Computation Practices of the Assyrian Merchants During the Nineteenth Century BCE

Cécile Michel

Chapter 10

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Abstract - Among Old Assyrian school texts from Aššur and Kaneš, a group of lenticular tablets presents conversion exercises. Their statements, as well as the numerical data employed, are very similar to those given by merchants in their letters. For their everyday activities, the Assyrians used the Mesopotamian weight system to quantify the metal – gold, silver, tin and copper – objects of their commercial exchanges. Weighed silver was often used as a means of payment. In order to compute the value of a given metal in silver, it was necessary to multiply or to divide the quantity of metal by the specified conversion ratio.

As we do not have tablets showing the intermediary steps, we do not know, up to now, how the merchants carried out the conversions. A systematic analysis of the data given by the Assyrians when purchasing metals and the results they obtained gives clues about the computation methods they might have been using. These appear to be very different from those utilized by contemporaneous Babylonian administrators.

Abbreviations

AKT 1	see Bilgiç and Günbattı (1995)
AKT 5	see Veenhof (2010b)
AKT 6b	see Larsen (2013)
BIN 4	see Clay (1927)
BIN 6	see Stephens (1944)
CCT 2	see Smith (1924)
CCT 3	see Smith (1925)
CTMMA 1	see Larsen (1988)
Kt ?/k	Tablet found during the 1948- Turkish excavations at Kültepe and preserved in the Anadolu Medeniyetleri Müzesi, Ankara.
Prag I	see Hecker et al. (1998)
TC 2	see Thureau-Dangin (1928)
TC 3	see Lewy (1937)

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10.1 Introduction

The hundreds of thousands of cuneiform tablets found in Near Eastern archaeological sites allow scholars to reconstruct the history of this area during the last three millennia before our era. This huge collection of written sources highlights different cultures of quantification and computation according to the period, the geographical area or the milieu under consideration.[2]

During the first centuries of the second millennium BCE, Assyrian merchants were involved in a long-distance trade with Central Anatolia, where they left thousands of tablets, mainly linked to commercial accounting. These private archives have been found at Kültepe, ancient Kaneš, 22 km north-east of Kayseri. For their everyday activities, the Assyrians used the Mesopotamian weight system to quantify the metal – gold, silver, tin and copper – objects of their commercial exchanges. A close look at the notation of quantities shows that merchants always preferred to use fractions of the higher metrological unit. Weighed silver was often used as a means of payment. In order to compute the value of a given metal in silver, it was necessary to multiply the quantity of metal by the specified conversion ratio, or to divide it by the conversion ratio.
A group of school texts on lenticular tablets presents exercises for such conversions. Their statements, as well as the numerical data employed, are very similar to those given by the merchants in their letters sent to their correspondents in Anatolia.

As we do not have tablets showing the intermediary steps, we do not know, up to now, how the merchants carried out the conversions. With this chapter, I propose a systematic analysis of the computations carried out by Assyrian merchants when purchasing or selling metals. The results given by merchants are often correct, but there are also errors, *i.e.* incorrect results, or approximations, including rounding upwards or downwards.[3] A detailed examination of the results obtained allows us to present some hypotheses on their culture of computation. Did the Assyrians convert their data into the sexagesimal place value notation to carry out their computations, like the contemporaneous Babylonian administrators? Or did they compute directly with fractions? Are the notations of quantity linked to the merchants' computational practices?

[2] Chemla, Keller and Christine Proust (forthcoming), and the contributions of Michel et al., Rost, Sauvage, Lecompte, Proust and Middeke-Conlin in this volume.
[3] The systematic analysis of errors in the accounting of Babylonian administrators was studied by Middeke-Conlin (2015).

After a general presentation of the documentation studied and the notations used to express numbers and quantities, I will analyse the conversions carried out by the merchants, first in the school texts, then in the texts of practice, in order to present the various clues I have found on the computational methods employed by the Assyrian merchants.

10.2 Numbers and Quantities in the Old Assyrian Archives

The Old Assyrian textual corpus constitutes the first large group of private archives in the history of humanity.[4] These sources document the activities of merchants engaged in long-distance trade between the cities of Aššur, on the Tigris (modern Iraq), and of Kaneš, central Anatolia (site of Kültepe, near the modern city of Kayseri, modern Turkey). These merchants used a metrological system of weights to quantify the metals that they traded from the East to the West and back.

10.2.1 The Archives of the Assyrian Merchants at Kaneš and Their Context

At the beginning of the second millennium BCE, some Assyrian merchants left Aššur and settled in forty or so towns in Anatolia, in order to develop long-distance trade. Only three of these settlements have been excavated. Kültepe, the ancient town of Kaneš, was the administrative centre of this trade network.[5]

The Assyrian merchants' archives, concentrating on the first half of the nineteenth century BCE, were preserved in their houses located in the lower town at Kaneš. Up to now, the archives, which belonged to two or three generations of merchants, include some 22 500 cuneiform tablets.[6] The local palace, seat of the Anatolian authority, did not yield any tablets, and the central Assyrian administration in both Kaneš and in Aššur have not been recovered. Thus, all that can be reconstructed for the Old Assyrian period relies on private archives with a strong focus on trade. In Aššur, only two dozen texts dating to this period were found in later levels.

4 For general studies on the Old Assyrian archives see Veenhof (2003); Veenhof (2013), Michel (forthcoming a).
5 Veenhof (2008a); Veenhof (2010a); Larsen (2015).
6 For a detailed inventory of the Old Assyrian archives and a complete bibliography of this corpus, see Michel (2003, 2006a, 2011, 2015).

These Old Assyrian written sources represent the first evidence of a complex, long-distance commercial system. In Aššur, merchants bought tin, originating from the Far East, presumably Uzbekistan, and textiles. Some of the textiles were imported from Babylonia, and the rest were produced locally by the merchants' wives and daughters. They also acquired robust black donkeys able to transport the merchandise all the way to Anatolia, a journey of six weeks across some 1 300 kilometres.[7] Once in Anatolia, the Assyrians sold their goods for cash or on credit, via the local trade in copper and wool.[8] On the return journey, they brought gold and silver to Aššur. These precious metals were used to purchase merchandise in order to set up new caravans, were hoarded or invested into real estate and slaves. Despite the numerous and often high taxes that had to be paid to the Assyrian and Anatolian authorities and *en route*, the Old Assyrian trade was extremely profitable. The tin was sold for twice its purchase price in Anatolia, and the textiles three times their cost.

The merchant archives deal with the purchase and the sale of merchandise, as well as the computation of the various taxes. One of the main operations carried out by the merchants is the conversion of the value of one metal into another, often silver. Texts also testify to a great variety of joint ventures, for which it was necessary to calculate the dividends and the share of each associate, taking into account profits and losses. All these data may be reconstructed from the variety of cuneiform texts which were recovered. The Old Assyrian texts include letters and legal texts, among which are many commercial and financial contracts, and various types of personal memoranda. All these are usually referred to as text of practice, which means that these archives served a practical purpose. The other types of texts found in the merchants' archives are rare literary and historical texts, incantations, and school texts.

Old Assyrian school texts are quite scarce, about twenty tablets were found in Aššur and in Kaneš; some of these are still unpublished. School texts reflect the school curriculum followed by some educated merchants. Thirteen small round tablets bear conversion exercises (see below),[9] and four others consist mostly of lexical and metrological lists written over several columns on large tablets.[10]

The following study is based on both school texts and texts of practice. Before studying their content, I will first explain the notations used by nineteenth century BCE Assyrian merchants to express numbers and quantities.

7 Michel (2008a); Veenhof (2008b).

8 Dercksen (1996).

9 Donbaz (1985). There are more similar school texts, still unpublished, which are preserved both in Berlin and Ankara.

10 Hecker (1993); Michel (2006b); Michel (2008b). The tablet (Kt 00/k 12) is a list of personal names arranged according to the theophoric element by which they start (Donbaz 2004, 185). A long list contains expressions and phrases quite common in merchants' letters (Kt v/k 7 + u/k 31, Hecker 1993; Michel 2010a).

10.2.2 The Notation of Numbers and Quantities as Illustrated by a List of Weights

Among the school texts excavated at Kaneš, a large metrological list, dated to the eighteenth century BCE, presents a progressive list of weights from 1 *gin* to 60 or 100 *gu*.[11] This list, which has been the subject of another article,[12] is very different from those found in Southern Mesopotamia; it has no systematic order. Its author's intention was both to help in memorizing the weight system and to practice the different notations of integers and fractions. This list of weights reproduces the notation of numbers and quantities that were in use during the Old Assyrian period. The Assyrian merchants favoured the use of fractions in their notation and invented new signs to express 1/6 and 1/4.

1/6 1/4 1/3 1/2 2/3 5/6

The weight system, the first metrological system used by the merchants, is the standard Mesopotamian weight system.

