Incentives under Upstream-Downstream Moral Hazard Contract

Patrice Loisel, Bernard Elyakime

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Résumé : Cet article étudie les caractéristiques d'un contrat de production d'un bien public environnemental avec aléa moral entre deux producteurs privés initialement non-associés mais dépendants par un flux physique, l'un étant en amont de l'autre. La performance de la fonction de paiement est étudiée en détail, afin de clarifier la façon dont le contrat avec aléa moral fonctionne. Après avoir présenté et calculé le contrat nous déduisons les fonctions de paiement non-linéaires incitatives. Les résultats du producteur en aval dépendent du résultat du producteur en amont. Les fonctions de paiement héritent donc de cette structure de dépendance : le paiement du producteur en amont ne dépend que de ses résultats alors que le paiement du producteur en aval dépend de ses propres résultats ainsi que de ceux du producteur en amont. Dans certains cas, le comportement de la fonction de paiement aval peut entraîner sa non-acceptabilité par les contractants. Pour y remédier nous proposons l'adjonction d'une contrainte d'acceptabilité : un bunching d'un nouveau type apparaît pour le paiement du producteur aval.

Abstract : This paper explores the characteristics of an upstream-downstream moral hazard contract between two private initially non-associated producers in a spatialized and flow dependence, one upstream of the other, to provide an environmental public good. The performance of the payment function is studied in detail, in order to clarify how the moral hazard contract operates. After having drawn up and calculated the contract, we derive the incentive non-linear payment functions. The results of the downstream producer depend on the result of the upstream producer. Payments producers thus inherit this dependency structure : payment for the upstream contractor only depends on his results whereas payment for the downstream contractor depends on his own results and on those of the upstream contractor. In some cases, the behavior of the downstream payment function can lead to a possible non-acceptability of the payment function by contractors. To remedy the situation we add an acceptability constraint : a new type of bunching appears for the payment of the downstream producer.

Keywords : Public environmental policy, Environmental good, Partnership, Spatialized contract, ancillary statistics, Bunching.

JEL : D8, Q5.

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1 Introduction

The implementation of producers' practices in compliance with physical or ecological environment is crucial for the environmental public policy (among others Baumol and Oates, (1988), Perman et al. (2003), Vachon and Klassen (2006)). Indeed, farmers' good practices (e.g. respectful of the physical or ecological environment) generated in the rural land have an increasing interest as demonstrated in Dale and Polaski (2007). Studying economic incentive instruments that encourage the physical or ecological environment is also a topics in industrial economics, e.g. Laffont and Tirole (1996). This paper presents the study of an incentive economic instrument allowing a public regulation of industrial or agricultural practices respectful of the physical or ecological environment.

The moral hazard contract seems to be well-suited to the implementation of an environmental public regulation of this type by two agents (Holmström (1979), Shavell (1979), Gabel and Sinclair-Degagné (1993), Salanié (1997), Laffont and Martimort (2002)) : it proposes a payment to the contractors suited to the level of the contractual-production that they generate and therefore to the choice of actions that they have accomplished. Nevertheless Goldsmith and Basak (2001) examined difficulties concerning the implementation of a moral hazard agricultural contract using an imperfect and costly environmental performance indicator. Although society could remunerate such productions of positive externalities in the framework of sustainable development, that remuneration is not simple to implement. Moreover, concerning the incentive contract, the classic result of the moral hazard relationship between a principal and an agent, without any repetition of this relationship and with a single contracting agent, has been known since the works of Holmström (1979) and Shavell (1979). By moving away from the optimum sharing of revenue between the principal and the agent, the principal can incite the agent to achieve a better level of effort by remunerating him accordingly. Holmström (1979) and Grossman and Hart (1983) demonstrated the advantage of using a signal based on the effort furnished in a moral hazard contract. The more information the principal has to draw up the contract, the more precise the contract will be (ie the more the actions will be precise and the more the principal will stand to gain). Therefore, the nature of the information selected when designing the contract seems essential to establish the properties of the incentive contract and that is what we are interested in here. Pursuing the exploration of how to characterize a moral hazard contract based on private actions of a collective of producers in order to get a global result, we seek to build a partnership between the producers, through a well-informed incentive contract.

We consider a specific new context, related to a spatialized and flow dependence between two private producers. One is upstream of the other and each one acts on his own area. These actions are carried out in the framework of a public moral hazard contract binding the two producers to deliver an environmental public good. More precisely, the downstream producer is dependent on the upstream producer by an additive or a multiplicative flow. A first example is the management of a non-polluted river flow. Another example is related to the productive practices allowing to preserve an animal or vegetal biodiversity, or the agricultural soils in order to avoid their erosion in a catchment area, Le Bissonnais (2013). This upstream-downstream dependence induces a specific incentive contract in which the result of the upstream producer will influence the result of

the downstream producer. For example an upstream farmer, who acts on his agricultural soil to decrease the water streaming, allows less efforts from the downstream farmer to avoid the soil erosion than if he were alone, Elyakime (2000). We highlight the consequences of using a moral hazard contract in the chosen situation. The incentive requires a payment for the downstream contractor that depends not only on his result, which is perfectly natural, but also on the result of the upstream contractor. The latter is relevant, as any other variable (generally named ancillary statistics) influencing the contractual results, in accordance with the Sufficient Statistic Theorem as demonstrated in Holmström (1979) or studied in Sinclair-Desgagné (2009). In this unusual contractual configuration, it is necessary to study consequences of the upstream result variable on the behavior of the payment functions.

This paper is constructed as follows : after having defined in section 2 the moral hazard contract binding two producers interacting to meet the needs of the contract, we define the payment functions for each of them. We then detail different classes of models in section 3 : contract with a multiplicative effect, contract with an additive or a mixed effect. We complete the study with examples and interpretation, before discussing results and concluding in section 4 and 5.

2 Upstream-downstream moral hazard contract

In a model of public economics, we consider a principal (a public regulator) who establishes a contract with two private producers. The first producer is located upstream of the second one. The two producers are in a flow dependence, as an upstream-downstream flow dependence. These producers act in the framework of a single annual upstream-downstream contract to produce an environmental public good. The upstream producer's actions yield an output, like the downstream ones. The two producers are supposed initially non-associated and more precisely never to be bound by an upstream-downstream incentive contract. So, they are supposed not to know the characteristics of a contract of this type. Moreover, if one of them refuses the contract, it is not implemented.

