Retail distribution in the city: analyzing point patterns in non-uniform network space

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RESUME. La distribution des commerces dans la ville doit être analysée en prenant en compte la structure réticulaire de l'espace urbain. De surcroît, les réseaux viaires des villes contemporaines varient en intensité et en configuration dans l'espace. Le concept d'hétérogénéité dans un semis de points est souvent associé à l'hétérogénéité de la distribution spatiale de son intensité. Cet article développe une nouvelle proposition méthodologique pour analyser une distribution de points sur un réseau non uniforme. L'analyse configurationnelle des réseaux et les théories fractales fournissent des paramètres locaux de la distribution spatiale du réseau et du semis de points. La combinaison de ces deux valeurs nous informe sur la relation entre les distributions des points et du réseau. L'homo/hétérogénéité d'un semis de points est redéfinie par rapport à la distribution spatiale locale du réseau. Une application de la méthode pour un cas d'étude réel de distribution commerciale est présentée dans la seconde partie de l'article.

ABSTRACT. Retail distribution in the city has to be analyzed taking into consideration the network structure of urban space. What is more, street networks in contemporary cities vary in intensity and configuration over space. The concept of heterogeneity referred to point patterns is often associated to the heterogeneity of its intensity distribution. This paper develops a new methodological proposal for analyzing the distribution of points on a non-uniform network. From the configurational studies of networks and fractal theories, we derive local parameters of network and point pattern distribution. The combination of these two values give us an information about the relation between the points and the network distribution. The homo/heterogeneity of a point pattern is redefined in relation to the relative local network pattern. An application of this method to a real case-study of retail distribution is presented in the second part of the paper.

MOTS-CLES : distribution des commerces, réseau viaire urbain, analyse des semis de points, réseau hétérogène, dimension fractale, atteignabilité, fonction K de Ripley

KEYWORDS: retail distribution, urban street network, point pattern analysis, heterogeneous network, fractal dimension, reach, Ripley's k-function

1. Introduction: spatial point patterns on networks

When we want to study the spatial distribution of retail activity in a city, the problem that we face is a typical problem of *point pattern analysis on network* where points represent retail activity distribution, while the network is the spatial domain where events occur, namely the urban street network.

The increasing awareness of the importance of spatial relations in every discipline, stimulated the creation of *ad hoc* solutions, with a consequent large variety of methods for spatial pattern recognition (Perry et al. 2002). Each of them studies a particular aspect of spatial relations, which is necessary for the comprehension of the spatial behavior of the phenomenon under investigation. Dale et al. (2002) analyzed a large number of methods arising from several disciplines in order to identify similarities from both mathematical and conceptual points of view. They conclude their work affirming that despite the partial redundancy of these methods, their combination is required for a better comprehension of different aspects of spatial structures. This remark legitimates the need to explore new approaches in point pattern analysis and to couple them with more traditional ones, with the aim of identifying new aspects of spatial distributions of events. In this work, we will try to understand if a new approach, strongly building on well-established techniques, could lead us to a better understanding of the spatial relation between network and point distributions.

Resuming and describing all methods for spatial patterns analysis on networks, is an ambitious task which requires an abstraction from any particular domain of application and deepest knowledge of each method; this goal goes beyond our scope. Within this paper we are more interest in the applications of some particular methods with a heuristic approach, for exploratory data analysis (EDA). As suggested by Perry et al. (2006) "*spatial statistics have most utility when used alongside a process-based or mechanistic investigation of the underlying processes driving the spatial patterns observed, whether experimental or model-based*".

For this reason, in the second part of this paper we will recapitulate only a subset of these statistical methods. More particularly, we will focus on local analyses only, with the aim of characterizing local distributions of points as they are observed from each of them. Our underlying research question is in fact the characterization and explanation of retail distributions as observed from any retail location in the city, given the street network connecting retail activities among them and given strong heterogeneities in the street network. Ripley's local K function (1979) and its adaptation to network space (Okabe 2012), retained our attention: its implementation process is the starting point of other network-based analyses, based on configurational and fractal approaches, which lead to indicators describing different aspects of spatial patterns.

In the third part of this paper we will describe the basic principles of fractal and network configuration analyses and highlight the similarities and differences with some phases of K-function analysis on networks. Domain implications in spatial distribution of retail activities in the city will be explored.