60 60 180

gu_2 → mana → gin_2 → še

(≈30 kg) (≈ 500 g) (≈8.33 g) (≈0.046 g)

In the list, the scribe used subtractive notation repeatedly. For example (col. ii:13), the quantity 1 *mana* 19 *gin* is expressed with the subtractive notation 1 1/3 ma-na la_2 1 gin_2, thus '1 1/3 *mana* minus 1 *gin*'. Later, the scribe writes (col. iii:18) 20 la_2 1 ma-na, '20 *mana* minus 1 *mana*', thus '19 *mana*', and (col. iv:19) 20 la_2 1 gu_2, '20 minus 1 *gu*', thus '19 *gu*'.

The Assyrian merchants did not use the Mesopotamian sexagesimal system in which a vertical wedge corresponds to 1 or 60. Instead, they used an additive and decimal system in which 60 was written with six times the sign for 10. The numbers 100 and 1000 were expressed by their Akkadian names, *meat* and *lim* respectively. When using the metrological system of weights,

[11] Kt t/k 76+79; Michel (1998). The reverse of this tablet bears a lexical list enumerating various metals, stones and plants. The fragment (k/- 129) also belongs to the lexical list genre; it gives names of stones, animals, aromatics, and multiplication formulas using a-ra_2 'multiply' in a broken context.

[12] Michel (forthcoming b).

they preferred using fractions of the higher unit instead of integers of the lower unit, for example, they wrote 1/3 *mana* instead of 20 *gin*.[13]

10.3 Conversions

The most frequent computations carried out by the merchants were conversions. The scribe had to calculate the weight of silver necessary for the purchase of a fixed weight of another metal.[14] Two different operations were carried out depending on the value of the metal to be converted into silver. When the value of the other metal was higher than that of silver, as for gold, the merchant had to do a multiplication, while when the value of the other metal was lower than that of silver, as for tin, he had to do a division.
Such computations are found both in school texts and in texts of practice, *i.e.* letters and personal memoranda.

10.3.1 Conversions in School Texts

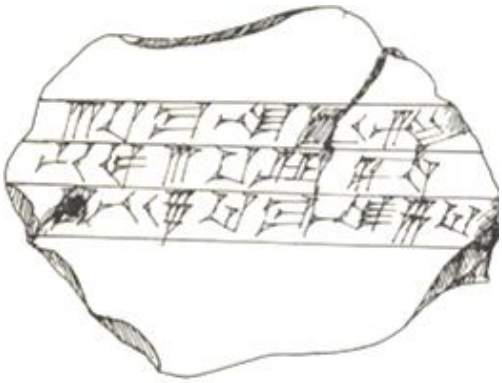
The vast majority of the Old Assyrian school tablets found in Aššur and Kaneš present a round shape which was quite frequent for such exercises as they were easy to produce; they contain conversion exercises. Only some have been published up to now.
In the following examples, the first two lines include the statement of the exercise and the last line gives the solution, *i.e.* the result of the conversion of the initial quantity of gold into silver. To reach this result one simply needs to multiply the quantity of gold by the price of one *gin* of gold to find its price in silver. The intermediary steps of the operation are never given on the tablets.

[13] Such use of fractions is a constant in merchants' milieus, as witness for example the Middle Ages written sources (Benoit 1992). According to Patrice Baubeau (personal communication), it is only in the middle of the 1970s that the New York Stock Exchange abandoned fractions for decimal numbers, because of its computerization.
[14] Michel (1992); Michel (2008b)

Provenance	Preservation	Inventory No.	Publication	No. in the present chapter
Aššur	Vorderasiatisches Museum, Berlin	A 1001 = Ass 14479 (too damaged)	Donbaz (1985)	
Aššur	Vorderasiatisches Museum, Berlin	Ass 13058e = 13058i	Donbaz (1985: 16)	**10.3.1.C**
Aššur	Vorderasiatisches Museum, Berlin	Ass 13058f	Donbaz (1985: 5, 16)	**10.3.1.A**
Aššur	Vorderasiatisches Museum, Berlin	Ass 13058g	Donbaz (1985: 16)	**10.3.1.B**
Aššur	Vorderasiatisches Museum, Berlin	Ass 13058k	Donbaz (1985: 16)	**10.3.1.F**
Aššur	Vorderasiatisches Museum, Berlin	Ass 13058l	unpublished	
Aššur	Vorderasiatisches Museum, Berlin	Ass 13058m	unpublished	
Aššur	Vorderasiatisches Museum, Berlin	Ass 13058n	unpublished	
Aššur	Vorderasiatisches Museum, Berlin	Ass 13058o	unpublished	
Aššur	Vorderasiatisches Museum, Berlin	Ass 13058p	unpublished	
Kaneš	Anadolu Medeniyetleri Müzesi, Ankara	Kt a/k 178	Hecker (1996)	**10.3.1.D**
Kaneš	Anadolu Medeniyetleri Müzesi, Ankara	Kt 84/k 3	Donbaz (1985:, 7; no copy)	**10.3.1.E**
Kaneš	Anadolu Medeniyetleri Müzesi, Ankara	Several, unknown	unpublished	

Table 10.1 Old Assyrian conversion school exercises

10.3.1.A



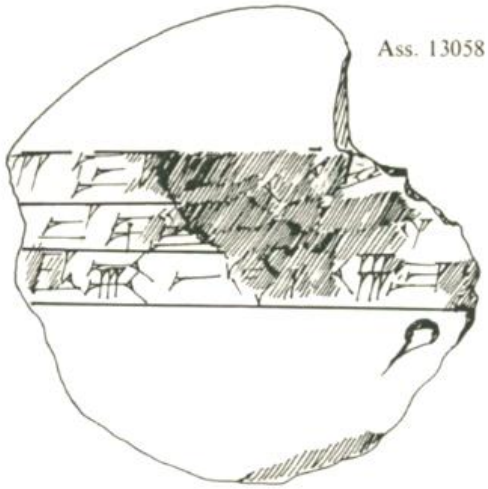
5[!*] 1/3 *ma-na* ku_3-gi
$hu\text{-}sa_2$ 3 1/3 gin_2-ta
ku_3-bi_4 17 2/3 *ma-na* 6 2/3 [gin_2]

*The number on the copy is written '3'.

'5 1/3 *mana* of red gold at
3 1/3 *gin* (of silver) each (*gin* of gold), its (value in) silver:
17 2/3 *mana* 6 2/3 *gin*'

The result is correct.
The operation is the following: (5 +1/3) *mana* × (3+1/3) = 17 2/3 *mana* + 1/9 *mana* = 17 2/3 *mana* 6 2/3 *gin*

10.3.1.B

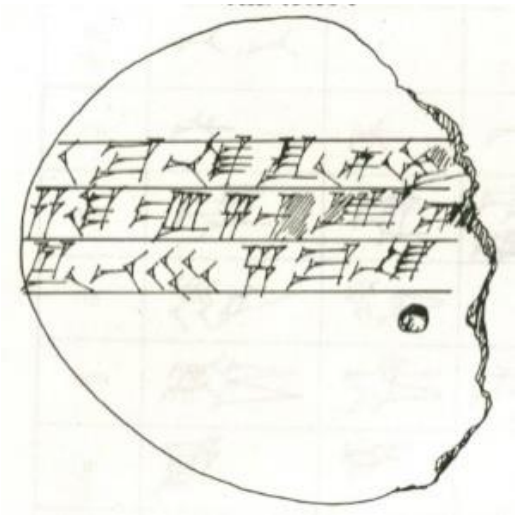


[1]4 *ma*-[*na* ku_3-g]i
[*li*]*q-tum* [5 1/2 gin_2-t]a
ku_3-bi 1 gu_2 17 *ma*-[*na*]

'[1]4 *ma*[*na* of go]ld of [*li*]*qtum* quality
at [5 1/2 *gin* (of silver) eac]h (*gin* of gold)
its (value in) silver: 1 *gu* 17 *mana*'

With the restitutions given in the broken parts of the tablet, the result is correct.
The operation is the following: (14) *mana* × (5+1/2) = 70 *mana* + 7 *mana* = 1 *gu* 17 *mana*

10.3.1.C



10 *ma-na* ku$_3$-gi
za-ki-um 5 1/2 gin$_2$-ta
ku$_3$-bi$_4$ 55 *ma-na*

‘10 *mana* of pure gold
at 5 1/2 *gin* (of silver) each (*gin* of gold)
its (value in) silver: 55 *mana*’

The result is correct.
The operation is the following: (10) *mana* × (5+1/2) = 50 *mana* + 5 *mana* = 55 *mana*

10.3.1.D

«MAŠ (or 1/2)»
10 *ma-na* ku$_3$-gi
za-ki-um 5 1/2 gin$_2$-ta
ku$_3$-babbar[^{áp}-*šu*] 55 *ma-na*

‘10 *mana* of pure gold
at 5 1/2 *gin* (of silver) each (*gin* of gold)
its (value in) silver: 55 *mana*’

This text, which was found in a private house at Kaneš, shows the same exercise as the previous one unearthed at Aššur. The first line with only a MAŠ or 1/2 sign does not make sense.

