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Marie-Laure Cabon-Dhersin[*], Nicolas Drouhin[†]

July 28, 2020

Abstract

We consider a model of price competition in a homogeneous good, with soft-capacity constraints, in the special case of a Sone-Geary production function that implies a minimum firm size and leads to a U-shaped average cost function. We study free entry and obtain a Chamberlin-like result: zero profit and a positive markup at equilibrium.

Key words: price competition, soft-capacity constraint, tacit collusion, returns to scale, free entry.

Code JEL: L13, D43

[*]Corresponding author, Normandie Univ., Unirouen, CREAM, Rouen, France. (marie-laure.cabon-dhersin@univ-rouen.fr)

[†]Normandie Univ., Unicaen, CREM, UMR CNRS 6211, Caen, France (nicolas.drouhin@unicaen.fr)

1 Introduction.

We consider a model of price competition with soft-capacity constraints in line with Cabon-Dhersin and Drouhin (2014, 2020). Firms rely on two substitutable factors of production that are chosen sequentially: in the first stage, a capacity factor and in the second, when the firms compete on price, a variable factor to match incoming demand. This framework produces some non-standard results: the non-cooperative equilibrium corresponds to a collusive outcome and price tends to increase with the number of firms.

In Cabon-Dhersin and Drouhin (2020), we described a general calibration method and a textbook example (a Cobb-Douglas production function) that illustrates these results. In this example, whatever the returns to scale, entries (i.e. increases in the number of firms) not only leads to an infinite number of atomistic firms with positive mark-up, but also never drive the profit to zero.

While it is interesting from a theoretical point of view, we believe this property is questionable, not necessarily realistic and is in no way a general property of the model, but stems rather from the homotheticity of the Cobb-Douglas production function. In this article, we investigate an alternative case with a Stone-Geary production function in which a minimum firm size emerges.

We show that in this case, the number of firms at equilibrium is finite, providing a nice setting in which most of the properties of the original model of monopolistic competition (Chamberlin, 1933) are recovered, without assuming product differentiation and within a model that fully accounts for strategic interactions between firms.

2 A production function with a minimum firm size

We assume that the production function is a special case of the Stone-Geary class of utility functions:

$$f(z,v) = A \cdot \left((z - z_0)^{1-\alpha} v^{\alpha}\right)^{\rho} \tag{1}$$

with z_0, the minimum level for the fixed factor, interpreted as the "minimum firm size", $\rho > 0$, the "asymptotic scale elasticity of production", and, $\rho\,\alpha < 1$ and $\rho\,(1-\alpha) < 1$. Stone-Geary functions have been used extensively in consumer theory to provide a utility representation of the preferences of consumers with subsistence levels for some consumptions. In consumer theory, because utility is an *ordinal* concept, quasi-concavity is a sufficient condition for there to be an interior solution to the consumer choice program. The Stone-Geary function is quasi-concave over its full domain. In producer theory in contrast, the production function is *cardinal* by nature. The notion of "returns to scale" captures this "cardinal" nature of the producer decision. However, essentially because only monopoly models admit an equilibrium when returns to scale are increasing, the possibility of "internal" increasing returns has been ignored in the modern literature on oligopoly markets. Going back to the Stone-Geary formulation, we can compute the local scale elasticity of production:

$$\eta(z,v) = \frac{f_z(z,v)z + f_v(z,v)v}{f(z,v)} = (1-\alpha)\rho\frac{z}{z - z_0} + \alpha\rho \tag{2}$$

We can see that η depends only on z and decreases from $+\infty$ when z tends to z_0 and decreases down to ρ, the "asymptotic elasticity of production", when z tends to $+\infty$. Thus, when ρ is strictly lower than one, there is a threshold in z for which the returns to scale are increasing under and decreasing above, generating an elegant U-shaped average cost function, as illustrated in Beattie and Aradhyula (2015). Because our general model of price competition (Cabon-Dhersin and Drouhin, 2020) disentangles the existence of equilibrium from the nature of the returns to scale, the Stone-Geary production function with a minimum firm size can rightfully be used within this model to investigate some industrial organization problems.