A new methodology will be then proposed in section 4 to analyze the relation between the spatial distribution of the network and points, inheriting properties from the three aforementioned approaches. This simple method provides a new local indicator measuring hot- and cold- spots in relation to the network predisposition to support the presence of the point distribution.

The application of this methodology to a real case study of retail activity on an urban street network will show us its peculiarities and limitations in section five. Conclusions and research perspectives will complete the paper.

2. Point pattern analysis: some methods

First of all, we need to define the subject of this study: a spatial point process is a *"stochastic mechanism which generates a countable set of events* x_i *in the plane"* (Diggle 2003, p. 43); point pattern is a realisation of a process (Gatrell et al. 1996), which means a simple collection of points distributed in a specific spatial domain. The analyzed region can have a particular size and shape: in urban studies, often this space coincides with the urban street network. Point patterns are realizations of point processes. In the simplest case, each point (or '*event'*, Diggle 2003) is defined by its spatial coordinates; when additional attributes are considered, we refer to 'marked' point patterns (Gatrell et al. 1996). In this paper, we will consider the simple point pattern defined on an urban street network in order to evaluate different methodological approaches. Further works could consider marked information (i.e. retail category).

Traditionally statistical methods are divided in those using areas as units of calculus (e.g. the quadrat-based methods described by Greig-Smith 1952 Moran's I 1950 and others) and statistics using distances (Clark and Evans 1954 and others). The former are preferred when analyzing events occurring in a discrete space, i.e. administrative units, segments of a river system, etc. and/or where the precise location of the events within this partitioning of space is unavailable or irrelevant. The latter are more adapted when events are described by their precise localization and no *a priori* space partitioning can be assumed. Since retail locations in the city are precisely known[1] and we do not want to recur to any pre-established partitioning of space, we will focus here on distance-based methods.

The first step when analyzing point distribution in space is exploring its density calculated as the ratio between the number of points and the size of the analyzed space. Intensity is the inverse function of density and is also usually employed. It describes the average spacing between the event points. Since retail activities are located on a network constrained space, our denominator (numerator for intensity)

[1] our case study application has been realized using the detailed geodatabase from the local Chamber of Commerce.

will be the length of the analyzed street network (or sub-network). This procedure gives us a general information about the whole study area; applying the same ratio to subspaces, we can investigate how density and intensity vary in space. Local measures of density/intensity on networks can be obtained with several methods as the Kernel density (Okabe 2012), or the moving window on network (Steenberghen 2010) adaptation of the quadrat method.

These approaches inform us about the density/intensity of the point pattern insisting on a selected space or subspace; comparing the local intensity to its average on the whole space of study, give us an information of local high/low concentrations. This process is implemented for example in LINCS analysis (Local Indicators of Network-Constrained Clusters, see Yamada and Thill 2004, 2007, Fusco and Araldi 2016).

If we want to go further, analyzing the reciprocal position of points, other techniques need to be used: nearest neigh neighborhood (NN, Clark 1954) the n-th NN, pair correlation function Ripley's K function and others. These methods are characterized by the study of the relative distribution of points around each occurrence with a distance based function summarizing the general spatial structure.

Ripley's K function retains our attention in this paper for several reasons: while NN, n-th NN, pair correlation functions study points with specific spatial conditions, K function analyzes patterns of all points at different scales. This function is obtained considering the mean of the count of events within disks centered on each point; this count is divided by the overall density in the study region. Increasing the radius, we obtain a function describing the point pattern at different scales. K function considers as reference *n* realizations of a Poisson spatial process with known probability using a Monte Carlo sampling scheme. The definition of this process and its properties are widely discussed in literature (ex Stoyan and Penttinen 2000). The purpose of the *n* realizations is to obtain significant upper and lower bounds for different radii of analysis: if the empirical K function of the point distribution falls inside this envelope, the hypothesis of randomness is satisfied. If the empirical value is higher than the upper bound for a given radius, our point pattern presents significant spatial clustering behavior at this scale of analysis; if it falls below the lower one, we can infer the presence of regularity in our distribution.