10.3.1.E

5 *ma-na* ku$_3$-gi *liq-tum*	5 *mana* of gold of *liqtum* quality
15 gin$_2$-ta ku$_3$-babbar ^{áp}-*šu*	at 15 *gin* (of silver) each (*gin* of gold),
1 gu$_2$ 15 *ma-na*	its (value in) silver (is) 1 *gu* 15 *mana*

The result is correct.
The operation is the following: 5 *mana* × 15 = 75 *mana* = 1 *gu* 15 *mana*

The next exercise is a little more complicated since the scribe had to calculate the weight of silver necessary for the purchase of copper which is quantified by a top-pack (*elītum*), whose weight was 30 *mana* according to other texts.[15] The computation was presumably done by a rule of three, which might have been part of the arithmetical training of apprentice merchants. In order to solve this exercise, the scribe had to multiply the given quantity of copper by the exchange rate, and divide the result by the weight of an *elītum*-package. This tablet was found in Aššur; copper was rated at between 90 and 100 *gin* of copper for 1 *gin* of silver. Here the rate is slightly above, *i.e.* 50 *gin* of copper for 1 *gin* of silver, which is very expensive compared to the highest rates attested in Anatolia for refined copper (60 *gin* of copper for 1 *gin* of silver).[16]

10.3.1.F

2^{l} gu$_2$ 14 *ma-na* urudu
e-li-a-tum 1/2 *ma-na*
5 gin$_2$-ta ku$_3$-bi$_4$ 2 *ma-na*
6 1/2 gin$_2$ 15 še

‘2 *gu* 14 *mana* of copper
the package-*elitum* at 1/2 *mana*
5 *gin.* Its (value in) silver: 2 *mana*
6 1/2 *gin* 15 *še*’

15 For the weight of the top-pack (*elītum*), see Veenhof (1972: 13, 25-27).
16 Dercksen (1996: 159).

The correct result is 2 1/2 *mana* 6 1/3 *gin*.
The operation is the following:

(2 *gu* + 14 *mana*) × (1/2 *mana* + 5 *gin*)÷30 *mana* =
(4 *mana* + 14/30 *mana*) × (1/2 *mana*+5 *gin*) =
2 *mana* + 7/30 *mana* + 20 *gin* + 70/30 *gin* =
2 *mana* + 36 *gin* + 1/3 *gin* =
2 1/2 *mana* 6 1/3 *gin.*

The scribe could have computed keeping the fractions as I did, or he could have used other methods to find the correct result. There is no indication about the way he reached his solution.

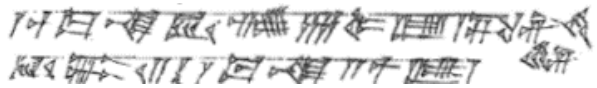
10.3.2 Conversions Operated by Merchants in Their Everyday Activities

Such school texts can be seen as on-the-job training. Indeed, conversions were the most common operations carried out by the merchants. A comparison of these exercises with computations made in the merchants' accounts confirms this fact.

No. in the present chapter	Inventory no.	Publication	Provenance	Preservation
10.3.2.A	AO 8264	TC 3, 43:6-7 copied by Julius Lewy	Kaneš	Musée du Louvre, Paris
10.3.2.B	BM 113286	CCT 3, 5a:4-6 copied by Sidney Smith	Kaneš	British Museum, London

Table 10.2 Old Assyrian texts analysed in section 10.3.2

10.3.2.A Extract of a letter



1 1/2 *ma-na* ku_3-gi 8 1/4 gin_2-ta *ni-dí-/in*
ku_3-bi 12 1/3 *ma-na* 2 1/2 gin_2

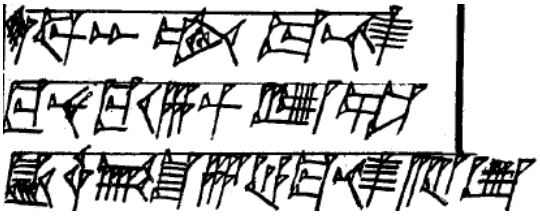
‘1 1/2 *mana* of gold at 8 1/4 *gin* (of silver) each (*gin* of gold) we sold,
its (value in silver): 12 1/3 *mana* 2 1/2 *gin*’

The authors of this letter explained to their correspondent how they sold his gold. They specify the quantity of gold at their disposal, the rate on the market and the price of silver they obtained. The result is correct. The operation, which is not given on the tablet, is the following:

(1+1/2) *mana* × (8+1/4) =
8 *mana* + 4 *mana* + 1/4 *mana* + 1/8 *mana* =
12 *mana* + 22 1/2 *gin* = 12 1/3 *mana* 2 1/2 *gin*

As in the school exercises, we observe that the conversion ratio between the gold and the silver is given with fractions, here 8 1/4 *gin* of silver for 1 *gin* of gold.
In general, computation which consisted in converting a quantity of gold into silver, corresponding to a simple multiplication, is correct. This is less true when the metal to be converted into silver has a value inferior to that of silver, like tin, or copper, presumably because the operation to be carried out is a division of the weight of the metal by the conversion ratio. This is the case in the following example:

10.3.2.B Extract of a letter



2 gu_2 10 *ma-na*
ku-nu-ku 16 1/2 gin_2-ta
ku_3-babbar-bi-*šu* 7 5/6 *ma-na* 2 2/3 gin_2

‘2 *gu* 10 *mana* of (tin under) seal at
16 1/2 *gin* (of tin) each (*gin* of silver)
its (value) in silver: 7 5/6 *mana* 2 2/3 *gin*’

The result is an approximation of the correct amount.
The operation is the following:

(2 *gu* + 10 *mana*) ÷ (16 + 1/2) =
(2 *gu* + 10 *mana*) ÷ 33/2 =
(2 *gu* + 10 *mana*) × 2/33 =
7 *mana* + 16 *gin* + 12/33 *gin* + 36 *gin* + 12/33 *gin* =
7 5/6 *mana* 2 24/33 *gin*

2/3 *gin* is the best approximation among the fractions known by the merchants for 24/33 *gin*, a fraction unknown to the Assyrians.

10.3.3 Computations Carried out in such Letters: A Combination of Conversions and Additions

Many letters, described by M. Larsen as caravan accounts, were sent from Aššur to Kaneš giving detailed accounts linked to the purchase activities in Aššur.[17] The following letter was found in 1993 in the archives of Ali-ahum and his son Aššur-taklāku.[18] Its authors explain to an anonymous merchant and to a man called Šū-Bēlum how they spent the gold that they received from them: they converted the gold into silver, and with the silver they bought textiles, tin and donkeys, and paid taxes and the personnel of the caravan which departed for Kaneš with someone called Bazia.

No. in the present chapter	Inventory No.	Publication	Provenance	Preservation
10.3.3.A	Kt 93/k 335	Unpublished	Kaneš	Anadolu Medeniyetleri Müzesi, Ankara

Table 10.3 Old Assyrian text analysed in Sect. 10.3.3

[17] Larsen (1967: 97-140). For the Old Assyrian letters see Michel (2001).
[18] About 1000 tablets were found during the 1993 excavations in one or two much destroyed houses. I thank the former Tahsin Özgüç and the current director of Kültepe excavations, Fikri Kulakoğlu, for their permission to publish this archive.

10.3.3.A Letter. Photo Cécile Michel ©Archaeological mission of Kültepe

Ob. *a-na* dam-gar$_3$ *u$_3$ Šu-be-lim*
2 *qi$_2$-bi-ma um-ma Ah-ša-lim*
I-ku-pi$_2$-a u$_3$ Ba-zi-a-ma
4 2 2/3 *ma-na* ku$_3$-gi *ku-nu-ki-ma*!
Ba-zi-a ub-lam ku$_3$-gi
6 8 2/3 gin$_2$-ta *ta-dí-in*
ku$_3$-bi 23 ma-na 6 2/3 gin$_2$
8 ša$_3$-ba 28 tug$_2^{hi-a}$ sig$_5$ diri
42 tug$_2^{hi-a}$ *qá-áb-li-<ú>-tum*
10 ù 46 tug$_2^{hi-a}$ *ša qá-tim*
šu-nigin$_2$ 1 *me-at* 16 tug$_2^{hi-a}$
12 *qá-dum* : *ša li-wi-tim*
ku$_3$-bi-*šu-nu* 9 1/3 *ma-na* 9 1/2
gin$_2$
14 2 gu$_2$ *an-na ku-nu-ku*
14 gin$_2$-ta ku$_3$-bi
16 8 1/2 *ma-na* 4 1/4 gin$_2$
lo.e.21 *ma-na* an-na^{ak} /*qá-tim*
18 13 1/2 gin$_2$-ta
rev. ku$_3$-bi 1 1/2 *ma-na* 3 1/2 gin$_2$
20 6 anše^{hi-a} *ṣa-lá-mu*
ku$_3$-bi 2 *ma-na* 12 gin$_2$
22 1/3 *ma-na* 1 gin$_2$ ku$_3$-babbar
ú-nu-sú-nu ù ú-/kul-ta-áš-nu
24 2/3 *ma-na* 5 gin$_2$ ku$_3$-babbar
ša 3 *sà-ri-dí* : *a-dí*
26 *Ha-hi-im* : *a-na ta-ba-lim*
ni-dí-in 11 1/2 gin$_2$
28 ku$_3$-babbar *wa-ṣí-tám*
ni-dí-in lu-qú-ut-ku-nu
30 *gam-ra-*[*ku*?]*-nu-tí*
3 5/6 gin$_2$ ku$_3$-babbar
32 *té-ṣú-bi$_4$* : *Ba-zi-a*
ú-lá il$_5$-qé
u.e. *mì-ma a-nim*
ku-nu-ki-ni
le.e. *Ba-zi-a i-ra-dí-a-ku-nu-ti*

