Job creation in a dual labor market: a constructivist approach

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Abstract

In this paper, I review matching models of dual labor markets from a theoretical point of view and describe the consequences of the most common sets of assumptions on job creation. I assert that two poles arise in the literature depending on the modeling of fixed-term contracts. Some papers assume that fixed-term contracts are flexible in the sense that firm-worker matches may costlessly separate any time. Others assume that a fixed-term match is rigid and cannot split before reaching its stipulated termination date, regardless the undergone shocks. Modeling fixed-term contracts as utterly flexible tends to make fixed-term contracts the only vehicle of job creation, while open-ended contracts only appear as converted expiring fixed-term contracts. This counter-factual result encourages the use of *ad hoc* hiring rules that ensure that job creation involves both contracts. On the contrary, modeling fixed-term contracts as rigid makes fixed-term contracts less attractive and leaves more room for job creation to involve open-ended contracts. Substitution effects between contracts can be considered in these frameworks. I build up a model with rigid fixed-term contracts and heterogeneous productivity of matches assumption by assumption and find major robustness issues. Introducing the convertibility of fixed-term contracts into open-ended ones flips over the ranking of contracts at the hiring stage with respect to productivity. Enabling matches to optimize the average duration of fixed-term contracts leads to highly counter-factual results: the shortest and the least productive fixed-term matches have the highest probabilities to be converted to open-ended contracts. The highlighted robustness issues and counter-factual predictions contaminate recent papers studying labor market dualism and heterogeneity in workload fluctuations.

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*Le Mans Université (GAINS-TEPP, IRA) & Cepremap (ENS-Paris). E-mail: normann@dynare.org

Introduction

In Western Europe, fixed-term employment has considerably expanded over the last decades. Fixed-term contracts are now pervasive in job creation flows. As the celebrated paper Shimer (2012) shows, job creation flows mostly account for unemployment fluctuations. Understanding why agents prefer hiring through one type of contract is thus essential. A few papers tried to theorize the contractual choice at the hiring stage using matching models. Surprisingly, though, if papers have reviewed this literature, none has assessed the commonly used assumptions and their consequences over job creation. I try to bridge that gap.

I find that two main modeling strategies arise. Some papers model fixed-term contracts as highly flexible; firm-worker matches may split at zero cost any time. On the contrary, other papers assume that fixed-term contracts are rigid; separations cannot occur before reaching the stipulated end date. Matching models with flexible fixed-term contracts tend to predict job creation flows with fixed-term contracts only, which is counter-factual. As a result, some papers introduce *ad hoc* hiring rules with a settled share of open-ended contracts in job creation. Matching models with rigid fixed-term contracts, on the contrary, generally leave room for both open-ended and fixed-term contracts in job creation. However, I show that models with rigid fixed-term contracts lack robustness when it comes to the ranking of contracts in job creation. Overall, my results suggest that a matching model with realistic implications would strike a balance between flexible and rigid fixed-term contracts, avoiding both *ad hoc* hiring rules and robustness issues rigid fixed-term contracts bring along.

Why do flexible and rigid fixed-term contracts shape job creation differently? As the modeling of open-ended contracts displays little variability across papers, the differences stem from the modeling of fixed-term contracts. To be more specific, assumptions impact the surplus of fixed-term matches, which in turn shapes the way job creation resorts to open-ended and fixed-term contracts. Flexible fixed-term contracts deliver a higher surplus than rigid fixed-term contracts, as rigid fixed-term contracts may force some matches to stay together after an adverse productivity shock, for example. Thus, in uncertain environments, job creation would naturally favor flexible fixed-term contracts to rigid fixed-term contracts.

I propose a constructivist thought experiment to delineate the consequences of both rigid and flexible fixed-term contracts on job creation. This approach enables to circumscribe how each assumption shapes job creation, building a model brick by brick. I consider a classic continuous-time framework à-la Mortensen and Pissarides (1994), where one-job firms and workers meet according to a matching function. New firm-worker pairs are heterogeneous in productivity and undergo adverse productivity shocks that occur at a given rate. Firms and workers negotiate wages through Nash bargaining; hiring decisions only depend on the joint surplus of the match. Open-ended contracts stipulate a firing cost. If fixed-term contracts are flexible, there is no reason to consider open-ended contracts at the hiring stage. Open-ended contracts may only arise as converted expired fixed-term contracts. A way to impose both fixed-term and open-ended contracts in job creation is to direct a given share of new firm-worker pair to each type of contract, as some papers do in the literature. If fixed-term contracts are rigid, the ranking of open-ended and fixed-term contracts in job creation is now unclear. A fixed-term match cannot separate after an adverse productivity shock, whereas an open-ended match may be able to do so at some cost.

My approach also shows how robustness issues arise in models with rigid fixed-term contracts. Assume first that fixed-term contracts cannot be converted into open-ended contracts and that their average duration is fixed. New firm-worker pairs that draw a high productivity want to make the most out of it and a fixed-term contracts may be too short in that regard; the contract may reach its stipulated end before the production opportunity has come to an end. In this case, open-ended contracts are more relevant. The productivity draw needs to be high enough to compensate the match for future firing costs. If the drawn productivity is not high enough to compensate future firing costs, a fixed-term contract is a good option as it enables quickly going back to searching for high productivity draws and generates some positive surplus for a while. Open-ended contracts lock up highly productive matches, whereas fixed-term contracts are good a compromise for intermediate productivities.

Now, assume that fixed-term matches may be converted into open-ended matches once expired if it is profitable to do so. Fixed-term contracts are now better than open-ended contracts to deal with high productivity draws. If an adverse productivity shock occurs before the fixed-term contract reaches its termination date, it is not converted. Otherwise, the contract is converted into an open-ended contract. If firing costs are not too high, fixed-term contracts are not relevant for intermediate productivities. It is better to pay firing costs and split immediately than bearing with a depressed productivity up to the end date of the contract. As a result, making expired fixed-term matches convertible into open-ended ones reverses the ranking of contracts in job creation. Most productive matches end up in fixed-term contracts, while open-ended contracts cover less productive matches. This new ranking of contracts in job creation leads to a counter-factual result if any new match can maximize the joint surplus of a fixed-term match with respect to its average duration. As the optimal average duration increases with the productivity of the match, the least productive fixed-term matches are the most likely to be converted into open-ended contracts.

Many papers review the literature. The closest paper is Dolado et al. (2002), which presents models and stylized facts of labor market dualism. Beyond including new papers, I focus on matching models, whereas the scope of Dolado et al. (2002) is wider. Boeri (2011) describes the evolution of labor market institutions over time and reviews the empirical literature assessing the impact of employment protection on unemployment. It also builds a theoretical model to assess the impact of labor market institutions on unemployment. Bentolila et al. (2019) depicts theoretical and empirical insights about the impact of employment protection on dual labor markets.

The first section reviews the literature of matching models of dual labor markets. The second section builds a variety of matching models with rigid fixed-term contracts and highlights some robustness issues. The third section concludes.

1 Literature review

In this section, I review the literature dealing with labor market dualism using matching models. Note that there are also models describing labor demand when firms face adjustment costs[1]. I first delineate two polar categories of papers with respect to the modeling of fixed-term contracts. Then, I review the papers and their approaches towards job creation in a dual labor market.

1.1 Flexible or rigid fixed-term contracts?

Considering papers with both fixed-term and open-ended contracts, the chosen specifications to model open-ended contracts look alike. In general, open-ended contracts stipulate a firing cost that needs to be paid in case of a split. Some academics also include a cost-less separation channel in their models. Papers mostly differ on the fixed-term side of the labor market. In that regard, most models can be classified in two main categories: models with rigid fixed-term contracts and models with flexible fixed-term contracts.

[1]See Nickell (1978, 1979, 1986); Bentolila and Bertola (1990); Bertola (1992); Saint-Paul (1996); Risager and Sørensen (1997); Wasmer (1999); Dolado et al. (2002); Caggese and Cuñat (2008)

Rigid fixed-term contracts cannot split before the stipulated expiry date is reached, or it is expensive to do so. Flexible fixed-term contracts, on the contrary, can be terminated any time at no cost. What does the actual law say about it? In France, a fixed-term worker may leave his position for an open-ended contract without compensating the firm. Otherwise, regardless gross misconducts or exceptional situations, the unilateral end of a fixed-term contract before its due date is costly to the requester. Concerted terminations are possible and entail no transactions, though.

Both the rigid and flexible approaches to the modeling of fixed-term contracts are relevant. On the one hand, the fact that most fixed-term contracts reach their expiry date in the data supports the idea that fixed-term contracts are rigid; concerted terminations seldom occur. In France, Milin (2018) mentions that only 2 % of fixed-term contracts splitted before their stipulated termination date in 2017. On the other hand, firms and workers negotiate wages through Nash bargaining in most models. It makes hiring and firing decisions efficient for both the firm and the worker. In this case, as all separations are mutually beneficial, assuming that fixed-term matches may split at zero cost any time is consistent with the legal possibility of concerted termination.

The flexible and rigid approaches towards fixed-term-contracts lead to different trade-offs at the hiring stage. In a basic environment with stochastic productivity and random search, flexible fixed-term contracts dominate open-ended contracts in terms of surplus. Indeed, a fixed-term match facing an adverse productivity shock may costlessly split if the contract is flexible, whereas splitting an open-ended match incurs a firing cost. Thus, in a framework with flexible fixed-term contracts, firm-worker pairs always opt for fixed-term contracts at the hiring stage if they are free to do so. Without additional constraints, job creation only occurs through fixed-term contracts. Costain et al. (2010) is a polar example of unconstrained job creation with flexible fixed-term contracts. In what follows, I also review papers with constrained job creation.

At the other end of the spectrum, rigid fixed-term contracts blur the ranking of contracts in job creation. In a model with random search and stochastic productivity, rigid fixed-term contracts are no longer systematically delivering a higher total surplus than open-ended contracts. After an adverse productivity shock, a fixed-term match cannot split and must endure a low surplus for a while. Thus, in very bad times, open-ended contracts could be better; splitting at some cost may be better than bearing a fixed-term match with a depressed productivity. As Cahuc et al. (2016, 2019) and Rion (2019, 2020) show, rigid fixed-term contracts lead to coexisting fixed-term and open-ended contracts at the hiring stage. While flexible fixed-term contracts lead to a counter-factual unconstrained job creation through fixed-term contracts only, rigid fixed-term contracts enable job creation to occur through both fixed-term and open-ended contracts. Rigid fixed-term contracts enable the study of substitution effects between hiring fixed-term contracts and open-ended contracts. This nice feature of models with rigid fixed-term contracts comes at the cost of robustness issues as I demonstrate below. More specifically, in models with heterogeneous productivity, the ranking of contracts in job creation dramatically changes when expiring fixed-term matches can be converted into open-ended matches.

1.2 Flexible fixed-term contracts

I now review models with flexible fixed-term contracts. I distinguish what I call unconstrained and constrained job creation. Unconstrained job creation consists in choosing the contract that maximizes the joint surplus of the new match. Constrained job creation, on the contrary, makes the choice of the contract inconsistent with the highest joint surplus.

Costain et al. (2010) also builds a continuous-time framework with random search and matches face aggregate and idiosyncratic productivity shocks. Aggregate shocks follow a discrete Markov process while idiosyncratic productivity shocks are i.i.d. Open-ended contracts stipulate a firing cost, whereas fixed-term ones can be terminated any time. Fixed-term contracts reach their stipulated end date at a given a exogenous rate and may be converted into open-ended contracts at that moment if it is profitable. As discussed above, job creation is not constrained and, thus, only occurs through fixed-term contracts. Sala et al. (2012) roughly adopts a similar approach in a discrete-time model where idiosyncratic productivities of matches are i.i.d across periods. Job creation obeys the same mechanisms.

Blanchard and Landier (2002) builds a continuous-time model. Firms may immediately hire workers and new matches start at a given productivity under an *entry-level* contract with firing costs. The new matches then face i.i.d productivity shocks with a given arrival rate. The first productivity shock leads either to a split with firing costs or a conversion into a *regular* contract. Regular contracts also involve firing costs, but these firing costs only turn up in the bargaining of wages; regular contracts undergo no productivity shocks and costlessly split at an exogenous rate. When the firing costs of entry-level contracts are lower than the firing costs of regular contracts, dualism arises. In this case, using my terminology, entry-level contracts are flexible fixed-term contracts while regular contracts are open-ended contracts. The stronger bargaining position of regular workers lures firms away from converting entry-level jobs. As a result, firms intensively resort to entry-level contracts.