When Ripley's K function is applied on a point pattern on network, the traditional planar approach needs to be revisited: spatial relation between points are distorted because of the network spatial constrain which reduce the spatial domain where events occur. In this case, the procedure presented above needs to consider network-based subspaces instead of Euclidean disks. Many studies demonstrated statistically how the planar techniques can generate false clusters or hot-/cold-spots when applied to a network constrained space; Yamada and Thill (2004) use a data set of traffic accidents in Buffalo in order to demonstrates how the general planar K function often overestimates clusters compared to the network based methods. The same conclusion has been reached when applied to crime data (Lu et Chen 2007).

Whether in planar or in network domain, another aspect needs to be considered. The main assumption of complete spatial randomness is that the intensity of events will not vary across the region (Diggle 2003). Several studies, in the last decade, considered the heterogeneity of the point pattern distribution on network. Two main solutions have then been proposed. Okabe (2012), considering car accidents on road networks, proposes a ‘physical’ modification of the network space in order to consider the variation of the traffic density on each axis. The second solution takes in consideration the variation of the intensity of the point process at a macro-scale within the study area: the kernel density or different a priori intensity parameters are used in the K function (Baddeley et al. 2000).

The discussion of the best method to implement when considering spatial heterogeneity goes beyond the purpose of this study. It seems to us, however, that the aforementioned solutions focus more on the heterogeneity of point patterns on networks than on the heterogeneity of the underlying network spatial distribution. One could say that the network heterogeneity is considered inasmuch as the K function is obtained by the ratio between the count of points in a selected sub-space and the sub-space size. But this is precisely a way of focusing on variation of intensity in space, by assuming that the phenomenon under study is independent on network heterogeneity. What we want to explore in this work is precisely the relation between point pattern and network pattern heterogeneities. This forces us to first analyze them separately and only later explore the relations between the two.

What we have to retain from this discussion is the radial-centered, network-based approach used by the K function to count events at different distances and informing us on the distribution of the points around any selected event. In the next section, we will see how this approach is adopted in other methods, namely fractal analysis and network configuration analysis.

3. Patterns analysis: similarities with different approaches

3.1 Fractal analysis

As introduced in the previous section, when analyzing the spatial distribution of a human settlement, its social and economic functioning, independently by the scale of observation we can observe an apparently irregular distribution; geographers, sociologists, economists before other disciplines had to face this particular distribution (Frankhauser 1998). Under the apparent irregularity of the spatial distribution, a more complex spatial ordering has often been identified. Self-organization phenomena have been associated to these complex form and distributions.

Fractals have been demonstrated to be a valid approach to the analysis of these irregular distributions. The impossibility to find a regular pattern within the usual Euclidean geometry of discrete dimension (the line, the plane, the 3-D volume) does not mean that regularities cannot be detected in a space possessing a fractal dimension. Fractal analysis tries first to find a singular parameter describing a repeating pattern at different scales, a parameter capable of accounting for the apparent irregularity of a geometrical features spatial distribution. This parameter is the fractal dimension of the distribution. In some cases, braking points in the fractal dimension can be found over the scale range. The purpose of the analysis is then to find a simple set of parameters in order to explain the complexity of the non-homogenous spatial organization. Starting from Mandelbrot (1977), researchers widely apply the fractal approach on urban systems and agglomerations developing new morphological parameters (Fotheringham et al. 1989, Batty and Longley1994, White 1993, Frankhauser 1994, 1998. etc).

Based on the large scientific literature of urban fractal forms, we will focus our discussion on a particular formulation of fractal analysis. In this domain, analysis can have a global or local approach; in the first group, we can find grid, dilatation and correlation methods (Batty and Longley 1994, Frankhauser 1994). These methods are characterized by the determination of a unique value describing the whole distribution regularity. More interesting for our purpose are the local methods based on the radial analysis and on *curves of scaling behavior*.

Fractal approach to network analysis has been applied in Albert and Barabasi (2002), Song et al. (2005) and Bejan (1996). In many of these studies, the fractal analysis is applied to topological networks (WWW, proteins, cellular networks etc.) using the box counting method on topological measure of distances between nodes. In this work, we are more interested in its metrical implementation because of our case study represented by a real street network.

