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Mixing by coherent vortices in a two-layer stratified fluid

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Abstract :

Here we present results of experiments on mixing at a density interface conducted in a Taylor-Couette device at Reynolds numbers such that the Taylor vortices are laminar to chaotic. The Laser Induced Fluorescence (LIF) technique has been used to obtain quantitative measurements of the spatio-temporal evolution of the density. It is shown that the entrainment rate and mixing efficiency are an order of magnitude smaller than in the turbulent vortex regime and the established entrainment laws do not apply in this laminar-chaotic vortex regime. The vertical transport in the homogeneous layers is smaller than the interfacial flux and several density interfaces appear at different heights. The interactions between the interfaces control the subsequent density flux, and hence the mixing efficiency.

Résumé :

Nous présentons ici les résultats d'expériences concernant le mélange au niveau d'une interface de densité, expériences réalisées au sein d'un dispositif Taylor-Couette à des nombres de Reynolds tels que les vortex de Taylor sont laminaires à chaotic. La technique de fluorescence induite par laser (LIF) a été utilisée afin d'obtenir des mesures quantitatives de l'évolution spatio-temporelle de la densité. On montre que la vitesse d'entrainement et l'efficacité du mélange sont un ordre de grandeur plus petit qu'en régime turbulent et que les lois d'entrainement établis ne s'appliquent plus dans ces régimes de vortex laminaire-chaotic. Le transport vertical dans les couches homogènes est plus faible que le flux interfacial, d'où l'apparition de plusieurs interfaces de densité à différentes hauteurs. Leurs interactions mutuelles modifient l'intensité du flux de densité, et donc l'efficacité du mélange.

Key-words :

Mixing, stratification, laminar-turbulent

1 Introduction

Mixing in stratified fluids has received considerable attention because of its importance in geophysical flows. These flows are generally turbulent. A fairly large number of experimental and theoretical studies have considered different aspects of turbulent mixing in stratified flows. Phillips (1971) and Posmentier (1977) established a model predicting the variations of the mixing efficiency, that is expressed by the flux Richardson number Ri_f, as a function of the buoyancy gradient, expressed in non-dimensional form by the Richardson number Ri_o. The buoyancy flux first increases with buoyancy gradient and then, beyond a critical of Ri_o the buoyancy gradient limits the vertical transport of density. This leads to a decrease in mixing efficiency and the formation of stronger density interfaces separated by homogeneous layers. Linden (1979), E & Hopfinger (1986) and Billant & Chomaz (2000) have found good agreement with this model predictions by studying mixing across a density interface by turbulence produced externally to the interface. A modified version of the Phillips-Posmentier model proposed by Balmforth *et al.* (1998) suggests that the mixing efficiency should increase at large Ri_o number to prevent interface sharpening without limit. The main feature of the Balmforth *et al.* (1998) model is the assumption that the size of the turbulent vortices involved in mixing is not fixed; at large Ri0 energy is injected by the mixing device at smaller scales i.e. the eddy turnover time decreases at large Ri_o. Experimental support of this model is provided by Guyez *et al.* (2007), showing that the mixing efficiency indeed increases at very large Richardson number. The experiments also show a large variability in mixing efficiency for the same experimental conditions. Here we consider mixing at a density interface by low Reynolds number laminar-chaotic Taylor vortices. This regime is of interest as it can occur in many industrial processes. Most of the general purpose pharmaceutical companies and biologists face mixing problems for which turbulent flows can't be used as it may destroy the activated molecules (DNA, proteins, bloods ...). It is shown here that although the general mixing behaviour appears similar to that of turbulent Taylor vortices it is quantitatively quite different. The experimental setup is presented in Section 2. The results containing the spatio-temporal density evolution, the entrainment rate and the mixing efficiency are analysed in Section 3.