[1-3]Say to the *tamkārum*-merchant and to Šū-Bēlum: thus (say) Ah-šalim, Ikuppiya and Bazia.
[4-6]Bazia brought here 2 2/3 *mana* of sealed gold. The gold has been sold at 8 2/3 *gin* (of silver) each (*gin* of gold); [7]its (value in) silver: 23 minas 6 2/3 *gin*.
[8-10]Thereof 28 extra good quality textiles, 42 medium quality textiles and 46 ordinary textiles; [11-12]in all 116 textiles including those for wrapping, [13]their (value in) silver: 9 1/3 *mana* 9 1/2 *gin*.
[14-16]2 talents of sealed tin at 14 *gin* (of tin) each (*gin* of silver); its (value in) silver: 8 1/2 *mana* 4 1/4 *gin*. [17-19]21 *mana* of hand tin at 13 1/2 *gin* (of tin) each (*gin* of silver); its (value in) silver: 1 1/2 *mana* 3 1/2 *gin*.
[20-23]6 black donkeys, their (value in) silver: 2 *mana* 12 *gin*. 1/3 *mana* 1 *gin* of silver: their harnesses and their fodder. [24-29]We gave 2/3 *mana* 5 *gin* of silver (as salary) to 3 donkey drivers for transport to Hahhum. We gave 11 1/2 *gin* of silver as export tax.
[29-33]Your merchandise has been spent for you. Bazia did not take the 3 5/6 *gin* of silver as supplement
[34-36]All this, under your seal, Bazia is leading to you.

The three senders of this letter, including the man in charge of the caravan, first exchanged the gold they received into silver, then purchased tin, textiles and donkeys. The transactions may be summarized as follows:

Conversion of gold into silver

2 2/3 *mana* of sealed gold at 8 2/3 *gin* of silver per gin of gold.
Value in silver: 23 mana 6 2/3 gin. The result is correct.
The operation is the following:

(2+2/3) *mana* × (8+2/3) =
16 *mana* + 16/3 *mana* + 4/3 *mana* + 4/9 *mana* =
22 *mana* + 2/3 *mana* + 4/9 *mana* =
23 *mana* 6 2/3 *gin*

Purchase of textiles

28 extra-good quality textiles, 42 medium quality textiles, 46 ordinary textiles. Total: 116 textiles, plus textiles for wrapping.
Value in silver: 9 1/3 *mana* 9 1/2 *gin*.

Purchases of tin

2 *gu* of sealed tin at 14 *gin* of tin per *gin* of silver.
Value in silver: 8 1/2 *mana* 4 1/4 *gin.*
The correct result is 8 1/2 *mana* 4 2/7 *gin*.
1/4 *gin* is the closest approximation among the fractions known by the Assyrians for 2/7 *gin*.
The operation is the following:

(2 *gu*) ÷ (14) =
8 *mana* + 8/14 *mana* =
8 *mana* + 1/2 *mana* + 1/14 *mana* =
8 1/2 *mana* 4 2/7 *gin*

21 *mana* of hand tin at 13 1/2 *gin* of tin per *gin* of silver.
Value in silver: 1 1/2 *mana* 3 1/2 *gin.*
The correct result is 1 1/2 *mana* 3 1/3 *gin.*
1/2 *gin* is an error for 1/3 *gin*.

The operation is the following:

(21 *mana*) ÷ (13 + 1/2) =
21 *mana* × 2/27 =
42/27 *mana* =
1 *mana* + 15/27 *mana* =
1 1/2 *mana* 3 1/3 *gin*

Purchase of donkeys and other expenses

6 black donkeys, their harnesses and their fodder.
Value in silver: 2 *mana* 12 *gin* + 1/3 *mana* 1 *gin*.
Salary of 3 donkey drivers.
Value in silver: 2/3 *mana* 5 *gin.*
Export tax.
Value in silver: 11 1/2 *gin.*

Total of the expenses

Expenses	***mana***	***gin***
Textiles	9 1/3	9 1/2
sealed tin	8 1/2	4 1/4
(unsealed) tin	1 1/2	3 1/2
donkeys	2	12
harnesses and fodder	1/3	1
donkey drivers	2/3	5
export tax		11 1/2
Total amount	**23**	**6 3/4**

The total of the expenses indicated on the tablet amount to 23 *mana* 6 3/4 *gin* of silver. The quantity of silver obtained by selling the gold, was 23 *mana* 6 2/3 *gin*, which means that the expenses exceeded the silver available by 1/12 *gin* or 15 *še* (+ 15 *še* of silver).
If we take into account the error made by the scribe in converting the unsealed tin into silver and thus correct 1/2 *gin* into 1/3 *gin*, then the total expenses are 23 *mana* 6 7/12 *gin.* With this corrected value, the silver obtained by the sale of the gold has been spent, except for 15 še, which represents about 0,7 g (- 15 *še* of silver). Note that 2/3 *gin* is the best approximation among the fractions known by the merchants for 7/12 *gin*. The difference between the two results is thus 30 *še* of silver.

10.4 Analysis of Conversions

A systematic analysis of the conversions of metals into silver, and the results obtained by the merchants can help us to better understand how they carried out their computations. The following tables bring together several examples, classified according to the metal to be converted. Only the examples which give incorrect results are edited, the others are summarized in the tables. The first section presents transactions on gold, a metal more expensive than silver. Such conversions can be solved with a simple multiplication. The second section groups transactions on tin, a metal cheaper than silver. Such conversions imply a division.

10.4.1 Conversions of Gold into Silver

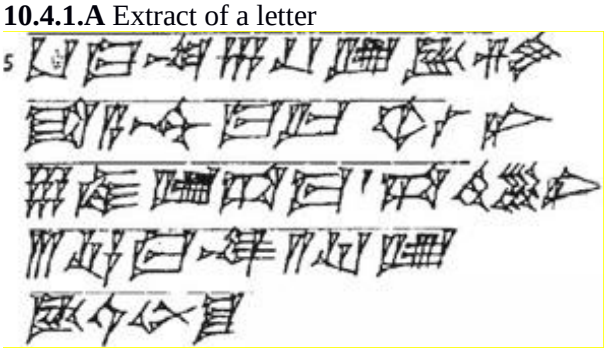
The nine text extracts analysed in this section are referenced in the table below. I choose to study some texts presenting different conversion ratios often expressed with fractions.

No. in the present chapter	Inventory No.	Publication	Provenance	Preservation
10.4.1.A	NBC 3952	BIN 6, 65: 5-9 Copied by Ferris Stephens	Kaneš	Yale Babylonian Collection, New Haven
10.4.1.B	Kt 93/k 769	Unpublished (l. 3-5)	Kaneš	Anadolu Medeniyetleri Müzesi, Ankara
10.4.1.C	BM 115048	CCT 3, 22a:7-9 Copied by Sidney Smith	Kaneš	British Museum, London
10.4.1.D	BM 115048	CCT 3, 22a: 10-12 Copied by Sidney Smith	Kaneš	British Museum, London
10.4.1.E	Kt 93/k 335	Unpublished (l. 4-7)	Kaneš	Anadolu Medeniyetleri Müzesi, Ankara
10.4.1.F	A0 8264	TC 3, 43: 6-7 Copied by Julius Lewy	Kaneš	Musée du Louvre, Paris
10.4.1.G	Prague I 435	Prag I 435: 4-7 Copied by Lubor Matouš	Kaneš	Charles University, Prague
10.4.1.H	AO 8229	TC 3, 36: 3-6 Copied by Julius Lewy	Kaneš	Musée du Louvre, Paris
10.4.1.I	A0 8264	TC 3, 43: 8-9 Copied by Julius Lewy	Kaneš	Musée du Louvre, Paris

Table 10.4 Old Assyrian texts analysed in Sect. 10.4.1

In the three following examples, either the scribe made an error, or he gave an approximate result.