Many papers with flexible fixed-term contracts impose constraints on job creation. A wide-spread constraint is that an exogenous and settled share of new jobs is directed to each contract. In this manner, the model is able to replicate the presence of both fixed-term and open-ended contracts at the hiring stage, whereas there would be only fixed-term contracts in an unconstrained framework. Cahuc and Postel-Vinay (2002) builds a discrete-time model with this hiring rule. Firms and workers meet through a random-search process and matches are heterogeneous in productivities. Matches face i.i.d productivity shocks each period. Fixed-term contracts last one period and may be converted into open-ended contracts if it is profitable to do so. Open-ended contracts involve firing costs. In such a framework, new firm-worker pairs would systematically opt for fixed-term contracts; open-ended contracts would only emerge as converted fixed-term contracts without the hiring rule. Bentolila et al. (2012) uses the same job creation rule as Cahuc and Postel-Vinay (2002), whereas fixed-term contracts are rigid in its framework. Créchet (2018) assumes that a given share of created jobs may end up in open-ended contracts only, while the other created jobs may freely choose between open-ended and fixed-term contracts. Sala and Silva (2009) constraints job creation in a different way. The paper is very close to Sala et al. (2012) described above. The only difference is that fixed-term matches are less productive than open-ended contracts all else equal. The authors assume that job creation only occurs through fixed-term contracts and open-ended jobs stem from converted fixed-term contracts. Yet, as Rion (2020) show in a comparable framework, unconstrained job creation either involves both fixed-term and open-ended jobs, or open-ended jobs only.

How is it possible to endogenously obtain both fixed-term and open-ended contracts in job creation with flexible fixed-term contracts? Intuitively, introducing some mechanisms that lower the surplus of fixed-term contracts in specific cases should work. Cao et al. (2010) introduces on-the-job search, for example. The model is standard in many aspects; time is discrete, search is random, matches are heterogeneous in productivity which is i.i.d across periods. Fixed-term contracts last one period and may be converted into open-ended contracts at expiry. Firms and workers negotiate wages through Nash bargaining. Without any additional assumption, fixed-term contracts would always dominate open-ended contracts in terms of joint surplus. Here, the departing assumption is that workers can search on the job if they pay a search cost. The opportunity cost of searching is higher for workers with high wages, whereas on-the-job search strengthens the outside option of workers that are paid less. Firing costs makes fixed-term wages lower than open-ended wages all else equal. As a result, fixed-term workers have more incentive to search on-the-job than open-ended workers. Consequently, highly productive new firm-worker pairs tend to enter into an open-ended contract to prevent on-the-job search and benefit from their high productivity draw, while less productive matches have less to loose if the worker searches on the job and ends up leaving.

Giving up on random search enables to get both contracts naturally involved in job creation. Berton and Garibaldi (2012) opt for a continuous-time model with directed search. Workers are heterogeneous in their immediate utility of unemployment. Matches are subject to productivity shocks that occur at a given Poisson rate. Open-ended contract stipulate a firing cost, whereas fixed-term contracts may split any time for free. Wages are all the same, regardless the type of contract, and only depend on productivity. The resulting equilibrium leaves room for both contracts in job creation. From the worker's point of view, fixed-term contracts deliver a higher job-finding rate and a short time earning some wage, whereas open-ended contracts are more difficult to reach but offer long-lasting earnings. Typically, workers with low flow unemployment utility tend to go for fixed-term contracts, while workers with high unemployment-utility flow are ready to wait longer for an open-ended contract. On the firm's side, open-ended contracts grant firms with a high job-filling rate and a low flexibility, whereas fixed-term contracts yield a low job-filling rate and a high flexibility.

1.3 Rigid fixed-term contracts

In this section, I review papers with rigid fixed-term contracts. Again, I distinguish constrained and unconstrained job creation.

The closest paper to the thought experiment I propose in introduction is the model of Rion (2019). Firms and workers meet according to matching function in a continuous-time framework. Firm-worker pairs are heterogeneous in productivity and receive i.i.d shocks in productivity at a given rate. Open-ended contracts stipulate a firing cost while fixed-term contracts split following an exogenous termination rate. A fixed-term match cannot split before a termination shock hits regardless its productivity. Note that fixed-term matches cannot be converted into open-ended ones when expiring. Job creation is unconstrained; a new-firm worker pair draw a productivity from a given distribution and optimizes the choice of the contract accordingly. Most productive draws end up in open-ended contracts; fixed-term contracts may be too short to take full advantage of the initial high productivity draw. When the productivity is not high enough to consider paying firing costs in the future, fixed-term contract are a good fit. Firms and workers can go back to search for good matches before long and benefit from a positive surplus. As I show below, Rion (2019) is not robust. Making fixed-term matches convertible into open-ended matches dramatically changes the contractual layout of job creation.

In the same vein of models with heterogeneous productivities, the model of Rion (2020) is a discrete-time model and is very similar to Rion (2019). A few departures arise as productivities of matches are i.i.d across periods and fixed-term matches are less productive than open-ended ones all else equal. Agents strike a balance between productivity and flexibility. Most productive matches end up in open-ended matches; the agents are ready to pay firing costs in order to avoid the *ad hoc* productivity loss a fixed-term contract would incur. When the initial productivity draw is lower, matches favor fixed-term contracts. Again, the contractual framework of job creation would change much if expiring fixed-term matches were convertible.

Other models focus on heterogeneity in workload fluctuations. Cahuc et al. (2016, 2019) build a continuous-time model where firms and workers randomly meet according to a matching function. Matches face adverse productivity shocks that occur at heterogeneous rates. Open-ended contracts stipulate a firing cost. Firm-worker pair can optimize the duration of the contract and cannot split before its stipulated end. Fixed-term matches can be converted into an open-ended contract once expired. The optimal duration of fixed-term contracts decreases with the arrival rate of adverse productivity shocks; the more frequent the shocks, the shorter fixed-term contracts. The riskiest contracts end up in fixed-term contracts. On the contrary, open-ended contracts cover low-risk matches. The least risky fixed-term matches spared by adverse productivity shocks are worth converting into open-ended matches.

In some papers with rigid fixed-term contracts, job creation is constrained. Bentolila et al. (2012) builds a continuous-time random-matching model. Matches undergo i.i.d productivity shocks that occur at a given rate and start at a settled high productivity. Open-ended matches renegotiate wages through Nash-bargaining each time a productivity shock occurs, whereas fixed-term matches keep the initially negotiated wages up to expiry, which occurs at a given rate. Expiring fixed-term matches can be converted into open-ended matches. The paper focuses on the role labor courts play in open-ended job destruction and its impact on the labor market. If an open-ended match faces an adverse enough shock, the firm sends an advance notice to the worker and the match splits after the firing permission is issued, which takes time. Firing permission is issued at a given Poisson rate. Once it has arrived, the firm is bound to pay a firing cost. Between the notification and the firing permission issuance, the match delivers the lowest possible productivity and the worker is paid the average wage in the economy. Job creation is constrained; new-matches are directed towards each type of contract with a given probability. If job creation was unconstrained, since new matches deliver all the same productivity, new firm-worker pairs would simply opt for the type of contract that delivers the highest joint surplus. As a result, job creation would always occur through either open-ended or fixed-term contracts.

2 Rigid fixed-term contracts: a constructivist approach

The following sections build assumption by assumption various models with rigid fixed-term contracts to delineate robustness issues and the assumptions responsible for them. The first subsection describes the common core of these models. The second subsection focuses on models with heterogeneous processes of workload fluctuations. The last subsection describes models with heterogeneous productivities and the robustness issues that arise in models with rigid fixed-term contracts.

2.1 Initial assumptions

In this subsection, I build a basic model as a starting point. I shall extend this model over various dimensions and see the way job creation changes in the next subsections. I am particularly interested in the contractual composition of created jobs and the ranking of contract types in job creation.

Time is continuous. Firms as well as workers are identical *ex ante*. Firms post vacancies to attract workers and may employ one worker. I assume that the number of new matches per unit of time follows a function with constant returns to scale and the number of vacancies and job seekers as inputs. Meeting rates only depend on the labor market tightness θ, which is the ratio of the number of vacancies and job seekers. Firms meet unemployed workers with probability $q(\theta)$, while unemployed workers face a probability $p(\theta) \equiv \theta q(\theta)$ of finding a vacancy. A firm-worker pair produces y per unit of time, whether it be under an open-ended or a fixed-term contract. Matched firms and workers face a shock that makes their associations unproductive with probability λ per unit of time. This sudden drop in productivity may stem from a demand drop as well as a failure from the firm, the worker or the match itself.

Newly matched firms and workers maximize their expected joint surplus using either fixed-term or open-ended contracts.

Open-ended jobs stipulate a wage and may end through two channels: one involving firing costs and one which does not. The adverse productivity shock involves a split with firing costs F. The firing cost is a red-tape, dead-weight cost. It is not a firm-worker transfer such as a severance payment. In real life, open-ended matches may also end without entailing the payment of a firing cost (quits and retirement for example). I denote s the probability of cost-less separations per unit of time.

Fixed-term contracts stipulate a duration and a wage. For now, I assume that the duration of fixed-term contracts is fixed in expectation. I model it through a memory-less job destruction rate δ. In other words, the duration of fixed-term contracts is δ^{-1} in expectations and follows an exponential law with parameter δ. One may think it is unrealistic, as fixed-term contracts specify a duration and not a probabilistic job destruction rate. A Poisson process for fixed-term job destruction does not change conclusions qualitatively speaking, while it makes expressions more tractable and elegant than definite durations. Note that a productivity shock does not end a fixed-term contract in contrast with open-ended contracts. The firm has to pay the initially negotiated wage up to the end of the contract.

At this point, the trade-off is shaped. The new firm-worker pairs choose between a long open-ended contract with a costly separation in case of an adverse productivity shock, and a short fixed-term contract with a cost-less separation and potentially unproductive times.

I assume that the hiring decisions are jointly efficient to rule out incentive compatibility constraints and other theoretical difficulties related to firm-worker asymmetry. Newly matched firms and workers choose the contract that maximizes their expected joint surplus; the wage is then pinned down according to a Nash-bargaining rule. Note that the wage is set once for all and does not change until the match splits. I shall relax this assumption later in the paper.

I also assume that the vacancy-posting activity brings no profit to firms. Otherwise, as there are no barrier to entry, new comers would compete until profits are washed out. This so-called free-entry condition is classic in the literature (Mortensen and Pissarides, 1994). I denote V the present discounted value of a vacancy.

$$V = 0 \tag{1}$$

I denote J^p and W^p the firm's and worker's expected surpluses from an open-ended contract with wage w. J^f and W^f are their fixed-term counterparts. I denote S^p and S^f the joint surpluses from an open-ended contract and a fixed-term contract. The joint surpluses do not depend on the wage as it is a simple transfer between the firm and the worker. U is the present discounted utility from unemployment. The joint surpluses verify

$$S^p = J^p - V + W^p - U$$
$$S^f = J^f - V + W^f - U$$

The joint surplus associated with open-ended contracts does not include firing costs. A disagreement over the negotiation of wages does not incur the payment of firing costs for the firm. Firing costs do not belong to the firms' outside option while agents negotiate wages.

A firm employing a worker under an open-ended contract with wage w earns $(y - w)$ per unit of time. These flows stop if the match splits, whether it be with or without firing costs. An open-ended match splits without firing costs with probability s. An adverse productivity shock strikes with probability λ and involves the payment of firing costs F. As for the worker, he earns w per unit of time until the match ends.

$$rJ^p = y - w + s(V - J^p) + \lambda(V - F - J^p)$$
$$rW^p = w + (s + \lambda)(U - W^p)$$

I assumed that open-ended matches split when an adverse productivity shock hits. It has to be consistent to do so, though; separations need to be jointly efficient. It implies that the firm-worker pair renegotiates a wage w after the adverse productivity shock. The joint surplus associated with continuing the match must be negative for the separation to be jointly efficient. Under which conditions is it the case? I denote J^c and W^c the surpluses of a firm and a worker carrying on the match after an adverse productivity shock.

$$rJ^c = -w + s(V - J^c)$$
$$rW^c = w + s(U - W^c)$$

The associated joint surplus S^c is defined by

$$S^c = J^c - (V - F) + W^c - U$$

In contrast with the surplus of a new open-ended match, the surplus of a continuing match includes the firing cost. If the firm and the worker cannot agree on a new wage, splitting now involves paying the firing cost; the firm-worker pair formed a while ago at that point.