We consider a contract with a risk-neutral principal, an upstream and a downstream risk-averse producers. Let x be the result of support X produced by the upstream producer. Let y be the downstream result of support $\mathcal{Y}$. Results x and y are assumed to be verifiable, stochastic and measurable. Result x is distributed with F_1 (supposed known density f_1) depending on the action variable a_1 of the upstream producer. Result y is distributed with F_2 (supposed known density f_2) positively depending on the action a_2 of the downstream producer and the upstream result x. The actions a_1, a_2, of support A_1 and A_2, are supposed not observable.

The gross social gain that comes from producers' actions is denoted by $G(x, y)$. The utilities of producers are indirectly depending on the action variables through the payment given to the producers. Moreover, the costs of the actions are directly depending on action variables.

This is a classic stylized situation : the result, obtained by the upstream producer's action,

affects the downstream producer's stand output, and so could increase or decrease the results of the latter's actions.

2.1 The contractual framework

We could base the incentive solely on the result observable at the output from each producer. But in a moral hazard contract, the more information is used in the contract, the more the protection actions will be precise and the more the principal will stand to gain. For the upstream producer, the payment will be based on the upstream result $x : t_1(x)$. For better knowing the specific influence of the downstream producer's action a_2 on result y, we are proposing a downstream producer's payment which also depends on the upstream result $t_2(y, x)$.

We consider the utility function $u_1(\cdot)$ ($u_1^{'} > 0$, $u_1^{''} < 0$) for upstream producer 1 (resp. $u_2(\cdot)$ for downstream producer 2) who receives the payment $t_1(\cdot)$ (resp. $t_2(\cdot)$ for downstream producer 2) and pays the cost $w_1(a_1)$ for the action a_1 (resp. $w_2(a_2)$ for downstream producer 2 and for the action a_2). So we have the net utility $u_1(t_1(x)) - w_1(a_1)$ (resp. $u_2(t_2(y, x)) - w_2(a_2)$). In a first step we suppose no externality on the stand income (no effects of the producers' actions on their respective utilities), so the public moral hazard program is written as :

$$\max_{a_1,a_2,t_1(.),t_2(.,.)} \int_x \int_y [(G(x,y) - (1+\gamma)(t_1(x) + t_2(y,x)) + u_1(t_1(x)) - w_1(a_1) + u_2(t_2(y,x)) - w_2(a_2)]dF_2(y|x,a_2)dF_1(x|a_1) \quad (1)$$

under respectively the participation constraints detailing for each producer and the principal :

$$\int_x \int_y (G(x,y) - (1+\gamma)(t_1(x) + t_2(y,x)))dF_2(y|x,a_2^*)dF_1(x|a_1^*) \geq E_0 \quad \text{(PC0)}$$

$$\int_x u_1(t_1(x))dF_1(x|a_1^*) - w_1(a_1^*) \geq E_1 \quad \text{(PC1)}$$

$$\int_x \int_y u_2(t_2(y,x))dF_2(y|x,a_2^*)dF_1(x|a_1^*) - w_2(a_2^*) \geq E_2 \quad \text{(PC2)}$$

and each producer's incentive constraint which are therefore at their economic optimum with respect to their respective actions a_1 and a_2 :

$$a_1^* = \arg\max_{a_1} \int_x u_1(t_1(x))dF_1(x|a_1) - w_1(a_1) \quad \text{(IC1)}$$

$$a_2^* = \arg\max_{a_2} \int_x \int_y u_2(t_2(y,x))dF_2(y|x,a_2)dF_1(x|a_1^*) - w_2(a_2) \quad \text{(IC2)}$$

where γ is the social cost of one unit of public funds. In this contract, we move the upstream-downstream flow dependence of the producers in the conditional distribution. The suggested payment thus inherits this dependence structure.

2.2 The payments

By using the first-order approach (Rogerson (1985), Jewitt (1988), Sinclair-Degagné (1994), Mirrlees (1999), Carlier and Dana (2005)) let us consider the Lagrangian (see in appendix A) with the Lagrange multipliers λ_0 for the principal's participation constraint (PC0), λ_1, λ_2 for the producer's participation constraints (PC1), (PC2) (with λ_0,λ_1 and $\lambda_2 \geq 0$) and μ_1, μ_2 for the incentive constraints imposed on each producer 1 and 2 (IC1), (IC2). We deduce the first-order optimality conditions relative to payments t_1 and t_2 :

$$\frac{(1+\lambda_0)(1+\gamma)}{u_1^{'}(t_1(x))} = 1 + \lambda_1 + \mu_1 \frac{f_{1,a_1}}{f_1}(x|a_1)$$
$$\frac{(1+\lambda_0)(1+\gamma)}{u_2^{'}(t_2(y,x))} = 1 + \lambda_2 + \mu_2 \frac{f_{2,a_2}}{f_2}(y|x,a_2)$$

The upstream producer's payment function t_1 explicitly depends on his result x whereas the downstream producer's payment function t_2 depends on both y and x. The payment functions are, with μ_1, $\mu_2 > 0$, as demonstrated in (Holmström, 1979 ; Shavell, 1979) :

$$t_1(x) = (\frac{1}{u_1^{'}})^{-1}[\frac{1+\lambda_1+\mu_1\frac{f_{1,a_1}}{f_1}(x|a_1)}{(1+\lambda_0)(1+\gamma)}]$$
$$t_2(y,x) = (\frac{1}{u_2^{'}})^{-1}[\frac{1+\lambda_2+\mu_2\frac{f_{2,a_2}}{f_2}(y|x,a_2)}{(1+\lambda_0)(1+\gamma)}] \quad (2)$$

The payment t_2 depends on the probability density function $f_2(y|x,a_2)$. As result y is supposed distributed with f_2 positively depending on the action a_2 of the downstream producer and the upstream result x, downstream result y is positively conditioned by the upstream result x. Consequently, this downstream result includes the product of the upstream producer's effort. So if the downstream payment is decreasing with respect to the upstream result x, this effect will be partly neutralized. Conversely, if the downstream payment is increasing, the effect is amplified. Hence, it is necessary to analyze the downstream payment t_2's behavior with respect to the upstream result x.