10.4.1.A Extract of a letter



1/3 *ma-na* 6 1/3 gin_2 ku_3-gi
ša A-šur$_3$*-ma-lik-ma ub-la*$_2$*-ni*
9 la_2 1/6 gin_2-ta-*ma* : *ta-dí-in*
3 5/6 *ma-na* 2 2/3 gin_2
ku_3-babbar^{ap}2-*šu*

‘1/3 *mana* 6 1/3 *gin* gold
that Aššur-malik brought, you sold
at 9 minus 1/6 *gin* (silver) each (*gin* gold); 3 5/6 *mana* 2 2/3 *gin*
its (value in) silver’

The operation is the following respecting the subtractive notation:

(1/3 *mana* + 6 1/3 *gin*) × (9-1/6) =
(3 *mana* + 54 *gin* + 3 *gin*) – (1/18 *mana* + 1 *gin* + 1/18 *gin*) =
(3 5/6 *mana* + 7 *gin*) – (4 1/3 *gin* + 1/18 *gin*) =
3 5/6 *mana* 2 2/3 *gin* – 10 *še*

Using this method of computation, the scribe has rounded the correct result up, giving up the – 10 *še*.

The operation is the following when converting the subtractive notation into a positive one:

(1/3 *mana* + 6 1/3 *gin*) × (9-1/6) =
(1/3 *mana* + 6 1/3 *gin*) × (53/6) =
(1/18 *mana* + 1 *gin* + 1/18 *gin*) × 53 =
(4 *gin* + 1/3 *gin* + 10 *še*) × 53 =
3 *mana* + 51 *gin* + 2/3 *gin* + 5/6 *gin* + 20 *še* =
3 5/6 *mana* 2 1/2 *gin* 20 *še*

If the scribe had used this method, then we would have presumably had the following result on the tablet: 3 5/6 *mana* 2 1/2 *gin*, the scribe would have probably rounded the correct result down, dropping the quantity of silver given in the smaller unit, 20 *še*.

There is a strong presumption that the scribe used the first method to compute this conversion, thus respecting the subtractive notation for the conversion ratio and computing with fractions. Instead of giving the conversion ratio with an additive notation, *i.e.* 8 5/6 *gin*, the scribe gave it with a subtractive notation: 9 minus 1/6 *gin*. Such a process may have a simple explanation: such a notation is intended to simplify the computation, which was to multiply the weight of

gold by the conversion ratio. Since the original weight of gold is noted with twice the fraction 1/3, it was presumably simpler to multiply it by 9 instead of by 8, 9 being divisible by 3.

10.4.1.B Extract of a letter. Photo Cécile Michel, ©Archaeological mission of Kültepe

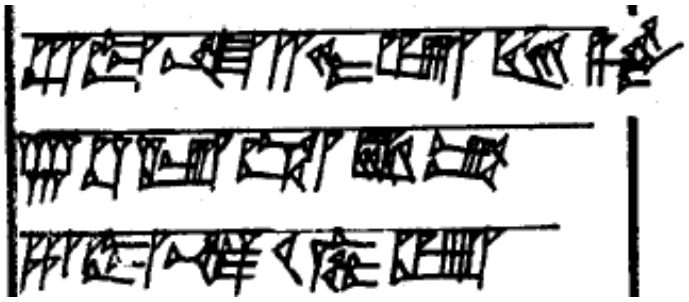
[1] 1/3 *ma-na* ku_3-gi *E-na-A-šùr*
ub-lam 8 gin_2-ta *ni-dí-šu* ku_3-[b]i
10 2/3 *ma-na* 1 gin_2

‘[1] 1/3 *mana* gold Enna-Aššur brought here. We sold it at 8 gin (silver) each (gin of gold); its value in silver: 10 2/3 *mana* 1 *gin*’

The operation is the following:
(1+1/3 *mana*) × 8 *gin* =
10 2/3 *mana*

The scribe curiously added 1 *gin* to the result – was he distracted?

10.4.1.C Extract of a letter



2/3 *ma-na* 2 1/4 gin_2 ku_3-gi
7 1/3 gin_2-ta ku_3-bi
5 *ma-na* 10 la_2 1/4 gin_2

‘2/3 *mana* 2 1/4 *gin* gold
At 7 1/3 *gin* (of silver) each (*gin* of gold); its value in silver: 5 *mana* 10 minus 1/4 *gin*’

The operation is the following:

(2/3 *mana* + 2 1/4 *gin*) × (7 + 1/3) =
(2/3 *mana* + 2 1/4 *gin*) × 22/3 =
(2/9 *mana* + 2/3 *gin* + 1/12 *gin*) × 22 =
4 *mana* + 8/9 *mana* + 14 *gin* + 2/3 *gin* + 1 *gin* + 5/6 *gin* =
5 *mana* 9 5/6 *gin*
The given result, 5 *mana* 10 minus 1/4 *gin*, is incorrect; the scribe omitted 15 *še*, *i.e.* the difference between 1/6 and 1/4 *gin*. He provided the result using a subtractive notation because there is no sign to express 3/4.

Text	Amount of gold	Conversion ratio	Value in silver	Correct result	Difference
10.4.1.D	1/2 *mana* 1/4 *gin* sig_5 (good quality)	9 la_2 1/3 *gin*	4 1/3 *mana* 2 1/6 *gin*	yes	
10.4.1.A	1/3 *mana* 6 1/3 *gin*	9 la_2 1/6 *gin*	3 5/6 *mana* 2 2/3 *gin*	3 5/6 *mana* 2 2/3 *gin* - 10 še	+ 10 *še*
10.4.1.E	2 2/3 *mana*	8 2/3 *gin*	23 *mana* 6 2/3 *gin*	yes	
10.4.1.F	1 1/2 *mana*	8 1/4 *gin*	12 1/3 *mana* 2 1/2 *gin*	yes	
10.4.1.B	1 1/3 *mana*	8 *gin*	10 2/3 *mana* 1 *gin*	10 2/3 *mana*	+ 1 *gin*
10.4.1.G	1 *mana*	7 2/3 *gin*	8 la_2 1/3 *mana*	yes	
10.4.1.C	2/3 *mana* 2 1/4 *gin*	7 1/3 *gin*	5 *mana* 10 la_2 1/4 *gin*	5 *mana* 9 5/6 *gin*	- 15 *še*
10.4.1.H	2/3 *mana*	7 1/3 *gin*	4 5/6 *mana* 3 1/3 *gin*	yes	
10.4.1.I	1/2 *mana kupuršinnum*	6 2/3 *gin*	3 1/3 *mana*	yes	

Table 10.5 Table showing samples for the conversion of gold into silver arranged according to a decreasing conversion ratio

10.4.2 Conversions of Tin into Silver

The twenty-five text extracts analysed in this section are referenced in the table below. I choose to study some examples presenting various conversion ratios often expressed with fractions.

No. in the present chapter	Inventory No.	Publication	Provenance	Preservation
10.4.2.A	Kt 93/k 769	Unpublished (l. 13-14)	Kaneš	Anadolu Medeniyetleri Müzesi, Ankara
10.4.2.B	BM 113286	CCT 3, 5a:4-6 Copied by Sidney Smith	Kaneš	British Museum, London
10.4.2.C	66.245.10	CTMMA 1, 75:6-8 Copied by Mogens Larsen	Kaneš	Metropolitan Museum of Art, New York
10.4.2.D	AO 8229	TC 3, 36:41-42 Copied by Julius Lewy	Kaneš	Musée du Louvre, Paris
10.4.2.E	Kt 93/k 521	Unpublished (l. 32-34)	Kaneš	Anadolu Medeniyetleri Müzesi, Ankara
10.4.2.F	AO 8629	TC 3, 24:31-34 Copied by Julius Lewy	Kaneš	Musée du Louvre, Paris
10.4.2.G	Kt 93/k 763	Unpublished (l. 9-19)	Kaneš	Anadolu Medeniyetleri Müzesi, Ankara

10.4.2.H	AO 8264	TC 3, 43:14-15 Copied by Julius Lewy	Kaneš	Musée du Louvre, Paris
10.4.2.I	BM 113258	CCT 2, 2:33-36 Copied by Sidney Smith	Kaneš	British Museum, London
10.4.2.J	NBC 1695	BIN 4, 30: 15-17 Copied by Albert Clay	Kaneš	Yale Babylonian Collection, New Haven
10.4.2.K	Ank. 2804	AKT 1, 18:7-9 Emin Bilgiç *et al.*	Kaneš	Anadolu Medeniyetleri Müzesi, Ankara
10.4.2.L	Prague I 435	Prag I 435:8-9 Copied by Lubor Matouš	Kaneš	Charles University, Prague
10.4.2.M	Kt 93/k 335	Unpublished (l. 14-16)	Kaneš	Anadolu Medeniyetleri Müzesi, Ankara
10.4.2.N	AO 8676	TC 2, 6:6-9 Copied by François Thureau-Dangin	Kaneš	Musée du Louvre, Paris
10.4.2.O	AO 8683	TC 2, 14:11-14 Copied by François Thureau-Dangin	Kaneš	Musée du Louvre, Paris
10.4.2.P	AO 9329	TC 3, 134:3'-5' Copied by Julius Lewy	Kaneš	Musée du Louvre, Paris
10.4.2.Q	Kt 93/k 335	Unpublished (l. 17-19)	Kaneš	Anadolu Medeniyetleri Müzesi, Ankara
10.4.2.R	Prague I 704	Prag I 704:18-20 Copied by Lubor Matouš	Kaneš	Charles University, Prague
10.4.2.S	NBC 1687	BIN 4, 27: 24-26 Copied by Albert Clay	Kaneš	Yale Babylonian Collection, New Haven
10.4.2.T	NBC 1692	BIN 4, 29: 22-24 Copied by Albert Clay	Kaneš	Yale Babylonian Collection, New Haven
10.4.2.U	Kt 93/k 511	Unpublished (l. 7-8)	Kaneš	Anadolu Medeniyetleri Müzesi, Ankara
10.4.2.V	Kt 93/k 337	Unpublished (l. 17-18)	Kaneš	Anadolu Medeniyetleri Müzesi, Ankara

Table 10.6 Old Assyrian texts analysed in section 10.4.2

In the seven following examples, either the scribe made an error, or he gave an approximate result, sometimes rounding the result.