A little algebra provides an expression for S^c.

$$S^c = F - \frac{rU}{r + s}$$

The following assumption needs to hold for the surplus of a continuing open-ended match to be negative.

Assumption 1. $F < rU/(r + s)$

If the firing cost is too high, it may be better to carry the burden of a zero productivity until a cost-free separation shock hits instead of immediately paying the firing cost.

A fixed-term contract reaches its end with probability δ per unit of time. The match faces an adverse productivity shock with probability λ, in which case the productivity of the match reaches zero. It bears no consequence on the worker, who earns the initially negotiated wage up to a job destruction shock.

$$rJ^f(y, w) = y - w + \lambda(J^f(0, w) - J^f(y, w)) + \delta(V - J^f(y, w))$$
$$rW^f(w) = w + \delta(U - W^f(w))$$

Using the free-entry condition (1) and the expressions of surpluses above, a little algebra leads to the following joint surpluses.

$$S^p = \frac{y - \lambda F - rU}{r + s + \lambda}$$
$$S^f = \frac{y}{r + \delta + \lambda} - \frac{rU}{r + \delta}$$

2.2 Heterogeneous workload fluctuations

In this section, I extend the basic framework with heterogeneous arrival rates of adverse productivity shocks λ. I also assume that new firm-worker pairs optimize the fixed-term job destruction rate δ. Then, I broaden the model to make possible conversion of expired fixed-term contracts into open-ended ones. Appendix A.1 includes all the proofs of the following propositions.

2.2.1 Basic framework

As in Cahuc et al. (2016) and Cahuc et al. (2019), I assume first that jobs are heterogeneous in the arrival rate of adverse productivity shocks. Cahuc et al. (2016) assumes that firms pay a sunk cost to draw a shock arrival rate λ from a distribution G and then maintain a vacancy. Once the match splits, the firm has to pay to draw a new value of the shock arrival rate λ. For the sake of simplicity, I assume that a job-specific arrival rate of productivity shocks λ is drawn from distribution G when a firm-worker pair forms. Another difference with Cahuc et al. (2016) is that durations follow an exponential law instead of being fixed. In the current model, agents optimize the expected duration of fixed-term jobs instead of the duration itself. Qualitatively speaking, it keeps job creation as in Cahuc et al. (2016) and eases the following exposition of robustness issues. With these assumptions, the value of a vacancy verifies

$$rV = -\gamma + q(\theta)(1 - \eta) \int \max \left[S^p(\lambda), S^f_o(\lambda), 0 \right] dG(\lambda)$$

where γ is the maintaining cost of the vacancy, η is the worker's share of the joint surplus and $S^f_o(\lambda)$ is the optimized joint surplus of fixed-term contracts with shock arrival rate λ.

$$S^f_o(\lambda) = \max_{\delta \geq 0} S^f(\lambda, \delta)$$

Joint surpluses now depend on the arrival rate of productivity shocks λ.

Considering open-ended jobs, I assume that there are no cost-less separations to stick to the framework of Cahuc et al. (2016): for now, $s = 0$. I consider the case $s > 0$ later in this section. Thus, the joint surplus of an open-ended job with an arrival rate of adverse productivity shocks λ boils down to

$$S^p(\lambda) = \frac{y - \lambda F - rU}{r + \lambda}$$

Proposition 1 describes the behavior of S^p.

Proposition 1. *S^p is continuous and decreasing. Moreover, $S^p(0) = y/r - U > 0$ and $\lim_{\lambda \to +\infty} S^p(\lambda) = -F < 0$. Thus, there exists a unique λ^p such that $S^p(\lambda^p) = 0$.*

The higher the probability of adverse productivity shocks, the less relevant open-ended contracts are. As λ increases, open-ended contracts shorten, which brings forward firing costs.

In addition to the arrival rate of productivity shocks, the surplus associated with fixed-term contracts also depends on the fixed-term job destruction rate δ.

Proposition 2. *The function $\delta \mapsto S^f(\delta, \lambda)$ reaches a maximum at $\delta^*(\lambda)$ such that*

$$\delta^*(\lambda) = \lambda \left(\sqrt{\frac{y}{rU}} - 1 \right)^{-1} - r$$

Moreover, $\delta^*(0) = -r < 0$ and $\lim_{\lambda \to +\infty} \delta^*(\lambda) = +\infty$. δ^* being continuous, there exists a unique $\underline{\lambda}$ such that $\delta^*(\underline{\lambda}) = 0$.

δ^* increases with the shock arrival rate λ. The more frequently adverse productivity shocks strike, the less the optimal fixed-term contract lasts in expectation. The optimal duration finds a middle ground between too short contracts that are still productive when they expire and too long contracts that lock down the agents in unproductive matches for some time. Figure 2.2.1 displays δ^* in function of λ.

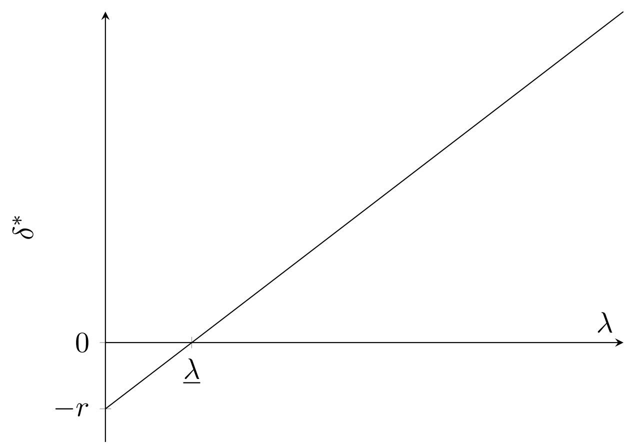

Figure 1: Optimal duration and arrival rate of adverse productivity shocks

Proposition 3 delineates the optimized surplus S_o^f for fixed-term contracts.

Proposition 3. *S_o^f is continuous and decreasing. Moreover, $S_o^f(0) = y/r - U$ and $\lim_{\lambda \to +\infty} S_o^f(\lambda) = 0$. Thus, $S_o^f > 0$.*

Describing job creation boils down to comparing joint surpluses for both types of contracts. Proposition 4 sums it up.

Proposition 4. *There exists λ^* such that $S^f(\lambda^*) = S^p(\lambda^*)$ and job creation takes place through open-ended contracts for matches with $\lambda \leq \lambda^*$, and through fixed-term contracts otherwise. Figure 2 sums up job creation.*

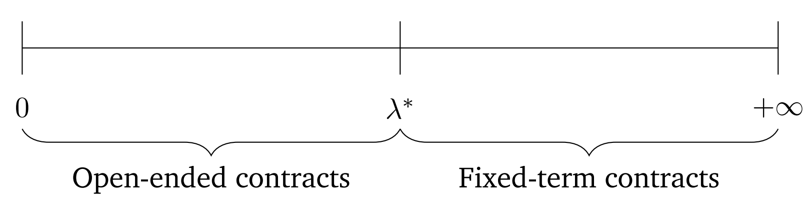

Figure 2: Arrival rate of productivity shocks and job creation

Proposition 4 describes an intuitive scenario for job creation. Matches that face a high risk of becoming unproductive quickly are better off signing a fixed-term contract. At the other end of the spectrum, matches with low arrival rates of adverse productivity shocks prefer endorsing an open-ended contract. Figure 2.2.1 displays joint surpluses and the associated thresholds.

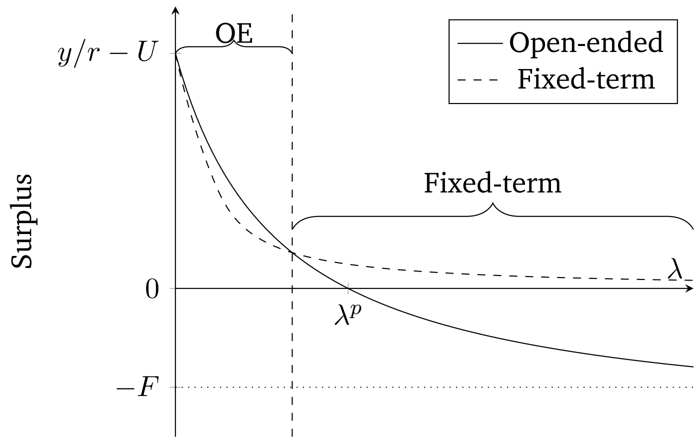

Figure 3: Surpluses and arrival rate of adverse productivity shocks

The ranking of contracts may change as soon as cost-less separations occur in open-ended matches, *id est* when $s > 0$. Without adverse productivity shocks, the longest the contract, the better it is; an open-ended contract may split with probability s whereas a fixed-term contract can be endless. Thus, fixed-term contracts are better than open-ended ones as the probability of adverse productivity shocks tends towards zero. Overall, cost-less separations of open-ended matches and job-to-job transitions impact job creation a lot. As far as I know, the literature has not tackled this issue yet in a dual labor market framework.

2.2.2 Conversion of fixed-term contracts into open-ended contracts

Now, I assume that expiring fixed-term contracts are converted into open-ended ones if it is profitable to do so. If I keep the model as it is right now, open-ended contracts and fixed-term contracts are not distinct; open-ended contracts are converted fixed-term contracts with zero duration. Thus, there is no room left for job creation through open-ended contracts. To avoid this trivial situation, I assume that each creation or conversion incurs a cost c to the firm. The surplus of open-ended matches with a probability of an adverse shock λ now writes

$$S^p(\lambda) = \frac{y - \lambda F - rU}{r + \lambda} - c$$

The following proposition describes its behavior.

Proposition 5. *S^p is continuous and strictly decreasing in λ. Moreover, $S^p(0) = \frac{y}{r} - U - c$ and $\lim_{\lambda \to +\infty} S^p(\lambda) = -F - c$. Hence, there exists a unique λ^p such that $S^p(\lambda^p) = 0$.*

As for fixed-term contracts, they are worth converting if no adverse productivity shock has struck before the termination shock. The conversion of fixed-term contracts into open-ended contracts entails the renegotiation of the wage according to a Nash-sharing rule. Thus, the conversion decision is efficient for both the firm and the worker. The joint surplus from a fixed-term contract with destruction probability δ and an adverse shock probability λ is

$$S^f(\lambda,\delta) = \frac{y+\delta S^p(\lambda)^+}{r+\delta+\lambda} - \frac{rU}{r+\delta} - c \tag{2}$$

Proposition 6. *The function $\delta \mapsto S^f(\delta,\lambda)$ reaches a maximum at $\delta^*(\lambda)$ such that*

$$\delta^*(\lambda) = \begin{cases} \lambda\left(\sqrt{\frac{\lambda(F+c)+r(U+c)}{rU}}-1\right)^{-1} - r & \text{if } \lambda \le \lambda^p \\ \lambda\left(\sqrt{\frac{y}{rU}}-1\right)^{-1} - r & \text{otherwise} \end{cases}$$

δ^* is increasing in λ and there exists a unique $\underline{\lambda}$ such that $\delta^*\left(\underline{\lambda}\right) = 0$.

Figure 2.2.2 displays the optimal job destruction rate δ^*. For all $\lambda > \lambda^p$, δ^* is the same as in the previous case without convertible fixed-term contracts. It makes sense as the associated fixed term jobs will not be converted into open-ended contracts; it is not profitable to do so. On the contrary, if $\lambda \le \lambda^p$, the conversion to an open-ended contract takes place if no adverse productivity shock occurs before the fixed-term contract expires. In this case, the optimal job destruction rate exceeds its no-convertibility counterpart, the dashed line on the graph. Why are short fixed-term contracts less attractive when they are convertible? Short contracts loose flexibility with respect to the no-convertibility case. They are more likely converted into open-ended contracts, which brings in contracting and firing costs. Long fixed-term contract, on the contrary, postpone these costs, which makes them more appealing.

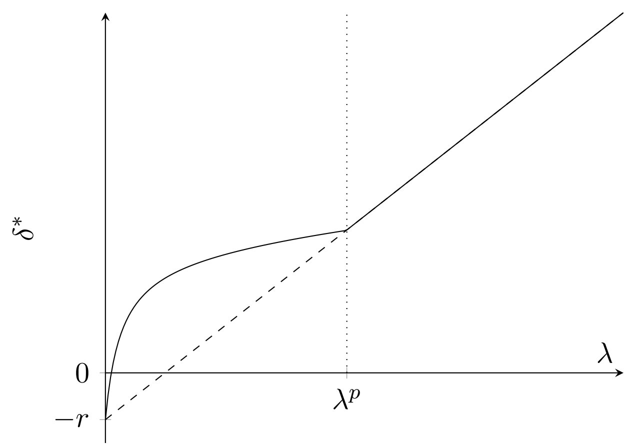

Figure 4: Optimal duration and arrival rate of adverse productivity shocks

The solid line is the curve of the optimal fixed-term job destruction rate δ^* when fixed-term contracts are convertible; the dashed line when they are not.