3 Upstream-downstream payment behavior

In order to analyze the behavior of payments t_1 and t_2 it is necessary to define the probability densities relative to the respective results derived from applying the contract. Several distributions can be used depending on the type of problem studied.

The upstream producer's payment function t_1 will be increasing depending on the upstream result x if F_1 satisfies the MLRP condition, that is to say $\frac{\partial}{\partial x}\frac{f_{1,a_1}}{f_1}(x|a_1) > 0$, as shown in Gross-

man and Hart, 1983. Likewise, the downstream producer's payment function t_2 will be increasing depending on the downstream result y if F_2 satisfies the MLRP condition, that is to say $\frac{\partial}{\partial y}\frac{f_{2,a_2}}{f_2}(y|x,a_2) > 0$.

We assume that the downstream result y is given by : $y = \theta(x,a_2)z_a$ where θ is an increasing function with respect to the upstream result x and the downstream action a_2, z_a is a random variable of distribution H. Hence the downstream distribution F_2 is generated by distribution H (of density h) of the ratio of the downstream result y and $\theta(x,a_2)$: $F_2(y|x,a_2) = H(\frac{y}{\theta(x,a_2)})$ of support $\mathcal{Y} = R^+$. z_a can be defined as a reduced downstream result with support $\mathcal{Z} = R^+$. In this case $\theta(x,a_2)$ is easily interpretable : the expectation of downstream result y is proportional to $\theta(x,a_2)$.

In order to analyze the behavior of the downstream producer's payment with respect to the producer's output, using the change of variable $z = \frac{y}{\theta(x,a_2)}$, we can express $\frac{f_{2,a_2}}{f_2}$ as a function of the reduced downstream result z_a :

$$\frac{f_{2,a_2}}{f_2}(y|x,a_2) = -\frac{\theta'_{a_2}}{\theta}(1 + [\frac{h'}{h}z](z_a)) \tag{3}$$

Then, using the relation $\theta(x,a_2)dz = dy$ for fixed x, we deduce the derivative of $\frac{f_{2,a_2}}{f_2}(y|x,a_2)$ with respect to downstream result y which determines the sign of $\frac{\partial t_2}{\partial y}(y|x,a_2)$:

$$\frac{\partial}{\partial y}\frac{f_{2,a_2}}{f_2}(y|x,a_2) = -\frac{\theta'_{a_2}}{\theta}[\frac{h'}{h}z]'(z_a)\frac{\partial z}{\partial y} = -\frac{\theta'_{a_2}}{\theta^2}[\frac{h'}{h}z]'(z_a)$$

So, knowing the increase of the function θ with respect to the downstream action a_2, to ensure that the downstream distribution result F_2 satisfies the MLRP condition, we assume :

$(\mathcal{H}_1) : [\frac{h'}{h}z]'(z) < 0$ for all reduced downstream result $z > 0$.

In parallel, in order to check whether the first-order approach is valid, we consider the Jewitt's conditions. We consider utility function u that satisfies : $u(u'^{-1}(1/z))$ in concave in z (for example $u(z) = z^{1-\pi}$ with $\frac{1}{2} < \pi < 1$) and concerning distribution F :
$(\mathcal{J}_1) : \int_0^y F(y|x,a_2)dy$ is non increasing convex in a for all $y \in \mathcal{Y}$
$(\mathcal{J}_2) : \int_0^{+\infty} xdF(y|x,a_2)$ is nondecreasing concave in a
$(\mathcal{J}_3)$: (Concave Monotone Likelihood Ratio Property) the likelihood ratio $\frac{f_{a_2}}{f}(y|x,a_2)$ is nondecreasing and concave in y for all value of a.
To ensure the Jewitt's conditions, we consider the following condition on distribution H :

$(\mathcal{H}_2) : [\frac{h'}{h}z]'' > 0$ for all reduced downstream payment z.

With additional conditions on θ, Jewitt's conditions are satisfied (Proof in Appendix B) :

Proposition 3.1 *Let H be a distribution of support $\mathcal{Y} = R^+$ satisfying the hypothesis $(\mathcal{H}_1), (\mathcal{H}_2)$, let reduced downstream result $z_a = \frac{y}{\theta(x,a_2)}$ where θ is a non-negative, increasing function with respect to the upstream result x and downstream action a_2, concave in a_2. Then the downstream distribution result F_2 defined by $F_2(y|x,a_2) = H(z_a)$ satisfies the MLRP and the Jewitt's conditions.*

Moreover, using the relation $\theta'_x(x,a_2)zdx + \theta(x,a_2)dz = 0$ for fixed y, we deduce the derivative of $\frac{f_{2,a_2}}{f_2}(y|x,a_2)$ with respect to upstream result x :

$$\begin{aligned}\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x,a_2) =& -(1+[\frac{h'}{h}z](z_a))[\frac{\theta'_{a_2}}{\theta}]'_x - [\frac{h'}{h}z]'(z_a)\frac{\partial z}{\partial x}\frac{\theta'_{a_2}}{\theta} \\ =&(1+[\frac{h'}{h}z](z_a))\frac{\theta'_{a_2}\theta'_x - \theta''_{a_2,x}\theta}{\theta^2} + [\frac{h'}{h}z]'(z_a)z_a\frac{\theta'_{a_2}\theta'_x}{\theta^2}\end{aligned}$$

which determines the sign of $\frac{\partial t_2}{\partial x}(y|x,a_2)$. If $\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x,a_2) < 0$ then $\frac{\partial t_2}{\partial x}(y|x,a_2) < 0$, the more the upstream producer's output variable is favorable, the less the downstream contracting producer will be remunerated with payment t_2. But, at the opposite, if $\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x,a_2) > 0$ then $\frac{\partial t_2}{\partial x}(y|x,a_2) > 0$ and the more the upstream producer's output variable is favorable, the more the downstream contracting producer will be remunerated with payment t_2. These two types of downstream payment t_2's behavior with respect to the upstream result x are an unusual contractual specificity of an incentive contract due to the flow dependence. Thus, we analyze the sign of the derivative of $\frac{f_{2,a_2}}{f_2}(y|x,a_2)$ with respect to upstream result x for different distributions.