2 Experimental conditions

Experiments were conducted in a Taylor-Couette device where only the inner cylinder rotates. This device was previously used by Caton *et al.* (2000) to study flow regimes in presence of a linear stratification. The radii of the inner and outer cylinders are respectively $a = 4$ cm and $b = 5$ cm giving a gap $d = b - a = 1$ cm. The Reynolds number based on the rotation speed Ω of the inner cylinder is $Re = \Omega ad/\nu$, where ν is the fluid viscosity. In the experiments $364 < Re < 1025$. The fluid depth in the gap is $h = 56$ cm. In order to reduce optical distortions, the device is placed inside a rectangular tank filled with water. The annular gap was filled with salt water below and fresh water above, with the interface at mid-height $h/2$. The initial interface thickness is closed to 20 mm and the buoyancy jump is:

$$\Delta B = g\Delta\rho/\overline{\rho}, \tag{1}$$

where g is the gravitational acceleration, $\Delta\rho$ the difference of density between the layers and ρ the mean density of the flow. A sketch of a typical experiment is presented in Figure 1. The spatio-temporal evolution of the density was measured by means of the Laser Induced Fluorescence (LIF) technique using a 2 mm thick, vertical laser light sheet of an adequate power. The light emitted by a fluorescent dye (rhodamine 6G) is proportional to the dye concentration. Hence if a known concentration of dye is mixed with the salted water before filling the gap, then the intensity of the fluorescence will also be proportional to the concentration of salt. A camera (Jai CV-M4+CL, B&W, 1024*1380 pixels) records the intensity of the fluorescence in the space defined by the intersection of the laser plane and the fluid. It is fitted with a high-pass filter to only keep the fluorescent light.

For all the experiments, the Richardson number Ri_o can be measured at any density interface and at any instant of time:

$$Ri_o = \frac{\Delta B(t) d_\vartheta(t)}{U_\vartheta'^2} \tag{2}$$

d_ϑ is the vortex height and U'_ϑ is the rms velocity of the Taylor vortices. Ri_o evolves in time as flow regimes change. It is easily measured in these laminar-chaotic regimes as the separations between the vortices are highlighted by buoyancy gradients. For U'_ϑ, we use the data obtained by Wereley & Lueptow (1998) obtained in a Taylor-Couette flow of similar size. At each instant of time the values of Ri_f and Ri_0 are then calculated:

$$Ri_f(t) = \frac{d_\vartheta(t)\,\Delta B(t)\,U_e(t)}{U_\vartheta'^3} = E(t)\,Ri_o(t) \tag{3}$$