As in Cabon-Dhersin and Drouhin (2020), we compute $\hat{v}$, the quantity of variable factor, as an implicit function defined by $y = f(z, v)$:

$$\hat{v}(y, z) = \frac{y^{\frac{1}{\alpha\rho}}}{A^{\frac{1}{\alpha\rho}}(z - z_0)^{\frac{1-\alpha}{\alpha}}} \tag{3}$$

Thus, when n firms operate in the market, the profit function $\hat{\pi}$ can be written:

$$\hat{\pi}(p, z, n) = p\frac{D(p)}{n} - p_z z - p_v \frac{\left(\frac{D(p)}{n}\right)^{\frac{1}{\alpha\rho}}}{A^{\frac{1}{\alpha\rho}}(z - z_0)^{\frac{1-\alpha}{\alpha}}} \tag{4}$$

with D the demand function for the whole market, p, the market price, p_z the price of the fixed factor, and p_v the price of the variable factor.

3 Equilibrium of the game for a given number of firms

To solve the soft-capacity constrained price competition problem with this Stone-Geary production function for a finite number of firms n, we build on our previous work (Cabon-Dhersin and Drouhin, 2020) and define:

1. $\bar{p}(z, n)$, the price that solves the equation $\hat{\pi}(p, z, n) = \hat{\pi}(p, z, 1)$, the threshold under which firms have no interest in undercutting their rivals in the second stage, when costs are convex (Dastidar, 1995, 2001),

2. $\hat{p}(z, n)$, the price that solves $\hat{\pi}(p, z, n) = -p_z z$, the threshold under which the variable profit is negative in the second stage,

3. $p^*(z, n)$, the price that maximizes $\hat{\pi}(p, z, n)$ for a given z and n, the purely collusive price,

4. $\bar{z}(p, n)$, the level of the fixed factor that maximizes $\hat{\pi}(p, z, n)$ s.t. $p \leq \bar{p}(z, n)$ (i.e.

p is a Nash equilibrium in the second stage),

5. $z^*(p,n)$, the level of the fixed factor that maximizes $\hat{\pi}(p,z,n)$

and using

Proposition 1 (Cabon-Dhersin and Drouhin (2020)'s Proposition 3)**.** *The unique outcome in which all n firms choose the same fixed factor level, $z^C(n)$, in the first stage and quote the same price, $p^C(n)$, in the second, with $p^C(n)$ being a solution of the program,*

$$\mathcal{P}_1(n) \left\{ \begin{array}{ll} & \max_p \hat{\pi}(p,z,n) \\ s.t. & z^C(n) = \bar{z}(p,n) \\ & \hat{\pi}(p,z,n) \geq \hat{\pi}(\hat{p}(z,n), \underset{\tilde{z}}{\operatorname{argmax}}\, \hat{\pi}(\hat{p}(z,n),\tilde{z},1),1) \\ & \hat{\pi}(p,z,n) \geq 0 \end{array} \right.$$

is a Subgame Perfect Nash Equilibrium (SPNE) of the game. Moreover, $\hat{\pi}(z^C(n), p^C(n))$ is the Payoff Dominant SPNE of the game.

The three constraints in Program $\mathcal{P}_1(n)$ avoid possible deviations during strategic interactions. The first constraint ensures that the price corresponds to a Nash equilibrium in the second stage. The second ensures that the fixed factor is high enough to prevent rivals investing massively to trigger a limit pricing strategy (i.e a Non-Existence of a Limit Pricing Strategy (NELPS) constraint). The third ensures a positive profit, because firms are always free to choose $z = 0$ in the first stage to avoid negative profits.

4 Free-entry equilibrium

In this section, we extend the model to endogenize the number of firms operating in the market (free-entry equilibrium). Let us add an initial stage 0 in which firms have the choice to leave or enter the market, and then play a soft-capacity constrained game of price competition in the subsequent stages. Of course, if $(p^C(n), z^C(n))$ is a solution of program $\mathcal{P}_1(n)$, all firms operating in the market make a positive profit and there is no incentive to leave. What does an outside firm decide? The firm will decide to enter the market if an equilibrium exists with $n+1$ firms in the market (i.e if Program $\mathcal{P}_1(n+1)$ has a solution with a positive profit).

Definition 1. *n^e is the free-entry equilibrium number of firms if and only if:*

$$\begin{cases} (z^C(n^e), p^C(n^e)) \text{ is the solution of program } \mathcal{P}_1(n^e) \\ \text{and} \\ \mathcal{P}_1(n^e+1) \text{ has no solution} \end{cases}$$

Proposition 2. *Assuming that:*
i) the production function is a Stone-Geary function such as equation (1), with $\rho < 1$
ii) there is an equilibrium for at least one $n \geq 2$
iii) the demand function is characterized by a choke price p_{max} (i.e. $\forall p \geq p_{max},\ D(p) = 0$).
There is always a finite free-entry equilibrium value for the number of firms.

