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The three worlds of welfare capitalism revisited

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The three worlds of welfare capitalism revisited

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Abstract

We introduce a new way to model the Bismarckian social insurance system, stressing its corporatist dimension. Comparing the Beveridgean, Bismarckian and Liberal systems according to the majority voting rule, we show that for a given distribution of risks inside society, the Liberal system wins if the inequality of income is low, and the Beveridgean system wins if the inequality of income is high. Using a utilitarian criterion, the Beveridgean system always dominates and the Bismarckian system is preferred to the Liberal one.

JEL Classification: D63; D72; H53
Keywords: social insurance systems · political economy · Bismarck · Beveridge · inequality · redistribution

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1 Introduction

This paper compares the three main systems of welfare capitalism – Beveridgean, Bismarckian, Liberal – as analyzed by Esping-Andersen (1990) from both a positive and a normative perspective. To do this, we introduce a new way to model a Bismarckian type of social insurance to account for the fact that Bismarckian systems are organized around groups of agents. We aim to focus on the redistributive design of these different regimes and compare the preferred systems from both perspectives.

The background for our considerations is the following: In many countries with a Bismarckian system, such as Germany, Austria, France or Belgium, a variety of social protection funds for illness, occupational injury, family or pension cover specific groups of people. For instance, the set of French social insurance funds refers to professional groups such as railway and public transportation system employees, seamen, civil servants, agricultural workers, entrepreneurs, etc. For occupational injury, the German insurance system is similarly organized on a professional group basis: Specific employer’s mutual insurance associations cover the commercial, agricultural or the public sector as well as railway workers, firefighters and local authority employees etc. There are other examples in Bismarckian countries where the formation of groups are a result of the agents’ choice. For instance, in Belgium or Germany, people can choose from a (large) range of health insurance funds. These funds are organized on the level of geographic coverage, employers, craft guilds, etc.

The recognition of this organizational and strongly corporatist feature of the Bismarckian system goes back to the seminal work of Esping-Andersen (1990): “corporatism was typically built around occupational groups seeking to uphold [...] status distinctions and used these as the organizational nexus for society and economy.” (p. 60). To be precise, Esping-Andersen (1990) clustered welfare states as “conservative”, “social-democratic” and “liberal” regime types. In line with the established economic literature we retain for the first two systems the nomenclature of “Bismarckian” and “Beveridgean” systems.

1 German Social Security Law, Book Nr. VII
2 In Germany, before the amendment to the Social Security Law in 1996, people had to insure themselves according to the selection criterion of the health insurance funds. Therefore, these funds covered only people who exactly matched their selection criterion, e.g. they lived in a specific geographic region, they worked for a specific employer or in certain craft guilds etc. Nowadays, people can choose which fund they want to be insured in, cf. German Social Security Law, Book Nr. V. Further source: www.prospeur.org
As well as other dimensions, one important aspect that distinguishes these systems is their degree of income redistribution. Firstly, Liberal systems are associated with a very low degree of income redistribution since they mainly encourage private insurance. Secondly, Beveridgean systems, based on the principles of universality, uniqueness and uniformity of benefits, are associated with a high degree of income redistribution. This is due to proportional tax rates but flat benefits. Finally, Bismarckian systems are associated with a lower degree of income redistribution. Most often, they have been modeled in the literature as a global insurance system, organized by the state, where individuals pay taxes proportional to their incomes, but independent of their risks, and receive benefits proportional to their income.

The problem with this way of modeling the Bismarckian system is that it ignores the “corporatist” attribute of such systems: If individuals are differentiated by income and by risk, then a pooling of individuals with specific risks inside each fund leads to “intra-group horizontal redistribution” in the Bismarckian system, i.e. it leads to redistribution from low-risk to high-risk agents inside each fund. As a consequence, each fund is characterized by its specific average risk. Transferred to the level of individual preferences, this implies that individuals who bear a high risk may benefit from a low average group risk. In a similar vein, the introduction of both income and risk heterogeneity of individuals leads to another kind of horizontal redistribution inside the Beveridgean systems: Here, redistribution is based on individual risk and on the distribution of risks inside the entire society. In our terminology, this type is called “global horizontal redistribution” and it complements the usual vertical income redistribution of the Beveridgean system. Again, transferred to the level of individual preference, poor and/or high-risk individuals benefit from the Beveridgean system. The Liberal system is characterized by neither horizontal nor vertical redistribution, since it consists of a private insurance mechanism, with a contribution rate that is proportional to individual risk and income. In the following, we provide a model which accounts for all of these redistribution patterns by analyzing individual and aggregate preferences for the three systems.

There are two strands of literature which are related to our model. The first strand has determined both the type and the size of social insurance or social security systems, respectively, and therefore refers to the explicit redistribution of risk among individuals. The second strand has been concerned with the determination of the type of social insurance system, i.e., whether it is a Bismarckian or Beveridgean system, and the size of the system, i.e., the total amount of redistribution.

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3In our model, individuals can be thought of being differentiated with respect to risk along a horizontal axis and with respect to income along a vertical axis. The notion of “horizontal” refers to the redistribution of risk (i.e. from low-risk to high-risk people) with two aspects: within the entire society, or within groups. Accordingly, “vertical” refers to redistribution of income (i.e. from rich to poor people). See also Section 3.2.
The distinction between Bismarckian and Beveridgean systems, see Casamatta et al. (2000a) (for social insurance) and Pestieau (1999) (for social security). They analyze the optimal size of the system (in terms of the tax rate) with the type of system chosen at the constitutional stage. Their main result is that the degree of redistribution affects the political support for the size of the system. Rossignol and Taugourdeau (2004) study both the level of tax rate and the type of system within a probabilistic model of electoral competition. They proved that the chosen social insurance system is that which minimizes the contribution rate for a high relative risk aversion, and that the reverse is true for a low relative risk aversion. Moreover, Conde-Ruiz and Profeta (2007) provide an OLG model of social security where the size and the type of system is determined simultaneously, yet issue-by-issue. They find that the key determinant which shapes their analytical result is income inequality: The Beveridgean system can be supported by a coalition of low and high income individuals.

We complement this first strand of literature in two ways. Firstly, the Bismarckian system is modeled as a corporatist one, which enables us to distinguish it more clearly from the Liberal system. Secondly, the choice of the system is determined alternatively according to a positive and a normative criterion, that we are able to compare.

The second strand of literature this paper refers to analyzes the link between income inequality and the level of redistribution inside society. Indeed, in our model, the degree of inequality of income and the distribution of risk crucially affect the choice of an agent, which affects the choice of the system for both positive and normative criteria. The link between income inequality and redistribution has first been highlighted in the well-known Meltzer and Richard (1981) general equilibrium model of a labor economy where the share of redistributed income is determined by majority voting. Their main finding is that if mean income rises relative to the income of the median voter, then redistribution increases. In other words, a more unequal income distribution leads to more redistribution. In addition to the standard redistributive mechanism from rich to poor, insurance motives have also been introduced in the analysis of welfare policies. For instance, Moene and Wallerstein (2001) show that the redistributive and the insurance mechanisms work in opposite directions in the sense that support for social

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4See also Cremer et al. (2007) for the effect of myopic and non-myopic individuals on social security.