The following proposition details the behavior of the optimized joint surplus of fixed-term jobs $S^f_o(\lambda) = \max_{\delta \ge 0} S^f(\lambda,\delta)$.

Proposition 7. *S^f_o is continuous and decreases with λ from $S^f_o(0) = y/r - U - c$ to $\lim_{\lambda \to +\infty} S^f_o(\lambda) = -c$. Hence, there exists a unique $\overline{\lambda}$ such that $S^f_o\left(\overline{\lambda}\right) = 0$.*

Job creation occurs as the following proposition depicts.

Proposition 8. *There exists a unique λ^* such that $S^f(\lambda^*) = S^p(\lambda^*)$.*

- *If $S_o^f(\lambda^p) \leq 0$, then $\overline{\lambda} \leq \lambda^p \leq \lambda^*$ and job creation only occurs through open-ended contracts and for matches such that $\lambda \in [0, \lambda^p]$*

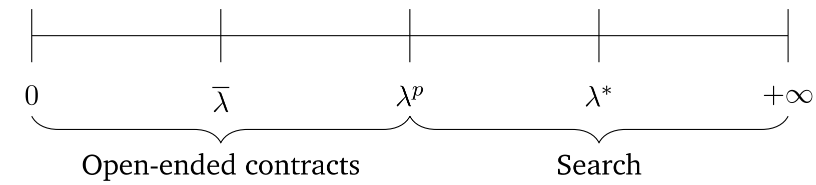

Figure 5: Arrival rate of productivity shocks and job creation: the case $S_o^f(\lambda^p) \leq 0$

- *If $S_o^f(\lambda^p) > 0$, then $\lambda^* < \lambda^p < \overline{\lambda}$. Job creation occurs through open-ended contracts when $\lambda < \lambda^*$, and through fixed-term contracts when $\lambda^* < \lambda < \overline{\lambda}$.*

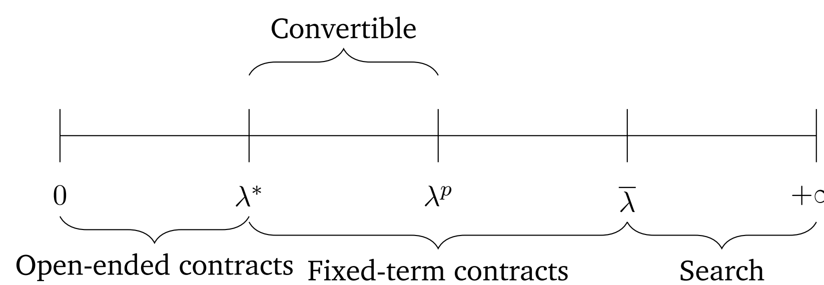

Figure 6: Arrival rate of productivity shocks and job creation: the case $S_o^f(\lambda^p) > 0$

The convertibility of fixed-term contracts does not change the overall ranking of contracts with respect to the arrival rate of adverse productivity shocks. Low arrival rates of adverse productivity shocks lead to hires under open-ended contracts. Least spared matches opt for fixed-term contracts. The only change boils down to the future of fixed-term contracts. Fixed-term contracts with a low arrival rate of adverse productivity shocks end up being converted if they remain productive up to expiry. Figures 5 and 6 show the joint surpluses in the two cases Proposition 8 outline.

The next section shows that the model with heterogeneous arrival rates of adverse productivity shocks may lead to counter-factual results if the productivity of new matches is no longer fixed. More specifically, regardless the arrival rate of productivity shocks, the shortest fixed-term contracts are the most likely to be converted into open-ended contracts when the productivity of matches is heterogeneous.

2.3 Heterogeneous productivities: the rise of robustness issues

In this section, I introduce heterogeneous productivities starting for the baseline model described above. I show that the convertibility of fixed-term contracts into open-ended contracts dramatically changes the ranking of contracts in job creation. Without convertibility, most productive matches operate under open-ended contracts, whereas fixed-term contracts encompass intermediate productivities. When fixed-term contracts are convertible, on the contrary, most productive matches end up in fixed-term contracts, while open-ended jobs are relegated to middle productivities. Then, I assume that productivity shocks are i.i.d and that new matches optimize the job destruction rate of fixed-term jobs. I find that the higher the productivity, the lower the optimized job destruction rate of fixed-term contracts. Again, highest productivities lead to fixed-term jobs while open-ended contracts cover intermediate productivities. It leads to a counter-factual result; the shortest fixed-term contracts are the most likely to be converted into open-ended contracts. The contractual ranking in job creation Rion (2019) and Rion (2020) is not robust to the introduction of convertibility. Introducing heterogeneity in the arrival rate of productivity shocks does not alter this finding. Cahuc et al. (2016) and Cahuc et al. (2019) also lead to the counter-factual finding that the shortest fixed-term contracts are the most likely to be converted into open-ended contracts if the productivity of new matches is random instead of settled.

2.3.1 Basic framework

Starting from the basic framework delineated above, what happens with job creation if the productivity of jobs is heterogeneous? I assume that new firm-worker pairs draw their productivity y job from a distribution with cdf G.

The match chooses the contract that maximizes the joint surplus, going back to searching being also an available option. Nash bargaining ensures that the chosen contract also maximizes the firm's and worker's surpluses. If firms pay γ per unit of time to maintain a vacancy, the present discounted value of a vacancy verifies

$$rV = -\gamma + q(\theta)(1-\eta)\int \max\left\{S^p(y), S^f(y), 0\right\} dG(y) \quad (3)$$

where η is the worker's share of the joint surplus and joint surpluses now depend on the productivity of the match y.

I shall define a few useful thresholds to describe job creation in this framework. I define profitability thresholds y^c and y^f for open-ended and fixed-term contracts.

$$S^p\left(y^c\right) = 0$$
$$S^f\left(y^f\right) = 0$$

I assume that splits of fixed-term jobs occur more frequently than cost-less separations of open-ended jobs. Mathematically speaking, it boils down to $s < \delta$. This assumption is in line with the data in Western European countries. Under this assumption, the surplus of open-ended jobs has a higher slope than the surplus of fixed-term jobs.

$$\frac{\partial S^p}{\partial y} = \frac{1}{r+s+\lambda} > \frac{1}{r+\delta+\lambda} = \frac{\partial S^f}{\partial y}$$

As a result, there exists a threshold y^* that breaks even the surplus of open-ended jobs and the surplus of fixed-term jobs.

$$S^p\left(y^*\right) = S^f\left(y^*\right) \quad (4)$$

Open-ended jobs dominate fixed-term ones as productivity exceeds y^*.

The ranking of the thresholds y^c, y^f and y^* shapes job creation. Proposition 9 makes it clearer.

Proposition 9. *These assertions are equivalent*

1. $y^* > y^f$
2. $y^* > y^c$
3. $y^c > y^f$

Using the free-entry condition (1), the definition of V (3) and integrations by parts, the job creation condition arises.

$$\frac{\gamma}{(1-\eta)q(\theta)} = \frac{1}{r+s+\lambda}\int_{\max(y^c,y^*)}^{+\infty}(1-G(y))dy + \frac{1}{r+\delta+\lambda}\int_{y^f}^{\max(y^f,y^*)}(1-G(y))dy \tag{5}$$

The next proposition describes job creation.

Proposition 10. *Considering an equilibrium* $\left(\theta, y^c, y^f, y^*\right)$

- *If $F > rU/(r+\delta)$, job creation only occurs through open-ended contracts. In this case, $y^* \leq y^f \leq y^c$; open-ended contracts are hired when $z \in (y^c, +\infty)$ as figure 7 displays.*

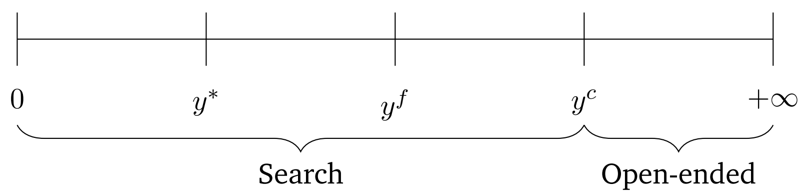

Figure 7: Hiring open-ended contracts only

- *Otherwise, job creation is dual. Fixed-term contracts are hired when $z \in (y^f, y^*)$ and open-ended contracts are hired when $z \in (y^*, +\infty)$. Figure 8 sums it up.*

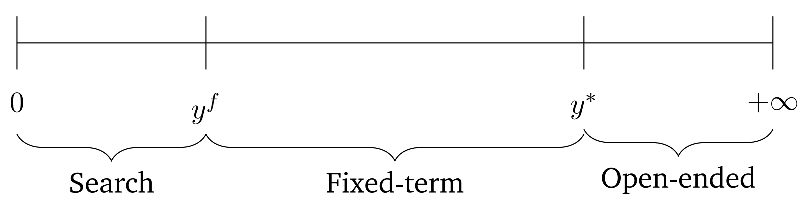

Figure 8: Dual job creation

In this framework, open-ended contracts serve to lock up highly productive matches. The match must be productive enough for immediate gains to exceed the expected firing costs. Open-ended contracts make the best out of a high productivity draw, as they last longer than fixed-term contracts. Fixed-term contracts provide a proper option when productivity is not that high; they strike a balance between going back to searching for a high productivity draw and producing for a while. Rion (2019) models job creation in the same manner. The only difference is that productivity shocks are i.i.d and job destruction is endogenous. Productivity shocks are not necessarily adverse and do not systematically lead to a split of open-ended contracts.

2.3.2 Convertible fixed-term contracts

Now, I assume that fixed-term contracts are converted into open-ended ones if it is profitable to do so. Fixed-term contracts are worth converting if no adverse productivity shock has struck before the termination shock. I also assume that the conversion of fixed-term contracts into open-ended contracts entails the renegotiation of the wage according to a Nash-sharing rule. Thus, the conversion decision is efficient for both the firm and the worker. I denote w and w' the negotiated wages when fixed-term jobs are created and converted. The firm's and worker's surpluses from a fixed-term contract with productivity y write

$$rJ^f(y,w) = y - w + \lambda(J^f(0,w) - J^f(y,w)) + \delta\left(\max\left[J^p(y,w'),0\right] - J^f(y,w)\right)$$
$$rW^f(y,w) = w + \lambda(W^f(0,w) - W^f(y,w)) + \delta\left(\max\left[W^p(y,w'),U\right] - W^f(y,w)\right)$$

The joint surplus of a fixed-term contract with productivity y boils down to

$$S^f(y) = \frac{y + \delta S^p(y)^+}{r+\delta+\lambda} - \frac{rU}{r+\delta}$$

The definition of y^c and y^f remain the same. The following proposition describes job creation.

Proposition 11. *Consider an equilibrium,*

- *If $F > rU/(r+\delta)$, job creation only occurs through fixed-term contracts as Figure 9 sums it up.*

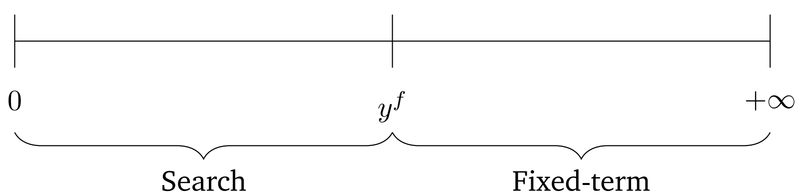

Figure 9: Hiring fixed-term contracts only

- *Otherwise, job creation is dual. There exists y^* such that fixed-term contracts are hired when $y > y^*$ and open-ended contracts are preferred $y \in (y^c, y^*)$. Figure 10 sums it up.*

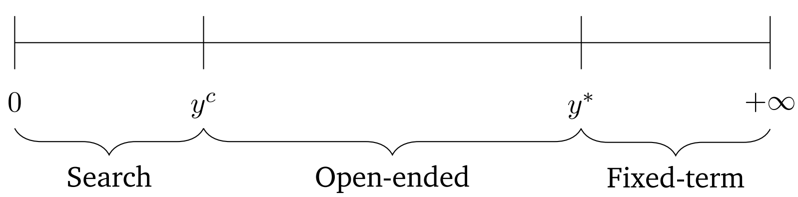

Figure 10: Dual job creation

The conversion option reverses the roles of open-ended and fixed-term contracts in job creation. Open-ended contracts no longer lock up high productivity draws in a better way than fixed-term contracts. Fixed-term contracts provide more flexibility. If an adverse productivity shock occurs before a termination shock, no conversion occurs. Otherwise, the conversion enables to take advantage of the high productivity draw through an open-ended contract. Is there any room left for open-ended contracts? Fixed-term contracts are rigid in some sense; the match cannot split after an adverse productivity shock. If firing costs are small enough or fixed-term contracts are long enough, agents find expensive not to split in case of an adverse productivity shock, while open-ended contracts allow such separations at a cost. As a result, open-ended contracts are relevant for intermediate productivities when the separation cost is not too high or fixed-term contracts are too long. Therefore, the results from Rion (2019) about job creation do not resist to introducing conversion of fixed-term matches.