If we consider the radial analysis, we can find similarities with the statistical method of K function: they both consider a specific point called center of analysis and for different radii *r* they count the number of elements (i.e. built cells on a regular grid for radial analysis, number of events for K function) falling inside each disk. The fractal law that we obtain through this analysis is given by a regression line on a log-log diagram, plotting the radius of calculus *r* and the corresponding number of elements *N(r)*:

$$N(r) = r^D \rightarrow \log N(r) = D_r * \log r \quad (1)$$

From this equation, we obtain the fractal dimension D_r; this parameter resumes the trans-scalar spatial organization of the geometrical pattern around the center of analysis. As suggested by Frankhauser (1998, p. 217), this local method is a detailed approach in studying variation of local spatial patterns "*by comparing the results obtained for different counting centers"*; additionally, he described this approach as an instrument for research for the analysis of empirical structures through their comparison and classification.

As we can see, this procedure is equivalent to the one described above in Ripley's K function in a Euclidean space. The counting method with disks of increasing radii is applied by fractal analysis in order to identify the parameter synthetically describing the apparently irregular distribution analyzed. Like the K function, we will need to apply the local fractal approach in the network space: we will see in the following section how the network approach will preserve the same structure of equation (1). When calculated locally on the street network itself, fractal parameter D_r will then describe how the spatial domain size grows with the radius of analysis around each point; this information determines a fundamental property of the spatial domain where point patterns take place.

3.1 Network configurational analysis

The same principle of non-homogeneity in the spatial distribution of urban, social and economic patterns discussed before, can be investigated on networks.

Bavelas (1948) identified the importance that central places in social networks have on human communication: a central localization is characterized by power, influence and control properties on the whole network. Based on this theory, several indicators of network properties have been studied (ex: Leavitt 1951; Freeman, 1977 etc.). This approach, has been later applied to urban systems and street networks. Within a street network, the notion of centrality takes a wider connotation ranging from visibility, popularity, passage and attraction notions, influencing economical and functional properties of the space. The central idea of configurational analysis of street networks is that the network elements possess configurational properties derived from the spatial relations that they establish with all other network elements within the whole study area (global properties) or a subspace of it (local properties within a given radius). The network configuration is seen as an underlying organizer of patterns of potential movement and encounters. When buildings and urban functions are integrated in the configurational analysis of the street network, increasingly comprehensive analyses of the configuration of urban space can be performed.

When we study the relations between urban network configurational properties and the localization of social and economic activities within a city, *The social logic of space* (Hillier and Hanson 1984) and *Space is the machine* (Hillier 1996) are unanimously considered the milestones of this research field and, more particularly, of the analytical methods of Space Syntax (SSx). Porta et al. (2006a, 2006b) identified two approaches of configurational analysis based on two different representation of the network: the dual approach applied in space syntax studies, where street segments are modelled as nodes and intersections as arcs (Hillier et Hanson, 1984; Hillier 1996); and the primal approach, base of Multiple Centrality Assessment (MCA), with an opposite representation (Batty 2005; Porta et al. 2006a; Svetsuk 2010). This approach has been widely applied in the last few years to different urban phenomena in order to study their spatial distribution, which is highly heterogeneous: property prices, office rents, land use intensity, retail distribution, etc.

Like with statistical and fractal analysis, we now focus our attention on a particular indicator within configurational analysis. When we count the number of nodes (links, destinations, etc.) attainable from a certain location, we are studying an accessibility measure which is a function of the connectivity of a point within the network: *Reach* in MCA, *Node Count* in SSx. Their formulation is of interest when they are calculated locally within a given radius:

$$\text{Reach}_r[i] = \Sigma_{d[i,j] \leq r}(j) \qquad [2]$$

Their implementation procedure is -another time- the same we encountered in Ripley's K function and fractal local radial analysis. Configurational analysis, unlike fractal analysis compares the centrality value in the space at a selected scale (given radius r). Some studies try to compare the relation of these properties at different scales or at different times (Barthelemy 2011). Unlike K function and fractal radial analysis, though, configurational indicators were conceived in the first place for network analysis and don't need any particular adaptation to network environments.