5In addition, see Romer (1975) and Roberts (1977) on whose results Meltzer and Richard (1981) build upon.
insurance spending declines with increased income inequality. Finally, Kim (2007) extends the analysis of redistribution based on insurance motives by introducing a distribution of risks inside the society, where the level of risk depends on the agent’s sector of activity. The main result of this model is that political demand for unemployment insurance is clearly influenced by both the distribution of risks and income.

As already indicated, our model provides a complete differentiation of individuals along three dimensions: income, individual risk and group risk. This is a key point of our analysis. In the previous literature, Casamatta et al. (2000b) introduce heterogeneity of individuals by a one dimensional differentiation with three discrete levels of income but the same probability of receiving income or relying on social benefits. Casamatta et al. (2000a) and Conde-Ruiz and Profeta (2007) differentiate along two dimensions, namely age (working young vs. retired old) and the level of income (continuous in Casamatta et al., 2000a, discrete in Conde-Ruiz and Profeta 2007). A related double differentiation of individuals with regard to income and likelihood of illness is found in Gouveia (1997) who analyzes the outcome of majority voting over the public provision of a private good (in particular, health care).

We concentrate on the case of insurance systems that cover unemployment, occupational injury or health risks. Individuals earn a wage income in the good state of the world and receive insurance benefits in the bad state of the world. Furthermore, they are members of a group which is characterized by a group-specific risk distribution. This implies that groups can be ranked according to the average risk of its members. We incorporate into our analysis a Liberal insurance system reflecting an actuarial fair private insurance, a Beveridgean system involving redistribution for the entire society and a Bismarckian system comprising redistribution between high-risk and low-risk individuals within a group. In a two stage model, first, the system of insurance is decided and second, the level of the tax rate is determined. The choice of the tax rate and the choice of the system are determined according to a positive criterion, then compared to a normative one.

In the following we show that by majority voting, the Liberal system wins if the inequality of income is low and the Beveridgean system wins

Moene and Wallerstein (2001) focus on the impact of income inequality on the support of welfare spending when welfare benefits are targeted towards the employed or the unemployed. See also Iversen and Soskice (2001) for a similar model analyzing social policy preferences which depend on different types of skill investments reflecting unemployment risks. Benabou (2000) analyzes the impact of inequality and redistributive policies that enhance efficiency within a stochastic growth model.
if the inequality of income is high. Employing a utilitarian criterion, the Beveridgean system dominates both the Bismarckian and Liberal systems but the Bismarckian system is preferred to the Liberal one.

This paper is organized as follows: Section 2 introduces the model. In Section 3 we analyze the pairwise preferences of individuals and determine the type of welfare system chosen by majority voting. In Section 4 we analyze the outcome of a utilitarian social planner and compare the results of both criteria. We conclude in Section 5.

2 The model

The society is divided into groups which are denoted by \( k = 1, \ldots, M \) and there are \( N_k \) members per Group \( k \). There are \( N \) agents in the society with \( N = \sum_{k=1}^{M} N_k \). An agent \( i \) of Group \( k \) has an income \( w_i \) and a risk \( p_i \) to lose this income. A high level of \( p_i \) implies that agent \( i \) is risky in terms of bad health or unemployment, for instance. Each group \( k \) is characterized by a specific distribution function of risk \( f_k \). To concentrate our analysis on the heterogeneity of the distribution of risk, we suppose that the distribution function of income \( g \) is similar in each group. Moreover, for the sake of the readability of our results, we assume that the distribution of incomes and risks are independent. Therefore, groups are heterogeneous with respect to risks but homogeneous with respect to income distribution. We now describe the distribution of income and risk in more detail.

2.1 Distributions of income and risk

The distribution of income for each group is represented by the probability density function \( g \) defined on \( [w_{inf}; w_{sup}] \) with average income \( \bar{w} = \int w g(w) dw \). The function \( g \) is positively skewed such that median income \( w_m \) is lower than average income \( \bar{w} \). Income levels can then be ranked as \( 0 \leq w_{inf} \leq w_m \leq \bar{w} \leq w_{sup} \).

The distribution of risk depends on the group \( k \) and implies a group-specific risk probability density function \( f_k \) defined on \( [0; 1] \). This function \( f_k \) is positively skewed, as well, and produces a particular intra-group average risk \( \bar{p}_k \), where \( \bar{p}_k = \frac{1}{N_k} \int p f_k(p) dp \) and \( N_k = \int f_k(p) dp \) with \( f_k \geq 0 \). Let \( f \) be the risk probability density function of the entire society, i.e. \( f = \sum_{k=1}^{M} f_k \).

These groups could be professional groups (e.g. the service sector, the agricultural sector, the industrial sector etc.) or other types of groups.
We normalize \( N = 1 = \int f \). The average risk in the entire society is \( \bar{p} = \int pf(p)dp \).

We assume that the intra-group average risks are ranked as

\[
\bar{p}_1 < \bar{p}_2 < \bar{p}_3 < \ldots < \bar{p}_M
\]  

(1)

In addition, we postulate that \( p_{m,k} = p_m \) for every \( k \), i.e. the median risk of each group \( p_{m,k} \) corresponds to the median risk in society \( p_m \), even if the distribution of risk inside each group is different.

How can we justify these two assumptions? It is clear that there is a majority of low-risk people in each group. It is reasonable to assume that the groups are mainly differentiated by the distribution of their high-risk members. This implies that the groups have different average risks \( \bar{p}_k \) (i.e. \( \bar{p}_1 < \ldots < \bar{p}_M \)), but approximatively similar median risks \( p_{m,k} \) (i.e. \( p_{m,k} = p_m \) for every \( k \)).

Finally, based on the positive skewness of function \( f_k \), we now postulate

\[
\forall k, p_m < \bar{p}_k
\]

which implies \( p_m < \bar{p} \).

In the following, we will present empirical justification for the relationship between median risk and average risk.

### 2.2 Empirical evidence

It is a well-known stylized fact that income distributions in many developed countries exhibit positive skewness, see, e.g. [Neal and Rosen (2000)](#). To establish the positive skewness of the risk distribution we can refer to the same line of argument as before: We want to show that there is a majority of low-risk and a minority of high-risk members in each group. In our model, risk refers to the probability of having to rely on (social) insurance benefits due to unemployment or illness. We provide for each of these risk factors an empirically observable proxy.

For unemployment we compare median and average duration of unemployment using data from OECD countries for the years 2000 to 2010.\(^8\) We find that the proportion of countries where median duration of unemployment is clearly smaller than average duration is substantial for the whole

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\(^{8}\)Early contributions to the literature analyzing functional forms of earnings capacities are [Staehle (1942), Miller (1955), or Harrison (1981)](#). For reasons of comparability across all OECD countries we chose unemployment rates of male work force.
time period, see Figure 1. Overall, less than 5% of total observations exhibit a reverse relationship with average risk lower than median risk.[10]

![Figure 1: Median vs. average duration of unemployment in OECD countries, proportions of countries relative to all OECD countries, 2000–2010, male work force. Source: OECD Labour Market Statistics, own calculations.](image)

For illness our basic hypothesis is that people affected by chronic health problems or disability bear a higher risk of having to rely on insurance payments. Since these people constitute a minority in society, average risk will be lower than median risk. Indeed, data from OECD (2010b) shows that the self-assessed prevalence of chronic health problems or disability is lower than 15% on OECD average for the whole working age population. Even for age group 50–64, the proportion of people with self-assessed chronic health problems or disability is lower than 25% on average and only for few countries a little higher than 30%. Given a minority of people bearing a high risk due to chronic health problems and disabilities, the majority of people has quite a low risk.

