt{5}$).

2) So, the formula (1) establishes a link between π, e and ϕ. The first two had already been related in the famous Euler formula ($e^{i\pi} + 1 = 0$), where they also articulated with three other important numbers in mathematics, the imaginary i, the number 1, neutral element of the multiplication, and 0, the neutral element of the addition. The novelty here was the presence of ϕ, the golden number, which appears twice in the left limb of the formula (1).

We have explicited the formula but how to know whether it is true or not (which means : what is the proof ?) and how could this formula have appeared in Ramanujan's head ?

We will first note the total originality of Ramanujan's formula of 1913, of which he could not know either the demonstration. It was only in 1917, while in London, that he discovered the work of a certain Rogers dating from 1894 and who could anticipate it. But it was impossible for him to know this work in his distant Indian province. Moreover, his formula was really only rigorously demonstrated for the first time sixteen years later, by George Neville Watson (see [Wat 29]).

So the problem is : how did Ramanujan build this formula ? When one asked Ramanujan these kinds of questions (from which his ideas or results came from ?), he responded by referring to the goddess Namagiri, under the dictation of whom he wrote. Obviously, this is not an answer and no one can be content with it.

If one strives to overcome this mythology and to explain the creation of Ramanujan, one can formulate the following hypotheses :

1) One have known, since the Greeks, this remarkable property of the number ϕ which is that one can approach it by the continuous well known fraction :

$$\frac{\sqrt{5}+1}{2} = 1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\dots}}}} = 1,618033988... \qquad (4)$$

2) In 1894, the British mathematician, Leonard James Rogers (1862-1933) (see [Rog 94], 329), working on the expansion of certain infinite products, had constructed a kind of generalization of the formula (4) associated with the approximation of the golden ratio, in the form of the following continuous fraction :

$$F(q) = 1 + \frac{q}{1+\frac{q^2}{1+\frac{q^3}{1+\frac{q^4}{1+\dots}}}} \qquad (5)$$

where the n^{th} numerator of (4) is replaced by $q^n, 0 \leq n < \infty$.

A variant of formula (1), known of Ramanujan, corresponds exactly to $F(q)$:

$$\frac{e^{-2i\pi/5}}{\sqrt{\frac{5+\sqrt{5}}{2}} - \phi} = 1 + \frac{e^{-2\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\frac{e^{-8\pi}}{1+\dots}}}} \qquad (6)$$

which right member is the same as in formula (5) when $q = e^{-2\pi}$.

According to the Indian mathematician Gaurav Bhatnagar ([Bha 15]), it is likely that Ramanujan has made this kind of reasoning. He probably generalized the continuous fraction associated to ϕ, and then evaluated the successive results when truncating the continuous fraction at a certain level.

To obtain a recurrence, Bhatnagar supposes that Ramanujan was able to think of adding to q an additional parameter z, replacing the formula (5) by :

$$c(z,q) = 1 + \cfrac{zq}{1 + \cfrac{zq^2}{1 + \cfrac{zq^3}{1 + \cfrac{zq^4}{1+...}}}}, \tag{7}$$

which leads to the recurrence relation :

$$c(z,q) = 1 + \frac{zq}{c(zq,q)} \tag{8}$$

that the Indian mathematician could have evaluated gradually like that :

$$c_0(z,q) = 1, \quad c_1(z,q) = 1+\frac{zq}{1}, \quad c_2(z,q) = 1+\frac{zq}{1+\frac{zq^2}{1}} = 1+\frac{zq}{1+zq^2} = \frac{1+zq+zq^2}{1+zq^2}, \text{etc.}$$

Then, calling the numerator $H_n(z,q)$ and the denominator $H_{n-1}(zq,q)$, he probably found a new recurrence relation and assumed that the solution could be a series development of powers of the type :

$$H(z,q) = \sum_0^\infty a_k z^k.$$

Taking $a_0 = 1$, he probably also found that :

$$H(z,q) = \sum_0^\infty \frac{q^{k^2}}{(1-q)(1-q^2)...(1-q^k)} a_0 = \sum_0^\infty \frac{q^{k^2}}{(q;q)_k} z^k$$

with $(q;k)_k = (1-q)(1-q^2)...(1-q^k)$.

What interessed Ramanujan was $c(1,q)$, with :

$$c(1,q) = \frac{H(1,q)}{H(q,q)},$$

i.e., in other terms :

$$1+\frac{q}{1+\frac{q^2}{1+\frac{q^3}{1+\frac{q^4}{1+...}}}}=\frac{\sum_0^\infty \frac{q^{k^2}}{(q;k)_k}}{\sum_0^\infty \frac{q^{k^2+k}}{(q;q)_k}}. \tag{9}$$

The continuous Rogers-Ramanujan'fraction is so the quotient of these two sums.