2.3.3 Optimized duration and i.i.d productivity shocks

The following analysis shows that the models with heterogeneous arrival rate in adverse productivity shocks as well as heterogeneous productivities of matches lead to counter-factual results when some assumptions are relaxed: the shortest fixed-term contracts are the most likely to be converted into open-ended ones and the most productive matches are hired under fixed-term contracts.

I relax the assumption that productivity shocks are necessarily adverse. Now, I assume that they are independent and identically distributed and drawn from a law with cumulative distribution function G. Consequently, Assumption 1 is no longer needed to ensure that separating open-ended matches be efficient after a productivity shock. Continuing open-ended contracts now display non-trivial surpluses. The joint surpluses writes

$$S^c(z) = (J^c(z) - [V - F]) + (W^c(z) - U)$$

In contrast with the surplus of new open-ended contracts, firing costs belong to the outside option of firms with continuing open-ended workers. The associated firm's and worker's surpluses are

$$rJ^c(y) = y - w^c(y) + s\left(V - J^c(y)\right) + \lambda \int \left(max\left[J^c\left(y'\right), V - F\right] - J^c(y)\right) dG\left(y'\right)$$
$$rW^c(y) = w^c(y) + \lambda \int \left(\max\left[W^c\left(y'\right), U\right] - W^c(y)\right) dG\left(y'\right) + s\left(U - W^c(y)\right)$$

where $w^c(y)$ is the negotiated wage through Nash bargaining. Therefore, the joint surplus boils verify

$$(r + s + \lambda)S^c\left(y\right) = y - rU + \left(r + s\right)F + \lambda \int \max\left[S^c\left(y'\right), 0\right] dG\left(y'\right)$$

In this framework, as we shall demonstrate later, the possibility to convert expiring fixed-term contracts into open-ended ones makes open-ended hires irrelevant. To this extent, I introduce a fixed hiring and transformation cost c, which corresponds to the administrative cost of hiring a worker or transforming an expired fixed-term contract into an open-ended one. Consequently, the surplus of a new open-ended contract with productivity y becomes

$$S^p(y) = S^c(y) - F - c$$

The duration of fixed-term contracts is endogenously chosen to maximize the joint surplus when a worker-firm pair forms and conversions into open-ended contracts become possible when fixed-term contracts end. The surplus associated with a job creation through a z-productivity fixed-term contract denoted as $S_o^f(z)$ now includes the choice of the instantaneous expiration probability δ and verifies

$$S_o^f(y) = \sup_{\delta \geq 0} S^f(y, \delta) - c$$

where $S^f(y, \delta)$ denotes the joint surplus of a fixed-term contract with productivity y and expiration probability δ.

Firm's and worker's surpluses of a fixed-term contract become

$$rJ^f(y,\delta) = y - w^f(y) + \lambda \int \left(J^f(y',\delta) - J^f(y,\delta)\right) dG(y') + \delta \left(\max\left[J_0^p(y), V\right] - J^f(y,\delta)\right)$$
$$rW^f(y,\delta) = w^f(y) + \lambda \int \left(W^f(y',\delta) - W^f(y,\delta)\right) dG(y') + \delta \left(\max\left[W_0^p(y), U\right] - W^f(y,\delta)\right)$$

The resulting joint surplus of a fixed-term contract with productivity y and job destruction rate δ write

$$(r + \lambda + \delta) S^f(y,\delta) = y - rU + \delta S_0^p(y)^+ + \lambda \int S^f(y',\delta) \, dG(y') \tag{6}$$

The job creation condition is now

$$\frac{\gamma}{(1-\eta) q(\theta)} = \int \max\left[S^p(y), S_o^f(y), 0\right] dG(y)$$

The following proposition characterizes the optimal duration of fixed-term contracts.

Proposition 12. *Let y be in the support of G.*

- *If $Ey/r < \alpha \equiv U + \int S^p(y')^+ \, dG(y')$, the optimal expiration rate δ^* verifies*

$$\delta^*(y) = \begin{cases} +\infty & \text{if } x(y) \leq 0 \\ \lambda \frac{1+\sqrt{1+x(y)}}{x(y)} - r & \text{if } 0 < x(y) < \frac{\lambda}{r}\left(2 + \frac{\lambda}{r}\right) \\ 0 & \text{otherwise} \end{cases}$$
$$\text{where } x(y) = \frac{y - r\alpha - (r+\lambda)\left(S^p(y)^+ - \int S^p(y')^+ \, dG(y')\right)}{r\alpha - Ey}$$

- *Otherwise, $\delta^*(y) \in \{0, +\infty\}$*

$Ey/r - \alpha$ is the expected surplus of a fixed-term contract with zero probability of destruction after a productivity shock. The gain is the present discounted value of expected production Ey/r, while the losses α are the present discounted values of a return into unemployment U and the expected value of a conversion into a open-ended contract $\int S^p(y')^+ \, dG(y')$. These two events do not occur under a zero probability of fixed-term job destruction, which explains why they appear as losses. Conversely, a productivity shock has a probability one to hit such an eternal fixed-term contract, which explains the role of the *expected* surplus as a hiring criterion.

If $Ey/r \geq \alpha$, a no-end fixed-term contract is expected to be profitable considering the impact of productivity shocks throughout its existence. Therefore, if the immediate surplus of a new match — namely the surplus before any productivity shock hits — is not too low, a zero probability of destruction is optimal. Otherwise, an immediate destruction is preferable and an infinite probability of destruction is chosen.

Conversely, if $Ey/r < \alpha$, a no-end fixed-term contract has an expected negative surplus after a productivity shock. This encourages firms to shorten the stipulated durations of fixed-term contracts in order to avoid productivity shocks. Intuitively, if the productivity of the match is neither too low nor too high to opt for an infinite or zero probability of job destruction, there is room for optimization in terms of durations. The contract must be long enough to benefit from the current level of productivity but short enough to avoid losses associated with a productivity shock, which is expected to be detrimental. In that regard, an increase in the probability of shock occurrence pushes up the destruction probability for a given productivity y.

The probability of destruction decreases with the productivity of the firm-worker pair. Importantly, proposition 12 and its ramifications are still valid if the match chooses the duration instead of the destruction probability of a fixed-term contract as is the case in Cahuc et al. (2016).

According to proposition 12, the next assumption is necessary to rule out labor markets with polar durations of fixed-term contracts.

Assumption 2. $Ey/r < \alpha$

The following proposition states the optimal choice between fixed-term and open-ended contracts in function of the productivity of the match.

Proposition 13. *Under Assumption 2,*

- *If $c = 0$, job creation only occurs through fixed-term contracts*
- *If $0 < c < \frac{\lambda}{r}\left(\alpha - \frac{Ey}{r}\right)$, job creation is dual if and only if $x\left(y^c\right) < x^* \equiv (2+\beta)\,\beta$, where $\beta = \sqrt{\frac{c\lambda}{r\alpha - Ey}}$. Otherwise, job creation only occurs through fixed-term contracts.*

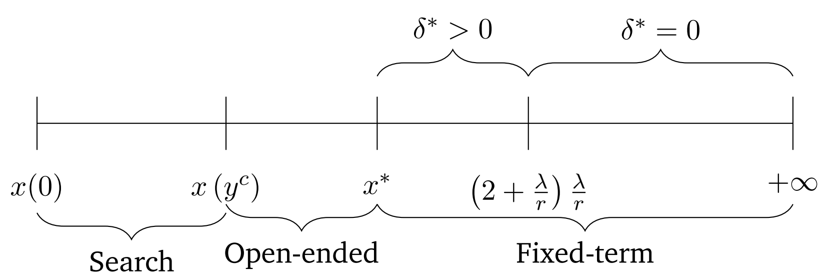

Figure 11: Dual job creation when $0 < c < \frac{\lambda}{r}\left(\alpha - \frac{Ey}{r}\right)$

- *If $c \geq \frac{\lambda}{r}\left(\alpha - \frac{Ey}{r}\right)$, job creation is dual if and only if $x\left(y^c\right) < x^* \equiv \frac{\lambda}{r} + \frac{c(r+\lambda)}{r\alpha - Ey}$. Otherwise, job creation only occurs through fixed-term contracts. fixed-term contracts have a zero probability of destruction.*

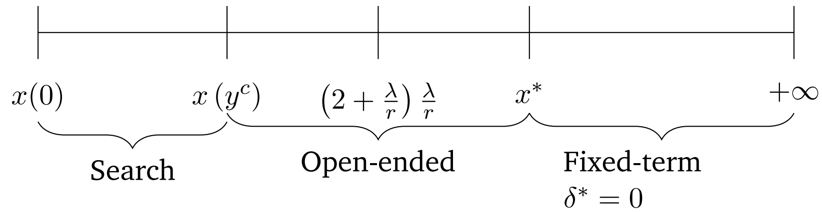

Figure 12: Dual job creation when $c \geq \frac{\lambda}{r}\left(\alpha - \frac{Ey}{r}\right)$

When there is no hiring cost, a new open-ended contract is equivalent to a new fixed-term contract with zero duration. Consequently, fixed-term contracts at least weakly dominate open-ended contracts at the hiring stage.

When the hiring cost is positive the job creation scheme is reversed with respect to the basic model with heterogeneous productivities. The endogenous choice of fixed-term contracts' job-destruction probability is not responsible for this plot twist. The possibility to convert an expiring fixed-term contract into a open-ended one accounts for this inversion. Indeed, on one hand, the flexibility provided by fixed-term contracts has expanded, as it now enables long-term relationships through both long fixed-term contracts and conversion into open-ended contracts. Avoiding the supplementary contracting cost constitutes the only motivation to directly hire through open-ended contracts instead of converting a fixed-term contract.

The firing cost is implicitly taken into account in the program of fixed-term matches because of the possibility of conversion to an open-ended contract at expiry. Consequently, fixed-term contracts cease to constitute waiting devices in opposition to productive open-ended contracts. The possibility to convert fixed-term contracts into open-ended ones makes fixed-term contracts sufficiently more flexible than open-ended contracts to cope with both high and low productivities at the hiring stage. The only caveat of fixed-term contracts with a finite duration is a superior administrative costs. In case of conversion, the contracting cost is paid twice. Firms and workers no longer strike a balance between productivity and flexibility as in the previous model. The sole compromise takes place between flexibility and hiring costs.

When productivity is moderate, the match can opt for a short fixed-term contract. On one hand, this provides a flexibility gain: if an adverse productivity shock occurs, the contract will end up quickly. The shorter the contract, the thinner this advantage. On the other hand, if an open-ended contract is beneficial, the contracting cost is paid twice. The shorter the contract, the heavier this drawback. Consequently, open-ended contracts tend to be preferred to very short fixed-term contracts whenever the former are beneficial. Conversely, when productivity is high, the hiring costs become small compared to the flexibility gains a longer fixed-term contract provides. As productivity converges towards infinity, a no-term fixed-term contract is even better than a open-ended contract. Somehow, the rigid open-ended contract is more flexible than a no-end fixed-term contract, which constitutes a better device to lock a firm-worker pair with a high productivity as there is a zero probability of separation.

When the hiring cost is high, the scope for short fixed-term contracts is reduced and open-ended contracts turn out to be more attractive. Finite fixed-term contracts are irrelevant as they impose paying twice hiring costs. Job creation only occurs through no-end fixed-term contracts and open-ended contracts. The limit point is the expected difference in surplus a finite fixed-term contract provides when a productivity shock occurs compared to a no-end fixed-term contract, namely $-\lambda\left(Ey/r - \alpha\right)/r$. This situation is not realistic. Therefore, a high contracting cost is incompatible with a proper fit of the data, hence the following additional assumption.