Proof: It is sufficient to prove that $n = +\infty$ cannot be an equilibrium value in the general model of price competition with soft-capacity constraint when the production function is a Stone-Geary function. The average cost of production is

$$AC(p, z, n) = p_z z \left(\frac{D(p)}{n}\right)^{-1} + p_v \frac{\left(\frac{D(p)}{n}\right)^{\frac{1}{\alpha\rho}-1}}{A^{\frac{1}{\alpha\rho}}(z-z_0)^{\frac{1-\alpha}{\alpha}}} \tag{5}$$

Because of the choke-price assumption, the equilibrium price is always strictly lower than p_{max} and demand is finite and strictly positive. Moreover, when n tends to infinity, $D(p)/n$ tends to zero and the fixed factor z tends to z_0. We deduce that: $\lim_{n\to+\infty} AC(p, z, n) = +\infty$. (The first term of equation (5) tends to $+\infty$ and the second term is necessarily positive). Thus, when the number of firms tends to infinity, the profit becomes negative, contradicting the conditions of proposition (1). $\square$

5 Illustration: a Chamberlin-like model

Let us examine a numerical example of the properties of the soft-capacity constrained price competition model with a Stone-Geary production function. The results recalled in Proposition (1), make the equilibrium quite easy to compute because the equilibrium of the non-cooperative game is a solution of the constrained optimization program. In the following, we use the general calibration procedure outlined on page 106 of Cabon-Dhersin and Drouhin (2020).

We assume that the demand function is linear ($D(p) = b(p_{max} - p)$, with $b = 1$, and $p_{max} = 10$. For the Stone-Geary function, we assume that $\alpha = .7$, $\rho = .9$ and $A = 1$. Finally, we normalize the factor prices p_z and p_v to one. The results are summarized in Figure 1.

The left side shows the price and average cost as a function of the number of firms. The price pattern is very similar to the one illustrated in Cabon-Dhersin and Drouhin (2020). When the number of firms is small (here $n \in \{2, 3\}$), the NELPS constraint is binding, meaning that the firms need to account for the possibility of being excluded from the market by a firm that overinvests in the first stage. Between, $n = 4$ and $n = 15$ the "Nash Equilibrium constraint in the second stage" is binding, meaning that $p^C(n) = \bar{p}(z^C, n)$. This constraint becomes less effective as the number of firms increases, meaning that prices also increase. Beyond $n = 15$, neither constraint is binding, such that the equilibrium price is purely collusive and that $\bar{z}(p, n) = z^*(p, n)$. Because in our example the returns to scale are slightly decreasing in the domain, the price also decreases slightly as the number of firms increases. Finally, the curve marked by red Xs shows the effective average cost for each firm (Equation (5)), which illustrates how the average profit goes to zero for $n^e = 30$.

At this point, it is important to note that the average cost for the firms in our model, as described by Equation (5), differs from the average cost function in most standard models. In standard models, the cost function does not depend on the type of competition in the market for output. It is just the cost of the optimum combination of factors. In our model however, firms account for the need to have a Nash Equilibrium in the second stage. Since in the following we use the usual "average cost" function as a benchmark, we refer to it as the "long-term" average cost.

It is easy to compute the conditional factor demand $z(y, p_z, p_w)$, $v(y, p_z, p_w)$ and the long-term average cost in the Stone-Geary case:

$$z(y, p_z, p_w) = \left(\frac{y}{A}\right)^{1/\rho} \left(\frac{p_v}{p_z}\right)^{\alpha} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + z_0$$

$$v(y, p_z, p_w) = \left(\frac{y}{A}\right)^{1/\rho} \left(\frac{p_z}{p_v}\right)^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}$$

$$LTAC(y, p_z, p_v) = y^{1/\rho-1} A^{-1/\rho} p_v^{\alpha} p_z^{1-\alpha} \frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}} + p_z \frac{z_0}{y}$$

The graph on the right of Figure 1 has the same y-axis (price and cost) as the one of the left but the x-axis is y, the equilibrium output level of each firm for different numbers of firms. Of course, y tends to decrease as the number of firms operating in the market increases. The main interest of this representation is that it allows us to trace out both the equilibrium price/output or average cost/output combination for each number of firms and the inverse demand function and long-term average and marginal cost functions. We can thus describe the dynamics of entry in the traditional "industrial organisation" way.