3.1 Multiplicative effect on the downstream result

We first consider the case where the expectation of the downstream result depends on the upstream result and the downstream producer's action in a multiplicative way : $\theta(x,a_2) = \alpha(x)\beta(a_2)$. Hence, $\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x,a_2) = L_{H0}(z_a)\frac{\theta'_{a_2}\theta'_x}{\theta^2}$ where the function L_{H0} is defined by $L_{H0}(z) = [\frac{h'}{h}z]'z$. Knowing that function θ increases with respect to the downstream action a_2 and the upstream result x, if the condition $(\mathcal{H}_1)$ is satisfied then $L_{H0}(z) \leq 0$ for all z. So, the behavior of function L_{H0} implies behavior of type $(\mathcal{B}_0)$ for the downstream payment t_2 :

$(\mathcal{B}_0)$ $\frac{\partial t_2}{\partial x}(y|x,a_2) \leq 0$ *for all upstream result x and downstream result y.*

So, the more favorable the upstream producer's output, the lower the downstream payment t_2.

The contract with payment t_2 is highly incentive and acceptable. We notice that, in this case, the behavior of the downstream payment is not distribution dependant.

In the case of a multiplicative effect on the downstream result, the upstream result and the downstream action effects are uncoupled. In this case, the distribution of downstream result can be rewritten $F(y|x, a_2) = H(\frac{y/\alpha(x)}{\beta(a_2)})$. So the decrease of upstream result x is linked to the increase of downstream result y, MLRP condition ensures both behaviors : downstream payment t_2 increasing with respect to y and decreasing with respect to x.

Considering the protection of animal species in a catchment area by two upstream-downstream agricultural or industrial producers, due to population dynamics the expectation of the downstream population y depends on the upstream population x and the downstream producer's action a_2 in a multiplicative way. The results obtained above are applicable to the relevant contract.

3.2 Additive effect on the downstream result

We now consider the case where the expectation of the downstream result y depends on the upstream result x and the downstream producer's action a_2 in an additive way : $\theta(x, a_2) = (\alpha(x) + \beta(a_2))^k, 0 < k < 1$.

From the expression of θ, we deduce : $\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x, a_2) = L_{Hk}(z_a)\frac{\theta'_{a_2}\theta'_x}{\theta^2}$ where the function L_{Hk} is defined by $L_{Hk}(z) = \frac{1}{k}(1 + \frac{h'}{h}z) + (\frac{h'}{h}z)'z$. Knowing that function θ increases with respect to the downstream action a_2 and the upstream result x, the sign of $L_{Hk}(z_a)$ determines the sign of $\frac{\partial}{\partial x}\frac{f_{2,\,a_2}}{f_2}(y|x, a_2)$ and therefore determines the increase or decrease downstream producer's payment t_2 with respect to the upstream result x. The sign of L_{Hk} depends on distribution H. Contrary to the previous case, the derivative $\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x, a_2)$ may not be of constant sign, hence it generates another behavior for the downstream payment.

Distribution H may generate a second type of payment behavior, for example $H_I(z) = \exp(\frac{-b}{z^p}),\ z \geq 0$ with $b > 0,\ p > 0$, then $L_{H_Ik}(z) = \frac{-p}{k}(1 + \frac{b(kp-1)}{z^p})$. So, if $kp \geq 1$ the downstream payment t_2 is of type $(\mathcal{B}_0)$ as in the multiplicative case. But, if $kp < 1$, it exists a unique $z_0 = (b\,(1 - kp))^{\frac{1}{p}} > 0$ such that $L_{H_Ik}(z) > 0$ for $z < z_0$ and $L_{H_Ik}(z) < 0$ for $z > z_0$. Similar behavior (as for the case $kp < 1$ with distribution H_I) may be obtained with distribution $H_{II}(z) = 1 - \frac{1}{(1 + bz)^p}$. These distributions H_I and H_{II} satisfy the hypothesis $(\mathcal{H}_1), (\mathcal{H}_2)$, hence the Jewitt's conditions. The distribution H_I with $kp < 1$ and the distribution H_{II} are typical cases of a more general situation where behavior of the downstream payment t_2 is of a new type :

Proposition 3.2 *Let H be a distribution of support $\mathcal{Y} = R^+$ satisfying the hypothesis $(\mathcal{H}_1), (\mathcal{H}_2)$, let reduced downstream result $z_a = \frac{y}{\theta(x,a_2)}$ where $\theta(x, a_2) = (\alpha(x) + \beta(a_2))^k, 0 < k < 1$ with*

α and β are non-negative and respectively increasing function with respect to the upstream result x and downstream action a_2. If $L_{Hk}(z) = 0$ for a unique z_0, $L_{Hk}(z) > 0$ for $z < z_0$ and $L_{Hk}(z) < 0$ for $z > z_0$ then behavior of the downstream payment t_2 is of the following type :

($\mathcal{B}_1$) *For all upstream result x, it exists a unique $y_0(x, a_2)$ such that the $\dfrac{\partial t_2}{\partial x}(y|x, a_2) > 0$ for $y < y_0(x, a_2)$ and $\dfrac{\partial t_2}{\partial x}(y|x, a_2) < 0$ for all $y > y_0(x, a_2)$ with $y_0(x, a_2) = \theta(x, a_2)z_0$.*

In our example we show that type of downstream payment ($\mathcal{B}_0$) or ($\mathcal{B}_1$) may depend of the parameters of the distribution : the downstream payment t_2 is of type ($\mathcal{B}_0$) if $kp \geq 1$ and the downstream payment t_2 is of type ($\mathcal{B}_1$) if $kp < 1$.

In case of an additive effect and contrary to the previous multiplicative effect case, the upstream result x and the downstream action a_2 are coupled in their effects. Thus even if downstream payment t_2 increasing with respect to y, downstream payment t_2 may be increasing with respect to x.