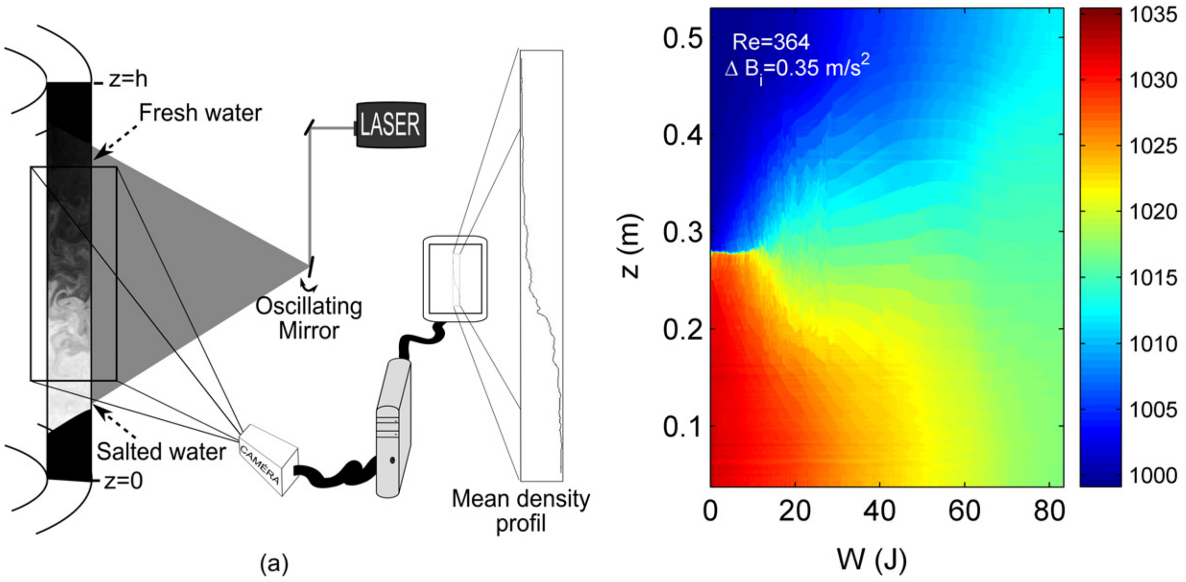

Figure 1: Left : Sketch of the experimental setup - Right : Spatio-temporal evolution of the density (blue for the fresh water and red for the salted water) at low Reynolds number. The time evolution t is measured in term of the energy W used to rotate the cylinder during the corresponding time : $W = M\Omega t$, M being the torque applied by the cylinder on the water.

E is non-dimensional form of the entrainment rate U_e , evaluated at a density interface localised at the height z, divided by U'_ϑ. The entrainment velocity is given by:

$$U_e(z,t) = \frac{1}{\Delta\rho(t)} \int_z^h \frac{\partial \rho(z,t)}{\partial t} dz \tag{4}$$

3 Results and discussion:

3.1 Spatio-temporal evolution of the vertical density profile

The spatio-temporal evolution of the density depends on Reynolds number and on the initial buoyancy jump ΔB_i. Density interfaces appear at all levels between the vortices at low and moderate Reynolds number ($Re < 1000$). The intensity of the density jump decreases with distance from the main interface. During a typical experiment, mixing occurs at different stages. First, the main interface seem to control the mixing and has a stratification much higher (about five times) than that of the other interfaces. All secondary interfaces are issued from splitting of the main interface and finally occupy the whole fluid depth. The birth of a new interface is periodic and starts with a wavy excitation of the interfaces. When the main interface breaks, the mixing seems to be more disorganised as a large number of density interfaces appear within a height of about 10 cm. In this part of the flow, the interface positions change very rapidly (due to splitting and fusion phenomena) whereas, on the outside, the former interfaces are not disturbed. Gradually the flow reaches a more stable state. The vertical density profile is then similar to a stair case of steps of different heights. Moreover, it is observed that, for the same Reynolds number, a reduction of the initial buoyancy jump leads to the destruction of the main interface as soon as the cylinder begins to rotate. Between all vortices a stratified interface appears. These observations lead to some initial comments. When mixing occurs in the laminar-chaotic regime, the entrainment speed (at which the fluid is taken through a density gradient) is of the same order as the transfer speed (at which fluid is exchanged between homogeneous vortices). Thus the initial homogenous layers above and below the interface are not able to remain homogeneous, contrary to what is observed in the turbulent regimes (Guyez *et al.* (2007)). Nevertheless, when the initial interfacial buoyancy jump is large, the main interface is maintained until the stratification is small enough to be destroyed. In all the cases, the two-layer experiments and linearly stratified experiments lead to the formation of similar density profiles. Complete mixing then takes a very long time as low density gradients result in very low density fluxes.