Assumption 3. $0 < c < \frac{\lambda}{r}\left(\alpha - \frac{Ey}{r}\right)$

The necessary theoretical foundations are elaborated enough to demonstrate the inadequacy of such a model to match data. Under assumptions 2 and 3, if job creation is dual, the resulting equilibrium is such that the shortest fixed-term contracts are the most likely to be converted into open-ended contracts, which is at odds with the data. The only way to remedy this problem is to consider equilibria with job creation through fixed-term contracts exclusively. Open-ended job creation solely occurs through conversion of fixed-term contracts. This is not relevant considering the ridiculous empirical probabilities of the latter event. Interestingly, notice that introducing heterogeneous firms in terms of shock arrival rate λ does not change these findings. To this extent, the model of Cahuc et al. (2016) critically relies on the hypothesis that new jobs have a fixed productivity. In this manner, a one-to-one link can be established between the distribution of the productivity shock arrival rate λ and the distribution of fixed-term matches' durations. Assuming heterogeneous productivities for new matches leads to the same counter-factual conclusions for each value of the shock arrival rate λ.

Conclusion

In this paper, I have reviewed random-matching models accounting for labor market dualism. I have shown that the contractual layout of job creation crucially depends on the modeling of fixed-term contracts.

When fixed-term matches can costlessly split in response to shocks, fixed-term contracts are flexible; job creation tends to favor fixed-term contracts. One way leave room for open-ended jobs in the labor market equilibrium is to make expiring fixed-term contracts convertible into open-ended contracts, or constrain job creation. In that regard, many papers direct a settled share of new matches to each type of contract, which prevents studying contractual substitution effects in job creation.

On the contrary, when fixed-term matches cannot split before their stipulated end date regardless undergone adverse shocks, fixed-term contracts are rigid. In that case, job creation may leave room for both contracts. Job creation does not only occur through fixed-term contracts and there is no need for *ad hoc* job creation rules to get both types of contracts involved. Contractual substitution effects are not shut down at the hiring stage.

The nice features of models with rigid fixed-term contracts come at a high price; the resulting models lack robustness. In models with heterogeneous productivities, the ranking of contracts in job creation dramatically changes if fixed-term matches become convertible into open-ended ones. If firm-worker pairs optimize the duration of fixed-term contracts, the shortest fixed-term contracts are the most likely to be converted into open-ended contracts at expiry. As for models with heterogeneous processes of workload fluctuations, they critically depend on the assumption that the productivity of new matches is settled. Relaxing this assumption leads to the aforementioned counter-factual conclusions.

Overall, future models should carefully consider the modeling of fixed-term contracts and navigate through the pros and cons of both modeling strategies. Building a robust model of job creation in a dual labor market that takes seriously contractual substitution effects is still to be done.

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A Workload fluctuations

A.1 Proofs

Proposition 1

Proof. I rule out trivial cases where open-ended contracts are never relevant and assume open-ended contracts are profitable when there are no adverse productivity shocks.

Assumption 4. $y > rU$

S^p is continuous and decreases with λ. Indeed, the derivative of S^p with respect to λ goes by

$$\frac{\partial S^p}{\partial \lambda} = \frac{-y + rU - rF}{(r+\lambda)^2} < \frac{-(y - rU)}{(r+\lambda)^2} < 0$$

Moreover, $S^p(0) = y/r - U$ and $\lim_{\lambda \to +\infty} S^p(\lambda) = -F < 0$. As a result, there exists a unique λ^p such that

$$S^p(\lambda^p) = 0$$

$$\text{id est } \lambda^p = \frac{y - rU}{F}$$

□

Proposition 2

Proof. Consider the derivative of S^f with respect to δ.

$$\frac{\partial S^f}{\partial \delta} = -\frac{y}{(r+\delta+\lambda)^2} + \frac{rU}{(r+\delta)^2} > 0$$

$$\text{i.e. } \delta < \lambda \left(\sqrt{\frac{y}{rU}} - 1 \right)^{-1} - r \equiv \delta^*(\lambda) \text{ for all } \lambda \geq \lambda^p$$

Note that δ^* is linear in λ with a positive slope and, thus, is continuous and increasing in λ. In addition, $\delta^*(0) = -r < 0$ and $\lim_{\lambda \to +\infty} \delta^*(\lambda) = +\infty$. Thus, there exists a unique $\underline{\lambda}$ such that $\delta^*(\underline{\lambda}) = 0$.

$$\underline{\lambda} = r \left(\sqrt{\frac{y}{rU}} - 1 \right)$$

□

Proposition 3

Proof. For a given λ, 2 cases arise in the program $S_o^f(\lambda) = \max_{\delta \geq 0} S^f(\lambda, \delta)$

- If $\delta^*(\lambda) > 0$, *id est* if $\lambda > \underline{\lambda}$ then $\delta^*(\lambda)$ is the unique maximizer of $\delta \mapsto S^f(\delta, \lambda)$ and $S_o^f = S^f(\delta^*(\lambda), \lambda)$.

- Otherwise, if $\delta^*(\lambda) \leq 0$, *id est* if $\lambda \leq \underline{\lambda}$, $\delta^*(\lambda)$ does not comply with the non-negativity constraint over the optimal δ. Note that $\delta \mapsto S^f(\delta, \lambda)$ is continuous and decreasing over the interval $[\delta^*(\lambda), +\infty]$. Hence, $\delta = 0$ leads to the highest possible value of the surplus and respects the constraint $\delta \geq 0$.

Therefore, the optimized surplus of fixed-term jobs $S_o^f(\lambda) = S^f(\lambda, \delta^*(\lambda))$ verifies

$$S_o^f(\lambda) = \begin{cases} \frac{y}{r+\lambda} - U & \text{if } \lambda \leq \underline{\lambda} \\ \frac{\left(\sqrt{y}-\sqrt{rU}\right)^2}{\lambda} & \text{otherwise} \end{cases}$$

S_o^f decreases with λ. Moreover, $S_o^f(0) = y/r - U > 0$ and $\lim_{\lambda \to +\infty} S_o^f(\lambda) = 0^+$. Thus, $S_o^f > 0$. □

Proposition 4

Proof. Now, I consider the function $\Delta = S^f - S^p$. If $\lambda \leq \underline{\lambda}$, Assumption 1 entails that

$$\Delta(\lambda) = \frac{\lambda(F-U)}{r+\lambda} < 0$$

Thus, $S^f(\lambda) > S^p(\lambda)$ for all $\lambda \leq \underline{\lambda}$.
If $\lambda > \underline{\lambda}$, Δ can be expressed as

$$\Delta(\lambda) = \frac{F\lambda^2 - 2\sqrt{rU}\left(\sqrt{y}-\sqrt{rU}\right)\lambda + r\left(\sqrt{y}-\sqrt{rU}\right)^2}{\lambda(r+\lambda)}$$

The sign of Δ boils down to the sign of the numerator, which is a second-degree polynomial function. It has two positive roots λ_1^* and λ_2^*.

$$\lambda_1^* = \sqrt{rU}\frac{\sqrt{y}-\sqrt{rU}}{F}\left(1-\sqrt{1-\frac{F}{U}}\right)$$
$$\lambda_2^* = \sqrt{rU}\frac{\sqrt{y}-\sqrt{rU}}{F}\left(1+\sqrt{1-\frac{F}{U}}\right)$$

It is possible to locate these roots with respect to the thresholds $\underline{\lambda}$ and λ^p.

$$\lambda_1^* = \sqrt{rU}\frac{\sqrt{y}-\sqrt{rU}}{F}\left(1-\sqrt{1-\frac{F}{U}}\right) < \sqrt{rU}\frac{\sqrt{y}-\sqrt{rU}}{F}\left(1-\left(1-\frac{F}{U}\right)\right) = \underline{\lambda}$$
$$\underline{\lambda} = \sqrt{rU}\frac{\sqrt{y}-\sqrt{rU}}{F}\left(1-\left(1-\frac{F}{U}\right)\right) < \sqrt{rU}\frac{\sqrt{y}-\sqrt{rU}}{F}\left(1+\sqrt{1-\frac{F}{U}}\right) = \lambda_2^*$$
$$\lambda_2^* = \sqrt{rU}\frac{\sqrt{y}-\sqrt{rU}}{F}\left(1+\sqrt{1-\frac{F}{U}}\right) < \sqrt{rU}\frac{\sqrt{y}-\sqrt{rU}}{F}\left(1+\sqrt{\frac{y}{rU}}\right) = \lambda^p$$

The ordering $\lambda_1^* < \underline{\lambda} < \lambda^* \equiv \lambda_2^* < \lambda^p$ implies that

$$\begin{cases} S^p(\lambda) > S^f(\lambda) > 0 & \text{if } \lambda \leq \lambda^* \\ S^f(\lambda) > S^p(\lambda) > 0 & \text{if } \lambda^* < \lambda < \lambda^p \\ S^f(\lambda) > 0 > S^p(\lambda) & \text{otherwise} \end{cases}$$

Job creation takes place accordingly.

□

Proposition 5

Proof. I rule out trivial cases where open-ended contracts are never relevant and assume open-ended contracts are profitable when there are no adverse productivity shocks.

Assumption 5. $y > r(U + c)$

S^p is strictly decreasing in λ.

$$\frac{\partial S^p}{\partial \lambda} = \frac{-y + r(U - F)}{(r + \lambda)^2} < \frac{-(y - rU)}{(r + \lambda)^2} < 0$$

Assumption 5 ensures that $y > r(U + c) > y > rU$.

Note also that $S^p(0) = \frac{y}{r} - U - c$ and $\lim_{\lambda \to +\infty} S^p(\lambda) = -F - c$. Hence, there exists a unique λ^p such that $S^p(\lambda^p) = 0$.

$$\lambda^p = \frac{y - r(U + c)}{F + c}$$

□

Proposition 6

Proof. Consider 2 cases for a given λ:

- If $\lambda < \lambda^p$, then $S^p(\lambda) > 0$ and

$$\frac{\partial S^f}{\partial \delta}(\lambda, \delta) = -\frac{\lambda(F + c) + r(U + c)}{(r + \delta + \lambda)^2} + \frac{rU}{(r + \delta)^2} > 0$$

$$\text{i.e. } \delta < \lambda \left(\sqrt{\frac{\lambda(F + c) + r(U + c)}{rU}} - 1 \right)^{-1} - r \equiv \delta^*(\lambda) \text{ for all } \lambda < \lambda^p$$

- If $\lambda \geq \lambda^p$, then $S^p(\lambda) \leq 0$ and

$$\frac{\partial S^f}{\partial \delta} = -\frac{y}{(r + \delta + \lambda)^2} + \frac{rU}{(r + \delta)^2} > 0$$

$$\text{i.e. } \delta < \lambda \left(\sqrt{\frac{y}{rU}} - 1 \right)^{-1} - r \equiv \delta^*(\lambda) \text{ for all } \lambda \geq \lambda^p$$

Now, I compute the derivative of δ^* with respect to λ to study its variations.

- If $\lambda \geq \lambda^p$, $S^p(\lambda) \leq 0$ and δ^* is linearly increasing in λ.

$$\delta^*(\lambda) = \frac{\lambda}{\sqrt{\frac{y}{rU}} - 1} - r$$

- Otherwise, $S^p(\lambda) < 0$, and δ^* writes

$$\delta^*(\lambda) = \frac{\lambda}{\sqrt{\frac{\lambda(F+c)+r(U+c)}{rU}} - 1} - r$$

The derivative of δ^* verifies

$$\begin{aligned}
(\delta^*)'(\lambda) &\propto \sqrt{\lambda(F+c)+r(U+c)} - \sqrt{rU} - \frac{\lambda(F+c)}{2\sqrt{\lambda(F+c)+r(U+c)}} \\
&\propto 2r(U+c) + \lambda(F+c) - 2\sqrt{rU}\sqrt{\lambda(F+c)+r(U+c)} \\
&\propto \left(\sqrt{rU} - \sqrt{\lambda(F+c)+r(U+c)}\right)^2 + rc > 0
\end{aligned}$$

As a result, δ^* is increasing in λ. Moreover, δ^* is continuous and $\delta^*(0) = -r$, $\lim_{\lambda\to\infty} \delta^*(\lambda) = +\infty$. Thus, there exists a unique $\underline{\lambda}$ such that $\delta^*(\underline{\lambda}) = 0$. □

Proposition 7

Proof. For a given λ, 2 cases arise in the program $S^f_o(\lambda) = \max_{\delta\geq 0} S^f(\lambda, \delta)$

- If $\delta^*(\lambda) > 0$, *id est* if $\lambda > \underline{\lambda}$ then $\delta^*(\lambda)$ is the unique maximizer of $\delta \mapsto S^f(\delta, \lambda)$ and $S^f_o = S^f(\delta^*(\lambda), \lambda)$.
- Otherwise, if $\delta^*(\lambda) \leq 0$, *id est* if $\lambda \leq \underline{\lambda}$, $\delta^*(\lambda)$ does not comply with the non-negativity constraint over the optimal δ. Note that $\delta \mapsto S^f(\delta, \lambda)$ is continuous and decreasing over the interval $[\delta^*(\lambda), +\infty]$. Hence, $\delta = 0$ leads to the highest possible value of the surplus and respects the constraint $\delta \geq 0$.