3.1 The local counting method

We saw so far three spatial analysis methods from different research traditions. Each of them is applied with different adaptations, purposes and meanings but the three share, at least initially, the same procedure. In order to better understand our proposal in the next chapter, we summarize some concepts discussed above: 1) Local measures of spatial patterns are based on the counting method applied to disks obtained by increasing radii around each point of the analysis; while configurational analysis focuses its attention on a particular radius r, the fractal approach and K function analyze the evolution of the count with increasing size of r; fractal analysis summarizes this evolution with the fractal dimension D_r but can also highlight breaking points where fractal dimension changes. 2) Statistical procedures like K function compares the empirical count to the expected outcome of a random process in order to determine if there is a spatial clustering behavior; this inferential approach is missing in fractal and configurational analyses, which remain more descriptive. 3) Network space is a particular domain: its non-Euclidean dimension influences the spatial distribution of events; moreover, empirical street networks in contemporary cities present strong heterogeneities; in statistical studies the focus is on heterogeneity of point intensity; configurational indicators are on the contrary conceived to highlight the differences in network properties within urban space.

The aim of the method implemented in this paper is to analyze the point pattern on the network separately from the network configuration. A comparable calculus will be used in order to observe similarities or differences between the two spatial distributions. In order to achieve this purpose, we avoid the use of density/intensity that combines these two sources information, hiding possible relations between point pattern heterogeneity and network heterogeneity.

Separating network and retail activity heterogeneities is the starting point of Piovani et al. (2017), as well. These authors use percolation theory (Stauffer and Aharony 1994) to analyze the variation of the share of clusterized street segments and retail activities, respectively, using different percolation thresholds. Although sharing the same goal of analyzing separately network and retail activity patterns, their approach remains global and is not a local counting method: the clustering behaviors of the two phenomena are analyzed and compared within a whole study area. On the contrary, our approach remains local by evaluating network and point pattern heterogeneities around each point. Street segments and retail points are not clustered and an average relation between the two phenomena over the whole study area is not sought for. A finer analysis is provided which highlights local specificities and possible pattern inversions.

4. Proposal

The methodology proposed in this paper is based on the counting method described above applied locally, at every point of analysis, with different radii. At first, L_r is calculated as the length of the network subset included in a reticular radius *r* from each event point. We then apply the same procedure to the pattern of event points: N_r represent the number of points at the given reticular radius *r*. In both cases, the range of *r* is limited between *r-min* and *r-max,* justified by considerations on the domain of analysis.

As we saw above, fractal analysis has been first applied to geometrical features in order to characterize their distribution pattern through its spatial dimension. When we analyze networks with a metrical fractal approach, this amounts to determining its local geometrical dimension. As we know from fractals theories a network has a geometrical dimension which is equivalent neither to the linear dimension nor to the bi-dimensional planar space. So when we plot the values of *r* and L_r in a log-log diagram, we recognize a linear distribution with slope α_L corresponding to the fractal dimension D_r we encountered in [1]. The parameter α_L is then a local indicator of spatial distribution of the network, varying between 1 and 2 : the lower limit corresponds to a purely linear structure, the higher, to an extremely dense homogeneous mesh.

The example in Fig. 1, show us a real application to a street network: α_L is close to 1 when r remains between 0 and 40 meters (corresponding to the block size). From 40 meters to 640 meters, if we consider the semi-linear network (blue in Fig.1) α_L is 1,13 while considering the regular mesh (red in Fig.1) α_L is equal to 1,97. The parameter α_L (or D_r in [1]), calculated for each point of the analysis, gives us the information about the proportionality factor of the evolution of the network around the considered point. Network heterogeneities impact the average slope α_L considerably. When a point falls within a densely meshed area of small spatial extend surrounded by more linear network extensions, the slope variates over *r*, with higher values for smaller radii and smaller ones later. The average value is relatively low if the dimension of the densely meshed area is much smaller than r_{max}.

The second step sees the same procedure applied to the point pattern. Once again we obtain the parameter $\alpha_N = D_r$; this time the point distribution can vary between 0 and 2. The application of this fractal approach to the point pattern gives us once again an indication of the local proportional factor. When $\alpha_N \rightarrow 2$ the number of points grows almost as in a two-dimensional homogeneous plane and many more event points are at larger distances than at smaller ones. When $\alpha_N \rightarrow 0$ far away points are so rare that the point of analysis is almost isolated. α_N is relatively low even when the point of analysis is within a dense point cluster, whose dimension is much smaller than r_{max}. In this case the cluster, and not the single point, is isolated from other clusters. But the effect of network density is not taken care of.