3.2 Entrainment rate and mixing efficiency

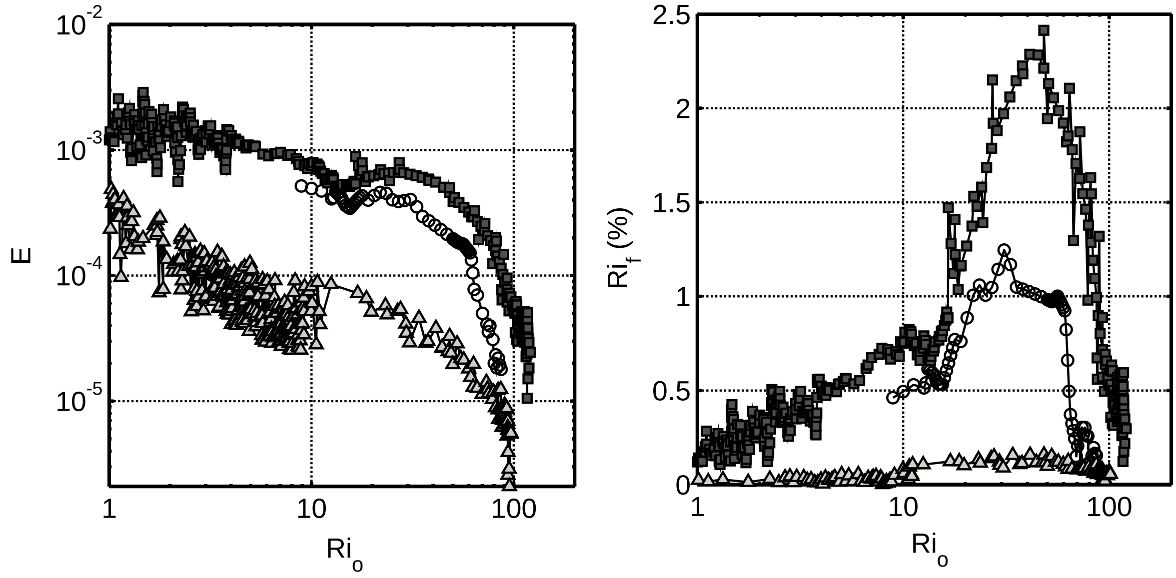

Figure 2: Variation of the Entainment speed E and Richardson flux number Ri_f as a function of the Global Richardson number Ri_o, measured at most stratified interfaces during experiments at $\triangle Re = 455$, $\circ Re = 727$ and $\square Re = 1025$. The initial buoyancy jump is $\Delta B_i = 0.35$ kg/m^3.

The entrainment coefficient E as a function of Ri_o is shown in Figure 2a. It is evident from this figure that no entrainment law of the form ${Ri_o}^{-n}$ emerges. In Figure 2a the evolution of the flux Richardson number as a function of the Ri_o is shown. For all Reynolds numbers $Re < 1025$, these curves exhibit the general bell shape form in accordance with the Phillips-Posmentier model. However, the detailed behaviour differs from that obtained with turbulent vortices; the fall-off beyond maximum mixing efficiency is more rapid (see figure 2a) and there is a strong dependency on Reynolds number. There is no increase in mixing efficiency at large Ri_o primarily because the Reynolds number based on an eddy scale smaller than d_ϑ is too small. Maximum mixing efficiency is reached when the Richardson number approaches 30. On the spatio-temporal images, maximum Ri_f corresponds to the moment when the stratification of the main interfaces decrease very rapidly. The maximum of Ri_f for $Re = 1025$ is still about ten times smaller than in the turbulent regime (Guyez *et al.* (2007)). A possible reason is the small scales disappear progressively in the laminar regimes. In order to produce mixing in the laminar-chaotic regime the vortices have to displace buoyant fluid elements over a height equal to their seize. The buoyancy flux is then limited by the kinetic energy available on the eddy scale. This is much less than in the turbulent case. Also the homogenisation inside the vortices due to diffusion lasts longer as there no small scales and hence less stretching of the fluid.

Another interesting result appears when we compare the curves of mixing efficiency measured for different interfaces in the same experiment (Figure 3). The value of maximum mixing efficiency changes with the interfaces considered and is highest for the interface with largest buoyancy jump.