Moreover, if we consider health expenditures as a proxy of the health risk, then it clearly appears that mean health expenditures are consistently higher than median health expenditures (Jung and Tran, 2010).

[10] Data from OECD (2010a). Estimation of median and average duration of unemployment and calculations of average duration of unemployment are available from the authors upon request.
2.3 The three systems

The agent $i$ earns with probability $(1 - p_i)$ an income $w_i$ which is subject to a payroll tax $t$, such that $(1 - t)w_i$ is his net of tax income. With probability $p_i$ the agent receives social insurance benefits $b_i$ which, in the case of a Beveridgean system (BE), are identical for all agents $b_i = b^{BE}$. In the case of the Bismarckian system (BI), social insurance benefits $b_i$ are proportional to individual income but the coefficient of proportionality is identical for all agents inside the group $k$, i.e. $b_i = b^{BI}_k(w_i) = c_k \cdot w_i$. Finally, in the case of a Liberal system (L), benefits that an agent receives in the bad state of the world are actuarially computed, based on both his risk $p_i$ and the wage $w_i$ that he would receive in the good state of the world, i.e. $b_i = b^L(p_i, w_i)$. No redistribution occurs in this last system.

Hence, under the Liberal system, the budget constraint for each agent $i$ is given by $(1 - p_i)tw_i = p_i b^L(p_i, w_i)$ which immediately implies

$$b^L(p_i, w_i) = \frac{1 - p_i}{p_i} tw_i$$

Under the Bismarckian system, the budget constraint in Group $k$ is

$$\frac{1}{N_k} \int \int ((1 - p)tw) f_k(p) g(w) dp dw = \frac{1}{N_k} \int \int p b^{BI}_k(w) f_k(p) g(w) dp dw$$

and since $b^{BI}_k(w_i) = c_k \cdot w_i$ it implies

$$\frac{1}{N_k} \int \int ((1 - p)tw) f_k(p) g(w) dp dw = \frac{1}{N_k} \int \int pc_k w f_k(p) g(w) dp dw$$

thus $c_k = \frac{1 - \overline{p}_k}{\overline{p}_k} t$, and finally

$$b^{BI}_k(w_i) = \frac{1 - \overline{p}_k}{\overline{p}_k} tw_i$$

Lastly, under the Beveridgean system, the social insurance budget constraint satisfies the identity $\int \int ((1 - p)tw) f(p) g(w) dp dw = \int \int p b^{BE} f(p) g(w) dp dw$, which implies

$$b^{BE} = \frac{1 - \overline{p}}{\overline{p}} tw$$

with $\overline{p} = \int pf(p)dp$.

The welfare function under the Beveridgean system, for an individual $i$ of risk $p_i$ if the tax rate is $t$, is now:

$$W^{BE}(t, p_i, w_i) = (1 - p_i) U((1 - t)w_i) + p_i U\left(\frac{1 - \overline{p}}{\overline{p}} tw\right)$$

Analogously, the group-specific welfare function for the Bismarckian system for a member $i$ of Group $k$, is

$$W^k_{BI}(t, p_i, w_i) = (1 - p_i) U((1 - t)w_i) + p_i U\left(\frac{1 - \overline{p}_k}{\overline{p}_k} tw_i\right)$$
and the welfare function of an agent $i$ under the Liberal system is:

$$W^L(t, p_i, w_i) = (1 - p_i)U((1 - t)w_i) + p_iU\left(\frac{1 - p_i}{p_i}tw_i\right)$$

We aim to determine the preferred system according to two alternative criteria, i.e. a positive one, majority voting and a normative one, utilitarian criterion. In both cases, the timing of decisions is as follows: In the first stage, the welfare system is chosen. In the second stage, the level of the tax rate is chosen, according to the studied criterion. We will solve these games by backward induction.

For the sake of simplicity we specify the utility function to be $U(x) = \ln x$.

3 Majority voting

3.1 Choice of tax rate

Maximizing the level of the welfare of a given agent $i$ with respect to the tax rate $t_i$ yields the same preferred tax rate under the three systems:

$$t^*_i = p_i \quad (2)$$

The preferred tax rate does not depend on income. Moreover, since agents are differentiated by their risk $p_i$, their preferences are single peaked with respect to the tax rate. As a result, according to the majority rule, the tax rates that are chosen in both the Beveridgean and Bismarckian systems are those preferred by the median voter, i.e.:

$$t^*_{BE} = t^*_m = p_m$$
$$t^*_{BI} = t^*_{m,k} = p_{m,k}$$

Since all groups have approximately similar median risks $p_{m,k}$ (i.e. $p_{m,k} = p_m$ for any $k$), the tax rate chosen by majority voting corresponds to the choice of the society’s median agent and is the same in both the Beveridgean and Bismarckian systems

$$t^*_{BE} = t^*_{BI} = t^*_m = p_m$$

In the Liberal system the choice of tax rate is made independently by each agent and corresponds to his personal level of risk$^{11}$

$$t^*_L = t^*_i = p_i$$

$^{11}$For the sake of simplicity we refer to the term “tax rate” also with regard to the Liberal system. “Contribution rate” would be a more precise term.
Incorporating the chosen tax rates in the welfare functions gives:

\[ W_{BE}(t^*_m, p_i, w_i) = (1 - p_i) \ln((1 - p_m) w_i) + p_i \ln \left( \frac{1 - p}{p} p_m w \right) \]  
(3)

\[ W_{BI}^k(t^*_m, p_i, w_i) = (1 - p_i) \ln((1 - p_m) w_i) + p_i \ln \left( \frac{1 - \bar{p}_k}{\bar{p}_k} p_m w_i \right) \]  
(4)

\[ W^L(t^*_i, p_i, w_i) = (1 - p_i) \ln((1 - p_i) w_i) + p_i \ln \left( \frac{1 - p_i}{p_i} p_i w \right) \]

\[ = \ln \left( (1 - p_i) w_i \right) \]  
(5)

3.2 Individual preferences on the system

Before determining the system that would be chosen by majority voting, we need to study individual preferences for the systems using the tax rates we have just determined. We focus on a pairwise comparison of the three systems to have a complete ranking of the systems for each agent.

3.2.1 Bismarck or Beveridge?

We start by comparing the Beveridgean and Bismarckian systems with the tax rates obtained by majority voting.

Proposition 1.
Agent \( i \) of Group \( k \) prefers a Beveridgean system to a Bismarckian one iff \( w_i < r_k \bar{w} \), where \( r_k = \frac{1 - \bar{p}}{1 - \bar{p}_k} \) is an increasing function of \( \bar{p}_k \), and thus of \( k \).