With a downstream payment t_2 of type ($\mathcal{B}_1$), for low values of downstream result y, the downstream producer's payment t_2 is increasing with respect to the upstream producer's result x. Hence, the upstream contractor not putting forth enough efforts, induces a decrease of the downstream contractor's payment. So assuming the characteristics of an upstream-downstream incentive contract not known by the two contractors, the downstream payment function could be not acceptable (for $y < \theta(x, a_2)z_0$). We remark that the contract is indeed optimal since it has been designed to be optimal for the needs of the public environmental agency. Nevertheless as the contractors mainly focus on payments, they are not necessary aware of this aspect and they might question these payments.

To avoid this questioning, in order to make the contract payment function acceptable, it is necessary to impose a non-increasing downstream payment t_2 with respect to upstream result x for a fixed downstream result y. So we should impose the supplementary condition :

$$\frac{\partial t_2}{\partial x}(y|x, a_2) \leq 0 \text{ for all upstream result } x \text{ and downstream result } y \qquad \text{(AC)}$$

This condition is equivalent to $\dfrac{\partial}{\partial x}\dfrac{f_{2,a_2}}{f_2}(y|x, a_2) \leq 0$. The condition (AC) is named acceptability constraint and is joined to participation and incentive contraints in the initial program (1).

The payment t_2 may be modified to ensure the non-increasing of downstream payment with respect to upstream result x according to :

Proposition 3.3 *With the notations of Proposition 3.2, if downstream payment t_2 is of type ($\mathcal{B}_1$) and y_0 invertible with respect to upstream result x then the downstream producer's payment T_2 ensures the non-increasing of downstream payment with respect to upstream result x :*
- for $y < y_0(x, a_2)$, $T_2(y, x) = t_2(y, x_0(y))$ where $x_0(y)$ is the unique solution of equation

$y = y_0(x_0, a_2)$.
- for $y > y_0(x, a_2)$, the payment $t_2(y, x)$ is acceptable, that is to say $T_2(y, x) = t_2(y, x)$.

The proof of this Proposition is in Appendix C. For low values of downstream result y ($y < y_0(x, a_2)$), the downstream payment $T_2(y, x)$ is constant with respect to upstream result x, whereas $t_2(y, x)$ is increasing with respect to y : there is bunching for low downstream result y. The complexity of this contract is only related to the nature of the distribution of the downstream result $F_2(y|x, a_2)$: the distribution of upstream result $F_1(x)$ has no impact.

This bunching is specific : MLRP condition is both satisfied for upstream and downstream payment distributions but the downstream payment may be constant on an interval of upstream result values. This new kind of bunching is the consequence of the interaction of the two producers.

Considering the preservation of agricultural soil (to avoid soil erosion) by two upstream-downstream farmers using new agricultural practices, due to the additivity of mud flows the expectation of the downstream flow y depends on the upstream flow x and the downstream producer's action a_2 in an additive way. The results obtained above are applicable to the relevant contract.

3.3 Mixed effect on the downstream result

We now focus on the case where the expectation of the downstream result both depends on the upstream result and the downstream action : $\theta(x, a_2) = exp(\xi a_2 x + \alpha(x) + \beta(a_2))$.

As in the case of an additive effect, the upstream result x and the downstream action a_2 are coupled in their effects. We consider the additional hypothesis :

$$(\mathcal{H}_3) : \lim_{z\to 0} 1 + \frac{h'}{h}z \geq 0, \lim_{z\to+\infty} 1 + \frac{h'}{h}z < 0 \text{ and } [\frac{h'}{h}z]'z \text{ increasing.}$$

In this case, downstream payment t_2 has a third behavior denoted $(\mathcal{B}_2)$:

Proposition 3.4 *Let H be a distribution of support $\mathcal{Y} = R^+$ satisfying the hypothesis $(\mathcal{H}_1), (\mathcal{H}_2), (\mathcal{H}_3)$, let reduced downstream result $z_a = \frac{y}{\theta(x,a_2)}$ where $\theta(x, a_2) = exp(\xi a_2 x + \alpha(x) + \beta(a_2))$ with $\xi > 0$ is a non-negative, increasing function with respect to upstream result x and downstream action a_2, concave in a_2. Then the behavior of the downstream payment t_2 is of the following type :*

($\mathcal{B}_2$) *For all upstream result x, it exists a unique $y_0(x, a_2)$ such that $\frac{\partial t_2}{\partial x}(y|x, a_2) < 0$ for all $y < y_0(x, a_2)$ and $\frac{\partial t_2}{\partial x}(y|x, a_2) > 0$ for all $y > y_0(x, a_2)$.*

The proof of Proposition 3.4 is given in Appendix D. If the downstream payment t_2 is of type $(\mathcal{B}_2)$, the acceptability constraint (AC) is not satisfied for $y > y_0(x, a_2)$. We illustrate this

case with distribution H_I and function $\theta(x, a_2) = \exp(1 + 4a_2x - 2a_2^2)$ with $\frac{1}{4} \leq x \leq \frac{1}{2}$ and $0 \leq a \leq \frac{1}{4}$. We have $\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x, a_2) = 4p(1 - \frac{b}{z^p}) - 16\frac{bp^2}{z^p}(x - a_2)a_2$, so $y_0(x, a_2) = [b(1 + 4p(x - a_2)a_2)]^{\frac{1}{p}} exp(1 + 4xa_2 - 2a_2^2)$ and $\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x, a_2) > 0$ for $y > y_0(x, a_2)$.

If y_0 is invertible with respect to result x then as in the previous case with Proposition 3.3, it is possible to obtain payments satisfying the acceptability condition (AC) :
- for $y < y_0(x, a_2)$, the payment $t_2(y, x)$ is acceptable, that is to say $T_2(y, x) = t_2(y, x)$.
- for $y > y_0(x, a_2)$ the downstream producer's payment becomes $T_2(y, x) = t_2(y, x_0(y))$ where $x_0(y)$ is the unique solution for equation $y = y_0(x_0, a_2)$. So payment $T_2(y, x)$ is constant with respect to upstream result x, whereas $t_2(y, x)$ is increasing with respect to x. So, there is bunching for high downstream result y.