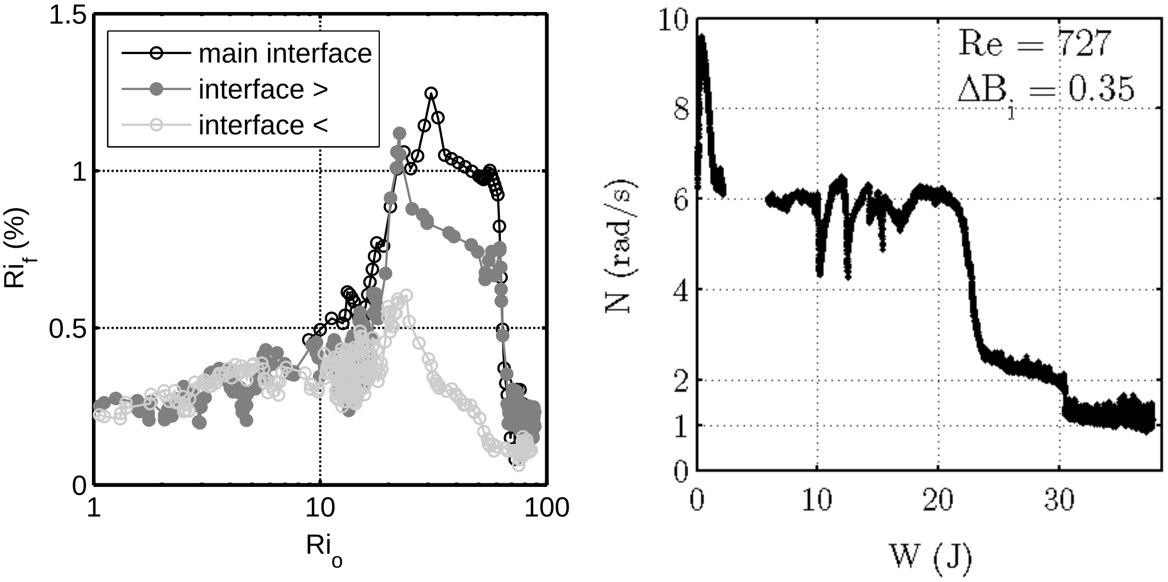

Figure 3: Left : Evolution of the Richardson flux number as a function of the Richardson number, measured at three different interfaces during an experiment at $Re = 727$ and $\Delta B_i = 0.35$ kg/m^3. Right : Stratification of the main interface versus the energy used $W (= M\Omega t)$.

This unexpected behaviour can be linked to the other observation. When representing the time evolution of the stratification, it appears that the stratification of the main interface does not decrease randomly. It tries to maintain its value during a multiple of the rotation speed of the inner cylinder (Figure 3). Here, the dips between 6 and 4 rad/s correspond to a change of interface. Indeed, the higher value of the stratification is linked to different interfaces as the mixing time increases. The coupling of the main interface and the rotation speed is only observed when several density interfaces are present at the same time, namely at low Reynolds number. The stratification of the other interfaces is not coupled to the rotation speed. It is supposed that the density flux across these other interfaces adjusts itself in such a way that the stratification of the main interface can be kept at a specified value. The coupling between the stratification and the rotation speed is then a parameter that should be taken into account for the determination of the mixing efficiency. Further investigation are needed to better understand what are the mechanisms which lead to this coupling.

4 Conclusion:

Below a certain Reynolds number when eddies are laminar to chaotic, mixing is strongly Reynolds number dependent. The mixing efficiency is considerably less than in the turbulent regime because the energy available for mixing is much less. Furthermore, the absence of small scale structures increases the time needed for the homogenisation inside the vortices. Several density interfaces develop above and below the main interface as the exchange of fluid between eddies is reduced (Akonur & Lueptow (2003)). This is demonstrated by the spatio-temporal evolution of the density that shows a stair case density profile. The same density structure is obtained when the initial density stratification is linear (Caton *et al.* (2000)). An interesting result is that the $Ri_f - Ri_o$ relation is affected by the presence of surrounding density interfaces. Maximum of efficiency is measured at the interface of largest buoyancy jump; the others exhibit lower value of Ri_{f-max}. This behaviour seems to be a consequence of the adjustment of the secondary interfaces to allow a coupling between the stratification of the main interface and the rotation speed of the inner cylinder. The complete destruction of the stratification is then very long which is consistent with Janiaud *et al.* (2000) who found that in these regimes, the effective diffusion coefficient is very small.

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