This agent prefers the Bismarckian system iff \( w_i > r_k \bar{w} \).

Proof. From Equations (3) and (4)

\[ W_{BE}(t^*_m, p_i, w_i) > W_{BI}^k(t^*_m, p_i, w_i) \]

\[ \iff p_i \ln \left( \frac{1 - \bar{p}}{\bar{p} p_m} \right) > p_i \ln \left( \frac{1 - \bar{p}_k}{\bar{p}_k} p_m w_i \right) \]

which is equivalent to

\[ \frac{1 - \bar{p}}{\bar{p}} w > \frac{1 - \bar{p}_k}{\bar{p}_k} w_i \]

i.e. equivalent to \( w_i < r_k \bar{w} \), where \( r_k \) is clearly an increasing function of \( \bar{p}_k \), and \( \bar{p}_k \) is an increasing function of \( k \) according to Inequality (1).
Note that the coefficient $r_k$ is a measure of the average risk $\bar{p}_k$ in Group $k$, relative to the average risk of society, $\bar{p}$. If the average risk in Group $k$ coincides with the society’s average risk, then $r_k$ is equal to 1. If the average risk in Group $k$ is lower (higher) than the society’s average risk, then $r_k$ is strictly smaller (larger) than 1. Since $r_k$ is an increasing function of $k$, we can write:

$$ r_1 < r_2 < \ldots < r_j < 1 < r_{j+1} < \ldots < r_M $$

With the usual way of modeling the Bismarckian system, there is only one group, i.e. $M = 1$. In this case, $\bar{p}_1 = \bar{p}$ so that $r_1 = 1$. It immediately implies that an agent of income $w_i$ prefers a Beveridgean system if $w_i < \bar{w}$ and a Bismarckian one if $w_i > \bar{w}$. It is a particular case of our Proposition 1.

With the more realistic way of modeling the Bismarckian system that we adopt here, the Bismarckian system is particularly interesting for agents who belong to low-risk groups, i.e. to Group $k$ with $k$ low. Agents who bear a high risk benefit from a group with a low mean risk because of the intra-group horizontal redistribution.

Both the Beveridgean and the Bismarckian systems imply horizontal redistribution (i.e. from low-risk to high-risk agents), but the only system with vertical redistribution (i.e. from rich to poor agents) is the Beveridgean one. Thus, poor agents prefer Beveridge to Bismarck.

An individual $i$ prefers a Beveridgean system if his income $w_i$ is such that $w_i < r_k \bar{w}$, as shown in Figure 2. In each group, there may be a proportion of agents who prefer the Beveridgean system and another that prefer the Bismarckian one. The proportion of agents who prefer the Beveridgean system increases with the average risk of the group. As a consequence, according to the ranking of the $\bar{p}_k$, the proportion of agents who prefer a Beveridgean system is the lowest in Group 1 and the highest in Group $M$. Note that the individual choice of the system only depends on the group the individual belongs to, and on his individual income $w_i$, but not on his individual risk $p_i$. Finally, every agent of Group $k$ prefers a Beveridgean system if

$$ w_{sup} < r_k \bar{w} $$

which is more likely to be true for high $k$ (that is for a high average risk), whereas every agent prefers a Bismarckian system if

$$ w_{inf} > r_k \bar{w} $$

which is more likely to be true for low $k$ (that is for a low average risk).
3.2.2 Bismarck or Liberal?

Now we compare the Bismarckian and Liberal systems. An agent $i$ of Group $k$ prefers a Bismarckian system if

$$W^B_{k}(t^*_m, p_i, w_i) > W^L_{k}(t^*_i, p_i, w_i)$$

which leads to the following proposition:

**Proposition 2.**

Agent $i$ of Group $k$ prefers a Bismarckian system to a Liberal one iff $p_i > \hat{p}_k$, where $\hat{p}_k$ only depends on the group of the agent, and is an increasing function of $k$.

This agent prefers the Liberal system iff $p_i < \hat{p}_k$.

**Proof.** See Appendix A. □

Note that the choice between L and BI does not depend on the income earned by the agent in the good state of the world, because in both systems there is no vertical redistribution.

A Bismarckian system implies intra-group horizontal redistribution (i.e. from low-risk to high-risk agents) in opposition to the Liberal system.
Figure 3: Individual preferences: L or BI?

a result, high-risk people prefer the Bismarckian system to the Liberal one. For a given agent of risk \( p_i \), the Bismarckian system is more interesting if the other agents of the group are low risk. If \( k \) is low (i.e. \( \bar{p}_k \) low), Group \( k \) is a very low-risk group. It is then interesting to have a Bismarckian system for an agent of this group, as it appears in Figure 3.

3.2.3 Beveridge or Liberal?

Now we compare the Beveridgean and Liberal systems. An agent \( i \) prefers the Beveridgean system iff

\[
W^{BE}(t^*_m, p_i, w_i) > W^L(t^*_i, p_i, w_i)
\]

which leads to the following proposition:

**Proposition 3.**

Agent \( i \) prefers a Beveridgean system to a Liberal one iff \( w_i < \hat{w}(p_i) \) where \( \hat{w}(p_i) \) is an increasing function of \( p_i \), with \( \hat{w}(0) = 0 \) and \( \hat{w}(1) = +\infty \).

This agent prefers the Liberal system iff \( w_i > \hat{w}(p_i) \)
**Proof.** See Appendix B. □

Figure 4 presents the partition of the population between those who prefer a Liberal system and those who prefer a Beveridgean system. The preference depends both on the income and the risk supported by the agent. The curve representing the function $\hat{w}$ characterizes the boundary between both regimes. Therefore, a combination of income and risk on the boundary makes the agent indifferent to both regimes.

An agent $i$ of income $w_i$ and risk $p_i$ prefers a Beveridgean system against a Liberal one iff $w_i < \hat{w}(p_i)$, where $\hat{w}$ is an increasing function of $p_i$. Agents with a sufficiently high income and relatively low risk prefer the Liberal system. Agents with a sufficiently low income and relatively high risk prefer the Beveridgean system because they benefit from vertical and horizontal redistributions.

A Beveridgean system implies both global horizontal and vertical (i.e. from rich to poor agents) redistribution. Both high-risk and/or poor agents have an incentive to choose a Beveridgean system to benefit from redistribution. Conversely, low-risk and/or rich agents benefit more from supporting their preferred tax rate. In addition, these agents do not benefit from redistribution. Therefore, they are in favor of a Liberal system.

![Figure 4: Individual preferences: L or BE?](image-url)
3.2.4 Summary of individual preferences

Overall, there are three types of redistribution mechanisms which essentially determine individual preferences. They are summarized in Table 1.

<table>
<thead>
<tr>
<th>Redistribution mechanism</th>
<th>Effective in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical redistribution</td>
<td>BE</td>
</tr>
<tr>
<td>Global horizontal redistribution</td>
<td>BE</td>
</tr>
<tr>
<td>Intra-group horizontal redistribution</td>
<td>BI</td>
</tr>
</tbody>
</table>

Table 1: Summary of redistribution mechanisms

Figure 5 gives an overview of the partition functions for individual preferences for a given Group $k$. Firstly, the Beveridgean system is clearly preferred by agents who are characterized by a combination of very low income and very high risk. However, Beveridge is also preferred by poor agents who support a relatively small risk if the “income effect” of a high vertical redistribution dominates.