10.4.2.A Extract of a letter. Photo Cécile Michel, ©Archaeological mission of Kültepe

5 gu_2 3 *ma-na* an-/na
17 gin_2-ta ku_3-bi 17 5/6 *ma-na*

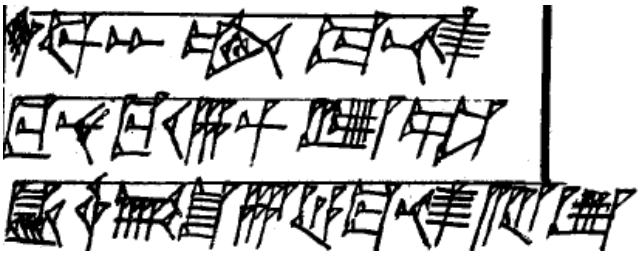
‘5 *gu* 3 *mana* tin at 17 *gin* (tin) each (*gin* of silver) ; its (value in) silver : 17 5/6 *mana*’

The operation is the following:

(5 *gu* + 3 *mana*) ÷ 17 =
17 *mana* + 11/17 *mana* + 10 *gin* + 10/17 *gin* =
17 *mana* + 48 *gin* + 24/17 *gin* =
17 *mana* 49 7/17 *gin*

The result given by the scribe is a rounding up; he added 10/17 *gin* to reach 5/6 *mana*.
It is possible to reverse the problem: the scribe knows the quantity of silver available and wants to compute how much tin he has to give to obtain this amount of silver, the correct result is 5 *gu* 3 *mana* 10 *gin* tin. Then the scribe gave up the quantity of tin given in the smaller unit, which is 10 *gin*.

10.4.2.B Extract of a letter.



2 gu_2 10 *ma-na*
ku-nu-ku 16 1/2 gin_2-ta
ku_3-babbar^{pi}-*šu* 7 5/6 *ma-na* 2 2/3 gin_2

‘2 *gu* 10 *mana* of sealed (tin)
At 16 1/2 *gin* (tin) each (*gin* of silver); its (value in) silver: 7 5/6 *mana* 2 2/3 *gin*’

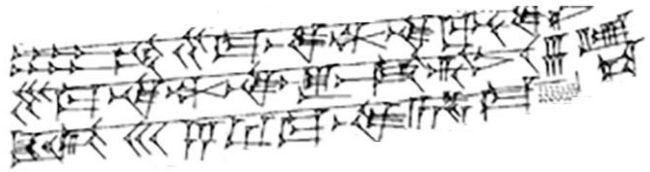
The operation is the following:

(2 *gu* + 10 *mana*) ÷ (16 + 1/2) =
(2 *gu* + 10 *mana*) × 2/33 =
(3 *mana* + 21/33 *mana* + 18 *gin* + 6/33 *gin*) × 2 =
6 *mana* + 42/33 *mana* + 36 *gin* + 12/33 *gin* =
7 *mana* + 52 *gin* + 24/33 *gin* =
7 5/6 *mana* 2 24/33 *gin*

The result given by the scribe is an approximation, 2/3 *gin* is the closest fraction known by the Assyrians for 24/33 *gin*.

As for the previous example, it is possible to reverse the problem: the scribe knows the quantity of silver available and wants to compute how much tin he has to give to obtain this amount of silver, the correct result is 2 *gu* 9 5/6 *mana* 9 *gin* tin. Here, the scribe added 1 *gin* of tin to reach a quantity expressed only with *gu* and *mana*.

10.4.2.C Extract of a letter.



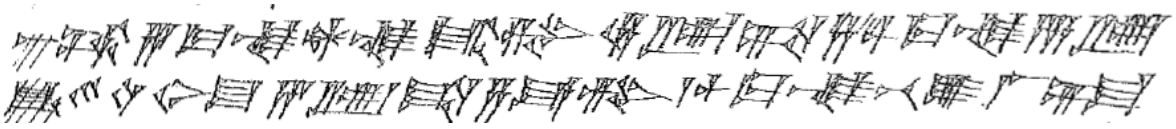
8 gu_2 40 *ma-na* an-na *ku-nu-ki-ni*
50 *ma-na* an-na^{ak} *qa*$_2$-*tim* 16 gin_2-ta
ku_3-bi 35 2/3 *ma-na*

‘8 *gu* 40 *mana* tin under our seal
50 *mana* hand tin at 16 *gin* (tin) each (*gin* of silver); its (value in) silver: 35 2/3 *mana*’

The operation is the following:
(8 *gu* + 40 *mana* + 50 *mana*) ÷ 16 =
(9 *gu* + 30 *mana*) ÷ 16 =
35 *mana* + 5/8 *mana* =
35 *mana* + 37 1/2 *gin* =
35 1/2 *mana* 7 1/2 *gin*

The result given by the scribe is rounded up: he added 2 1/2 *gin* to reach 40 *gin*, *i.e.* 2/3 *mana*. When reversing the problem: the scribe knows the quantity of silver available and wants to compute how much tin he has to obtain this amount of silver, the correct result is 9 1/2 *gu* 2/3 *gin* tin. Here, the scribe gave up the fraction of *gin* (2/3) in order to have a quantity expressed only with *gu*.

10.4.2.D Extract of a letter.



1 gu_2 15 *ma-na* an-na *qa*$_2$*-tim*
14 gin_2-ta 4 1/2 *ma-na* 7 gin_2
ku3-babbar^{ap}2-*šu*

1 *gu* 15 *mana* hand tin at 14 *gin* (tin) each (*gin* silver); its (value in) silver 4 1/2 *mana* 7 *gin*

The operation is the following:

(1 *gu* + 15 *mana*) ÷ 14 =
5 *mana* + 5/14 *mana* =
5 *mana* + 21 *gin* + 3/7 *gin* =
5 1/3 *mana* 1 3/7 *gin*
The scribe made an error in his computation, the result is incorrect.

10.4.2.E Extract of a letter. Photo Cécile Michel, ©Archaeological mission of Kültepe

52 1/2 *ma-na* 5 gin_2 an-na
iz-ku-am 8 1/2 gin_2-ta ku_3-bi
6 *ma-na* 11 1/6 gin_2

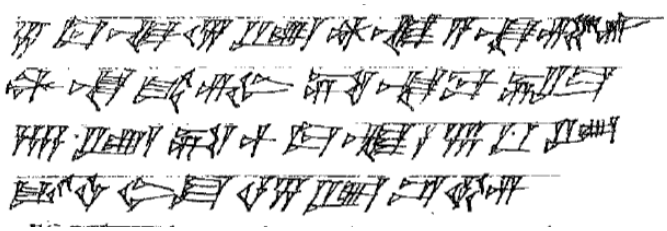
‘52 1/2 *mana* 5 *gin* he cleared
at 8 1/2 *gin* (tin) each (*gin* silver); its (value in) silver: 6 *mana* 11 1/6 *gin*’

The operation is the following:

(52 1/2 *mana* + 5 *gin*) ÷ (8 + 1/2) =
(52 1/2 *mana* + 5 *gin*) × (2/17) =
(3 *mana* + 1/17 *mana* + 2 *gin* + 1/17 *gin*) × 2 =
6 *mana* + 7 *gin* + 1/17 *gin* + 4 *gin* + 2/17 *gin* =
6 *mana* 11 3/17 *gin*

The result given by the scribe is an approximation, 1/6 *gin* is the closest fraction known by the Assyrian merchants for 3/17 *gin*.
When reversing the problem: the scribe knows the quantity of silver available and wants to compute how much tin he has to give to obtain this amount of silver, the correct result is 52 1/2 *mana* 4 5/6 *gin* 15 *še* tin. Here, the scribe added 15 *še* of tin to reach a quantity expressed only in *mana* and *gin*.

10.4.2.F Extract of a letter.