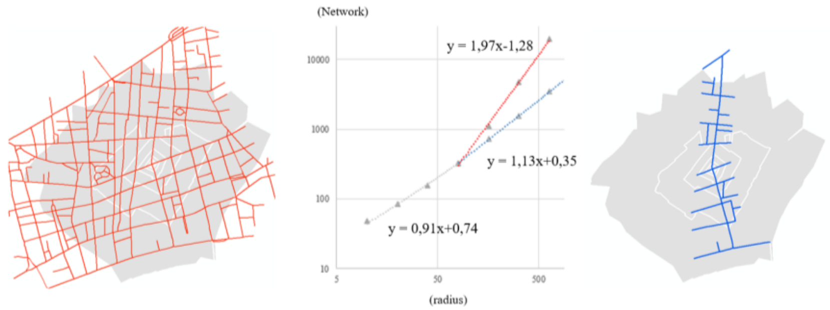

Fig.1 The Fractal Dimensions obtained from a dense meshed (red) and a corarse(blue) grid.

The next step is thus crucial for the analysis: if we compare α_L and α_N we can analyze the two different behaviors of network and point pattern distribution around each point. If they have the same fractal dimension (α_N - α_L = 0), the point distribution follows the network distribution; in other words, the distribution of points is homogeneously distributed on the network, independently on its density.

More interesting is the case when α_N - $\alpha_L \neq 0$: if the difference is positive, the point distribution grows more rapidly with *r* than the network distribution. This is often the case of isolated points which are not too far away from point clusters, even after considering the effect of network density variation. On the contrary, a negative difference characterizes a high local concentration of event points with a local count that grows less rapidly than the network length.

Before discussing limitations and potential evolutions of this approach, we describe a real case study of retail distribution on an urban street network, as well as the interpretation of the results obtained.

5. Application to the analysis of retail in the city

We applied the proposed method to the Nice metropolitan area in South-Eastern France. The city of Nice and its neighboring municipalities are characterized by strong heterogeneity in network distribution, due to physical constraints and interpenetration of urban fabric from different historical periods (Araldi and Fusco 2017). Retail distribution in urban space is relatively heterogeneous, too. We implemented the three steps method on an 18,000-point dataset distributed over a 2,300 kilometer long street network. A maximum radius of 640 meters has been considered in the calculus in order to account for possible interaction between commerce locations given the constraints of pedestrian movement. The minimum radius is 40 meters, below which network geometry becomes linear. The radius of analysis is increased as a geometrical progression of factor 2: 40 m, 80 m, 160 m, 320 m, 640 m. For each analysis point we can thus calculate five values of L_r and N_r.

Within these values, the linear regressions of L_r and N_r on *r* in a log-log plot have an average R^2 of 0,989 and 0,936, respectively, with standard deviations of

0,0254 and 0,0579 for the 18,000 series. Linear regressions on log-log plots are thus relatively well suited for the network length count, less for the retail distribution. A few outliers need a contextual clarification: the presence of shopping malls containing several retailers are modeled with the superposition of several points at the same location in our original geodatabase; this particular event generates erroneous α_N due to the presence of N_{40}>>1, reducing the α_N value and the R^2 of the linear fit.

In order to analyze our results, we will focus our discussions on the small area of Saint-Laurent-du-Var, west of Nice, where almost all the variations and combinations of our pattern distributions are represented (Figures 2 and 3). This urban area presents physical (river and sea front respectively east and south) and infrastructural constraints (railways and motorways longitudinal disposition, following the seafront), a small densely meshed village, a less densely meshed sea-front and low-density, tree-like street networks in the rest of the area. From a simple visualization of the retail pattern, we can easily recognize two agglomerations: in the north area of the selected region the cluster corresponds to the center of Saint Laurent-du-Var village while in the south, a more recent commercial development around a shopping mall. Outside these clusters, an apparently homogenous distribution of retailers.