Figure 5: Overview of partition functions for individual preferences
Secondly, the Liberal system is preferred by agents with a low risk, from the “quite rich” to the very rich agents because low-risk agents are against horizontal redistribution and rich agents are against vertical redistribution. However, a Liberal system is also preferred by poor agents who have a very low risk: If this “low-risk effect” dominates the income effect from vertical redistribution inside the Beveridgean system, then these agents also prefer Liberal to Beveridge. The additional advantage of a Liberal system is that the tax rate is not chosen by a decision-maker, but is the one preferred by the agent.

Thirdly, agents are in favor of a Bismarckian system if they are sufficiently rich and have a level of risk beyond a certain threshold since the Bismarckian system features intra-group horizontal redistribution but no vertical redistribution.

The impact of a higher group risk on the partition space of individual preferences can be seen by considering preferences of Group \( k+1 \). Compared to Group \( k \), the indifference curves of agents in this group will move for \( BE = BI \) upwards and for \( BI = L \) to the right. This is indicated in Figure 5. As a consequence, the space where the Bismarckian system is preferred becomes smaller because a Bismarckian system is more favorable for lower group risk.

### 3.3 Choice of the system under majority voting

Before studying the choice of a utilitarian planner we first focus on a simple positive decision rule: majority voting. We restrict our analysis to pairwise comparisons of the choice of the systems. Note that there is not necessarily unanimity within a group regarding a preferred system.

#### 3.3.1 Bismarck or Beveridge?

In the following we define as the “poor” those agents whose income is lower than the average income \( (w_i < \bar{w}) \) and as the “rich” those agents who have a higher-than-average income \( (w_i > \bar{w}) \). We study the impact of a “mean preserving spread” (hereafter referred to as MPS)\(^{12}\). This means that rich people become richer, poor people become poorer, but the average income remains unchanged.

Recall that the indicator of risk of Group \( k \) relative to society’s risk, \( r_k \), is ranked as follows: \( r_1 < ... < r_j < 1 < r_{j+1} < ... < r_M \)

\(^{12}\)Note that an MPS is related to second-order stochastic dominance, this is well defined in \cite{Mas-Colell1995}, chapter 6.D.
Proposition 4.

(i) If the inequality of income is low, i.e. here, if \( r_j \bar{w} < w_{\text{inf}} < w_{\text{sup}} < r_{j+1} \bar{w} \), then in Groups 1, 2, ..., \( j \), there is unanimity in favor of the Bismarckian system, and in Groups \( j + 1 \), ..., \( M \), there is unanimity in favor of the Beveridgean system.

(ii) A Mean Preserving Spread (MPS) of the distribution of incomes implies a lower political support for the Bismarckian system in Groups 1, ..., \( j \), and a lower political support for the Beveridgean system in Groups \( j + 1 \), ..., \( M \).

(iii) With a sufficiently large MPS, there is a majority in favor of the Beveridgean system.

Proof. See Appendix C.

The decisive factor which determines the type of system is income inequality.

An interpretation of this Proposition is as follows:

(i) If the inequality of income is low, the impact of vertical redistribution can be neglected. In this case, comparing BI and BE systems means that two different types of horizontal redistribution are compared: intra-group horizontal redistribution in the BI system and global horizontal redistribution in the BE system. The intra-group horizontal redistribution is more favorable for groups 1, ..., \( j \) because for them \( \bar{p}_k < \bar{p} \) holds true. Therefore, each member in these groups prefers a BI system.

The reverse is true for groups \( j + 1 \), ..., \( M \): They benefit more from global horizontal redistribution since \( \bar{p} \) is not greater than \( \bar{p}_k \). Consequently, all agents in these groups prefer a BE system.

(ii) Let us consider Groups 1, ..., \( j \) after an MPS. The rich agents of these groups will still prefer the BI system. The same is true for the “rather” poor people, because they benefit from their low intra-group risk \( \bar{p}_k \) in the BI system. The poorest people become even poorer with the MPS, so that they finally prefer BE because it allows vertical redistribution.

For agents who belong to Groups \( j + 1 \), ..., \( M \), again, the reverse is true. With an MPS, the poor agents of these groups will still prefer BE, and also the “rather” rich people. The richest agents become even richer with the MPS, so that they finally prefer BI because it does not imply vertical redistribution. Therefore, if a majority of people belongs to Groups 1, ..., \( j \), then BI is adopted by majority voting. BE is adopted if the reverse is true.

(iii) In case of a very large inequality of income (i.e. very large MPS), the effect of vertical redistribution dominates horizontal redistribution: people
with an income \( w_i \) lower than \( \bar{w} \) almost all prefer BE. Then, there is a majority for BE since \( w_m < \bar{w} \).

This effect is in line with the result from Meltzer and Richard (1981) which states that when the share of redistributed income is determined by majority voting, a more unequal income distribution leads to more redistribution.

### 3.3.2 Bismarck or Liberal?

We now focus on the choice between a Bismarckian and a Liberal system. The inequality of income has no impact on the political support of the Liberal system against the Bismarckian one, because in both systems the social benefit is proportional to the income, i.e. there is no vertical redistribution.

Recall we have assumed that the median voter is the same in each group, i.e. \( p_{m,k} = p_m \) for every \( k \).

**Proposition 5.**

According to the majority voting criterion a Liberal system is adopted against a Bismarckian one.

**Proof.** We can set

\[
  h_k(p_m) = W_k^{BI}(t^*_m, p_m, w_i) - W_k(\tilde{t}_i^*, p_m, w_i) = p_m \ln \left( \frac{1 - \bar{p}_k}{\bar{p}_k} \frac{p_m}{1 - p_m} \right)
\]

Since \( p_{m,k} = p_m \) for every \( k \), and \( p_m < \bar{p}_k \) for every \( k \), then \( h_k(p_m) < 0 \).

From Proposition 2 and its proof, we are then able to state that for all agents \( i \) with \( p_i \leq p_m \), that represent at least 50% of the voters of each group, the Liberal system is preferred. \( \square \)

The main advantage of the Liberal system is that agents can choose their individually preferred tax rate. The main advantage of the Bismarckian system is intra-group horizontal redistribution; however, this advantage only applies to a minority of people. As a result, the Liberal system is preferred by a majority of agents.

### 3.3.3 Beveridge or Liberal?

Let us now study the majority choice between the Beveridgean and Liberal systems.

According to Proposition 3, we know that an agent of income \( w_i \) and risk \( p_i \) prefers the Beveridgean system against the Liberal one if \( w_i < \bar{w}(p_i) \).
The function $\hat{w}$ depends on $p_m$, $p$, and $\pi$. In the following proposition, we show that the political support for BE (against L) depends on the level of inequality of income in the society.

**Proposition 6.**

(i) If the inequality of income is low, here more precisely, if $\frac{\text{wmin}}{\text{w}} > \eta$, where $\eta$ only depends on the distribution of risks, and $0 < \eta < 1$, then a majority of agents prefers a Liberal system to a Beveridgean one.