Considering the preservation of a non polluted river by two upstream-downstream producers, due to the complexity of river water flows, the expectation of the downstream result y depends on the upstream result x and the downstream producer's action a_2 in a mixed way. The results obtained above are applicable to the relevant contract.

3.4 Examples and interpretation

We illustrate the three $(\mathcal{B}_0)$, $(\mathcal{B}_1)$ and $(\mathcal{B}_2)$ type behaviors of the downstream payment t_2 previously highlighted. We consider downstream probability distribution F_2 with expected downstream result proportional to $\theta(x, a_2)$ generated by the distribution H_I.

First, $\theta(x, a_2) = (1 + x + a_2)^k, 0 < k < 1,$ *for* $x \geq 0$ *and* $a_2 \geq 0$:

Figure 1 and Figure 2 respectively represent the isovalue (in the plane (x, y)) of downstream producer's payment for the case $kp \geq 1$ and $kp < 1$. On both figures, an arrow indicates the direction of increasing payments.

In case $kp \geq 1$, using the direction of increasing payments in Figure 1, we deduce that for a constant result y, the payment t_2 decreases with increasing result x. Hence in this case the payment t_2 is acceptable.

In case $kp < 1$ for a small x (such that $\theta(x, a_2)z_0 < y$) using the direction of increasing payments in Figure 2, we deduce that, for a constant y, the payment t_2 decreases with increasing x, hence t_2 is acceptable. But for a high value of x (such that $\theta(x, a_2)z_0 \geq y$) we see that for a given y, the payment t_2 (in dash line) increases with increasing x hence t_2 is not acceptable. Contrarily, the payment T_2 (in solid line) doesn't increase and is constant with respect to x.

Both, the distribution H that generates the distribution of the reduced downstream result and the expression $\theta(x, a_2)$ (which is proportional to the expectation of the downstream result y)

control the type of payment, with or without modification.

Second, $\theta(x, a_2) = exp(1 + 4a_2x - 2a_2^2)$*, for* $\frac{1}{4} \leq x \leq \frac{1}{2}$ *and* $0 \leq a \leq \frac{1}{4}$:

Figure 3 represents the isovalue of downstream payment t_2. In this case, for a small value of x (such that $\theta(x, a_2)z_0 \leq y$) we then deduce that for a given y, the payment t_2 (in dash line) increases with increasing x, hence t_2 is not acceptable. Contrarily, the payment T_2 (in solid line) doesn't increase and is constant with respect to x.

In all cases, looking at the Figures we confirm this corollary of the Propositions :

Corollary 3.1 *In the plane* (x, y)*, restricted to the isocontours curves of the downstream payment* $T_2(y, x)$*, downstream result* y *is a nondecreasing function with respect to upstream result* x*.*

Proof : On isocontours curves of the downstream payment, $T_2(y, x)$ is constant, hence $\frac{dT_2}{dx} = \frac{\partial T_2}{\partial y}\frac{dy}{dx} + \frac{\partial T_2}{\partial x} = 0$. From $\frac{\partial T_2}{\partial y} > 0$ and $\frac{\partial T_2}{\partial x} \leq 0$, we deduce the result. □

In conclusion, we obtained very different behaviors with the previous parametrizations : no bunching in payment of type ($\mathcal{B}_0$), hence by construction the downstream payment function is acceptable, contrary to the payment functions of type ($\mathcal{B}_1$) or type ($\mathcal{B}_2$). Thanks to the acceptability constraint, we obtain two cases of bunching in payment either for low downstream result ($\mathcal{B}_1$) or for high downstream result ($\mathcal{B}_2$). In these last two cases, the bunchings induce the acceptability of the downstream payment function.

4 Discussion

We have considered two producers of an environmental public good in a spatialized and flow dependence involved in a collective action. This action is carried out in the framework of a moral hazard contract binding the two producers. One is upstream of the other and each one acts on his area. The question was thus how to build a partnership between these two producers through a well-informed incentive contract.

The additive, multiplicative or mixed models show that the upstream-downstream moral hazard contract binding two producers interacting to obtain an overall result can be resolved. It is undoubtebly preferable for society to take into account precise information to build an incentive contract, and taking into account the additional upstream information on which the downstream result depends is very profitable. Hence, the non-linear payment to the downstream producer will depend both on his own result and on the result of the upstream producer. These models thus make it possible to build a necessary partnership of the two producers so that they produce together the public good sought by the principal.

An incentive contract is by construction optimal and gives an optimal payment function, but we stressed that the payment of the downstream producer could have a problematic behavior compared to the results of the upstream producer and so could be disputed by the contracting agents within the framework of our working hypotheses. For some configurations the incentive payment to the second producer (the downstream one) could be non acceptable by the contractors. Everything will depend on the probability distribution of the results obtained through the application of the upstream-downstream contract. Thanks to a new constraint named acceptability constraint, the non-linear incentive payment of the downstream producer will not depend on the results of the upstream producer on a given interval, inducing a new type of bunching with the results of the upstream producer [1], in an upstream-downstream moral hazard contract case study. That could clearly improve the acceptability of the contract payment functions by the two contractors. This characteristic concerns any producer in relation to another producer and any action in accordance with the analyzed moral hazard contract. These results generalize those already known and explained for example in Salanié (1997). Moreover, if we consider producers' utilities impacted by their actions, with a negative externality $h_1(x)$ for the upstream stand (respect. $h_2(y)$ for the downstream stand) increasing with respect to upstream result x (respect. y for the downstream stand), previous Propositions and characteristics remain valid [2].

Thanks to the new acceptability constraint, the use of the upstream-downstream incentive contract with bunching is relevant in various situations. A first example is the management of a non-polluted river flow. Another example is related to the productive practices allowing the preservation of animal or vegetal biodiversity or the agricultural soils in order to avoid their erosion in a catchment area. In these various situations and configurations, studying an incentive upstream-downstream contract binding three or more producers is an interesting research outlook.

5 Conclusion

In exploring how to use a moral hazard contract to carry out a collective action by two producers to produce an environmental public good in a spatialized and flow dependence, an upstream-downstream moral hazard contract is studied. As the nature of the information selected when designing the contract seems essential to establish the properties of the incentive contract we have sought to build a partnership between the producers through a well-informed upstream-downstream incentive contract.