5 *ma-na* 15 gin$_2$ an-na *a-na ṣe$_2$-er*
an-na *qa$_2$-tim ta-na-pa$_2$-al*
8 gin$_2$-ta 1/2 *ma-na* 9 1/3 gin$_2$
ku$_3$-babbar^{ap}2-*šu*

‘5 *mana* 15 *gin* tin you will pay over the hand tin
at 8 *gin* (tin) each (*gin* silver); its (value in) silver: 1/2 *mana* 9 1/3 *gin*’

The operation is the following:

(5 *mana* + 15 *gin*) ÷ 8 =
37 *gin* + 1/2 *gin* + 1 *gin* + 7/8 *gin* =
1/2 *mana* 9 3/8 *gin*

The result given by the scribe is an approximation, 1/3 *gin* is the closest fraction known by the Assyrians for 3/8 *gin*. When reversing the problem: the scribe knows the quantity of silver available and wants to compute how much tin he has to give to obtain this amount of silver, the correct result is 5 *mana* 14 2/3 *gin* tin. The scribe would have then rounded up adding 1/3 *gin* tin to reach 15 *gin* tin.

10.4.2.G Extract of a letter. Photo Cécile Michel, ©Archaeological mission of Kültepe

1 gu$_2$ 52 *ma-na* 7 gin$_2$-ta
ta-di$_2$-in 1 gu$_2$ 7 la$_2$ 1/6 gin$_2$-ta
ta-di$_2$-in : ku$_3$-babbar^{pa}2-*šu*
24 2/3 *ma-na* 6 gin$_2$
…
[1] gu$_2$ 1 *ma-na* 7 gin$_2$-ta
ta-di$_2$-in : ku$_3$-bi
8 2/3 *ma-na* 3 gin$_2$
50 *ma-na* 6 1/4 gin$_2$-ta *ta-di$_2$-/in*
ku$_3$-bi 8 *ma-na*

‘1 *gu* 52 *mana* (tin) at 7 *gin* (tin) each (*gin* silver) was sold, 1 *gu* (tin) at 7 minus 1/6 *gin* (tin) each (*gin* silver) was sold; its (value in) silver: 24 2/3 *mana* 6 *gin*. (…)
[1] *gu* 1 *mana* was sold at 7 *gin* (tin) each (*gin* silver); its (value in) silver: 8 2/3 *mana* 3 *gin*. 50 *mana* at 6 1/4 *gin* (tin) each (*gin* silver); its (value in) silver: 8 *mana’*.

This letter shows several operations.

a) The first is the following:

(1 *gu* + 52 *mana*) ÷ 7 =
16 *mana*

b) The second is the following:

1 *gu* ÷ (7-1/6) =
1 *gu* ÷ (41/6) =
60 *mana* × (6/41) =
8 2/3 *mana* 6 34/41 *gin*
The addition of these two quantities of silver gives 24 2/3 *mana* 6 34/41 *gin*. The scribe gave up the fraction of *gin* from the second result, *i.e.* 34/41 *gin*.

c) The third operation is the following:

(1 *gu* + 1 *mana*) ÷ 7 =
8 *mana* + 4/7 *mana* + 8 *gin* + 4/7 *gin* =
8 2/3 *mana* 2 6/7 *gin*
The scribe rounded up 2 6/7 *gin* to reach 3 *gin*. When reversing the problem: the scribe knows the quantity of silver available and wants to compute how much tin he has to give to obtain this amount of silver, the correct result is 1 *gu* 1 *mana* 1 *gin*. The scribe then would have rounded down this quantity, giving up 1 *gin*.

d) The fourth operation is the following:

50 *mana* ÷ (6 + 1/4) =
50 *mana* ÷ (25/4) =
50 *mana* × (4/25) =
8 *mana*

The result is correct.

Text	Amount of tin	Conversion ratio	Equivalent in silver	Correct result	Difference
10.4.2.A	5 *gu* 3 *mana*	17 *gin*	17 5/6 *mana*	Correct value: 17 *mana* 49 7/17 *gin* The amount of silver corresponds to: 5 *gu* 3 mana 10 *gin* tin	+ 10/17 gin rounding up - 10 *gin* tin
10.4.2.B	2 *gu* 10 *mana*	16 1/2 *gin*	7 5/6 *mana* 2 2/3 *gin*	Correct value: 7 5/6 *mana* 2 24/33 *gin* The amount of silver corresponds to: 2 *gu* 9 5/6 *mana* 9 *gin* tin	Approx. + 1 *gin* tin
10.4.2.C	9 1/2 *gu*	16 *gin*	35 2/3 *mana*	Correct value: 35 1/2 *mana* 7 1/2 *gin* The amount of silver corresponds to: 9 1/2 *gu* 2/3 *gin* tin	+ 2 1/2 *gin* rounding up -2/3 *gin* tin
10.4.2.H	2 *gu* 20 *mana*	15 *gin*	9 1/3 *mana*	yes	
10.4.2.I	2 *gu* 16 *mana*	15 la_2 1/6 *gin*	9 *mana* 10 *gin*	Correct value: 9 *mana* 10 10/89 *gin* The amount of silver corresponds to: 2 *gu* 15 5/6 *mana* 8 1/3 *gin* tin	- 10/89 gin rounding down + 1 2/3 *gin* tin
10.4.2.J	3 *gu* 37 1/2 *mana*	14 1/2 *gin*	15 *mana*	yes	
10.4.2.K	9 *gu* 30 *mana*	14 1/4 *gin*	40 *mana*	yes	
10.4.2.L	2 *gu* 10 *mana*	14 *gin*	9 *mana* 17 *gin*	Correct value: 9 *mana* 17 1/7 *gin* The amount of silver corresponds to: 2 *gu* 9 5/6 *mana* 8 *gin* tin	- 1/7 *gin* rounding down + 2 *gin* tin
10.4.2.M	2 *gu*	14 *gin*	8 1/2 *mana* 4 1/4 *gin*	Correct value: 8 1/2 mana 4 2/7 *gin* The amount of silver corresponds to: 1 5/6 *gu* 9 5/6 *mana* 9 1/2 *gin* tin	Approx. + 1/2 *gin* tin
10.4.2.N	2 *gu* 10 *mana* + 9 mana	14 *gin*	9 5/6 *mana* 6 la_2 1/6 *gin*	Correct value: 9 5/6 *mana* 6 minus 2/7 *gin* The amount of silver corresponds to: 2 *gu* 19 *mana* 1 2/3 *gin* tin	Approx. - 1 2/3 *gin* tin
10.4.2.D	1 *gu* 5 *mana*	14 *gin*	4 1/2 *mana* 7 *gin*	Correct value: 4 1/2 *mana* 8 4/7 *gin* The amount of silver corresponds to: 1 *gu* 4 1/2 *mana* 8 *gin* tin	1 4/7 *gin* error + 22 *gin* tin
10.4.2.O	15 *mana*	14 *gin*	1 *mana* 4 1/3 *gin*	Correct value: 1 *mana* 4 2/7 *gin* The amount of silver corresponds to: 15 *mana* 2/3 *gin* tin	Approx. - 2/3 *gin* tin
10.4.2.P	6 *gu* 58 2/3 *mana*	13 2/3 *gin*	30 1/2 *mana* 8 *gin*	Correct value: 30 1/2 *mana* 8 2/41 *gin* The amount of silver corresponds to: 6 *gu* 58 1/2 *mana* 9 1/3 *gin* tin	- 2/41 rounding down + 2/3 *gin* tin
10.4.2.Q	21 *mana*	13 1/2 *gin*	1 1/2 *mana* 3 1/2 *gin*	Correct value: 1 1/2 *mana* 3 1/3 *gin*	Error 1/2 for 1/3 *gin* + 1/6 *gin* tin

10.4.2.R	40 1/2 *mana*	13 1/2 *gin*	3 *mana*	yes	
10.4.2.S	2 *gu*	10 *gin*	12 *mana*	yes	
10.4.2.T	1 *gu* 2 *mana* 15 *gin*	*9 gin*	6 5/6 *mana* 5 *gin*	yes	
10.4.2.E	52 1/2 *mana* 5 *gin*	8 1/2 *gin*	6 *mana* 11 1/6 *gin*	Correct value: 6 mana 11 3/17 The amount of silver corresponds to 52 1/2 mana 4 5/6 *gin* 15 še tin	Approx. + 15 *še* tin (rounding)
10.4.2.F	5 *mana* 15 *gin*	8 *gin*	1/2 *mana* 9 1/3 *gin*	Correct value: 1/2 mana 9 3/8 *gin* The amount of silver corresponds to 5 mana 14 2/3 *gin* tin	Approx. + 1/3 *gin* tin rounding up
10.4.2.U	4 *gu* 20 *mana*	8 *gin*	32 1/2 mana	yes	
10.4.2.G.a	1 *gu* 52 *mana*	7 *gin*	16 *mana*	yes	
10.4.2.G.c	1 *gu* 1 *mana*	7 *gin*	8 2/3 *mana* 3 *gin*	Correct value: 8 2/3 mana 2 6/7 gin The amount of silver corresponds to: 1 gú 1 mana 1 *gin* tin	+ 1/7 *gin* rounding up - 1 *gin* tin
10.4.2.G.b	1 *gu*	7 la_2 1/6 *gin*	8 2/3 *mana* 6 *gin*	Correct value: 8 2/3 *mana* 6 34/41 *gin* The amount of silver corresponds to 59 5/6 mana 4 1/3 *gin*	- 34/41 *gin* rounding down + 2/3 *gin* tin
10.4.2.G.d	50 *mana*	6 1/4 *gin*	8 *mana*	yes	
10.4.2.V	2 gú 20 mana 15 *gin*	6 *gin*	23 1/3 mana 2 1/2 *gin*	yes	

Table 10.7 Samples for the conversion of tin into silver arranged according to a decreasing conversion ratio

10.4.3 General Observations and Clues on Computation Methods

The vast majority of the results of conversion computations are correct, or given with an approximation, including roundings up or down. There are rare cases where the result is incorrect.