Below $n = 15$, the average cost is higher than the long-term average cost, because one of the two first constraints of Program $\mathcal{P}_1(n)$ is binding. Above $n = 15$, the program outcome is purely collusive. The average cost is equal to the long-term average cost. Firms continue to enter as long as the profit remains positive. For $n^e = 30$, profits are so close to zero that entry becomes unprofitable for outside firms. This is the standard condition of (approximate) tangency between the average revenue function (i.e. the demand function) and the long-term average cost function. Because, the free-entry equilibrium occurs in the decreasing part of the average cost function, the mark-up is positive, a situation that was described for the first time by Chamberlin (1933) (Figure 17 on page 99 in the 7th edition, 1956). The conditions required for this to occur have been the subject of many discussions since (see for example, Parenti et al. (2017) in the case of differentiated products).

In Cabon-Dhersin and Drouhin (2020), we built a general model of price competition for homogeneous goods that produced the same results: entry never reduces the mark-up to zero because of the sequential choice of production factors and the convexity of the resulting short-term cost. The present short article illustrates this property in the case of a U-shaped average cost function.

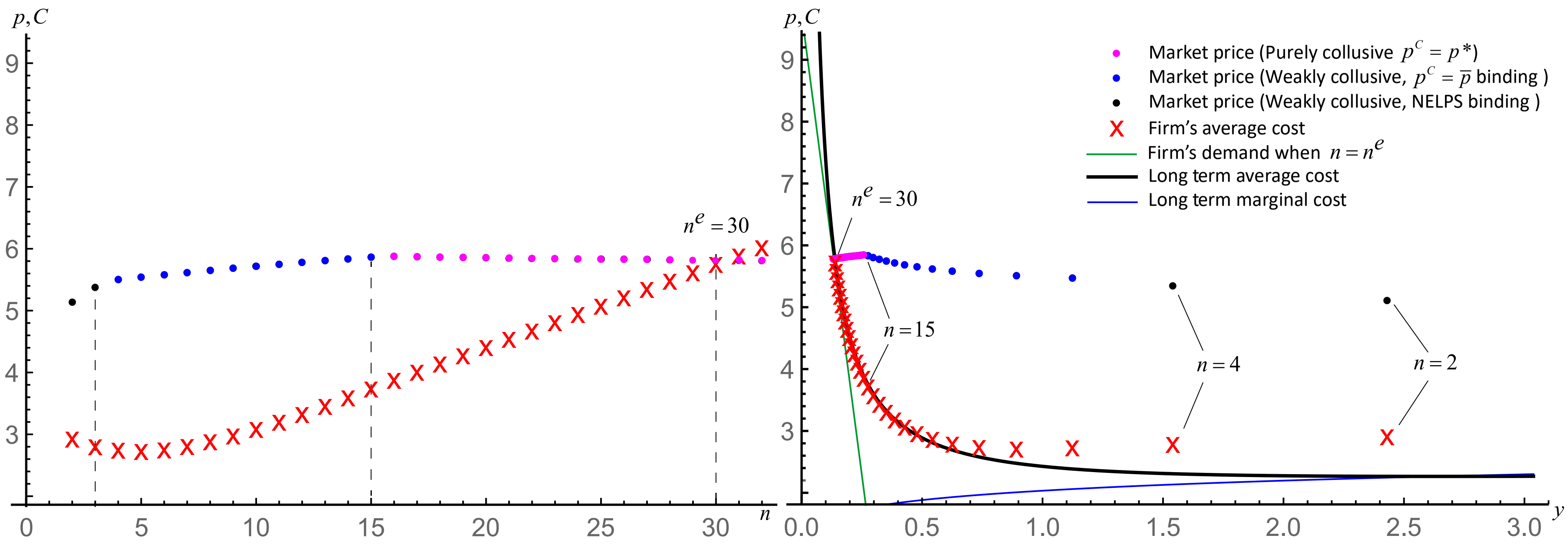

Figure 1: Price and costs as a function of the number of firms and their output level ($\rho = .9$, $\alpha = .7$, $A = 1$, $p_{max} = 10$, $b = 1$, and $p_z = p_v = 1$.)

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