(ii) A Mean Preserving Spread (MPS) of the distribution of income implies a higher political support for the Beveridgean system.

(iii) With a sufficiently large MPS, there is a majority in favor of the Beveridgean system.

**Proof.** See Appendix D. $\square$

Again, there is a precise interpretation of this proposition.

(i) If the inequality of income is low, vertical redistribution does not matter. In this case, BE vs. L means global horizontal redistribution vs. no redistribution at all. Global horizontal redistribution is in favor of high-risk agents which are a minority. In turn, there is a majority for the L system.

(ii) The higher the inequality of income, the stronger the vertical redistribution. Poor agents are in favor of vertical redistribution. Since they constitute more than 50% of the population, the support for BE increases.

(iii) With a sufficiently large inequality of income, the effect of vertical redistribution dominates, so poor agents will be in favor of BE.

4 Utilitarian criterion

In this section, we focus on a normative criterion, analyzing the choice of a utilitarian social planner.

4.1 Choice of tax rate

The Liberal system is characterized by total liberty of choice for any individual. Similarly to the majority voting analysis, the individually preferred tax rate is applied, i.e.

$$t_L^* = t_i^* = p_i$$

Under the Beveridgean system, the utilitarian planner determines the common optimal tax rate, $t_u^{BE}$, by maximizing the average of the individual
welfares. Since

\[ U_{BE} = \int \int W_{BE}(t_{u}^{BE}, p, w) f(p) g(w) dp dw \]

\[ = \int \int \left[ (1 - p) \ln((1 - t_{u}^{BE}) w) + p \ln \left( \frac{1 - p}{p} t_{u}^{BE} \frac{w}{\bar{w}} \right) \right] f(p) g(w) dp dw \]

\[ = (1 - \bar{p}) \ln(1 - t_{u}^{BE}) + \bar{p} \ln \left( \frac{1 - \bar{p}}{\bar{p}} t_{u}^{BE} \right) + (1 - \bar{p}) \ln \bar{w} + \bar{p} \ln \bar{w} \]

where \( \bar{w} \) stands for the mean of \( \ln w_i \). Then \( \max_{t_{u}^{BE}} U_{BE} \) implies \( t_{u}^{BE} = \bar{p} \).

Under the Bismarckian system, the utilitarian planner determines the optimal tax rate of Group \( k \), \( t_{u,k}^{BI} \), by maximizing the average of the individual welfares of Group \( k \). Since

\[ U_{BI,k} = \frac{1}{N_k} \int \int W_{BI,k}(t_{u,k}^{BI}, p, w) f_k(p) g(w) dp dw \]

\[ = \frac{1}{N_k} \int \int \left[ (1 - p) \ln \left( (1 - t_{u,k}^{BI}) w \right) + p \ln \left( \frac{1 - p}{p} t_{u,k}^{BI} \frac{w}{\bar{w}} \right) \right] f_k(p) g(w) dp dw \]

\[ = (1 - \bar{p}_k) \ln(1 - t_{u,k}^{BI}) + \bar{p}_k \ln \left( \frac{1 - \bar{p}_k}{\bar{p}_k} t_{u,k}^{BI} \right) + \bar{p}_k \ln t_{u,k}^{BI} + \bar{p}_k \ln \bar{w} \]

then maximizing \( U_{BI,k} \) yields the optimal tax rate \( t_{u,k}^{BI} = \bar{p}_k \).

4.2 Choice of the system

The Beveridgean system yields the following social welfare:

\[ U_{BE}^* = (1 - \bar{p}) \ln(1 - t_{u}^{BE}) + \bar{p} \ln \left( \frac{1 - \bar{p}}{\bar{p}} t_{u}^{BE} \right) + (1 - \bar{p}) \ln \bar{w} + \bar{p} \ln \bar{w} \]

\[ = (1 - \bar{p}) \ln(1 - \bar{p}) + \bar{p} \ln \left( \frac{1 - \bar{p}}{\bar{p}} \right) + (1 - \bar{p}) \ln \bar{w} + \bar{p} \ln \bar{w} \]

\[ = \ln(1 - \bar{p}) + (1 - \bar{p}) \ln \bar{w} + \bar{p} \ln \bar{w} \quad (6) \]

The Bismarckian system produces the social welfare:

\[ U_{BI}^* = \sum_{k=1}^{M} N_k U_{BI,k}^* \]

\(^{13}\)Generally, in Bismarckian countries, tax rates differ across funds.
where

\[ U_{BI,k}^* = (1 - \bar{p}_k) \left[ \ln (1 - \bar{p}_k) + \ln w \right] + \bar{p}_k \ln \left( \frac{1 - \bar{p}_k}{\bar{p}_k} \right) + \bar{p}_k \ln \bar{p}_k + \bar{p}_k \ln w \]

thus

\[ U_{BI}^* = \sum_{k=1}^{M} N_k [\ln (1 - \bar{p}_k)] + \ln w \]  
(7)

The Liberal system gives the social welfare:

\[ U_L^* = \iint W^L(t^*; p; w) f(p) g(w) d\rho d\theta \]
\[ = \iint \ln ((1 - p) w) f(p) g(w) d\rho d\theta \]
\[ = \ln w + \ln (1 - p) \]  
(8)

The comparison of the different welfare functions leads to the following proposition.

**Proposition 7.**

A utilitarian planner has the following preferences: \( BE > BI > L \)

**Proof.** Comparing (6) and (7) gives

\[ U_{BE}^* - U_{BI}^* = \ln(1 - \bar{p}) + (1 - \bar{p}) \ln w + \bar{p} \ln w - \left[ \sum_{k=1}^{M} N_k [\ln (1 - \bar{p}_k)] + \ln w \right] \]
\[ = \left[ \ln(1 - \bar{p}) - \sum_{k=1}^{M} N_k [\ln (1 - \bar{p}_k)] \right] + \bar{p} \ln w - \ln w > 0 \]

due to the Jensen inequality.

Comparing (7) and (8) gives

\[ U_{BI}^* - U_L^* = \left[ \sum_{k=1}^{M} N_k [\ln (1 - \bar{p}_k)] + \ln w \right] - \left[ \ln w + \ln (1 - p) \right] \]
\[ = \sum_{k=1}^{M} N_k [\ln (1 - \bar{p}_k)] - \ln (1 - p) > 0 \]

\( U_{BI}^* - U_L^* \) is positive due to the Jensen inequality.  \( \square \)
Under the utilitarian criterion, the Beveridgean system is always preferred even if the distribution of risk is strongly asymmetric in favor of low-risk agents (say, 85% of agents have a risk lower than the average risk).

If we compare BE and BI using a utilitarian criterion, two effects are clearly in favor of BE:
- the vertical redistribution that benefits poor people more than it hurts rich people,
- the global horizontal redistribution that benefits agents belonging to high-risk groups more than it can hurt agents belonging to low-risk groups.

Note that even if there is no inequality of income, BE is still preferred because of this second effect.

Similarly, if we compare BI and L, the intra-group horizontal redistribution is only at work in BI so that it is preferred by the utilitarian policy-maker.