The upstream-downstream moral hazard contract is calculable. Incentives in an upstream-downstream moral hazard contract are generated without difficulties with a non-linear payment. But the incentive downstream payment function is not necessarily acceptable within the framework of our working hypotheses. Indeed, everything will depend on the probability distribution of the results obtained through the application of the upstream-downstream contract.

1. An exogenous constraint on $t_2(x, y)$ could induce its decrease and so an acceptable contract.
2. The upstream producer's gross utility (respect. downstream) becomes $u_1(t_1(x) - h_1(x))$ (resp. $u_2(t_2(y, x) - h_2(y))$. The main modification is a translation in the payment : $h_1(x)$ (respect. $h_2(y)$) is added to payment T_1 or t_1 (resp. T_2 or t_2). The principal thus pays the value of the externality.

Thanks to the right financial incentives, the use of an upstream-downstream incentive contract with an acceptability constraint allows to regulate industrial or agricultural practices respectful of the physical or ecological environment. This work could serve as a basis for studying various environmental contracts.

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A The Lagrangian of the upstream-downstream contract

Let us consider the Lagrangian with the Lagrange multipliers λ_0 for the principal's participation constraint, λ_1, λ_2 for the participation constraints and μ_1, μ_2 for the incentive constraints imposed on upstream producer 1 and downstream producer 2 :

$$\mathcal{L}(a_1, a_2, t_1(.), t_2(.,.)) =$$
$$\int_x \int_y [G(x,y) - (1+\gamma)(t_1(x) + t_2(y,x)) + u_1(t_1(x)) - w_1(a_1) + u_2(t_2(y,x)) - w_2(a_2)] dF_2(y|x,a_2) dF_1(x|a_1)$$
$$+ \lambda_0 (\int_x \int_y (G(x,y) - (1+\gamma)(t_1(x) + t_2(y,x))) dF_2(y|x,a_2) dF_1(x|a_1) - E_0)$$
$$+ \lambda_1 (\int_x u_1(t_1(x)) dF_1(x|a_1) - w_1(a_1) - E_1) + \lambda_2 (\int_x \int_y u_2(t_2(y,x)) dF_2(y|x,a_2) dF_1(x|a_1) - w_2(a_2) - E_2)$$
$$+ \mu_1 (\int_x u_1(t_1(x)) dF_{1,a_1}(x|a_1) - w_1'(a_1)) + \mu_2 (\int_x \int_y u_2(t_2(y,x)) dF_{2,a_2}(y|x,a_2) dF_1(x|a_1) - w_2'(a_2))$$

B Proof of the Proposition 3.1

From expression of distribution F, assuming θ concave in a, we successively check the Jewitt's conditions :

- $\int_0^y F_{a_2}(y|x,a_2)dy = -\theta'_{a_2} \int_0^{z_a} zdH(z) < 0$, $\int_0^y F_{a_2a_2}(y|x,a_2)dy = -\theta''_{a_2a_2} \int_0^{z_a} zdH(z) + \frac{{\theta'_{a_2}}^2}{\theta} z_a^2 h(z_a)$ then from concavity of θ, $\int_0^y F_{a_2a_2}(y|x,a_2) \geq \frac{{\theta'_{a_2}}^2}{\theta} z_a^2 h(z_a)$, so $(\mathcal{J}_1)$ is satisfied.

- $\int_{\mathcal{Y}} ydF(y|x,a_2) = \int_{\mathcal{Y}} ydH(\frac{y}{\theta}) = \theta \int_Z zdH(z)$ then $\int_{\mathcal{Y}} ydF_{a_2}(y|x,a_2) = \theta'_{a_2} \int_Z zdH(z) >$

0 and $\int_{\mathcal{Y}} y dF_{a_2 a_2}(y|x,a_2) = \theta''_{a_2 a_2} \int_Z z dH(z) < 0$, so $(\mathcal{J}_2)$ is satisfied.

- from $(\mathcal{H}_1)$, $\frac{f_{2,a_2}}{f_2}(y|x,a_2) = -\frac{\theta'_{a_2}}{\theta^2}(1 + [\frac{h'}{h}z](z_a))$ is nondecreasing. Moreover, from $(\mathcal{H}_2)$, we deduce the concavity of the likelihood ratio. □

C Proof of the Proposition 3.3

To take into account the acceptability condition (AC), we introduce a non-negative Lagrange multiplier η in the Lagrangian.
The corresponding first order condition gives : $\eta(y,x)\frac{\partial T_2}{\partial x}(y,x) = 0$ with $\eta(y,x) \geq 0$ and $\frac{\partial T_2}{\partial x}(y,x) \leq 0$. Hence either $\frac{\partial T_2}{\partial x}(y,x) < 0$ then $\eta(y,x) = 0$ (which corresponds to the case $y > \theta(x,a_2)z_0$) either $\frac{\partial T_2}{\partial x}(y,x) = 0$ (which corresponds to the case $y < \theta(x,a_2)z_0$). In the last case we verify that payment T_2 is increasing with respect to result y.
To ensure this behavior, payment T_2 must be increasing with respect to y on the curve $y = \theta(x,a_2)z_0$ that is $f_{2,a_2}/f_2$ must be increasing with respect to y at constant $z = z_0$ or equivalently with respect to x, hence from Equation (3) :

$$\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(\theta(x,a_2)z_0|x,a_2) = (\theta'_{a_2}\theta'_x - \theta''_{a_2 x}\theta)(1 + [\frac{h'}{h}z](z_0)) = \frac{1}{k}(1 + [\frac{h'}{h}z](z_0))\theta'_{a_2}\theta'_x > 0 \square$$

D Proof of the Proposition 3.4

$\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x,a_2) = -(1 + \frac{h'}{h}z)\xi + [\frac{h'}{h}z]'z\frac{\theta'_{a_2}\theta'_x}{\theta^2}$. From $1 + \frac{h'}{h}z \geq 0$ in the vicinity of $z = 0$ and $(\mathcal{H}_1)$ we deduce that $\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x,a_2) < 0$ in the vicinity of $z = 0$. From behavior of h for large values of z, we deduce that $\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x,a_2) > 0$ for sufficiently high value z. From the increase of $[\frac{h'}{h}z]'z$ with respect to z, we deduce the existence of $z_0(x,a_2)$ such that $\frac{\partial}{\partial x}\frac{f_{2,a_2}}{f_2}(y|x,a_2)$ negative for $z < z_0$ and positive for $z > z_0$, thus $y_0(x,a_2) = z_0(x,a_2)\theta(x,a_2)$. □

References

Baumol W. and W. Oates. 1988. The Theory of Environmental Policy. Eds Press, Cambridge, UK.