But these may be also expressed as products. We have :

$$H(1,q)=1+\frac{q}{(1-q)}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}+...$$

which is of the form $1+q+...+$ etc. One can then multiply each member by $(1-q)$, then eliminating the first two elements of the right member, and simplyfy the followyin expressions, then obtaining :

$$H(1,q)(1-q)=\frac{q^4}{(1-q^2)}+\frac{q^9}{(1-q^2)(1-q^3)}+...$$

But one can go on, multiplying now the two members by $(1-q^4)$. We get then :

$$H(1,q)(1-q)(1-q^4)=1+q^6+\frac{q^9}{(1-q^2)(1-q^3)}+...=1+q^6+...\text{etc..}$$

And so on. The two members now may be multiplied by $(1-q^6)$, etc. Finally, we can expect something like :

$$H(1,q)(1-q)(1-q^4)(1-q^6)(1-q^9)(1-q^{11})(1-q^{14})...=1.$$

The presence of the numbers 1, 4, 6, 9, 11, 14, etc. seems to indicate that the powers are of the type $5m+1$ et $5m+4$ for m = 0, 1, 2, 3, etc. So Ramanujam has probably conjectured that :

$$H(1,q)=\sum_0^\infty \frac{q^{k^2}}{(q:k)_k}=\prod_{m=0}^\infty \frac{1}{(1-q^{5m+1})(1-q^{5m+4})} \tag{10}$$

A similar method can be applied to $H(q,q)$, which leads to :

$$H(q,q)(1-q^2)(1-q^3)(1-q^7)(1-q^8)(1-q^{12})(1-q^{13})... = 1.$$

Then, Ramanujan probably conjectured that :

$$H(q,q) = \sum_0^\infty \frac{q^{k^2+k}}{(q;q)_k} = \prod_{m=0}^\infty \frac{1}{(1-q^{5m+2})(1-q^{5m+3})}. \tag{11}$$

But the equations (10) et (11) are precisely a variant of which is called now the Rogers-Ramanujan relations – since :

$$F(q) = \frac{H(1,q)}{H(q,q)}.$$

We know also that, if $q = e^{2\pi i\tau}$, then $H(q,1), H(q,q)$, as well as their quotient $F(q)$, are modular functions of τ, since they have integral coefficients, the theory of complex multiplication implies that their values for τ an imaginary quadratic irrational are algebraic numbers that can be evaluated explicitly. So we get the right member of the formula (5) – and also this explanation of formula (1), which is easy to get from it :

$$F(q) = \frac{e^{-2i\pi/5}}{\sqrt{\frac{5+\sqrt{5}}{2}} - \phi}.$$

Therefore, nothing mystical has intervened in this creation. If we summarize our analysis, we can argue that 1) Ramanujan has made a generalization of a formula well known since the Greeks, then 2) introduced a new element to allow him to calculate a first recurrence ; 3) After which, thanks to his calculating capacities and prodigious intuitions of numbers, he found other recurrences which had led him to a very general formula completely new.

As we can see, this kind of creativity includes both mimetic elements (the initial formula of ϕ), of which a small variation is first constructed, then, by introducing new non-mimetic elements, finally arrives at the surprising formula. This explains also that Ramanujan has not a real proof of his formula, essentially constructed through successive intuitions and conjectures, after many obstacles and impasses have been overcome, as in the labyrinthian process described by Ehrenzweig..

5.2 The Grothendieck case

We would like now to comment a general view about the conditions of mathematical creativity, which happens to have been theorized by one of the most brilliant mathematicians of the XXth century, Alexander Grothendieck. Speaking, in *Récoltes et semailles* of the "productive" period of its mathematical activity, that is to say that which extends over about twenty years between 1950 and 1969, Grothendieck writes :

"It was years of intense creativity. During this long period of my life, almost all of my time and energy was spent on so-called "piece work" : the painstaking work of assembling and honing, required for construction of all the rooms of the houses that a voice (or a demon ...) inside me to build, according to a master of work that it blew me as and when the work advanced. Taken by the tasks of "trade" : those in turn stonemason, bricklayer, carpenter, even plumber, carpenter and cabinetmaker - rarely did I take the time to note black on white, was not in broad strokes, the master plan invisible to all (as it appeared later on) except for me, who in the course of days, months, and years guided my hand with a sleepwalking certainty. (42).

The word "somnambulist", suggested to the author by the title of a book by Arthur Koestler, is, I believe, very precisely what Anton Ehrenzweig had in mind. Note also, as in Ramanujan, the impression that the creation was dictated by a kind of inner voice, in this case a "demon", rather than a goddess - question of culture, no doubt.