- On several occasions, the conversion ratios or the results are given with a subtractive notation. For example, the two first examples concerning the conversion of gold into silver give a conversion ratio with a subtractive notation: In the first case, the conversion ratio is 9 minus 1/3 *gin* (Text 10.4.2.D.). The quantity of gold to be converted being 1/2 *mana* 1/4 *gin*, it would have been easier to multiply this quantity by 8 2/3 *gin*, thus using an additive notation.
- But in the second case, the conversion ratio is 9 la_2 1/6 *gin* (10.4.2.A.). This notation simplifies the computation since the quantity of gold to convert is 1/3 *mana* 6 1/3 *gin*, thus expressed with two times a third.

In the samples concerning the conversion of tin, on two occasions we also find a conversion ratio with a subtractive notation:

- In the first case, it is 15 la_2 1/6 *gin* (10.4.2.I.) and the quantity of tin to convert is 2 *gu* 16 *mana*. 2 *gu*, at least, is easily divided by 15, since it corresponds to 2 × 60 *mana*.
- In the second occurrence, the conversion ratio is noted as 7 la_2 1/6 *gin* (10.4.2.G.b). In this case, the operation would have been easier with a conversion ratio given in an additive notation: 6 5/6 *gin*.

Note that the use of a subtractive notation can be the consequence of the absence of a specific sign to express a fraction. For example, in the computed value of silver in text 10.4.2.C. we find 10 la_2 1/4 *gin*, because there is no sign to express the fraction 3/4.

In many cases, the scribe rounded the result down or up, either by giving up the fraction of the smaller unit:

- 34/41 *gin* (10.4.2.G.b)
- 1/7 *gin* (10.4.2.L.)
- 10/89 *gin* (10.4.2.I.)
- 2/41 *gin* (10.4.2.P.)

or by adding a small quantity in *gin* to express the result only with *gu*, *mana*, or at least with a round number of *gin*:

- + 2 1/2 *gin* to reach 2/3 *mana* (10.4.2.C.)
- + 10/17 *gin* to reach 5/6 *mana* (10.4.2.A.)
- + 1/7 *gin* to reach 3 *gin* (10.4.2.G.c)

In several other occurrences, the scribe gives an approximate result, choosing the nearest known fraction of *gin*:

- 2/3 *gin* for 24/33 *gin* (10.4.2.B.)
- 1/3 *gin* for 2/7 *gin* (10.4.2.O.)
- 1/3 *gin* for 3/8 *gin* (10.4.2.F.)

- 1/4 *gin* for 2/7 *gin* (10.4.2.Q.)
- 1/6 *gin* for 2/7 *gin* (10.4.2.N.)
- 1/6 *gin* for 3/17 *gin* (10.4.2.E.)

It is interesting to see the merchants' hesitation on the way to render the fraction 2/7, the best approximation among the fractions known by the merchants being 1/4. 1/3 is a good approximation for 3/8 and 1/6 is a good approximation for 3/17.

In Table 10.2, which gives the conversions of tin into silver, the second last column proposes the results of the computation of the price in silver (first line) and of the quantity of tin bought with the amount of silver specified in the text (second line). The last column gives both the difference for the price of silver, and below for the quantity of tin computed. When doing the computation on the quantity of tin, the difference is usually small and generally corresponds to an integer or to a known fraction. In some cases, a small quantity of tin has been omitted, to round it to the higher unit:

- 10 *gin* (10.4.2.A.)
- 2/3 *gin* (10.4.2.C.)
- 1 2/3 *gin* (10.4.2.N.)
- 2/3 *gin* (10.4.2.O.)
- 1 *gin* (10.4.2.G.c)

In the other occurrences, a small quantity of tin has been added to reach a round number

- 1 *gin* to reach 10 *mana* (10.4.2.B)
- 1 2/3 *gin* (10.4.2.I.)
- 2 *gin* (10.4.2.L.)
- 1/2 *gin* (10.4.2.M.)
- 2/3 *gin* (10.4.2.P.)
- 1/6 *gin* (10.4.2.M.)
- 15 *še* (10.4.2.E.)
- 1/3 *gin* (10.4.2.F.)
- 2/3 *gin* (10.4.2.G.b.)

The difference between the quantity of tin given by the scribe and that computed is between 15 *še* (0.68 g) and 1 2/3 *gin* (13.9 g). The quantity 15 še corresponds to the smallest weight recorded at Kültepe.[19] One may note that the quantities of tin bought are never given with a quantity of *gin* lower than 5 *gin*, while for silver, we find fractions of *gin*. This is quite logical since silver is much more expensive than tin. The results computed above suggest that the quantity of tin bought was defined in advance, and the price in silver then computed according to the announced conversion ratio. However, the adjustment was presumably done on the quantity of tin and not on the quantity of silver.

[19] Michel (forthcoming b).

All these observations give clues about the way the Assyrian merchants computed in order to convert the value of one metal into another.
They always preferred to use fractions of the higher unit instead of integers of the lower unit. They used a limited number of fractions: 1/6, 1/4, 1/3, 1/2, 2/3, 5/6. They did not know the fraction 1/5 which is attested in contemporaneous Babylonian texts.[20] Such a use of fractions implies that they were carrying out computations with fractions on each measurement unit separately. The fact that in some cases they used a subtractive notation could also suggest that they computed starting from the higher measurement unit and ended their computation with the smallest measurement unit.
A systematic analysis of the computations shows that errors are rare. The variations are usually below 1 *gin* of tin, or between 1 and 2 *gin* of tin: most of the time, they correspond to a rounding if we consider that the last adjustments were made on the quantities of tin.

10.5 Conclusion

This chapter analysed the computations carried out by merchants when converting the value of a quantity of one metal into another, according to a given conversion ratio. While carrying out such operations, the Assyrians presumably did not use the SPVN system as the Babylonians did in the palace administrations at the same time. Indeed, Babylonian administrators, when carrying out such computations, used first to convert their data into the sexagesimal place value notation (SPVN) using metrological tables. They carried out computations on SPVN numbers and then converted the result back into the correct metrological system with the help of metrological lists.[21] Instead, the Assyrians computed with the metrological values, including fractions, starting with the largest measurement units. Such a hypothesis can only be confirmed by using a much larger sample of texts.

The texts give the baseline data, *i.e.* the quantity of gold or tin and the conversion ratio, and the result obtained, *i.e.* the corresponding value in silver. However, there is no trace of the computation, and the steps of the calculation always remain invisible. These intermediary calculation steps could have been written down on another type of media, for example wooden tablets coated with wax. Such a media was already in use during the late third millennium BCE, and there are at least two Old Assyrian texts which refer to writing boards with wax: *ina ṭuppim ša iskurim*, 'on a wax tablet'(ATK 5 11:21-22), and 1 *ṭuppum ša iskurim*, '1 wax tablet' (AKT 6b 468:12-13), in an inventory of a private chapel.[22]
Another possibility, suggested in earlier publications, is that the merchants used a computing tool.[23] Such an instrument could be either a tool on which the merchants were computing, like an abacus, or a complement to mental computation on which they would reproduce the results

[20] Michel (1992).
[21] Proust (2008, 2010).
[22] First published by Barjamovic and Larsen (2008) as Kt 94/k 670.
[23] Michel (2006b, 2008b, 2010b). It has been proposed that such calculating instruments could take the form of the objects identified as game boards with 61 holes, of which several exemplars, dating to the first half of the second millennium BCE have been discovered. On such a tool, we could imagine that the inner row of holes could have served to note fractions, while the exterior row would have been devoted to integers.

of the intermediary steps. Since they computed with integers and fractions, such an instrument would allow both to be represented. It is interesting to note that such a computing instrument, the *nikkassū*, is mentioned just before the tablet coated with wax in the inventory of the private chapel quoted above (AKT 6b 468:12).[24]

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[24] Dercksen (2015).

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