4.3 Do the positive results meet the normative recommendations?

In order to compare the results obtained under both criteria, we need to evaluate the impact of every redistribution mechanism (vertical and horizontal) either with our positive or normative criterion.

A utilitarian planner is in favor of vertical redistribution since it benefits low income agents more than it hurts high income agents. This argues for BE rather than for BI or L. Similarly, the majority voting rule supports any vertical redistribution because high income agents are a minority in the society. Again, this argues for BE.

On the one hand, a utilitarian planner is also in favor of any horizontal redistribution since it benefits high-risk agents more than it hurts low-risk agents. This argues for BE or BI rather than L. On the other hand, the majority voting rule does not support any horizontal redistribution because high-risk agents constitute a minority in the society. This argues for L.

In addition, the utilitarian planner gives priority to global horizontal redistribution compared to intra-group horizontal redistribution since the first one is a broader type of redistribution. This argues for BE against BI. The intra-group horizontal redistribution is preferred under a majority voting rule if and only if there is a majority of agents in “good groups”, i.e. Group $k$ such that $p_k < p$. This last case argues for BI.

As a consequence of all these redistribution effects, the utilitarian planner prefers a more redistributive system, i.e. BE. The preferred system under a
majority voting rule depends on the relative sizes of income inequality and risk inequality as well as on the proportion of agents belonging to “good groups”.

More precisely:

– the lower the income inequality, the higher the political support for L,
– the higher the income inequality, the higher the political support for BE,

so that the majority voting choice corresponds to the utilitarian choice if the inequality of income is sufficiently high.

Moreover, the higher the proportion of agents belonging to “good groups”, the higher the political support for BI. More precisely, when comparing BE and BI, a majority voting rule leads to adopt BI if the inequality of income is low, which is in opposition to the utilitarian choice.

Finally, when only BI and L are compared, the utilitarian criterion leads to prefer BI since it allows horizontal redistribution. This is in opposition to the result with majority voting, since with $p_m < \bar{p}$ only a minority of the society benefits from horizontal redistribution.

5 Conclusion

We have studied the three main types of welfare capitalism within a simple economic model which incorporates specific groups. In particular, we have introduced a more accurate way to model the corporatist Bismarckian system, taking into account the fact that this system allows intra-group horizontal redistribution, as outlined by Esping-Andersen (1990).

For the choice of the welfare system using the majority voting rule, we have shown the influence of the inequality of income, distribution of risk and the group structure. Under a utilitarian criterion, the Beveridgean system is always preferred. Moreover, the Bismarckian is always preferred to a Liberal one.

The utilitarian preference for the Beveridgean system may explain the evolution of the Bismarckian countries towards mixed systems incorporating an increasing part of Beveridgean characteristics.

This paper offers preliminary results which allows us to state that the main results concerning the choice of the welfare system are crucially modified under the new way of modeling the Bismarckian system. This is a first step in a research program that should be encompassed by the development of new studies incorporating this new way of modeling.
Appendix

A Proof of Proposition 2

According to (4) and (5), \( W_k^{BI}(t^*_m, p_i, w_i) > W_L(t^*_i, p_i, w_i) \) is equivalent to

\[
(1 - p_i) \ln (1 - p_m) + p_i \ln \left( \frac{1 - \bar{p}_k}{\bar{p}_k} p_m \right) > \ln (1 - p_i) \quad (9)
\]

We set

\[
h_k(p_i) = (1 - p_i) \ln (1 - p_m) + p_i \ln \left( \frac{1 - \bar{p}_k}{\bar{p}_k} p_m \right) - \ln (1 - p_i)
\]

We have

\[h_k(0) = \ln (1 - p_m) < 0 \text{ and } h_k(1) \to +\infty\]

and

\[
h'_k(p_i) = \frac{1}{1 - p_i} + \ln \left( \frac{1 - \bar{p}_k}{\bar{p}_k} \frac{p_m}{1 - p_m} \right);
\]

\[
h''_k(p_i) = \frac{1}{(1 - p_i)^2} > 0
\]

\( h_k \) is a convex function on \([0, 1)\), with \( h_k(0) < 0 \) and \( \lim_{p \to 1} h_k(p) = +\infty \) so that there is clearly a unique \( \hat{p}_k \) such that \( h_k(\hat{p}_k) = 0 \). Note that \( \hat{p}_k \) depends only on \( \bar{p}_k \) and \( p_m \).

In addition, \( h_k(\hat{p}_k) = \ln (1 - p_m) + \hat{p}_k \ln \left( \frac{1 - \bar{p}_k}{\bar{p}_k} \frac{p_m}{1 - p_m} \right) - \ln (1 - \hat{p}_k) = 0 \)

According to the implicit function theorem, \( \frac{\partial \hat{p}_k}{\partial p_k} = -\left( \frac{\partial h_k}{\partial \hat{p}_k} \right)^{-1} \times \frac{\partial h_k}{\partial p_k} > 0 \), and \( \hat{p}_k \) is an increasing function of \( k \), so that \( \hat{p}_k \) is an increasing function of \( k \).

B Proof of Proposition 3

According to (4) and (5) we have

\[
H(p_i, w_i) = W^{BE}(t^*_m, p_i, w_i) - W^L(t^*_i, p_i, w_i)
\]

\[
= (1 - p_i) \ln [(1 - p_m) w_i] + p_i \ln \left( \frac{1 - \bar{p}}{\bar{p}} p_m w_i \right) - \ln [(1 - p_i) w_i]
\]

\[
= \ln (1 - p_m) - \ln (1 - p_i) + p_i \ln \left( \frac{1 - \bar{p}}{\bar{p}} \frac{p_m}{1 - p_m} \frac{w_i}{w} \right)
\]
Moreover, $H (p_i, w_i) = 0 \iff \ln \left( \frac{1-p}{p} \frac{p_m}{1-p_m} \frac{w_i}{m_i} \right) = \frac{1}{p_i} \ln \left( \frac{1-p_i}{1-p_m} \right) $

i.e. $H (p_i, w_i) = 0 \iff \frac{1-p}{p} \frac{p_m}{1-p_m} \frac{w_i}{m_i} = \left( \frac{1-p_i}{1-p_m} \right)^{1/p_i}$

Then, an agent $i$ prefers BE to L iff $H (p_i, w_i) \geq 0$, i.e. iff $w_i \leq \hat{w}(p_i)$, where

$$
\hat{w}(p_i) = \frac{1-p}{p} \frac{p_m}{1-p_m} \frac{w_i}{m_i} \left( \frac{1-p_m}{1-p_i} \right)^{1/p_i}
$$

Let us show that $\hat{w}(p_i)$ is an increasing function of $p_i$, with $\hat{w}(0) = 0$ and $\lim_{p \to 1} \hat{w}(p) = +\infty$

$$
\hat{w}(p_i) = C \times \exp \left( a(p_i) \right), \text{ where } C = \frac{1-p}{p} \frac{p_m}{1-p_m} w_i > 0 \text{ and } a(p_i) = \frac{1}{p_i} \ln \left( \frac{1-p_m}{1-p_i} \right)
$$