When a spatial pattern of points presents strong heterogeneity in its distribution as in our case study, the application of K function in its traditional or modified version, can only support the evidence of clustering behaviors. What we are asking to our method is to go beyond this detection guiding the analyst in the understanding of the spatial relations of these clusters with the heterogeneity of the underlying network, at the point level, without any spatial aggregation.

As introduced in the previous section, we first need to analyze separately the two patterns of point and network; in both cases the linear model assumed, is not always the best representation of the empirical distribution. The goodness of fit of the linear model in Saint-Laurent-du-Var is represented in Fig. 2 where R^2 values are mapped. Low values of R^2 correspond to two different shapes of L_r departing from linearity: small local retail concentration surrounded by less dense areas (Fig 2.a) or isolated retailers around which other retailers can be reached only at further distances (Fig2.b). In the first case α_L is close to or lower than 1; in the second, α_L is close to or higher than 2 (Fig.3). When the method is applied to the network, this behavior is weaker but persists.

With this information, we can now analyze the distribution of the two slope parameters, represented in in Fig.3.1 and Fig.3.2. The two retail clusters presented above are well identified by low values of α_N (Fig.3.2). This parameter can be read as a local degree of “isolation/agglomeration” of the pattern.

The same interpretation can be given to α_L: higher values correspond to connective long axes, cul-de-sac irregular regions, adjacent or surrounded by denser meshed areas; on the contrary when the local network of a point is characterized by a higher street density than the farthest areas, α_L is determined by lower values.

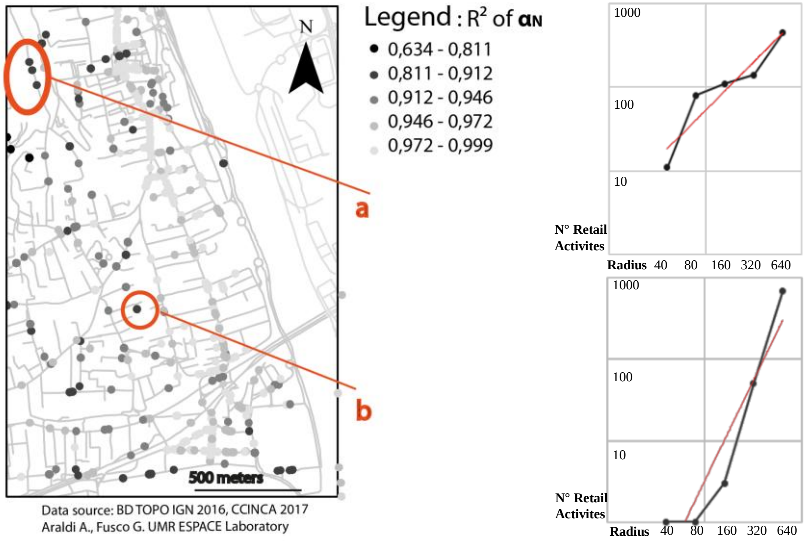

Fig.2 Distribution of R^2 of the linear regression of N(r); a and b represent the function N(r) of two different point having R^2 lower than the average value

While Figure 3.1 and 3.2 inform us on the two spatial distributions separately, their difference, which we will call local fractal deviation[2], helps us characterize their local relations (Fig.3.3). We can then recognize three behaviors:

1-When the fractal dimensions of the point pattern and of the network are similar, we can recognize a point pattern homogeneously distributed on the local sub-network; high retail concentrations are normally further than r_{max} (Fig.3d).

2- If the local fractal deviation is negative (α_N-α_L<0), we identify local high concentrations in well meshed grids Fig.3e.

3- If it is positive (α_N-α_L>0), we often have isolated retail activities (or very small concentration) in not particularly meshed sections of the street network (Fig.3f); from these isolated patterns, high retail concentrations can be reached at around r_{max}, explaining the increase of the slope α_N.

Fig3.3, shows the importance of our approach in reading relations between point pattern and network: while the two retail clusters could be easily recognized even without sophisticated methods, the rest were just classified as low concentration areas. Our approach gives us the possibility to distinguish between homogenous areas and isolated or small local clusters.

[2] deviation and not difference because the fractal dimension of the street network has the role of a reference pattern.