Carlier, G. and R. A. Dana, 2005. Existence and Monotonicity of Solutions to Moral Hazard Problems. Journal of Mathematical Economics 41 : 826-843.

Dale V. and S. Polasky. 2007. Measures of the effects of agricultural practices on ecosystem services. Ecological Economics 64 (2) : 286-296.

Elyakime B. 2000. B. L'érosion des terres agricoles et sa gestion préventive : un cadre d'action collective et interactive, Région et Développement, 12, 105-120.

Gabel H. L. and B. Sinclair-Desgagné. 1993. Managerial incentives and environmental compliance. Journal of Environmental economics and management 24 : 229-240.

Goldsmith, P.D. and R. Basak. 2001. Incentive Contracts and Environmental Performance Indicators. Environmental and Resource Economics 20 (4) : 259-279.

Grossman, S. and O. Hart. 1983. An Analysis of the Principal-Agent Problem. Econometrica 51 (1) : 7-45.

Holmström, B. 1979. Moral Hazard and Observability. The Bell Journal of Economics 10 (1) : 74-91.

Jewitt, I. 1988. Justifying the First-Order Approach to Principal-Agent Problems. Econometrica 56 (5) : 1177-1190.

Laffont J.J. and D. Martimort. 2002. The theory of incentives : the principal agent model. Eds Princeton University Press.

Laffont J.J. and J. Tirole, 1996. Pollution permits and environmental innovation. Journal of Public Economics 62 (1-2) : 127-140.

Le Bissonnais Y. 2013. Webinaire organisé par l'Association Française pour l'Etude des Sols. https ://vimeo.com/82442543

Mirrlees, J. A. 1999. The Theory of Moral Hazard and Unobservable Behavior : Part I. Review of Economic Studies 66 : 3-21.

Perman R., Y. Man, J. McGilvray and M. Common. 2003. Natural ressource and environmental economics. Third edition Eds Pearson Education Limited.

Rogerson, W. P. 1985. The First-Order Approach to Principal-Agent Problems. Econometrica 53 : 1357-1367.

Salanié B. 1997. The Economics of Contracts : A primer. Eds The MIT Press.

Shavell, S. 1979. On Moral Hazard and Insurance. Quaterly Journal of Economics 93 : 541-562.

Sinclair-Desgagné, B. 1994. The first-order approach to multi-signal principal agent problems. Econometrica 62 (2) : 459-465.

Sinclair-Desgagné, B. 2009. Ancillary Statistics in Principal-Agent Models, Econometrica 77 : 279-281.

Vachon S. and R. D. Klassen 2006. Extending green practices across the supply chain. The impact of upstream and downstream integration. International Journal of Operations and Production Management 26 (7) : 795-821.

Figure 1: isovalues of downstream payment for k $p \geq 1$

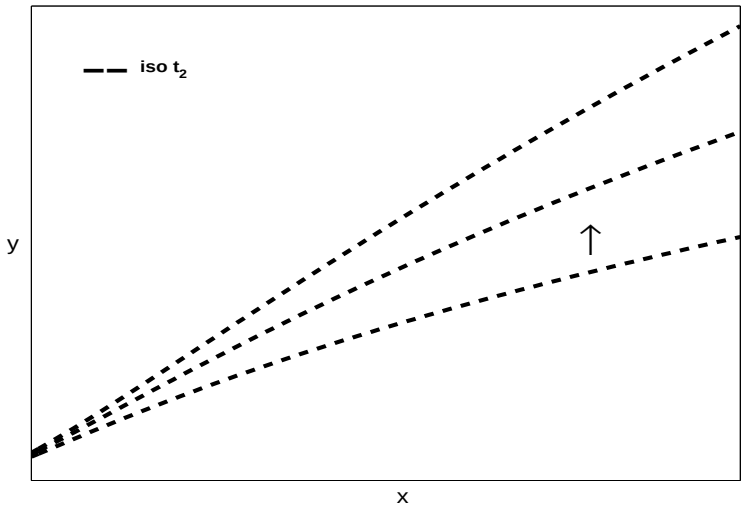

Figure 2: isovalues of downstream payment for k p < 1

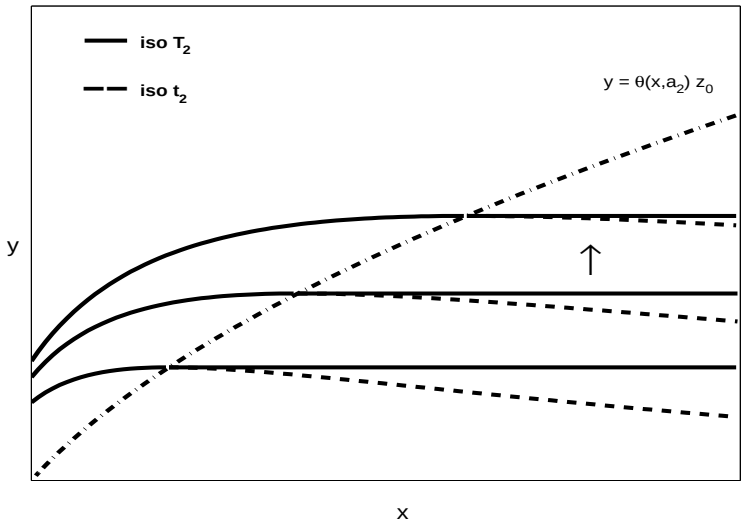

Figure 3: isovalues of downstream payment of type (iii)