Clearly, we can write that $a(0) = \lim_{p_i \to 0} a(p_i) = -\infty$, and $a(1) = \lim_{p_i \to 1} a(p_i) = +\infty$, thus $\hat{w}(0) = 0$ and $\hat{w}(1) = +\infty$

We just have to show that $a(p_i)$ is an increasing function of $p_i$.

$$
a(p_i) = \frac{1}{p_i} \ln (1 - p_m) - \frac{1}{p_i} \ln (1 - p_i)
$$

$$
a'(p_i) = - \frac{1}{p_i^2} \ln (1 - p_m) + \frac{1}{p_i} \ln (1 - p_i) + \frac{1}{p_i (1 - p_i)}
$$

$$
p_i^2 a'(p_i) = - \ln (1 - p_m) + \ln (1 - p_i) + \frac{p_i}{(1 - p_i)}
$$

$$
= - \ln (1 - p_m) + \ln (1 - p_i) + \frac{1}{1 - p_i}
$$

Let us show that $p_i^2 a'(p_i) \geq 0$ for any $p_i \in [0; 1]$. We set $b'(p_i) = p_i^2 a'(p_i) = - \ln (1 - p_m) + \ln (1 - p_i) + \frac{1}{(1 - p_i)}$ where $b'(p_i) = \frac{1}{1 - p_i} + \frac{1}{(1 - p_i)^2} = \frac{p_i}{(1 - p_i)^2} > 0$ and $b(0) = - \ln (1 - p_m) > 0$

Thus $b(p_i) > 0$ on $p_i \in [0; 1]$, so that $p_i^2 a'(p_i) > 0$ and $a(p_i)$ is an increasing function of $p_i$. \qed

### Proof of Proposition 4

(i) According to Proposition 1, an agent $i$ of Group $k$ prefers BE to BI iff $w_i < r_k \overline{w}$ where $r_1 < \ldots < r_j < r_{j+1} < \ldots < r_M$.

By assumption, $r_j \overline{w} < w_{\inf}$, then for any agent $i$ of Group $k$ with $k \leq j$, we have $w_i \geq w_{\inf} > r_j \overline{w} \geq r_k \overline{w}$. Thus, there is unanimity in favor of BI in Groups 1, 2, ..., $j$.

Similarly, by assumption, $w_{\sup} < r_{j+1} \overline{w}$, then for any agent $i$ of Group $k$ with $k \geq j + 1$, we have $w_i \leq w_{\sup} < r_{j+1} \overline{w} \leq r_k \overline{w}$. Thus there is unanimity in favor of BE in Groups $j + 1$, ..., $M$. 

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(ii) Impact of an MPS.

- For \( k \leq j \), the agent \( i \) prefers BE iff \( w_i < r_k \bar{w} \), where \( r_k < 1 \).

With an MPS, the proportion of people with \( w_i < r_k \bar{w} \) increases, so that the political support for BE increases.

- For \( k \geq j + 1 \), the agent \( i \) prefers BI iff \( w_i > r_k \bar{w} \), where \( r_k > 1 \).

With an MPS, the proportion of people with \( w_i > r_k \bar{w} \) increases, so that the political support for BI increases.

(iii) Impact of a sufficiently large MPS.

- For \( k \leq j \), with a large MPS, the proportion of people of income \( w_i \in [r_k \bar{w}; \bar{w}] \) becomes very small, so that the proportion in favor of BE becomes arbitrarily near that of people of income \( w_i < \bar{w} \).

- For \( k \geq j + 1 \), with a large MPS, the proportion of people of income \( w_i \in [\bar{w}; r_k \bar{w}] \) becomes very small, so that the proportion in favor of BI becomes arbitrarily near that of people of income \( w_i > \bar{w} \).

Finally, whatever the group, if the MPS is sufficiently large, then the proportion of people in favor of BE is arbitrarily close to the proportion of people of income \( w_i < \bar{w} \). Since the median income \( w_m \) is lower than \( \bar{w} \), we can conclude that with a sufficiently large MPS, there is a majority of people in favor of BE against BI.

D Proof of Proposition 6

(i) For any agent \( i \) of risk \( p_i \) and income \( w_i \):

\[
H(p_i, w_i) = W^{BE}(t^*_m, p_i, w_i) - W^L(t^*_i, p_i, w_i) \\
= \ln (1 - p_m) - \ln (1 - p_i) + p_i \ln \left( \frac{1 - \bar{p}}{\bar{p}} \frac{p_m}{1 - p_m} \right) + p_i \ln \left( \frac{\bar{w}}{w_i} \right) \\
= \tilde{h}(p_i) + p_i \ln \left( \frac{\bar{w}}{w_i} \right)
\]

where \( \tilde{h}(p_i) = \ln (1 - p_m) - \ln (1 - p_i) + p_i \ln \left( \frac{1 - \bar{p}}{\bar{p}} \frac{p_m}{1 - p_m} \right) \)

For an individual of income \( \bar{w} \) and risk \( p_i \), \( \tilde{h}(p_i) \) is the difference of welfares under BE and L.

\( \tilde{h}'(p_i) = \frac{1}{(1-p_i)^2} > 0 \), and \( \tilde{h}(0) = \ln(1 - p_m) < 0 \), and

\( \tilde{h}(p_m) = p_m \ln \left( \frac{1 - \bar{p}}{\bar{p}} \frac{p_m}{1 - p_m} \right) < 0 \) because \( p_m < \bar{p} \).
\( \tilde{h} \) is a convex function with \( \tilde{h}(0) < 0 \) and \( \tilde{h}(p_m) < 0 \), thus \( \tilde{h}(p_i) < 0 \) for all \( p_i \leq p_m \). \( \tilde{h} \) is a continuous function, then \( \max_{0 \leq p \leq p_m} \tilde{h}(p) < 0 \).

Setting \( \eta = \exp \left[ \max_{0 \leq p \leq p_m} \tilde{h}(p) \right] \), we have then \( 0 < \eta < 1 \). By assumption, \( \eta < \frac{w_{\text{inf}}}{\bar{w}} \), thus \( \max_{0 \leq p \leq p_m} \tilde{h}(p) < \ln \left( \frac{w_{\text{inf}}}{\bar{w}} \right) \).

For every \( p_i \), with \( p_i \leq p_m \), we have \( H(p_i, w_i) = \tilde{h}(p_i) + p_i \ln \left( \frac{w_i}{w_m} \right) \leq \max_{0 \leq p \leq p_m} \tilde{h}(p) + \ln \left( \frac{w_i}{w_m} \right) < 0 \)

Then, any agent \( i \) such that \( p_i \leq p_m \) prefers \( L \) to \( \text{BE} \), i.e. a majority of people are in favor of \( L \).

(ii) An MPS implies that \( \ln \left( \frac{w_i}{w_m} \right) \) increases for a majority of people because \( w_m < \bar{w} \), thus it increases the political support for the Beveridgean system.

(iii) With a sufficiently large MPS of the distribution of income, we have \( p_i \ln \left( \frac{w_i}{w_m} \right) \geq p_i \ln \left( \frac{w_i}{w_m} \right) > \tilde{h}(p) \) for 50% of the population. Then, clearly \( H(p_i, w_i) > 0 \) for a majority of people. □
References