Therefore, the optimized joint surplus of fixed-term contracts S^f_o verifies

$$S^f_o(\lambda) = \begin{cases} S^f(0,\lambda) & \text{if } \lambda \leq \underline{\lambda} \\ S^f(\delta^*(\lambda),\lambda) & \text{otherwise} \end{cases} \tag{7}$$

Now, I pinpoint the derivative of S^f to study its variations.

- If $\lambda \leq \underline{\lambda}$, (7) and (2) yield

$$\frac{\partial S^f_o}{\partial \lambda} = \left.\frac{\partial S^f}{\partial \lambda}\right|_{(0,\lambda)} = -\frac{y}{(r+\lambda)^2} < 0$$

- If $\lambda > \underline{\lambda}$, (7) and (2) lead to

$$\frac{\partial S^f_o}{\partial \lambda} = \left.\frac{\partial S^f}{\partial \lambda}\right|_{(\delta^*(\lambda),\lambda)} + \frac{\partial \delta^*(\lambda)}{\partial \lambda}\left.\frac{\partial S^f}{\partial \delta}\right|_{(\delta^*(\lambda),\lambda)}$$

For a given λ, the proof of proposition 6 shows that $\delta^*(\lambda)$ nullifies the derivative of $\delta \mapsto S^f(\delta, \lambda)$

$$\left.\frac{\partial S^f}{\partial \delta}\right|_{(\delta^*(\lambda),\lambda)} = 0$$

Hence, the derivative of S_o^f with respect to λ boils down to

$$\frac{\partial S_o^f}{\partial \lambda} = \left.\frac{\partial S^f}{\partial \lambda}\right|_{(\delta^*(\lambda),\lambda)}$$

– If $\lambda < \lambda^p$, then $S^p(\lambda) > 0$ and

$$\frac{\partial S_o^f}{\partial \lambda} = -\frac{y + \delta^*(\lambda) S^p(\lambda)}{(r + \delta^*(\lambda) + \lambda)^2} + \frac{\delta^*(\lambda)}{r + \delta^*(\lambda) + \lambda} \underbrace{\frac{\partial S^p}{\partial \lambda}}_{<0} < 0 \tag{8}$$

– Otherwise, if $\lambda \geq \lambda^p$, then $S^p(\lambda) \leq 0$ and

$$\frac{\partial S_o^f}{\partial \lambda} = -\frac{y}{(r + \delta^*(\lambda) + \lambda)^2} < 0$$

Thus, S_o^f is decreasing and continuous. Moreover, $S_o^f(0) = S^f(0,0) = y/r - U - c$ and $\lim_{\lambda \to +\infty} S_o^f(\lambda) = -c$. Hence, there exists a unique $\overline{\lambda}$ such that $S_o^f\left(\overline{\lambda}\right) = 0$. □

Proposition 8

Proof. Consider λ such that $0 < \lambda \leq \underline{\lambda}$ and $\Delta \equiv S_o^f - S^p$.

$$\Delta(\lambda) = S_o^f(\lambda) - S^p(\lambda) = \frac{\lambda(F - U)}{r + \lambda} < 0$$

Δ is continuous, negative on $]0, \underline{\lambda}]$. In addition, $\Delta(0) = 0$ and $\lim_{\lambda \to +\infty} \Delta(\lambda) = F > 0$. Consequently, there must exist at least one $\lambda^* > \underline{\lambda}$ such that $\Delta(\lambda^*) = 0$.

First, I rewrite Δ for $\lambda > \underline{\lambda}$, $\lambda \neq \lambda^p$. To do this, I rewrite S_o^f. As stated in the proposition 6, δ^* nullifies $\partial S^f / \partial \delta$, which entails

$$S^p(\lambda)^+ = \frac{1}{r + \lambda} \left(y - \left(1 + \frac{\lambda}{r + \delta^*(\lambda)} \right)^2 rU \right)$$

This leads to the following expression for S_o^f.

$$S_o^f(\lambda) = \frac{y}{r + \lambda} - \left(1 + \frac{\delta^*(\lambda)}{r + \lambda} \left(1 + \frac{\lambda}{r + \delta^*(\lambda)} \right) \right) \frac{rU}{r + \delta^*(\lambda)} - c$$

As a result, Δ boils down to

$$\Delta(\lambda) = \frac{\lambda}{r + \lambda} \left(F - U \left[1 - \left(\frac{\delta^*(\lambda)}{r + \delta^*(\lambda)} \right)^2 \right] \right)$$

and its derivative writes

$$\Delta'(\lambda) = \frac{r}{\lambda(r+\lambda)}\Delta(\lambda) + \frac{2\lambda r U \left(\delta^*\right)'(\lambda)\delta^*(\lambda)}{(r+\lambda)\left(r+\delta^*(\lambda)\right)^3}$$

Note that

$$\Delta'\left(\lambda^*\right) = \frac{2\lambda^* r U \left(\delta^*\right)'\left(\lambda^*\right)\delta^*\left(\lambda^*\right)}{\left(r+\lambda^*\right)\left(r+\delta^*\left(\lambda^*\right)\right)^3} > 0$$

Thus, λ^* is unique. Otherwise, there would exist one λ^* such that $\Delta^*\left(\lambda^*\right) = 0$ and $\left(\Delta^*\right)'\left(\lambda^*\right) < 0$. As a result, $\Delta(\lambda) \leq 0$ for $\lambda \leq \lambda^*$ and $\Delta(\lambda) > 0$ otherwise. 2 cases arise:

- if $S_o^f\left(\lambda^p\right) \leq 0 = S_o^f\left(\overline{\lambda}\right)$, as S_o^f is decreasing, $\lambda^p \geq \overline{\lambda}$. Moreover, $\Delta\left(\lambda^p\right) \leq 0$, which implies that $\lambda^p \leq \lambda^*$. Overall, $\overline{\lambda} \leq \lambda^p \leq \lambda^*$. Job creation only occurs through open-ended contracts and encompasses matches with $\lambda \in [0, \lambda^p]$.
- if $S_o^f\left(\lambda^p\right) > 0 = S_o^f\left(\overline{\lambda}\right)$, as S_o^f is decreasing, $\lambda^p < \overline{\lambda}$. Moreover, $\Delta\left(\lambda^p\right) > 0$, which implies that $\lambda^p > \lambda^*$. Overall, $\lambda^* < \lambda^p < \overline{\lambda}$. Job creation occurs through open-ended contracts when $\lambda < \lambda^*$, and through fixed-term contracts when $\lambda^* < \lambda < \overline{\lambda}$. Fixed-term matches such that $\lambda^* < \lambda < \lambda^p$ and that do not face an adverse productivity shock are converted into open-ended matches.

□

A.2 Additional graphs

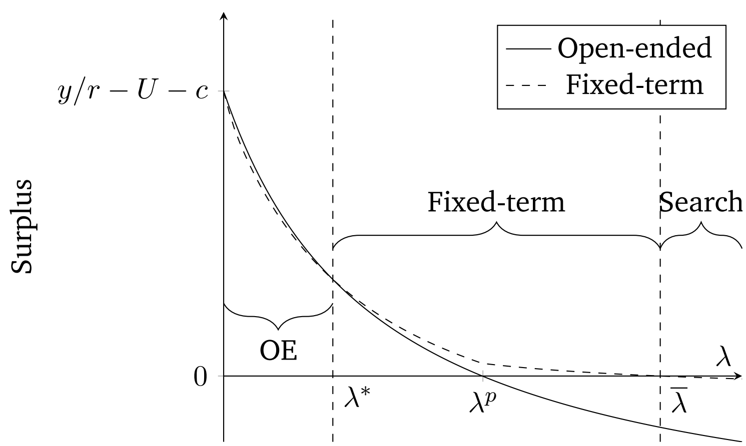

Figure 13: Arrival rate of productivity shocks and job creation: dual case

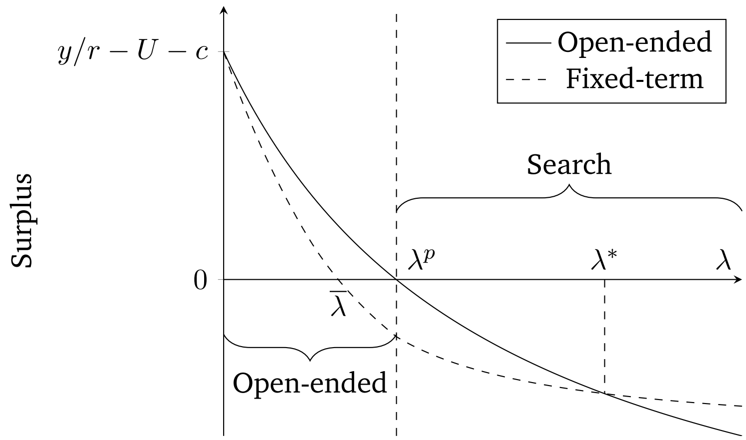

Figure 14: Arrival rate of productivity shocks and job creation: open-ended jobs only

B Heterogeneous productivities

B.1 Proofs

Proposition 9

Proof. I denote $\rho^p = 1/(r+s+\lambda)$ and $\rho^f = 1/(r+\delta+\lambda)$. As mentioned above, $\rho^p > \rho^f$.

- Assume that $y^* > y^f$. (4) implies that $\rho^p y^* = \left(\rho^p - \rho^f\right) y^* + \rho^f y^* = \rho^p y^c + \rho^f \left(y^* - y^f\right)$. Since $y^* - y^f > 0$, the latter equality implies $y^* > y^c$.
- Assume that $y^* > y^c$. Again, jointly with algebraic manipulations, (4) implies that $\rho^f y^c = -\left(\rho^p - \rho^f\right) y^c + (\rho^p - \rho^f) y^* + \rho^f y^f > -\left(\rho^p - \rho^f\right) y^c + (\rho^p - \rho^f) y^c + \rho^f y^f > \rho^f y^f$, which entails that $y^c > y^f$.
- Assume that $y^c > y^f$. Algebraic manipulations and (4) imply that $\left(\rho^p - \rho^f\right) y^* = \rho^p \left(y^c - y^f\right) + \left(\rho^p - \rho^f\right) y^f > \left(\rho^p - \rho^f\right) y^f$, which implies $y^* > y^f$. □

□

Proposition 10

Proof.
- If $F \geq rU/(r+\delta)$, $y^c \geq y^f$. Referring to proposition 9, the latter inequality entails $y^* \leq y^f \leq y^c$. (5) entails that only open-ended contracts weigh in job creation for productivities $y > y^c$.
- Otherwise, $y^c < y^f$ and proposition 9 ensures that $y^f < y^c < y^*$. (5) entails that job creation is dual. For productivities $y^f < y < y^*$, job creation occurs through fixed-term contracts. Productivities y such that $y > y^*$ involve open-ended contracts.

□

Proposition 11

Proof.
- If $F \geq rU/(r+\delta)$, then $S^f(y^c) \geq 0 = S^p(y^c)$. Since S^f is increasing in y and $S^f\left(y^f\right) = 0$, y^f and y^c verify $y^f \leq y^c$.

  For all $y \leq y^c$, S^p and S^f are linear in y and

$$S^p(y) = \frac{y - y^c}{r + s + \lambda}$$
$$S^f(y) = \frac{y - y^f}{r + \delta + \lambda}$$

Using these expressions, one may demonstrate that fixed-term contracts deliver a higher joint surplus than their open-ended counterparts for productivities $y \leq y^c$.

$$S^f(y) = \frac{y^c - y^f}{r + \delta + \lambda} + \frac{y - y^c}{r + \delta + \lambda} > \underbrace{S^f(y^c)}_{\geq 0} + \underbrace{\frac{y - y^c}{r + s + \lambda}}_{= S^p(y)} \geq S^p(y)$$

The first inequality inequality stems from the fact that $s < \delta$, which ensures that $(r+s+\lambda)^{-1} > (r + \delta + \lambda)^{-1}$.