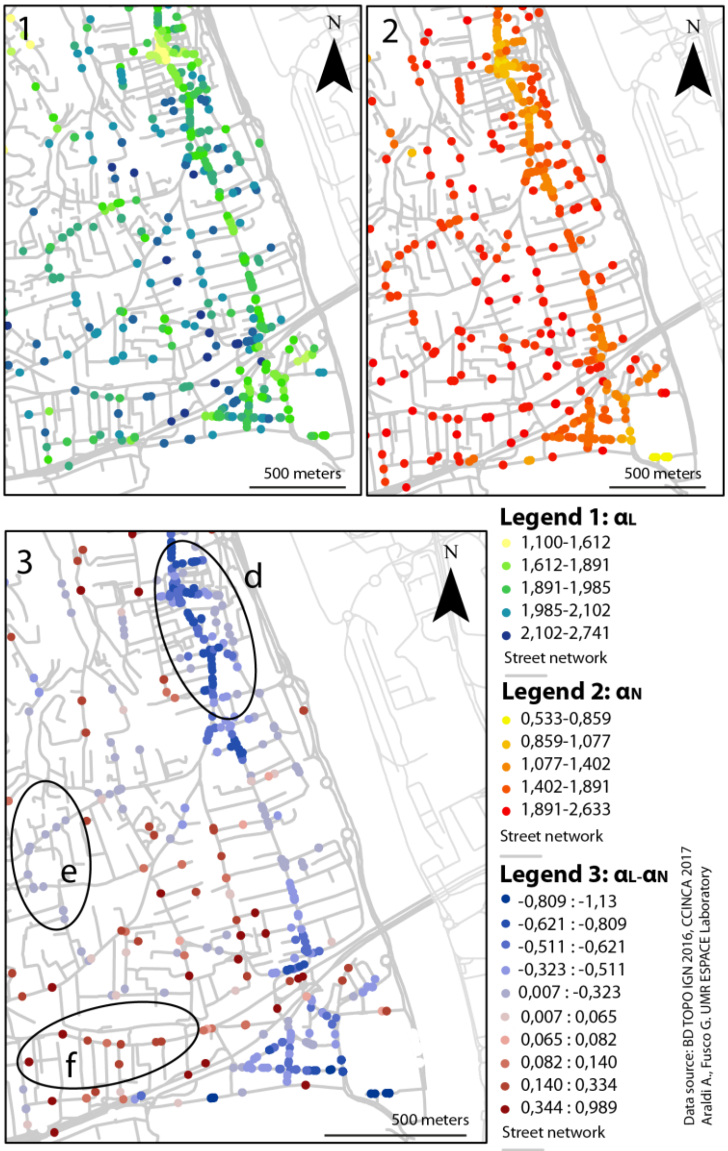

Fig.3 In the three maps we can analyze the spatial disposition of the indicators implemented: in 1 the fractal parameter of the network α_L, in 2 the fractal parameter of the retail distribution α_N and in 3 the difference α_N-α_L.

6. Conclusion and perspectives

Our proposal should be conceived just as the starting point of a new methodology of point pattern analysis on heterogeneous street networks, which could be applied to different domains (retail activity, crime, road accidents, etc.). Its core is the local count of elements falling within increasing network radii around each event point, a common phase of K function analysis, fractal radial analysis and reach indicator in configurational analysis. Event point pattern and network are analyzed separately and later combined. Relations between point pattern heterogeneity and network heterogeneity can thus be investigated at the a very local level, without needing a global partitioning or clustering of features. Different relationships between point patterns and network patterns can be highlighted at a local level. Within their work on London, Piovani et al. (2017) show that street segments and retail activities have very similar percolation thresholds and that the latter cluster more intensively than the former at all scales of analysis. Our analysis on the metropolitan area of Nice shows that the local fractal deviation of retail activities referred to the street network can be positive, negative or null, highlighting different local behaviors. A more fine-grained analysis of the relation between street network morphology and retail activity distribution becomes thus possible, as shown by our case study.

As already pointed out, the method needs further developments. A calculus of significance levels of local fractal deviations could be implemented, as in statistical approaches. Marked point patterns could also be analyzed, as already proposed by local cross-K function analysis.

Acknowledgements.
This research was carried out thanks to a research grant of the Nice Côte d'Azur Chamber of Commerce and Industry (CIFRE agreement with UMR ESPACE).

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