Stressing the blockages of this "innate in us" creativity that university education produces, Grothendieck also argues for not confusing "creativity" and simple "productivity without restraint". For him, full creativity implies being in touch, without blocking, with his own creative force :

"The presence, in the life of such a person, of a continuous creativity, is the sign of a" contact " continuous, so piecemeal and imperfect as it is, with the creative force in it. (...) In my intellectual pursuits and in particular, in my mathematical work, with modest "donations" (but a considerable investment), it seems to me that this "contact" with the force in me, ie also , the tacit and deep knowledge that I had, have been almost intact. That is to say, that I almost "worked" on all my means (creators) in this area (very fragmentary it is true) of my life, almost without loss, diversion or blockages of energy through the usual "rubbing effects." (p.572)

And it is the awareness of a blockage or helplessness that releases, according to him, prisoner creativity. In this sense, the weight of history seems to him one of the blockages that must be freed, and the famous formula of Bernard de Chartres, taken up by Pascal or Newton, and that "we are dwarves, but dwarfs perched on giant shoulders "seems to him more than a hypocrisy, a perfect absurdity :

"If each generation was" smaller "in format than the previous ones, it has been a long time since the human species was extinguished, out of breath, reduced to a derisory mass of homunculi ! I know that creativity in man is no less today (and, no doubt, greater) than a hundred years ago, or a hundred centuries ago. I know too well, to speak only of mathematics, that such ideas and works of people that I knew well, without excluding me from their number, would have been the very honor of the greatest mathematician of the past. . And I also know that my motivation in doing mathematics, and no more surely that of most of my old friends in the mathematical world, lies in "the hope of solving some of the problems" bequeathed by my predecessors ! If it were otherwise, our science would be powerless to renew itself - it would cease to be creative "(659-660).

Hence the idea that if his mathematician friends had given free rein to their creativity, many problems would be solved and the theories that Grothendieck had initiated, but left in the works, would now have reached the age of rigorous formalization. This intense creativity, which Grothendieck reproaches to some of his friends (Deligne, Serre) for having, as it were, denied, he sees it as a childish or even feminine property (the "yin" side), quickly repressed or overshadowed by the dominant male (the "yang" side), bringing "a state of imbalance (...) where the qualities "yin"or "feminine" are extirpated mercilessly" (p 853).

The causes of such renunciation, Grothendieck seems to attribute both a degradation of the mathematical mind and a loss of self-respect that also reflects a certain spirit of the time. "And it is in the loss of self-respect that I recognize the root of the loss of respect for others, and for the living work that has come out of his hands or those of the Creator" (p. 911). As if the human creativity was only the trace of a superior creativity of which the man would be, basically, only a more or less conscious agent. It is not enough, therefore, to create, to let be unhindered its nature "yin". Still, it is necessary to remain in permanent contact with this almost metaphysical creative force of which one is in some way the agent :

"It is not this feature in itself that distinguishes my" style "personal approach to mathematics from that of any other. It seems to me, indeed, that even among mathematicians, it is not so rare that this original (or "dominant") background note is yin. What is exceptional in my case (it seems to me) is that in my discovery process and especially, in my mathematical work, I have been all my life fully faithful to this original nature, without no hint of making alterations or corrections, whether by virtue of the desiderata of an interior censor (which in any case has never seen anything but fire, so we would be far from suspecting a sensitivity and a creative approach "feminine" in a business "between men" like mathematics !), or for the sake of complying with the canons of good taste in force in the outside world, and more particularly, in the scientific world. There is no doubt for me that it is thanks above all to this fidelity to my own nature, in this limited domain of my life at least, that my mathematical creativity has been able to unfold fully and unhindered, like a tree. vigorous, firmly planted in open ground, unfold freely to the rhythm of nights and days, winds and seasons. It was so, even though my "gifts" are rather modest, and the beginnings were not announced under the best auspices "(p. 925).

Indeed, Grothendieck, a modest student at the University of Montpellier, had painfully earned his degree in mathematics. However, in a few years, if we believe him, a spectacular metamorphosis has made him, suddenly, the genius that everyone knows. In fact, it seems that, from his studies, the originality of the researcher was manifested in the fact that, unwilling to attend classes and record results from others, he tried, in his corner, to rebuild mathematics alone, thus finding spontaneously, for example, the theory of the Lebesgue integral, which does not seem within the reach of anyone. As in the case of Ramanujan, the learning phase is ignored, while obviously, it is undoubtedly during this one that the methods and procedures were established so fruitful that allowed the hatching and development of the subsequent work. The mystery, therefore, remains : to stay in touch with one's creative force is one thing, but it must be educated and channeled to achieve actual receptive creations. The path by which these remarkable teachings have been acquired and accumulated thus remains partly unknown.

6 Conclusion

Without trying to be exhaustive, we have tried to go through the vast field of creativity and wanted to draw a small panorama of the rational methods by which this one can be both explained and simulated. Nevertheless, it seems to us that, if the past, very often, explains the present and anticipates the future, nevertheless, in a certain number of cases of high intellectuality, the explanation by the existence of prior knowledge and a montage favoring their association is not enough to explain the originality of certain productions. In these cases, it is necessary to recognize a particular power for their authors or, in any case, an aptitude to face the difficulties without stopping to keep in touch with their own creative force which is as well that of the child that they have not ceased to be, or of the universe itself in their raw state, of which they are, finally, particularly successful expressions.

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