As a result, the only hires with a productivity lower than y^c occur through fixed-term contracts with a minimal productivity y^f.

As for productivities $y > y^c$, using the linearity of S^f and S^p, one may show that fixed-term contracts still dominate open-ended ones.

$$S^f(y) = \underbrace{\left(1 + \frac{s}{r + \delta + \lambda}\right)}_{>1} \frac{y - y^c}{r + s + \lambda} + S^f(y^c) > S^p(y) + \underbrace{S^f(y^c)}_{\geq 0} > S^p(y)$$

- Otherwise, $S^f\left(y^c\right) < 0 = S^p\left(y^c\right)$. S^f being increasing in y and $S^f\left(y^f\right) = 0$, y^f and y^c verify $y^f > y^c$. Moreover, S^p and S^f are in linear in y and for all $y > y^c$

$$\frac{\partial S^f}{\partial y} = \left(1 + \frac{s}{r + \delta + \lambda}\right) \frac{1}{r + s + \lambda} > \frac{1}{r + s + \lambda} = \frac{\partial S^p}{\partial y}$$

Thus, there exists $y^* > y^f$ such that $S^p\left(y^*\right) = S^f\left(y^*\right)$. Consequently, the ranking of joint surpluses verify

$$\begin{cases} S^p(y) > 0 > S^f(y) & \forall y^c < y < y^f \\ S^p(y) > S^f(y) > 0 & \forall y^f < y < y^* \\ S^f(y) > S^p(y) > 0 & \forall y > y^* \end{cases}$$

Job creation takes place accordingly.

□

Proposition 12

Proof. Let me denote $z(y) = y - r\alpha - (r+\lambda)\left(S_0^p(y)^+ - \int S_0^p(y')^+ \, dG(y')\right)$. Notice that y is increasing in y. Algebraic manipulations deliver another expression of S^f as follows

$$S^f(y,\delta) = \frac{1}{r+\delta+\lambda}\left(z(y) - \frac{\lambda}{r+\delta}(r\alpha - Ey)\right) + S_0^p(y)^+ \tag{9}$$

Differentiating this expression with respect to δ yields

$$\frac{\partial S^f}{\partial \delta} = -\frac{1}{(r+\delta+\lambda)^2}\left(z(y) - \frac{\lambda}{r+\delta}(r\alpha - Ey)\right) + \frac{1}{r+\delta+\lambda}\frac{\lambda}{(r+\delta)^2}(r\alpha - Ey)$$

Consequently, provided that $r+\delta > 0$,

$$\frac{\partial S^f}{\partial \delta} \geq 0 \Leftrightarrow z(y)(r+\delta)^2 - 2\lambda(r\alpha - Ey)(r+\delta) - \lambda^2(r\alpha - Ey) \leq 0$$

Studying the variations of S^f boils down to assessing the sign of a second-degree polynomial in $(r+\delta)$.

- If $r\alpha = Ey$, $sign\left(\frac{\partial S^f}{\partial \delta}\right) = sign\,(-z(y))$. In this case, $\delta^*(y) \in \{0, +\infty\}$ if $z(y) \neq 0$. Otherwise, any non-negative δ maximizes S^f.
- If $r\alpha > Ey$, $sign\left(\frac{\partial S^f}{\partial \delta}\right) = sign\,(P_y(r+\delta))$, where $P_y(X) = -x(y)X^2 + 2\lambda X + \lambda^2$. Several subcases arise.
  - If $x(y) \leq -1$, $\partial S^f/\partial\delta \geq 0$ and $\delta^*(y) = +\infty$
  - If $-1 < x(y) < 0$, P_y has two negative roots and, thus, is positive on $(0,+\infty)$. Consequently, S^f is increasing in δ over the latter interval and $\delta^*(y) = +\infty$
  - If $x(y) = 0$, P_y is linear and positive over $(0,+\infty)$ and $\delta^*(y) = +\infty$.
  - If $x(y) > 0$, P_y has one negative and one positive root, the latter verifying $r + \delta_0 = \lambda\frac{1+\sqrt{1+x(y)}}{x(y)}$. Either $\delta_0 < 0$, in which case S^f decreases over $(0,\infty)$ and $\delta^*(y) = 0$, or $\delta_0 \geq 0$, in which case S^f attains a maximum at $\delta^*(y) = \delta_0 \in (0,\infty)$.
    When do we have $\delta_0 \geq 0$? The latter condition is equivalent to $r(1+x(y)) - \lambda\sqrt{1+x(y)} - (r+\lambda) \leq 0$, which is - again - a second-degree polynomial in $\sqrt{1+x(y)}$ with roots -1 and $(\lambda+r)/r$. Consequently, $\delta_0 \geq 0$ if and only if $-1 \leq \sqrt{1+x(y)} \leq (\lambda+r)/r$. Since $x(y) > 0$, this is true whenever $x(y) \leq \frac{\lambda}{r}\left(2 + \frac{\lambda}{r}\right)$.
- If $r\alpha < Ey$, $sign\left(\frac{\partial S^f}{\partial \delta}\right) = -sign\,(P_y(r+\delta))$. One may revisit the cases tackled above.
  - If $x(y) \leq 0$, S^f is decreasing in δ over $(0,+\infty)$ and $\delta^*(y) = 0$
  - If $x(y) > 0$, P_y has one negative and one positive root, the latter verifying $r + \delta_0 = \lambda\frac{1+\sqrt{1+x(y)}}{x(y)}$. Either $\delta_0 < 0$, in which case S^f increases over $(0,\infty)$ and $\delta^*(y) = +\infty$, or $\delta_0 \geq 0$, in which case S^f attains a minimum at $\delta_0 \in (0,\infty)$ and $\delta^*(y) \in \{0, +\infty\}$.

□

Proposition 13

Proof. The choice between a fixed-term and a open-ended contract at the hiring stage can be summed up in the sign of the function Δ defined as $\Delta(y) = S_0^f(y) - S_0^p(y)^+$. The latter can be rewritten as

$$\Delta(y) = S^f\left(y, \delta^*(y)\right) - c - S_0^p(y)^+$$

Using (9), the previous equation becomes

$$\Delta(y) = \frac{r\alpha - Ey}{r + \delta^*(y) + \lambda}\left(x(y) - \frac{\lambda}{r + \delta^*(y)}\right) - c$$

Several cases arise when $c > 0$

- If $x(y) \leq 0$, $\delta^*(y) = +\infty$ and $\Delta(y) = -c < 0$. Hiring only takes place through open-ended contracts under the constraint that $y \geq y^c$.
- If $0 < x(y) < \frac{\lambda}{r}\left(2 + \frac{\lambda}{r}\right)$, using the definition of δ^* spelled in proposition 12, one may rewrite $\Delta(y)$ as

  $$\Delta(y) = \frac{r\alpha - Ey}{\lambda}\left(\frac{x(y)}{1 + \sqrt{1 + x(y)}}\right)^2 - c$$

  Algebraic manipulations entail that

  $$sign\left(\Delta(y)\right) = sign\left(x(y) + 1 - \beta\sqrt{1 + x(y)} - (1 + \beta)\right)$$

  $$\text{where } \beta = \sqrt{\frac{c\lambda}{r\alpha - Ey}}$$

  The right-hand side of the equation above is a second-degree polynomial in $\sqrt{1 + x(y)}$. Therefore, since $x(y) > 0$, $\Delta(y) \geq 0$ if and only if $x(y) \geq (2 + \beta)\,\beta$.
- If $x(y) \geq \frac{\lambda}{r}\left(2 + \frac{\lambda}{r}\right)$, $\delta^*(y) = 0$ and $\Delta(y) = \frac{r\alpha - Ey}{r + \lambda}\left(x(y) - \frac{\lambda}{r}\right) - c$. Thus, $\Delta(y) \geq 0$ if and only if $x(y) \geq \frac{\lambda}{r} + \frac{c(r+\lambda)}{r\alpha - Ey}$.

  Notice that the latter condition is always fulfilled in this specific case if and only if $\beta \leq \frac{\lambda}{r}$. If $\beta \leq \frac{\lambda}{r}$,

  $$\begin{aligned} \frac{\lambda}{r} + \frac{c(r+\lambda)}{r\alpha - Ey} &= \frac{\lambda}{r} + \beta^2\left(1 + \frac{r}{\lambda}\right) \\ &\leq \frac{\lambda}{r} + \beta^2 + \beta \\ &\leq \frac{\lambda}{r} + \left(\frac{\lambda}{r}\right)^2 + \frac{\lambda}{r} \\ &\leq \frac{\lambda}{r}\left(2 + \frac{\lambda}{r}\right) \end{aligned}$$

  Conversely, if $\frac{\lambda}{r} + \frac{c(r+\lambda)}{r\alpha - Ey} = \frac{\lambda}{r} + \beta^2\left(1 + \frac{r}{\lambda}\right) \leq \frac{\lambda}{r}\left(2 + \frac{\lambda}{r}\right)$, algebraic manipulations directly entail that $\beta^2 \leq \left(\frac{\lambda}{r}\right)^2$, which proves that $\beta < \frac{\lambda}{r}$.

We now have all the necessary information to circumscribe the equilibria with dual job creation.

- If $\beta < \frac{\lambda}{r}$, $\Delta(y) \geq 0$ if and only if $x(y) \geq (2+\beta)\,\beta$. In other words, fixed-term contracts are hired whenever $x(y) \geq (2+\beta)\,\beta$. x being increasing in y, there is room for job creation through open-ended contracts if and only if $x\left(y^c\right) < (2+\beta)\,\beta$.

- If $\beta \geq \frac{\lambda}{r}$, $\Delta(y) \geq 0$ if and only if $x(y) \geq \frac{\lambda}{r} + \beta^2\left(1+\frac{r}{\lambda}\right)$. Job creation is dual if and only if $x\left(y^c\right) < \frac{\lambda}{r} + \beta^2\left(1+\frac{r}{\lambda}\right)$. In this case, all fixed-term contracts have a zero probability of job destruction.

To end the proof, notice that $\beta < \frac{\lambda}{r}$ is equivalent to $c < \frac{\lambda}{r}\left(\alpha - \frac{Ey}{r}\right)$. Moreover, when $c = 0$, revisiting each point above ensures that hiring through a fixed-term contract is weakly preferable to hiring through a open-ended contract. $\square$

B.2 Additional graphs

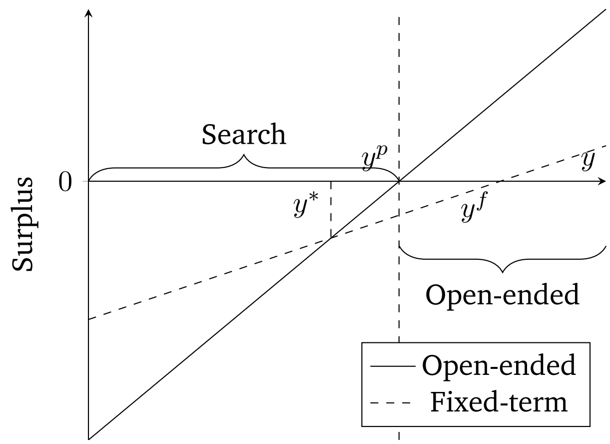

Figure 15: Job creation with open-ended jobs only

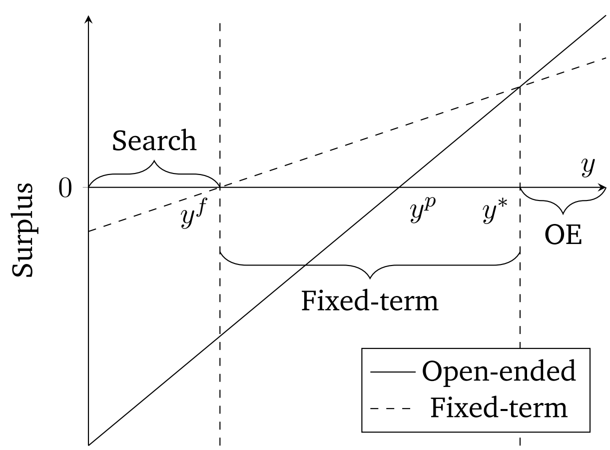

Figure 16: Dual job creation